ADVISOR 2
A Study of Industrial Marketing Budgeting
Part 2: Change Models, Distribution Channel Models, Uses

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This paper represents the results of efforts by many people. Professors Jean-Marie Choffray of ESSEC, John Little and Alvin Silk of MIT were all influential in formulating and performing this analysis. In addition, Jessie Abraham, Donald Barefoot, Barbara Bolshon, Sandy Fitchet, Gregory Resker and Mary Ann Ritter contributed significantly to developing these results. We are indebted to Donald Gluck of duPont for stimulating the start of the ADVISOR Project, to Harry Darling of ANA for helping it to prosper and to all of the representatives from the participating companies for their time, support and creative input.
This paper reviews descriptive models of annual marketing budgeting changes for industrial products, a model for distribution channel selection and issues of use of ADVISOR results.

The change in marketing budgets is related to changes in market share, changes in product plans and changes in the number of competitors modified by the number of customers, their concentration and the size of the advertising budget.

The decision to use a direct channel of distribution (primarily salesforce) is affected by the size of the firm, the size of an average order, the stage in the product's life cycle, the complexity of the product, the fraction of the product's sales made-to-order and the purchase frequency of the product.

A discussion of the use of the ADVISOR models both for marketing decision making and for industrial marketing researchers and model builders is included.
1. Introduction

The first part of this paper (Lilien [7]) focused on the motivation for and objectives of the ADVISOR 2 studies: we search for quantitative guidelines for industrial marketing budgeting decisions.

The main premise of the ADVISOR studies is that there are a small set of general product and market factors which, on an industry-wide basis, are the major determinants of industrial marketing budgeting. The models, derived from an empirical study of practice, can be of importance both to researchers as well as to practitioners.

Part I of this paper reviewed the background of ADVISOR 2, the data used and the norm models, those models used to describe the level of spending. In this part, we review the change models, models of the determinants of year-to-year changes in the marketing budget. We also review a model for distribution-channel selection. Finally, we assess the usefulness of ADVISOR results and implications for further work.

2. Change Model Structure

The primary concept of the ADVISOR change models is that year-to-year changes in the marketing budget (advertising, in particular) are primarily the result of changes in the internal or external environment, modified by the level of certain key variables.

We are only concerned with trying to model moderate changes in the budget. Small changes in advertising are essentially noise, and reflect changes in media costs, allocation and accounting peculiarities -- they are not real, planned changes and should be excluded from consideration. Very
large changes reflect a total product reevaluation and should be considered as outliers for the purpose of this analysis. A normal, year-to-year change, then, has an upper and lower limit.

These concepts imply that we should include variables such as
- changes in market share
- changes in plans
- changes in the number of competitors

and subject the model to modifications for the number of customers, customer concentration, and the size of the advertising budget.

We can then model the change as having an S-shaped response as in Exhibit 1.

A convenient way to represent a relationship such as that in Exhibit 1 is as follows:

Define $\Delta \text{ADV} = \frac{\text{ADV}_{75} - \text{ADV}_{74}}{\text{ADV}_{74}}$

Suppose ADV has a (logical) upper limit of $U$ and a lower limit of $L$.

$\Delta^* = \frac{\text{ADV} - L}{U - L} \in (0,1)$

Assume we then take

$$\logit(\Delta^*) = \log\left(\frac{\Delta^*}{1-\Delta^*}\right)$$

and postulate that

$$\logit(\Delta^*) = a_0 + a_1x_1 + a_2x_2 + \ldots + a_nx_n,$$

where $x_1, \ldots, x_n$ are the independent variables described above. This transformation will produce a relationship like that displayed in Exhibit 1; it is a multivariate logistic function:

$$\Delta \text{ADV} = L + \frac{U - L}{1 + \exp\left(-\left(a_0 + a_1x_1 + a_2x_2 + \ldots + a_nx_n\right)\right)}$$

The coefficients $a_0, \ldots, a_n$ can be estimated by a maximum likelihood procedure, by weighted least squares (or by ordinary least squares as long
as $\Delta^*$ is not close to 0 or 1 for many observations).

The next section operationalizes this model, looking at changes in advertising, changes in advertising allocation and changes in marketing as well. Note that we should not anticipate a strong marketing relationship, where marketing = advertising + (personal selling + technical services).

$= A + PSTS$

PSTS is not a tactical variable. Thus, if we model

$$\frac{M_t - M_{t-1}}{M_{t-1}} = f(a_o + a_1 x_1 + \ldots a_n x_n)$$

we might expect that $PSTS_t = (1+\alpha) PSTS_{t-1}$, where $\alpha$ is a random variable, (or long-term time trend) distributed across products and not related to key, annual changes in independent variables. Thus

$$M_t - M_{t-1} = \frac{ADV_t + (1+\alpha) PSTS_{t-1} - (ADV_{t-1} + PSTS_{t-1})}{ADV_{t-1} + PSTS_{t-1}}$$

Both in ADVISOR 1 and here we find that $10^\cdot ADV \cdot PSTS$ as a rule. Assuming this is the case for $t-1$, we get

$$\frac{M_t - M_{t-1}}{M_{t-1}} = \frac{ADV_t - ADV_{t-1} + 10\alpha ADV_{t-1}}{11ADV_{t-1}}$$

$$= \frac{ADV_t - ADV_{t-1}}{11ADV_{t-1}} + \frac{10}{11}\alpha$$

$$= \frac{1}{11} (\Delta ADV) + \frac{10}{11}\alpha$$

$$= K_1 \cdot \Delta ADV + K_2 \alpha \quad \text{where} \ K_1, K_2 \text{ are constants.}$$
EXHIBIT 1

Postulated Change Model Form

\[
\frac{\text{ADV}_{75} - \text{ADV}_{74}}{\text{ADV}_{74}}
\]

points excluded from analysis (as 'noise')

independent variable
Assuming \( \alpha \) is distributed across the population, independent of the variables affecting \( \text{ADV} \) in equation (1), it serves as an additional error term and we expect a model of \( \frac{M_t - M_{t-1}}{M_{t-1}} \) to be related to the same variables as the advertising model but to have greater error of estimation. This follows because the noise portion of the equation includes the error from \( \Delta \text{ADV} \) plus the new noise component, \( K_2 \alpha \).

