Economic Strategy for Developing Nuclear Breeder Reactors

Part II

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5. INITIAL FORECASTS OF THE EFFECTS OF BREEDER REACTORS

An outline of a strategy for future decisions is far removed from assessing the results from using particular rules in the past. But the construction of a first breeder reactor prototype is not so far removed into the future that initial decisions on number and types of such plants -- and thus on the whole breeder program -- can be avoided. The technology of the first fast breeder prototype, at least insofar as core configuration and coolant are concerned, is being decided now with the allocation of initial research funds among laboratories and experiments to that end. These decisions have been based upon limited forecasts of the final results of the research, and on standards associated with defense needs for domestic uranium or with the preservation of stocks of this natural resource wherever it might be located. They can be put into perspective by an initial assessment of the effects from fast breeders, and compared with those from application of the rules proposed in Chapter 3.

The effects from any one of the research programs of interest will have to be found in the pattern of adoption of breeders by electricity generating companies. A successful project results in large numbers of breeder reactors installed each year by private equipment manufacturing companies for the generating companies. The effects are the economic gains from doing so -- the consumers' surplus from these specific reactor types over and above the cost of their construction and fuel during the plant lifetime.
Initial forecasts are based upon the information available in 1967 for successful installation of capacity in the middle 1980's and thereafter. The information on costs of capacity is scant. There are some estimates of what reactors will sell for, which may or may not be implicit forecasts of costs (given that prices may or may not be different from costs). In contrast, there is a great deal of expectational information on capacity to produce -- on equipment and fuel cycle performance over the lifetime of 1985 design reactors. This information can be used to derive the production functions $M = f(K,F)$ where $M$ is a measure of capacity to produce thermal energy and $K,F$ are capital and fuel inputs to make that capacity effective; with various assumptions as to prices for $K$ and $F$, the limits on production as shown by this function determine the minimum costs for providing various amounts of $M$. The procedure is first to find the production function, and then use factor prices to define unit costs from the function.

Forecasts of the magnitude of demand for capacity are made a number of ways. But the emphasis here is on finding a demand function $M = f(P,Y)$ where $P$ describes various prices for capacity by reactor and boiler manufacturers and $Y$ is a general variable relating to sizes of markets for electricity. The data for finding the function are from additions to capacity in light water reactors; assuming that the demand for breeders does not differ in kind -- that the reticence shown in introducing light water reactors as reflected in its demand will reappear to the same degree when it comes time to consider commercial fast breeder reactors -- then the relevant future values of $P,Y$ can be used to forecast future additions to capacity demand.
The difference between demands for capacity and the costs of providing them make up the economic gains from reactors. Some part of this difference can be attributed to the introduction of new breeder reactors, as shown in Figures 1 through 6 in the preceding chapters. Estimates are made of this specific gain from the cost and demand functions, given that the pricing policies of new firms follow the policies established earlier by the thermal reactor manufacturers.

These are initial estimates and are only as reliable as this procedure and the underlying data. Even with a priori arguments of first quality, the estimates have no life expectancy beyond the date of arrival of new and different data. Then what follows is no more than a first impression of the effects of breeders, and a demonstration of an economic analysis that could be repeated as new evidence is developed.

The Production Functions of Fast Breeder Reactors

Reactors added to electric generating systems in the 1980's will be governed by technical limits on output from various inputs that can be expressed as "production functions" $MW_t = f(K,F)$ for thermal megawatts of capacity $MW_t$ dependent on physical units of capital equipment $K$ and on the inventory of nuclear fuel $F$. The limits are set by the state of the art for any type of reactor: a particular type, using materials with limited conductivity and durability, is capable of delivering only so much thermal energy from atomic fission to the turbine generator. Breeder reactors, as a new state of the art, will have new and different production functions in the sense that new combinations of capital and fuel will produce desired levels of capacity.
If liquid metal, steam, and gas-cooled fast breeders are all developed, then there will be three new and different production functions; it would then be possible to obtain capacity from a relatively larger number of combinations of capital and fuel.

Some of the additional combinations could be most interesting. One could turn out to be the least cost combination: with input prices expected to be \( P_K^* \) and \( P_F^* \), amounts of capital and fuel \( K^* \) and \( F^* \) produce \( MW_t \) at least total expenditure, and a particular breeder type might attain \( K^*, F^* \) for \( MW_t \) while the others did not. But capital and fuel prices are not known for certain. Another breeder might be the only type able to produce \( MW_t \) for some \( K, F \) combinations that could be least cost combinations but that do not include \( K^*, F^* \) (this would be the case if capital and fuel prices turn out to be different from \( P_K^* \) and \( P_F^* \)). If these conditions are realized, the breeder would add to the economy by reducing the cost of resources used to generate electricity or by reducing the risk that these costs are going to be very high.

In the 1960's, rather than the 1980's, these combinations can only be investigated by predicting the 1980 production functions. Prediction faces the risk that excellent research results will reduce \( K \) for given \( F \) and \( MW_t' \), and that research failures will result in higher values of \( K \) than called for in present design studies. But predictions can be made from the highly detailed engineering analyses of the 1985 breeders which account for those risks. The analyses -- called "design studies" and descriptive of plant operations as well as equipment -- can be considered indicators of "central tendency" for \( MW_t, K, \) and \( F \) for 1980 plants even though both higher and lower
values of $K$ are possible because they serve to direct the research now taking place. The design is the best present estimate of plant capability for given amounts of $K$, $F$. Deviations from the estimate can be expected to decrease or increase research costs, as shown in the preceding chapter, so as to adhere to these results.

There is information throughout the current design studies for estimating the parameters of the production functions. All functions seem to be within the bounds of the general form

$$MW_t^S F_b^\pi = \alpha K^B F^\psi$$

with $MW_t$ the capacity or maximum sustainable output of thermal energy at any time during reactor lifetime and $F_b$ the bred fuel output from capital $K$ and fuel $F$, subject to reductions or increases in magnitude as shown by the factor $\alpha$. This form is plausible because it implies that the marginal products of capital in either capacity ($\partial MW_t/\partial K$) or breeding ($\partial F_b/\partial K$) depend upon the amounts of the other input and of the other output (a pattern of marginal products which has generally been observed to follow from variations in output temperature and fuel rod design). Rewriting the relation so that

$$MW_t = \alpha K^B S F_b^\psi$$

-- treating bred fuel as equivalent to input fuel -- then information is required for estimating $B'$ and $\psi'$ for the different types of breeder reactors.

The more useful data come from engineering studies of performance at various coolant temperatures and pressures. These show the tradeoffs of fuel
for capital in the design or \( \partial K / \partial F \) for a given reactor size MW. But differentiating K with respect to F in the general form of the production function results in:

\[
(1) \quad \frac{\partial K}{\partial F} = -\psi'K_1/F_1
\]

Information is obtained as well from separate engineering studies -- or chapters in the designs of the 1985 systems -- on the capital and fuel requirements for various sizes of a type of reactor. The studies of scale produce values \( K_1, F_1 \) for MW and \( K_2, F_2 \) for MW for two sizes MW and MW of any reactor. These values can be inserted in the production function definition -- so that MW = \( \alpha K_1^\beta' F_1^{\pi'} \) and MW = \( \alpha K_2^\beta' F_2^{\pi'} \) -- and solving these for the unknown exponents \( \beta' \) and \( \pi' \) results in:

\[
(2) \quad \beta' \log(K_1/K_2) + \psi' \log(F_1/F_2) = \log(MW_1/MW_2).
\]

The equations (1) and (2) provide sufficient detail to solve for \( \beta' \) and \( \psi' \).

Estimates of \( \beta' \) and \( \psi' \) are described here for each fast breeder type. They are based on this procedure, which can be called a "sensitivity" analysis in the variables of interest for determining costs of construction and operation of future breeder reactor capacity. They are not the result of deliberate tests of performance with more capital and less fuel, for example, but rather follow from comparisons of roughly similar designs in all respects other than capital/fuel ratios. As such, they provide a synopsis of today's views on technology for the 1980's and 1990's.
The Liquid Metal Fast Breeder Reactor

All reactors, to this point in time, have been built around a core made up of fuel rods in which uranium or plutonium fission takes place. The core may be cylindrical or pancaked in shape, and the number and size of rods can vary with core temperature and the means for removing the heat energy. In "fast breeder reactors," as contrasted with "thermal reactors," the lack of moderating material in the core results in reduced probability of fissioning of uranium ($^{233}U$, $^{235}U$), but increased probability of nonfission capture of neutrons by $^{238}U$ so as to "breed" fissionable plutonium $^{239}Pu$. The bred plutonium, or an original inventory of the same material, then provides heat energy.

Liquid sodium is one coolant that can be used to transfer the heat from fast fission from the core to a steam source for power generation. Sodium is molten at $200^\circ F$ and boils at $1600^\circ F$, so that a wide range of heat-transfer temperatures is possible at environmental pressures. It has excellent heat absorption characteristics, but is corrosive to many metals and reacts so strongly to water that the combination is explosive under pressure.

As a result of these characteristics, the flow diagram for a sodium-cooled fast reactor is complex. The heat transfer subsystem from the core to the steam generator is divided into a number of independent and self-contained loops so that the failure of sodium flow at one location will not disrupt the entire transportation network. Most loops have back-up and bypass routes, as well, so that any malfunction resulting from sodium corrosion is isolated and separated from the continued operation of other components in that loop. Typically, the heat is routed by sodium from the core through primary transfer loops to three or more secondary sodium loops, and then transferred from there to a number of separate steam generators; each primary
or secondary sodium loop is self-contained and the heat exchanger is made up of a shell of one material in which is contained the tube of the second material.

There have been four major design studies of 1980 liquid metal fast breeder reactors with capacity close to 2500 thermal megawatts, and a number of article-length analyses either of smaller reactors or of particular design variations. Each of the major designs has a set of elaborate heat transfer loops -- in particular, six separate sodium loops for the transfer of heat to two separate steam generators. But no particular design has a specific requirement for a core configuration or even a fuel rod; the configuration

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[1] The four design studies for a large liquid metal fast breeder reactor sought to provide 1,000 electrical megawatts of capacity in each case. Some of the designs involved higher levels of thermal efficiency than others, so that the thermal megawatts of capacity varied around 2500 thermal megawatts. The four studies are:


These are referred to as the "major designs"; but there are others of importance or interest as to particular parameters. The study by K. P. Cohen and G. L. O'Neill, "Safety and Economic Characteristics of a 1000 Megawatt Electric Fast Sodium-Cooled Reactor Design," (ANL 6700, 1965) contributes to the evaluation of system reliability. The study by W. Hafele, D. Smidt, and K. Wirtz, "The Karlsruhe Reference Design of a 1000 Megawatt Electric Sodium-Cooled Fast Breeder Reactor," (ibid.) also contributes to the analysis of accidents which disrupt system reliability. Studies of smaller liquid metal reactors by General Electric contribute to the analysis of scale: cf. H. E. Dodge, et.al., Conceptual Design of a 565 Megawatt Electric Fast Ceramic Reactor (GEAP-4226, April 1963), and K. N. Horst, et.al., Core Design Study for a 500 Megawatt Fast Oxide Reactor (GEAP-3721, December 28, 1961).
and arrangement of rods, and their life cycle from fabrication to removal from the core, do not differ in kind from those in the major designs of other types of reactors.\(^2\) The dimensions of these elaborate capital systems \(K\) and less elaborate fuel requirements \(F\) can be found by cross reference in the four designs, and by comparison with earlier designs of smaller liquid metal breeders.

The General Electric study, and that of Allis-Chalmers, indicate capital expenditures in great detail.\(^3\) Consider capital \(K\) to be items of equipment for the production of heat energy in the form of steam at 1000\(^\circ\)F and 2500 p.s.i.a., where these items include all systems in the reactor and for the transfer of heat to sodium intermediate loops and then to the steam generator. Then the relevant equipment includes (a) the reactor vessel and internals, (b) primary sodium pumps, drives, and piping, (c) intermediate

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\(^2\)This is not to deny the existence of different configurations in the design of the fuel core. For example, there are striking differences among the four studies themselves, as shown in the view of the Reactor Engineering Division, Chicago Operations Office, U. S. Atomic Energy Commission, An Evaluation of Four-Design Studies of a 1000 Megawatt Electric Ceramic-Fueled Fast Breeder Reactor (C00-279, December 1, 1964), in the diagram of reactor core arrangements of the four design studies on the first page. But these core configurations are not specific to the liquid metal fast reactor; in fact, the configurations in the gas and steam cooled reactors discussed below are more similar to those of Combustion Engineering Corporation and General Electric than these two designs are to the Westinghouse and Allis-Chalmers designs for the liquid metal reactor.

\(^3\)The implications of particular constraints in these studies cannot be known with any exactitude. But it would seem quite likely that they stand in the way of finding the least cost combination \(K_1,F_1\) for given MW\(_1\) even with the factor prices stated in the AEC Guide to Nuclear Power Cost Evaluation. Both the contraints on capital and on fuel set the levels of utilization of these two inputs, rather than allowing the ratio of marginal products of the inputs to be equated to the ratio of factor prices. Of more concern is the possibility that the observation of \(F_1\) is not the minimum amount required to produce MW\(_1\) with any stated \(K_1\) -- the specification of maximum core outlet temperature may require more \(F_1\) than is necessary. Then variation of the \(K,F\) combination would not lead to an approximation of the least cost combination of inputs, because the observed combination may include too much of both inputs. This possibility is left open here, although the analysis proceeds as if it did not hold.
heat exchangers, (d) secondary sodium pumps, drives, and piping, and (e) final heat exchangers including the steam generating system. But it does not include the turbine generator building, the generator unit and its accessory electrical equipment. The total of expenditures for the relevant equipment, termed $\Sigma P Q$ for component prices $P$ multiplied by respective quantities $Q$, comes to $84.6 \cdot 10^6$ and the total number of components $\Sigma Q$ is 467.\footnote{Cf., the General Electric Company, Liquid Metal Fast Breeder Reactor Design Study, \emph{op.
\textit{cit.}}, Section 2.8, "Economics Data." The number of components is estimated from counting items of equipment in Table 2.8.2.1, "Reactor Equipment Cost Summary," and -- where the Table is incomplete -- in the plant layout charts and diagrams shown throughout the report.} Not all components are of the same size, nor do they involve the same amount of fabrication and engineering of the same metals; weighting each by its price -- so that those with higher prices are assumed to be larger -- the average size of a component is $\Sigma P Q / \Sigma P = 84.6(10^6)/46.9(10^6) = 1.80$ units of capital. Then total capital comes to $(\Sigma P Q / \Sigma P)\Sigma Q = (1.80)(467) = 841$ units.

