WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

DIRECT ASSESSMENT OF CONSUMER UTILITY FUNCTIONS:
von Neumann-Morgenstern Utility Theory
Applied to Marketing

by

John R. Hauser[*] and Glen L. Urban[**]

Working Paper 843-76   March 1976

MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

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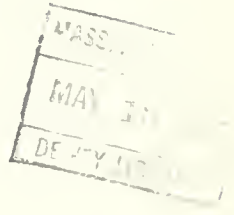
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[*] Assistant Professor of Marketing and Transportation
     Graduate School of Management, Northwestern University

[**] Associate Professor of Management Science
     Sloan School of Management, M.I.T.

ABSTRACT

The design of successful products and services requires an understanding of how consumers combine perceptions of product attributes into preferences among products. This paper briefly reviews the existing methods of expectancy value, preference regression, conjoint analysis, and logit models with respect to underlying theory, functional form, level of aggregation, stimuli presented to consumers, measures taken, estimation method, and specific strengths for use in marketing. Building on this comparison von Neumman-Morgenstern theory is presented for directly assessing consumer preferences. This method, new to marketing, has the advantage of axiomatic specification of functional form enabling it to explicitly identify and incorporate risk phenomena, attribute interactions, and other non-linearities. Preferences are measured on an individual level with "indifference" questions. Its disadvantage is the measurement task to which consumers are asked to respond.

This paper summarizes representative results of von Neumann-Morgenstern theory and discusses measurement and estimation of the resulting consumer preference functions. Its advantages and disadvantages for use in marketing are carefully discussed and application situations are identified where it is a promising method. A specific empirical example is presented for the design of a new service. New empirical results are then given comparing von Neumann-Morgenstern theory to the selected existing techniques of least squares and monotonic preference regression, logit analysis, and a null model of unit weights.

Since consumer preference is critical to the success of products and services, considerable research has been applied to the task of determining how consumers combine perceptions of product attributes into preference. Early work was directed at applying psychological concepts developed by Fishbein [5]. In many of these applications a linear additive function of directly stated "importances" of product attributes and ratings of product attributes were used to predict a preference measure (Wilkie and Pessemier [23]). In contrast, Carroll [4] used regression to fit a utility function to stated preference by specifying the location of an "ideal point " based on the assumption of a utility function form. Work in conjoint analysis used monotonic analysis of variance to estimate "importances" based on stated rank order preferences with respect to various prespecified product attributes (Green and Wind [8]). Stochastic modeling of observed choice with the logit form also has been used to estimate the impor- tances of attributes (McFadden [15]).

Another technique that is directed at the problem of assessing utility functions is von Neumann-Morgenstern utility theory [22]. Although this technique has been applied to many prescriptive decision situations (Keeney, 1973 [12]), it has only recently been proposed for application to marketing (Hauser and Urban [9]). The purposes of this paper are to (1) develop a comparative structure to position von Neumann-Morgenstern relative to existing methods, (2) present some new comparative empirical experience, and (3) assess the usefulness of von Neumann-Morgenstern utility theory in marketing.

EXISTING TECHNIQUES

Before describing an empirical application of von Neumann-Morgenstern utility theory, existing techniques will be positioned in a comparative structure which examines the theoretical base, assumed form of the utility function, level of aggregation, measurement requirements, and estimation methods of each technique. See Table One.

TABLE ONE

COMPARISON OF EXISTING TECHNIQUES OF DETERMINING "IMPORTANCES"

	EXPECTANCY VALUES	PREFERENCE REGRESSION	CONJOINT ANALYSIS	LOGIT MODEL
UNDERLYING THEORY	PSYCHOLOGY	STATISTICS	MATHEMATICAL PSYCHOLOGY	STOCHASTIC CHOICE
FUNCTIONAL FORM	LINEAR	LINEAR AND NON-LINEAR	ADDITIVE	LINEAR IN PARAMETERS
LEVEL OF AGGREGATION	INDIVIDUAL	GROUP	INDIVIDUAL	GROUP
STIMULI PRE-SENTED TO RESPONDENT	ATTRIBUTE SCALES	ACTUAL ALTER-NATIVES OR CONCEPTS	PROFILES OF ATTRIBUTES	OBSERVED ACTUAL ALTERNATIVES
MEASURES TAKEN	ATTRIBUTE IMPORTANCES	ATTRIBUTE RATINGS AND PREFERENCE	RANK ORDER PREFERENCE	OBSERVED BEHAVIOR
ESTIMATION METHOD	DIRECT CONSUMER INPUT	REGRESSION	MONOTONIC ANALYSIS OF VARIANCE	MAXIMUM LIKELIHOOD

Expectancy Value Models

Several multiattribute models have been proposed based on psychological theories of attitude formation (Fishbein [5], Rosenburg [17]). Although not equivalent, the models are conceptually similar in that they define an attitude towards an object as a linear additive function of an individual's reactions to an object on an attribute scale multiplied by a measure of the effect of that attribute in the overall attitude formation.

These models have received considerable attention and have been subject to various extensions (Wilke and Pessemier [23] and Ryan and Bonfield [20]). For

purposes of discussion we will adopt Wilke and Pessemier's multi-attribute model formulation:

(1)
$$\hat{P}_{ij} = \sum_{k} \lambda_{ik} \tilde{x}_{ijk}$$

where: $\tilde{x}_{ijk}$ = individual i's belief as to the extent to which an attribute k is offered by choice alternative j

λ_{ik} = "importance" weight specified by individual i for attribute k

$\hat{P}_{ij}$ = predicted attitude of individual i for choice alternative j

Although methods of measurement vary, specific individual estimates of λ_{ik} and $\tilde{x}_{ijk}$ are obtained from consumers. The predicted attitude $\hat{P}_{ij}$ is correlated to a measure of the overall attitude to access validity. This overall measure usually is preference for the choice alternative.

