DATA ENVELOPMENT ANALYSIS AS A NEW MANAGERIAL AUDIT METHODOLOGY - TEST AND EVALUATION

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Abstract

Data Envelopment Analysis (DEA) is a linear programming based technique that was developed to evaluate relative efficiency of nonprofit and public sector Decision Making Units (DMU's) that use multiple inputs to produce multiple outputs. In this study, DEA is evaluated and tested for use as a managerial audit tool to identify and measure inefficiencies among a set of DMU's. Based on three applications of DEA, this technique is found to be a useful technique for allocation of audit resources and for analytic review of operating efficiency when applied to a specific set of audit situations and when interpreted with recognition of DEA's particular strengths and limitations. The value of DEA is further found to extend to a class of for profit managerial audits in addition to the nonprofit and public sector types of audits.
Managerial audits designed to evaluate the effectiveness and efficiency of the operations of an organization have gained increased acceptance and have been increasingly used by managers in government and business. These managerial audits, also referred to as "comprehensive audits" [7], operational audits", "operation reviews" [9], were first actively used by the U.S. General Accounting Office [11] and have since been utilized by other governments, e.g., Canada, Australia, and Israel as well as by various state audit agencies. Corporate internal auditors have increasingly been required to complete managerial audits [9] and regulatory agencies have hired management consulting firms to conduct such audits for utilities (see for example [12]). It has been suggested by J. Burton [1] that managerial audits should be included as an integral part of a financial audit by CPA's to increase the value of their service and to better justify the increasing cost of such audits to management. Trends toward increased use of managerial audits suggest that any methodologies which help to achieve the objectives of these audits will be of value.

In this paper the use of Data Envelopment Analysis is evaluated as a managerial audit technique. Data Envelopment Analysis (DEA) is a linear programming based technique developed by A. Charnes, W. W. Cooper, and E. Rhodes (CCR) [5] [6] to evaluate the relative efficiency of public sector Decision Making Units (DMU's) that use multiple inputs to produce multiple outputs. The mathematical integrity of DEA and its consistency with microeconomic theory have been documented by CCR [5] [6]. Relying on the soundness of this theoretical foundation, DEA has been used to evaluate various public sector DMU's such as educational institution [2] [3] and armed forces recruiting office [8]. The purposes is to investigate how DEA results can be interpreted and used in a managerial audit context to evaluate the efficiency of DMU's and to define the application where DEA is most
appropriate compared with more traditional audit technique to assess organization efficiency. In addition, this investigation serves to clarify and illustrate the strengths and weaknesses of DEA and suggests that DEA can be effectively applied in many for profit business settings for managerial audit purposes in addition to its original intended use for public sector and other nonprofit organization evaluations.

**Efficiency versus Effectiveness - Defined**

Before proceeding, I should clarify the types of audit objectives for which we consider use of DEA in this paper. The managerial audit may attempt to evaluate effectiveness, the ability of a DMU to set and meet goals, and efficiency, the use of inputs to produce the desired outputs. I do not consider the effectiveness objective in this paper but rather assume that the outputs selected by the DMU are consistent with their effectiveness criteria, i.e., that they are producing goods or services that are consistent with the goals. Rather, the emphasis is on the assessment of DMU's technical efficiency. A DMU is defined here to be technically inefficient if a) the DMU could produce the same level of the outputs it produced with fewer inputs than it used or b) the DMU could have produced more outputs than it produced with the same level of inputs used.

**Efficiency Evaluation of multiple output-multiple input organizations**

The characteristics of DEA that prompt interest in evaluating public sector and nonprofit organizations are as follows

1. Ability to simultaneously consider multiple outputs and inputs in evaluating efficiency.
2. The production function, i.e., efficient input-output relationship need not be known.

These characteristics are particularly useful for nonprofit/public sector evaluation because such organization produce multiple outputs which cannot be
adequately evaluated with more traditional measures such as profit and return or investment, as profit maximization or cost minimization is but one of a broader set of goals. For example, a nonprofit hospital may produce multiple outputs including treatment of a variety of patient types, research, training nurses and medical students, and community education. Most of these outputs do not have competitive market prices and the amount of inputs needed to efficiently produce these outputs is generally not known in any detail. An evaluation of hospital efficiency needs to consider the amount of resources used to provide all these outputs when the efficient output input relationships are not known. Hence, DEA appears to be well suited to evaluate the efficiency of such organizations and consequently it is evaluated as a managerial audit tool for such applications. The applicability of DEA in a for profit audit application is also considered at a latter point in the paper.

