AN ANALYSIS OF DOUBLE ENTRY*

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Introduction

Double entry bookkeeping has been known and used at least four and one-half centuries. Yet it has never been analyzed.[1] We describe the process and use the results, but haven't analyzed it.

Once double entry, the concepts, and the incorporated operations are understood, it becomes possible to improve and extend it. Perhaps the present form and methods are best. We do not know. At present we have no choice. Analysis will reveal the nature of double entry, the possible operations, the possible processes, and their results. With this knowledge it will no longer be necessary to perform double entry by following an ancient recipe. We can choose what we do and how we do it.

A Definition of Double Entry

The definition of double entry presented and discussed here is a representation of the concept behind the activities of the modern accountant which are called double entry accounting. This definition is not suggested as the reasoning of any individual accountant. It is offered, however, as a definition which will produce the results presently obtained, when combined with present methods and techniques of accounting.

Double entry accounting is a method by which dollar units of account are classified according to their source and their present use. The balance sheet

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[1] An excellent attempt was made by Charles E. Sprague in his Philosophy of Accounts (New York: The Ronald Press Company, 1922).

is a statement of sources and uses of dollar-units-of account. The asset side shows uses while the equity side shows sources. Journal entries also show source and use. For example, when an asset is purchased for cash, the journal entry shows cash, the source; and the asset purchased, the use. If the asset were acquired for credit, accounts or notes payable (some creditor) would be the source. The income statement can also be considered a statement of sources and uses of assets.

The important elements of the definition are:

1. dollar-units-of-account

2. the idea that each dollar-unit-of-account can be classified twice

3. the classification categories source and present use.[1]

Each of these will be discussed separately and related to traditional methods of double entry.

Dollar-units-of-account

One of the functions of money is a unit of account.[2] By emphasizing this aspect of money, the value of a unit and its function as a unit of exchange is ignored and attention is focussed upon the number of these units. This is an important aspect of double entry. No evaluation of the dollar-unit-of-account is inherent in the present day practice of the double entry concept;

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[1] See for example, Thomas M. Hill and Myron J. Gordon, Accounting: A Management Approach (rev. ed.; Homewood, Illinois: Richard D. Irwin, Inc., 1959), p. 4.

[2] See for example Alfred W. Stonier and Douglas C. Hague, A Textbook of Economic Theory (London: Longmans Green and Co., 1956), p. 365.

no mechanism for doing this exists.[1] Whatever number is introduced is accepted only as a number to be manipulated. What value it represents is not important.[2]

Another aspect of the dollar-unit-of-account is homogeneity. One dollar of account is like any other and, as a consequence, they can be aggregated. If, on the other hand, we used as units of account several different types (apples, oranges, and lemons), aggregation would be impossible, except in a vector notation, unless a substitution ratio between unit types existed.

The assets, liabilities, and net worth of the organization are not the focal point of double entry. The focal point is the dollar-units-of-account. The manner in which the dollar-units-of-account are counted describes, in a certain fashion, the assets, liabilities, and net worth of the organization. It can be argued that these, not dollar-units-of-account, are being manipulated. However, this argument requires some qualifications. First they are not being manipulated in the accounts, instead some symbolic representation of them is being manipulated. Second, the symbolic representation chosen is quite unusual. There are many characteristics of an asset, a liability, or net worth which can be representative either separately or in combination with other things. Out of all these, only one characteristic is chosen, and that one is the number of dollars given up to acquire the asset, or the number of dollars received from a source (liability or net worth).

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[1]Valuation is an important element of accounting of course. This discussion is about the double entry concept which underlies double entry bookkeeping, an important accounting process.

[2]All units of account are treated identically, i.e., they are regarded as indistinguishable. There are two possible inferences from this. One is that in double entry all units of account are regarded equally in value. The other is that in double entry, valuation is irrelevant to the operation (although not irrelevant to the reason the operations are performed). I adopt the latter view.

