A Dynamic Model of the
Regulated Firm under Uncertainty*

by

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A Dynamic Model of the
Economic Effect under Uncertainty

Part I: Introduction
A Dynamic Model of the Regulated Firm

Under Uncertainty

1. Introduction

Much of the work done in recent years on the theory of the regulated firm has been influenced by the work of H. Averch and L.L. Johnson. The Averch-Johnson model (the A-J model) employs a static framework for a monopoly firm producing a single homogeneous good, and facing perfect markets for the two inputs (labour and capital) it uses. The major proposition emerging from this model, is that when the monopoly firm is subjected to rate-of-return regulation, it uses a capital-labour ratio higher than the one that minimizes cost for the level of output it elects to produce. This result and several corollaries have been extended and clarified by several authors.

The A-J model has been extremely fruitful in providing analytical insights, and has the added advantage of being simple. However, the simplicity is achieved at the expense of omitting some of the important aspects of regulatory practice. Many of these abstractions from reality have been

1. In (1).
2. See Baumol and Klevorick (6) for a good survey of the literature. A more complete and up to date exposition is provided by Bailey (2).
pointed out earlier and a few have been incorporated into the analysis by other researchers e.g. problems of regulatory lag, \(^3\) the choice of a different objective function such as revenue maximization, \(^4\) and the implications for maximization of social-welfare. \(^5\) However, several important aspects of the regulatory problem have received relatively limited attention in the literature.

The A-J model and its extensions ignore the problem of uncertainty. With the exception of the work of Myers and Leland, \(^6\) the received theory has surprisingly little to say on the subject. The firm in the A-J model does not face the problems of uncertain input prices or unpredictable demand. When the demand is uncertain, the firm is faced with the problem of adjusting its combination of inputs so as to satisfy demand, provided, of course, it is profitable to do so. This would require us to distinguish between relatively more flexible factor, labour, and the durable input, capital; this, in turn, has implications for the allocation decision. \(^7\)

The static view of the regulatory problem taken by A-J ignores an important regulatory issue. In the A-J model,

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3. See Bailey and Coleman (3).
4. See Bailey and Malone (4).
5. For instance, Kleverick (10) and Sheshinski (15).
6. In (13) and (14), and (12) respectively.
7. The effect of the greater flexibility of labour relative to capital, on the allocation decisions of the monopoly firm, will be the subject of a separate paper.
the firm remains continuously in the optimum position, once it has chosen its inputs and consequently output and price. There is no scope for changing demand conditions through time and technological change. A related issue is the nature of the regulatory constraint. The A-J assumption that the fair rate-of-return constraint is binding, implies that the firm earns exactly the fair rate-of-return on its rate base at all points in time. This is clearly inconsistent with the observed behaviour of regulated firms, some of which do earn more than the allowed rate-of-return, causing regulators to take appropriate action by reducing prices, while others earn less than the fair rate and consequently petition for a rate increase.

An important feature of the actual regulatory process not captured by the A-J version, is that regulators fix the price the firm can charge based on the rate-of-return criteria, rather than specifying an explicit rate-of-return constraint. Adjustments of the price downwards or upwards, are undertaken on the basis of the past profit performance of the firm i.e., whether or not it has earned more or less than the allowed rate-of-return. Thus, regulators act

8. A detailed discussion is provided by Klevorick (11).
9. This is a stylization of observed practice. The "reduction" process consists of denials of further proposed rate increases. In a period of rising factor costs, this in effect reduces price in real terms.
through the specification of the price that the firm can charge, rather than the rate-of-return it can earn. It should be noted that this mechanism is consistent with the fact that the rate-of-return constraint is not binding at all points in time, unlike in the A-J model.

The objective of this note is to study the behaviour of the regulated firm in a dynamic context. The model used here, attempts to capture the price-setting function of the regulators, by specifying a dynamic formula for rate revision. Uncertainty is explicitly introduced through the state-preference framework that is employed here. Thus, the uncertainty is idealized, since it is assumed that, there are as many independent securities available as the number of future contingencies, or, alternatively, there are securities for every possible future contingency (Arrow-Debreu securities). In either case, there exists a price today, for a unit of consumption received in particular future states of nature. While this is an oversimplification of the problem of uncertainty, it is hoped that some understanding can be gained of the behaviour of the regulated firm, in an uncertain setting.

This note studies the implications of such a model for the input and output decisions of the regulated firm. An analogue of the A-J theorem in the dynamic context, under uncertainty, is proposed and proved. The capital stock decisions of the regulated and unregulated firms are
compared, when regulation is "effective," i.e. the price set by the regulators is no more than what an unregulated monopolist would have chosen, under the same demand and cost conditions. The *ex post* efficiency of allocation of inputs of the two firms are compared.

Section 2 describes the firm's environment together with a characterization of the uncertainty. The regulator's actions are modelled in Section 3. The next section describes the regulated firm's problem and characterizes its optimal policy. Section 5 considers the unregulated firm's optimal policy and compares it with that of the regulated firm.

2. The Firm's Environment

In the state-preference framework employed here to describe uncertainty, conditions in the future are characterized by listing the possible "states of nature" that can occur in the future.\(^\text{10}\)

Out of the set of possible states of nature, only one can occur at a point in time. The realized values of the random variables are contingent on the state of nature occurring, and uncertainty of the variables under study can be described by listing all possible contingent values.\(^\Rightarrow\)

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10. For a detailed discussion, see Hirshleifer (7).
Thus, given that a particular state of nature occurs, the values of the relevant variables are known with certainty. By considering all future periods relevant for purposes of decision making, and the possible states of nature in each time period, one can obtain a complete picture of the uncertainty regarding the variables under study. The states of nature in the future are indicated by \((\theta, t)\) where \(\theta\) ranges from 1 to \(N_t\), and \(t\) from 1 to \(T\).

The regulated firm is assumed to be a monopolist producing a single homogeneous good. At the beginning of period \(t\), the regulatory commission fixes a price \(p_t\) on the basis of past experience, that the firm is allowed to charge. Given the price \(p_t\), the demand obtaining in state \((\theta, t)\) in time \(t\) is \(q_{\theta,t}(p_t)\). It is assumed that \(q_{\theta,t}'(p_t) < 0\). The revenue obtained by the firm, given that state \((\theta, t)\) occurs is \(p_t \cdot q_{\theta,t}\), if the firm satisfies the entire demand.

