CONSUMPTION AND LIQUIDITY CONSTRAINTS

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ABSTRACT

This paper studies the consumption and savings decisions of an individual who knows there is a positive probability that he will become liquidity constrained at a later date. It also studies the effect on these decisions of a higher variability of income on any given date when lifetime income is assumed to be constant. In other words, it studies the effect of an increase in the "timing" variability of income. I show that this increase generally leads the individual to keep more assets and more liabilities.
I. INTRODUCTION

This paper studies the consumption and savings decisions of an individual who knows there is a positive probability that he will become liquidity constrained at a later date. It also studies the effect on these decisions of a higher variability of income on any given date when lifetime income is assumed to be constant. In other words, it studies the effect of an increase in the "timing" variability of income. I show that this increase generally leads the individual to keep more assets and more liabilities.

The point of departure of this paper is that a number of people have financial assets and liabilities simultaneously. For instance, Table 1 presents the average overlap of assets and liabilities as a percent of the maximum of assets and liabilities for various classes of people. These 1973 data come from the Denver Income Maintenance experiment.\(^1\)

Interestingly the overlap increases with net worth and home ownership. The overlap of assets and liabilities is puzzling because the rate of interest paid on loans is usually much higher than the rate of interest received on assets of comparative maturity and risk.\(^2\) For instance, in July 1983 the rate on new mortgages is around 13% while the rate on 30 year Treasury notes is 11%. The rate on consumer installment credit is 18% while the rate on money market accounts is under 10%. In a sense these rates of interest must be different since their difference must cover the costs of intermediation. In this paper I argue that the simultaneous presence of assets and liabilities in an individual's portfolio suggests that, with positive probability the individual will want to borrow in the future but will only be able to do so on terms more unfavorable than those on current borrowings. This inability to secure debt on terms similar to terms on existing debt will be called a liquidity constraint. This definition of liquidity constraints, which is presented more formally in Section II, is quite similar to the concept used in
TABLE 1

Mean ratio of the minimum of financial assets and financial liabilities to the maximum of assets and liabilities.

<table>
<thead>
<tr>
<th>Net worth</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 0</td>
<td>.050</td>
</tr>
<tr>
<td>Between 0 and 5000\$</td>
<td>.133</td>
</tr>
<tr>
<td>Between 5000\$ and 10,000\$</td>
<td>.141</td>
</tr>
<tr>
<td>Over 10,000\$</td>
<td>.230</td>
</tr>
<tr>
<td>Renters</td>
<td>.136</td>
</tr>
<tr>
<td>Homeowners</td>
<td>.171</td>
</tr>
</tbody>
</table>
the empirical work of Hayashi (1982), and Runkle (1983). It is however distinct from other definitions which deserve mention. The first is that an individual is liquidity constrained whenever he cannot borrow at the rate at which he could lend. This presumably makes all borrowers liquidity constrained. The second definition is that someone is liquidity constrained when he can't borrow at any interest rate. This makes only the victims of usury laws liquidity constrained. This latitude in definitions parallels the latitude of the definitions of "involuntarily unemployed" expounded by Lucas (1979). The purest definition of liquidity constraint is probably that an individual is liquidity constrained if his expected marginal rate of substitution between consumption today and consumption tomorrow is above the cost to society of providing him with a unit of consumption tomorrow in all states of nature. However, such a definition in contrast to the one presented here is not very operational.

After exploring the causes of the simultaneous presence of assets and liabilities in Section II, I assume the individual does indeed borrow and lend. I then study in Section III the effect on consumption of an increase in the variability of the timing of income. Conclusions are presented in Section IV.

II. LIQUIDITY CONSTRAINTS.

In this Section I consider an individual who will live at least three periods. His utility function at zero is given by:

\[ V = E_0 \sum_{i=0}^{T} \rho^i U(\bar{c}_i) \]  

(1)

where \( \rho \) is a discount factor, \( U \) is a concave function \( \bar{c}_i \) is the vector of consumptions at \( i \), while \( E_0 \) takes expectations conditional on information available at zero. The individual has access to the following financial
instruments: a short term bond in period zero which costs $1 in period zero and pays \((1+r_0)\) dollars in period 1, a short term bond in period 1 which pays \((1+r_1)\) dollars in period 2, and a long term bond in period zero which pays \((1+\frac{r_o}{1+r_1})\) dollars in period 2. The individual can also borrow short in period zero by paying \((1+r_0)_0\) in period 1 for each dollar borrowed in period zero. \(\beta_0\) is bigger than one since it covers the cost of intermediation. It is important to emphasize that \(\beta_0\) does not include a default risk premium. The individual considered here does not plan to default. If defaulting were a distinct possibility, the individual might borrow and lend simultaneously simply because his loans would provide some insurance against those states of nature which induce default.

By paying \((1+r_1)\beta_1\) in period 2, the individual can borrow one dollar in period 1. He is also allowed to borrow long in period zero by paying respectively \(\phi_1\) and \(\phi_2\) dollars in periods 1 and 2 for each dollar received in period zero. Finally, there is a futures market. By promising to pay \(f\) dollars in period 1, an individual is assured of receiving then a claim on $1 in period 2. Arbitrage by individuals who hold both short and long bonds insures that \(f\) is equal to \((1+r_0)/(1+\frac{r_o}{1+r_1})\). Let \(S_0, S_1, L_0, D_0, D_1, D\) and \(F\) denote the holdings of short bonds at zero and one, long bonds at zero, short debt at zero and one, long debt and future contracts respectively. Financial instruments for later periods are neglected to preserve simplicity. The budget constraints in the first three periods are given by:

\[
\begin{align*}
\bar{P}_0 \bar{C}_0 + S_0 + L - D_0 - D &= y_0 \\
\bar{P}_1 \bar{C}_1 + D_0 (1+r_0)\beta_0 + D\phi_1 + S_1 &= D_1 + y_1 + S_0 (1+r_0) + L(\frac{1+r_0}{1+r_1}) \\
&+ F(\frac{1}{1+r_1} - \frac{1+r_0}{1+\frac{r_o}{1+r_1}})
\end{align*}
\]
\[ \bar{P}_2 \bar{C}_2 + D\phi_2 + D_1(1+r_1)\beta_1 = y_2 + S_1 (1+r_1) \]

where \( y_1 \) and \( \bar{P}_1 \) are income in 1 and the vector of prices in 1 respectively. Note that I ignore the advantages of certain assets as means of payments. These advantages are an alternative rationale for the overlap of assets and liabilities shown in Table 2. The theory developed here is really concerned with the overlap between nonmonetary financial assets and liabilities. \(^3\)