Dual Classification

There exists in any firm only one set of dollar-units-of-account not two, but we show, in double entry accounting, what appears to be two sets of units of account.[4] This is typically shown as an equation of either the form A = L + P or A = E. This is not a paradox. The explanation is that each unit of account is classified according to two different characteristics. Thus, given the number of dollar-units-of-account of an organization, we can classify them according to their present disposition (i.e., they were used to acquire these assets) and we can also classify them according to their source (creditors, stockholders, or earnings).[1] Thus it is apparent that the statements A = L + P and A = E are not equations, but identities.[2]

Classification Categories

The classifications, source[3] and present use, are two characteristics that all dollar-units-of-account have in common. While any one dollar cannot be identified with a specific category or a classification, the total dollars can be divided among the categories of each classification. Thus the identification of a specific dollar-unit-of-account with a specific classification category is unnecessary.

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[1] See for example, Hill and Gordon, p. 4; and Robert N. Anthony, _Management Accounting_ (Homewood: Richard D. Irwin, Inc., 1960), pp. 46-47.

[2] See for example, Billy E. Goetz, _Management Planning and Control_ (New York: McGraw-Hill Book Company, Inc., 1949), p. 155; and Maurice Moonitz and Charles C. Staehling, _Accounting: An Analysis of its Problems_, Vol. 1 (Brooklyn: The Foundation Press, 1952), p. 43.

[3] Source is a general not a descriptive term. A longer and more accurate title would be original or reclassified source. Basically source means original entry into the firm. Successive re-entries due to asset turnover are ignored. On the other hand retained earnings can be reclassified and this obscures the original source. As used here, source means original or reclassified source.

[4] Sprague recognized this implicitly but William Morse Cole in his _Accounts: Their Construction and Interpretation_ (Boston: Houghton Mifflin Co., 1908), p. 14 appears to be the first author to clearly state this relationship.

Assignment to a classification category is accomplished by either of two processes, partitioning or counting. In partitioning the total is divided among the classification categories. By counting, on the other hand, the classification category totals are calculated and the classification total is determined from this. In this latter process the total number of dollar-units-of-account are not known initially so it is important that each dollar-unit-of-account is counted once and only once in each of the two classifications.

Analytic Representation of Double Entry

Partitioning a Set

Analytically double entry is the partitioning of a point set of dollar-units-of-account according to two different characteristics. A partitioning of a set B into the partitions b_i, $i = 1,\ldots, p$, $j = 1,\ldots, p$, is defined by the following three relations:

$$1. \quad B = \bigcup_i B_i \tag{1}$$

$$2. \quad B_i \cap B_j = 0, \, i \neq j \tag{2}$$

$$3. \quad B_i \subseteq B \tag{3}$$

Relation 1 states that all the elements of the set B are in all of the partitions taken together. Relation 2 states that each element of B is in <u>only</u> one of the partitions. Relation 3 states that each partition contains all or some of the elements in B and no others. These *three* relations state that every element of the set B is included once and only once in each partition.

Let

> D = the set of dollar-units-of-account of an organization
>
> $a_i \equiv$ a partitioning of the set according to the present use of the dollars of account, $i = 1, \ldots, p$
>
> $e_j \equiv$ a partitioning of the set according to the source of the dollars of account, $j = 1, \ldots, s$.

Thus, a partitioning of a set (D) of dollar-units-of-account according to the characteristic present embodiment of the dollars a_i, $i = 1, \ldots, p$ (for p different embodiments or assets) and another partitioning of the same set according to the characteristic source of the unit-dollars-of-account e_j, $j = 1, \ldots, s$ (for s different sources) can be made.

Since
$$\bigcup_i a_i = D$$

and
$$\bigcup_j e_j = D,$$

it follows that
$$\bigcup_i a_i \equiv \bigcup_j e_j$$

This relationship is the same as the balance sheet in which the sum of the assets and liabilities are represented as being equal.[1] As mentioned before, they are not only equal but identical because of the nature of the double entry concept.