The firm's output in period \(t\) depends on the stock it has at the beginning of the period, \(K_t\), and the labor it hires during the period \(L_t\). It is assumed that decisions regarding the labor to be employed are completely flexible and can be made once the state of nature is revealed. The firm's production function remains stable through time.\(^{11}\)

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\(^{11}\) It would be straightforward to introduce a variable corresponding to technology (analogous to capital stock), as proposed by Kleverick (11), which results from investment in research (analogous to investment).
production function is written as

\[ Q = f(K, L), \quad K \geq 0, \quad L \geq 0. \]

(1) \[ \frac{\partial Q}{\partial K} > 0, \quad \frac{\partial Q}{\partial L} > 0, \quad \frac{\partial^2 Q}{\partial K^2} < 0, \quad \frac{\partial^2 Q}{\partial K \partial L} > 0, \quad \frac{\partial^2 Q}{\partial L^2} < 0. \]

Incorporating the usual assumptions of positive, but declining, marginal productivity of both factors, as well as increasing marginal productivity of one factor, when the other factor is increased. For purposes of our analysis, it is more convenient to express the firm's production opportunities, in terms of the labour-requirements function.

\[ L = L(Q, K) \quad Q \geq 0 \quad K \geq 0 \]

(2) \[ L_1 = \frac{\partial L}{\partial Q} > 0, \quad L_2 = \frac{\partial L}{\partial K} < 0, \quad L_{11} = \frac{\partial^2 L}{\partial Q \partial K} > 0, \quad L_{12} = \frac{\partial^2 L}{\partial Q^2} < 0, \quad L_{22} = \frac{\partial^2 L}{\partial K^2} > 0 \]

The firm faces perfect markets in hiring labour, and in adding to its stock of capital. The decision to be made by the firm at the beginning of period \( t \), is its level of investment \( I_t \); it has a capital stock of \( K_{t-1} \) at the end of period \( (t-1) \) which can be increased, but not decreased, at the beginning of period \( t \). It is assumed that capital

12. The properties of the labour-requirements function follow from those of the production function. For example,

\[ \frac{\partial^2 L}{\partial K \partial Q} = \frac{\partial}{\partial K} \left( \frac{\partial L}{\partial Q} \right) = \frac{\partial}{\partial K} \left( \frac{1}{\partial Q} \frac{\partial Q}{\partial K} \right) = -\frac{1}{(\partial Q/\partial K)^2} \frac{\partial^2 Q/\partial K \partial Q}{\partial Q} < 0 \]

\[ \frac{\partial^2 L}{\partial Q^2} = \frac{\partial}{\partial Q} \left( \frac{\partial L}{\partial Q} \right) = \frac{\partial}{\partial Q} \left( \frac{1}{\partial Q} \frac{\partial Q}{\partial K} \right) = -\frac{1}{(\partial Q/\partial K)^2} \frac{\partial^2 Q/\partial Q \partial K}{\partial Q} > 0 \]
does not depreciate.\(^{13}\) Hence, \(I_t > 0.\)

The firm starts in period \(t\) with a capital stock \(K_t\) and a price specified by the regulators of \(p_t\). \(^{14}\) The state of nature is revealed to be \((\Theta, t)\), the demand is \(q_{\Theta, t}(p_t)\) at the given price. The firm would have to hire labour \(X(q_{\Theta, t}, K_t)\) if it wished to satisfy the demand fully. However, for given \(K\) and \(p\) at high enough values of \(q_{\Theta, t}(p_t)\), it may not be profitable for the firm to hire the labour required to produce the quantity \(q_{\Theta, t}(p_t)\) at the fixed wage rate \(w\). This will happen when \(\frac{\partial X}{\partial q} > p/w\). Thus, the maximum profitable output is a function of \(K_t\) and \(p_t\), for given \(w\). This level is defined to be \(\bar{q}(K_t, p_t).\) \(^{15}\)

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13. It is relatively simple to incorporate a fixed depreciation rate for capital which would modify the constraint from one on net investment to one on gross investment. However, no additional insights would be forthcoming.

14. It is assumed in this model that the period between regulatory reviews is the same as the production period i.e., the period in which the capital investment decision is made. In many cases, the production period is shorter (e.g., one year) than the period between reviews (e.g., three years). In such a situation, the qualitative conclusions of the present model would be preserved. The regulated firm acts as a profit maximizer, taking price as given, between the regulatory reviews, but keeping in view the fact that capital stock decisions made in the interim will affect price specified by the regulators at the next review. The longer the period between regulatory reviews, relative to the production period, the less the regulated firm deviates from the behaviour of a firm that optimizes value, with given price. In the limiting case when the period becomes infinitely large, compared to the production period, the firm takes price as given and optimizes value. It should be noted that this is somewhat similar to the result of Bailey and Coleman (3), that the extent of overcapitalization of the regulated firm varies inversely with the regulatory lag.
The possible states of nature in period $t$ can thus be divided into two sets, one containing those states where demand is fully satisfied and the other including those where excess demand exists. Assuming that states of nature are arranged in the order of increasing demand, one can define a number $M_t$ corresponding to the state of nature in which demand is just met. The number $M_t$ in period $t$ is a function of $p_t$ and $K_t$. As the price $p_t$ increases, demand in all states of nature falls, and the maximum profitable output $\bar{q}(K_t, p_t)$ increases. Demand can be satisfied in more states of nature and the number $M_t$ thus increases. For increases in the capital stock $K_t$, the maximum profitable output increases.\[16,17\]

The "operating profit" in state $(\theta, t)$ is given by

$$p_t \theta - \omega Z(\theta_t, Q)$$

If demand is satisfied, i.e., $(\theta, t) \leq M_t$.

15. It is assumed that the wage rate $\omega$ is constant and certain through time. $\bar{q}(K, p)$ is a concave function of $K$ and $p$. At $\bar{q}$, $\delta Z(K, p)/\omega = p/\omega$. As price increases, $\delta Z(K, p)/\omega$ increases, but since $\delta Z(K, p)/\omega < 0$, $\bar{q}$ increases less than proportionately. As $K$ increases, $\delta Z(K, p)/\omega$ decreases, but since $\delta Z(K, p)/\omega < 0$, the increase in $\bar{q}$ to compensate for the increase in $K$ is less than proportionate.

16. It is assumed that the demand functions in the various states of nature do not intersect i.e. demand functions are monotonically increasing in the index of the state of nature, $\theta$, at every price.

17. As $K$ increases, $\delta Z/\delta K$ decreases in every state of the world, as $\delta Z/\delta K < 0$. As $\delta Z/\delta K$ increases with $\theta$ and consequently with the number of the state of the world, the value of $M_t$ increases so as to make $\delta Z/\delta K|_{\theta_t + 1} = p/\omega$.\[16,17\]
\[
\phi_{0,t} = \text{defined to be the market value as of time 0, the beginning of the first period, of a contract that pays one unit of consumption in state } (\theta, t). \text{ The value of contingent income in state } (\theta, t) \text{ as of the beginning of the decision horizon is given by }
\]
\[
\phi_{0,t} \left[ p_t \ q_{0,t} - \omega \ L(q_{0,t}, K_t) \right], \quad (\theta, t) \leq M_t
\]
\[
\phi_{0,t} \left[ p_t \ C(K_t, p_t) - \omega \ L(Q(K_t, p_t), K_t) \right], \quad (\theta, t) > M_t
\]

The present market value of the firm's net operating income in period \( t \) is given by
\[
\sum_{\theta=1}^{N_t} \phi_{0,t} \left[ p_t \ q_{0,t} - \omega \ L(q_{0,t}, K_t) \right]
\]
\[
+ \sum_{\theta=1}^{N_t} \phi_{0,t} \left[ p_t \ C(K_t, p_t) - \omega \ L(Q(K_t, p_t), K_t) \right]
\]

where \( N_t \) represents the state of nature with the highest demand. It is assumed that the price of a unit of capital stock is unity. Thus, all prices and costs are scaled to units of the price of capital.