3. Change Model Calibration

The structure of the change models leads us to:

\[
\Delta A = \text{ADVP}_t = \frac{(\text{Advertising}_t - \text{Advertising}_{t-1})}{\text{Advertising}_{t-1}}
\]

with

\[
A^*_t = \frac{(\text{ADVP}_t + .55)}{1.57}
\]

(taking empirical upper and lower bounds from frequency plots).

Then we look at

\[
\text{(3)} \quad \text{logit} (A^*) = \ln \left( \frac{A^*}{(1-A^*)} \right) = \sum a_i x_i
\]

where \( \{x_i\} \) are independent variables. We refer to the dependent variables of interest as:

- \( \text{ADVP} \) = proportional change in advertising
- \( \text{MKTP} \) = proportional change in marketing
- \( \text{PERSP} \) = proportional change in personal advertising
- \( \text{IMPERSP} \) = proportional change in impersonal advertising

To estimate the parameters of model (3), the dependent variables are screened in order to remove both very large and very small proportional
changes. Large changes in advertising were removed by restricting ADVP to the range (-.55, 1.0), logical break-points from the frequency plots. This eliminates cases of unusually large proportional increases or decreases in advertising or marketing budgets, generally part of a total product re-evaluation.

Small changes were removed by an "epsilon-cut" procedure: cases where the dependent variable fell in the range \( (\epsilon, \frac{-\epsilon}{1+\epsilon}) \) were eliminated from analysis. The assumption underlying this procedure is that small dependent variable changes are not "real"; they may be accounting peculiarities, trends, or reflect changes in media costs and not necessarily reflect a "real" change. Several values for epsilon were tried, ranging between .05 and .20. Exhibit 2 plots the value of \( R^2 \) versus various values of \( \epsilon \); an apparent elbow occurs around \( \epsilon = .175 \), suggesting that the model is no longer improved by cutting out additional points, and that more severe restrictions reduce the amount of information.
EXHIBIT 2

$R^2$ vs $\varepsilon$ for logit(A*) Model

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$R^2$</th>
<th>$n$</th>
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<tr>
<td>.20</td>
<td>.400</td>
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<td>.175</td>
<td>.388</td>
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<td>.15</td>
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<td>.125</td>
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<td>90</td>
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<tr>
<td>.10</td>
<td>.320</td>
<td>102</td>
</tr>
</tbody>
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elbow, chosen value for $\varepsilon$
Another key issue dealing with the nature of a proportional change was addressed in the construction of the advertising model. If the advertising budget for a product is 'small' to begin with (i.e. the denominator of $ADVP_t = (ADV_t - ADV_{t-1})/ADV_{t-1}$ is small), any proportional or percentage increase in the budget in year $t$ will naturally tend to be large, regardless of the magnitude of changes in independent variables, (and vice-versa).

To capture this effect an advertising budget variable, ADVDUM, was incorporated as an independent variable. (A product's advertising budget was defined to be 'small' if it was less than the sample median). A negative coefficient for this variable, ADVDUM, was postulated and found to hold.

Implicit in this formulation is that the relationship between the independent variables and advertising change are the same over both the large and small advertising budget products. We would expect that the small budget cases would have more inherent noise, however, due to the size of their denominators. This hypothesis was tested by splitting the data set into two samples (high and low advertising), running the ALOGIT model on both samples separately, and comparing the resulting coefficients and $R^2$s. No significant differences in the coefficients were noted, though the $R^2$ was .27 for the low advertising sample while it was .57 for the high sample.

The ADVISOR 2 data base was modified somewhat for this evaluation. We had data for four years, 1972 to 1975. Thus we have three changes, 1975-1974, 1974-1973, 1973-1972. Only two are useable, however, if we postulate that advertising changes between 1974 and 1975 are affected by environmental changes that occurred between 1973 and 1974. This does permit us to look at two sets of changes: 1975-1974, and 1974-1973.
Such a data base was constructed, doubling the number of cases in the ADVISOR 2 sample. Note that this is done here and not in the norm models, since year-to-year levels (advertising dollars) are highly correlated within a product, while changes are not. We therefore have a total of 250 cases with which to begin analysis.

The variables which we wish to include in the advertising, advertising allocation, and marketing models are:

- lagged changes in market share
- changes in marketing plans
- lagged changes in the number of competitors.

We also expect this relationship to be modified by

- stage in the product life cycle
- customer concentration
- the number of users of the product
- the number of salesmen
- lagged advertising budget size (dummy variable)

The above five variables are expected to influence the level of change; for example, the higher the number of users, the greater the change expected in advertising.

Before using these 250 points, an additional set of analyses were performed: this involved testing the assumption that there was no fundamental difference (i.e., in the environment) between 1975-1974 and 1974-1973, and that these data could therefore be grouped. This possibility was tested in the following way. First, a dummy variable was constructed with a value of 0 for 1975-1974, and 1 for 1974-1973. The coefficient of this
dummy variable, if significantly different than zero, would represent an intercept-shift (the same relationship holds, but it is higher or lower one year than the other). The coefficient was not found to be different than zero.

Finally, the entire model was run on each half of the data separately, and the coefficients were compared. No significant differences in the level of the coefficients was noted. We thus conclude that we may analyze all 250 cases together.

Appendix 1 gives the definitions of the variables used in the models while Exhibits 3-6 give the results of the regressions. Appendix 2 gives the associated correlation matrices and Exhibit 7 compares the results.