The quantity of fuel required for 2500 MW\textsubscript{t} is investigated in all four design studies. Total fuel mass is divided between "the core" and "the blanket" surrounding the center of fission. But the latter has qualities and functional use different from the former, with the plutonium in the core used both to produce heat energy and neutrons for converting the blanket uranium to more plutonium. The plutonium creates and sustains fission and is necessary in a fast environment for breeding. Capacity to produce thermal megawatts is determined, then, by the kilograms of fissile plutonium in place at all times during the lifetime of the capital equipment. The initial plutonium loading $M_0$ provides this capacity when the equipment is installed (since this amount is capable of producing 2500 MW\textsubscript{t}). But this loading lasts only two to three years, and capital is in operable condition twenty to thirty years
so that the inventory required to provide lifetime capacity is the present or initial sum of all required future loadings -- that is, the sum of the present discounted values of the volumes required to be installed for continuous capacity over the equipment lifetime.

The time perspective for planning productive capacity is from the initial installation of equipment over the lifetime of that equipment. The planner has to consider capital and fuel in the same dimension -- that quantity of each input factor required to provide thirty-year capacity to produce a certain amount of thermal energy. The capital capacity is indicated by the equipment installed during construction along with certain items which have to be replaced before the thirty-year lifetime is complete. The fuel required is the sum of annual loadings of fuel and blanket uranium, designated \( N_i \) for \( i = 1 \) to \( i = 30 \). But these annual loadings do not all take place at the time of the initial installation of the equipment; rather they can be postponed with consequent savings of investment capital shown by the annual rate of return \( r \) on such invested capital. Each loading \( N_i \) can be discounted by \( (1 + r)^i \), so that fuel inventory at the time plans are made for construction equals \( \sum_i (1 + r)^{-i} \).

This amount does not yet take account of the gains from breeding. During each inventory life, \( B \) units of fissionable material are produced as a ratio of those consumed; then \( B^{1/t} \) units are produced annually given that \( t = \) length of life of an inventory of core fuel and \( (B^{1/t} - 1) \) of that inventory can be removed as excess each year. At the end of the first year, the necessary core inventory is \( N_o - N_o(B^{1/t} - 1) \) and the present value of this amount is \( N_o - N_o(B^{1/t} - 1)/(1 + r) \). At the end of the second year, the fuel core has again increased by \( (B^{1/t} - 1) \) and a three-year plan requires
\[ F = N_0 - N_0 \left( B^{1/t} - 1 \right) / (1 + r) - N_0 \left( B^{1/t} - 1 \right) / (1 + r)^2. \]

The fuel requirements for the thirty-year reactor life are \( F = N_0 \left[ 1 - (B^{1/t} - 1) \Sigma (1 + r)^{-1} \right]. \)

According to the General Electric design analysis, 2357 kilograms of fissile plutonium are installed on the day the plant ceases to be a construction project (or, according to the Allis-Chalmers design, 2910 kilograms are installed).\(^5\) This volume of fuel is to be loaded again and again over the reactor lifetime, so that it is always in place; but at the same time the breeding of new plutonium provides more fuel in place than this amount installed.\(^6\) For the General Electric case, the value of fissionable fuel in the core increases at the rate of \( (B^{1/t} - 1) = 0.081 \) per year. The present value of all future loadings declines at a rate of at least 10 per cent in each year from the time at which the initial loading takes place, given that the minimum opportunity costs for fuel in use are at least this great, so that loadings in year \( t \) are equivalent to the initial loading discounted by 10.0 per cent for each intervening year. The sum of thirty loadings over the reactor lifetime is approximately \( F = \left( 2357 \right) \left[ (1 - 0.081) \Sigma (1.1)^{-1} \right] \) or 558 kilograms. Then \( MW_t = 2500, K = 841 \) and \( F = 558 \) is an approximate single observation of capacity and capacity input requirements for the general LMFBR production function.

\(^5\) The differences in fuel inventories can be accounted for in terms of temperature and pressure conditions in the core, which utilize fuel more intensively in the first case, and less intensively in the second case. If the technologies assumed in the design studies are the same, then the two values of \( F \) are observations for fuel and capital in different combinations producing the same amount of output. That is, the design studies show \( F_1, F_2 \) for \( MW_t = \lambda = f(K, F) \), an isoquant of minimum amounts of capital and fuel to produce capacity set at \( \lambda \).

A tradeoff of fuel for capital is involved in changing the "specific power" of the system, as measured in energy kilowatts per kilogram of fissile plutonium. Higher specific power can be attained by increasing the thermal conductivity of the fuel or the fuel and coolant temperatures; either course of action adds to thermal stress and corrosion in the core assembly and to the capital items in the shielding and heat transfer systems. Both reduce the fuel inventory. The requirements for more capital at higher power are shown in the Combustion Engineering study in the increase in fuel assembly components and the decrease in the radius and length of each component -- where, as a first approximation, the increased fabrication implied by more and smaller rods is equivalent to an increase in components. The corrosion and deterioration due to increased temperatures and pressures require higher quality components or more frequent replacement of components in reactor vessels and internals. In the Combustion Engineering study, these new requirements altogether are equivalent to a 11.3 per cent increase in these components given a 100 per cent increase in specific power. The reduction in fuel inventory from increased specific power is shown in both the General Electric and Allis-Chalmers studies. By "doubling the core power density and specific power, simultaneously reducing the number of batches to keep the same refueling

7 The "specific power" of a reactor design is defined as thermal megawatts of capacity divided by the equilibrium fissile fuel inventory. In economic terms, specific power is the average productivity of fuel MWt/F.

8 Figure IV-27 in the Combustion Engineering study shows that a 100 per cent increase in specific power over the base value of 175 KW/KG causes an increase in capital costs of .19 mills per kilowatt hour. This is 11.3 per cent of the imputed costs of 1.68 mills with an 80 per cent load factor and all factor prices unchanged. Then an increase in costs must take place from increased quantities of inputs equal to 11.3 per cent of the original value of these inputs.
schedule -- [and accepting] a penalty in the physics area ..." the input plutonium content is used up more completely and thus the inventory to produce a given capacity is reduced; the net effect in the General Electric design is that fuel inventory is reduced approximately 9.5 per cent.⁹

The tradeoff of fuel for capital \( \partial K/\partial F \) can be shown by the results of these changes to reach higher specific power. If the increase in fuel assemblies and reactor internals as shown by Combustion Engineering is applied to the number of units of capital shown in the General Electric design, then 28.4 more units of capital are required to increase power by 100 per cent. This reduces fuel by 51.8 units, so that \( \partial K/\partial F = .55 \) or 55 units of capital are added for each 100 unit reduction in fuel.

The capital and fuel requirements at different thermal energy outputs have not been investigated as part of any one design study. General statements have been made on the advantages of large scale: The AEC design handbook is based on output increasing at a faster rate than inputs, so that larger plants have lower capital/output and fuel/output ratios than smaller plants; but the Westinghouse design study, in considering both small and large versions of a liquid metal reactor, shows no decrease in inputs per megawatt of capacity at large scale.¹⁰

The only detailed studies of different sizes of the same reactor are two General Electric designs, those for

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the 2500 MWₜ liquid metal fast reactor and a 1395 MWₜ reactor based on the same coolant systems and core configuration. The conceptual design for the first indicates, as shown above, \( K = 841 \) and \( F = 558 \); the second design provides most if not all the information necessary for constructing comparable estimates of \( K \) and \( F \) for \( MWₜ = 1395 \).

The 1395 MWₜ design is based on the same technology -- the study was completed in 1963, rather than 1964, but it has the same flow diagram as the larger reactor.¹¹ Four primary sodium pumps move the liquid coolant through the bottom and then out of the top of the reactor core to four intermediate heat exchangers which also contain sodium; the secondary loops then carry the heat energy to a once-through steam generator or to steam reheater loops. Such a system differs only in the number and size of loops or other components, when compared to the 2500 MWₜ General Electric liquid metal design, but not the techniques of heat transfer, the use of an intermediate transfer network, or the design of the reactor vessel and internals. Sodium is transported out of the reactor at 1050°F and 26 p.s.i. at the rate of 60 \( \times 10^6 \) lb./hour in the smaller reactor, and at the same temperature and pressure -- but in larger quantity, at 86 \( \times 10^6 \) lb./hour -- in the larger reactor.¹² The "product," evidenced by heat energy transported by the sodium to pass-through steam generators, would seem to be the same.

The costs of units of capital, defined as \( (ΣP^*Q^*) \) for the small reactor for comparability with capital \( (ΣPQ) \) in the larger system

¹¹ Compare Figure 2.4 in GEAP 4418, the 2500 MWₜ design study, with Figure 9 in GEAP 4226, the basic 1395 MWₜ design study. The flow diagrams are the same, at least in these simplified versions of the two systems.

¹² The basis for differences in rates of flow is in the size and number of components to achieve greater heat transfer so as to make the capacity level for the larger reactor.
described above, are $62.9 \cdot 10^6$ for components and construction of the system. The number of components $\Sigma Q^*$, and the sum of prices $\Sigma P^*$, are not shown in the study, but a sample of prices for reactor equipment is identical to the sample in the design for the larger reactor,\(^{13}\) so that $\Sigma P^*$ is assumed to be equal to $\Sigma P$ for the larger design. Given the similarity in the designs of the two systems, the number of components should not differ either, so that both $\Sigma P^* - \Sigma P$ and $\Sigma Q^* - \Sigma Q$. The difference between systems should be in the size of components with the smaller average size occurring in the smaller reactor. Then capital $(\Sigma P^*Q^*/\Sigma P^*)\Sigma Q^*$ equals $(\Sigma Q^*Q^*/\Sigma P)(\Sigma Q)$, which is $[(62.9)/(46.9)](467)$ or 625 units.

The smaller reactor uses more fuel for producing a given amount of heat energy, it seems. The initial core loading of fissionable plutonium, equal to 1200 kilograms, is larger per megawatt of capacity than that in the 2500 MW\(_t\) design reactor. This smaller machine does not "breed" at as high a rate because capture of neutrons by fertile uranium is less complete in a smaller core; thus the additional fuel loadings are larger relative to the first core inventory. The burden of a low rate of breeding, in fact, shows clearly in

$$(B^1/\tau - 1) = .042$$

rather than the .081 expected for the full scale 2500 MW\(_t\) system.\(^{14}\) The impact of these two factors is large: total lifetime inventory,

\(^{13}\)This is the case at least for those basic items such as steel plate and croloy for which price comparisons are possible. Cf., GEAP-4418, Table 2.8.2.1 and GEAP-4226, Table IV.

\(^{14}\)As calculated from information in the General Electric analysis for the formula $B = [h(r - 1) + 1]$ with $h = 24 b E_m/(1 + \alpha)/103 E_f x e$, and $r$ the breeding rate. The variables in $h$ are: $b =$ total fissions/core fissions, $E_m =$ maximum core burnup, $m =$ maximum/average power, $\alpha =$ capture/fission, $E_f =$ fission energy yield, $x =$ fast fission factor, $e =$ fission weight/total fuel weight.
viewed from the day in which plant construction is begun, is \(1200[1 - .042(9.43)]\) or 729 kilograms.

The values of \(K\) and \(F\) for \(MW_t\) in the two reactors, and of \(\partial K/\partial F\) for the larger reactor, provide information for a first approximation to the LMFBR production function. The two designs indicate for the equation

\[
[\beta'(\log K_1/K_2) + \psi'(\log F_1/F_2) = \log(MW_1/MW_2)],
\]

that \(\beta'(1290) + \psi'(-1161) = 2534\). The tradeoff of capital for fuel on specific power \(\partial K/\partial F = -\psi'K/\beta' F = -55/100\) or, for the design value of 2500 \(MW_t\), \(\psi'(841)/\beta'(558) = 55/100\). Solving these two equations yields \(\beta' = 2.9\) and \(\psi' = 1.1\) or a description of the function as \(MW_t = K^{2.9}F^{1.1}\).

Even the first approximation provides a characterization of the liquid metal breeder. The extensive heat absorption ability of liquid sodium comes to fruition in the large scale reactor to a much greater extent than in the small scale; this is shown by the very large sum of exponents, so large that a doubling of inputs increases output capacity sixteen times over. Small additions of capital make large additions to thermal energy capacity, as shown by the values of the marginal product of capital in the ranges of \(F, K\) in the two design studies; this marginal product, or \(\partial MW_t/\partial K = \beta MW_t/K = 2.9 \ MW_t/K\) increases over the range from 1395 \(MW_t\) to 2500 \(MW_t\). If capital is added to a given level of fuel, output increases by a greater amount up to technical limits set by permissible metal temperatures and pressures.

A second approximation to the liquid metal production function has to account for deviations of output from that for which the reactor is designed. The realized outputs from spaced runs with a given amount of equipment and fuel are very seldom the same, because driving the core at limit thermal
conditions causes distortion in the shape of the fuel rods consequent to
the energy output. There is always some chance of melting the fuel when
there are random increases in temperature or transient changes in power,
and the latter involve the risk of excess reactivity rendering fission
uncontrollable.\(^\text{15}\)