The model has been used by a number of market researchers. One of the more successful applications is reported by Bass and Talarzyk [2]. They predict rank order preference for frequently purchased consumer goods based on rank ordering of the importances of each scale and belief ratings of 1 to 6 on pre-defined attribute scales. Correct first preference prediction occurred in 65 to 75% of the cases over 6 product classes. This compared favorably to a naive model which assigned all choices proportional to market share and produces 35-55% first preference prediction. Other researchers have experienced varying success and a range of fits has been reported. Sheth and Talarzyk [16] report lower correlations of actual and predicted preference from .1 to .4 [14] while Ryan and Bonfield [20] report correlations as high as .7 and .8 for an extended version of Fishbein's model.

The advantages of these models are the relatively simple consumer measurement task and the idiosyncratic measurement which allows for individual differences in the importance parameters. A disadvantage is that the model is quite sensitive to the consumer's ability to directly supply an accurate importance parameter. Furthermore, the arbitrary linear functional form does not allow

non-linear effects to be modeled and requires a complete and independent set of attributes.

Preference Regression:

Statistical procedures have been used to recover importances (Carroll [4], Urban [19]. In these approaches a measured preference value is used as a dependent variable and attribute ratings are treated as independent variables. This is in contrast to expectancy value models where importances are directly stated by consumers. Regression is used to fit an importance parameter for the case of a linear additive function. The regression approach allows non-linearity and interactions in the functional form. For example, in Carroll and Chang's model linear, quadratic, and quadratic with pairwise interaction forms are available. Carroll and Chang's procedure is idiosyncratic while Urban regresses across choice alternatives and individuals.

Consumers provide attribute ratings and preference values (rank order or constant sum) for existing brands or for concept descriptions of new brands. If rank order preference is provided, monotonic regression is used to estimate the parameters. If constant sum preference data is collected, standard regression procedures may be followed.

Although the regression approach can be used to specify individual parameters, the measurement requirements indicated above realistically limit the number of observations per individual to less than ten. Therefore, the degrees of freedom available usually indicate the need to estimation across individuals in a group. In these cases care must be taken to assure that the individuals included in the group are homogeneous with respect to their underlying utility parameters. Clustering and segmentation methods are available to carry out this task (Hauser and Urban [8]).

In the linear case, the model is similar to equation (1) except that

λ_{ik} becomes λ_k, where λ_k is the importance for attribute k in the group.

$$(2) \qquad\qquad P_{ij} = \sum_k \lambda_k x'_{ijk} + \varepsilon_{ij}$$

P_{ij} is the observed preference of real or simulated product j for individual i, and x'_{ijk} are the perceptual attribute levels. In most cases x'_{ijk} represents a reduced space set of co-ordinates of the attributes obtained from factor analysis or non-metric scaling of the perception data consisting of attribute ratings or similarly judgements, respectively. ε_{ij} is the error term.

This model has not been as widely used as the expectancy value model, but has undergone considerable testing (Green and Rao [7], Urban [21]). Srinivasan and Shocker [18] have developed an alternative fitting procedure utilizing linear programming to minimize the errors in predicting pariwise preference rank orders by a linear function of attributes.

The advantage of preference fitting methods is that the estimation provides a direct link from preference to the importance weights. It allows flexibility in functional form and uses generally available computer programs. Its disadvantages are that in the individual case degrees of freedom are limited and in the group case importance weights must be estimated across consumers with estimation techniques that require prior grouping for homogeneity.

Conjoint Analysis

Conjoint analysis draws upon work of mathematical psychologists such as Krantz, Luce, Suppes, and Tversky [14]. Green and Wind [8] and Johnson [10] and other market researchers have taken a special case of this theory and applied it to estimating consumer preference functions.

The conjoint analysis model considers observed rank order preference as a function of a set of prespecified independent variables. In the additive case:

(3a) $$P_{ij} = \sum_{k,\ell} \lambda_{ik\ell} \, x^{*}_{jk\ell} + \varepsilon_{ij}$$

where $\lambda_{ik\ell}$ is the value individual i places on having the kth attribute at the ℓth level and $x^{*}_{jk\ell}$ is a (0, 1) variable which indicates whether stimulus j has the kth attribute at the ℓth level, and ε_{ij} is the error term. The function is idiosyncratic. Sufficient degrees of freedom are obtained at the individual level by presenting the consumer with many (n $\approx$ 30) stimuli. Each stimulus is a statement of a factorial combination or profile of the attributes ($x^{*}_{jk\ell}$). These may be presented on a card with one profile per card. The consumer's task is to rank order the cards with respect to his or her preference. In most analyses the number of attributes is large (6 to 10) and the consumer is presented with a fractional factorial design. In practice, this limits the utility function to the additive case even though in theory the conjoint model could be more complex (Krantz, Luce, Seppes, Tversky [14]). The importance weights are estimated by monotonic analysis of variance techniques.

Conjoint measurement has been used by Green and Wind [8] for brand choice for frequently purchased goods and for flight transportation carriers, and by Johnson [10] for automobile and "hard goods" brand choice. Reported fits are quite good. Johnson reports a first preference recovery of 45%.

One strength of conjoint measurement is that it is based upon measurement axioms which allow estimation of the preference function based on observing certain preference judgements. Furthermore, it is idiosyncratic, which allows for individual differences in the preference functions. One primary disadvantage is that the measurement task is based on rankings of hypothetical attribute profiles. This means attributes of the product must be pre-specified. While this provides an advantage in that more instrument variables can be defined, the issues of perception are not investigated as they are in the preference

regression approach where reduced space attribute ratings are processed as independent variables. In the usual measurement scheme the model is assumed to be linear or additive. This may be an oversimplification of the choice process and places a large burden on the researcher to pre-specify a linearly independent and complete set of attributes.