3. The Regulatory Environment

The regulatory commission uses the latest available information on costs and demand and sets prices so that the firm's current rate-of-return - based on current capital
stock, demand and cost conditions - is reduced to the "fair" level. The rule followed by the regulators is that period t's output at prices fixed for period (t+1) will result in the firm's earning exactly the fair rate-of-return on the capital stock in period t. This is only an adjustment mechanism, and as such, the fair rate-of-return is not achieved at any point in time, except by chance. In order to implement this rule, the regulators examine the firm's costs, output, and capital stock in period t and set the price for period (t+1) according to the following formula.

18. This is a case of lagged regulation with the period of the lag being one period. Bailey and Colenan (3) show that the A-J effect of overcapitalization is diminished when the regulatory lag is introduced; the extent of overcapitalization varies inversely with the length of the lag. Baumol and Klevorick (6) sketch out the implications of regulatory lag, in a model where the firm makes decisions regarding capital, labour, and investment in research, to increase the stock of knowledge. In a model, where the period between reviews is stochastic, Klevorick (11) investigates the relation between the investments in capital stock and research, and the expected period between reviews. He concludes that without making detailed assumptions regarding the demand, production and regulatory reaction functions, it is not possible to predict the direction of the effect of regulatory lag, on the firm's allocation decisions. For other references on this question, see Ronbright (7), Baumol (5) and Wein (16). Also see footnote (14).

19. This is similar in spirit to the views of Baumol and Klevorick (6), regarding the working of regulation. To quote, "regulation proceeds, roughly by taking the latest information available on company costs and demands, and resetting prices so that the firm's current rate-of-return - based on its current capital stock, technology, demand conditions, and so forth - is reduced to the "fair" rate level." See also Klevorick (11), p. 58. and Davis (3), p. 271, for similar opinions.

Apart from the fact that, modelling the action of
where $p_{t+1}$ is the regulator's price for period $(t+1)$, $s$ is the fair rate-of-return, $q_t$ is the actual output in period $t$, and $w_X(q_t, K_t)$ represents the actual labour costs in period $t$. Thus $p_{t+1}$ represents the price that allows the firm to earn exactly a proportion $s$ of its capital stock $K_t$, provided, of course, that the output turns out to be $q_t$. Thus, the distribution of the price $p_t$ is given by

\[
\begin{align*}
\begin{cases}
  \frac{sK_t + w_X(q_t, K_t)}{q_t} & \text{with probability } \Pi_{\Theta, t} \text{ for } (\Theta, t) \leq t, \\
  \frac{sK_t + w_X(k_t, k_t^*)}{q_t} & \text{with probability } 1 - \sum_{\Theta = 1}^{H_t} \Pi_{\Theta, t}
\end{cases}
\end{align*}
\]

where $\Pi_{\Theta, t}$ is the probability of occurrence of state $(\Theta, t)$.  

Regulators as setting the price the firm can charge, rather than specifying a rate-of-return constraint, is more realistic, in the context of uncertainty, there is another argument in favour of price-setting regulation. Ex ante, the regulators cannot set the "fair" rate-of-return, since the rate-of-return is a stochastic variable. Setting the central parameter (say, the mean) does not precisely constrain the firm's actions; there is no way of determining ex post, whether the firm followed the regulator's rate-of-return formula. The firm can "cheat" by actually aiming ex ante for a higher rate-of-return, and claiming ex post that this was the result of an outcome on the positive side of the distribution of returns. This is similar to the "cheating" described in Myers (1974), p. 314. In price setting, the firm can still earn more or less than the "fair" rate-of-return. However, by constraining price explicitly, the regulators take away the degree of freedom the firm needs, in order to be able to "cheat."
In addition to this formula, if regulation is to be "effective," the regulated firm's price should not be greater than what an unregulated monopolist would charge. It will be assumed that the horizon of the firm in making its decision is $T$ periods.

4. The Firm's Decision

At the beginning of the first period, the firm's decision is to optimize the present market value of its "operating profits," net of the present market value of investment costs of the programme in succeeding periods. The firm's objective function is

$$\max I_T = \sum_{t=1}^{T} \left[ \sum_{\sigma=1}^{N} \phi_{\sigma,t} \left( P_t q_{\sigma,t} - \omega I (q_{\sigma,t}, K_t) \right) \right]$$

$$+ \sum_{\sigma=H_t+1}^{N} \phi_{\sigma,t} \left( P_t \bar{S}(k_t, p_t) - \omega I (\bar{S}(k_t, p_t), K_t) - \alpha_{t-1} I_t \right)$$

(8) where $\alpha_t = \sum_{\sigma=1}^{N} \phi_{\sigma,t}$, $\alpha_0 = 1$

such that $I_t = K_t - K_{t-1} \geq 0$

$$P_{k,t} = \begin{cases} \frac{sk_t + w I (q_{\omega,t}, K_t)}{q_{\phi,t}} & \text{with probability } \Pi_{\omega,t} \\ \frac{sk_t + w I (\bar{S}(k_t, p_t), K_t)}{\bar{S}(k_t, p_t)} & \text{with probability } 1 - \sum_{\sigma=1}^{\omega} I_{\sigma,t} \end{cases}$$

20. The relationship between $\Pi_{\omega,t}$ and $\phi_{\omega,t}$ is not one of strict proportionality, but is determined by the supply of securities and the preferences of the various participants in the market.
Define
\[ B_{0,t} (K_t, p_t) = p_t q_{0,t} - \omega L (q_{0,t}, K_t) \]
(9)
\[ B_t (K_t, p_t) = p_t \bar{q} (K_t, p_t) - \omega L (\bar{q} (K_t, p_t), K_t) \]
\[ p_{\Theta, t+1} = \frac{s K_t + \omega L (q_{0,t}, K_t)}{q_{0,t}} \]
\[ p_{t+1} = \frac{s K_t + \omega L (\bar{q} (K_t, p_t), K_t)}{\bar{q} (K_t, p_t)} \]

and substitute in the above. Assuming that the states of nature are finely enough divided, the summation signs can be replaced by integral signs. 21

The solution to the above problem can be obtained by dynamic programming. We define \( V_t (K_t, p_t) \) as the maximum present value of future cash flows discounted to the beginning of period \( t \), given that the state of the system at the beginning of period \( t \) is \( \{K_{t-1}, p_{t-1}\} \). \( V (K_{t-1}, p_{t-1}) \) is the maximum present value across states of nature, discounted to the beginning of period \( t \) of an optimal policy starting at time \( t \) with the price set for the period being \( p_t \), and the capital stock at the end of the previous period being \( K_{t-1} \). \( V (K_{t-1}, p_{t}) \) is defined recursively by the following equation.