First note that the models fit reasonably well (except for marketing) and as we will see, the relationships are quite interpretable. The marketing equation was postulated in the previous section to be likely to have a poorer fit but to be consistent with the advertising equation. This is essentially what has occurred, and we will not discuss the model further. We reason that year-to-year changes in the marketing budget, stimulated by short-term changes in the environment, are likely to be reflected in the advertising component. We discuss the effects listed in Exhibit 7, column by column.

LFCYC: Later on in the product life cycle when a change occurs in advertising, it is likely to be in personal media, in support of salesmen serving known customers. The variable has no measurable effect on the level of the advertising change, however.

Change in PMSD: A decrease in the market share is a signal to increase all types of advertising.
LSLMEN: When the number of salesmen is high, change in advertising is most likely to be reflected in increases in the personal media, supporting the salesman.

Change in NCOM: Increases in the number of competitors signals an increase in advertising, and, likely, an increase in selling activity. This increase is seen mainly through the personal media.

Changes in CPLANS: If product plans are more aggressive in the current year than during the past year, advertising in increased. This increase is seen mainly through the impersonal media.

LUSERS: If the number of users is high (all other things being equal), impersonal media has a tendency to rise as does total advertising dollars, although to a lesser degree.

CONC: If the fraction of sales to the three largest customers is high, advertising (along with impersonal component) tends to decrease. This is consistent with the relationship with users.

ADVDUM: If the advertising budget is large, proportional changes in advertising will tend to be small. Conversely, if the advertising budget is small, percentage increases in advertising will be large, with these increases occurring in the impersonal portion of the advertising budget.

The models developed here are interesting, intuitively understandable and internally consistent. Most independent variables affecting advertising tend to affect only its personal or impersonal component. But are these effects real? The argument at the end of Part 1 of the paper holds here as well. We do a reasonable job measuring the difficult to analyze year-to-year fluctuations in advertising.
EXHIBIT 3

Advertising Change Model

\[
\text{LOGIT(ADVP)} = \text{ALOGIT} = -0.193 \\
+ 2.124 \times \text{CPLANS} \quad (4.0) \\
- 1.390 \times \text{PMSD} \quad (3.0) \\
- 1.147 \times \text{CONC} \quad (2.0) \\
+ 1.172 \times \text{NCOM} \quad (1.8) \\
+ 0.029 \times \ln(\text{USERS 2}) \quad (0.8) \\
- 0.532 \times \text{ADVDUM} \quad (2.1)
\]

\[R^2 = 0.39\]
\[SE = 1.03\]
\[F = 6.86\]
\[N = 72\]

EXHIBIT 4

Marketing Change Models

\[
\text{LOGIT(MKTP)} = \text{MLOGIT} = +0.084 \\
+ 1.338 \times \text{NCOM} \quad (2.2) \\
- 0.485 \times \text{CONC} \quad (1.2) \\
- 0.444 \times \text{PMSD} \quad (0.9) \\
- 0.279 \times \text{ADVDUM} \quad (1.5)
\]

\[R^2 = 0.17\]
\[SE = 0.62\]
\[F = 2.24\]
\[N = 48\]
EXHIBIT 5

Impersonal Advertising Change Model

\[
\text{LOGIT(IMPERS)} = \text{ILOGIT} = +.194
\]

\[
+1.136 \times \text{CPLANS} \quad (2.1)
\]

\[
-1.128 \times \text{PMSD} \quad (2.7)
\]

\[
+ .098 \times \ln(\text{USERS 2}) \quad (2.9)
\]

\[
- .124 \times \ln(\text{SLMEN}) \quad (1.3)
\]

\[
- .497 \times \text{CONC} \quad (0.7)
\]

\[
- .151 \times \text{LFCYC} \quad (0.6)
\]

\[
- .221 \times \text{ADV Dum} \quad (0.7)
\]

\[
R^2 = .31
\]

SE = .81

F = 3.16

N = 58

EXHIBIT 6

Personal Advertising Change Model

\[
\text{LOGIT(PERS)} = \text{PLOGIT} = -2.608
\]

\[
+ .582 \times \text{LFCYC} \quad (1.8)
\]

\[
-1.875 \times \text{PMSD} \quad (1.8)
\]

\[
+ .170 \times \ln(\text{SLMEN}) \quad (1.7)
\]

\[
+1.082 \times \text{NCOM} \quad (1.3)
\]

\[
R^2 = .22
\]

SE = 1.00

F = 2.63

N = 44
### Change Model Comparison

<table>
<thead>
<tr>
<th></th>
<th>LFCYC</th>
<th>PMSD</th>
<th>LSLMEN</th>
<th>NCOM</th>
<th>CPLANS</th>
<th>LUSERS</th>
<th>CONC</th>
<th>ADVDUM</th>
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<tbody>
<tr>
<td>PERSP</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
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<tr>
<td>ADVERP</td>
<td>-</td>
<td></td>
<td>+</td>
<td>+</td>
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<tr>
<td>IMPERSP</td>
<td>-</td>
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<td>-</td>
<td>+</td>
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<td>MKTP</td>
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Several issues can be raised however. The models are simple and do not explain an overwhelming portion of data variance. Several more complex model forms were tried (with various variable transformations) with no significant change in fit. Scattergrams do not reveal patterns of non-linearity. We conclude:

1. Our task is difficult as, in a sense, we try to explain residual variation (what's left over, on a year-to-year basis, from the norm models). Thus we cannot expect our models to have as high explanatory power as the norm models.

2. The effects identified are of the right sign, are consistent across models and do provide a level of understanding about the dynamics of industrial marketing budgeting not available before.

4. Distribution Channels Model

The data collected in ADVISOR 2 includes measures of directness of channels of distribution. Using the same motivation as developed earlier, we seek a relationship between product, market and environmental characteristics and the distribution channel selection decision. Our objective is, again, to develop norms to guide the channel selection decision.