Logit Models

Theoretical work on stochastic choice as represented in the Logit model can be applied to marketing (Ashton [1]). This random utility model (McFadden [15]) predicts choice probabilities by observing perceptions of all relevant choice alternatives and estimating underlying preference functions to best predict choice. The multinomial logit model posits choice as a result of maximizing preference where preference is a combination of an observable part and a random part. Under specific assumptions (McFadden [15]) this yields:

$$(4) \qquad L_{ij} \ = \ \exp{(\hat{P}_{ij})} \ / \ \sum_{n} \exp{(\hat{P}_{in})}$$

where L_{ij} is the probability that individual i chooses alternative j. In practice, the preference is assumed to be a linear function of attributes of each alternative:

$$(5) \qquad \hat{P}_{ij} \ = \ \sum_{k} \lambda_{k} \ x_{ijk}$$

where λ_{k} are the importance weights for attribute k and x_{ijk} are the observed attributes for individual i and stimulus j on attribute k. In this model, choice (0, 1) and the attribute levels are directly observed and importances (λ_{k}) are estimated to meet the maximum likelihood conditions. To achieve sufficient degrees of freedom, researchers have assumed that the same parameters apply for all consumers. Therefore homogeneity within the group must be assured by segmentation analysis or assumed to be true. Although

preference is linear in the attributes (eq. 5), note that the probability of choice itself is non-linear in the attributes (eq. 4).

Multinominal logit models have been most extensively used in transportation modeling (Ben-Akiva [3]). In marketing, Silk and Urban [17] report good fits of the multinomial logit model for observed store choice of consumer brands as a function of constant sum preference for brands.

The primary advantages of random utility models is the axiomatic specification of choice probabilities. This allows calibration of "revealed preference" by observing choice behavior and observed attribute values. This is also a potential disadvantage because other market forces such as distribution and promotion affect choice and often these effects on consumer preference cannot be separated without direct measurement of stated preference. Other disadvantages are that the importance weights are not idiosyncratic and the preference function usually is restricted to be linear.

Discussion

Each of the existing techniques produce estimates of importances of attributes, but their methods are quite diverse in their theoretical bases, functional forms, level of aggregation, measurement, and estimation (see Table 1). Each has its strengths, its weaknesses, and particular applications where it is the best possible technique.

Expectancy value is useful for exploratory or diagnostic work because the respondent's task is simple and can be applied with a large number of attributes. In addition, the specific measurement allows for individual variations in consumers and for possible segmentation. However, an arbitrary linear functional form is assumed and prior specification of the attributes must be made.

Preference regression circumvents the questions of direct specification of importance weights by statistically estimating the importance weights based

on stated preference. This estimation, combined with perceptual reduction of the product's attribute space, allows the issues of psychological positioning to be effectively addressed (Urban [21]). But individual importances are sacrificed. Thus prior segmentation on homogeneity of preference parameters is required. The functional form could be linear or non-linear, but usually the linear form is chosen. Thus decreasing returns and attribute interaction are not modeled.

Conjoint analysis allows consideration of a pre-specified set of attributes so instrumental variables such as price, package, and brand name can be defined. This makes conjoint analysis a useful tool for physical design of products. The importances help define a best combination of product attributes. However, conjoint measurment requires relatively extensive measurement -- individual's ranking of many abstract alternatives. Careful prior measurement is required to assure that the attributes adequately describe choice alternatives, are independent, and are relatively small in number. Eight to ten attributes are usually the limit since the number of abstract alternatives grows exponentially in the number of attributes.

Logit models are based on observed choices rather than stated preference so they provide an alternative view of attribute importances for marketing decision. This is particularly useful if resources are not available for more extensive measurement. The Logit technique requires the functional form of the preference functions and attributes to be specified prior to estimation. Applications have tended to use linear function (Ben-Akiva [3]).

All of the above techniques are extremely powerful marketing tools when used in the proper context, but there is yet another technique that has not been applied to marketing -- von Neumann-Morgenstern utility theory. It offers potential advantages in the specification of functional form, the consideration

of risk in the choice process, and the idiosyncratic estimation of complex

preference parameters.

We first describe some of the underlying utility theory, what steps are

necessary to apply the theory to marketing, and then present an empirical

example of measuring preference for health care delivery systems. Finally, we

compare the values of the importance weights and the predictive ability of the

von Neumann-Morgenstern based technique to some other selected techniques and assess

the usefulness of von Neumann-Morgenstern theory in marketing.

A VON NEUMANN-MORGENSTERN BASED METHODOLOGY FOR DESCRIBING PREFERENCES

Underlying Theory

Von Neumann-Morgenstern [22] postulated a set of axioms to deal with

rational decision making (choice) under uncertainty. The axioms have two im-

portant implications. First, they imply the existence of a unique preference

function, $u(\cdot)$, with the property that u (alternative 1) > u (alternative 2)

if and only if alternative 1 is preferred to alternative 2. Second, they

imply that this function is cardinal in the sense that if the characteristics

(attributes) of the alternatives are uncertain then E [u (alternative 1)] >

E [u (alternative 2)] if and only if "uncertain" alternative 1 is preferred to

"uncertain" alternative 2. E [alternative j] means the mathematical expected

value of u (alternative j). These axioms, existence theorems, and uniqueness

theorems are useful because we can only measure a function if it exists and is

unique. The theorems tell us when it exists and is unique and under what

testable conditions specific functional forms are appropriate. This theory

is complementary to conjoint theory which examines when certain forms are

measurable, and stochastic choice theory (e.g., logit) which examines how
preference once measured predicts actual choice.

We briefly summarize representative, but important, results here and list
in the bibliography, papers which contain complete derivations of these and
other results. See particularly Farquhar [5] for a survey of utility theoretic
results.

Functional Form

Based on specific assumptions of independence, utility theorists have
derived specific utility function forms. For example, Keeney shows that under
"utility independence" (defined in the next section) the von Neumann-Morgenstern
axioms lead to a preference function that is a special polynomial called quasi-
additive (Keeney, 1974 [11]).

$$(6) \quad C(x_1, x_2, \ldots x_k) = \sum_k \lambda_k u_k (x_k) + \sum_k \sum_{\ell>k} \lambda_{k,\ell} u_k(x_k) u_\ell(x_\ell)$$

$$+ \ldots + \lambda_{1,2,3,\ldots K} u_1(x_1) u_2(x_2) \ldots u_k(x_k)$$

where $C(\cdot)$ = preference function of attributes x_1 to x_k

$u_k(x_k)$ = utility of attribute k at level x_k

λ_k = importance coefficient for attribute k

$\lambda_{k,\ell}, \lambda_{k,\ell,m} \lambda_{1,2,3\ldots K}$,etc. = importance coefficient for interactions of attributes k and ℓ, of attributes k, ℓ and m, etc. up to interaction of all attributes.

