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A DYNAMIC SALES CALL POLICY MODEL*

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1. Introduction

In spite of the fact that personal selling is the largest single item in the marketing budgets of most firms, only a handful of analytical attacks upon personal selling decision problems have been reported in the literature. While certain characteristics of personal selling decision problems, sales managers, and management scientists have contributed to this dearth of progress, the time seems ripe for a concerted effort on the part of management scientists to assist the sales manager in the solution of certain sales management problems.

This paper will present a model framework for the analysis of a particular sales management problem - that of specifying call norms for current and potential customers. Consideration will first be given to the nature of the sales call policy decision problem. Then the dynamic call policy model will be developed. Attention will next turn to the application of the model to hypothetical data which will illustrate the nature of the call policies recommended by the model as well as provide insight into its practical computability.


2 For a consideration of the factors which have contributed to this lack of progress see D.B. Montgomery and F.E. Webster, Jr. "Application of Operations Research to Personal Selling Decisions", Journal of Marketing, (January, 1968), pp. 50-57.
2. The Decision Problem

Periodically a salesman must make a decision as to how much sales effort to expend upon a particular customer or class of customers. The decision often takes the form of a specification of the number of sales calls which should be made on the customer during some period of time. This decision is termed the call norm for this customer (or class of customers).

In establishing a call norm for a customer we shall view the salesman's objective as maximizing the total profit contribution to the firm which will result from his sales effort on a particular customer. Thus we are taking the firm's viewpoint in developing the salesman's call norms rather than the viewpoint of the salesman in terms of maximizing his own income. However, if the salesman's compensation is proportional to the profit contribution he makes to the firm, then the two viewpoints would coincide in the sense that both the salesman and the firm would select the same call norm.\(^3\)

In the present paper we concentrate upon situations in which there may be a continuing relationship between the salesman and the customer. That is, we shall focus on situations in which the customer has a recurring need for the type of products offered by the salesman.

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The success of a salesman with a customer in any time period would seem to depend upon at least the following key elements:

1. The prospect's sales potential.
2. The profit contribution per unit of sales.
3. The salesman's past history of sales to the customer. Has the customer ordered recently? If he did, did he order very much in terms of his sales potential?
4. The sales effort expended on the customer in the past several periods.
5. The current period sales effort expended upon the customer.
6. The customer's response to sales effort.
7. The cost of each sales call.

The sales call policy model developed below is an attempt to synthesize the above elements into a realistic, albeit abstracted, representation of the problem. Implicit in the model is the assumption that the remainder of the firm's marketing mix (e.g. price, advertising, product line, etc.) remains constant or changes in such a manner that there will be no repercussions on the call policy. In addition, competitive sales activity is not modeled explicitly, although differing competitive climates might be encompassed by considering their effects upon the parameters in the model.
The model develops an optimal call policy for a particular customer. In practice, however, the firm would want to stratify current and potential customers into relatively homogeneous groups for the purpose of establishing call norms. Such a procedure eases the input and computational burden of the model for firms having a substantial number of accounts. The bases for such a stratification of customers will generally be sales potential and the responsiveness of the customer to such factors as current sales effort, sales effort in the recent past, and the history of purchases from the salesman.

3. The Call Policy Model

The model focuses upon the number of sales calls to allocate to a given customer over some period (e.g. month, quarter, year, etc.). The central elements in the model are two probabilities -- the probability that the customer will order and the conditional probability associated with the size of his order given that he orders from the firm.

Before outlining the nature of these probabilities, it is useful to specify two salient characteristics of the customer which will influence the success of sales calls in the current period. The first characteristic is a measure of the history of sales to this customer. In the present model sales history is measured as the exponentially smoothed ratio of the customer's
purchases to his potential. That is,

\( H_t = a(X_{t-1}/P) + (1 - a) H_{t-1} \)

where \( H_t \) = smoothed history of sales to the customer at the beginning of period \( t \) (measured as the ratio of customer purchases to his potential)

\( X_t \) = unit sales to the customer during period \( t \)

\( P \) = sales potential of the account (in units)

\( a \) = smoothing constant which determines the weight given to recent sales \((0 < a < 1)\).