Outline of the paper

The following section 2 briefly describes the DEA technique and ways it can be applied using standard linear programming computer codes. Section 3 describes an application of DEA to an artificial data set where the efficient and inefficient DMU's are known a priori. We use this artificial data set to investigate DEA's ability to identify inefficient DMU's compared to the known inefficient DMU's. This approach is adopted because in field application of DEA as in [3] and [8] the truly inefficient DMU's are not known and hence, the accuracy of the DEA results cannot strictly be evaluated as I attempt to do here. When DEA locates inefficient DMU's, these DMU's are strictly inefficient compared to other DMU's in the observation set. However, I also find that even in this somewhat simple example, all the inefficient DMU's are not identified as such. In addition, comparing the detailed DEA results to the known inefficiencies indicates alternative paths to improving efficiency
of inefficient DMU's but it does not single out the specific path that directly moves the DMU to the "known" efficient production function.

Section 4 describes the field testing of DEA in a set of teaching hospitals where the inefficient hospitals are not known, as will be the case in most applications of efficiency evaluation techniques like DEA. A panel of hospital experts, including the management of one of the identified inefficient hospitals, are used to evaluate the DEA results. In addition, the DEA technique is compared with the widely used ratio analysis approach to determine its relative capabilities in identifying inefficient DMU's. The DEA results are found to be accurate based on the experts' judgment. In addition, the traditional use of ratio analysis is found to be incapable of locating inefficiencies identified via DEA.

Section 5 discusses an application of DEA in a for profit setting and considers ways that this technique may be appropriate in this sector. The branches of a savings bank are evaluated using DEA and the results suggest ways in which DEA may be useful in a managerial audit of businesses with multioffice organization and which need to consider multiple outputs and inputs in their evaluation of operating efficiency.

Section 6 concludes by discussing how DEA might be appropriately used as a managerial audit tool, how it relates to other traditional techniques with respect to their relative strengths and weaknesses and the areas that need to be researched in further developing this tool for managerial audit application.

2. Data Envelopment Analysis - a new methodology for measurement of relative technical efficiency.

Data Envelopment Analysis generalizes the usual output to input ratio measure of technical efficiency in terms of a fractional linear program which can be summarized as follows [5] [6].
Objective:

$$\max h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

where \( o \) is the Decision Making Unit (DMU) being evaluated in the set of \( j = 1, \ldots, n \) DMU's.

Constraints:

Less than \( Unity \):

$$1 > \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} ; \quad j = 1, \ldots, n$$

(1)

Positivity Constraints:

$$0 < u_r ; \quad r = 1, \ldots, s$$

$$0 < v_i ; \quad i = 1, \ldots, m$$

Data:

Outputs:

$$y_{rj} = \text{observed amount of } r^{th} \text{ output for the } j^{th} \text{ DMU}$$

Inputs:

$$x_{ij} = \text{observed amount of } i^{th} \text{ input for the } j^{th} \text{ DMU}$$

The data used for each DMU are the \( y_{rj} \) outputs; and the inputs, \( x_{ij} \).

The \( u_r, v_i \) values are objectively determined from the data in terms of the above model. DEA is designed for an \textit{ex post} evaluation of how efficient each DMU was with the actual inputs \( x_{ij} \) used to produce its outputs \( y_j \) without explicit knowledge of the input-output relationships it used. The weights in the form of the \( u_r \) and the \( v_i \) are also not known or given \textit{a priori}. They are, instead, calculated as \( (u_r, v_i) \) values to be assigned to each input and output in order to maximize the efficiency rating -- \( h^*_o \) -- of the DMU being evaluated. That is, the solution sought is the set of \( (u_r, v_i) \) values that will give the DMU being rated the highest efficiency ratio, \( h^*_o \), but not result in an input-output ratio exceeding 1 when applied to any DMU in the observation set.
Applying DEA to a set of DMU's results in an efficiency rating for each DMU of 1 (relatively efficient) or less than 1 (relatively inefficient). These ratings, however, represent relative efficiencies obtained from a piecewise linear production possibility frontier comprised of the most efficient units in the set of \( j = 1, \ldots, n \) DMU's.