The utility of an attribute, $u_k(x_k)$, is a non-linear function of the attribute
level x_k. These non-linear functions can also be derived from basic assumptions.
For example, if risk aversion with respect to x_k does not depend upon the amount
of x_k already guaranteed, then the "constantly risk averse" form is the only
possible form. I.e.,

(7) $u_k(x_k) = a+b \exp(-r\, x_k)$, where r is the risk aversion

coefficient (r > 0 risk averse, r < 0 risk seeking, r = 0 risk neutral).

The utility theoretic equations in 6 and 7 are idiosyncratic so each

individual is modeled separately. The utility theory form allows non-linear

and interaction effects of attributes in the modeling of choice. One real

advantage of the utility function is that the risk aversion coefficient allows

explicit measurement and inclusion of risk phenomena.

Measurement

The parameterization of risk phenomena is based on measures obtained by

presenting respondents with lotteries.   This task is simple in concept, but

difficult in practice. We explain here the concept. The stimulus is a game

in which the respondent determines when he would be indifferent between a certain

outcome and a gamble based on two uncertain events. For example, Figure one

is a schematic of a lottery given to a consumer for a choice of medical services.

He or she must consider joining a health plan in which the waiting time to see

a doctor is in question. In plan 1 the waiting time is known to be 20 minutes.

In plan 2 the time will be either 10 minutes or 60 minutes, but it is not

certain which will occur. The task is to set the probability so that the

respondent is indifferent between the certain event and the lottery.

At a 99.9% chance of 10 minutes and a 0.1% chance at 60 minutes, most

people would choose the lottery. At a 0.1% chance of 10 minutes and a 99.9%

chance of 60 minutes, most people would prefer the certain wait of 20 minutes.

The respondents' task is to continually narrow this range until he or she can

select a probability, p, such that at a slightly higher probability, $(p + \Delta)$

of 10 minutes, he or she prefers the lottery and at a slightly lower

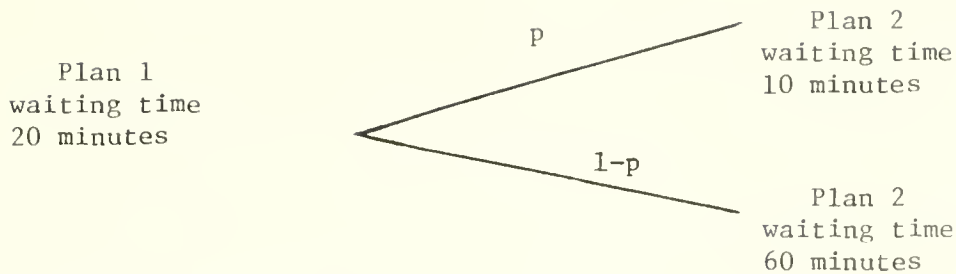probability $(p - \Delta)$ he or she prefers the certain wait.

```
                                                p                    Plan 2
                                                                   waiting time
   Plan 1                                                          10 minutes
 waiting time
 20 minutes
                                        1-p
                                                                     Plan 2
                                                                   waiting time
                                                                   60 minutes
```

Figure 1:   Schematic of Risk Aversion Question

By considering this lottery we can now define the conditions necessary
for utility independence that were required in deriving equation (6). If the
lottery was asked assuming that all other attributes (e.g., quality, price)
were at "high" levels, utility independence would exist if the indifference
probability did not change when all other attributes were changed to be at "low"
levels.

In order to parameterize a multiattribute utility function, measures must
also be taken to reflect tradeoffs between attributes. Figure Two presents a
tradeoff between waiting time and price for a health plan. The consumer must
set the level of price that will make him or her indifferent between the two
plans (A and B).

Plan A   Plan B

Waiting time 20 minutes   Waiting time 30 minutes
Price $10   Price ___ ___ ___

Figure 2:   Schematic of Trade-Off Question

If the answer to this question is the same with other variables such as
quality at a "high" level in both plan A and B as with quality at a "low" level
in both plan A and B, "preferential independence" is said to be satisfied.

Estimation

In utility theory measures are taken so that the parameters can be directly calculated. In the constantly risk averse form one lottery is conducted for each attribute to calculate the risk aversion coefficient (see equation 7). Tradeoff and other lottery questions are asked until the number of parameters equal the number of observations. Additional observations may be taken to test assumptions such as utility independence (i.e., an additional lottery for each attribute) or to assure consistency by repeated measures. (A representative calculation is shown in appendix two.)

Utility theory is substantially different from previous methods for esti- mating importances (see Table 1). It is based on explicit theory for functional forms, idiosyncratically considers non-linear and additive effects of attributes, includes consideration of risk, obtains measures by lotteries and tradeoffs, and directly calculates the functional parameters from the measured responses.

USE IN MARKETING

Utility theory has many attractive features, but has not been applied to marketing problems. In marketing we want to describe the consumer choice process so that we can design a product or service which the market will view as attractive and buy. Alternatively we may wish to influence choice by changing the consumer's utility function. For example, a possible marketing strategy for a public transit authority might be to increase the perceived importance of costs of driving autos to encourage use of mass transit. This is a different problem than the usual utility theory application to a situation characterized by one decision maker or a small group of decision makers. In these applications utility theory helps the decision maker rationally evaluate the alternatives and quantitatively incorporate any uncertainty he has about

the outcome of any decision. The applications emphasize prescribing a solution (Keeney [12]) rather than describing choice behavior.

In applying utility theory to marketing , several issues are important and require modification to usual utility theoretic approaches. First, marketing reflects many diverse decision makers with varying preferences. It is necessary to measure preferences for a sufficiently large sample of the population to insure that the distribution of preferences is correctly characterized. Thus the measurement must be administered by a standardized personal interview and aggregate market representations developed.