The second characteristic is termed the customer's remembered sales effort. It reflects the fact that past sales effort may help pave the way for success in the current period. However, the effect of past sales effort will also tend to diminish with time - i.e. forgetting will occur. Thus the impact of past sales effort on success in the current period will be measured by remembered sales effort which is given by

\( E_t = g(E_{t-1} + S_{t-1}) \)

where \( E_t \) = sales effort from past periods remembered at the beginning of period \( t \).

\( S_t \) = sales effort expended on the customer during period \( t \) (number of sales calls).

\( g \) = parameter representing the fraction of previous sales effort retained each period \((0 \leq g < 1)\).
When the call policy model is formulated as a Markov sequential decision process, the remembered sales effort and sales history variables will define a two-dimensional state space for the customer.

**Probability of Ordering**

In discussing the probability that the customer will order, we consider current customers separately from potential customers. For a current customer, the probability of his ordering in the current period is taken to be a function of his sales history, remembered sales effort, and the current sales effort allocated to him. The probability is given by

\[
P_t(H_t, E_t, S_t) = c(1 - \exp{-bH_t}) + (1 - c)(1 - \exp{-d(E_t + S_t)})
\]

where \( c \) = parameter reflecting the relative impact of sales history versus remembered and current sales effort \((0 \leq c \leq 1)\).

\( b, d \) = parameters reflecting diminishing returns to sales history as well as remembered and current sales effort \((b \geq 0, d \geq 0)\).

In this formulation the probability of obtaining an order is composed of two components: one reflecting sales history and the other reflecting sales efforts, both current and past. Each component experiences a diminishing return to the factor of which it is a function. The question might arise as to why one component of the order probability
should be a function of the sales history, $H_t$. The reasoning behind this formulation is that sales history reflects the success of past sales effort. The formulation in Equation (3) enhances the chances of success in the current period according to how successful past efforts have been, as indicated by the level of $H_t$. Further, the component which is responsive to sales effort allows for a substitution of remembered sales effort and current sales effort. Thus, if a salesman has built up a cumulative history of sales calls to the customer, less current period effort will be required to yield a given likelihood of obtaining an order.

The order probability for prospects is developed below. One reason for treating prospects differently from current customers is that by definition, prospects have no sales history. Furthermore, it would seem that additional information should be taken into account in our representation of prospects. We shall first specify the functional representation of a prospect's order probability and then discuss its implications. Let

$P_{tn} = \text{PMAX} (n, E_t)[1 - \exp \{-q(E_t + S_t)\}]$

$= [\exp \{-knE_t\}][1 - \exp\{-q(E_t + S_t)\}]$

where $n =$ the number of previous periods in which the prospect has been called upon and in which he has not ordered. $n = 0, 1, \ldots, N$
$$P_{tn} \quad \text{the probability that a prospect who has been called on and yet has not ordered for n periods will order in period t. Note that t corresponds to n+1.}$$

$$\text{PMAx(n,E}_t\text{)} \quad \text{the maximum order probability for a prospect having a remembered sales effort of E}_t\text{ and having been called upon for n previous periods without buying.}\quad (0 \leq \text{PMAx(n,E}_t\text{)} \leq 1.0).$$

$$k \quad \text{parameter reflecting the reduction in the maximum probability of an order as n and/or E}_t\text{ increase.}$$

$$q \quad \text{parameter reflecting diminishing returns to current and remembered sales effort.}$$