The three studies of the large liquid metal reactor have been con-
cerned with forced outage and other deviations from target levels of operation;
necessarily decisions have been made to set lower targets so as to reduce
these deviations.\(^\text{16}\) The departures resulting from transient temperature
\(^\text{15}\)The changes in reactivity with respect to temperature and power determine
the stability of the system. Temperature-induced reactivity changes \(\partial \rho / \partial T\),
with \(\rho\) defined as the per cent deviation from steady state levels of fission
activity, are random events; if a temperature increase is not to increase
fission, so as to increase temperature and fission once again and render
the system unstable, then values of \(\partial \rho / \partial T < 0\) are required. In some con-
ceivable fast sodium-cooled reactors, increases in temperature reduce fuel
and sodium density, and reduced density implies \(\partial \rho / \partial T < 0\). But in other
conceivable designs for this reactor with a compact core of minimum possible
size fissioning at high temperature, the fast neutron energy spectrum may
cause \(\partial \rho / \partial T\) to have the opposite sign. That is, it is possible that (1)
\(\partial \rho_1 / \partial T < 0\), where \(\rho_1\) results from reduced sodium density, but the mag-
titude of this negative effect decreases as the compactness of any core in-
creases; (2) \(\partial \rho_2 / \partial T > 0\) where \(\rho_2\) is the effect of sodium in "degrading
the neutron energy spectrum" so as to increase fission from the available
neutrons; (3) the size of \(\partial \rho / \partial T\) becomes larger with larger reactor size.
Thus for larger reactors with compact cores, the sum total effect may be
either \(\partial (\rho_1 + \rho_2) / \partial T < 0\) or \(\partial (\rho_1 + \rho_2) / \partial T > 0\). These reactors with
extremely compact cores operating at higher temperatures are prone to
disaster; they are not the bases for the 1985 reactors outlined here, but
there is a finite chance that the core of the reactors being considered
could be rendered more compact in an accident, so that this unstable be-
behavior was realized.
\(^\text{16}\)Decreasing the volume of fuel in the core, and increasing the capacity of
the sodium coolant and transfer systems so as to maintain sodium outlet
temperatures, results in the undesirable larger positive values of \(\partial \rho / \partial T\).
(Cf., The General Electric study, Liquid Metal Fast Breeder Reactor Design
Study, \textit{op.cit.}, Section 3.3.2, "Evaluation Techniques For Survey Data and
Their Graphic Representation, Survey of Fast Reactor Cores"; cf., also
(footnote continued on following page)
changes which have been found acceptable by the originators of the designs are a function of the design temperature and level of operation. In
\[ MW_t = \alpha K^\beta F^\gamma \] the constant term \( \alpha \) takes the form \( \alpha_0 e^{-x} \), where \( x \) equals the temperature variant \( \frac{T}{T_0} \times (1 + \xi) \), with \( \xi \) the temperature-induced elasticity of reactivity \( T_\rho \rho / \rho T \). When actual temperature \( T \) equals design temperature \( T^* \), \( \alpha = 6.2 \times 10^{-9} \) and \( MW_t = 6.2(10^{-9})K^2.9F^{1.1} \), the design values of inputs and capacity output. But when a transient results in a 50\(^0\) increase in temperature at \( T^* = 1000^0F \) and the coefficient of reactivity is \(-.005\), then capacity is reduced by 5 per cent. A 25 per cent deviation in temperature substantially reduces the effectiveness of the reactor -- but by much less than from a melting accident which would take place in a less conservative design. Temperature deviations in this reactor are expected to result in penalties on available capacity of small magnitude, at least the safety rod and fast shut-down systems in the General Electric design can be expected to go into effect for changes in temperatures of any importance. Then this approximation of the production function is
\[ MW_t = \alpha K^\beta F^\gamma = 6.2(10^{-9})K^2.9F^{1.1}e^{-\left( \frac{T}{T_0}(1 - .005) \right)} \]
for instantaneous capacity \( MW_t \), capital \( K \), fuel inventory \( F \), coolant outlet temperature \( T \), and \( \xi \) equal to \(-.007\) in this case.

K. P. Cohen, and G. L. O'Neill, op.cit., Section 3.2, "Reactor Design.") But this design variant also increases the rate of production of new fissile fuel so as to decrease fuel inventories. There is a gain in expected design productivity, then, from reducing the certainty of this design productivity. The General Electric tradeoff appears to be an addition to reactivity = \( .006 \) for every one per cent and an increase in the breeding ratio by as much as 20 per cent.
The Steam Cooled Fast Breeder Reactor

The uranium-plutonium rods making up the core of any fast reactor can be put together to transfer heat energy directly to steam, rather than to an intermediate sodium loop for re-transfer to steam. The design of the steam core would not differ greatly from those in the sodium reactor studies; oxides of the two fuels in five-foot rods can be arranged in hexagonal clusters to allow the pass-through of pre-heated steam in the same manner as the pass-through of liquid sodium discussed above. But the design of the heat energy transport system greatly departs from that for the liquid sodium reactor.

Descriptions of flow from the water source to steam entering the reactor, and then from the reactor to the turbine generators, indicate the great simplicity of the direct steam system. Steam condensers in series or parallel to the turbine generator send a mixture of water and steam first through low pressure and then high pressure feed pumps to pass-through steam heaters. Steam emerges from the high pressure heaters at approximately 500°F and, in reactors of 1200 MWt or more, at the rate of more than 2.5 x 10⁶ lbs./hour. A drum-type or Loeffler boiler then mixes this steam with hotter steam from the reactor and transmits it to a circulator at close to 600°F and at the pressure of approximately 1200-1500 p.s.i.a. By the time that the discharge from the circulator reaches the reactor, it is at more than 600°F and 1500 p.s.i.a.; it enters the reactor from below and exits from the top at 950°F and about the same pressure. This steam is then passed directly through the turbine generator to produce electricity. There are no intermediate heat transfer loops -- rather, the reactor coolant is used to turn the blades of the turbine.
The simplicity of this system results in greatly reduced expenditures on capital equipment to construct and operate any proposed steam cooled breeder reactor (SCBR). Capital costs are lower than for a liquid sodium reactor of equal capacity, as a result of the elimination of primary and secondary heat transfer systems, but also as a result of reductions in shielding and back-up equipment to guarantee the integrity of the sodium system. In other terms, for the SCBR production function \( MW_t = \alpha K^{\beta} F^{\gamma} \), the value of \( K_1 \) for given \( MW_1 \) and \( F_1 \) is smaller than for the LMFBR. The value of \( F_1 \), or the inventory of fissile plutonium in the core, is another matter. Steam is not as good a heat conductor as liquid sodium, nor are the energy-flux conditions in a steam environment as conducive to breeding, so that more fuel is needed for the SCBR than for liquid sodium coolant systems.

Design studies of an 870 MW\(_t\) steam cooled breeder provide a single observation of capacity, capital, and fuel to show these effects. Comparison with a design for a 2500 MW\(_t\) steam system -- which is in bare outline form -- allows some inferences on economies of scale and ultimately some basis for assessing differences from the liquid metal reactor.

Most of the studies of the smaller SCBR were done in the late 1950's and early 1960's, by the United Nuclear Corporation, on the basis of technology then available or expected to be available by the early 1970's.\(^{17}\) The early dates of the designs do not result in technical lags behind those of other

reactor types, because the technology of this system was as well developed at that earlier time. The direct steam cycle through the reactor to the turbine has been based upon extensive experience with boiling water thermal reactors; the requirements for raising steam temperatures above those in the LWR's are no more difficult than similar goals for the LMFBR. Developmental problems lie ahead in fuel rod exposure to high temperatures and long periods in core inventory, since there has been little experience with prolonged burnup of fuels in a fast reactor; but these problems arise for the liquid metal and gas reactors and the solutions assumed in the SCBR studies are not different from those assumed elsewhere.

The "present-day" design in the U.N.C. studies calls for capacity to produce 870 megawatts of thermal energy for steam turbine generators that are to operate at 36.4 per cent net efficiency, and the "1975 design" in the same studies calls for 825 thermal megawatts to produce the same amount of electricity output. The thermal capacity provides steam at 945°F and 1400 p.s.i.a. in the 1960's, and 1060°F and 2000 p.s.i.a. by 1975 (so that, by the middle 1970's, the characteristics of heat energy are approximately the same as those designated for the liquid metal fast breeder reactor).

Capital expenditures $P*Q*$ to provide the capacity in the "present day" design are $35.2(10^6)$ for fabrication and construction of components; the total of prices for these items $P*$ is $17.1(10^6)$ and the total number of items $Q*$ is 59. For comparability with other reactor systems, three calculations must be made. First, the price-weighted average number of units $(P*Q*)/(P*)Q*$ is $(35.2/17.1)(59)$ or 121. Second, this amount has to be "scaled" so that a single unit of capital is the same size as in the liquid metal fast breeder. If an item requires the same amount of engineering
and fabrication per square foot in all systems, then any difference in expense among systems is due to difference in size. Units all the same size, then, have the same prices -- or an item in the SCBR is the same size as in the IMFBR if the two have the same price ratio. The amount of capital in the steam cooled breeder is based upon an average price of $P*/Q* = 0.30(10^6)$, while the unit of capital in the IMFBR is priced at $0.10(10^6)$; since capital in the first reactor is relatively large in price terms, then the 121 units increase by $(0.30/0.10)$ to 363 units comparable to those in the IMFBR. Third, costs for constructing this reactor are estimated by the United Nuclear Corporation designers to be considerably less in 1975 than in 1967; technical improvements in components, if not reduced factor prices, reduce the number of components from 363 to 278 by the time construction of the SCBR or the IMFBR begins. It is estimated that $K = 278$ is the required amount of capital in this design of a steam cooled breeder reactor with capacity of 825 thermal megawatts.

The inventory of fuel to provide such capacity begins with a loading of 1370 kilograms of fissile plutonium and with approximately seven times this much fertile uranium in the core and surrounding blanket for breeding more plutonium. Breeding is limited in this design because the intensity of neutron radiation is reduced by the steam coolant and because high steam temperatures and pressures require fuel rod claddings which reduce neutron absorption by the fertile uranium. Also, the coolant reduces the active

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18. The assumption is supported by G. Sofer, et.al., Conceptual Design in Economic Evaluation of a Steam Cooled Fast Breeder Reactor (op.cit., NDA 2148-5), p. 117, Sections 1, 2, 3.

19. This assumes an extremely modest rate of progress: 2 per cent per annum from "learning by doing" the direct cycle system in improved versions of the PWR thermal reactor.

lifetime of fuel in the core; burnup is expected to be limited to 60,000 MW-days per metric ton of core fuel, so that any one core is useful for only about 2.3 years. Limited results with breeding and burnup increase the inventory of fuel over the thirty-year lifetime of the reactor. With the ratio of produced to utilized fissionable material $B$ equal to 1.16 the annual rate of reduction of inventory $B^{-1/t}$ for a lifespan $t$ of one core is 4.6 per cent. Carrying changes of 10 per cent reduce the present value of future loadings as well. Total inventory in present value terms is the accumulation of $1370(1 - B^{1/t} - 1)$ discounted by 10.00 per cent for each year, or $$(1370)(1 - .046) \sum_{t=1}^{30} (1.1)^{-t} = 775$$ kilograms of fuel. The estimate of lifetime fissile fuel requirements completes this observation: for 825 MW$_t$ of instantaneous capacity in a 1975 steam cooled breeder reactor, $K = 278$ and $F = 775$ over the equipment lifetime of thirty years.

The tradeoff of capital for fuel $\partial K/\partial F$ is not shown in the design studies of the 825 MW$_t$ reactor. But there are indications of the effects from increasing specific power. The "near term reactor" in the U.N.C. analysis has specific power of 505 kilowatts per kilogram of fissile plutonium, while the "1975 reactor" has specific power of 577 KW/Kg; if the effects on $K$ and $F$ of other differences between the two reactors can be accounted for, then the residual effect must be due to the variation in the specific power.

The first reason for differences is that the "1975 reactor" benefits from technical progress which reduces $K$ and $F$, while the "near term reactor" does not. The fruits of progress imply a substantial reduction in $K$: "The use of vapor suppression type of containment and ... simplification in starting, shutdown, refueling, and fuel handling equipment" is estimated to

$^{21}$G. Sofer, et.al., _op.cit._, p. 117.
reduce expenditures substantially. The net effect (1) of the higher specific power which increases capital requirements, and (2) the technical progress which reduces these requirements, is an 8 per cent reduction in expenditures. Reductions in capital from technical progress alone should exceed 2 per cent per annum if there is even a modest annual increase in productivity of capital from "learning by doing" with the direct cycle in the light water thermal reactors installed from 1967 to 1975. The elimination of 18 per cent of actual capital, so that K of 363 in 1967 terms is equal to 278 in 1975 terms, can be attributed to technical progress separate and independent from changes in specific power.

The second reason for differences in K is possible variations in the scale or size of the 1967 and 1975 reactors. Total thermal capacity in the near-term plant is 876 and in the later plant is 825 so that there would appear to be larger scale in the first plant. But the difference is a result of increased thermal efficiency from the higher temperatures and pressures which cause the higher specific power in the second plant -- that is, a result of producing a smaller amount of steam of higher quality which can in turn produce the same amount of electricity -- so that the "outputs" of the two plants are the same. There is no need for an adjustment for scale of operations.

Then for 1975 technical conditions as the standard of reference, a steam reactor with specific power of 505 has 278 capital units, but the reactor with power of 577 has 334 units. Both experience the same rate of progress, but the second has the 18 per cent rate of progress for the decade reduced to 8 per cent to meet the requirements from higher power.
Increasing specific power reduces the inventory of fuel required for any desired level of thermal capacity. This is more than a matter of defining "higher specific power" as "greater output capacity per unit of fuel"; it is a variation of design characteristics to attain steam temperatures and pressures, and integrity for fuel rod cladding, that approach the limits allowed by technology. These temperatures are conducive to more complete consumption of fuel, and thus to a longer life for any core inventory; but higher temperatures reduce breeding because they require fuel rod cladding materials that absorb neutrons. In the United Nuclear designs, the prolonged burnup at higher temperature is the dominant effect. Core fuel under the lower specific power conditions lasts 33,000 MW days per ton of fuel on the average, when the lifetime inventory is approximately 775 kilograms. Burnup increases substantially at the higher specific power to 60,000 MW-days, while the breeding ratio is reduced only by .02, so that the present value of total lifetime fissile fuel inventory is reduced from 775 to 397 kilograms.

Increasing specific power has the expected effects on the stocks of both capital and fuel then. Capital is increased by 58 units, and fuel is decreased by 378 units, so that $\mathcal{E}_F = +58/-378$ or -12.7 per 100 units of fuel.

The 825 MW reactor is reasonably well defined, both in terms of an observation of $K,F$ and of variations in specific power for other combinations of $K,F$. The 2500 MW steam reactor has not been defined in detail, however. One consequence is that there are no studies of the effects of size on capital and fuel requirements in the range of 1200 MW to 2500 MW comparable to those for the liquid sodium reactor to show some indication of economies of scale. One study by Kernforschungszentrum Karlsruhe shows the outlines...
of a design for a 2500 MWt steam cooled reactor but only points to the need for further analysis of the tradeoffs between breeding, fuel burnup, and core reliability to attain efficient levels of capital and fuel. There is some indication that minimum capital costs are close to $119.0(10^6) and an initial fuel inventory is likely to be 3323 kilograms of plutonium when capacity is 2517 megawatts of thermal energy. Two assumptions can lead to estimates of units of capital \( (\Sigma P'Q' / \Sigma P')\Sigma Q' \) from the first statistic. First, if the expenditures on turbine generator components and structures are equal to the $43.9(10^6) shown in the 2500 MWt liquid sodium design reactor, then expenditures on the heat energy source \( \Sigma P'Q' \) are $119.0(10^6) - $43.9(10^6) = $75.1(10^6). Second, if these expenditures \( \Sigma P'Q' \) are for the same number of components at the same charge per square foot of fabrication and engineering as in the 825 MWt steam cooled reactor, then units of capital \( (\Sigma P'Q' / \Sigma P')\Sigma Q' = (\Sigma P'Q' / \Sigma P^*)\Sigma Q^* \) where \( P^*, Q^* \) are those shown in the United Nuclear design studies of the smaller steam cooled fast breeder. This assumption should be modified: the Karlsruhe design shows six independent coolant circuits -- each with a Loeffler boiler as the last link before circulation of the steam through the reactor core -- rather than the two circuits shown in the United Nuclear designs, so that the number of units of capital in the larger design \( \Sigma Q' = \Sigma Q^* + 26 = 85 \), where the additional 26 units result from this three-fold increase in circuitry. If the prices of all units of a given size are the same in the two reactors, then the units of capital are \( (\Sigma P'Q' / \Sigma P^*)\Sigma Q' = (75.1/17.1)85 = 375 \). In capital comparable in size to those for the liquid

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22 R. A. Mueller, et.al., _op. cit._

23 This is not an unrealistic assumption, given that the temperature and pressure conditions for steam here are approximately those in the liquid sodium fast breeder reactor.
metal fast breeder reactor, for post-1975 technological conditions, increases to 750 units.