A manufacturer must assess channel alternatives to determine if they meet established objectives and fit well with the company, product, competitive environment, and users. Normally, firms need only analyze a subset of all possible channel structures as being feasible. The number of alternatives which remain may still be fairly large, however, as seen in Exhibit 8 (taken from Risley [11], p.68).
EXHIBIT 8

Industrial Distribution Channel Alternatives

Manufacturer-Seller

- Mfgr's Agents
- Mfgr's Branches
- Export

Industrial Distributors

Contractors

Franchised Dealers

Industrial Users and OEMs

EXHIBIT 9

The Six Most Common Industrial Channels

Producers of Industrial Goods

Manufacturer's Representative

Manufacturer's Representative

Manufacturer's Sales Branches

Manufacturer's Sales Branches

Industrial Distributor

Industrial Distributor

Industrial Customers
Exhibit 9 depicts the six most common channels for industrial products (taken from Haas [4], p.153). Diamond's [3] study of 167 industrial manufacturers in 220 product lines found that these six basic channels accounted for all sales in his sample. It should also be noted that of the six basic channels, two represent "captive" or company-internal channels, while the remaining four alternatives are "independent" or company-external channels (three of which utilize industrial distributors as an intermediary). This distinction points up a key question: should the distribution channel be composed of captive or independent units?

In a review of current research, Barefoot [1] concludes that some current approaches to channel evaluation are valuable tools which, when used effectively, can assist industrial marketing managers in improving the process of distribution channel selection. The current approaches are mainly conceptual, however, and do not provide sound, quantitative guidelines for decision-making.

Two key reasons for the lack of definitive, quantitative research in distribution channel selection for industrial markets are the complexity of the decision and the difficulty in specifying its key determinants. We therefore, propose a simple, hierarchical decision making approach, where initial stages of the decision process constrain the following stages. This process is presented in Exhibit 10.

In the first stage, a decision is made about whether a product's distribution should be handled by captive or independent channel members.

Given that choice, the next question is whether to use direct or indirect distribution. If it is decided in stage one to distribute a
Hierarchical Decision Model

Product-Market Characteristics

Stage 1
Captive or Independent?
- Captive
- Independent

Stage 2
Direct or Indirect?
- Direct
- Indirect

Stage 3
Select Specific Channel

Selection Made
product through independent resellers, the stage two choice will have already taken place implicitly (i.e., indirect distribution). If company-owned distribution is specified, however, the industrial marketer must determine whether a direct sales force or some form of captive intermediary is warranted.

The third stage involves selecting a specific channel structure. If the output of the first two stages of the model are "captive" and "direct", respectively, then the third stage is determined. However, if the first two stages yield "captive" and "indirect", or "independent" and "indirect", the industrial marketer must determine the most appropriate channel which fits that particular description.

Our analysis treats only the first two stages of the proposed hierarchical model, since the output of the third stage often follows directly from the specifications of stages one and two. Lewis [7] and Haas [4], note that the choice of specific distribution channels is often constrained by legal, competitive, and other environmental constraints. In some instances, the marketer has no choice at all at this level of decision making. There is a tendency for competitive products to be sold through the same channels, and a similar tendency holds for related noncompetitive products in the same general field. Sometimes there are accepted industry channels that cannot be easily modified.

The model we develop is concerned primarily with stage 1 of the analysis above. (The ADVISOR 2 data did not provide a sufficient number of products with company-owned resellers to allow meaningful analysis at stage 2). We are concerned, at stage 1 with discriminating between a captive
or internal (generally salesforce) channel versus an independent or external channel (industrial distributor). Our model will then be a discriminant analysis model, with key independent variables including:

- size of the firm
- size of an average order
- technical purchase complexity
- stage in product life cycle
- fraction of units standard, carried in inventory
- purchase frequency
- number of customers

Barefoot [1] gives details of the literature review process that lead to the selection of the list above as key discriminating variables.

5. Distribution Channels Model Calibration

Linear discriminant analysis (our analysis tool) assumes, for optimality, that the discriminating variables have a multivariate normal distribution and that the covariance matrices of the two groups be equal (Morrison [10]).

According to Heyck and Klecka [5], these assumptions do not need to be strongly adhered to in practice, as the technique is very robust. In practice, unless one wishes to make statistical inferences, relatively major deviations from these assumptions pose few problems. As an applied decision-making tool, as in this instance, discriminant analysis can provide useful descriptive information.

To develop the discriminant function, the two groups were designated as those products which had at least 90 percent of their distribution handled
through captive (group one) or independent (group two) channels. This yielded group memberships of fifty-five and thirty-five, respectively. The remaining thirty-five cases, which exhibited more of a balanced distribution strategy, were held out of the analysis. The results of the discriminant analysis are described in Exhibit 11. This Exhibit contains all variables postulated to be important except for the number of customers, (whose effect was not significantly different from zero).

EXHIBIT 11

**Standardized Discriminant Function**

\[ D = 0.585(\text{Size of firm}) + 0.361(\text{Size of average order}) \\
+ 0.309(\text{Technical-purchase complexity}) - 0.552(\text{Stage in product life cycle}) - 0.264(\text{Degree of standardization}) \\
- 0.206(\text{Purchase frequency}) \]

The canonical correlation associated with the discriminant function is 0.518. This measure of association explains how closely the discriminant function and a dummy variable which defines the group membership of each case are related. The canonical correlation squared (i.e., 0.268) can be interpreted as the proportion of variance in the discriminant function which is explained by the groups.

A summary of the discriminant function and key associated statistics can be found in Appendix 3.
The classification phase of the discriminant analysis revealed that 70 percent of the cases used in the development of the discriminant function were correctly classified. This represents a success rate of prediction significantly greater than that expected by chance (i.e., 55/90 = .61).