Let \( C_1 \) and \( P_1 \) be the quantity consumed and the price at 1 of the numeraire good. Using (2), (3) and (4) to substitute for \( C_1, C_2 \) and \( C_3 \) in (1), one obtains the following first order conditions which take into account that \( S_0, S_1, D_0, D_1, D, \text{and } L \) must be nonnegative:

\[
\frac{\partial V}{\partial S_0} = -\frac{1}{P_0} \frac{\partial U}{\partial C_0} + \rho (1+r_0) E_0 \frac{1}{P_1} \frac{\partial U}{\partial C_1} \leq 0 \quad S_0 \frac{\partial V}{\partial S_0} = 0 \quad (5)
\]

\[
\frac{\partial V}{\partial L} = -\frac{1}{P_0} \frac{\partial U}{\partial C_0} + \rho (1+r_0) E_0 \frac{1}{P_1(1+r_1)} \frac{\partial U}{\partial C_1} \leq 0 \quad L \frac{\partial V}{\partial L} = 0 \quad (6)
\]

\[
\frac{\partial V}{\partial D_0} = \frac{1}{P_0} \frac{\partial U}{\partial C_0} - \rho (1+r_0) \beta_0 E_0 \frac{1}{P_1} \frac{\partial U}{\partial C_1} \leq 0 \quad D_0 \frac{\partial V}{\partial D_0} = 0 \quad (7)
\]

\[
\frac{\partial V}{\partial D} = \frac{1}{P_0} \frac{\partial U}{\partial C_0} - \phi_1 \rho E_0 \frac{1}{P_1} \frac{\partial U}{\partial C_1} - \phi_2 \rho \frac{2}{E_0} \frac{1}{P_2} \frac{\partial U}{\partial C_2} \leq 0 \quad D \frac{\partial V}{\partial D} = 0 \quad (8)
\]

\[
\frac{\partial V}{\partial \theta} = E_0 \left( \frac{1}{1+r_0} - \frac{1+r_0}{1+r_1} \right) \frac{1}{P_1} \frac{\partial U}{\partial C_1} = 0 \quad (9)
\]

\[
\frac{\partial V}{\partial S_1} = -\frac{1}{P_1} \frac{\partial U}{\partial C_1} + (1+r_1) \rho E_1 \frac{1}{P_2} \frac{\partial U}{\partial C_2} \leq 0 \quad S_1 \frac{\partial V}{\partial S_1} = 0 \quad (10)
\]

\[
\frac{\partial V}{\partial D_1} = \frac{1}{P_1} \frac{\partial U}{\partial C_1} - (1+r_1) \beta_1 \rho E_1 \frac{1}{P_2} \frac{\partial U}{\partial C_2} \leq 0 \quad D_1 \frac{\partial V}{\partial D_1} = 0 \quad (11)
\]

The last two equations are obtained by differentiating the expected sum of
the utilities from period 1 on. From these expressions it is easy to establish that the individual never holds simultaneously short assets and liabilities. In particular:

**Proposition 1:**

1) \( S_1 > 0 \implies D_1 = 0 \)

2) \( D_1 > 0 \implies S_1 = 0 \)

**Proof.**

1) From (5) or (10)

\[
\frac{1}{p_1} \frac{\partial U}{\partial C_1} - \rho(1+r_1)E_1 \frac{1}{p_{i+1}} \frac{\partial U}{\partial C_{i+1}} = 0
\]

So:

\[
\frac{1}{p_1} \frac{\partial U}{\partial C_1} - \rho(1+r_1)E_1 \frac{1}{p_{i+1}} \frac{\partial U}{\partial C_{i+1}} = \frac{1}{p_1} \frac{\partial U}{\partial C_1} (1-\beta_1) < 0
\]

And: \( D_1 = 0 \)

2) From (7) or (11)

\[
\frac{1}{p_1} \frac{\partial U}{\partial C_1} = \rho(1+r_1)E_1 \frac{1}{p_{i+1}} \frac{\partial U}{\partial C_{i+1}}
\]

So:

\[
-\frac{1}{p_1} \frac{\partial U}{\partial C_1} + \rho(1+r_1)E_1 \frac{1}{p_{i+1}} \frac{\partial U}{\partial C_{i+1}} = \rho(1+r_1)(1-\beta_1)E_1 \frac{1}{p_{i+1}} \frac{\partial U}{\partial C_{i+1}} < 0
\]

And: \( S_1 = 0 \)

If the individual can borrow in period 1 on terms roughly equal to those offered on long term loans in period zero:

\[
\frac{\phi_1}{(1+r_0)\beta_0} + \frac{\phi_2}{(1+r_0)\beta_1} = 1
\]

That is the value of the payments in the two periods discounted at the rate of
interest plus the commission one has to pay on short loans equals one. It
turns out that even when \( \frac{\phi_1}{(1+r_o)S_o} + \frac{\phi_2}{(1+\ell_o)B_1} \) is somewhat smaller than one,
so that long term loans have a more attractive rate of interest than short
loans, long term loans will be zero when assets are positive in period zero,
as the following proposition demonstrates.

**Proposition 2:** \( S_o > 0, \frac{\phi_1}{1+r_o} + \frac{\phi_2}{(1+\ell_o)B_1} > 1 \Rightarrow D = 0 \)

**Proof.** Using (5) and (9):

\[
\rho E \frac{1}{P_1} \frac{\partial U}{\partial C_1} = \frac{1+\ell_o}{1+r_o} \rho E \frac{1}{P_1(1+r_1)} \frac{\partial U}{\partial C_1} = \frac{1}{P_o(1+r_o)} \frac{\partial U}{\partial C_o}
\]

At time zero, the individual knows that with probability \( \pi_1 \), (11) will
hold with equality in period 1, with probability \( \pi_2 \), (10) will hold with
equality, and with probability \( 1-\pi_1-\pi_2 \):

\[
\rho E \frac{1}{P_2} \frac{\partial U}{\partial C_2} < \frac{1}{P_1(1+r_1)} \frac{\partial U}{\partial C_1} < \beta_1 \rho E \frac{1}{P_2} \frac{\partial U}{\partial C_2}
\]

Therefore:

\[
E_o \frac{1}{P_1(1+r_1)} \frac{\partial U}{\partial C_1} \leq \beta_1 \rho E_o \frac{1}{P_2} \frac{\partial U}{\partial C_2}
\]

with equality only when \( \pi_1 = 1 \).

So:

\[
\frac{\partial V}{\partial D} \leq \frac{1}{P_o} \frac{\partial U}{\partial C_o} - \phi_1 \rho E_o \frac{1}{P_1} \frac{\partial U}{\partial C_1} - \frac{\phi_2}{\beta_1} E_o \frac{1}{P_1(1+r_1)} \frac{\partial U}{\partial C_1} = \frac{1}{P_o} \frac{\partial U}{\partial C_o} [1 - \frac{\phi_1}{1+r_o} - \frac{\phi_2}{\beta_1(1+\ell_o)}]
\]

Q.E.D.