The second statistic in the Karlsruhe study -- the initial fuel inventory -- is not the only indicator of the behavior of the fuel cycle. The proposed design is to operate at the technical limits of temperature and pressure, so as to guarantee burnup of core fuel of between 50,000 and 100,000 MW-days per ton. This prolongation of exposure in the core increases the lifetime of the fuel elements to approximately 3.5 years but reduces the breeding ratio somewhat; the consequent ratio of fissionable fuel produced in the core and blanket to that consumed is 1.086. Then the rate of discount \( B^{1/t} = (1.086)^{3.03} \) and the inventory over the plant lifetime of thirty years is \((3323)(1 - .236)\) equal to 2540 kilograms of plutonium.

These values of \( K \) and \( F \) can be used along with those from the United Nuclear designs to show the extent of economies of scale. The Karlsruhe design indicates \( K' = 750, F' = 2540 \), and \( MW'_t = 2517 \) while the smaller reactor indicates \( K'' = 334, F'' = 397 \), and \( MW''_t = 825 \). In the general form of the production function \( MW_t = aK^{B'F'} \), the two observations show the coefficients of \( B \) and \( \psi \) as \( \log(2517/825) = B'\log(750/334) + \psi'\log(2540/397) \), or \( 4.844 = 3522B' + 8060\psi' \). The estimate is a first indication that economies of scale are limited: the sum of \( B' \) and \( \psi' \) is somewhat less than one, so that a doubling of capital and fuel leads to less than twice as much capacity. A second indication, leading to separate estimates of \( B' \) and \( \psi' \), confirms the first. The second indication is \( \partial K/\partial F = \psi'K/B'F = -.127 \), or \( \psi'(334)(10^2) = B'(127)(397) \) with \( K = 334 \) and \( F = 397 \). The equation can be put together with the estimate of \( \partial K/\partial F \) to provide two equations with two unknowns, \( B' \) and \( \psi' \), that can be solved for each unknown. The
two equations solve for $\Psi' = 0.47$ and $\beta' = 0.30$. The first approximation to the steam cooled fast reactor production function is then

$$M_{w_f} = K^{0.30} F^{0.47},$$

a description that points to diseconomies of scale, in contrast to the economies observed for the liquid metal fast breeder reactor.

The Gas Cooled Fast Breeder Reactor

The direct steam cycle for transferring heat from the reactor is only one type of gas cooling system; others using helium, carbon dioxide, or sulphur dioxide can deliver roughly comparable amounts of heat to steam generators or heat exchangers. One of these shows remarkably promising productivity. The helium gas system is of great interest because it is well advanced towards adoption in a sophisticated thermal reactor demonstration plant, and because it contrasts sharply in performance with the steam-cooled reactor so that it provides distinctly alternative results.

Helium is introduced into a coolant system from outside of the reactor through filters and compressors, and it enters the reactor container and finally the core by means of large gas compressors at 500°F and 1050-1060 p.s.i.a.\(^2\)

The gas is blown up into the fuel where its temperature is

\(^2\)These values are "representative" design values derived from the two most detailed design studies of a gas cooled fast breeder reactor: Oak Ridge National Laboratory, Reactor Division, Gas Cooled Fast Reactor Concepts (ORNL-3642, September 1964); and General Atomics Division, The General Dynamics Corporation, A Study of a Gas Cooled Fast Breeder Reactor, Initial Study, Core Design Analysis and System Development Program, Final Summary Report (GA-5537, August 15, 1964).
increased to 1150°F and its pressure is reduced approximately to 1015-1020 p.s.i.a., and then is blown down through four steam generators surrounding the core. The blowers, core, and generators are all contained in a single vessel of pre-stressed concrete or steel and this vessel is contained within a secondary reinforced concrete structure. The helium emerges from the two pressure vessels at 500°F and 1000 p.s.i.a., for purification and storage, and the steam emerges at 950° - 1000°F and 2400 p.s.i.a. for transfer to the turbine-generators. The circuit is completed outside of the pressure vessel by condensing the steam and running the condensate through feed pumps for re-entry to the steam generator intake, and by taking the helium from storage to the gas compressors for re-entry as well.

This design immediately shows its differences with the steam-cooled reactor layout. The largest components -- gas compressors, fuel core, steam generators, primary and secondary pressure vessels -- include three that are not of comparable magnitude in the steam cooled reactor. The compressors in a 2500 MWt gas reactor are so much larger than the pumps in the steam system that the technology extends beyond the steam experience and beyond present experience with all large pumps, for that matter. The steam generators are not found in the direct cycle steam-cooled reactor at all. Moreover, the primary and secondary pressure vessels are much larger and more complex because of the necessity to contain the fission products and the high-pressure helium when an accident opens the gas piping or the heat-transfer surfaces.

The design differs somewhat as well from those for the liquid-metal fast breeders. It is simpler: the four gas-to-steam heat transfer loops in the primary vessel are once-through steam generators, not primary to intermediate loops carrying more coolant to final loops containing steam.
System reliability is sought through particular design variants not found in the liquid sodium reactor. The helium reactor is designed to pour steam and water into the core, if the coolant is voided by accident, to prevent core meltdown.\(^{25}\) As a result, the safety techniques and routings in the helium design are much more straightforward. Also the containment of compressors, core, and heat transfer surfaces in the pressure vessel in the helium reactor is substantially more complete. It is prompted by the high pressures under which the helium operates, as compared to the ambient pressure of liquid sodium, and it adds to the capital components over and above those for containment in the liquid sodium reactor.

In terms of economic performance, where such performance is measured by \(\text{MW}_t = \alpha K^B \Psi^F\), the helium reactor has characteristics in contrast with both of the other reactors that reflect these design differences. The simplicity of the system reduces the number of units of capital \(K\). But the pressure requirements increase the sizes of units of capital, particularly in containment. Fuel utilization is more complete since helium does not reduce neutron energy flux as much as steam. On the other hand, this gas is a relatively poor heat transfer medium, so that the thermal energy capability of a kilogram of plutonium -- that is, the specific power of the fissile fuel -- is lower than for either steam or sodium reactors at the same turbine

\(^{25}\)The liquid metal reactor cannot rely on this technique because of the volatility of sodium in water, so that elaborate additions to capital are made in the LMFBR to reduce the probability of voiding and elaborate auxiliary safety rod or sodium storage systems are constructed to cut off reactivity increases if voiding ever does take place. Cf. K. P. Cohen, and G. L. O'Neill, "Safety and Economic Characteristics of a 1000 Megawatt Electric Fast Sodium Cooled Reactor Design," \textit{op. cit.}, and General Atomics Division, \textit{The Study of a Gas Cooled Fast Breeder Reactor}, \textit{op. cit.}
throttle temperature and pressure. This requires an increase in the plutonium content in a fuel loading for any given level of thermal megawatts. The net effects of these design characteristics on the relative requirements of capital and fuel for $\text{MW}_t$ can be shown by defining $\beta'$ and $\psi'$ for a 1975-1980 helium cooled fast breeder reactor.

Calculations of approximate $\beta'$ and $\psi'$ cannot be made from a single design study, or even from a group of related studies similar to those by General Electric on the liquid metal fast breeder. Detailed core analyses for gas-cooled reactors have been completed by the General Atomic Division of General Dynamics Corporation which provide the parameters for estimating fuel inventories for a wide variety of designs.\textsuperscript{26} The study of Gas Cooled Fast Reactor Concepts by Oak Ridge National Laboratory specifies the capital components to go with fuels and different gas coolants for different reactor capabilities.\textsuperscript{27} A consistent or single design has to be constructed de novo from the characteristics for parts of a helium-cooled 2600 $\text{MW}_t$ fast reactor shown in these two sources. Then this design can be varied for size and for specific power. First estimates of $K_0$, $F_0$, $\text{MW}_0$, and then of the effects on $K$, $F$, and $\text{MW}_t$ of scale and of varying specific power follow from this single design and its variants.

The most important design feature is the integrated primary and secondary containment vessel, with the gas compressors, fuel core, and gas-to-steam heat transfer surfaces inside the primary vessel. The ORNL study shows the

\textsuperscript{26} General Atomic Division, \textit{A Study of a Gas Cooled Fast Breeder Reactor}, \textit{op.cit.}

\textsuperscript{27} The Oak Ridge National Laboratory, \textit{Gas Cooled Fast Reactor Concepts}, \textit{op.cit.}
costs of these components as more than $37.9 \times 10^6$ for capability of 2600 MW_t. With the addition of helium transfer, purification, oil removal and low pressure leak-off recovery systems (all outside of the containment vessels) total capital costs are $52.1 \times 10^6$ for 85 components. The average size of a unit of capital $\frac{\text{EPQ}}{\text{EP}}$ equals $\frac{52.1 \times 10^6}{32 \times 10^6} = 1.63$, so that the total amount of capital is $(1.63)(85) = 139$ units of roughly equal size.

For size comparable to that of a unit in the liquid metal reactor, this is 522 units, because the average price per unit is 3.76 times that in the liquid metal reactor.

The inventory of fuel required to complement these units of capital can be found after specifying the particular uranium-plutonium mixture of interest and the specific power of the mixture. Oxides of plutonium are to be fabricated to perform at specific powers close to 800 thermal kilowatts per kilogram of this fuel; at least fuel requirements are not reduced by higher specific powers, for fabrication requirements increase at such a rate as to compensate for any further decrease in oxide content beyond this level.\(^\text{28}\) An initial inventory for 782 KW/Kg at 2600 MW_t is shown as $N_0 = 3350$ in the GA analysis. The lifetime inventory is equal to $N_0 \left[1 - B^{1/t} \left(-1\right)\right] \sum_{t=1}^{30} \left(1 + r\right)^{-t} = 351$ kilograms, given that the annual reduction of in-place requirements is 11.7 per cent because of a ratio of fissionable products produced to consumed

\(^{28}\)Cf. the General Atomic Division, A Study of a Gas Cooled Fast Breeder Reactor, op. cit., p. 40, where it is argued that "raising the rate can lead to increased fuel cost if pursued too far. This arises because, although inventory charges are inversely proportional to rating, a component of the fuel fabrication cost increases with rating (because there is a limit to the power available per unit total length of fuel element, set by internal heat conduction, regardless of the coolant median) so somewhere there is an optimum as far as total fuel cost is concerned. The question is dealt with greater length in Section 12, where it is shown there is little or no economic incentive to use ratings much over some 800 KW/Kg." Ibid.
of 1.44, a core lifetime of 3.3 years, and an annual use charge of 10 per cent for plutonium. The gain is so great in produced fuel in later years as to more than compensate for the use charge even in present value terms. The gas breeder produces more fission fuel than it consumes. Then $K_0 = 522$, $F_0 = -351$, and $MW_0 = 2600$ for a design based on the ORNL specifications for a large helium-cooled fast reactor.

Redesigning this reactor to decrease the specific power increases the fuel inventory and reduces the capital requirements. The variant allows an estimate of $\mathcal{F}K/\mathcal{F}F$, the tradeoff of capital for fuel in the helium-cooled reactor's production function. The increase in the fuel input is 228 kilograms in a reactor lifetime as the specific power is reduced from 800 kW/Kg to 718 kW/Kg.\textsuperscript{29} The reduction in capital is more difficult to assess. It appears from these studies of design variants that the simplification of the core, and the reduction of helium temperature and pressure, as a result of 782-718/782 per cent power reduction, has the same effect on capital requirements as that per cent reduction in thermal megawatt capacity. The ORNL design variants for the helium-cooled fast reactor show an average reduction of 0.6 per cent in units of capital contained in the primary and secondary pressure vessels for each 1.0 per cent reduction in thermal capacity over the range 1200 to 2600 MW$_t$; on this basis, the 522 units of capital are reduced by 24 units, as a result of the 7.8 per cent power reduction.

\textsuperscript{29}The fuel cycle analyses show that a reduction from specific power of 782 KW/Kg to 718 KW/Kg involves a decrease in burnup from reduced core temperatures sufficient to reduce $B$ from 1.17 to 1.04. This increases the present value of inventory from -351 Kg to -123Kg, for a decrease in specific power of 64 KW/Kg. It is assumed that there is no further increase in inventory from a reduction in specific power from 800 to 782 Kg -- or that the inventory required at these two specific powers is roughly the same (as in the previous footnote).
Then, from the reduction in specific power, capital is reduced by 24 units and fuel is increased by 228 units, or \( \frac{\partial K}{\partial F} = -\frac{11}{100} \).

Most of the effects of the scale of output are shown by other design variants. A reduction in scale, from 2600 MW\textsubscript{t} to 1317 MW\textsubscript{t} as one variant in the ORNL study, brings capital in the pressure vessels down by approximately 50 per cent. Since the requirements for peripheral equipment -- purification and storage components -- are not changed substantially, then the number of units of capital is 405 rather than 498 at specific power of 718 KW/Kg. The effects on fuel inventory are shown by the fuel cycle design details for the 1317 MW\textsubscript{t} helium fast reactor variant. The initial inventory is 2038 kilograms for this size of the general design at specific power of 646 KW/Kg; with an overall breeding ratio of 1.5 and a core lifetime of 3.6 years, the thirty-year requirement of fuel in present value terms is -18 kilograms. The initial inventory is reduced to an amount close to 1789 kilograms, and the lifetime inventory to -187 kilograms, by increasing specific power to 718 KW/Kg. Then this smaller reactor utilizes slightly more fuel over the inventory lifetime than does the larger reactor -- -187 kilograms rather than -351 kilograms -- or as noted in the General Atomic study, "fuel cycle cost [i.e., inventory per unit of capacity] does not vary greatly with reactor size, for cores of 5000 liters or larger."\textsuperscript{30}

These estimates can be used to outline the dimensions of the production function. The tradeoff of the two inputs \( \frac{\partial K}{\partial F} = -\psi'K_1/\beta'F_1 = -\frac{11}{100} \), which with \( K_1 = 498, F_1 = -123 \), at MW\textsubscript{t} = 2600 and specific power of 718 KW/Kg is equal to \(-\psi'(498)/\beta'(-123) = -\frac{11}{100} \). A second equation follows from the

\textsuperscript{30} General Atomic Division, \textit{op.cit.}, p. 162.
effects of scale; the ratios of the observations of inputs and of outputs in
the production function are \( \log(2600/1317) = \beta' \cdot \log(498/405) + \psi' \cdot \log(-123/-187) \)
which reduces to \( 2954 = 1897\beta' - 1820\psi' \). Solving these two equations for
\( \beta' = 1.46 \) and \( \psi' = -0.04 \), the first approximation to the production function
for the helium-cooled fast breeder reactor appears to be

\[
MW_t = k1.46F^{-0.04}
\]

for thermal megawatts of capacity in terms of thirty years of capital and
fuel requirements.