A predictive test was performed classifying the thirty-five firms which were held out of the original analysis. This is a severe test of the model's classification abilities, as the majority of cases in this subset have at least 30 percent of their distribution going through the secondary channel mode. Twenty-two of the thirty-five cases in the subsample were correctly classified, yielding a rate of 62.9 percent successfully predicted. Although this success rate is lower than the 70 percent recorded with the initial which was utilized to develop the discriminant function, it is still substantially better than a rate which could be achieved by chance (Appendix 4).

To check on the significance of the model's predictive capabilities, a normal approximation to the binomial distribution of distribution mode (i.e., \( p = \text{proportion captive}, \ (1-p) = \text{proportion independent} \)) was used to test if 62.9 percent is statistically greater than the proportional chance criterion of 50 percent. For the subsample of thirty-five cases which were held out of the original analysis, it was found that .63 is greater than .50 at a 94 percent level of significance. Thus, it appears that the model works.

Checks on the normality of the data and the lack of equality of covariance matrices resulted in deviation from theoretical assumptions (Barefoot [1]). Thus, linear discriminant analysis is not the optimal decision rule in this instance; however, its application provides the industrial
marketing manager with valuable information with which to improve distribution system decision making. Since the data collected for ADVISOR 2 was never collected specifically for this purpose, we should not be surprised. Further refinements in the analysis would only be possible after a considerable amount of additional work and collection of additional data, calling for another specially designed study.

The standardized discriminant function is easy to interpret. The mean score is zero with a standard deviation of one. Therefore, any single score represents the number of deviations that a product is away from the mean for all cases. The standardized coefficients represent the magnitude of the contribution of their associated variables to the discriminant function, with their signs indicating whether that contribution is positive or negative. We interpret the results in Exhibit 11 as follows:

"Size of firm" has the largest positive contribution to the discriminant score (which corresponds to captive distribution as the dependent variable). This means that as a firm grows larger, it is better able to support a company-owned distribution system.

"Size of average order" also exhibits a positive effect on captive distribution. The captive option is dominated by direct sales, since 52 percent of all unit sales in the ADVISOR 2 sample are made via direct sales, as compared to 4.8 percent through company-owned resellers. Thus, as the average order size increases, the captive (essentially direct) option becomes more economical.

"Technical-purchase complexity" also contributes positively to the discriminant function value. This variable is based on the manufacturer's
impression of the importance of technical service in a particular case's product category and on the perceived amount of analysis which the buyer must perform prior to purchasing the product. The more important technical service is to a product's success and the more important the buyer views the purchase, the more likely a manufacturer will be to sell it through company-owned distribution channels.

"Stage in product life cycle" is important: the negative sign indicates that as the product goes from growth to maturity, the chances become less and less that captive distribution is the appropriate strategy. Thus, a new or growing product is more likely to use a captive form of distribution than one which has leveled off in sales. This result is intuitively appealing, as it could imply that a manufacturer with a product which is relatively new to the market would rather internally control the marketing task of creating awareness, stimulating a perceived need, and educating potential users. When the marketing communications task is largely completed, and the product enters the mature stage of its life, independent channels take over.

"Degree of standardization" also has a negative effect. This variable is derived from the fraction of a product's sales which is standard or carried in inventory. A product which is complex, unique, or made-to-order, is more frequently sold through direct means than through independent middlemen, unless the manufacturer has a highly skilled and dependable external distribution channel available.

Another variable which has a negative effect is "purchase frequency", indicating that as purchase frequency increases, the likelihood of using captive distribution channels decreases. As a product becomes more of a
common purchase item, with less personal selling necessary to secure a sale, it becomes advantageous for a manufacturer to sell through distributors or other convenient outlets.

In order to use these results, as described in the next section, we need a classification function, which gives the likelihood of a given mode of distribution given a discriminant score.

If we let:

\[ D = \text{our discriminant score} = \sum a_i x_i \text{ where } \{a_i\} \text{ are the coefficients and } x_i \text{ are the discriminating variables}, \]

\[ q_1 = \text{probability of captive} \]

\[ q_2 = 1 - q_1 = \text{probability of independent channel} \]

\[ P_r(D/\text{Captive}) = \text{empirical ordinate of the (likelihood) density function of } D, \text{ given a captive channel, then} \]

the classification problem is to find \( P_r(\text{Captive}/D) \), which, by Bayes rule, is

\[ P_r(\text{Captive}/D) = \frac{P_r(D/\text{Captive}) q_1}{q_1 P_r(D/\text{Captive}) + q_2 P_r(D/\text{Independent})} \]

Thus, we can easily calculate a classification score using our discriminant function.

The first stage of the proposed three-stage model, which has been developed here, seems quite useful but has some limitations. There are problems with meeting the assumptions required for a linear classification procedure. The most critical of these assumptions was that of equal covariance matrices between the two groups.

The equal covariance assumption is rarely satisfied in practice, although when the two matrices are close it makes little difference in the
results if equality is assumed. According to Lachenbruch [6], when the
covariance matrices are quite different (as in this case), the preferred
classification rule involves using a quadratic (as opposed to linear) dis-
criminant function. In practice, however, deviations from the normality
assumption tend to affect this function rather seriously. Quadratic dis-
crimination analysis is not robust to non-normality, particularly if the
distribution has longer tails than the normal. This suggests that such an
approach to discriminating between groups in this analysis would be risky.

A possible method of circumventing these problems is to employ
nonparametric classification rules. Such methods make no assumptions
about the form of the distribution or equivalence of group covariance
matrices. To date, however, no methods are widely available, and one must
develop the classification rule for each application separately. Relatively
little has been done in applying such rules to discriminant problems
(Lachenbruch [6]), and nonparametric methods require larger samples than
are needed for parameter estimation. Therefore, nonparametric discrimina-
tion might be appropriate if a larger study were undertaken.