In the first place, notice that we have introduced the concept of a prospect's age as measured by n. We postulate that the longer a prospect is called upon and yet remains a prospect, the lower the likelihood that he will order from the firm. Furthermore, the older the prospect is in this sense (i.e. the larger his n) and the more intensively he has been called upon with no success (i.e. the higher his E}_t\text{), the greater the diminution in the firm's prospects for winning his as a customer. Consequently, in Equation (4) we have represented the maximum probability of winning the prospect as a customer in period t as an exponentially declining function of nE}_t\text{.}

However, remembered sales effort should also have some positive effect upon the prospect's order probability in period t. The fraction of PMAx(n,E}_t\text{) that will be realized in period t is taken to be a positive function of both current and remembered sales effort. The function also exhibits diminishing returns.
In sum, we have postulated a model in which the length of time a prospect has remained a prospect and the intensity of cumulative sales effort to him will yield information on our likelihood of converting him from a prospect into a customer. In most cases we will only require information on \( n = 0, 1, \ldots, N \) for some small number \( N \). For example, if the period is a quarter, then \( N = 4 \) is probably sufficient. Once a prospect gets to \( N \), he is no longer aged, but remains in the prospect class specified by \( N \).

**Probability of Order Size**

If a customer orders, the size of his order is taken to be a Poisson random variable having mean sales rate \( \lambda_t(P, H_t, E_t, S_t) \) which is viewed as an increasing function of the customer's sales potential \( (P) \), his sales history \( (H_t) \), his remembered sales effort \( (E_t) \), and current period sales effort \( (S_t) \). More explicitly, the mean sales rate is given by

\[
(5) \quad \lambda_t(P, H_t, E_t, S_t) = P \left[ 1 - \exp \left\{ -fr H_t - (1 - f)s(E_t + S_t) \right\} \right]
\]

where

\[
\lambda_t(P, H_t, E_t, S_t) = \text{Poisson parameter in the order size distribution as a function of } P, H_t, E_t, \text{ and } S_t.
\]

\( f = \text{parameter representing the relative importance of sales history versus current and remembered sales effort } (0 \leq f \leq 1.0). \)

\( r, s = \text{parameters representing diminishing returns to sales history and sales effort} \) i.e., they determine the extent to which increasing \( H_t \) and \( E_t + S_t \) causes \( \lambda_t \) to approach its asymptotic value of \( P \) \( (r > 0, \ s > 0) \).
Two interesting aspects of this formulation should be noted. In the first place, it is assumed that a good recent sales history to the customer can be substituted for some current and remembered sales effort. Secondly, note that we have specified the customer's sales potential as an upper bound to the expected size of his total orders in a given period. Thus, his actual purchases may be significantly in excess of his sales potential, P.

It should be emphasized at this point that the procedure used to develop an optimal call policy is appropriate for any functions describing $\lambda_t$, $P_t$, and $P_{tn}$. No convexity or concavity restrictions are needed. The functions specified in Equations (3), (4), and (5) are presented for concreteness and because they represent a reasonable initial representation of the process. It may be anticipated that field study will lead to suitable modifications to the model. The present representation is sufficient to explore the behavior of the model and the computational efficiency with which it may be used.

For any current customer we may now define the probability of ordering any amount $X_t$ as a function of $P_t$, $H_t$, $E_t$, and $S_t$. Thus, for $X_t = 0$ we have

\begin{align*}
(6) \quad P(X_t = 0 \mid P_t, H_t, E_t, S_t) &= 1 - P_t(H_t, E_t, S_t) \\
&\quad + P_t(H_t, E_t, S_t) \exp \{-\lambda_t (P_t, H_t, E_t, S_t)\}
\end{align*}
and for $X_t = 1, 2, \ldots$ we have

\[ P(X_t | P_t, H_t, E_t, S_t) = \frac{P_t(H_t, E_t, S_t) [\lambda_t(P_t, H_t, E_t, S_t)]^{X_t}}{X_t!} \exp(\lambda_t(P_t, H_t, E_t, S_t)) \]

Equation (6) is the sum of the probability of no order and the product of the order probability and the Poisson probability that the order quantity is zero. The latter term reflects an intention to order on the part of the customer but for some reason (perhaps a short term sales decline or excess inventory situation) the customer does not require any additional units of the product during the period. The larger $\lambda_t$, the smaller will be the probability of this event. Note that for a potential account we would substitute $P_{tn}$ for $P_t$ in equations (6) and (7).