**Safety Characteristics of Steam-Cooled and Gas-Cooled Breeder Reactors**

As shown in the case of the liquid metal reactor, the capacity of
fast breeder reactor concept can vary from the level called for in the de-
sign, as a result of transient temperature changes in the reactor core. In
the liquid metal cooled reactor designs, both \textit{a priori} and experimental
analyses have shown that, at certain very high fuel temperatures, incursions
of more temperature or power will add to reactivity so as to lead to further
temperature and reactivity increases. A departure from design temperature
can lead to dis-equilibrium operation of the fuel core, with no self-adjustment
back to equilibrium. The core can be shut down before it melts as a result
of dis-equilibrium; this shut-down one way or another takes the operating
value of \( MW_t \) to zero. But the designs for working reactors avoid as much
as possible this extreme result. Rather, the output potential is cur-
tailed in favor of smaller variations, in keeping with the production function

\[
MW_t = \alpha K F^\beta \psi', \quad \text{with} \quad \alpha = \alpha_o \cdot e^{-\Delta T/T}(1 + \xi)
\]

as the productivity effects
from any transient temperature \( T \) greater than design temperature \( T^* \). A similar characterization of the effects of temperature changes can be made for both the steam cooled and gas cooled reactor designs.

Experiments with reactivity and temperature point to similarities in the behavior of steam and gas in the fuel core, and to contrasts with that of liquid metal. Temperature surges can be expected to occur infrequently in helium or steam during circulation of the coolant; for one, the occurrence of gas bubbles, as in liquid sodium, cannot lead to local hot spots which become temperature surges. But coolant temperature will increase much more rapidly when gas pumping systems stop, because the lower heat absorption of these goals places more reliance on coolant flow to reduce core temperature. Then the way to achieve reliability of gas and steam reactors is to prevent voiding of the coolant from the core.

The adverse results from voiding are greater when the fraction of the core given over to coolant is smaller. This fraction is 30 per cent larger in the steam reactor than in the gas reactor, for coolant temperature conditions in the NDA and Oak Ridge design studies. The poor results are also more pronounced when the total pumping power required to circulate the coolant through the core is larger; this power is 60 per cent less in the steam than in the gas cooled reactor. If there is an outage in coolant system pumping, the gas reactor core is subject to the larger temperature increase: more fuel per unit volume, and more reliance on pumping to remove heat from this fuel, result in more in-core heat in such circumstances.

\[^{31}\text{Cf. M. Dalle-Donne, "Comparison of Helium, Carbon Dioxide, and Steam as Coolants of 1000 MW Megawatt Electric Fast Reactor" (Euratomgesselschaft für Kernforschung mb II, 1965), Table I.}\]

\[^{32}\text{Ibid, Table I.}\]
The temperature change reduces plant capability more in the helium than in the steam cooled reactor. For any given number of degrees of temperature change, there is more induced change in reactivity in the helium and plutonium core. The reactivity effect $\partial \rho / \partial T$ is parceled into $\rho_1$ and $\rho_2$ in both designs. The value for $\partial \rho_1 / \partial T$, the decrease in reactivity $\rho$ from decreased coolant density at higher temperatures, is ten times less in helium because its density does not vary greatly; the value of $\partial \rho_2 / \partial T$ is positive and large in both systems. Then the overall change in reactivity in the gas reactor is more likely to be positive, since it could be dominated by the value of $\partial \rho_2 / \partial T$ over that of $\partial \rho_1 / \partial T$ in this case. The design has to hold back this reactor to a greater extent — in other words, the gas reactor must be shut down more rapidly so that runaway expansive reactivity-to-temperature feedbacks do not take place.

The tradeoffs over the range of present values of specific power for temperature and pressure safety are likely to favor steam and liquid metal over gas. The temperature coefficients of reactivity are similar for the first two, and negative under most temperature conditions; but those for gas-cooled

33Substantial temperature changes from complete loss of pumping power, and thus from voiding of coolant in the core, are usually prevented from going too high by shutting down of the reactor. There has to be some way devised, in fact, for shutting down quickly enough to prevent melting of the core and, at the limit, causing a nuclear explosion. Present plans in both gas and steam reactors center on emergency routings which flood the core with water from the steam systems, which will reduce temperature but, by greatly reducing neutron leakage out of the core, will also increase reactivity. For safety -- or reliability of thermal megawatts of capacity -- there must be absorption of the additional neutrons, presumably by using core additives or "poisons" such as boron or gadolinium. But these additives affect the design level for production of thermal energy, so that a tradeoff -- yet to be specified -- has to be made of design thermal power for decreased probability of zero power for extended periods in the reactor lifetime. Cf. J. W. Hallam, R. K. Haling, P. Killian, and G. T. Peterson, "The Flood Safety of Steam and Gas Cooled Reactors," (U.S. Atomic Energy Commission, Contract AT(04-3)180, PA 13, 1965), Figure 1; cf., also, G. Sofer, et al., op. cit., pp. 79-82 and General Atomic Division, op. cit., p. 72 et seq.
reactor designs are positive under a wider range of such conditions because of the lack of a "cancelling effect" from substantial resonance absorption of neutrons at higher temperatures (here shown as $\mathcal{O}_0 / \mathcal{O} T$, the "Doppler Effect"). Then the necessary shut-down time for the first two reactors is likely to be less, either because of less chance of meltdown or of having to close down the reactor at $T > T^*$ to prevent such core meltdown. This is shown by considering in $MW_t = \alpha K F^\psi$ that $\alpha = \alpha_0 e^{+x}$ with $x = -(\Delta T/T)(1 + \xi)$, the product of the temperature variant and one plus the elasticity of power with respect to temperature, as in the liquid metal production function.

The lowest values of $\xi$ in the range 1200 to 2500 $MW_t$ are for the steam cooled and liquid metal breeders with the gas cooled reactor reaching much higher values. To scale capacity in terms of relative reliability, consider $\xi = -0.013$ for the steam cooled reactor and $\xi = -0.008$ for the gas reactor; then $e$ is 3 per cent larger for the first reactor than for the other reactor type. The production function for the steam cooled breeder reactor is then, with $\alpha_0$ to scale for outputs less than 2500 $MW_t$,

$$MW_t = (8.62)K^0.30F^0.47e^{-\Delta T/T}(1 - .013)$$

while the gas cooled fast breeder is

$$MW_t = (.169)K^{1.46}F^{-0.04}e^{-\Delta T/T}(1 - .008)$$

The two types of fast reactors require different amounts of both $K$, $F$ for given thermal megawatts of capacity, and with differing degrees of reliability.

$^{34}$Cf. M. Dalle-Donne, op. cit., Table 4 for Doppler Coefficients and K. Cohen, and C. P. O'Neill, op. cit., p. 91 and Table 2 for Doppler Coefficients for the liquid metal fast breeder reactor shown in the General Electric Design Study.
A Summary View of 1980-1990 Production Functions

There are indications even in an outline of production functions that the technologies of the different breeder reactors are quite different. This is shown both in individual observations of $\text{MW}_t$, K and F, and in the effects of increasing inputs K, F on the scale of output $\text{MW}_t$.

The design studies on which the functions have been based were not "optimized," in the sense of showing the least-cost combinations of capital and fuel for given capacity and factor prices, but they were in search of a best design in this sense. They seem to indicate, at the least, minimum amounts of capital for one or another given amount of fuel and given thermal capacity and core temperature. But, as limited as this may be, the combinations of K and F chosen for 2500 to 2600 $\text{MW}_t$ in the three types of breeders show differences in technology. The liquid sodium system seems more "capital using" than the steam cooled system, given that both more capital and a higher ratio of capital to fuel is required in the first of these types. The gas system seems more "capital using" than either, in the K/F definition of that term, given that it is a net fuel producer from the beginning.

In moving from small to large scales of operation, the liquid metal fast breeder requires much smaller additions to capital and fuel than do the other two reactors. If capacity is increased to the 2500 $\text{MW}_t$ level from half this amount in a design reactor, the accompanying increase in capital and fuel is no more than 49 per cent (given that the increase in inputs $\Delta$

---

35 Even though the core temperature may dictate a suboptimal combination of these two types of inputs.
equal $\Delta^{b+\psi} = 2.0$, and $\beta' + \psi' = 4.0$, then $\Delta = 1.49$). The increase in requirements for the steam cooled reactor is 147 per cent of the half scale capital and fuel when moving to this higher level of capacity (for $\beta + \psi = 0.9$, then $\Delta = 2.47$). The gas cooled reactor produces capacity with additions to inputs between these two patterns, since an increase of 65 per cent of $K$, $F$ is required for a 100 per cent increase in $MW_t$. The economies of full scale accruing to liquid metal are large, to gas are smaller but existent, and to steam are non-existent.

These are initial impressions. A study of the behavior of these production functions given expected ranges of factor prices can indicate the differences in technologies more clearly. The terms of reference are the costs of producing output -- of additional capacity to product thermal energy -- from the different systems.

**From Production Functions to Marginal Costs of Fast Breeder Capacity**

The companies with these production functions are in the business of designing and constructing fast breeder reactors for sale. The sales involve costs for design and construction, and these costs are determined by the amount of new capacity that can be provided from given amounts of inputs. But the production functions are not the only determinants of costs for the reactor manufacturers. The levels of prices for capital components and for uranium and plutonium fuel have as important effects upon the costs of this new capacity.

Once the technology and basic design for the type of reactor is available, a plant can be constructed at various sizes from one megawatt to more than 2500 thermal megawatts; the cost of adding one plant's capacity
of any size within the technology is the "marginal cost of capacity." The choices of the reactor manufacturer at each size center on specific temperature and pressure conditions within wide limits set by the general designs. To minimize costs of providing capacity -- where this depends not only on capital but also the present value of the stock of fuel -- a design ratio of inputs has to be determined and then the scale of all inputs found.

The decision depends both on factor prices and the dimensions of the production functions: Total costs \( C = P_K K + P_F F \) depend on capital and fuel prices \( P_K, P_F \) and the quantities \( K, F \) constrained by the production function \( MW_t = \alpha K^\beta F^\psi \). The least-cost combination of \( K, F \) is given by the partial differentials of \( C^* = P_K K + P_F F + (MW - \alpha K^\beta F^\psi) \); where these partials are equal to zero; the level of minimum total costs in keeping with this combination is defined as

\[
C^{**} = \alpha^{-1/\beta'} + \psi' MW_t^{1/\beta'+\psi'} \left[ P_F (P_F^\beta/P_K^\psi')^{-\beta'/\beta'+\psi'} + P_K (P_F^\beta/P_K^\psi')^{\psi'/\beta'+\psi'} \right]
\]

The marginal costs of providing thermal capacity in this technology are then \( C^{**} \) for the marginal plant, or \( C^{**}/MW \) per kilowatt for this last plant in the production line.

The marginal costs of additional thermal capacity in a Liquid Metal Fast Breeder Reactor in the 1980's should follow from the constraints in the production function outlined in the previous pages and from factor prices forecast in the General Electric and Westinghouse design studies. The production function values \( \alpha = 6.2 \times 10^{-9}, \beta' + \psi' = 4.0 \) have been found in the sensitivity analysis of fuel power and plant size. The unit price of
capital $\Sigma P/\Sigma Q = \$10^5$ is derived from the same studies, apparently from straightforward extrapolation of present prices of components. The price per kilogram of fuel, not so straightforward, follows from adding fabrication, recovery, and uranium costs to the price of $\$10$ per gram for plutonium where all of these are forecast from very slight alterations of present conditions. These costs apply to the axial and radial blankets of uranium required for breeding around the plutonium core as well, so that total expenditures on all fertile and fissile material are divided by the number of kilograms of plutonium in the initial blanket to approximate a unit expenditure on fuel. This fuel price is $\$21.2$ per gram, or $\$21.2(10^3)$ per kilogram of plutonium.

These estimators $\alpha$, $\beta$, $\psi$, $P_K$, $P_F$ show what it would cost to construct the General Electric type of Liquid Metal Fast Breeder Reactor in the middle 1980's. With the general design of this plant as outlined, then one more of the smallest plants (156 MWt) would cost $\$228$ per thermal kilowatt of capacity. A second alternative might be to construct a 312 MWt plant; here the cost is $\$135$ per thermal kilowatt for this slightly larger plant. Moving to even larger scales reduces the cost from $\$135$ to $\$29$ per kilowatt at the 2500 MWt limit of technology.

Economies of scale in the liquid metal reactor are not duplicated in the Steam Cooled Breeder Reactor. For the same mid-1980's conditions as in the design LMFBR, the steam cooled breeder shows much lower costs per kilowatt for small plants but higher costs per unit of capacity for the largest plant. These estimates are shown in Table 14 -- for the 156 megawatt thermal reactor, the steam cooled breeder promises marginal costs of $\$16$, but for larger reactors these costs increase to $\$28$ at 1250 MW and $\$37$ per
Table 14

The Marginal Costs of Capacity in a Single Reactor

<table>
<thead>
<tr>
<th>Thermal Megawatts of Capacity</th>
<th>Marginal Costs, $ per Thermal Kilowatt of Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LMFBR</td>
</tr>
<tr>
<td>156</td>
<td>228</td>
</tr>
<tr>
<td>312</td>
<td>135</td>
</tr>
<tr>
<td>625</td>
<td>81</td>
</tr>
<tr>
<td>1250</td>
<td>48</td>
</tr>
<tr>
<td>2500</td>
<td>29</td>
</tr>
</tbody>
</table>

thermal kilowatt at 2500 MW. Increasing marginal costs follow from the more stringent limits on production in larger sized plants -- from $\beta' + \psi' = .77$ rather than 4.0 -- but not from differences in factor prices, since a unit of capital or fuel is assumed to cost the same in the two types of reactor.\(^{36}\)

The Gas Cooled Breeder Reactor promises costs at the margin which show some of the advantages of the liquid metal breeder at large scale, but also the good performance of the steam cooled reactor at small scale. The costs at 2500 MW, the largest size within the range of the general design studies, are $22$ for the last thermal kilowatt. This is somewhat less than the amount per kilowatt estimated for the liquid metal fast breeder of

\(^{36}\) The General Electric factor prices are used in estimating marginal costs for the steam cooled breeder. This is appropriate because the units of capital and fuel have been scaled for compatibility with the LMFBR in estimation of the production function.
that size, and 30 per cent less than the marginal costs for the steam cooled breeder of that size. Marginal costs are higher in small scale gas reactors, as in the liquid metal design type, but not to the same extent. The marginal costs at 156 MW$_t$ are $55 per thermal kilowatt in the gas cooled reactor as compared to $217 in the liquid metal reactor. Moving back from optimal thermal capacity to lower levels of capacity in these two types of fast reactor increases costs at the margin because a number of capital items have to be present in the system regardless of size. The number of capital items held constant is not as large in the gas cooled as in the liquid metal reactor but is greater than in the steam cooled breeder which can be scaled down to size in all items. As a result, the gas cooled breeder has marginal costs that are between those of the liquid metal and steam fast reactors at all sizes.