Furthermore, in marketing the attribute measures are often psychological rather than physical. In prescriptive utility theory the performance measures are quantifiable (e.g. tons of hydrocarbons released into the air). In consumer choice, hard to quantify psychological measures, (e.g., quality of a health care plan) become important. In decision making, a manager may learn to think of quantity in health care as the number of MD's available, but will the consumer? It is imperative in marketing to measure, characterize, and quantify how consumers perceive the alternative products or services. Thus psychometric techniques must be used prior to utility assessment to identify a complete set of performance measures which include both psychological and physical measures. The use of psychological attributes with the utility lotteries and tradeoffs increases the burden and cost of measurement. When an individual manager's career may rest on the outcome of a major decision, he will make available the necessary time (e.g., 4-8 hours) to have his utility function assessed. But will the consumer? Usually one hour would be the maximum time for a market research interview. In a short 45-60 minute interview the consumer must be motivated and educated to the lottery and tradeoff questions necessary for assessment and respond to the assessment and verification questions. Furthermore, the tasks cannot be too onerous or too complex, but

must involve the consumer so that he gives thoughtful answers which reflect reality. The cost of measurement will be substantial since a reasonable sample (e.g., n $\geq$ 100) must be taken to represent the diversity of consumers and allow an estimate of market response.

Even with an adequate sample and a carefully refined measurement instrument, response errors can be expected. There may be errors in measurement of perception, understanding the tasks, mathematical model specification, neglecting important effects, or random fluctuations in preferences.
Thus the parameters we obtain are only estimates of the true parameters. Ideally redundant questions should be asked, but the measurement cost and time constraints preclude this.

Each of these issues of diverse consumers, psychological performance measures, measurement burden, and measurement error are non-trivial issues in applying von Neumann-Morgenstern theory to marketing. We will present how we addressed these issues in a particular problem of health service marketing. We feel that this example highlights the issues and suggests a set of possible solutions. Hopefully this example will facilitate discussion and encourage researchers to develop more and better techniques to address these issues in applying von Neumann-Morgenstern theory.

DIRECT EMPIRICAL ASSESSMENT OF VON NEUMANN-MORGENSTERN PREFERENCE FOR HEALTH CARE DELIVERY SYSTEMS

Health Maintenance Organizations (HMO) have been proposed as a method of reducing costs and increasing availability and quality of health services. Although some HMO's have been successful, a major problem is gaining sufficient enrollment. MIT was developing an HMO and provided the managerial setting. This marketing problem was addressed as a product design and communication problem. We will discuss how utility theory was applied in this case and provide empirical comparisons to alternative methods of estimating the importance of attributes.

This discussion is restricted to student response. (See Hauser and Urban [9] for a managerial description of the case and initial consideration of Faculty and Staff by non-utility theoretic methods.)

Data was obtained by one hour interviews with a randomly selected sample of eighty MIT students. The survey included measures of general health attitudes and practices and specific questions to estimate attribute importances.

The first task was to identify the attribute or performance measures. Group discussions with students clearly indicated the high degree of psychological involvement. For example, concern was expressed about the level of trust in the doctors, the red tape and "hassle" at the clinic, the friendliness of personnel, and the personalness of care. As a result of these discussions 16 attitude scales were developed. In a preliminary student questionnaire, students rated their existing health care and 3 concepts (MIT HMO, Harvard Community Health Plan, and Massachusetts Health Foundation) with respect to these statements by recording their level of agreement or disagreement on a 5 point scale. Factor analysis of this data led to the definition of the four under-lying psychological factors. These factors explained 55% of variance in the data. The raw scales and the factor that they were most highly correlated with are shown in Appendix I. The factors were named "quality", "personalness", "value" (benefit vs. price), and "convenience". These four underlying factors were used as attribute or performance measures (x_k) in the utility model and the factor scores were used as attribute values for alternate models.

We began with warmup questions to train the respondents to the meaning of the lotteries. Then each student answered five lottery and three tradeoff questions (see Figure 1 and 2 for simplified prototypes). Utility and prefer-ential independence assumptions were investigated by repeated administration of the lottery and tradeoff questions. Rank order preferences were recorded

for the three new HMO alternatives and the respondent's existing health
service. The questionnaire closed with demographic questions.

Results

Importance weights were obtained by estimation of a special form of the
quasi-additive function shown in equation 6 that is called the multiplicative
form:

$$(8) \qquad 1 + \Lambda \, C(x_1, x_2, \ldots x_K) = \prod_{k=1}^{K} (1 + \Lambda \lambda_k \, u_k \, (x_k))$$

where Λ is the interaction coefficient

and $\Lambda > 0$ implies complimentality

$\Lambda = 0$ implies no interaction (i.e., additive)

$\Lambda < 0$ implies substitution

λ_k = importance coefficients

$u_k(x_k)$ = utility of attribute x_k (see equation 7).

Table Two shows the average normalized weights $(\lambda_k / \Sigma \lambda_k)$ for the sample.

Quality has the highest coefficient followed by value and convenience,
with personalness having the lowest value. There was considerable individual
variation. The interquartile ranges were for: quality +12.5% to -18% of the
median, personalness +45% to -31%, value +14% to -29%, and convenience +17%
to -23% of the median. The risk aversion coefficients (r in equation 7) were .
rank ordered similarly to   the importance coefficients (r_1 = .693 for
quality, r_2 = .332 for personalness, r_3 = .424 for value, and r_4 = .310 for
convenience). This suggests the hypothesis that the more important a
performance measure is, the less willing a consumer is to take a chance on
its level. The full interquartile interval for the interaction coefficient (Λ)
was between -.99 and -.93, indicating substitution between attributes for most
consumers.