21. The solution for an optimum in the discrete case is analogous to that in the continuous case. However, the necessary conditions in the latter case are less tedious to calculate and easier to interpret. While the "continuous" assumption is unrealistic in view of the delineation of states, the discrepancy is not large for finely divided states. It should be noted that the relations obtained in this case can be translated into the discrete case though the calculations are messy.
\[ V_t(K_{t+1}, \ell_t) = \max_{I_t \geq 0} \int_0^N \phi_{\theta, t} \left[ \beta_{\theta, t} (K_t, \ell_t) \right] d\theta \]

\[ + \int_0^N \phi_{\theta, t} \left[ \phi_{\ell, t} (K_t, \ell_t, t) \right] d\theta + \int_0^N \phi_{\ell, t} \left[ V_{t+1} (K_t, \ell_t, t+1) \right] d\theta + \int_0^N \phi_{\ell, t} \left[ V_{t+1} (K_t, \ell_t, t+1) \right] d\theta \]

\( \Phi_{\theta, t} \) is the market value as of the beginning of period 1 of a contract that delivers one unit (of consumption) in \((\theta, t)\), i.e., in state of nature \( \theta \) in period \( t \). \( \phi_{\theta, t} \) is the market value as of the beginning of period 1 of a contract that delivers one unit with certainty (in every state of nature) at the beginning of period \( t \). \( \phi_{\ell, t} / \phi_{\ell, t-1} \) is the market value as of the beginning of period \( t \) of a contract that delivers one unit in \((\theta, t)\) i.e., in state of nature \( \theta \) in period \( t \).

The first two terms in the above expression, represent the market values as of the beginning of period \( t \), of operating income across all states of nature in time period \( t \); the third term represents the investment costs incurred, and the last two terms are the present market values as of the beginning of period \( t \), of optimal policies from \((t+1)\) onwards, given that the state of the system at the beginning of period \((t+1)\), is inherited as a result of the present decision, and the outcome of the uncertainty in period \( t \).

It should be noted that \( V_{t+1} (K_t, \ell_t, t+1) \) represents the present market value as of the beginning of period \((t+1)\), and weighting it by \( \phi_{\ell, t} / \phi_{\ell, t-1} \) discounts it back from state \((\theta, t)\) to the beginning of period \( t \).

After substituting the reaction function of the regulators, the functional equation can be written as

\[ V_t(K_{t-1}, \ell_t) = \max_{K_t \geq K_{t-1}} \int_0^N \phi_{\theta, t} \left[ \phi_{\ell, t} (K_t, \ell_t) \right] d\theta + \int_0^N \phi_{\ell, t} \left[ \phi_{\ell, t} (K_t, \ell_t) \right] d\theta \]
Defining $X_t = K_{t-1}$, and dropping subscripts $t$ except where needed for purpose of clarity, we have

$$V_t(X,\rho) = \frac{\max}{K > X} \left( \int_{K}^{M(K,\rho)} B_t(K,\rho) d\theta + \int_{H(K,\rho)}^{N(K,\rho)} \frac{\partial \phi}{\partial \theta} V_{t+1}(K,\rho) d\theta \right)$$

(12a)

$$- (K - X) + \int_{1}^{M(K,\rho)} \frac{\partial \phi}{\partial \theta} V_{t+1}(K,\rho) d\theta + \int_{H(K,\rho)}^{N(K,\rho)} \frac{\partial \phi}{\partial \theta} V_{t+1}(K,\rho) d\theta$$

Define

(12b) $$H_t(K,\rho) = \int_{1}^{M(K,\rho)} \frac{\partial \phi}{\partial \theta} [B_t(K,\rho)] + \int_{H(K,\rho)}^{N(K,\rho)} \frac{\partial \phi}{\partial \theta} [\bar{B}_t(K,\rho)] d\theta$$

Thus,

$$V_t(K,\rho) = \frac{\max}{K > X} \left\{ X + H_t(K,\rho) \right\} = X + \frac{\max}{K > X} H_t(K,\rho)$$

If the optimum $K$ without the constraint that $I \geq 0$, happens to be less than $X$, then $K^* = X$. Hence, $K^* = \max_{K > 0} \{ X, K' \}$ where $K'$ solves the problem $\max_{K > 0} H_t(K,\rho)$.

In making its optimal decision, the firm selects $I_t^* = K_t^* - K_{t-1}$ to maximize $V_t(K,\rho)$. From an inspection of the expression for $V_t(K,\rho)$, it is clear that the firm's decision is not myopic. To determine its optimal investment for period $t$, the firm must know its investment in period $(t+1)$, as $V_{t+1}(\cdot)$ enters the optimization problem. $V_{t+1}(\cdot)$ in turn depends on $V_{t+2}(\cdot)$, and so on. While an actual problem can be solved using dynamic programming methods, it is difficult
to derive any analytical results without making further assumptions. In order to study the optimal decision of the firm in period $t$, the specific assumption made here is that the firm will undertake investment in both periods $t$ and $(t+1)$, in all states of nature i.e., $k_{t+1}^* \geq k_t$ and $k_t^* \geq k_{t-1}$. This assumption delinks the decision made in period $t$ from that in $(t+1)$, except to the extent that price in period $(t+1)$ is affected by investment in period $t$.

Thus, the problem of optimization of $V_t(X,p)$ becomes one of optimization of $H_t(K,p)$, while from the relation between $V_{t+1}(.)$ and $H_{t+1}(.)$, we can write

\[
V_{t+1}(K, p_{t+1}) = K + H_{t+1}(K_{t+1}^*, p_{t+1}, t_{t+1})
\]
\[
V_{t+1}(K, p_{t+1}) = K + H_{t+1}(K_t^*, p_{t+1})
\]

By definition,

\[
\int_0^{N_t} \phi_{0,t} \, dt = \alpha_t
\]

Substituting into $H(K,p)$, and collecting terms, we have

\[
H_t(K_t^*, p) = \max_{k_t \in X_t} \frac{\alpha_t - \alpha_{t-1}}{k_{t-1}} K_t + \int \frac{M(K_t, p)}{\alpha_t} \left[ \phi_0(K_t, p) \right] \, d\theta + \int \frac{N}{\alpha_t} \left[ \phi_1(K_t, p) \right] \, d\theta
\]

(15)

\[
+ \int \frac{\phi_0}{\alpha_{t-1}} H_{t-1}(K_{t-1}^*, p_{t-1}) \, d\theta
\]

For a maximum, we must have

\[
\frac{\partial H_t(K_t, p)}{\partial K_t} = \frac{\alpha_t - \alpha_{t-1}}{k_{t-1}} K_t + \int \frac{M(K_t, p)}{\alpha_t} \left[ \frac{\partial \phi_0(K_t, p)}{\partial K_t} \right] \, d\theta
\]