The Partitioning Operations

To perform double entry the operations specified in the definition and equation (1) must be performed and the constraint in the definition and equation (2)

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[1] It is not possible to specify any general relation of the type $a_i \equiv e_j$ except in very special circumstances. For example, if a creditor lends the organization $100 in cash, if the firm has no other cash and no other creditor, and if $a_1 = \$100$ and $e_4 = \$100$ represent the two elements of partitioning process, for this transaction, we can say $a_1 \equiv e_4$.

must be observed. Once an initial partition has been made, there are two methods of recalculating the partitions. One is to repartition the dollar units of account after each change or series of changes has occurred. This is done when a set of accounts is established in terms of aggregates instead of entering the effect of every individual transaction. The other is to change only the partitions affected. This will produce the same result as repartitioning, but less work is involved. This procedure is usually followed today in double entry accounting, once the books are opened.

To change only the partitions affected we must be able to do certain things. There must be negative as well as positive quantities,[1] the partitions affected must be identified, and the amount of the change specified. Furthermore, it is desirable but not essential that a check on the integrity of the double entry identity be possible.

It is possible to perform double entry calculations if these tools are available. That is, these are the minimum necessary elements for performing double entry. There are several possible forms or transformations of these tools. Each transformation will give the same results and will be applied in accordance with the concept of double entry. However, the procedural rules for each transformation may vary.

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[1]With negative and positive quantities the only operation required in double entry is addition.

The Traditional Form of Double Entry

Given the definition of double entry and the analytical model stated above, the question arises "How does this relate to what is done in practice today?" Today, practice is simply the method by which the objectives of the definition are accomplished and the relations implied by the identity are maintained. However, this is not all of the extant mechanics. There is an additional part which accumulates what can be called interperiod information. The balance sheet is the general result of double entry accounting: the profit and loss and funds flow statements are examples of interperiod information. The accounting practice of today is simply one method by which double entry is implemented. This section will be devoted to discussing this practice.

For the practice of double entry the following are needed:

1. A system which embodies the concept and provides for:

    a. A recordation of the transactions by effect on the partitions.

    b. A means of accumulating these effects by partition.

    c. A means of recording details of aggregate changes in partitions (this is an optional feature actually, but necessary in current practice).

These requirements are satisfied by the traditional method of accounting.

The chart of accounts fulfills the definition by establishing the two types of partitions. It also establishes the analytical identity; before any transactions the identity is $0 \equiv 0$. The journal(s) in conjunction with the chart of accounts fulfill part a. In them every transaction is recorded along with the effect and the partitions affected. Part b is embodied in the ledgers. The ledgers actually contain the same data as the journals. However they are arranged differently. In the journals the data are arranged by

transaction in chronological order. In the ledgers the data *are* arranged by account and by debit or credit in chronological order.[1] The details of changes in partitions, part c, are formally incorporated into the double entry system for only one account--the net worth or proprietorship account. The chart of accounts are increased by the number of additional accounts necessary for this desired detail. One additional account is used for each desired subdivision.

The difference between the basic partitions and these subpartitions is clearly established by the convention that the former are known as real accounts and the latter as nominal accounts. It is interesting that no similar effort has been made to establish the relationship between the nominal accounts and the' individual real accounts. If the relationship were established, it would clearly reveal that these nominal accounts are related to the net worth or proprietorship account and no other. This relationship is a consequence of the nature of these accounts--devices for collecting the detail about changes in the associated real account. A great deal of confusion could be avoided if this relationship were clearly reflected in the account structure perhaps by the use of a prefix, e.g., Proprietorship--Rent Expense, Proprietorship--Sales.