TABLE TWO

IMPORTANCE ESTIMATES AND GOODNESS OF FIT

MIT STUDENTS

| | Normalized Importance Weights | | | | Preference Recovery | |
Method	Quality λ_1	Personal λ_2	Value λ_3	Convenience λ_4	1st choice	all choices
Utility Assessment						
Raw Importance Weights	.30	.19	.26	.25	.50	.47
Marginal Weights	.31	.25	.25	.19		
Preference Regression						
Least Squares	.32	.09	.38	.21	.47	.51
Monotonic	.34	.08	.31	.27	.45	.45
Logit Analysis	.34	.16	.34	.16	.43	.47
Unit Weights	.25	.25	.25	.25	.40	.44

The utility independence assumption was tested by repeated administration of the lottery questions at alternate levels of attributes. The utility independence assumption held exactly for quality in 51% of the cases, personalness 39%, value 55%, and convenience 53% of the cases. Satisfaction of the assumption was defined as within ± 10 percentage point deviation in the probability on repetition of the lottery. Quality met this utility independence condition in 66% of the cases, personalness in 71%, convenience in 68%, and value in 70% of the cases. Although this represents the first time utility independence has been tested for a consumer population, these results seem reasonable. We might point

out however, that expectancy value, preference regression, conjoint analysis, and logit implicitly assume utility independence with their choice of functional form.

The multiplicative form (equation 8) also requires pairwise preferential independence. This was tested with similar success and in 60% of the cases preferential independence assumptions were met exactly.

In order to test the "goodness" of the estimates we used the criterion of correct recovery of the stated rank order preferences. Table Two reports that when the attribute ratings are substituted in equation 8, the estimated utility function correctly predicts first preference among the 3 new HMO and 1 existing care alternatives 50% of the time. The correct prediction of 1, 2, 3 and 4th choice choice occurs in 47% of the cases. This is the percentage of occurrence of diagonal entries in the matrix of predicted and actual rank order of the four alternatives. These fits are satisfactory for a first attempt, but clearly indicate the existence of measurement errors in the utility theory measurement.

The importance weights themselves do not reflect non-linearity, risk aversion, and interactions. In order to get a richer measure of attribute response, the total marginal response to each attribute was determined by the gradient at the point of the utility function represented by the attribute ratings of the student's first choice health plan. The differences between the linear weights and the marginal weights at the first choice plan are that personalness is given higher weight and convenience is given lower weight.

The utility results were compared to importance estimates obtained by other selected methods. Preference regression analysis was conducted by treating the rank order preference for the four alternatives as the dependent variable and the factor scores reduced from the ratings of each plan as the independent variables.

It was assumed that the students represented a homogeneous group and regression was done across health alternatives and individuals with a linear function of the four attribute factors (see equation 2). Regression was done by least squares and monotonic regression. In the least squares case the importance coefficients rank ordered the factors in terms of importance as quality, value, convenience, and personalness. The coefficients were similar to the average utility theory coefficients and the fits were equally good with utility fitting first preference better and the regression fitting overall choices better. The use of monotonic regression did not improve the fits, but did estimate the importance of convenience as slightly higher and value as slightly lower than least squares regression.

The logit model was applied to the data by treating first preference as an observed choice (see equation 4). The linear importance coefficients were similar to the regression values. The fits were not quite as good in terms of first preference or overall choices as the regression.

In examining the alternative methods it appeared that the fits did not vary substantially over the space of importance estimates. To test this further, unit weights were assigned to the four underlying factors. These weights were not as good in predicting choice. 40% correct first choice fit for equal weights versus 50% for utility theory and 44% correct overall choices for equal weights versus 51% for least squares regression. The equal weights model serves as a null model and the adequacy of the fits indicate that care should be taken in concluding that weights are not equal for these four factors of quality, personalness, value, and convenience.

On the basis of preference recovery, utility theory performed about as well as other methods. Another measure of goodness was calculated by examining the root mean squares error (RMSE) between predicted and actual market share of the four health care alternatives presented to the students. This is not as

powerful a measure of fit as the preference recovery, but in marketing, market shares take on special importance in making new product, advertising, and promotion decisions. The RMSE for the utility assessment performed substantially better in predicting choice of existing care. The other methods over-predicted the switching from existing care to other plans.

TABLE THREE

PREDICTED AND ACTUAL SHARE OF CHOICES

MIT STUDENTS

	Existing Care	Harvard Community Plan	MIT HMO	Mass. Health Foundation	RMSE Error
Actual	.34	.11	.42	.13	-----
Utility	.30	.08	.42	.20	.203
Pref. Regression					
Least Squares	.19	.19	.45	.18	.410
Monotonic	.20	.24	.41	.15	.414
Logit	.22	.23	.35	.20	.409

The reason for this can be seen by considering the marginal weights (see Table Two). The average marginal value for personalness was higher than the average raw weights and the marginal value for convenience was lower than the raw weights. Since the new alternatives rated relatively high on convenience and low on personalness, the utility model predicted relatively less switching to the new alternatives. Thus by including risk aversion and other non-linearities, the utility theory improved prediction of the managerially relevant market shares for the new alternatives.

CONCLUSION

Utility theory was investigated for the potential benefits of utility function specification, consideration of risk, and idiosyncratic estimation of complex preference parameters. The data presented here indicates utility theory is feasible for some consumer markets. Although the preference fits are not uniformly superior for utility theory, they are equally good. The importance of risk aversion is indicated by the superiority of utility theory in specifying the share of choices for the existing service alternatives. This emphasizes the value of more complex functions for combining attributes. The advantages of utility theory were obtained at a substantial cost. The measurement required a personal interview of 45 minutes and the execution of the difficult lottery questions. In fact, in consumer groups characterized by low education levels, it is doubtful that the lottery questions could be executed. We conclude utility theory is a valuable tool for a marketing scientist to have in his or her tool kit. It can be most effectively used if:

(1) risk aversion and interaction phenomena are deemed to be important in the choice decision

(2) a sufficient budget is available to carry out extensive personal interviews

(3) individual utility parameters are important to decisions, and

(4) consumers are well educated.

It would be particularly effective if the number of decision makers was small and the purchase decision large . For example, purchase of large computers, aircraft, automated machine tools or other industrial products would be situations where extensive measurement could be done and risk aversion, non-linearities, and interactions in attributes would be important in predicting choices.