Thus we may conclude that while not definitive, this approach has
considerable value. By framing the decision process in a three-stage
hierarchical structure, the key decision determinants are more clearly
defined, and the decision making process itself is simpler to conceptualize
and apply. A quantitative decision tool was developed founded on empirical
data which can be of significant help in analyzing distribution channel
options.
6. Using ADVISOR Results

The result of the ADVISOR project can be used in a variety of ways to help support industrial marketing decision-making.

The use of the ADVISOR models as a tool for Managerial Control, is outlined in Exhibit 12. Here characteristics for an existing product are collected and input to a computer program. The program feeds back budgeting guidelines which are then compared with the actual budget. (Exhibit 12 gives a sample par report. The ranges are production intervals with a user-specified tolerance limit). If the guidelines agree with the budget, no further analysis is performed. If they disagree, reasons for the differences are sought. If special situations exist specific to the product or company, then, again, no further review is indicated. If no such conditions are found, either the product budget is modified or a special product audit is undertaken. In this mode of use, the model acts as a control procedure for exception analysis -- to find those product cases most in need of more detailed review.

Another mode of use deals with developing spending levels for products with no sales history. If a sales potential-projection can be made, then this, along with other characteristics can be entered into the ADVISOR program to generate advertising guidelines for the new product.

Most companies develop long range product plans each year. ADVISOR allows one to generate communications programs consistent with those plans. Product and market forecasts can be developed as ADVISOR input, helping to generate consistent spending projections.

There are many other uses possible of ADVISOR such as determining which key market variables to monitor as indicators of communications budget
EXHIBIT 12

Mode of Use 1 -- Managerial Control for an Existing Product
EXHIBIT 13

PROJECT ADVISOR 2 - BUDGET GUIDELINES

PRODUCT NAME: SAMPLE CASE

SUMMARY OF RESULTS

PRODUCT RATIOS

<table>
<thead>
<tr>
<th></th>
<th>1. ACTUAL</th>
<th>2. CENTER</th>
<th>3. RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADVERTISING (K$)</td>
<td>20.000</td>
<td>24.000</td>
<td>19.200 - 28.800</td>
</tr>
<tr>
<td>ADVERTISING/MARKETING</td>
<td>.020</td>
<td>.025</td>
<td>.020 - .030</td>
</tr>
<tr>
<td>MARKETING (K$)</td>
<td>1000.000</td>
<td>950.000</td>
<td>760.000 - 1140.000</td>
</tr>
</tbody>
</table>

ADVERTISING CHANGE NORM

1975 - 1974

<table>
<thead>
<tr>
<th></th>
<th>1. ACTUAL</th>
<th>2. CENTER</th>
<th>3. RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADVERTISING</td>
<td>.010</td>
<td>.011</td>
<td>.009 - .013</td>
</tr>
<tr>
<td>IMPERSONAL MEDIA</td>
<td>.005</td>
<td>.004</td>
<td>.003 - .005</td>
</tr>
</tbody>
</table>

MEDIA ALLOCATION

<table>
<thead>
<tr>
<th></th>
<th>1. ACTUAL</th>
<th>2. CENTER</th>
<th>3. RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMPERSONAL (K$)</td>
<td>10.000</td>
<td>9.000</td>
<td>7.20 - 10.80</td>
</tr>
</tbody>
</table>

PROPORTION OF DISTRIBUTION CHANNELS INTERNAL

<table>
<thead>
<tr>
<th></th>
<th>ACTUAL</th>
<th>NORM</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERNAL</td>
<td>.70</td>
<td>1.00</td>
</tr>
</tbody>
</table>
changes; assessing the relative or joint effect of several variables on marketing budgets; and allocating scarce marketing resources over a multi-product line.

An important thing to note is that the ADVISOR study uses, detailed above, were developed by project-participants. These are ways that participating companies are currently using the results. Each participating company has a computer program in house which automatically generates ADVISOR norms, taking a set of 19 key questions as input.

ADVISOR results are being used by researchers in another context. As quantitative, descriptive models of practice, they are being used in developing market share response models, as follows.

The ADVISOR marketing budget model can be characterized as:

\[ M = f(x_1 \ldots x_n). \]

Assume that \( x_1 \) = lagged product sales and that all other quantities are known. Then we get:

\[ M(x_1) = f(x_1 \mid x_2=x_2 \ldots x_n=x_n) \]

Suppose that we wish to predict total competitive market spending in a product class. If we know \( f(x_1) \), the distribution of competitive sales (or, equivalently, market shares), then we can predict total industry marketing effort as

\[ (4) \quad M = \int M(x_1) f(x_1) \, dx. \]

This can be input as "them" in US/(US + Them) models of marketing response. (If distributions on other independent variables, \( x_2, x_4 \ldots \), are known as well, (4) easily generalizes).
9. **Conclusions**

The purpose of parts I and II of this paper have been to report the results of the ADVISOR 2 research project. A literature review (Lilien et al [9]) revealed that unambiguous guidelines for budgeting industrial advertising are not presently available. The study also showed that a few product, market and competitive characteristics are sufficient to generate a variety of industrial marketing budgeting norms.

Norm models which suggest industry budgeting guidelines were developed for advertising, marketing, and the advertising/marketing ratio, as well as for the split of advertising with its personal and impersonal components.

Change models, demonstrating how product and market factors influence annual changes in budgeting, were developed for advertising, marketing and for the personal and impersonal components of advertising.

A distribution channels model was created to relate the selection of a captive versus an independent channels strategy to product and market characteristics.

These results were combined in a computerized guidelines model, so that the results might be compared with and applied to products not included in the analyses.

We feel that the study has been quite successful. A total of twenty-two companies provided 131 data points. Although this was not enough to provide a significant hold-out sample for predictive testing, enough data were available to support most of our modeling requirements. Results might have been stronger with a larger data base, but we feel they would not have unearthed significant new findings.