The details of changes in partitions for other accounts are often accumulated. This is not a part of the double entry system but an appendage. Examples are the subsidiary accounts used to accumulate the detail of some control account. This control account is often accounts receivable or accounts payable. These control accounts are not a part of the double entry mechanism. This can be established by removing them. When this is done, the double entry process will not be affected in the slightest.

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[1]The data in special journals and ledgers are not strictly identical due to the intermediate step of aggregation performed in the special journals instead of the ledger. However, such a variation does not affect this analysis.

The ancient terms debit and credit and their functions must be discussed also as they are part of the structure of the traditional double entry methods. I have stated that double entry consists of a partitioning process and the subsequent incorporation of the effects of transactions into these partitions. These effects are not always either increases or decreases, in the total unit-dollars-of-account, even in the best of circumstances. Sometimes they are an increase to one account and a decrease to another. Consequently it is necessary, in the journals, to indicate in addition to the account (partition) affected and the effect (amount of change), the sign of this change. A glance at any set of books of account will show that all numbers are unsigned. This might be construed as an indication that all accounting quantities are positive: this is not so. The sign of any quantity in double entry is given by considering jointly the account affected and whether the quantity is described as a debit or a credit. The/well known relationships are the key to this:
following

Account	Increase	Decrease
Asset	dr	cr
Liability	cr	dr
Net worth	cr	dr
Revenue	cr	dr
Expense	dr	cr

The relationships between the analytical formulation of double entry, the definition of double entry, and the traditional-current methods of performing double entry have been described. This was done by first setting up conditions and then demonstrating how the traditional-current methods fulfill these conditions. In the next section these same conditions will be fulfilled by alternative methods which will achieve identical and occasionally more elaborate results.

Additional Forms of Double Entry

It is my contention that double entry exists separately from the means
utilized to implement it. If this is so, then it should be possible to perform
double entry accounting by other means than those employed traditionally. This
section is devoted to the illustration of other methods of performing double
entry.

Plus and minus will be used instead of debit and credit. The absolute
meaning of these will be used. This differs from the variable meaning of debit
and credit which changes with the context within which it is used. The mathematical
operation addition will also be used.

Journal Entries

There is more than one type of journal entry for the traditional approach[1]
and there are several possible alternative methods. Some of these are illustrated
in the following examples. In these examples two illustrative journal entries
will be used. In traditional form these are:

1)	Accounts Payable	5.00	
	Cash		5.00
2)	Cash	2.00	
	Accounts Receivable		2.00

These transactions illustrate the two general types of partition changes:
1) a change (either increase or decrease) in the total number of dollars of
account; 2) no change in the total number of dollars of account but a

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[1]For example some European journal entries are of the form:

Account		Amount
Dr.	Cr.	
Cash	Accounts pay.	5.00

transfer among one of the types of partitions. Examples:

A)

	Asset	Liability of Net Worth		+	-
1)	Cash				5.00
		Accounts Payable			5.00
2)	Cash			2.00	
	Accts. Rec.				2.00

B)

		Asset		L.&N.W.	
	Account	+	-	+	-
1)	Accounts Pay.				5.00
	Cash		5.00		
2)	Cash	2.00			
	Accounts Rec.		2.00		

C)

	Account			
	+	-		Amount
1)		Accounts Pay.		5.00
		Cash		5.00
2)	Cash			2.00
		Accounts Rec.		2.00

D)

	Asset		L.&N.W.		
	+	-	+	-	Amount
1)		Cash		Accts. Pay.	5.00
2)	Cash	Accts. Rec.			2.00

There are three things which must be indicated: 1) account (partition) affected, 2) how affected (increase or decrease; in the traditional system debit

or credit), and 3) amount.[1][2] In this respect these four illustrations and the traditional system are equivalent. However, there are side effects of each which are not equivalent. A primary loss is that old standby debits equal credits. In its place a substitute rule can be devised where necessary for each case. These are:

A) When both transaction accounts are on the same side of the account double line, the dollar amounts must be on the opposite sides of the amount double line, and when both transaction accounts are on opposite sides of the account double line, the amounts must be on the same side of the amount double line;

B) This rule pertains only to the dollar amounts. When the dollar amounts are on the same side of the double line, they must be of the opposite sign and when they are on opposite sides of the double line, they must be of the same sign;

C) No rule necessary;

D) No rule necessary.