The appropriateness of utility theory in marketing could be improved through further research. As cited earlier, utility theory methods directly calculate parameters and do not explicitly acknowledge the concept of measurement error. Research is needed to allow degrees of freedom to be obtained by/lotteries and including additional tradeoffs in the estimation process. Maximum likelihood methods suggest themselves as a likely candidate for processing this data. The costs of utility measurement are high, but research might indicate more efficient methods for data collection. For example, the findings reported here indicate that risk aversion correlates with importance. If this is true it might be exploited so that only a subsample would be required to answer the difficult lottery questions. If these research tasks can be accomplished, utility theory will be more attractive and appropriate for mass consumer markets. Our work indicates that in some situations utility theory has advantages over other methods of assessing importances. It deserves attention from marketing scientists.

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APPENDIX ONE

Attitude Scales and Performance Measures*

Quality

I could trust that I am getting really good medical care.

The plan would help me prevent medical problems before they occurred.

I could easily find a good doctor.

The services would use the best possible hospitals.

Highly competent doctors and specialists would be available to serve me.

The service would use modern, up-to-date treatment methods.

Personalness

I would get a friendly, warm, and personal approach to my medical problems.

No one has access to my medical record except medical personnel.

Not too much work would be done by nurses and assistants rather than doctors.

There would be little redtape and bureaucratic hassle.

Value

I would not be paying too much for my required medical services.

There would be a high continuing interest in my health care.

It would be an organized and complete medical service for me and my family.

Convenience

I would be able to get medical service and advice easily any time of the day and night.

The health services would be inconveniently located and would be difficult to get to.

I would have to wait a long time to get service.

*See Hauser and Urban [9] for detailed factor loadings.

APPENDIX II

Since many readers may not have been exposed to the procedures used to calculate the utility function parameters, this appendix gives a simple representative example of how indifference questions are used to calculate a 3-attribute consumer utility function. We will assume that utility independence has been verified for each attribute. These assumptions imply the utility function is of the following form:

$$(9) \quad u(x_1, x_2, x_3) = k_1 u_1(x_1) + k_2 u_2(x_2) + k_3 u_3(x_3) + K k_1 k_2 u_1(x_1) u_2(x_2)$$
$$+ K k_1 k_3 u_1(x_1) u_3(x_3) + K k_2 k_3 u_2(x_2) u_3(x_3)$$
$$+ K^2 k_1 k_2 k_3 u_1(x_1) u_2(x_2) u_3(x_3)$$

Our first task is to measure the uni-attributed scale functions, $u_\ell(x_\ell)$. Assume the consumer is considering purchasing an automobile and that x_1 represents miles per gallon, x_2 represents comfort, and x_3 represents purchase price. We will first scale x_1. Since preference increases with more miles per gallon (everything else the same), $u_1(x_1)$ is monotonically increasing in x_1. Since $u_1(x_1)$ is unique to a positive linear transformation we can arbitrarily select endpoints. Thus select the high end at $x_1^* = 40$ and the low end at $x_1^o = 10$. The consumer is now given a lottery. He is asked to consider car A, which is a very comfortable, $5000 car with guaranteed gas mileage of 25 mpg, and car B which is a very comfortable, $5000 car with uncertain gas mileage. In fact car B's mileage is so uncertain that p% of the new cars get 40 mpg and (1-p)% get 10 mpg. Unfortunately, our consumer cannot test the cars and is stuck with his purchase. He is asked to set p such that a slightly larger p would cause him to prefer car B and a slightly smaller p would cause him to prefer car A. (In practice probabilities are represented by areas of colored wheels.) The lottery is schematically represented in Figure A1:

Car A	Car B

x_2 = very comfortable x_2 = very comfortable

x_3 = \$5000 x_3 = \$5000

x_1 = 25 mpg

x_1 =

```
        P_1        40 mpg
       /
      <
       \  1-P_1
        \
         10 mpg
```
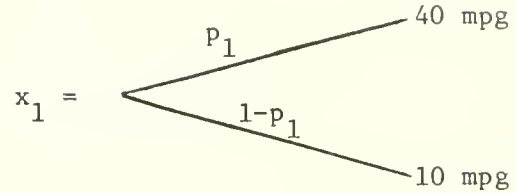
Set $u_1(x_1^*) = 1.0$ and $u_1(x_1^o) = 0.0$. Once the consumer selects p_1 we can use

the von Neumann-Morgenstern theorem to set the utility of car A to the utility

of car B. Algebraically this is given by:

u(25 mpg, very comfortable, \$5000) = $p_1 \cdot$u(40 mpg, very comfortable, \$5000)

$+(1-p_1) \cdot$u(10 mpg, very comfortable, \$5000).

Substituting equation 9 for $u(x_1, x_2, x_3)$ and cancelling terms yields:

(10) $u_1(25 \text{ mpg}) = p_1 u_1(x_1^*) + (1-p_1)\, u_1(x_1^o)$

Substituting for $u_1(x_1^*) = 1.0$ and $u_1(x_1^o) = 0$ yields:

$$u_1(25 \text{ mpg}) = p_1$$

If $u_1(x_1)$ is of the constantly risk averse form (equation 7 in text) we have one

equation in one unknown, i.e.:

$$u_1(x_1) = \frac{1 - e^{-r_1(x_1 - x_1^o)}}{1 - e^{-r_1(x_1^* - x_1^o)}}$$

$$u_1(25) = \frac{1 - e^{-r_1(25-10)}}{1 - e^{-r_1(40-10)}} = p_1$$

which can be solved for the risk aversion coefficient r_1 by setting $x = e^{-15r_1}$, and use of the quadratic substitution and formula. In this case r_1 is given by:

$$r_1 = \frac{1}{15} \ln\left(\frac{p_1}{1-p_1}\right)$$

Note that it was utility independence that allowed us to use equation 9. This form resulted in the cancellation that caused equation 10 to be independent of the selected levels of x_2 and x_3. The uni-attributed scale functions for comfort and purchase price are determined in a similar manner. For comfort, the analyst can either approximate the qualitative scale with a continuous scale or perform lotteries for each discrete point on the comfort scale. For purchase price be careful that $u_3(x_3)$ is monotonically decreasing in x_3.