These are forecast costs resulting from expected input factor prices and from the production functions implied by the major design studies. They may very well not be realized forecasts, given variations in either price or productivity conditions from those specified. Reasonable or probable variations in prices would reduce these costs. Variations in operating performance could increase them very slightly.
The price of a unit of capital is based on implicit assumptions of productivity change in the industries supplying components between the middle 1960's and the middle 1980's.\textsuperscript{37} A decline in components or material prices -- in 1965 dollars -- of one per cent per annum seems as likely as the negligible decline expected in the design studies. Then $\frac{\xi P}{\xi N} = 9(10^4)$ rather than $\frac{\xi P}{\xi N} = 10^5$ because of the "learning by fabricating" experience of suppliers of common components, similar to that most obvious in the early and middle history of the light water reactor programs.

The forecast price of a unit of fissile fuel in the inventory specified for the production function depends directly on assumptions as to fabrication and recovery charges, and as to the prices per gram of uranium and plutonium in the 1985-1999 markets for these fuels. The fabrication and recovery charges are not likely to vary from those compatible with competition in fabrication services and constant unit costs; these are the assumptions implicit in the General Electric fuel prices. But the prices of plutonium and uranium compounds in the fuel core may well vary from the $10$ per gram for the first and $8$ per pound for the second.

The purchases of initial inventories of plutonium and uranium for liquid metal fast breeders will take place in fuel markets quite different from those operating at present, and perhaps quite different from those envisioned for the mid-1980's. This can be seen even from a cursory review of

\textsuperscript{37}These forecasts are made in M. C. McNally, et al., Liquid Metal Fast Breeder Reactor Design Study (GEAP-4418), op. cit., Section 2.8.3. They were made on "a dollars/pound basis with the applicable unit cost varying with complexity and functional requirement ..." (2-200) on what seem to be very conservative expectations for improvements in these supplying industries.
forecast supply and demand conditions. 38

The supply conditions may not be different from the forecasts to date. The country's known reserves of uranium, and the stocks of plutonium in the hands of the Atomic Energy Commission, will be drawn upon to provide new inventory. The rate at which additions and withdrawals of natural or manufactured reserves takes place will depend upon the prices offered for fuel oxides or carbides by the equipment manufacturers or the electric generating companies. Natural reserves have accumulated in the last few years as a result of very few significant discoveries; "in the period 1952-58, U.S. uranium reserves have increased from approximately 4,000 tons to something in excess of 180,000 tons of U₃O₈ ... the large reserves of Ambrosia Lake

38 The demand conditions are discussed in more than cursory detail later in this chapter, since they derive directly from the demand for fast breeder reactors. These rough indications precede a formal analysis of the interactions between fuel prices and final demand prices for electricity. Extensive additional research has to be completed before a formal model of the two interrelated markets can be constructed for forecasting purposes. But the model might follow this outline: assume that the quantity of nuclear fuel "F" available for processing in any one year is

\[ F = a + bP_F + cZ, \]

where \( P_F \) is the price offered for newly-mined output, and \( Z \) is the stock of known reserves. The aggregate demand for these \( F = f(P_F, P_K, R) \) as determined by the prices of fuel and capital, and the revenues \( R \) from sale of the capacity made available by the fuel. The demand price is equal to the value

\[ V = \left( \frac{\partial R}{\partial F} \right) = \left( \frac{\partial R}{\partial MW} \right) \left( \frac{\partial MW}{\partial F} \right) \]

from the purchase of these supplies; when \( V = P_F \) the purchasers minimize capacity costs and set optimal (profit) levels of nuclear capacity. Then

\[ P_F = \frac{F - a - cZ}{b} \] from (1) and \( P_F = \left( \frac{\partial R}{\partial MW} \right) \left( \frac{\partial MW}{\partial F} \right) \) from (2) for each buyer, or

\[ \frac{F - a - cZ}{b} = \left( \frac{\partial R}{\partial MW} \right) \left( \frac{\partial MW}{\partial F} \right). \]

With \( \frac{\partial MW}{\partial F} = \frac{\psi MW}{F} \)

from the production function, \( F(F - a - cZ) = \psi MW(\frac{\partial R}{\partial M}) \) and \( F = F_0 \)

can be determined from the forecasts of (a) reserves, (b) the production function, and (c) final electricity revenues \( R \). The volume of fuel \( F_0 \)
can be then used to forecast price \( P_F \) from equation (1) or (2).
in New Mexico, Big Indian Wash in Utah, and the Gas Hills and Shirley Basin in Wyoming were being developed.\textsuperscript{39} An extrapolation of this early experience indicates that substantial additions will be made to reserves before the middle 1970's: "the number of discoveries that were made over a short period of time and with little exploration background strongly supports the position that there is much more uranium to be found in this country and that, given a market, it will be found."\textsuperscript{40} Industry and Atomic Energy Commission estimates of supply -- including those undiscovered but existing reserves -- center on 500,000 tons of uranium oxide which could be produced and sold at a price of $10 per pound or under.\textsuperscript{41} Further additions to supply can be had at higher prices: the assessments of reserves are generally based upon a direct relationship between greater expenditures on exploration and development in prospect of high uranium prices and additions to the supply of uranium oxide; they indicate a direct relationship $\log Z = a + bP$, where $Z$ is tons of uranium oxide, and $P$ is the price of this oxide per pound, with estimates close to $\log Z = -4.5 + .08P$ when only tonnage from "proved reserves" or "quite likely reserves" is included.\textsuperscript{42}

\textsuperscript{39}Atomic Industrial Forum, U.S. Uranium Reserves: A Report by the Committee on Mining and Milling (August 1966), page 2.
\textsuperscript{40}Ibid.
\textsuperscript{41}Ibid., where 425,000 tons are expected to be available and U.S. Atomic Energy Commission, The 1967 Supplement to the 1962 Report to the President on Civilian Nuclear Power (February 1967), page 6, where 525,000 tons are "reasonably assured" and "estimated additional" for less than $10 per pound U\textsubscript{3}O\textsubscript{8}.
\textsuperscript{42}These estimates are implicit in S. Golan, et al., "Uranium Utilization Patterns in a Free and Expanding Nuclear Economy Based on Sodium Cooled Reactors," (presented at an Atomic Industrial Forum Conference, November 27, 1962), Figure 5, Uranium Reserves and Resources in the U.S.A., page 11. But they are not inconsistent with the AEC and Atomic Industrial Forum estimates. Optimistic forecasts of future discoveries increase the supplies at relevant prices so that $\log Z = - .83 + 0.16P$, or at $10/pound, to 10^7$ tons.
There is less agreement on forecasts of the supply of plutonium fuel or the nature and extent of demands for these two inputs fuels. Both depend on forecasts of the demands for electricity in the last quarter of this century, and on the share of generating capacity in alternative fossil and nuclear fueled systems. There are a number of such forecasts, some of which differ by one-half from the highest conceivable values for either electricity generating capacity or the share of capacity dedicated to nuclear facilities.\textsuperscript{43} Even an estimate in the middle of the range shows that plutonium prices are not likely to stay as high as $10/\text{gram}$. The construction of additional thermal reactor capacity should lead to more than 100,000 MW\textsubscript{e} in place by the middle 1980's; each megawatt, given present patterns of in-core fission and energy conversion, should produce more than .3 kilograms of plutonium per year so that close to 30 metric tons of plutonium should be available each year for use either in defense systems or in new breeder reactors.\textsuperscript{44} By the mid-1980's there should be more than 150 tons from accumulation over the previous 15 years. Total supplies of natural and by-product fuel then would come to 450,000 tons of uranium and perhaps another 150 tons of plutonium.\textsuperscript{45}

\textsuperscript{43}The construction and use of long term demand forecasts is the subject of the following section; only a single forecast in the middle of those considered most conceivable is discussed here, because the resulting factor prices and marginal costs of capacity are not particularly sensitive to the forecast.

\textsuperscript{44}The projections of installed capacity range from 80,000 to 110,000 MW\textsubscript{e} in the Atomic Energy Commission, \textit{The 1967 Supplement to the 1962 Report to the President on Civilian Nuclear Power}, to more than 125,000 MW\textsubscript{e} by the General Electric Company (\textit{Ibid.}, page 14). The two numbers bracket the Edison Electric Institute forecast of 109,000 to 117,000 MW\textsubscript{e} but exclude the Westinghouse forecast of more than 150,000 MW\textsubscript{e} for total installed nuclear capacity. (\textit{Ibid.}) This last forecast seems less probable, particularly in the absence of theoretical apparatus to support it.

\textsuperscript{45}Where a ton of plutonium has energy equivalent to 600 tons of uranium, this would come to 90,000 additional tons of uranium.
at prices close to $8 per pound of uranium oxide and $10 per gram of plutonium.46

The demands for nuclear fuel are derived from final demand in the markets for electricity and from the costs of alternative fuel systems. The greater the utilization of energy in the form of electricity in the last quarter of the century, the greater the demands for nuclear fuel. But these demands also depend upon the prices of fossil and hydroelectric energy sources to produce this electricity through alternative means, and ultimately on the price of the nuclear fuel and capital themselves. The highest and least elastic demand -- ignoring the substitution of other inputs for nuclear fuel, and indirect substitution of other forms of energy for electricity resulting from higher fuel prices -- would consist of projected high output of electricity times the fuel/output ratio given by advanced technology. Electricity output has been widely and optimistically forecast at close to 8,000 billion kilowatt hours per year and this output would require 1500 thousand electrical megawatts of capacity by the year 2000. More than 600 thousand electrical megawatts of nuclear capacity would have to be constructed if most new capacity were to be dedicated to nuclear equipment.47 Different fast breeder reactor types require

46 At higher prices, the stocks increase -- for uranium, at the rate of a factor of 10 for every $1 per pound increase in the oxide price, for plutonium at an undetermined rate.

47 Cf. Atomic Energy Commission, op.cit., Table 4, for six separate and independent forecasts of electric utility generation in the United States in 1980 and 2000; cf. Table 6 for the Atomic Energy Commission capacity projections for nuclear plants that are required to generate this output from nuclear power sources. The assumption that nuclear plants must be the primary form of additional capacity is critical; it denies technical progress in and intensified competition from fossil fuel sources.
different amounts of plutonium, as has been shown in the basic designs for the 1,000 MWₑ plants; the present value of the stock of fuel in LMFBR is 558 kilograms for 1,000 MWₑ, but for the steam cooled breeder the stock is 2540 kilograms and the gas breeder stock is -350 kilograms. Capital/fuel ratios then vary with the type of reactor⁴⁸ but assuming a uniform distribution of plant sizes over reactor types, the fuel/capacity ratio might be close to 900 kilograms per 1000 MWₑ, so that the forecast additions to capacity would require 540,000 kilograms or 540 metric tons of plutonium if all additions were fast breeders over the period 1980-2000.

Supply in this very simple and highly nuclear-centered model of fuel markets then consists of more than 550,000 tons of uranium and 150 tons of plutonium at prices forecast in the design studies at the time of initiation of construction of breeder capacity in the middle 1980's. The plutonium content should equal 150 tons at the start, but should be increased by 30 tons per annum if thermal capacity installed before 1985 is maintained. Over the two decades from 1980 to 2000, then, the supply of plutonium should exceed 600 tons at forecast prices.

The demands for uranium for thermal capacity are equivalent to the expected supplies at approximately the forecast price. The maintenance of 100,000 MWₑ of thermal capacity installed before 1985 (by replacement with more sophisticated plants in the last 15 years of the century) should generate demands for

⁴⁸And, as has been shown in the estimation of the production function, with the specific power of the individual reactor itself.
roughly 2 tons per electrical megawatt or 200,000 tons of natural supplies. Uranium requirements for blanket supplies accompanying the plutonium core and fast breeders, and non-commercial demands, ought to fill out aggregate demands to rough equality with the 450,000 ton natural supplies at prices bringing forth these supplies.

This is a "strong" demand, "weak" supply forecast, however. Equally reasonable assumptions imply that plutonium demands may well fall short of forecast supplies. Reasonably optimistic demands for fast breeders should exceed 600 thousand megawatts of capacity, to be sure; but concentration on LMFBR and GCBR types -- and not SCBR reactors -- would reduce plutonium requirements to roughly 100 kilograms per 1000 MWe or 60 tons over a twenty-year period. On the supply side, faster construction of lightwater capacity adds to the annual additions of plutonium so as to accumulate more than 600 tons. Since the accumulation is assumed to take place as a by-product from energy production in thermal reactors, additions are price insensitive. As a consequence, the demands for this fuel could very likely fall short of supplies in the future. Even the optimistic forecasts of electricity and nuclear plant utilization might not lead to forecasts for enough fuel demand to clear the market at the $10/gram price; the price might well have to be lower, with the limit at a nominal level of $1 per gram.