The Markov Sequential Decision Process

In the probabilistic process defined above, the firm has a single control variable, sales calls ($S_t$), which may be set at a number of discrete values. The problem is to determine the optimal number of sales calls to make to a customer having some particular set of parameter values in equations (1)-(7) and currently having some sales history ($H_t$) and remembered sales effort ($E_t$). In this section we show that this problem may be cast in the form of a Markov sequential decision process and solved by means of Howard's policy and value iteration technique.\(^4\)

\[^4\text{See R.A. Howard, } \textit{Dynamic Programming and Markov Processes}, \textit{(Cambridge, Massachusetts: M.I.T. Press), 1960.}\]
The state of a customer at any time $t$ may be described by his values of $H_t$ and $E_t$. Thus, the state space in our model is two dimensional. If the model is to be computationally feasible, it will be necessary to transform intervals of $H_t$ and $E_t$ into some reasonable number of discrete states.

Consider first the sales history of the customer which we shall denote by $i$. Each possible value of $H_t$ is uniquely associated with one of the history states $i = 1, 2, ... I$. Let there be equal intervals of $H_t$ corresponding to each $i$ (except for state $I$ which will have associated with it any $H_t$ greater than some specified value). Now the sales history information we have about a customer at the beginning of any period $t$ will be his history state $i$, which will correspond to any $H_t$ in the interval which maps into $i$. In order to be able to assess the probabilities given in equations (6) and (7) when we only know $i$ and not $H_t$, we will associate with each $i$ the midpoint of the corresponding $H_t$ interval. We shall denote the midpoint by $HM_{it}$. The value $HM_{it}$ will be used in place of $H_t$ in the equations of the model for a customer in history state $i$.

Suppose we have a customer who at the beginning of period $t$ is in history state $i$ and who purchases $X_t$ units during period $t$. What will be his history state $i'$ at the beginning of period $t+1$? We first compute his value of $H_{t+1}$ as

(8) $H_{t+1} = a(X_t/P) + (1-a) HM_{it}$. 

Then his history state \( i' \) at \( t+1 \) will be the history state which corresponds to \( H_{t+1} \). Since we have chosen equal intervals of \( H_t \) for mapping \( H_t \) into history states, we may formally represent the correspondence between \( H_{t+1} \) and the history state \( i' \) by

\[
(9) \quad i' = \left[ 1.0 + \left( \frac{H_{t+1}}{SF_H} \right) \right] \\
= \left[ 1.0 + \left( \frac{\{a(X_t/P) + (1-a)HM_{it}\}}{SF_H} \right) \right]
\]

where \([\;]\) denotes the greatest integer less than function and \( SF_H \) denotes an appropriate scaling constant for history states. The term 1.0 in equation (9) reflects the fact that we have chosen to denote the lowest history state by \( i=1 \).

In formulating the Markov process we shall be interested in the probability that a customer in history state \( i \) having some remembered sales effort and receiving \( S_t \) sales calls during \( t \), will be in history state \( i' \) in \( t+1 \). That is, we need to know the transition probability \( P(i' \mid i, j, S_t) \) where \( j \) denotes the customer's effort state at \( t \). If each distinct value of sales to the customer in \( t(X_t) \) mapped into its own distinct history state, then this transition probability would simply equal the appropriate value from equation (6) or equation (7). However, we see from equation (9) that several values of \( X_t \) may lead to the same history state. Thus, the history state transition probability will be given by

\[
(10) \quad P(i' \mid i, j, S_t) = \sum P(X_t \mid i, j, S_t) \\
\forall X_t \} \quad \text{Eqn. (9) holds}
\]

where the terms on the right hand side are from equations (6) and (7).
with the obvious correspondence in notation and with suppression of the sales potential (\(P\)) since it is a parameter of the customer.