New guidelines are now available against which current marketing decision making can be checked. It is significant that these norms are both
quantitative and situation-specific — that is, they recognize the key underlying product and market characteristics, and their underlying interrelationships the understanding of which facilitates sensible budgeting practice.

A number of areas deserving further attention have become apparent during the study. First, our norm-model analysis is based on historical sales and is thus inappropriate for new products. A fruitful area for further work would be an ADVISOR type model for new products.

Similarly, an analysis discriminating between products that do advertise and products that don't advertise would resolve questions about when zero advertising seems appropriate. A special sample would need to be created for such an analysis.

An alternative approach to studying marketing mix development is to group products with similar marketing mixes and then determine differences between these groups (sort of a reverse ADVISOR analysis). The results of such an effort (Bolshon, Fitchet and Hansen [2]) largely confirm the analyses predicted here.

Another direction of current work is aimed at finding out not what industrial marketers are doing, but determining what they should do. Here we collect time series data on products going through a period of transition (changes in marketing or in the environment) and infer the sales response to changes in the market. This response model can then be used to develop profit-improving marketing plans. Initial results look encouraging and will be the subject of later publications.
There are clearly other related areas of study. For example, duplication of ADVISOR in a non-U.S. environment, (i.e., Europe) is currently underway and could be an important step toward establishing the cross-cultural generality of the results. The ADVISOR studies to date, however, have been an important first step in providing quantifying tools to support the industrial marketing budget-setting process.
REFERENCES


APPENDIX 1

CHANGE MODEL VARIABLES

Dependent Variables

ALOGIT = LOGIT(A*) = LN(A*/(1-A*))

MLOGIT = LOGIT(M*) = LN(M*/(1-M*))

ILOGIT = LOGIT(I*) = LN(I*/(1-I*))

PLOGIT = LOGIT(P*) = LN(P*/(1-P*))

ADVP = (ADV_t - ADV_{t-1}) / ADV_{t-1}

A* = (ADVP+.55)/1.57

MKTP = (MKT_t - MKT_{t-1}) / MKT_{t-1}

M* = (MKTP+.55)/1.57

IMPERSP = (IMPERS_t - IMPERS_{t-1}) / IMPERS_{t-1}

I* = (IMPERSP+.55)/1.57

PERSP = PERS_t - PERS_{t-1}

P* = (PERSP+.55)/1.57

Independent Variables

NCOM = \left[ \frac{\text{# of major competitors (t-1)} - \text{# of major competitors (t-2)}}{\text{# of major competitors (t-2)}} \right]

PMSD = \frac{\text{Market Share dollars (t-1)} - \text{Market Share dollars (t-2)}}{\text{Market Share dollars (t-2)}}

CPLANS = \frac{\text{PLANS}_t - \text{PLANS}_{t-1}}{\text{PLANS}_{t-1}}

CONC = \text{Fraction of industry dollar sales purchased by the industry's three largest customers.}

USERS2 = \text{Independent resellers (t-1) + users (t-1) + downstream specifiers (t-1)}

LUSERS2 = LN(USERS2)

SLMEN = \text{Personal Selling (t) + Technical Service (t)}

LSLMEN = LN(SLMEN)

LFCYC = \text{Stage in Lifecycle}

LFCYC = \begin{cases} 
0 \text{ if stage = 'growth'} \\
1 \text{ if stage = 'maturity'} 
\end{cases}

LADV = LN(ADV_{t-1})

ADVDUM = \text{Size of advertising budget}

ADVDUM = \begin{cases} 
0 \text{ if size = 'small' (LADV < 4.45)} \\
1 \text{ if size = 'large' (LADV > 4.45)} \\
\text{median (LADV) = 4.45} 
\end{cases}
**APPENDIX 2**

**CORRELATION MATRICES FOR CHANGE MODELS**

**Change in Advertising Model - Correlation Matrix**

<table>
<thead>
<tr>
<th></th>
<th>ALOGIT</th>
<th>NCOM</th>
<th>CONC</th>
<th>CPLANS</th>
<th>PMSD</th>
<th>LUSERS2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALOGIT</td>
<td>0.159</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NCOM</td>
<td>-0.281</td>
<td>0.006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONC</td>
<td>0.404</td>
<td>-0.146</td>
<td>0.090</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPLANS</td>
<td>-0.329</td>
<td>-0.054</td>
<td>0.139</td>
<td>0.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMSD</td>
<td>0.031</td>
<td>-0.050</td>
<td>0.353</td>
<td>0.018</td>
<td>0.232</td>
<td></td>
</tr>
<tr>
<td>LUSERS2</td>
<td>-0.177</td>
<td>-0.102</td>
<td>-0.197</td>
<td>-0.044</td>
<td>-0.023</td>
<td>0.270</td>
</tr>
</tbody>
</table>

**Change in Marketing Model - Correlation Matrix**

<table>
<thead>
<tr>
<th></th>
<th>MLOGIT</th>
<th>NCOM</th>
<th>CONC</th>
<th>PMSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLOGIT</td>
<td>0.328</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NCOM</td>
<td>-0.142</td>
<td>-0.162</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONC</td>
<td>-0.087</td>
<td>0.010</td>
<td>-0.265</td>
<td></td>
</tr>
<tr>
<td>PMSD</td>
<td>-0.173</td>
<td>0.010</td>
<td>-0.238</td>
<td>0.044</td>
</tr>
</tbody>
</table>
Change in Impersonal Advertising Model - Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>ILOGIT</th>
<th>CONC</th>
<th>CPLANS</th>
<th>PMSD</th>
<th>LFCYC</th>
<th>LUSER2</th>
<th>LSLMEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>ILOGIT</td>
<td></td>
<td>-0.064</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONC</td>
<td>-0.309</td>
<td></td>
<td>-0.36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPLANS</td>
<td>0.263</td>
<td>0.072</td>
<td>-0.098</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMSD</td>
<td>-0.218</td>
<td>-0.272</td>
<td>-0.167</td>
<td>-0.171</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LFCYC</td>
<td>0.193</td>
<td>-0.284</td>
<td>-0.048</td>
<td>0.230</td>
<td>0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LUSER2</td>
<td>-0.140</td>
<td>-0.451</td>
<td>-0.064</td>
<td>-0.045</td>
<td>0.107</td>
<td>0.384</td>
<td></td>
</tr>
<tr>
<td>LSLMEN</td>
<td>-0.160</td>
<td>-0.449</td>
<td>-0.040</td>
<td>0.050</td>
<td>0.050</td>
<td>0.304</td>
<td>0.720</td>
</tr>
</tbody>
</table>