These demands account for the present value of net plutonium inventory in breeders under extremely expansive conditions, but not for the initial inventories. These first core requirements No are greater than

\[ F = N_0 \left[ 1 - \beta \left( \beta^{1/2} - 1 \right) \Sigma (1 + r)^{-1} \right] \]

as has been seen. But the greater in-place requirements on the first round can more than be met from the large stock accumulated before 1985 given that fast breeders are accepted at an increasing rate. The equivalent of a plutonium futures market has to hold (footnote continued)
Prices of capital as low as $9(10^4)$ dollars and fuel of $13.1(10^3)$ dollars -- the latter as a result of a short-fall of demand equal to roughly $1/10$ of the stock of accumulated plutonium -- have differing effects upon the marginal costs of capacity in the three types of reactors. Liquid Metal Fast Breeder Reactors cost $80 \%$ as much at the margin if both of these price reductions are realized as in the case of the forecast prices. Lower fuel prices reduce marginal costs by roughly $24$ per thermal kilowatt for the small $156 \text{ MW}_t$ reactor, by $8$ per thermal kilowatt for the medium sized $625 \text{ MW}_t$ reactor, and only $3$ per kilowatt for the full-scale $2500 \text{ mega-watt thermal plant.}$ The lowering of price per unit of capital has less of an effect, since the reduction in costs at the margin for the smaller plant is only $14$ per kilowatt and for the larger plant is $2$ per kilowatt; the effects of both reductions are marginal costs varying between $190$ per kilowatt at $156 \text{ MW}_t$ and $24$ per kilowatt at $2500 \text{ MW}_t$(as shown in Table 15). The costs of the steam-cooled breeder system are not expected to be reduced at all. The low fuel efficiency in this system is responsible for large inventory requirements -- so large that the requirements would outstrip the accumulated volume of plutonium preceding the installation of any large number of steam cooled breeders. (Only if the steam system were integrated into a larger breeder program so that it provided a small

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these stocks for the first few years, then dedicate them in the middle years, then be replaced by newly-bred plutonium in the operating fast reactors in the last years of the century. This optimal pattern would follow from an Atomic Energy Commission policy of holding stocks so that the present values of all future sales prices were the same. To the contrary, a policy of dumping at extremely low prices in the 1980's, coupled with the absence of a market for secondhand plutonium, could lead to prices higher than $10/\text{gram}$ in the late 1990's. Irrational stock release and hindrance of markets could create "shortage"; but this seems not as conceivable, and far more reversible, than a simple supply-demand mechanism.
Table 15

Marginal Costs of Capacity From "Low" Fuel and Capital Input Prices

<table>
<thead>
<tr>
<th>Thermal Megawatts of Capacity</th>
<th>Marginal Costs, $ per Thermal Kilowatt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LMFBR</td>
</tr>
<tr>
<td>156</td>
<td>190</td>
</tr>
<tr>
<td>312</td>
<td>113</td>
</tr>
<tr>
<td>625</td>
<td>68</td>
</tr>
<tr>
<td>1250</td>
<td>40</td>
</tr>
<tr>
<td>2500</td>
<td>24</td>
</tr>
</tbody>
</table>

part of the capacity would this breeder experience low fuel costs.) A concurrent reduction in capital costs would make the marginal costs of steam breeder capacity little different from those shown in the previous table. Then the marginal costs are still $16 per kilowatt of capacity at 156 MWt and $37 per kilowatt at 2500 megawatts. For different reasons, the marginal costs of capacity in the gas-cooled system also change very little following from reduced factor prices. Costs are increased slightly as a result of a fuel price reduction because in these circumstances the gas system is a net producer of fuel faced with lower revenues from sales of nuclear fuel. The costs of the 156 MWt system at the margin are $56 rather than $55 per kilowatt and those of larger systems are increased by a few cents per kilowatt as well. The price reduction on units of capital has the effect of reducing the marginal costs of capacity in this
system as in every other; in this case marginal costs fall below those at either the new or old fuel prices to $50 per kilowatt for 156 MWt, $32 per kilowatt for 625 MWt, and $20 per kilowatt at the largest size of 2500 MWt.

The marginal costs of the three reactor types are brought closer together as a result of the hypothesized lower factor prices. The steam-cooled breeder, given exceedingly low capital expenditures for small capacity, continues to have forecast costs of capacity of $20 - $24 per thermal kilowatt for plants of less than 1250 thermal megawatts. The marginal costs of the other two systems are still higher, up to this size, but now the GCBR has lower costs at 1250 MWt and the LMFBR has 50 per cent lower costs at 2500 MWt. The reduced factor prices and marginal costs, to the extent that they occur, in part dispell the superiority of any particular reactor in costs at the margin over the whole range of relevant sizes.

There are variations in operating performance to account for in cost terms as well as variations in factor prices. The performance variations result from random and short-lived surges of temperature and pressure through the coolant system. The effects of temperature transients are shown in the exponential term \( e^{-\left(\Delta T/T\right)(1 + \xi)} \) in the production function, and these are transmitted to marginal costs as reduced factor productivity. Little can be done to forecast these effects. The temperature transients may be so frequent as to cause variations in capacity two-thirds of the time; more likely, their frequency would not extend beyond 10 per cent of
the time and their level beyond 500° in fuel cores attaining 4500°F. Then a 500° increase in temperature would reduce α by 12 per cent which would increase marginal costs by .3 per cent at all levels of capacity over the lifetime of a Liquid Metal Fast Breeder Reactor plant. The increase in costs in the Steam Cooled Breeder Reactor would be somewhat higher -- by 1.8 per cent of full utilization costs for the same lifetime frequency of temperature variation -- because of the greater relative importance of the constant term in setting the level of marginal cost. The gas cooled breeder reactor could experience costs .8 per cent greater because of these temperature transients. Then the lower range of expected marginal costs for adding to capacity in each type of reactor is shown by the estimates in Table 6-3, and the upper range is shown by the target costs in the design studies in Tables 6-1 and 6-2 where all estimates are increased to .3 per cent higher cost in the LMFBR, 1.8 per cent higher cost in the SCBR, and .8 per cent higher cost in the GCBR.

Other aspects of performance are far less predictable. The production functions may turn out to be a great deal different from those stated here -- either more or less output may be attained from given levels of capital and fuel inputs. But there is no way to assess results contrary to those implied by the design studies. The marginal costs of building and operating breeder capacity have to be assumed to be as shown by the forecast production functions and factor prices.

These are the cost effects from building a breeder reactor of one type or another; but how many breeders will be constructed in the 1980's and 1990's? The answer depends upon the demand for electric generating equipment, the demand for nuclear reactors, and the demand for specific breeder types.
Forecasts of the Demand for Nuclear Reactors

The contracts for fast breeder reactors which will follow on the successful operation of demonstration plants may encompass all of the new commitments for nuclear capacity and even some of those for fossil fuel plants. But there is some chance that they will be very few in number, and only for the rare plant built to the largest possible scale for base load electric generation in large cities. The choice of a nuclear plant, of a fast breeder, and of the type of fast breeder, will depend on the objectives of the electric generating companies and the prices of the various alternative sellers.

The buyer of electric generating facilities is a corporation of great complexity with objectives both so diverse and obscure that widely different purchase patterns for new equipment all seem appropriate. The equipment selected has evidenced the search for more efficient means to produce electricity and to meet both expanded and varying demands; at least the results of new plant and equipment have included an annual rate of productivity improvement of approximately 5.5 per cent since 1900, "more than three times the rate of increased productivity for the economy as a whole" and at the same time "production has increased at about twice the rate of increase in overall industrial production." Yet the companies have departed from an exclusive interest in the least cost plant. Thermal reactors purchased in the early and middle 1960's, at the demonstration or post-demonstration stage, had costs per kilowatt for construction and thermal efficiency

ratings which resulted in net expenditures per kilowatt hour of generation far greater than those in fossil fuel facilities which could have been constructed at the same location. The companies have constructed experimental or demonstration facilities in a number of instances other than when involved in new reactors. The new systems have generally had greater risk of temporary suspension of operations or even of long-term failure. This has been seen not only in the case of nuclear reactors -- in particular, in the suspension of operations at the high-temperature gas reactor plant in Pennsylvania or the liquid metal fast reactor in Detroit -- but also in the newer regional interchange systems where the result has been higher levels of system performance with higher risk of area-wide failure.

A rationalization of these diverse policies follows from treating electric generating companies as profit-making firms subject to severe restrictions on the decisions most greatly affecting such profits. The profits of the company generate the funds for survival; the greater their total 
\[ T = R - C \], with R equal to receipts and C expenditures on all inputs for providing MW of capacity, the greater the chance that questions of survival will be replaced by those on system growth. But state and local regulatory commissions and the Federal Power Commission set maximum levels for profits per unit of capital investment K so that \( (R - C)/K \leq r \), the allowed rate of return. Then purchases of equipment and fuel must be for the purpose of minimizing costs subject to constraints provided by technology and by regulations which emphasize buying capital K to increase profits \( R - C \). Consider costs \( C = f(p_1, K, p_2, F) \) subject to \( \lambda_1(K, F) \) for the generating company's production function and \( \lambda_2 \left( (R - C)/K - r \right) \) for profit regulation.
The minimum cost condition for the amount that the buyer will pay \( p_1 \) for capital \( K \) is not the equality of \( p_1 \) with value of marginal product 
\[ \left\{ \frac{\partial R}{\partial MW} \right\} \left( \frac{\partial MW}{\partial K} \right) \] but with the weighted average of this value product and the allowed rate of return \( r \). The weighting given to \( r \) depends on marginal changes in \((R - C/K)\) allowed as a result of more expenditures on capital inputs -- the greater the change in the allowed rate of return from additional dollars of capital costs, the greater the offer price for more equipment.\(^5\)

The analysis shows the difference between the purchase pattern of a generating company and any other company not under a profit constraint. More is used of capital relative to fuel in the regulated firm;\(^5\) in fact, the producing facilities are capital using and fuel saving even if the total costs of generation are increased thereby. With equipment as the favored input, those items promising lower fuel costs in the future would be even more favored. With research expenditures consisting only of capital -- or as costs amortized as capital costs according to the rules of the

\(^5\) Setting the partial differentials of \( C(p_1, K, p_2, F) + \lambda_1(K, F) + \lambda_2(R - C/K - r) \) equal to zero -- to minimize costs -- results in an expression for \( p_1 \) in terms of \( \left( \frac{\partial R}{\partial MW} \right) \left( \frac{\partial MW}{\partial K} \right) \), \( \lambda_1 \), \( \lambda_2 \), and \( r \) that is additive for the marginal product and the rate of return, with the additive weights given by the constraints \( \lambda_1 \), \( \lambda_2 \). Here \( \lambda_2 = -\frac{\partial C}{\partial r} \) the inverse of "profits allowed to be earned on costs."

\(^5\) The expression for the offer price for fuel \( p_2 \) is in terms of the sales value of this input's marginal product times the ratio of the constraints \(-\lambda_1/(1 + \lambda_2)\). The ratio of marginal products \( (\frac{\partial MW}{\partial K})/(\frac{\partial MW}{\partial F}) \) is not equal to \( p_1/p_2 \) as in the case of unconstrained profit maximization but rather is equal to \( p_1/p_2 \ast \left[ \lambda_2/(\lambda_2 - 1) \right] \left[ r/p_2 \right] \). Given, that \( \frac{\partial C}{\partial r} = -\lambda_2 \), then this last term is positive and relative capital - fuel factor prices are lower than in the absence of the constraint.
regulatory commissions -- then research programs promising reduced costs are the most favored of all.

Past forecasts of demand for nuclear generating plants have not taken account of these characteristics of the buyers. The procedure for forecasting demand for reactors has been to predict the total amount of electricity to be generated, and to follow this with an assessment of the nuclear share of output and thus of capacity. Most individual forecasts have not departed very far from this procedure. Total generation has been estimated from multiplying forecast future per capita electricity consumption by forecast population, where the two forecasts follow from extrapolating the trend of past values or from assuming that the growth of total kilowatt hours bears a direct relation to the growth of Gross National Product. The nuclear share has been derived from assuming that nuclear plants replace each potential fossil fuel plant with projected costs above those from nuclear operations: by adding up the capacities of these potential fossil plants with costs that are "too high," an estimate is obtained of the substitute nuclear capacity, and then of the nuclear share of total capacity.

The forecasts have not recognized much of an economic analysis even in the most complex of calculations. The explicit variables affecting total electricity generation have been population and per capita money income. An inverse relation between capacity and electricity price -- an implication of the marginal productivity theory -- has not usually been taken into account. Moreover, even though the general procedure has assumed that electricity capacity was free of any price effects, it has also assumed that the nuclear share of capacity was determined only by price since electricity generation
was 100 per cent nuclear wherever and whenever nuclear plant prices were lower, and 100 per cent fossil when fossil prices were lower.

Past forecasts have been less than accurate as well -- at least when those setting out estimates for early commitments are measured against actual commitments for capacity to be constructed before 1970. Total commitments by generating companies to nuclear plants in 1967 came to more than 21.1 thousand electrical megawatts to be constructed before the end of 1970. Three forecasts are representative of the tendency so far to underestimate these commitments. First, Mr. Philip Sporn predicted in 1959 that total generation from fission reactors would reach $150 \times 10^9$ kwh by 1975; if this production comes from base load plants operating 7,000 hours each year, then 21.3 thousand electrical megawatts of capacity will be required. This is a modest requirement -- so modest that no additional capacity would have to be built between 1970 and 1975. Second, K. M. Mayer in 1958 used the 1954 FPC regional projections

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53 Cf. the testimony of Mr. Philip Sporn in U.S. Congress, Energy Resources and Technology entitled The Role of Energy and of Electric Energy in the United States, pp. 76 et seq. His projection is based upon a demand for kilowatt hours of electricity and a nuclear portion of capacity to satisfy that demand. Total demand depends on a population "for 1975 ... of 240 million, which is the average of the two highest census bureau projections for that year," and Gross National Product at "a long term growth rate at 3.5% to 1975, or 850 billion 1957 dollars by that year." (Ibid.). The result is a projection of two thousand billion kilowatt hours as the country's electricity requirements in 1975. The author estimates that 150 billion kilowatt hours, or 7.5 per cent, will be generated by nuclear power to that date but that, "atomic power will be able, on an economic basis, to assume the burden" in the 1975-2000 so as to provide for 75 per cent of the increase in generation. This is the extent of the analytical underpinnings for the projection.

and "competitive cost thresholds" for nuclear power of 5-12 mills/kwh to predict nuclear generation of $7.8 \times 10^9$ kwh in 1965 and $44.6 \times 10^9$ kwh in 1970. If nuclear is assumed to be in the base load, then the projections of regional capacity to produce these kilowatt hours will be a total of 6,333 electrical megawatts in 1970; but this again comes to far less than presently outstanding contracts for 1970 nuclear capacity. Third, the Atomic Energy Commission, while generally providing the most optimistic estimates of the long-term share of nuclear in total power production, also has underestimated the short-term additions to nuclear capacity. The 1962 Report on Civilian Nuclear Power contains an estimate of 1,350 megawatts nuclear capacity for 1965 and 5,000 MW for 1970. The first has not been exceeded -- since installed capacity was 1,060 MW at the end of that year -- but capacity operating, under construction, or committed for future construction has exceeded the second amount four times over.\textsuperscript{55}

Any number of reasons can be found for the tendency to underpredict 1960 to 1970 growth in capacity. The exact cause cannot be researched in

Footnote 54 continued

sources, with the purchase of 100 per cent nuclear facilities if fossil costs exceed the nuclear costs. The author identifies "the nuclear competitive threshold in each region as that nuclear power cost necessary in order to displace one billion kilowatt hours per year of conventional generation." (page 162). The thresholds varied between 5 mills per kilowatt hour in Federal Power Commission region VIII and 12 mills per kilowatt hour in regions I and IV in 1965. The author forecasted that generating costs in that year would be approximately 11 mills per kilowatt hour in plants of 50 megawatt size or over and 16 mills per kilowatt hour in smaller plants, so that the competitive threshold would have been reached only in regions I and IV and duplicated only in region III.