The last two illustrations require no rule because each amount is stated once not twice.

In the first two cases we have rules which are no harder to apply than the traditional rule and which are no more deficient in what is left unsaid. That is, in all cases as in the traditional case a knowledge of the accounts and their real classification as asset, liability or net worth is assumed. Given this knowledge A, B, and D are usable. In illustration C an additional rule is needed. This rule is, when both transaction accounts are affected differently

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[1] There is an additional piece of information necessary. This information is not vital to double entry but to the associated record keeping. It is the chronology of events and can be provided by dating or numbering sequentially the journal entries.

[2] This was also clearly recognized by Cole, op. cit., p. 17.

(one increase and the other decrease), they must be of the same real type of account and conversely when both transaction accounts are affected in the same way (both increase or both decrease) they must be of different real types either asset or equity.

None of these implementations can be called a best or worst method in an absolute sense. Each may be best in a particular set of circumstances. I personally prefer D as an alternative to the traditional method because it emphasizes the partitioning process, it emphasizes that only one set of dollars are considered, and it has pedagogical benefits. I believe that it would be easier to teach accounting by relying upon known concepts of increase and decrease instead of obscuring matters by the use of debit and credit.

Ledgers

It was mentioned earlier that ledgers and journals contain exactly the same data, but arranged differently. Ledger data are arranged by account, by effect (increase or decrease), and chronologically. Journal data are arranged chronologically, by transaction, and by effect and account. The information in these two sets of records is identical. One is a rearrangement of the other. This relationship suggests two possible sets of operations not in use today. One relates to the production of the ledgers and the other to the detail obtained.

Traditionally we have generated ledgers by laboriously reproducing-- usually by hand--the data in the journals. Consider using the same physical data for both the journals and the ledgers. Much reproduction and a large source of error would be eliminated.

This two-fold use of the same set of data will be illustrated in the context of the traditional method. Consider a journal entry of the form

dr.	cr.	amount
Cash	Accounts Rec.	5.00

Further assume that each journal entry is physically separate from the others by the use of punched cards or magnetic tape. As a journal, these will be in chronological order. To generate a ledger or account balances the following procedure, the same for all accounts, will be followed. The Cash Account will be illustrated.

First, all journal entries which have either a debit or credit to cash will be separated from the other journal entries. Second these will be divided into those which are debits to cash and those which are credits to cash. Third these two divisions will each be placed in chronological order. At this point, by physical manipulation of some of the journal data, the two sides of the ledger account, cash, have been produced, except for beginning and ending balances. The two sides can be totaled separately and combined with the beginning balance to produce the ending balance.

Once the account balance has been calculated these journal entries must be combined with the others for the period, before the balance of any other account can be calculated. Except for a few accounts the journal entries of one account must be utilized in all the other accounts. Because of this interrelationship it is <u>impossible</u> to sort all journal entries into all ledger accounts simultaneously.[1]

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[1] An approximation to this occurs in the matrix formulation. Each cell of the matrix is one of the possible journal entries which can be made, thus every possible journal entry is represented in the matrix. Notice however that even here the data are not accumulated by account. They are accumulated by type of journal entry. These are then used to calculate account balances.

The account balances must be calculated sequentially so long as the journal entries remain a journal entry, i.e., each is not divided into the two parts, the debit and the credit. This is a possible disadvantage. It is offset by the advantage of being able to recreate the journal or calculating any ledger account at any time by regrouping the journal entries. If each journal entry were divided into two parts, the journal could be recreated only if there *was* an elaborate matching process.