The above formulation, involves a nonlinear-nonconvex programming problem. As is shown in [5] however, it may be restated into a dual linear programming problem (which can be used with any linear programming computer code) as follows:

\[
\begin{align*}
\text{Max} \quad & h' = \sum_{o} s \omega y_{ro} \\
\text{Subject to} \quad & 0 > \sum_{r=1}^{s} \mu_{y_{rj}} - \sum_{i=1}^{m} \omega_{i} x_{ij} ; \quad j = 1, \ldots, n \\
& l = \sum_{i=1}^{m} \omega_{i} x_{i0} \\
& o < \mu_{r}, \omega_{i} ; \quad r = 1, \ldots, s \quad i = 1, \ldots, m
\end{align*}
\] (2)

In order to include the condition that \( \mu_{r} \) and \( \omega_{i} \) are strictly greater than zero, it requires that

\[
\epsilon < \mu_{r}, \epsilon < \omega_{i} \quad \text{all} \quad r \quad \text{and} \quad i
\] (3)

where \( 0 < \epsilon \) is a positive constant which is so small that it cannot otherwise disturb any solution involving only real numbers. (Choice of an \( \epsilon \) value is further discussed in [6] [14]).

Note that to apply DEA, we only need to identify and obtain the relevant set of output and input data and that each output and input need only be measured in its natural physical units without the need to use a homogeneous measurement unit like dollars.
3. Application of DEA to a set of artificial DMU's

A set of artificial DMU's where a simple set of efficient production relationships was hypothesized to examine the capabilities of DEA. A set of fifteen DMU's was created, each of which uses three inputs to produce various levels of 3 outputs. Seven of these DMU's used the exact amount of the three inputs to jointly produce these three outputs as was required by the hypothesized efficient production function. Eight of these DMU's used more inputs than was required by the efficient production function, i.e., these eight DMU's were known to be inefficient. For convenience, the DMU's were given output and input names as would be found in a hospital. The artificial hospital data consisted of the following three outputs produced with the following three inputs during a one year period of time.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$ Regular patients treated per year (Reg. Pat.'s)</td>
<td>$x_1$ Full time equivalent staff used per year (FTE's)</td>
</tr>
<tr>
<td>$y_2$ Severe patients treated per year (Sev. Pat.'s)</td>
<td>$x_2$ Number of Hospital bed day available per year (Bed days)</td>
</tr>
<tr>
<td>$y_3$ Teaching of residents and interns in one year (Teach units)</td>
<td>$x_3$ Supplies in terms of dollars cost per year (Supply $'s)</td>
</tr>
</tbody>
</table>

The hypothesized efficient amount of inputs ($x_1$, $x_2$, $x_3$) required for each output ($y_1$, $y_2$, $y_3$) are reported in Exhibit 1 in the appendix. Exhibit 2 in the appendix reflects the data base that was developed and the inefficiencies that were introduced in hospitals H8 through H15. Exhibit 3 is an example of how the data in Exhibit 2 was developed and how it reflects the efficient production relationships hypothesized in Exhibit 1. Exhibit 3, for example, compares the development of data for efficient hospital H1 and inefficient hospital H15. We began by arbitrarily selecting output levels...
(y₁, y₂, y₃) for each hospital and arbitrarily adding inefficiencies in hospitals H8 through H15. H15 has the same output levels as H1 (see Exhibits 2 and 3), but H15 uses an inefficient level of FTE's (x₁), bed days (x₂), and supplies (x₃) in providing treatment for regular patients (y₁). The inefficiencies introduced are highlighted by the circled values in Exhibit 2. (The development of the data base is further discussed in [13] and [14]).