We now turn to a consideration of the customer's effort state, which we have denoted by \(j(j=1,\ldots,J)\). Each value of \(E_t\) will be uniquely associated with one of the effort states and we will take equal intervals on \(E_t\) in making the correspondence to the states \(j\). Again, in computing the probabilities in equations (6) and (7) when we only know \(j\) and not \(E_t\), we will associate with \(j\) a representative value of \(E_t\) which will be chosen as the midpoint of the corresponding \(E_t\) interval. The midpoint will be denoted by \(EM_{jt}\) and will be used in lieu of \(E_t\) in the model equations.

Now we could take the transition from effort state \(j\) at time \(t\) to effort state \(j'\) at \(t+1\) as deterministic simply by using

\[
(11) \quad j' = [1.0 + \left\{g(EM_{jt} + S_t)\right\}/SF_E]\]

where we have made use of equation (2), \(SF_E\) is the scaling factor for effort, and \([\ ]\) again denotes the greatest integer function. However, the effort state transition may also be taken as stochastic.

Consider the following approach. Let \(EU(j)\) denote the greatest value of \(E_t\) for which the effort state will be \(j\). Now, suppose for the purpose of determining the transition from \(j\) to \(j'\) that we assume that \(E_t\) is uniformly distributed over the interval \(EU(j-1)^+\) to \(EU(j)\) where we take \(EU(j-1)^+\) to be a value of \(E_t\)
just large enough to be classified in state $j$. We also take $EU(0)^+ = 0$. The above approach is consistent with our use of $\text{EM}_{jt}$ in our other computation since $\text{EM}_{jt}$ is the expected value of $E_t$ when the effort state is $j$ and $E_t$ is uniformly distributed over the interval. We should also note that in the transition from effort state $j$ into a new effort state $j'$ that $j'$ may only take on at most two distinct values. This results from the fact that $0 \leq g \leq 1.0$.

Now, compute

(12) $j_{\text{max}} = [1.0 + \{g(EU(j) + S_t)^+ / SF_E\}$

and

(13) $j_{\text{min}} = [1.0 + \{g(EU(j-1)^+ + S_t)\} / SF_E].$

Then if $j_{\text{max}} = j_{\text{min}}, P(j' = j_{\text{max}} \mid j, S_t) = 1.0$
if $j_{\text{max}} \neq j_{\text{min}},$ we have

(14) $P(j' = j_{\text{min}} \mid j, S_t) = \frac{EU(j_{\text{min}}) - g(EU(j-1)^+ + S_t)}{g(EU(j) - EU(j-1)^+)}$

since we assumed a uniform distribution of $E_t$ in state $j$.

We might note that except for an infinitesimally small term, $EU(j) - EU(j-1)^+ = SF_E$. Since $j'$ may take on at most two values, we have

(15) $P(j' = j_{\text{max}} \mid j, S_t) = 1 - P(j' = j_{\text{min}} \mid j, S_t).$
In order to use Howard's algorithm to find the optimal call policies for our model, we need to know the transition probabilities.

\[ P(i',j'|i,j,S^t) \quad i,i' = 1,2,\ldots, I \text{ and } j,j' = 1,2,\ldots, J \]

and their associated rewards

\[ R(i',j'|i,j,S^t) \quad i,i' = 1,2,\ldots, I \text{ and } j,j' = 1,2,\ldots, J. \]

The \( R(i',j'|i,j,S^t) \) represent the rewards (in our case profit contributions) associated with a customer who begins in state \( ij \), receives \( S^t \) sales calls, and ends up in \( i'j' \) at the beginning of period \( t+1 \). Since the history and effort states to which the customer goes at \( t+1 \) are independent random variables, we have

\[ (16) \quad P(i',j'|i,j,S^t) = P(j'|j,S^t) \cdot P(i'|i,j,S^t) \]

where the factors on the right hand side may be determined from the previous discussion.