In this model there are two conceivable reasons for the individual to hold
long term debt. The first is that he is speculating on changes in interest
rates. This is ruled out here by the presence of the costless futures market,
but, in any event, doesn't appear very realistic. The second reason is to prepare for the contingency that the individual will have to borrow in the future. If borrowing in the future is equally costly as borrowing today, borrowing in the future is preferable since the cost will generally have to be incurred with probability lower than one and, in addition, the individual doesn't incur the cost of borrowing the first period. As Proposition 3 illustrates, the premium on borrowing in period 1, \( \beta_1 \) must be quite high for borrowing and lending in period zero to be worthwhile.

\textbf{Proposition 3:} \( S_0 > 0, D > 0, \frac{\phi_1}{1+r_0} + \frac{\phi_2}{1+\ell_0} > 1 \Rightarrow \beta_1 > \frac{\phi_1}{1+r_0} + \frac{\phi_2}{1+\ell_0} \)

\textbf{Proof.} Using (5) and (8)

\[
E_0 \left( \frac{1}{P_1} \frac{\partial U}{\partial C_1} \right) = \frac{\rho \phi_2}{1+r_0-\phi_1} E_0 \frac{1}{P_2} \frac{\partial U}{\partial C_2}
\]

which implies that \( 1+r_0-\phi_1 \) is positive. Using (9):

\[
E_0 \frac{1}{P_1 (1+r_1)} \frac{\partial U}{\partial C_1} = \frac{\phi_2 (1+r_0)^\rho}{(1+r_0-\phi_1)(1+\ell_0)} E_0 \frac{1}{P_2} \frac{\partial U}{\partial C_2}
\]

From (10) and (11):

\[
\text{PE}_0 \frac{1}{P_2} \frac{\partial U}{\partial C_2} \leq E_0 \frac{1}{P_1 (1+r_1)} \frac{\partial U}{\partial C_1} \leq \beta_1 \text{PE}_0 \frac{1}{P_2} \frac{\partial U}{\partial C_2}
\]

Therefore:

\[
\beta_1 \geq \frac{\phi_2 (1+r_0)}{(1+r_0-\phi_1)(1+\ell_0)}
\]

with strict inequality unless the individual intends to borrow in all states of nature. \( \frac{\phi_1}{1+r_0} + \frac{\phi_2}{1+\ell_0} > 1 \) implies that \( \frac{\phi_2}{1+\ell_0} \) is bigger than \( \frac{(1+r_0-\phi_1)}{(1+r_0)} \), so that \( \beta_1 \) is bigger than one. Hence:
\[
\beta_1 (1 - \frac{\phi_1}{1+r_o}) \geq \frac{\phi_2}{1+\ell_o}
\]

\[
\beta_1 > \frac{\phi_1}{1+r_o} + \frac{\phi_2}{1+\ell_o}
\]

Q.E.D.

So, the premium as a percent of the principal paid on the short term debt must exceed the premium on the longer debt. This fact can be used to establish that the simultaneous presence of assets and liabilities in an individual's portfolio is prima facie evidence that he is likely to become liquidity constrained.

**Definition:** An individual is "liquidity constrained" in period 1 in the amount he borrows in period 1, \(D_1\), is smaller than it would be if \(\phi_1/1+r_o + \phi_2/1+\ell_o\) were equal to \(\beta_1\).

**Proposition 4:** \(D > 0, S_o > 0 \Rightarrow\) the individual has a positive probability of being liquidity constrained in period 2.

**Proof:** Under the conditions of the proposition, (12) holds. Unless incomes, prices and rates of return are distributed in an extremely unusual way, there will be states of nature for which \((\partial U/\partial C_1) [1/P_1(1+r_1)]\) is different from \([\phi_2 (1+r_o) P/ (1+\ell_o - \phi_1) (1+r_o)] E_1 (\partial U/\partial C_2) (1/P_1)\). This would happen in particular if there were states in period 1 in which the individual holds either short assets or liabilities. In this case there is a positive probability that \((\partial U/\partial C_1)[1/P_1(1+r_1)]\) will exceed \(\hat{\beta}_1 E_1 (\partial U/\partial C_2)(1/P_2)\) where \(\hat{\beta}_1\) is given by the premium on long debt, \(\phi_1/1+r_o + \phi_2/1+\ell_o\). Hence, there is a nonzero probability that \(\partial U_1/\partial D_1\) would be positive if \(\hat{\beta}_1\) were equal to \(\beta_1\).

Q.E.D.
In his empirical work Hayashi (1982) treats individuals as liquidity constrained when their consumption goes up by one dollar every time their income go up by a dollar. If the borrowing and lending rates of interest differ, there will be a set of realizations for income in which consumption responds in this way since neither saving nor borrowing will be worthwhile. The presence of liquidity constraints like those discussed here just makes the set larger. Runkle (1983) estimates individual Euler equations like (11) using the rate of interest on installment credit. Insofar as he finds that the inequality is reversed he finds that some people are unable to borrow as much as they want at this rate. They are liquidity constrained in a stronger sense than the one treated here.

III. THE EFFECT OF INCREASES IN THE TIMING VARIABILITY OF INCOME.

This section illustrates some comparative statics in a simplified version of the model presented in Section II. In particular it is assumed that $\tau$ is equal to 2 so that the individual has three periods to live. Prices and interest rates are constant so that the futures market is redundant. The real interest rate, $r$, is assumed to obey $\rho(l+r) = 1$. To focus on changes in liquidity constraints, I assume that lifetime resources are known in period zero so that $y_1(l+r) + y_2$ is equal to a constant $k$. Instead, $y_1$ is a random variable with probability density function $F(y_1)$. 

Bad times like troughs of the business cycles undoubtedly also have some effect on lifetime resources. On the other hand, recessions do tend to be followed by recoveries, so the effect of these events on lifetime resources may well be small. This would be particularly true if recessions were caused by intertemporal substitution of leisure on the part of optimizing individuals. In any event recessions have a very large effect on the ratio of
current income to "typical" income. This is the effect captured by the current model.

An increase in the timing variability of income is simply a mean preserving spread applied to \( F \). Such a change in variability can be viewed as analogous to a change in the amplitude of business cycles.

In subsection (a) I consider the case of one nondurable consumption good, while subsections (b) and (c) consider durable goods as well.