The efficiency rating results from DEA are summarized in table 1. Note that all the efficient DMU's (hospitals H1 - H7) were accurately located as relatively efficient (efficiency rating = 1.0). Among the inefficient DMU's (hospitals H8 - H15), six of these were accurately identified as inefficient and two were not located as inefficient (H10 and H13). This result illustrates the primary strength and limitation of DEA. All inefficient DMU's located will be strictly inefficient, i.e., there are other DMU's in the observation set which are more efficient than those identified as inefficient via DEA. At the same time, DEA will not necessarily locate all the inefficient units, because it is locating inefficiency by comparing DMU's within the entire data set. Indeed, there may be no technically efficient units in the data set. In the example in table 1 we know that H1 - H7 are technically efficient but we would not generally know this when using a technique like DEA, as the class of DMU's where DEA may be most helpful are those where the production function is not well defined. If the production function were known, other techniques for evaluating efficiency would be more direct and more appropriate.
<table>
<thead>
<tr>
<th>Efficient DMU's</th>
<th>DEA Efficiency Rating (E)</th>
<th>Efficiency Reference Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H3</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H4</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H5</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H6</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H7</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inefficient DMU's</td>
<td>DEA Efficiency Rating (E)</td>
<td>Efficiency Reference Set</td>
</tr>
<tr>
<td>H8</td>
<td>0.99</td>
<td>H4</td>
</tr>
<tr>
<td>H9</td>
<td>0.98</td>
<td>H1, H2, H6</td>
</tr>
<tr>
<td>H10</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H11</td>
<td>0.85</td>
<td>H4, H7</td>
</tr>
<tr>
<td>H12</td>
<td>0.99</td>
<td>H1, H4, H6</td>
</tr>
<tr>
<td>H13</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>H14</td>
<td>0.99</td>
<td>H1, H4, H6</td>
</tr>
<tr>
<td>H15</td>
<td>0.87</td>
<td>H4, H6, H7</td>
</tr>
</tbody>
</table>

-9-
The value of DEA with respect to managerial audits is that the units located as being inefficient are clearly inefficient compared to other DMU's in the data set and hence, a transfer of techniques from the more efficient units to the less efficient units can potentially improve the efficiency of the inefficient DMU's. In addition, DEA provides information which helps locate the inefficiencies. For example, H11 is found to be inefficient (efficiency rating is .85 which is less than 1.0) and DEA also indicates that the inefficiency is located most directly by comparison with the DMU's efficiency reference set, H4 and H7 as indicated in table 1. Hence, the auditor knows that H11 is inefficient and that its inefficiency can be studied most directly by comparing the H11 operation with those of the relatively efficient DMU's, H4 and H7. In this way, DEA focuses the search for sources and remedies for inefficiency.

One issue is whether other analytic techniques are already adequate and which may render DEA superfluous. Using this same artificial data set, Sherman [14] and [13] illustrates that the use of ratio analysis and regression econometric techniques were less powerful, and in some cases, misleading in identifying inefficiencies and efficient production relationships. In addition, other approaches do not provide insights into the magnitude of the inefficiency as is available with DEA.

Beyond the location of inefficient units, DEA provides the type of information illustrated in table 2 for the inefficient DMU, H11. This indicates how H11 is inefficient compared with H4 and H7 and the reduction of inputs that would make H11 efficient. DEA indicates that a weighted combination of H4 and H11 would result in a DMU that yields as much or more outputs as H11 but which uses less inputs (see Column E in table 2). These weights are shadow prices directly available from the DEA - Linear program results and are further described in [14] and [13]. Hence we learn from DEA
Table 2 - Hospital 11 DEA Results

<table>
<thead>
<tr>
<th>Input Output Vector</th>
<th>Objective Hospital H11</th>
<th>Efficiency Reference Set H4</th>
<th>Efficiency Reference Set H7</th>
<th>Efficiency Reference Set *(1.43)(B) + *(0.071)(C)</th>
<th>H11 Compared with composite (D) - (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_1$ FTE's</td>
<td>44.5</td>
<td>25</td>
<td>51.5</td>
<td>39.4</td>
<td>5.1</td>
</tr>
<tr>
<td>$x_2$ Bed Days</td>
<td>65,260</td>
<td>41,050</td>
<td>92,630</td>
<td>65,260</td>
<td>0</td>
</tr>
<tr>
<td>$x_3$ Supply $'$s</td>
<td>$265,000</td>
<td>$140,000</td>
<td>$270,000</td>
<td>$219,290</td>
<td>$45,710</td>
</tr>
<tr>
<td>Outputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_1$ Teaching</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>146.4</td>
<td>96.4</td>
</tr>
<tr>
<td>$y_2$ Severe Patients</td>
<td>3,000</td>
<td>2,000</td>
<td>2,000</td>
<td>30,000</td>
<td>-</td>
</tr>
<tr>
<td>$y_3$ Regular Patients</td>
<td>5,000</td>
<td>3,000</td>
<td>10,000</td>
<td>5,000</td>
<td>-</td>
</tr>
</tbody>
</table>

*shadow prices from linear program - DEA results (see [ ] detailed discussion of these shadow prices).
that Hll is inefficient and the amount of potential input reduction may be possible without decreasing its output levels. The auditor may choose to study Hll further to determine the reason for these inefficiencies, ways these inefficiencies can be reduced, or whether there are policy reasons for maintaining slack resources.