Interperiod Information

Interperiod information is the amounts and related causes of changes in individual accounts. To generate this account detail, account changes must be grouped by categories; types of changes. There are two methods for doing this presently available. One is by introducing additional accounts and the other is by manipulation of data.

The only account detail generated in the ledgers under traditional methods is net worth--proprietorship detail. This is done by the introduction of additional, nominal, accounts. Yet additional detail has been deemed so important in two cases, cash flow and funds flow, that one, the funds statement, is practically a third published financial statement and the other, the cash flow statement is considered highly desirable although less important. In each case all of the data necessary to produce these statements *are* available. No additional accounts are necessary as they are for the net worth--proprietorship detail. The example above will be extended to the production of a cash flow account.[1]

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[1] Note also A. Charnes, W. Cooper, and Y. Ijuri, "Breakeven Budgeting and Programming to Goals", *Journal of Accounting Research* (Spring 1963), pp. 33-36.

The cash flow statement shows by categories the cash inflow and outflow of the business. It is a type of reconciliation; a statement of the cause of the new account balance. The data in this statement is the same as the data in the account. To generate the statement from this data it must be arranged in the same manner as in the statement, i.e., by type of cause.

As before, all journal entries which are either a debit or a credit to cash must be separated from the remainder of the journal entries and the journal entries to cash must be separated into two groups: those which are debits to cash and those which are credits to cash. The debits are increases in cash and the credits are decreases.

If we examine the other half of the journal entry for each debit, we will find some account credited. This credited account is the source of the increase in the cash account. The same is true of the cash credit entries; the account debited on these represents the use of the cash.

At this point a statement of funds could be made by listing for sources all the amounts debited to the cash account and opposite them the account credited, and by listing as uses all the amounts credited to cash and opposite them the accounts debited. This would be a crude statement because many of the sources and uses would be repeated.

These redundancies can be eliminated. The cash entries are divided into two separate groups, debits to cash and credits to cash. Each of these groups must be subdivided. The basis for this subdivision is the other account of the journal entry. Thus the group of debits to cash will be subdivided into as many subdivisions as there are accounts credited. All credits to accounts receivable

will be grouped together, all credits to sales or proprietorship will be grouped together and so on. If it is desirable to aggregate two or more of these subdivisions, this can be done. Next, group totals must be calculated, then these must be put in whatever order is desired and listed as follows:

            group name   group total

This series of names and totals, with the beginning balances, constitute the source part of a cash flow statement. The very same procedure is applied to the accounts debited of the cash-credit journal entries to produce the uses part of the cash flow statement.

    This method is feasible and efficient. It illustrates the nature of this type of statement, its relation to the account, and to the journal. This procedure is of value only for the internal preparation of the cash flow statement. External preparation of the statement must be based of course upon available aggregated information that has been published. Notice that this procedure can be repeated for each account. It is a method of obtaining an analysis of any account which is as detailed as the original journal data.

Summary

Double entry may be defined as the classification of dollar-units-of-account according to source and present use. One not two sets of dollars are counted. The members of this set, not their value, are counted twice: each count is based upon a different classification. The accounting equation is an identity, not an equality.

Once the definition has been fulfilled for a set of unit-dollars-of-account the results are valid until there is a change either in the set or in the distribution to the partitions. The new partitions can be calculated in two ways.

The new set can be repartitioned. This is done rarely if ever because the change has *affected only* a few partitions. The change in the set can be counted and incorporated into the affected partitions. To do this it is necessary to identify the partitions (accounts) affected, the amount of the change, and whether, for each account, the change is an increase or a decrease. Such changes are usually noted and these are summed periodically to find the new partition values (account balances).

Traditionally this was done manually or by mechanization with ledgers, journals, and the debit-credit convention. Feasible alternatives exist and have been illustrated. Each of these alternatives have characteristics slightly different from the traditional method. Consequently, in certain situations, one of these may be preferred over the traditional method.

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