(a) The Case of One Consumption Good

Let \( C_i \) denote the consumption in period \( i \) and let the prices be normalized to one. Then, for \( y_i \) bigger than or equal to \( \overline{y} \), (10) holds with equality so that the consumption in periods 1 and 2 is given by:

\[
C_1 = C_2 = \frac{S_o (1+r) + k - \phi_1 (1+r) D - \phi_2 D}{2 + r}
\]

(13)

when \( y_1 \) is below \( \overline{y} \), (10) holds with a strict inequality; the individual doesn't lend in period 1. For \( y_1 \) slightly below \( \overline{y} \), the individual also doesn't borrow in period 1. I assume that the support of the distribution given by \( F \) doesn't include values of \( y_1 \) so low as to induce the individual to borrow in period 1. Then, for \( y_1 < \overline{y} \):

\[
\begin{align*}
C_1 &= S_o (1+r) + y_1 - \phi_1 D \\
C_2 &= k - y_1 (1+r) - \phi_2 D
\end{align*}
\]

(14)

Note that for \( y_1 = \overline{y} \), (14) must lead to the same consumption rule as (13) so that:

\[
\overline{y} = \frac{k + D (\phi_1 - \phi_2) - S_o (1+r)}{2 + r}
\]

(15)
I assume that in period 0 the individual borrows and lends. Thus (5) and (8) hold with equality and imply that:

\[
\frac{\partial U}{\partial C_0} (y_0 - S_0 + D) = E_0 \frac{\partial U}{\partial C_1} [1 - F(y)] \frac{\partial U}{\partial C_1} \left[ \frac{S_o (1+r) + k - D[\phi_1(1+r) + \phi_2]}{2 + r} \right]
\]

\[
+ \int_0^\infty f(y_1) \frac{\partial U}{\partial C_1} [S_o (1+r) + y_1 - \phi_1 D] dy_1
\]

(16)

\[
\frac{\partial U}{\partial C_0} (y_0 - S_0 + D) = \phi_1 \rho E_0 \frac{\partial U}{\partial C_1} + \phi_2 \rho^2 E_0 \frac{\partial U}{\partial C_2}
\]

\[= [1 - F(y)] \rho(\phi_1 + \rho \phi_2) \frac{\partial U}{\partial C_1} \left[ \frac{S_o (1+r) + k - D[\phi_1(1+r) + \phi_2]}{2 + r} \right]
\]

\[
+ \rho \int_0^\infty f(y_1)[\phi_1 \frac{\partial U}{\partial C_1} [S_o (1+r) + y_1 - \phi_1 D] + \rho \phi_2 \frac{\partial U}{\partial C_2} [k - y_1 (1+r) - \phi_2 D)]dy_1
\]

(17)

The solution of the individual's problem is the pair \((S_o, D)\) which satisfies (16) and (17) once (15) is used to substitute for \(\bar{y}\). Consider the distribution function \(G\) obtained by applying a mean preserving spread to \(F\). If \(g(y_1) = f(y_1)\) for \(y_1\) bigger is than or equal to \(\bar{y}\), \(G(\bar{y}) = F(\bar{y})\) and the solution to (16) and (17) doesn't change. If the change in the distribution of income doesn't affect the states in which the individual consumes his entire income, the individual is indifferent to these changes. Clearly, if the individual could borrow and lend freely at the rate of interest \(r\), no change in \(F\) would change the portfolio in period zero. On the other hand:

**Proposition 5:** Assume that \(G(\bar{y}) \neq F(\bar{y})\). Then, when \(\frac{\partial^3 U}{\partial C_1^3}\) is equal to zero, \(S_0\) and \(D\) will be higher when the distribution is given by \(G\) than when it is given by \(F\).

**Proof.** Figure 1 draws the marginal utility of consumption in period 1 assuming \(\frac{\partial^3 U}{\partial C_1^3}\) is zero for fixed \(S_0\) and \(D\) as a function of \(y_1\). This func-
tion is clearly convex around \( \bar{y} \). Therefore, a mean preserving spread applied to \( F \) which affects the probability of observing income below \( \bar{y} \) increases the mean of \( \frac{\partial U}{\partial C_1} \) by \( \theta \cdot \frac{5}{\theta} \).

Similarly, Figure 2 shows \( \frac{\partial U}{\partial C_2} \) as a function of \( y_1 \). This function is concave around \( \bar{y} \). A mean preserving spread applied to the distribution of \( y_1 \) thus reduces the mean of \( \frac{\partial U}{\partial C_2} \). Hence the mean of \( \phi_1 \frac{\partial U}{\partial C_1} + \rho \phi_2 \frac{\partial U}{\partial C_2} \) goes up by \( \psi \) which is strictly less than \( \phi_1 \theta \) as long as \( \phi_2 \) is bigger than zero.

Even when \( \phi_2 \) is equal to zero by the argument above (12), \( (1/\rho) \) is bigger than \( \phi_1 \) so that \( \rho \psi \) is smaller than \( \theta \).

Assuming the change in the distribution is small enough that one can approximate its effect by totally differentiating (16) and (17):

\[
A = \frac{dS_o}{dD} = -\theta
\]

\[
A = \rho \psi
\]

\[
\begin{align*}
A = & -\frac{d^2 U}{dC_o^2} - \left[1-F(\bar{y})\right] (1+r) \frac{d^2 U}{dC^2} + \int_0^y f(y_1) (1+r) \frac{d^2 U}{dC_1^2} dy_1 \\
& - \frac{d^2 U}{dC_1^2} \left[1-F(\bar{y})\right] \frac{(1+r)^2}{2+r} \frac{d^2 U}{dC_2^2} - \int_0^y f(y_1) \phi_1 \frac{d^2 U}{dC_1^2} dy_1 \\
& - \rho \int_0^y f(y_1) \phi_1 (1+r) \frac{d^2 U}{dC_1^2} dy_1 + \rho \int_0^y f(y_1) \phi_1 \frac{d^2 U}{dC_2^2} dy_1
\end{align*}
\]

where \( \frac{d^2 U}{dC^2} \) is the second derivative of \( U \) evaluated at the level of consumption given by (13). Thus:
\[ \frac{dS}{d} = S = \frac{1}{\Delta} \left\{ \frac{\partial^2 U}{\partial c_2^2} (\rho \psi - \Theta) + [1 - F(y)] \frac{\phi_1 + \rho \phi_2}{2 + r} \frac{\partial^2 U}{\partial c_1^2} \right\} \]

\[ + \rho \int_{0}^{\bar{y}} f(y) [\phi_1 \frac{\partial^2 U}{\partial c_1^2} (\psi - \phi_1 \Theta) - \Theta \rho \phi_2 \frac{\partial^2 U}{\partial c_1^2}] dy \]  

(19)

where \( \Delta \) is the determinant of \( A \) which the second order conditions require to be positive. So, assets increase.

On the other hand:

\[ \frac{dD}{d} = D = \frac{1}{\Delta} \left\{ \frac{\partial^2 U}{\partial c_2^2} (\rho \psi - \Theta) + [1 - F(\bar{y})] \frac{1 + r}{2 + r} \frac{\partial^2 U}{\partial c_2^2} \right\} \]

\[ + \rho \int_{0}^{\bar{y}} f(y) \left( \psi - \phi_1 \Theta \right) \frac{\partial^2 U}{\partial c_1^2} dy \]  

(20)

which is also positive.

Q.E.D.

The proposition is concerned only with the case in which the marginal utility of consumption is linear in consumption. However, assets and liabilities rise more generally. Exceedingly strong sufficient conditions are that \( \Theta \) be positive and that \( \psi \) be smaller than \( \phi_1 \Theta \). Suppose \( \partial^2 U/\partial c^2 \) is positive, then \( \Theta \) is positive as long as for some level of income below \( \bar{y} \), \( F(y) \neq G(y) \).