Based on our knowledge of the efficient output-input relationships, the information provided in table 2 must also be qualified with respect to the degree to which it can be literally interpreted. DEA results in table 2 directly indicate that a combination of two DMU's operating results would produce a composite DMU that is more efficient than Hll. This indicates one way for Hll to become more efficient. This is not, however, the only direction that Hll can choose or should choose to improve efficiency. For example, Hll was inefficient with respect to its use of FTE's and supplies used to provide severe and regular patients care (see Exhibit 2). The adjustment required to make Hll efficient based on the known production function differs from the adjustment suggested in table 2 as is indicated in table 3 below:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual Inputs and outputs of Hll</td>
<td>Efficient Inputs for Hll based on known production function</td>
<td>Adjustment Required to become efficient (B - A)</td>
<td>Adjustment indicated from table 2 based on DEA results</td>
</tr>
<tr>
<td>x₁</td>
<td>44.5</td>
<td>36.5</td>
<td>-8</td>
<td>-5.1</td>
</tr>
<tr>
<td>x₂</td>
<td>65,260</td>
<td>65,260</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x₃</td>
<td>265,000</td>
<td>200,000</td>
<td>-$65,000</td>
<td>-45,710</td>
</tr>
<tr>
<td>y₁</td>
<td>50</td>
<td></td>
<td></td>
<td>+96.4</td>
</tr>
<tr>
<td>y₂</td>
<td>3000</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>y₃</td>
<td>5000</td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3 compares the true adjustment required by the efficient production function for H11 to become efficient (Column C) with the DEA results (Column D). Column C in table 3 reflects the adjustments required for H11 to exactly fit the efficient output-input relationships we hypothesized. The two sets of adjustments in Columns C and D are both mathematically accurate ways for H11 to become efficient. Note however, that only the solution in Column D is available directly from DEA and that the "true" solution in Column C is not available. In addition, it may be impractical or impossible to make the adjustments suggested by Column D in table 3, i.e., can H11 actually increase teaching outputs to nearly three times its recent level? This suggests that the manager might use the DEA results to identify the presence of inefficiency and specific areas where the inefficiencies may lie. The DEA results also suggest alternative paths to improve efficiency, e.g., in the above case, H11 could either emulate H4 or H7 or it could aim for the composite input-output level suggested in table 3. The identification of the preferred and attainable paths to improve efficiency would naturally be based on managerial judgment. Should this lead to proposed paths that differ from the one derived from DEA, it is also possible to reapply DEA for a sensitivity analysis to determine if other paths proposed by management would improve the efficiency compared to the other DMU's in the data set.

4. Field Application of DEA to teaching hospitals

DEA was used to evaluate efficiency a group of seven teaching hospitals in Sherman [14]. The seven hospitals in the study were within a group of teaching hospitals that were identified as being "comparable" based on cluster analysis completed by the state rate setting agency and based on further review and adjustment which also reflected the opinion of hospital administrators about what constitutes "comparable" groupings. The state auditors developed a set of ratios to identify hospitals whose operating costs
were at a level which suggested that they were excessively costly and possibly poorly managed. Hospitals falling into this "relatively inefficient" category were then subject to an in-depth audit to determine if these costs were justifiable [10].

DEA was applied solely to the medical surgical area of hospitals using the following outputs produced with the following inputs during one year.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$ Patient days of care - age $&gt; 65$</td>
<td>$x_1$ Full time equivalent of nonphysician medical surgical staff</td>
</tr>
<tr>
<td>$y_2$ Patient days of care - age $&lt; 65$</td>
<td>$x_2$ medical surgical staff supply dollars</td>
</tr>
<tr>
<td>$y_3$ Number of nursing students in training during year</td>
<td>$x_3$ Bed days available</td>
</tr>
<tr>
<td>$y_4$ Number of interns and residents in training during the year</td>
<td></td>
</tr>
</tbody>
</table>

These outputs and inputs were selected in cooperation with a panel of hospital experts to reasonably represent the key inputs and outputs in this part of the hospital as described in [14]. The data was collected from the 1976 reports submitted to the state regulatory agency by each hospital.