It remains to determine the associated rewards. Let

\[ (17) \quad PC(X^t|S^t) = m X^t - uS^t \]

where \( PC(X^t|S^t) = \) the profit contribution which will result from the sale of \( X^t \) units to the customer as a result of \( S^t \) sales calls.

\[ m = \text{profit contribution per unit excluding the cost of sales calls ($/unit$)} \]

\[ u = \text{cost per sales call ($/call$)} \]

Once again, since several values of \( X^t \) may lead to the same history state \( i' \) in \( t+1 \), we will need to sum over the appropriate set of \( X^t \) in order to determine the expected reward associated with going
from state \(ij\) to state \(i'j'\). We have

\[
R(i',j'|i,j,S_t) = \sum_{j'} \left\{ \frac{P(j'|j,S_t)P(X_t|i,j,S_t)PC(X_t|S_t)}{P(i',j'|i,j,S_t)} \right\}
\]

Eqn. (9) holds

\[
= \sum_{j'} P(X_t|i,j,S_t)PC(X_t|S_t)
\]

Eqn. (9) holds

\[
P(i'|i,j,S_t)
\]

Note that the effort state transition probability drops out since the reward is not a function of that transition, but only of the history state transition.

The model is now in a form amenable to solution via one of Howard's algorithms. We shall utilize his iteration cycle with discounting. We incorporate discounting into the model for two reasons. First, we want to reflect the time value of the stream of profit contributions which will result. Second, we want to reflect an element of uncertainty concerning the duration of our relationship with a given customer. Both of these effects will be encompassed in the discount factor. It should be noted that in using this algorithm we are considering the steady-state solution of the process (i.e. the process has a large number of stages).
4. Some Computational Results

This section presents some computational results using hypothetical data which will serve to illustrate the nature of the optimal call policies recommended by the model. For this purpose we present three runs of a very small scale version of the model. We have also run much larger versions of the model which indicate that these smaller scale results are representative.

The parameter values for the three runs or cases are presented in Table 1. Case 1 is taken as a reference case. Case 2 has the same parameters as Case 1 except for a heavier weight given to sales history relative to current and remembered sales effort in equations (3) and (5). Case 3 is also the same as Case 1 except that now sales history is given a much smaller relative weight in equations (3) and (5) than current and remembered sales effort. The optimal sales call policies for these three cases are given in Table 2.
**TABLE 1**

Hypothetical Parameter Values 
Used in the Call Policy Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description*</th>
<th>Case 1</th>
<th>Case 2**</th>
<th>Case3**</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Weight given to recent sales in $H_t$ (1)</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>g</td>
<td>Fraction of previous sales effort retained (2)</td>
<td>0.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>Weight given to $H_t$ component of $P_t$ (3)</td>
<td>0.5</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>b</td>
<td>DRC*** of $H_t$ in $P_t$ (3)</td>
<td>8.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>DRC of $E_t + S_t$ in $P_t$ (3)</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>q</td>
<td>DRC of $E_t + S_t$ in $P_{tn}$ (4)</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>k</td>
<td>COEFFICIENT OF $nE_t$ in $P_{tn}$ (4)</td>
<td>0.07</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N</td>
<td>MAX, aging of prospects (4)</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P</td>
<td>Sales Potential (5)</td>
<td>10.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>f</td>
<td>Weight of $H_t$ in $\lambda_t$ (5)</td>
<td>0.5</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>r</td>
<td>DRC of $H_t$ in $\lambda_t$ (5)</td>
<td>4.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>s</td>
<td>DRC of $E_t$ in $\lambda_t$ (5)</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>m</td>
<td>Profit contribution per unit (17)</td>
<td>$250</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>u</td>
<td>Cost per sales call (17)</td>
<td>$20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Discount Rate = 1/(1+i)</td>
<td>0.75</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max, number of Sales Calls</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

* The number in parenthesis corresponds to the equation in which the parameter appears.