(16a)

\[
+ \frac{\partial M(K_t, p)}{\partial K_t} \frac{\partial M}{\alpha_{t-1}} \left[ \phi_0(K_t, p) \right] + \int \frac{N}{\alpha_t} \left[ \frac{\partial \phi_1(K_t, p)}{\partial K_t} \right] \, d\theta
\]

\[
- \frac{\partial M(K_t, p)}{\partial K_t} \frac{\partial M}{\alpha_{t-1}} \left[ \phi_0(K_t, p) \right] + \int \frac{N}{\alpha_t} \left[ \frac{\partial H_t(K_{t-1}^*, p_{t-1})}{\partial K_t} \right] \, d\theta
\]

\[
+ \frac{\partial M(K_t, p)}{\partial K_t} \frac{\partial M}{\alpha_{t-1}} \left[ H_{t-1}(K_{t-1}^*, p_{t-1}) \right]
\]
Multiplying by $\alpha_{t-1}$ throughout, and using $\bar{z}_1(0,K) = p/w$ and $\bar{M}(K,p) = \bar{B}(K,p)$, we have $p_{t+1} = p_{t+1}$, since $q_0 = \bar{q}(K,p)$ for $\theta = M$, we have

$$\left(\alpha_t - \alpha_{t-1}\right) - \int_0^M \phi_0 \left[ \omega \bar{z}_2(q_0,K) \right] d\theta - \int_0^N \phi_0 \left[ \omega \bar{z}_2(\bar{z}(K,p),K) \right] d\theta$$

$$+ \int_0^M \phi_0 \frac{\partial H_{t+1}(K_t,\bar{z}_{t+1})}{\partial p} \left\{ \Delta + \omega \bar{z}_2(q_0,K) \right\} d\theta$$

$$+ \int_0^N \phi_0 \frac{\partial H_{t+1}(K_{t+1},\bar{z}_{t+1})}{\partial p} \left\{ \frac{\partial \bar{z}_2(q_0,K)}{\partial K} + \omega \bar{z}_2(q_0,K) - \theta K + \omega \bar{z}_2(q_0,K) \right\} d\theta = 0$$

where $\bar{z}_1 = \frac{\partial \bar{z}}{\partial q}$, $\bar{z}_2 = \frac{\partial \bar{z}}{\partial K}$, $\bar{q}_1 = \frac{\partial \bar{z}}{\partial K}$. The sufficiency of this relation is difficult to establish without specifying the demand functions, the production possibilities and the uncertainty in greater detail. $(\alpha_t - \alpha_{t-1})$ represents the present value of the cost of adding a marginal unit of capital stock in period $t$, for one period. The second and third terms represent the reduction in labour costs discounted to present, due to a marginal increase in capital stock. The last two terms indicate the present values of changes in operating cash flows in future periods resulting from a change in the regulated price, which in turn result from a marginal increase in capital stock in period $t$.

5. The Unregulated Firm's Decision

The unregulated firm is free to choose both prices $p_t$. 
and investment for period $t$, $I_t$, at the beginning of period $t$, given the possible states of nature, the demand conditions, and the production possibilities. In making its optimal decision, the unregulated firm need not take into account its effect on the price in future periods as a regulated firm would. The unregulated firm can, thus, make decisions in a myopic fashion, provided, of course, that it would undertake positive investments in periods $t$ and $(t+1)$. The unregulated firm's problem is

\begin{equation}
\text{Max} \sum_{t=1}^{T} \left[ \sum_{\theta=0}^{N_0} \phi_{0,t}^{\theta} \left( b_{0,t}^{\theta}(K_t, \rho_t) \right) + \sum_{\theta=0}^{N_t} \phi_{0,t}^{\theta} \left( \bar{b}_t (K_t, \rho_t) \right) - \alpha_{t-1} I_t \right]
\end{equation}

where $\alpha_t = \sum_{\theta=1}^{N_0} \phi_{0,t}^{\theta}$, $\alpha_0 = 1$

such that $I_t = K_t - K_{t-1} \geq 0$

As in the case of the regulated firm, this is written as

\begin{equation}
\text{Max} \sum_{t=h}^{T} \left[ \int_{\mathcal{L}}^{N_t} \phi_{0,t}^{\theta} \left( b_{0,t}^{\theta}(K_t, \rho_t) \right) d\omega + \int_{\mathcal{L}}^{N_0} \phi_{0,t}^{\theta} \left( \bar{b}_t (K_t, \rho_t) \right) d\theta 
\right]
\end{equation}

where $\alpha_t = \int_{\mathcal{L}}^{N_t} \phi_{0,t}^{\theta} d\omega$, $\alpha_0 = 1$

such that $I_t = K_t - K_{t-1} \geq 0$
This is a dynamic programming problem to be solved recursively. We define \( W_t(K_{t-1}) \) as the maximum present value of future cash flows across states of nature discounted to the beginning of period \( t \), given that the capital stock at the end of period \( (t-1) \) is \( K_{t-1} \). The functional equation for this problem is defined by

\[
W_t(K_{t-1}) = \max_{P_t} \int \Phi_{\theta,t} \left[ \theta_{\theta,t} (K_t, \eta_t) \right] d\theta + \int \Phi_{\alpha,t} \left[ \beta_t (K_t, \eta_t) \right] d\omega \\
- (K_t - K_{t-1}) + \int \Phi_{\alpha,t} \left[ W_{t+1}(K_t) \right] d\omega
\]

(19)

where, as in equations (3),

\[
\Phi_{\theta,t} (K_t, \eta_t) = p_t \Theta_{\eta,t} - \omega \Xi (\eta_{\eta,t}, K_t) \\
\Phi_{\alpha,t} (K_t, \eta_t) = p_t \Xi (K_t, \eta_t) - \omega \Xi (\eta_{\eta,t}, K_t)
\]

In this case, the state of the system is defined by the capital stock alone, \( \{K_{t-1}\} \), rather than \( \{K_{t-1}, p_t\} \) as in the regulated case, since the unregulated firm is free to choose both \( p \) and \( K \). We define \( x_t = K_{t-1} \) and drop the subscripts \( t \) and rewrite \( W(K_{t-1}) \),

\[
W_t(x) = \max_{K \geq x, p_t} \int \Phi_{\theta,t} \left[ \theta_{\theta,t} (K_t, \eta_t) \right] d\theta + \int \Phi_{\alpha,t} \left[ \beta_t (K_t, \eta_t) \right] d\omega \\
- (K - x) + \int \Phi_{\alpha,t} \left[ W_{t+1}(K_t) \right] d\omega
\]

(20)

Defining \( J_t(K, p) \) as

\[
J_t(K, p) = \int \Phi_{\theta,t} \left[ \theta_{\theta,t} (K_t, \eta_t) \right] d\theta + \int \Phi_{\alpha,t} \left[ \beta_t (K_t, \eta_t) \right] d\omega \\
- \kappa + \int \Phi_{\alpha,t} \left[ W_{t+1}(K_t) \right] d\omega
\]

(21)

we can write \( W_t \) as
(22) \( W_t(X) = \max_{K \geq X} \left\{ X + J_t^*(K, p) \right\} = X + \max_{K \geq X} \left\{ J_t^*(K, p) \right\} = X + J_t^*(K, p) \)

If the optimum \( K \) without the constraint that \( I \geq 0 \) happens to be less than \( X \), then \( K^* = X \). Thus, \( K^* = \max\{X, K'\} \) where \( K' \) solves the problem \( \max_{K \geq X} J_t(K, p) \).