The DEA results identified two of the seven hospitals as inefficient. There exists no objective rating of hospital operations covering these hospitals. Hence, to verify these results, a panel of hospital experts familiar with these hospitals was asked to evaluate the DEA results. In addition, the results were discussed with management of hospital D, one of the two inefficient hospitals. These procedures (fully described in [14]) confirmed that the DEA results reflected real inefficiencies and in hospital D, for example, DEA specifically located an inefficient level of bed days and personnel used to produce the given level of outputs.

The experts agreed that the two hospitals identified were likely to be relatively inefficient and that the DEA results were reasonable. Management
of hospital D reviewed the results and agreed that their personnel levels and bed days available were somewhat higher than comparable hospitals. Hospital management reduced its bed days but has maintained its high personnel level for internal policy reasons.

The value of DEA in this context is best illustrated by comparison with the auditor's system of locating potentially inefficient hospitals. Table 4 summarizes the DEA results and lists two of the key ratios used by the auditor to locate hospitals that were potentially inefficient and, therefore, candidates for further review. The state audit agency had only identified hospital D as potentially inefficient because it was the only hospital in this group with costs over one standard deviation above the mean.

Table 4

Comparison of teaching Hospitals' Medical Surgical area

<table>
<thead>
<tr>
<th>Hospital**</th>
<th>DEA Efficiency Rating</th>
<th>Efficiency Reference Set (ERS)</th>
<th>MS Cost Per Patient Day</th>
<th>MS Cost Per Patient</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>-</td>
<td>$34</td>
<td>$408</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>-</td>
<td>38</td>
<td>418</td>
</tr>
<tr>
<td>C</td>
<td>1.0</td>
<td>-</td>
<td>39</td>
<td>429</td>
</tr>
<tr>
<td>D</td>
<td>0.88</td>
<td>A, D, 0</td>
<td>32</td>
<td>407</td>
</tr>
<tr>
<td>E</td>
<td>1.0</td>
<td>-</td>
<td>27</td>
<td>343</td>
</tr>
<tr>
<td>F</td>
<td>1.0</td>
<td>-</td>
<td>29</td>
<td>348</td>
</tr>
<tr>
<td>G</td>
<td>0.93</td>
<td>0</td>
<td>36</td>
<td>324</td>
</tr>
</tbody>
</table>

Average Cost $34.29 $382.43
Standard Deviation 4.27 42.51

The cost per patient day and cost per patient. DEA did not locate D as inefficient; however, it accurately identified hospital D and G as inefficient, an
insight not available to the auditor using financial ratios. Although these two hospitals were not the most costly, there still existed inefficiencies located by DEA which, if remedied, could make them less costly. Thus, DEA captures efficiency dimensions not covered in the financial ratio/analytic review technique used by the auditors. These inefficiencies would not generally be located through the use of financial ratios because such ratios cannot consider multiple outputs simultaneously. For example, a hospital that provides a high volume of teaching services would have higher operating costs which would translate into augmented patient cost. Teaching outputs and patient care are jointly produced outputs with the same set of inputs and the costs cannot readily be separated to determine the cost per teaching output versus the cost per patient. Hence, the ratio used tends to ignore certain outputs which, in this case, is the teaching output.

5. Use of DEA to evaluate efficiency of for profit organizations

A study of savings and loan bank branches suggests that DEA may be useful for managerial audits in a for profit setting within a single business. Sherman and Gold [15] applied DEA to the fourteen branches of a savings bank to attempt to locate relatively inefficient branches. While the savings bank had certain profitability criteria, it had no clear concept of the amount of resources that were required to produce the transactions that branches were responsible for. Many bank transactions require resources that were not directly correlated with profit measures. For example, the withdrawal of funds from a savings account reduces the funds available for loan or investment and, therefore, reduces profitability. Nevertheless, withdrawals are in a sense one of the outputs for which a branch is responsible, and if this and other transactions could be completed more efficiently, the resource cost might be reduced and profits increased. The branch profit measure used
by the bank was not sensitive enough to address the transaction efficiency so the DEA approach was applied to evaluate its potential in this setting.

Based on discussions with bank management, the inputs were defined to be full-time equivalents of branch personnel, supply costs, and rent. Rent was used as rough surrogate for square feet of space due to unavailability of the latter. (This was understood to be a surrogate for certain branches and visual tests of the results were performed to assure that this did not bias the results). The outputs were developed by segregating the numerous transactions completed at a branch into four levels of complexity such that the most complex transactions were those which were likely to be most time consuming. (See [15] for details.)