On the other hand, even if the marginal utility of consumption is a convex function of consumption the marginal utility of consumption in period 2 is a locally concave function of \( y \) around \( \bar{y} \). That is, for small \( \varepsilon \)'s, and \( \lambda \) between zero and one:

\[ \lambda \frac{\partial U}{\partial c_2} (\bar{y} + \varepsilon) + (1 - \lambda) \frac{\partial U}{\partial c_2} (\bar{y} - \varepsilon) < \frac{\partial U}{\partial c_2} (\bar{y}) \]
So, for mean preserving spreads that affect the distribution mainly near \( \bar{y} \), the marginal utility of consumption in period 2 falls so that \( \psi \) is below \( \phi_1 \theta \).

Conversely, if \( \frac{\partial^3 U}{\partial C^3} \) is negative, \( \frac{\partial U}{\partial C_1} \) is still a convex function of \( y_1 \) near \( \bar{y} \). So \( \theta \) is positive for mean preserving spreads whose main effect is near \( \bar{y} \). Moreover, in this case \( \frac{\partial U}{\partial C_2} \) is globally concave and \( \psi \) is less than \( \phi_1 \theta \) as long as for some \( y \) below \( \bar{y} \), \( F(y) \neq G(y) \).

In the standard consumption under uncertainty problem considered, for instance by Sibley (1975), the individual can borrow and lend freely at some fixed rate of interest. An increase in the timing variability of income doesn't affect consumption. An increase in income variability increases saving if and only if \( \frac{\partial^3 U}{\partial C^3} \) is positive. Here lifetime income is constant. But, an increase in the timing variability of income does make the individual more uncertain about the resources he'll have available in period 1. In the standard problem, one drawback of saving in period zero is that, if \( \frac{\partial^3 U}{\partial C^3} \) is negative, the marginal utility of consumption becomes extremely low when the realization of income is high. This is not a problem here since once income exceeds a certain level the marginal utility of consumption becomes constant. So the individual increases his assets under broader circumstances.

Note that when assets and liabilities both increase, the overlap between assets and liabilities, i.e., the minimum of assets and liabilities, increases as well.

The change in consumption in period zero is given by:

\[
dC_o = dD - dS_o = C \equiv \frac{1}{A} \left\{\left[1-F(\bar{y})\right] \frac{\partial^2 U}{\partial C^2} \frac{1+r-\phi_1-\rho \phi_2}{2+r} \left[\psi - (\phi_1+\rho \phi_2) \theta\right] \right. \\
\left. + \int_0^{\bar{y}} f'(y_1)[(\psi-\phi_1 \theta) \frac{\partial^2 U}{\partial C_1^2} (1-\rho \phi_1) + \theta \rho \phi_2 \frac{\partial^2 U}{\partial C_2^2}]dy_1 \right\}
\]  

(21)
The first term on the RHS of (21) is negative. The second term is also negative if $\frac{\partial^2 U}{\partial C_i^2}$ is constant. In that case the change in $E_o \frac{\partial U}{\partial C_1}$ from the mean preserving spread is equal to minus the change in $E_o \frac{\partial U}{\partial C_2}$.

So, $\psi$ is equal to $(\phi_1 - \phi_2)\theta$. Moreover, since $\rho \phi_2$ is strictly bigger than $(1 - \rho \phi_1)$, the second term on the RHS of (21) is strictly smaller than $F(y) \frac{\partial^2 U}{\partial C_1}$ $(\psi - \phi_1 \theta + \phi_2 \theta)(1 - \rho \phi_1)$ which is equal to zero. So, consumption falls. Clearly, since the first term on the RHS of (21) is negative, consumption still falls for moderately nonlinear marginal utility of consumption.

(b) A Durable Good which isn't Resold

This subsection assumes that there is another good, which will be called a durable. This good is bought in period zero. It is a durable only in that the maximum allowable debt may depend on the amount of this good that is bought. On the other hand, in this subsection the individual can't resell the good in later periods. Alternatively, the market for durables may be imperfect. The fall in price as the durable ages may exceed the fall in utility caused by the good's depreciation. This would induce the individual to hold the same amount of the durable under diverse circumstances.

The utility of the individual can be written as:

$$V = E_o \sum_{i=0}^{2} \rho^i U(C_i) + W(B_0)$$

(22)

where $B_0$ is the amount of the durable, while $W$ is a concave function. The budget constraints are given by:

$$C_0 + qB_0 = y_0 + D - S_0$$

$$C_1 - \phi_1 D = y_1 + S_0(1+r) - S_1$$

$$C_2 - \phi_2 D = k - y_1(1+r) + S_1(1+r)$$

(23)
So, if the individual holds both assets and liabilities, the first order conditions are given by (16), (17), and:

\[
\frac{\partial U}{\partial B} - q \frac{\partial U}{\partial C} = 0
\]  

(24)

Totally differentiating (16), (17) and (23) after a mean preserving spread is applied to \( F \):

\[
\begin{bmatrix}
\frac{\partial^2 W}{\partial B^2} + q \frac{\partial^2 U}{\partial C^2} & q \frac{\partial^2 U}{\partial C^2} & -q \frac{\partial^2 U}{\partial C^2} \\
q \frac{\partial^2 U}{\partial C^2} & -q \frac{\partial^2 U}{\partial C^2} & \\
-q \frac{\partial^2 U}{\partial C^2} & & \\
\end{bmatrix}
\begin{align*}
\frac{dB}{o} \\
\frac{dS}{o} &= -\theta \\
\frac{dD}{\rho \psi} \\
\end{align*}
\]  

So:

\[
\begin{align*}
\frac{dS}{o} &= \frac{\gamma \Delta}{\lambda} \left( \frac{\partial^2 W}{\partial B^2} + q \frac{\partial^2 U}{\partial C^2} \right) + \frac{1}{\lambda} \frac{\partial^2 W}{\partial B^2} \frac{\partial^2 U}{\partial C^2} (\rho \psi - \theta) \\
\frac{dD}{\rho \psi} &= \frac{\gamma \Delta}{\lambda} \left( \frac{\partial^2 W}{\partial B^2} + q \frac{\partial^2 W}{\partial C^2} \right) + \frac{1}{\lambda} \frac{\partial^2 W}{\partial B^2} \frac{\partial^2 U}{\partial C^2} (\rho \psi - \theta) \\
\end{align*}
\]

where \( \gamma \) and \( \beta \) are defined in (19) and (20), while \( \lambda \) is the (negative) determinant of the matrix on the LHS of (24). Thus, assets and liabilities rise under the conditions mentioned in the previous subsection. The change in the holdings of durables is given by:

\[
\frac{dB}{o} = \frac{\gamma \Delta}{\lambda} 
\]  

(26)