As in the case of the regulated firm, the firm's decision is not myopic, due to the restriction that investment has to be positive. We again make the assumption that the firm undertakes investment in both periods \( t \) and \( t+1 \), so that the problem of optimization of \( W_t(X) \) becomes one of optimization of \( J(K, p) \) and \( W_{t+1}(K) \) can be written as

(23) \( W_{t+1}(K) = K + J_{t+1}(K^*, p_t) \)

By definition, \( \int_0^{N_0} \phi \left( \theta, t \right) d\theta = \alpha_t \).

The firm's optimum strategy can be obtained by solving

\[
J_t(K^*, p^*) = \max_{K \geq X} \frac{\alpha_t - \alpha_{t-1}}{\alpha_{t-1}} K + \int_0^{\alpha_{t-1}} \frac{\gamma(K, p)}{\alpha_{t-1}} \left[ \beta_0(K, p) \right] d\theta
\]

(24)

\[
+ \int_{\alpha_{t-1}}^{\alpha_t} \left[ \beta_t(K, p) \right] d\theta + \int_{\alpha_t}^{N_0} \frac{\beta_t(K, p)}{\alpha_t} J_{t+1}(K^*, p^*) d\theta
\]

The first order conditions for a maximum are written as

\[
\frac{\partial J_t(K, p)}{\partial K} = \frac{\alpha_t - \alpha_{t-1}}{\alpha_{t-1}} + \int_0^{\alpha_{t-1}} \frac{\gamma}{\alpha_{t-1}} \frac{\partial \beta_t(K, p)}{\partial K} d\theta + \int_{\alpha_t}^{N_0} \frac{\beta_t(K, p)}{\alpha_t} \frac{\partial J_{t+1}(K, p)}{\partial K} d\theta
\]

(25a) = 0

or, \( \alpha_t - \alpha_{t-1} - \int_0^{\alpha_t} \beta_t[K, p] d\theta - \int_{\alpha_t}^{N_0} \beta_t[K, p] d\theta \)
The first order condition for capital stock implies, that the present value of the cost of adding a marginal unit of capital in period $t$, for one period, has to be set equal to the marginal decrease in labour costs as a result of this decision. There are no implications of the decision regarding capital stock for decisions in future periods (except insofar as capital stock cannot be reduced). The second equation indicates that the present value of marginal cost and marginal revenue have to be equated, across states. However, in periods where demand is not fully satisfied, demand is insensitive to small increases and hence, price equals marginal cost.

We now examine the capital and output decisions of the regulated and unregulated firms in order to compare them. The first order conditions for capital stock of the two firms, for a given price, are written as

\[(26b) \quad \frac{\partial T_t(K, C)}{\partial p} = \int_1^N \phi_0 \left[ - \frac{\partial \ln \phi_t(K, C)}{\partial p} \right] d\theta + \int_1^N \phi_0 \left[ \frac{\partial \ln \phi_t(K, C)}{\partial p} \right] d\theta = 0.\]

or,

\[\int_1^N \phi_0 \left[ q_0 + q_0'(r) \cdot p - \omega \chi_1(q_0, K) \cdot q_0'(r) \right] d\theta + \int_1^N \phi_0 \left[ \phi(t, K, p) \right] d\theta = 0.\]

The second equation indicates that the present value of marginal cost and marginal revenue have to be equated, across states. However, in periods where demand is not fully satisfied, demand is insensitive to small increases and hence, price equals marginal cost.

We now examine the capital and output decisions of the regulated and unregulated firms in order to compare them.
The last term in the equation for the regulated firm can be written as

\[ \int_{M(K,\rho)}^{N} \frac{\partial H_{t+1}(K_{t+1}^*, \tilde{r}_{t+1})}{\partial \rho} \left\{ \left( \frac{\dot{S}}{\dot{Q}} - \frac{\dot{S}K\tilde{Q}}{\dot{Q}^2} \right) + \left( \frac{\omega L\tilde{Q}}{\dot{Q}} - \frac{\omega L\tilde{Q}}{\dot{Q}^2} \right) + \omega \frac{L^2}{\dot{Q}^2} \right\} d\rho \]

(27)

But \((s/\tilde{Q} - sK\tilde{Q}_t / \tilde{Q}^2) > 0\) since \(\tilde{Q} > \tilde{Q}_t\), which follows from \(\tilde{Q}_t = \frac{\partial Q}{\partial K^*} < 0\). Similarly, \((wL / \tilde{Q} - wL\tilde{Q}_t / \tilde{Q}^2) > 0\) since \(\tilde{Q} > \tilde{Q}_t\) which is implied by \(\tilde{z}_t = \frac{\partial \tilde{z}(Q, K)}{\partial K^*} > 0\). Hence the last term can be written as

\[ \int_{M(K,\rho)}^{N} \frac{\partial H_{t+1}(K_{t+1}^*, \tilde{r}_{t+1})}{\partial \rho} \left\{ \tilde{a} + \frac{\omega L^2 (\tilde{Q}, K)}{\tilde{Q}} \right\} d\rho \]

where \(\tilde{a} = \left[ \left( \frac{\dot{S}}{\dot{Q}} - \frac{\dot{S}K\tilde{Q}}{\dot{Q}^2} \right) + \left( \frac{\omega L\tilde{Q}}{\dot{Q}} - \frac{\omega L\tilde{Q}}{\dot{Q}^2} \right) \right] > 0\)

The expression for the regulated firm can be written as

\[ (\alpha \tilde{r} - \tilde{z}_t) \left[ \int_{M(K,\rho)}^{N} \frac{\partial H_{t+1}(K_{t+1}^*, \tilde{r}_{t+1})}{\partial \rho} \left\{ \tilde{a} + \frac{\omega L^2 (\tilde{Q}, K)}{\tilde{Q}} \right\} d\rho \right] - \int_{M(K,\rho)}^{N} \frac{\partial H_{t+1}(K_{t+1}^*, \tilde{r}_{t+1})}{\partial \rho} \left[ 1 - \frac{\tilde{Q}}{\tilde{Q}} \frac{\partial H_{t+1}(K_{t+1}^*, \tilde{r}_{t+1})}{\partial \rho} \right] d\rho \]

(28)

The above expression indicates that the increment in terms of present value, of a marginal increase in capital stock in period, \(t\), for one period has to be equated to the decrease in the present value of labour costs in period \(t\), together with the present value of changes in revenue in succeeding periods. The changes in revenue in succeeding periods, have two components, both arising through the
regulatory price mechanism 1) a change associated with a
decrease in the price in the succeeding period, \( p_{t+1} \),
through a decrease in labour costs in period \( t \), in all
possible states of nature, and, 2) another due to an
increase in the price in period \( (t+1) \), caused, in turn, by
an increase in capital costs. The direction of the change
is dependent on the signs of \( \frac{\partial H_t(K_{t+1}^*, p_{t+1}, \theta, t+1)}{\partial p} \)
in the various states of nature in period \( t \).