The next task is to determine the relative tradeoffs, i.e., the ratio of k_2 to k_1 and the ratio of k_3 to k_1. To measure this the consumer is given a tradeoff question. He is asked to consider car C, which is a very comfortable, \$5000 car with gas mileage of 10 mpg, and car D, which is a very uncomfortable, \$5000 car with some unspecified gas mileage, G. This is shown schematically in Figure A2:

Car C	Car D
$x_3 = \$5000$	$x_3 = \$5000$
$x_2 = $ very comfortable	$x_2 = $ very uncomfortable
$x_1 = 10$ mpg	$x_1 = G$

The consumer's task is to select G such that if G were slightly larger he would prefer car D and if G were slightly less he would prefer car C. Suppose we scaled $u_3(x_3 = 5000) = 0.0$, $u_3(x_3 = 2000) = 1.0$, $u_2(x_2 = $ very comfortable$) = 1.0$, and $u_2(x_2 = $ very uncomfortable$) = 0.0$. Again, once the consumer selects G we can use the von Neumann-Morgenstern theorem to set the utility of car C equal to the utility of car D. Algebraically this gives:
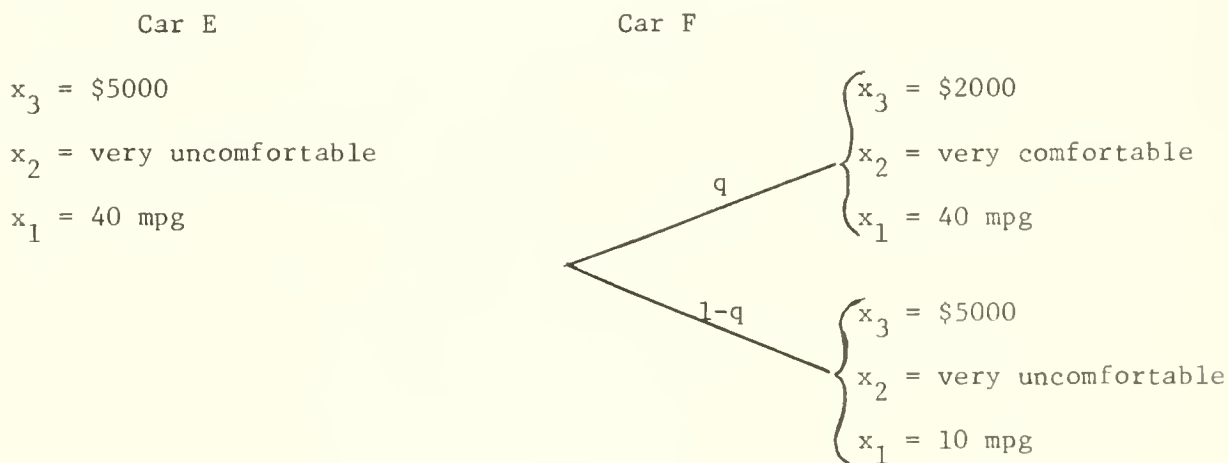
u(10 mpg, very comfortable, \$5000) = u(G mpg, very uncomfortable, \$5000).

Substituting equation 9 for $u(x_1, x_2, x_3)$ and cancelling terms yields (remember $u_3(\$5000) = 0.0$, etc.):

$$k_2 = k_1 \; u_1(G \text{ mpg})$$

which gives the relative value, k_2/k_1, directly since we have already measured $u_1(x_1)$ and can simply substitute for G. The relative value, k_3/k_1, is determined with a similar tradeoff question.

We now have the relative values, but in order to determine the absolute values we must ask one more question. Note that since equation 9 involves interactions, it is not enough to know the relative values. The absolute values of the k_ℓ's determine the form of the interactions ($\sum k_\ell < 1 \rightarrow$ complimentarity, $\sum k_\ell = 1 \rightarrow$ no interaction, $\sum k_\ell > 1 \rightarrow$ substitutibility (Keeney [11]). Therefore, we must measure the absolute values with a lottery or a tradeoff which has all attributes varying, not just one or two as was the case in previous questions. For ease of presentation we choose a simple lottery represented schematically in Figure A3:

Car E   Car F

x_3 = \$5000 x_3 = \$2000

x_2 = very uncomfortable x_2 = very comfortable

                                       q
x_1 = 40 mpg x_1 = 40 mpg

                                       1-q x_3 = \$5000

                                                x_2 = very uncomfortable

                                                x_1 = 10 mpg

Once the consumer selects q we can proceed as before to set the utilities equal to each other and substitute equation 9 for $u(x_1, x_2, x_3)$. By the von Neumann-

Morgenstern uniqueness theorem we can arbitrarily select the endpoints of $u(x_1, x_2, x_3)$. Therefore set u(40 mpg, very comfortable, \$2000) = 1.0 and u(10 mpg, very uncomfortable, \$5000) = 0.0. Substituting and cancelling terms yields the simple result of:

$$k_1 = q$$

This together with the ratio k_2/k_1 and k_3/k_1 gives the exact values of each of the k_ℓ's.

We need one more parameter, the interaction coefficient K, to specify the utility function. Since we have already set the upper end of our scale, i.e., u(40 mpg, very comfortable, \$2000) = 1.0 and since we now know k_1, k_2, k_3; K is determined by direct substitution in equation 9 yielding:

$$1.0 = k_1 + k_2 + k_3 + K(k_1 k_2 + k_1 k_3 + k_2 k_3) + K^2 k_1 k_2 k_3$$

which is a quadratic equation in K which can be solved directly. Keeney [11] shows that for equation 9 there is exactly one root of this polynomial in the relevant range: [-1, 0] if $\sum k_\ell > 1$ and (0, ∞) if $\sum k_\ell < 1$.

Thus three tradeoff and three lottery questions yield the seven parameters, k_1, k_2, k_3, K, r_1, r_2, r_3 for an interactive, non-linear, risk incorporating utility function. If one were to verify assumptions or estimate rather than solve the utility equations, more questions would of course be required.

We have given an arbitrary example in 3 dimensions. The measurement generalizes directly for more than 3 dimensions. We have simplified the algebra by using lottery and tradeoff questions with extreme values of the attributes. If one wants to tackle the complex algebra, it is conceptually straight forward to use lottery or tradeoff questions with less extreme values for the attributes.