So, purchases of durables fall if consumption were to have fallen in the
absence of durables. Equation (21) basically gives the change in consumption expenditure at \(0\). Since the utility of durables and nondurables is assumed separable, any change in total expenditure induces a change of the same sign in the purchases of both goods. This effect can be reversed if the ability of the individual to borrow depends on his purchases of durables. In the United States favorable credit arrangements can often be made in connection with purchases of durables. This makes it plausible that for those who borrow as much as they can:

\[
D = \mu q B_o
\]  

(27)

Then, the first order conditions (17) and (24) become:

\[
\frac{\partial W}{\partial B_o} - q(1-\mu) \frac{\partial U}{\partial C_o} - E_o \rho \left\{ \mu q \phi_1 \frac{\partial U}{\partial C_1} + \mu q \rho \phi_2 \frac{\partial U}{\partial C_2} \right\} = 0
\]  

(28)

The response of \(S\) and \(B_o\) to a mean preserving spread applied to \(F\) is given by:

\[
E \begin{bmatrix}
    dB_o \\
    dS_o
\end{bmatrix} = \begin{bmatrix}
    \rho \mu q \psi \\
    \theta
\end{bmatrix}
\]  

(29)

and:
\[
E = \begin{align*}
&\frac{\partial^2 w}{\partial B_o^2} + q^2 (1-\mu)^2 \frac{\partial^2 U}{\partial C_o^2} + 2q^2 \left\{ \frac{[1-F(y)](\phi_1 + \rho \phi_2)}{2 + r} \frac{\partial^2 U}{\partial C^2} \right\} \\
&\quad + \rho \int_0^y \! f(y_1) \left( \phi_1 \frac{\partial^2 U}{\partial C_1^2} + \rho \phi_2 \frac{\partial^2 U}{\partial C_2^2} \right) dy_1 \\
&\quad + \int_0^y \! f(y_1) \phi_1 \frac{\partial^2 U}{\partial C_1^2} dy_1 \\
&\quad - \frac{\partial^2 U}{\partial C_o^2} \frac{[1-F(y)] \mu q (\phi_1 + \rho \phi_2)}{\rho (2+r)} \frac{\partial^2 U}{\partial C^2} \\
&\quad + \frac{\partial^2 U}{\partial C_o^2} \frac{[1-F(y)] (1+r)^2}{2+r} \frac{\partial^2 U}{\partial C^2} \\
&\quad + \int_0^y \! f(y_1) (1+r) \frac{\partial^2 U}{\partial C_1^2} dy_1
\end{align*}
\]
Concavity guarantees that the determinant of $E$, $\Delta$, is positive. The change in $S_o$ is given by:

$$\text{d}S_o = -\frac{1}{\Delta} \left\{ \frac{\partial^2 U}{\partial C_0^2} q^2 (1-\mu)[\rho \mu \psi + (1-\mu)\theta] + \theta \frac{\partial^2 w}{\partial B_0^2} \right. $$

$$+ [1 - F(\overline{y})] \mu^2 q^2 \frac{\phi_1 + \rho \phi_2}{2 + r} \frac{\partial^2 U}{\partial C^2} (\phi_1 \theta - \psi)$$

$$+ \rho \mu^2 q^2 \overline{y} f(y_1) \left[ \phi_1 \frac{\partial^2 U}{\partial C_1^2} (\phi_1 \theta - \psi) + \rho \phi_2 \frac{\partial^2 U}{\partial C_2^2} \right] dy_1 \}$$

All terms except possibly the first one lead $S_o$ to go up under the conditions described previously. The first term might lead $S_o$ to fall if $\theta + \mu(\rho \psi - \theta)$ is negative. This requires that $\psi$ be negative and $\mu$ large. This second requirement lowers the importance of this term which vanishes when $\mu$ is one. The change in durable expenditure is given by:

$$\text{d}B_o = \frac{1}{\Delta} \left\{ \frac{\partial^2 U}{\partial C_0^2} q [\rho \mu \psi + (1-\mu)\theta] + [1-F(\overline{y})] (\frac{1+r}{2+r}) \mu q \frac{\partial^2 U}{\partial C_2^2} \right.$$

$$\left. + \mu q \overline{y} f(y_1) \frac{\partial^2 U}{\partial C_2^2} (\psi - \phi_1 \theta) dy_1 \right\}$$

This expression has an ambiguous sign. The first term on the RHS is ambiguous while the other terms (which represent the gains from debt in subsequent periods) are positive. Even for the case in which the second derivative of $U$ with respect to $C_1$ is constant, it is easy to construct situations in which durable consumption increases. This is true in particular if $F(\overline{y})$ is equal to .5, $r$ is .01, $\phi_1$ is .5, $\phi_2$ is .7 and $\mu$ is 1. Note that when $\frac{\partial^2 U}{\partial C_1^2}$ is constant, $B_o$ falls in the absence of (26). The limitations on borrowings
for purposes other than the purchase of durables thus can make durable consumption respond rather differently to increases in income variability. This result is in the spirit of Mishkin (1976) and the DRI model as reported in Eckstein et al. (1974) which make durable goods consumption depend on credit availability.

(c) A Perfect Market for Used Durables

The results of the previous subsection extend readily to a situation in which durables can be resold without penalty. Here, utility is obtained from the consumption of durables in each period:

\[ V = E_0 \sum_{i=0}^{2} \rho^i [U(C_i) + W(B_i)] \]  

Durables depreciate at the rate \( \delta \) so the budget constraints of an individual who borrows and lends at zero are given by:

\[ C_0 + qB_0 + S_0 - D = y_0 \]  
\[ C_1 + \phi_1 D + q[B_1 - (1-\delta)B_0] = y_1 + S_0 (1+r) \]  
\[ C_2 + \phi_2 D + q[B_2 - (1-\delta)B_1] = k - y_1 (1+r) + S_0 (1+r) + q \frac{(1-\delta)B_2}{1+r} \]

Note that for simplicity I assume that the individual can spend, in the last period, the money he will receive from selling the durables left over at the end of period 2. It is as if durables are rented in period 2.

In period 1, all the uncertainty is resolved and the first order conditions are given by:

\[ -\frac{\partial U}{\partial C_1} + \frac{\partial U}{\partial C_2} \leq 0 \]  
\[ S_1 \left( \frac{\partial U}{\partial C_1} - \frac{\partial U}{\partial C_2} \right) = 0 \]
\[ \frac{\partial W}{\partial B_1} - q \frac{\partial U}{\partial C_1} + q(1-\delta) \rho \frac{\partial U}{\partial C_2} = 0 \]  

(35)

\[ \frac{\partial U}{\partial B_2} - q[1 - \frac{(1-\delta)}{1+r}] \frac{\partial U}{\partial C_2} = 0 \]  

(36)

Therefore, if \( y_1 \) exceeds \( \bar{y} \), \( C_1 \) and \( C_2 \) are constant and equal to \( C \), while \( B_1 \) is equal to \( B_2 \). Equation (36) gives \( B_2 \) as \( t(C) \), where \( t \) is a strictly increasing function. So, using (32) and (33):

\[ C + q[1+r+\delta-\rho(1-\delta)]t(C) = k + S_o(1+r)^2 - D\phi_1(1+r) - D\phi_2 + qB_0(1-\delta)(1+r) \]

(37)

Therefore \( C \) is given by \( z[k + S_o(1+r)^2 - D\phi_1(1+r) - D\phi_2 + qB_0(1+\delta)(1+r)] \) where \( z \) is a strictly increasing function whose derivative will be denoted \( z' \).