The sign of \( \frac{\partial H_t(K_{t+1}^*, p_{t+1}, \theta, t+1)}{\partial p} \) will be positive
provided the regulator's formula implies a price less than
what the monopoly firm would have charged at the beginning
of period \( (t+1) \) and vice versa. Thus, while the
capital-labour combination chosen by the regulated firm,
will not, in general, equal that of the unregulated firm,
the direction of the shift is indeterminate, without
precise knowledge of the firm's production possibilities,
the demand curves it faces in the various states of nature,
and the description of uncertainty. This is quite different
from the A-J result in the static case and its dynamic
extension.\(^{22}\)

The main reason why the behaviour of the regulated firm
in the above discussion is indeterminate is that the
application of the regulatory formula in equation (7) could

\(^{22}\) See Davis (8) for a demonstration that the stationary
point of the dynamic regulatory process exhibits
overcapitalization in the A-J sense.
lead to prices that are "too high" i.e., higher than what an unconstrained monopolist would charge. However, if we assume that the regulatory process is "effective," so that regulators never set a price higher than what an unconstrained monopolist would charge even though the regulatory formula yields such a price.\textsuperscript{23} Under such circumstances, it can be shown that the regulated firm chooses an optimal capital stock, at least as large as that chosen by the unregulated firm, given the same price. Also, under such circumstances, the \textit{ex post} capital-labour ratio is greater for the regulated firm, than for the unregulated firm, for every state of nature that may obtain.

Under "effective regulation, \( \frac{\partial H_{li}(.))}{\partial p} = 0 \), for all \( 0 \).

Equation (28) can be written as

\[
(\alpha_t - \kappa_t) - \int_{0}^{M(k,p)} \phi_1 \left[ \omega \xi_2 (\theta, \kappa) \right] \left[ 1 - b_0 \right] d\theta
\]

\[
- \sum_{m(k,p)} \phi_0 \left[ \omega \xi_2 (\alpha(k,p), \kappa) \right] \left[ 1 - b \right] d\theta + c = 0
\]

where

\[
b_0 = - \frac{1}{q_0} \frac{\partial H_{li} (k_{t+1}^*, \tilde{r}_{t+1})}{\partial p} \geq 0
\]

\[
b_0 = - \frac{1}{q_0} \frac{\partial H_{li} (k_{t+1}^*, \tilde{r}_{t+1})}{\partial p} \geq 0
\]

\[
c = \int_{0}^{M(k,p)} \phi_0 \left[ \omega \xi_2 (\alpha(k,p), \kappa) \right] \left[ \tilde{a} \right] d\theta + \int_{0}^{M(k,p)} \phi_0 \left[ \omega \xi_2 (\alpha(k,p), \kappa) \right] \left[ \tilde{a} \right] d\theta \geq 0
\]

\textsuperscript{23} This is somewhat similar to the assumption in the standard A-l model, that the regulation is "binding," i.e., the rate-of-return allowed by the regulators is less than the monopoly rate-of-return that the firm could have earned, were it not regulated.
The relation in the above equation can be compared with the first order condition for the unregulated firm. Defining $K_r$ and $K_u$ as the optimal capital stock of the regulated and unregulated firms respectively, we can write for the same price,

$$\int_{0}^{M(K_r; f)} \hat{\phi}_0 \left[ \omega \hat{z}_2 (q_0, K_r) \right] [1 - b_o] d\omega - \int_{0}^{M(K_u; f)} \hat{\phi}_0 \left[ \omega \hat{z}_2 (c(K_r, f), K_r) \right] [1 - b] d\omega$$

$$\leq - \int_{0}^{M(K_u; f)} \hat{\phi}_0 \left[ \omega \hat{z}_2 (c(K_u, f), K_u) \right] d\omega - \int_{0}^{M(K_r; f)} \hat{\phi}_0 \left[ \omega \hat{z}_2 (c(K_r, f), K_r) \right] d\omega$$

(30)

This implies that

$$\int_{0}^{M(K_r; f)} \hat{\phi}_0 \left[ \hat{z}_2 (q_0, K_r) \right] d\omega + \int_{0}^{M(K_u; f)} \hat{\phi}_0 \left[ \hat{z}_2 (c(K_r, f), K_r) \right] d\omega$$

$$\geq \int_{0}^{M(K_u; f)} \hat{\phi}_0 \left[ \hat{z}_2 (q_0, K_u) \right] d\omega + \int_{0}^{M(K_r; f)} \hat{\phi}_0 \left[ \hat{z}_2 (c(K_u, f), K_u) \right] d\omega$$

(31)

For this to be true, the capital stock of the regulated firm has to be greater than that for the unregulated firm i.e., $K_r > K_u$. We can prove this by contradiction. Suppose this is not so. i.e. the optimum capital stock of the unregulated firm exceeds that of the regulated firm or, $K_u > K_r$. Since the prices specified are equal, it follows that the index of the last state of nature where demand is satisfied is greater for the unregulated firm than for the regulated.
a firm, i.e. \( M(K_u, p) > M(K_r, p) \), since \( \frac{\partial M(K_r, p)}{\partial K} > 0 \). Also, the maximum quantity produced by the unregulated firm is greater, i.e. \( \tilde{q}(K_u, p) > \tilde{q}(K_r, p) \) as \( \bar{\pi} > 0 \). Since \( \chi^*_2 > 0 \), \( \chi^*_2(q_0, K_u) > \chi^*_2(q_0, K_r) \).

For states of nature where demand is fully satisfied, \( q_0 \leq \tilde{q}(K_u, p) \), for \( a \leq M(K_u, p) \). Since \( \chi^*_1 < 0 \), \( \chi^*_2(q_0, K_u) \geq \chi^*_2(\tilde{q}(K_u, p), K_u) \).

We can write

\[
\int \phi^*_0 \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \phi^*_0 \left[ \chi^*_2(\tilde{q}(K_u, p), K_u) \right] d\omega = \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(\tilde{q}(K_u, p), K_u) \right] d\omega
\]

But,

\[
\int \phi^*_0 \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \phi^*_0 \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(\tilde{q}(K_u, p), K_u) \right] d\omega
\]

since, \( \chi^*_2(q_0, K_u) \geq \chi^*_2(\tilde{q}(K_u, p), K_u) \). We can write

\[
\int \phi^*_0 \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(\tilde{q}(K_u, p), K_u) \right] d\omega = \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(\tilde{q}(K_u, p), K_u) \right] d\omega
\]

But,

\[
\int \phi^*_0 \left[ \chi^*_2(q_0, K_u) \right] d\omega + \int \frac{M(K_u, p)}{M(K_r, p)} \left[ \chi^*_2(\tilde{q}(K_u, p), K_u) \right] d\omega
\]
since \( \frac{\partial}{\partial \kappa} (z_2(\bar{\beta}(K,p),K)) > 0 \).

Since \( \frac{\partial}{\partial \kappa} (z_2(\bar{\beta}(K,p),K)) > 0 \),

Hence, \( \int \phi_0 [z_2(y,\kappa)] \, d\theta + \int \phi_0 [z_2(\bar{\beta}(K_u,p),\kappa)] \, d\theta \).