Of the fourteen branches, six were found to be inefficient using DEA. One of these six was closed by management because it was not believed to be yielding adequate profits, without regard to the DEA results. Four of the six were believed to be relatively weak in operations and, therefore, the DEA results were consistent with management's "gut feel" but much more explicit. The other inefficient branch was believed to be a strong branch and management is investigating this to determine why the highly profitable branch is also technically inefficient. In contrast to the above examples, this study did not attempt to validate the DEA results, but rather assumes DEA to be reliable.

This application of DEA indicates how an internal audit group might adopt DEA to evaluate business operation where there are several offices with similar outputs operating in different locations, with different output levels, and with different mixes of outputs and inputs. Other examples where DEA may be useful are commercial bank branches, insurance claims offices, insurance brokerage offices, customer service networks supplying field service
networks to support the parent company products, or multiple offices of a CPA firm.

One may question whether a (for profit) business should use DEA, when the measures like profitability and return on investment are available and are widely acknowledged as key indicators of performances in contrast to the nonprofit case where such measures are less relevant. There appears to be at least two justifications for use of DEA in the for profit context. First, the financial ratios and analytic review techniques are highly dependent on use of dollar measures which can be biased because of inflationary factors, regional price differences, or methods of accounting for costs. DEA provides a means to evaluate performance based on physical units of inputs and outputs and can, therefore, suggest which DMU's are efficiently using their inputs without respect to the price they pay for these inputs and without regard to the cost accounting methods applied. As was illustrated in the hospital application, financial ratios may provide valid insights about profitability and costs, but DEA can provide additional insights about inefficiencies which, if remedied, may further increase profitability through improved efficiency.

The second reason DEA seems appropriate in the for profit setting is that the profit and ROI measures tend to reflect current operation and may even be penalized by expenditures for training, maintenance and repair, research and development which are incurred to promote future profitability. These other expenditures may result in outputs which are not reflected in current sales and are, therefore, not considered in the profit measure. These same outputs can, however, be included in a DEA evaluation. The application of DEA to evaluate savings bank branches represents the first application to a (for profit) business and it served to clarify certain problems and benefits that can be derived from such application. The true potential will be better
understood in application to other business organizations such as are listed above and which applications are now in the formative stage.

6. Conclusion - DEA as a Managerial audit tool

Based on the studies described in this paper and on the theoretical development of DEA in [4] [5] and [6], DEA appears to be a technique that is well suited for efficiency evaluations in certain managerial audit contexts. Our findings also suggest that DEA results are valid but with important qualifications which need to be considered in interpreting the results. I first summarize where DEA appears to be a useful managerial audit tool and follow with a summary of the strengths and limitations observed in the DEA results. Finally, areas of further research to improve and expand the usefulness of DEA for managerial audits are proposed.

How DEA can be used for Managerial Audits

DEA has been shown to be capable of achieving the following objective:

Location of relatively inefficient DMU's among a set of DMU's that use multiple inputs to produce multiple outputs where the efficient production function is not known, i.e., where inputs and outputs can be observed and measured, but the efficient amount of inputs required for each output is not known or readily determinable.

This type of information may be useful in a managerial audit context for two purposes: 1) audit resource allocation and 2) analytical review of operations.

Audit resource allocation. DEA can identify the relatively inefficient DMU's which are most likely to benefit from deeper investigation to pinpoint the source of these inefficiencies as a basis for developing recommendations for their reduction. In this context, DEA is an attention direction tool to allocate the limited managerial audit resources to areas where operating inefficiencies are known to exist.
Analytic review of operations. Where auditors are evaluating a set of DMU's to identify ways in which operations can be improved, DEA can be used to locate the relatively more and less efficient DMU's as a basis for studying and understanding the managerial techniques used in more efficient units and transferring these techniques to less efficient units.

As suggested in section 4, DEA can serve as a useful complement to traditional financial ratio analysis, and that it may prove more powerful than ratio analysis where the simultaneous consideration of multiple outputs and inputs is required for an efficiency assessment to achieve either of the above managerial audit purposes.

DEA is particularly applicable to public sector and nonprofit organization applications because they not only have multiple outputs and inputs, but also because their outputs can often be measured only in terms of physical units and, therefore, are not readily convertible to dollar output measures as are needed for many financial ratios. DEA is also of potential value in a for profit setting where either of the following conditions exist:

1. Outputs produced by a firm are not adequately incorporated into the profit measure because they represent investments in people or products that are expected to improve the future earning power of the firms, e.g., employee training, product development, customer relations development. In this case, DEA may serve as a performance measure that reflects performance related to a longer time period than that reflected in the profit and ROI measures.