When \( y_1 \) is below \( \bar{y} \), the individual spends all his resources in period 1. So (32) and (33) hold with \( S_1 \) equal to zero. Differentiating (32), (33), (35), and (36), one obtains:

\[ dC_1 = n(1+r)dS_o + q(1-\delta)ndB_o - (\phi_1n + \phi_2m)dD + [n - (1+r)m]dy_1 \]  

(38)

\[ dC_2 = n'(1+r)dS_o + q(1-\delta)n'dB_o - (\phi_1'n' + \phi_2'm')dD + [n' - (1+r)m']dy_1 \]  

(39)

\[ dB_1 = n''(1+r)dS_o + q(1-\delta)n''dB_o - (\phi_1'n'' + \phi_2'm'')dD + [n'' - (1+r)m'']dy_1 \]  

(40)

\[ n = \left[ \frac{\partial^2 W}{\partial B_1^2} + q(1-\delta)^2 \rho \frac{\partial^2 U}{\partial C_2^2} \frac{\partial^2 W}{\partial B_2^2} \right] / \lambda \]

\[ m = q(1-\delta)\rho \frac{\partial^2 U}{\partial C_2^2} \frac{\partial^2 W}{\partial B_2^2} / \lambda \]

\[ n' = q^2(1-\delta)\frac{\partial^2 U}{\partial C_1^2} \frac{\partial^2 W}{\partial B_2^2} / \lambda \]
\[ w' = \left[ \frac{\partial^2 w}{\partial b_1^2} + q^2 \frac{\partial^2 u}{\partial b_2^2} \right] \frac{\partial^2 w}{\partial b_2} / \lambda \]

\[ u'' = q \frac{\partial^2 u}{\partial c_1^2} \left[ \frac{\partial^2 w}{\partial b_2^2} + q^2 (1 - (1-\delta)p)^2 \frac{\partial^2 u}{\partial c_2^2} \right] / \lambda \]

\[ u'' = -q(1-\delta)p \frac{\partial^2 u}{\partial c_2} \frac{\partial^2 w}{\partial b_2} / \lambda \]

\[ \lambda = \left[ \frac{\partial^2 w}{\partial b_1^2} + q^2 \frac{\partial^2 u}{\partial c_1^2} \right] \left\{ \frac{\partial^2 w}{\partial b_2^2} + q^2 [1 - (1-\delta)p]^2 \frac{\partial^2 u}{\partial c_2^2} \right\} + q^2 (1-\delta)^2 \rho \frac{\partial^2 u}{\partial c_2^2} \frac{\partial^2 w}{\partial b_2^2} \]

for each realization of \( y_1 \). So, \( S_0 \) and \( B_0 \) raise consumption in the two periods while debt lowers it. Falls in \( y_1 \) always lower \( B_1 \) and raise \( C_2 \), but their effect on \( C_1 \) is ambiguous. One would expect \( C_1 \) to fall as in the previous subsection. However, particularly if \( \frac{\partial^2 w}{\partial b_1} \) is low, it is possible that the individual will sell so many durables that \( C_1 \) will actually rise.

The first order conditions at zero are given by (16), (17) and:

\[ \frac{\partial w}{\partial b_o} - q \frac{\partial u}{\partial c_o} + q(1-\delta)\rho E_o \frac{\partial u}{\partial c_1} = 0 \]  

(41)

Using (41) and (16):

\[ \frac{\partial w}{\partial b_o} + q[(1-\delta) - 1] \frac{\partial u}{\partial c_o} = 0 \]  

(42)

Now consider the effects of applying a mean preserving spread to \( F \). \( C_2 \) rises when \( y_1 \) falls. So the expected value of \( \frac{\partial u}{\partial c_2} \) falls both when \( \frac{\partial u}{\partial c_2} \) is linear and more generally when the effect of the mean preserving spread to the left of \( y \) is concentrated near \( y \). If \( C_1 \) falls monotonically with \( y_1 \), which is intuitively plausible, then, under the same conditions, \( E_o \frac{\partial u}{\partial c_1} \) rises. Then as in the previous section, \( \theta \), the change in \( E_o \frac{\partial u}{\partial c_1} \) is bigger than \( \rho \psi \) where \( \psi \) is
the change in $E_o(\phi_1 \frac{\partial U}{\partial C_1} + \rho \phi_2 \frac{\partial U}{\partial C_2})$. Moreover, $\phi_1 \theta$ is also bigger than $\psi$.

In the case in which $C_1$ rises monotonically when $y_1$ falls, the expected utility of consumption at 1 actually falls so that $\theta$ is negative. So, of course is $\psi$. However, in this case, since $E_o \frac{\partial U}{\partial C_2}$ falls also, $(\rho \psi - \theta)$ and $(\psi - \phi_1 \theta)$ are negative. In the case in which the utility function is strictly quadratic, the n's and m's are constant and $C_2$ always rises more than $C_1$ when $y_1$ falls. Hence, the fall in $E_o \frac{\partial U}{\partial C_2}$ exceeds the fall in $E_o \frac{\partial U}{\partial C_1}$ which ensures that $[\psi - (\phi_1 + \rho \phi_2) \theta]$ is negative. Assuming the mean preserving spread has a small effect, one can analyze its impact by differentiating (42) and (16) and (17):

$$
A^* = \begin{bmatrix}
\frac{dB_o}{\partial \theta} \\
\frac{dS_o}{\partial \theta} \\
\frac{dD}{\partial \theta}
\end{bmatrix} = \begin{bmatrix}
0 \\
-\theta \\
\rho \psi
\end{bmatrix}
$$

where:
The effect of the mean preserving spread on $B_0$ is ambiguous. On the one hand, the increase in the expected marginal utility of consuming in period 1 encourages the purchase of durables. On the other hand, the fact that the individual is encouraged to save in the form of financial assets tends to raise $\partial U/\partial C_0$, which lowers the attractiveness of durables. The change in $S_o$ is given by:

$$dS_o = \frac{1}{\Delta^*} \left\{ \frac{\partial^2 W}{\partial B^2_o} \frac{\partial^2 U}{\partial C^2_o} (\rho \psi - \theta) + \left[ \frac{\partial^2 W}{\partial B^2_o} + q^2 \delta \frac{\partial^2 U}{\partial C^2_o} \right] \left[ 1 - F(y) \right] \zeta (\phi_1 + \rho \phi_2) \frac{\partial^2 U}{\partial C^2_o} \left[ \psi - (\phi_1 + \rho \phi_2) \theta \right] \right\}$$