This contradicts equation (31) derived from the optimum conditions for the two firms. Hence the assumption made is false. Hence \( K_u < K_r \), or \( K_r > K_u \). Given that \( K_r > K_u \), and \( \frac{\partial}{\partial \kappa} (z_2(\bar{\beta}(K,p),K)) > 0 \), it follows that

\[
q_2(\bar{\beta}(K,p),K) > q_2(\bar{\beta}(K_u,p),K) , \quad \theta \leq M(K_u,p)
\]

Thus, the ex post capital-labour ratio is at least as large for the regulated firm as for the unregulated firm, in all states of nature.

The regulatory process in most cases is asymmetric.\(^{25}\) i.e.

\[25\text{. It can be shown that } \frac{\partial}{\partial \kappa} (z_2(\bar{\beta}(K,p),K)) > 0 \text{ for fairly general production functions. See Appendix for the proofs in the case of the Cobb-Douglas and CES production functions.}\]

\[25\text{. This term is defined and discussed in detail in Bailey}\]
one of the productive variables (labour) is excluded entirely from the constraint ceiling. In fact, it is this aspect of the regulatory mechanism, that causes the misallocation result of the A-J theorem in the static, certainty context, as well as in the model discussed here.

The theorem proved above compares the capital stock and input efficiency of the regulated firm with that of unregulated firm, at the same price. This is in contrast to the static A-J model where the capital-labour decisions are compared at the output the regulated firm elects to produce. The comparison is made with price held constant, in the present model, since price is specified \textit{ex ante}, but output is known only \textit{ex post}.

5. Conclusion

This paper has presented a model of the regulated firm, in an uncertain context. An attempt was made to overcome some of the shortcomings of the Averch-Johnson model of the regulated firm - rate-of-return regulation, problems due to uncertainty, and the static nature of the model. The model presented here, while more realistic than the Averch-Johnson version, is still not a complete characterization of regulation in practice.

\textit{(2) pp. 52-57.
The optimal behaviour of the regulated firm is examined, and contrasted with that of the unregulated firm operating under similar conditions. The model employs the state preference framework to describe uncertainty, and attempts to capture the price-setting function of regulators in a dynamic context. The rule followed by the regulators is that the price fixed for a period, when applied to the previous period's output results in the firm's earning the "fair" rate-of-return, on the previous period's capital stock. It is further assumed, that the firm can adjust its input of labour (or other "variable" inputs such as fuel) more "quickly," than it can adjust its capital stock: the firm decides on its investment in capital stock ahead of time, and adjusts "labour," to its optimal level, after the state of nature is revealed. Under these conditions, the capital stock employed by the regulated firm, is compared to that of the unregulated firm. The ex post input efficiency of the regulated firm is also examined.

In order to answer the above questions, analytically, rather than numerically, it was necessary to assume that the firms being compared, undertook investments in the period under study. However, by including specific production and demand conditions, numerical solutions can be developed for the firms' optimal programmes.

It was shown that in a dynamic context, under price regulation, based on past outcomes of demand and cost, the
regulated firm uses an inefficient combination of inputs for any given price level. Specifically, when regulation is "effective," (i.e. the regulators set a price no higher than what an unregulated monopolist would), the regulated firm employs a higher level of capital than is efficient. By efficient, is meant the input combination used by the unregulated firm at the same price level. Also, the ex post capital-labour ratio is at least as large the efficient combination.
Appendix

For the Cobb-Douglas production function,

\[ Q = K^\alpha L^\beta, \quad x_2 = \frac{Q^\gamma}{K^\alpha} \]

At the point when it is just profitable to increase production,

\[ \frac{\partial \bar{\xi}}{\partial \alpha} = \frac{1}{\beta} Q^{\gamma - \frac{\alpha}{\beta}} K^{-\frac{\alpha}{\beta}} = \frac{p}{\omega} \]

Thus, the maximum production, \( \bar{Q} \), is given by,

\[ \bar{Q} = \left( \frac{p K}{\omega} \right)^{\frac{\alpha}{\beta}} K^{\frac{\alpha}{\beta}} \]

The derivative of the labour-requirements function with respect to capital stock is given by

\[ x_2 = \frac{\partial \bar{\xi}}{\partial K} = -\frac{\alpha}{\beta} Q^{\gamma} K^{-\frac{(\alpha + \beta)}{\beta}} \]

Substituting (A3) into (A4), the derivative of the labour-requirements function with respect to capital stock, at the point of maximum production, is given by

\[ x_2 (\bar{\alpha} (K, p), K) = -\frac{\alpha}{\beta} \left( \frac{p K}{\omega} \right)^{\frac{\alpha}{\beta}} K^{-\frac{(\alpha + \beta)}{\beta}} \]
The question now is whether the sign of the derivative of the expression in \( (A5) \) with respect to capital stock, \( K \), is positive or at least, non-negative. Differentiating with respect to \( K \), we have

\[
\frac{\partial}{\partial K} \left[ \frac{L}{K} \left( \frac{K}{L} \right)^{\gamma-1} \right] = \frac{-\alpha(K+\gamma-1)}{\gamma(1-\gamma)} \left( \frac{\gamma+1}{\omega} \right)^{\gamma-1} \left( \frac{K}{1-\gamma} \right)^{\gamma} \geq 0
\]

since \( \alpha + \gamma \leq 1 \)

which confirms the assumption in the text, for the Cobb-Douglas case.

In the case of the (more general) CES production function, we can write

\[
(A7) \quad Q = \left[ a K^{-\beta} + b L^{-\gamma} \right]^{-\gamma} \quad , \quad \bar{Q} = \frac{1}{b} \left[ Q^{-\gamma} - a K^{-\beta} \right]^{-\gamma} = \frac{1}{b}
\]

Differentiating the labour-requirements function with respect to \( Q \), and setting it equal to \( p/w \), we can write the maximum profitable production, in this case as

\[
\frac{\partial \bar{Q}}{\partial Q} = \frac{Q^{-(1+\gamma)}}{b} \left[ \bar{Q}^{-\gamma} - a K^{-\beta} \right]^{-\gamma} \left( \frac{1}{\gamma} \right) = \frac{1}{\omega}
\]

\[
(A8) \quad \bar{Q} = \left[ \frac{1-c}{\alpha} \right]^{-\gamma} K
\]

where \( c = \left[ \frac{b/h}{o} \right]^{-\gamma} / \omega \)

Differentiating the labour requirements function with respect to capital stock, \( K \),
Substituting the maximum output in (A8), into (A9), we have

\[ \chi_2 (\alpha (k, p), K) = - \frac{a}{b} \left[ \left\{ \frac{1-c}{a} \right\} K^{-\beta} - \alpha K^{-\beta} \right] K^{-(1+\beta)} \]

To see the sign of \( \chi_{2,2} \) at the point of maximum profitable output, we differentiate (A10) with respect to \( K \)

\[ \frac{\partial}{\partial K} \left[ \chi_2 (\alpha (k, p), K) \right] = 0 \]

Thus, in the case of both the Cobb Douglas and CES production functions, the second derivative of the labour-requirements function, at the point of maximum profitable output, is non-negative.
References


10. Klevorick, Alvin, K. "The 'Optimal' Fair Rate of