** a - for a parameter value indicates that it has the same value for this case as for Case 1.

***DRC denotes diminishing returns constant (Note: strictly speaking $fr$ and $(1-f)s$ are the DRC's in equation (5).)
TABLE 2

Optimal Call Policies
for Cases 1, 2, and 3

Case 1 Reference Case

<table>
<thead>
<tr>
<th>Effort States</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>History States</td>
<td>1</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
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Entries are the optimal number of calls to make to a customer in each state.
Case 2  Heavy weight on $H_t$, Slight weight on $E_t + S_t$ in $P_t$ and $\lambda_t$

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Table 2, continued
Table 2, continued

Case 3  Slight weight on $H_t$, Heavy weight on $E_t + S_t$ in $P_t$ and $\lambda_t$

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While it is not our purpose to elaborate extensively on these hypothetical cases, a number of interesting features of the call policies may be noted. For example, consider the results for Prospect Age Class 2 in Case 1. The policy recommendation is that for a prospect who has been called upon for at least two periods and yet has not become a customer by purchasing, the firm should adopt a pulsing type of call strategy. To see this, consider a prospect in this class whose effort state is 4. The optimal policy is not to call upon that prospect during the current period. Then as the prospect's effort state diminishes due to forgetting he will eventually fall into one of the lower effort states, at which time the company should call upon him intensively. It should be noted that the single call which should be made when he is in effort state 3 will not be sufficient to sustain him in effort state 3 if he does not purchase as a result of that call.

Consequently, he will always ultimately fall toward state 2 if he persists in not purchasing. The model recommends a periodic pulse of sales effort for such prospects.

In case 2, we see that when a greater relative importance is attached to sales history, it generally becomes less attractive to call frequently on the account, particularly for large values of the history state. This is certainly what we would expect. Note, however, that in history state 1 and effort state 4 it is now profitable to make the maximum number of sales calls, 6, in contrast to the policy of 5 for this cell in Case 1. In this case, it is worth every effort to get the customer into a higher history state.
In Case 3 where the relative importance of current and remembered sales effort is much greater than that of sales history, we see that the call policy is independent of the history state. It might be noted that Case 3 would probably be representative of a highly competitive situation in which the salesman needs to continuously hustle in order to maintain the account as a customer. Case 2 is a situation in which continuing relationships are relatively easy to sustain.

An interesting contrast between Cases 2 and 3 occurs in the Prospect Age Classes. Now the call policy implications of the two cases reverse. Case 2 recommends more intensive calling than does Case 3 in several instances and it never recommends a lower level of calling in any instance. This was an unexpected result since the parameters of $P_{tn}$ for the prospect age classes are identical in both cases.

Howard's algorithm has proven to be computationally efficient for this model. The runs presented here converged in about three iterations and used an average of 0.13 minutes of IBM 360-65 time. Larger scale runs having 120 states compared to the 20 states in the present runs have run in about 1 minute on the model 65 and have never taken more than five iterations to converge on the optimal policy.
5. Conclusions

This paper has formulated and presented some preliminary results using a dynamic model for the determination of sales call policy. The approach of the model does not depend upon the particular functions which were used in this paper since it does not require any convexity or concavity restrictions on these functions.

Several issues remain for research on this model. Perhaps the first order of business is to try the model in actual selling situations. No doubt this will lead to revision of the model in order to make it more appropriate to particular empirical cases. Further, it would be interesting to examine the implications of the model for the size of the salesforce. In addition, it should be able to shed some light upon how a salesman can best allocate his time to the population of customers and prospects he faces at any point in time. One bonus of the discounted formulation is that it yields the present value of an optimal call strategy to a customer or prospect in any state or age class. This information should provide a starting point for the determination of how he should allocate his time to the population of prospects and customers.
ACKNOWLEDGEMENTS

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