$$+ \rho \frac{\partial^2 W}{\partial B^2_o} \int f(y_1) \left[ \frac{\partial^2 U}{\partial C^2_1} (n \phi_1 + m \phi_2) (\psi - \phi_1 \theta) - \rho \phi_1 \theta (n' \phi_1 + m \phi_2) \frac{\partial^2 U}{\partial C^2_2} \right] dy_1$$

$$+ \rho q^2 (1 - (1-\delta) \rho)^2 \frac{\partial^2 U}{\partial C^2_2} \int f(y_1) \left[ \frac{\partial^2 U}{\partial C^2_2} (\psi - \phi_1 \theta) (n \phi_1 + m \phi_2 - n'(1-\delta)) \right] dy_1$$

$$- \theta \rho \phi \frac{\partial^2 W}{\partial C^2_2} \left[ n' \phi_1 + m' \phi_2 - n'(1-\delta) \right] dy_1 \}$$

where $\Delta^*$ is the negative determinant of $A^*$. The first two terms always lead $S_o$ to rise. Instead, the last two integrals appear to have ambiguous signs. In particular, it seems that if $\theta$ is actually negative so that $C_1$ rises when $y_1$ falls, these terms could lead $S_o$ to fall. In this case, it would appear that a mean preserving spread applied to $F$ could actually lower the expected value of $\partial U/\partial C_1$. However, in the case in which the utility function is strictly quadratic, it can be shown that both these integrals actually raise $S_o$. This can be seen as follows. Let $g$ be the p.d.f. of the transformed distribution. Then, $\theta$ and $\psi$ are given by:

$$\theta = (n - (1+r) \ m) \frac{\partial^2 U}{\partial C^2_1} \Lambda$$
\[ \psi = \phi_1 \theta + \rho \phi_2 (n' - (1+r)m') \frac{\partial^2 u}{\partial c^2} \Lambda \]

\[ \Lambda = - \int_0^y [g(y_1) - f(y_1)](\bar{y} - y_1) dy_1 \]

where \( \Lambda \) is negative since \( g \) is obtained by applying a mean preserving spread to \( f \). So the third term in (45) is given by:

\[ \phi_2 \rho^2 \left( \frac{\partial^2 W}{\partial B_0^2} \right)^2 \Lambda \{[n'-(1+r)m'][n\phi_1 + m\phi_2] - [n-(1+r)m][n'\phi_1 + m'\phi_2]\} = \]

\[ \rho \left( \frac{\partial^2 W}{\partial B_0^2} \right)^2 \Lambda [n'm - nm'][\phi_1(1+r) + \phi_2] \]

which is negative. Similarly, the fourth term is given by:

\[ \phi_2 \rho^2 q^2 [1 - (1-r)\rho]^2 \left( \frac{\partial^2 U}{\partial c^2} \right)^3 \Lambda [n'm - nm'][\phi_1(1+r) + \phi_2 - (1+r)(1-\delta)] \]

which is also negative. While this argument is true only for linear marginal utilities, \( S_0 \) will rise also for moderately nonlinear ones.

The change in \( D \) is given by:

\[ dD = \frac{1}{\Delta^*} \left\{ \frac{\partial^2 W}{\partial B_0^2} \frac{\partial^2 U}{\partial c^2} (\rho\psi - \theta) + \left[ \frac{\partial^2 W}{\partial B_0^2} (1+r) + q^2 (1-\rho(1-\delta))(r+\delta)[1-F(\bar{y})]z' \frac{\partial^2 U}{\partial c^2} [\psi(\theta + \rho\phi_2)\theta] \right] \right\} 
\]

\[ + \left[ \frac{\partial^2 W}{\partial B_0^2} + q^2 (1 - (1-\delta)\rho)^2 \frac{\partial^2 U}{\partial c^2} \right] \int_0^y f(y_1)[(\psi - \phi_1\theta) \frac{\partial^2 U}{\partial c^2} n - \rho\phi_2\theta n', \frac{\partial^2 U}{\partial c^2} dy_1] \]

In the case in which consumption in period 1 falls as \( y_1 \) falls, debt rises unambiguously. The case in which \( C_1 \) does not respond monotonically to changes in \( y_1 \) is beyond the scope of this analysis. Note, however, that when \( dC_1/dy_1 \) is negative, all terms except the last one also lead to an
increase in $D$. Even when $\theta$ is negative, in the case of quadratic utility the last term is given by:

$$
\left. \left[ \frac{\partial^2 W}{\partial B^2} + \frac{q}{2} (1 - (1-\delta)\rho)^2 \right] \Lambda \left( \frac{\partial^2 U}{\partial C^2} \right)^2 \rho \phi_2 (n'm - nm')(1+r) \right.
$$

which is negative thus raising $D$.

Even when the market for durables is perfect, a mean preserving spread applied to $F$ tends to raise the marginal utility of consumption in the first period and lower it in the second period. When this happens assets which transfer resources from period zero to period one and liabilities which transfer resources from period two to period zero become more desirable.

IV. CONCLUSIONS

The analysis of this paper deserves to be extended in several directions. In the first place, it is of interest to see whether the tendency of assets and liabilities to increase when the timing of income becomes more variable extends to situations in which the utility function is nonseparable in durables and nondurables, in which the horizon has more than three periods and in which there are also other forms of uncertainty. More generally the portfolio problem of an individual who expects to be liquidity constrained with some probability deserves to be studied. This study may shed light on the empirical rate of return dominance of stocks over Treasury bills. The empirical studies of Hansen and Singleton (1983) and Prescott and Mehra (1982) suggest that the risk aversion of a "representative individual" is not sufficient to explain this dominance. Instead, it may be that stocks have a high expected return because their price movements tend to make them undesirable for those who are sometimes liquidity constrained.

Finally, the fact that assets and liabilities only overlap in the
portfolios of those who expect to be liquidity constrained holds promise for empirical research. In particular, it suggests that one may be able to measure the subjective likelihood that the individual will be liquidity constrained by studying his portfolio.
1 I wish to thank David Runkle for providing me the data on which this table is based.

2 The U.S. tax code is such that certain individuals might borrow because the after tax rate of interest on borrowing is below the after tax rate of return on certain tax shelters like trusts and real estate partnerships. This would not be a valid rationale if these individuals also held assets whose income is taxable at the normal rate. In any event, the analysis assumes that for the individual considered here, the rate of return on assets is below the interest rate on liabilities.

3 The rate of return on owner occupied houses depends on the utility derived from living in them. Thus they are excluded from this analysis.

4 Note that all the uncertainty gets resolved in period 1.

5 Kanbur (1982) discusses the general problem of the effect of mean preserving spreads applied to distributions when the payoffs (here consumption) have kinks as a function of the underlying random variable (here $y_1$). He considers not only the possibility raised here that the effect on the marginal utility of consumption of an additional unit of $y_1$ is different to the left and to the right of $y$, but also the possibility that $\partial U/\partial C$ differs on both sides of $\bar{y}$. 
REFERENCES