2. The desired evaluation of firms performance with financial ratios does not adequately consider their use of physical inputs to produce their outputs. The savings bank example discussed in section 5 is an example of how DEA can focus on operating efficiency as distinct from overall profitability as a way of locating operating improvements that may ultimately lead to increased profitability.
Strengths and Limitations of DEA

DEA has been shown to be theoretically sound and consistent with economic theory [4] and [5]. In addition, tests of DEA suggest that it provides reliable information in an application to a relatively uncomplicated hypothesized set of production relationships and for a set of teaching hospitals. While these tests of DEA and the underlying theoretical soundness do not necessarily assure its reliability in all field applications, they do provide insights not readily available from a study confined to the technique and its mathematical underpinning. These findings may be summarized as follows:

- DMU's identified as inefficient with DEA are strictly inefficient compared with other DMU's in the data set. In addition, when inefficient DMU's are located, they are found to be inefficient with respect to a narrower set of relatively efficient DMU's which helps to focus the investigation into the source and nature of the inefficiencies.

- DEA will not necessarily locate all the relatively inefficient units and at this point, we have no reliable method of determining how comprehensive DEA is in locating inefficient DMU's.

- DEA identifies alternative paths for improving the efficiency of an inefficient DMU up to the level of the relatively efficient DMU's in the data set. DEA does not, however, identify the one path that will move the inefficient DMU to the underlying efficient production relationship. Hence, managerial judgment is required to assess the improvement paths which are most appropriate for a particular DMU.
DEA has been tested using physical output and input measures and consequently has been limited in its use to assessment of technical output-input efficiency. Hence, when inefficiencies are identified, DEA indicates that the same outputs could have been produced with fewer inputs than were used. DEA does not, however, address issues of whether a firm is purchasing inputs at the lowest price or whether the input mix results in the lowest cost of producing a good or service. While it may well be possible to incorporate relative prices into the DEA model, this has not yet been tested and requires further research. (In addition, the use of nonphysical input and output measures may further weaken the results. For example, use of an index of quality or complexity of outputs as an output measure has been shown to lead to misleading results in [15]).

In general, the use of DEA for managerial audit purposes should be limited to the identification of inefficient DMU's and the general magnitude of these inefficiencies based on use of physical output and input measures. Before expanding the use of DEA beyond this for managerial audit purposes, these other uses need to be carefully researched for their theoretical soundness as well as for their performance in field tests.

Future research into use of DEA as a managerial audit tool

While much work is in process to expand the underlying DEA model as reflected for example in [4] and [5], there are a number of application-oriented areas of research which may help to improve DEA's capabilities in managerial audit applications which are briefly described below.

Output and Input identification and measurement

The procedures for identifying relevant outputs and inputs for a DEA efficiency assessment are critical to the relevance of the results. When
relevant outputs and inputs are either ignored or unmeasurable, the DEA results can be biased and possibly misleading. Further research to understand how to effectively identify and measure these outputs and inputs and the effects of errors in omission is needed. This will be particularly important in extending DEA to the for profit setting where the multiple outputs may have not been readily identified and measured in physical terms, due to the heavy reliance on financial measures of performance.

**Introduction of relative prices** - Testing of DEA with the introduction of relative price constraints in the linear program model may extend the results to evaluate price efficiency and input mix (allocative) efficiency as well as the already encompassed technical efficiency. This would expand the capabilities of DEA in the nonprofit applications and may make DEA substantially more attractive and powerful in for profit application where information about relative competitive market prices is readily available.

**Use of DEA in a time series as well as cross-section application.** The DEA applications to date concentrate on a cross-section of DMU's operating in one period. It may be possible to use multiyear or multiquarter data to detect trends in relative efficiency over time which may sharpen the managerial audit applications. For example, it may be possible to identify DMU's that were relatively efficient and which have recently become relatively inefficient and vice versa which would allow the audit to focus on dynamic as well as static sources of inefficiencies.

In conclusion, existing tests of DEA support its use in managerial audits of organization efficiency when interpreted in the ways suggested in this paper. While greater insights may also be available from DEA, these insights must be carefully subject to theoretical and field tests of the validity.
REFERENCES