Change in Personal Advertising Model - Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>PLOGIT</th>
<th>NCOM</th>
<th>PMSD</th>
<th>LFCYC</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLOGIT</td>
<td></td>
<td>0.151</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NCOM</td>
<td>-0.179</td>
<td></td>
<td>-0.067</td>
<td></td>
</tr>
<tr>
<td>PMSD</td>
<td>0.285</td>
<td>-0.091</td>
<td>0.118</td>
<td></td>
</tr>
<tr>
<td>LFCYC</td>
<td>0.238</td>
<td>-0.090</td>
<td>0.264</td>
<td>0.261</td>
</tr>
</tbody>
</table>
APPENDIX 3

DISCRIMINANT ANALYSIS RESULTS

Discriminant Function Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>Standardized</th>
<th>Unstandardized</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>0.585</td>
<td>0.00019</td>
<td>Size of Firm (in $Millions)</td>
</tr>
<tr>
<td>V2</td>
<td>0.309</td>
<td>0.147</td>
<td>Product Importance and technical service indicator variable</td>
</tr>
<tr>
<td>V3</td>
<td>0.361</td>
<td>0.00001</td>
<td>Size of average order (units)</td>
</tr>
<tr>
<td>V4</td>
<td>-0.552</td>
<td>-1.13</td>
<td>Stage in life cycle (Early=0, Late=1)</td>
</tr>
<tr>
<td>V5</td>
<td>-0.206</td>
<td>-0.0233</td>
<td>Purchase frequency (purchase/yr)</td>
</tr>
<tr>
<td>V6</td>
<td>-0.264</td>
<td>-0.719</td>
<td>Fraction of product sales standard, carried in inventory</td>
</tr>
<tr>
<td>Constant</td>
<td>--</td>
<td>-0.0388</td>
<td></td>
</tr>
</tbody>
</table>

Classification Function Coefficients

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>0.00012</td>
<td>-0.00015</td>
</tr>
<tr>
<td>V2</td>
<td>2.00</td>
<td>1.79</td>
</tr>
<tr>
<td>V3</td>
<td>-0.00001</td>
<td>-0.00001</td>
</tr>
<tr>
<td>V4</td>
<td>2.31</td>
<td>3.92</td>
</tr>
<tr>
<td>V5</td>
<td>0.225</td>
<td>0.259</td>
</tr>
<tr>
<td>V6</td>
<td>7.29</td>
<td>8.32</td>
</tr>
</tbody>
</table>

Canonical Correlation = 0.518
Wilk's Lambda = 0.732 (significant at 99.4% level)

Univariate F Ratios (with 1 and 88 degrees of freedom)

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V3</th>
<th>V5</th>
<th>V2</th>
<th>V4</th>
<th>V6</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>7.52</td>
<td>1.59</td>
<td>0.90</td>
<td>8.52</td>
<td>5.64</td>
<td>4.32</td>
</tr>
</tbody>
</table>
Group Covariance Matrices

Covariance Matrix for Group 1

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
<th>V6</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td></td>
<td>1552.8</td>
<td>570.1</td>
<td>-1.3*10^8</td>
<td>267243</td>
<td>-261.7</td>
</tr>
<tr>
<td>V2</td>
<td></td>
<td></td>
<td>-0.01</td>
<td>13582</td>
<td>-3.07</td>
<td>0.01</td>
</tr>
<tr>
<td>V3</td>
<td></td>
<td></td>
<td></td>
<td>18780</td>
<td>1.44</td>
<td>-0.02</td>
</tr>
<tr>
<td>V4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>909718</td>
<td>-10913</td>
</tr>
<tr>
<td>V5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.13</td>
</tr>
<tr>
<td>V6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Covariance Matrix for Group 2

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
<th>V6</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td></td>
<td>-1024.2</td>
<td>4780.4</td>
<td>-8*10^5</td>
<td>-3378.7</td>
<td>11.96</td>
</tr>
<tr>
<td>V2</td>
<td></td>
<td></td>
<td>-0.01</td>
<td>2317.0</td>
<td>-3.65</td>
<td>-0.18</td>
</tr>
<tr>
<td>V3</td>
<td></td>
<td></td>
<td></td>
<td>148.0</td>
<td>-0.58</td>
<td>-0.01</td>
</tr>
<tr>
<td>V4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-939.3</td>
<td>-1091.9</td>
</tr>
<tr>
<td>V5</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>-0.02</td>
</tr>
<tr>
<td>V6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Classification of "Hold-Out" Subsample

Predicated

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>Group 2</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Actual

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>13 ((0.5143))</td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td>8 ((0.4857))</td>
<td></td>
</tr>
</tbody>
</table>

\[ C_{\text{pro}} = a^2 + (1-a)^2 \]

yields

\[ C_{\text{pro}} = (0.5143)^2 + (0.4857)^2 = 0.5004 \]

where,

\( a = \) the proportion of individuals in Group 1
\( 1-a = \) the proportion of individuals in Group 2

Maximum Chance Criterion

\[ C_{\text{max}} = \max (a, 1-a) \]

yields

\[ C_{\text{max}} = \max (0.5143, 0.4857) = 0.5143 \]

Note: methodology taken from Morrison (27), p. 158.