\textsuperscript{55} Cf. U.S. Atomic Energy Commission, Civilian Nuclear Power: the 1967 Supplement to the 1962 Report to the President (February, 1967), pp. 19-20, where this early forecast has been revised in keeping with new commitments, but the forecast technique has not been noticeably revised.
detail here, but it is clear that both the relative prices of nuclear fuel and capital, and the effect these have on the nuclear share of capacity, were over-estimated. Nuclear prices were expected to result in generating costs greater than 6 mills per kwh and the nuclear share of demand was expected to be zero where fossil fuel plants resulted in lower costs. Both were wrong: nuclear plants have experienced lower prices in the last two to three years; but when they had higher prices than fossil plants, then nuclear plants were still chosen over the fossil-fueled facilities.

A more complete demand analysis is necessary to forecast even the most current behavior of buyers. The accuracy of longer term forecasts can be improved by constructing a model from all of this decade's experience with nuclear energy. The resulting analytical framework -- revised to account for the results of testing -- can then be used as the basis in forecasting nuclear capacity and production for 1980-2000.

Consider that total new nuclear and fossil fuel burning capacity M in megawatts installed in a given year depends upon the forecast average annual expenditure per kilowatt on plant and equipment P, and the forecast average price of fuel p per kilowatt hour generated in the lifetime of that equipment. The expectation is that \( M = f(P, p) \) is in accord with the law of declining demand for capital inputs in production -- or that installed facilities are more extensive where factor prices are lower.

The additions to capacity depend not only on the price for new capacity -- in this case, on capital and fuel expenditures in the equipment lifetime -- but also on the markets for electricity. Sales of electricity increase with the growth of population, with increased incomes of consumers or -- as a result of higher income -- with the growth of equipment
to use the energy; they are limited by higher prices for this output. Then the demand for capacity can be considered to depend on the long-run demand for electricity \( Q = f(P_e, Y, R) \) with \( P_e \) the price of electricity expected over the lifetime of the plant, \( Y \) the per capita real income, \( R \) the projected population in retail electricity markets; and the annual additions to capacity \( M \) would depend upon the year-to-year differences in \( E \) following from price, income, and population changes. This relation can be taken into account by including estimates of year-to-year changes (termed \( \partial Q/\partial t \) or \( \dot{Q} \) equal to \( \partial f(P_e, Y, R)/\partial t \)) in the demand for capacity so that \( M = f(P, p, \dot{Q}) \).

Additions to generating capacity each year are required not only to meet expanded demands for electricity, but merely to maintain the stock of generating equipment. Depreciation of present equipment results in reductions of stock each year which depend upon the amount of the previous accumulation of equipment; if this reduction is \( M^* \), the \( M^* = f(k) \) where \( k \) is the total accumulation of capital during previous years of electricity generation. The demand for capacity includes both \( M \) and \( M^* \) since replacement is required before expansion. Then total demand \( M = f(P, p, \dot{Q}, k) \) where the megawatts of new capacity include both those for expansion and those for replacement of worn out equipment.

The demand function \( M = f(P, p, \dot{Q}, k) \) can be estimated with data on prices, energy output and capital stock. Each observation could appropriately consist of behavior during a single year in the post World War II period, and least squares regressions of the functional relationships could be computed from these observations. This procedure results in choosing the functional form that minimizes the sum of squared deviations of equation from actual
values of M for the recent past. Forecasts of future capacity are then made by inserting estimates of the future values of these determining variables into the fitted demand function. Some estimates of future values are exogenous to this investigation, such as those for income and population, and are the product of intensive research elsewhere so that they can be used readily here; but the electricity price estimates have to be constructed from the average of past experience with technical change and consequent cost and price reductions in this industry.

The share of future demand for capacity going to nuclear plants depends on relative capital and fuel prices, but to a degree yet to be determined. Nuclear facilities should be installed when nuclear capital and fuel costs are less than those from fossil fuel plants with the same designated lifetime capacities. But nuclear facilities also are expected to be installed when their costs are not less because of preference for the higher capital content in nuclear plants -- and thus the higher rate base for regulated profits -- or because of preference for more research expenditures embodied in new facilities. The analysis at this stage must allow for less than perfect substitutability between types of production facilities; indeed, an attempt should be made to determine the rate of substitution of nuclear for coal or oil facilities as a result of changes in relative prices.

The assertion here is that the proportion of new nuclear to total capacity \( M_n / M \) at any one location depends upon the prices of nuclear equipment \( P_n \) and nuclear fuel \( p_n \), and upon the prices of the cheapest alternative fossil equipment \( P_f \) and fuel \( p_f \). A change in these relative prices should shift some new plants out of or into the nuclear class, but not all plants;
and the approximate extent of the shift can be determined from fitting the general demand relation \( \frac{M_n}{M} = f(P_n', P_n, P_f', P_f) \) to the experience with nuclear capacity additions in the period 1960-1970. The computed equation specifies the nuclear portion of capacity demand now being purchased, so as to provide a framework for forecasting nuclear facilities to be purchased in the period after introduction of breeders. Forecasts are obtained from inserting estimates of future prices into this equation and multiplying the resulting estimate of \( \frac{M_n}{M} \) by total capacity demand.

The initial forecast of the demand for breeder reactors does not follow the analytical framework in complete detail, because estimates have not been generated for each specified variable. A shortened version of the model in keeping with present information has three parts. The first shows the present demand function and projections of capacity installed in the 1980's and 1990's in keeping with this demand function. The second is a forecast of the nuclear portion of this new capacity. The third is an assessment of the change in the nuclear portion brought about by the introduction of fast breeder reactors in those two decades. The first is discussed here, the second and third in the next section of the chapter.

Present demand for thermal megawatts of capacity is obviously related to the prevailing conditions of factor prices and final electricity demand. These conditions may not hold in the future, but the functional relation between capacity demand and prices may be roughly the same now as in the last twenty years of the century so that the present demand function can be used with forecast future prices to forecast future additions to capacity. The measured demands are additions to thermal capacity in electricity-generating systems within any five-year period. New plants
added over this period are comparable because they are for the purpose of meeting future demands for electricity where the future is some years hence, and trade-offs of present for future plants can be made when there is lead time in meeting these electricity demands. The additions to capacity in particular systems making up a "power pool" should also all be roughly comparable; the factors affecting additions at one location in the pool should either be the same as at other locations, or else one of these acquisitions should be cancelled in favor of the least cost, highest output acquisition. Then the market for new capacity is roughly bounded by the limits on power pools and on a five-year period; the relevant markets in the last few years roughly conform to the nine power regions of the Edison Electric Institute in the periods 1958-1962, 1963-1967, and 1968-1972. The total megawatts of thermal capacity installed vary from 73 in Region IV in 1958-1962, to 15,586 in Region V in 1968-1972 (as shown in Table 16).

56. The Edison Electric Institute power generating regions are as follows:
   II. New York, New Jersey, Pennsylvania.
   III. Ohio, Indiana, Illinois, Michigan, Wisconsin.
   IV. Minnesota, Iowa, Missouri, North Dakota, South Dakota, Nebraska, Kansas.
   V. Delaware, Maryland, District of Columbia, Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida.
   VI. Kentucky, Tennessee, Alabama, Mississippi.
   VII. Arkansas, Louisiana, Oklahoma, Texas.
   IX. Washington, Oregon, California.

57. The amounts of capacity installed in each region have been compiled from the megawatt capacity ratings of all new plants shown in that period and region by the Federal Power Commission records. The installations for the period 1968-1972 have been compiled from forecasts and new contracts shown in industry surveys of future demands or from extrapolating the experience in new contracts for delivery in 1968-69 to the entire five-year period.
Table 16: Regional Additions to Thermal Capacity and Electricity Sales Factors, 1958-1972

<table>
<thead>
<tr>
<th>Region</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ions to</td>
<td>1,807</td>
<td>10,128</td>
<td>10,251</td>
<td>3,470</td>
<td>6,720</td>
<td>6,244</td>
<td>6,539</td>
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<tr>
<td></td>
<td>city 1962 (MWt)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>normal population, 1960</td>
<td>10,530</td>
<td>34,287</td>
<td>36,286</td>
<td>15,418</td>
<td>26,094</td>
<td>12,083</td>
<td>17,022</td>
<td>6,913</td>
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<td>of electricity, 1960 (WH)</td>
<td>2.547</td>
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<td>2.223</td>
<td>1.796</td>
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<td>1.789</td>
<td>1.538</td>
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<td>Capita</td>
<td>2424</td>
<td>2570</td>
<td>2385</td>
<td>2068</td>
<td>1833</td>
<td>1478</td>
<td>1809</td>
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<td></td>
<td>ions to</td>
<td>1,343</td>
<td>8,137</td>
<td>5,678</td>
<td>3,563</td>
<td>11,414</td>
<td>5,803</td>
<td>5,997</td>
<td>3,613</td>
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<td></td>
<td>city 1967 (MWt)</td>
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<tr>
<td></td>
<td>normal population, 1965</td>
<td>11,132</td>
<td>36,372</td>
<td>38,143</td>
<td>15,878</td>
<td>28,718</td>
<td>12,810</td>
<td>18,529</td>
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<td>of electricity, 1965 (WH)</td>
<td>2.360</td>
<td>1.895</td>
<td>1.664</td>
<td>2.045</td>
<td>1.613</td>
<td>0.879</td>
<td>1.611</td>
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<td>Capita</td>
<td>2979</td>
<td>3079</td>
<td>2964</td>
<td>2587</td>
<td>2354</td>
<td>1905</td>
<td>2217</td>
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<td>lal income, ($)</td>
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<tr>
<td></td>
<td>ions to</td>
<td>6,075</td>
<td>15,715</td>
<td>15,820</td>
<td>5,758</td>
<td>13,113</td>
<td>8,848</td>
<td>12,365</td>
<td>2,305</td>
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<td>city 1972 (MWt)</td>
<td></td>
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<td></td>
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<td>normal population, 1970</td>
<td>12,136</td>
<td>39,685</td>
<td>41,609</td>
<td>17,316</td>
<td>31,334</td>
<td>13,976</td>
<td>20,313</td>
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<td></td>
<td>of electricity, 1970 (WH)</td>
<td>2.422</td>
<td>1.985</td>
<td>1.592</td>
<td>2.070</td>
<td>1.672</td>
<td>0.693</td>
<td>1.586</td>
<td>1.448</td>
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<tr>
<td></td>
<td>Capita</td>
<td>3,456</td>
<td>3,572</td>
<td>3,438</td>
<td>3,001</td>
<td>2,736</td>
<td>2,210</td>
<td>2,572</td>
<td>2,873</td>
</tr>
<tr>
<td></td>
<td>lal income, ($)</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

source: Federal Power Commission, United States Bureau of the Census, and Bureau of Labor Statistics. The 1970 numbers are forecasts either equal to 5/2 of the difference between 1965 and 1967 observations or (in the case of electricity prices) equal to the 1967 observations.
The most direct explanation of variation in installed capacity is variations in demand for electricity output. If the demand for thermal capacity is in proportion to electricity output, then the determinants of electricity demand are those for capacity demand. The three main determinants are the size of the market for electricity and aggregate income in this market, and the price at which the electricity is sold. Relating the nine regional levels of population, per capita income, and prices of electricity sold in each of the five-year periods (in Table 6-4) to respective additions to thermal capacity shows the effect of these factors. The equation \( \log MW = \alpha + \beta \log P + \gamma \log(Y/N) + \delta \log N \) for capacity installations MW, price of electricity PE, per capita income Y/N, and population N, when fitted to these data by least squares is

\[
\log MW = 2.051 - 0.9836 \log (P \cdot 1000) + 0.9579 \log(Y/N) + 0.9039 \log(N).
\]

Each of the explanatory variables is statistically significant (in that the ratio of computed coefficients to standard errors of coefficients is greater than 2.0) and the three variables together explain approximately 69.8% of total variation in installations of thermal megawatts of capacity \( R^2 = .6979 \). This characterization of capacity demand of electricity companies is quite generally held: the size of the market -- in income and population dimensions -- if not the price of electricity are usually assumed to affect the demand for new capacity. But here the extent of the effects is limited in that price, income, and population elasticities are all less than 1.0, so that any percentage change in one of these variables is not matched by an equal percentage change in capacity additions.

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58 The "elasticity" of demand is defined as the percentage change in megawatt additions divided by the percentage change in the independent
This relation between capacity and determinants of final electricity sales should continue to hold true in the future. There is no reason for the various elasticities of demand to change, even though there are reasons to expect different electricity prices, per capita incomes and regional populations.

Prices of electricity have declined in the past, and it is expected that they will continue to decline between now and 1985 and thereafter. Past experience with electricity prices is given by the equation

\[ P = \alpha_0 + \alpha_1 T + \alpha_2 \text{WPI}, \]

where \( T \) is annual trend and \( \text{WPI} \) is the wholesale price index; least squares regressions of electricity price on trends and \( \text{WPI} \) in each region 1945-1965 show that the value of \( \alpha_1 \) was negative everywhere except in Region IX and that the annual rate of decline varied from .005 cents per kilowatt hour in Region II to .02 cents per kilowatt hour in Region IV. Electricity prices for 1985 and subsequent years can be assumed to be on the separate regression trend lines at the 1965 wholesale price index, at \( T = 41 \) and subsequent values.

Per capita income can be assumed to increase at a substantial rate. The United States has experienced additions to each person's income roughly equivalent to 2.4 per cent per annum in the last half-century and, with increased ability to approach potential full employment output this rate should be increased to close to 3 per cent in the remaining years of this

\[ (\text{cont'd., from p. 70). variable. If, for example, the price elasticity equals } \frac{\partial \text{MW}/\partial \text{MW}}{\partial P/P} \text{ then in the log MW equation this ratio is } \frac{3(\log \text{MW})}{3(\log P)} = \beta. \text{ The estimates of the other elasticities are the coefficients in the log regression equation; they are all less than 1 or greater than -1. Then the demands are, as a first approximation, inelastic.} \]
It is assumed that additions of this magnitude are made in each region to the present per capita income in that region. This preserves the present income distribution for the rest of the century, and probably underestimates demand effects (since relatively higher per capita incomes will likely occur at the larger centers of population).

The population in each power region -- the third independent variable determining future additions to capacity -- can be assumed to increase from roughly 200 million in the United States to more than 266 million in 1985, to more than 311 million in 1995 and to 367 million in 2005. These are forecasts shown by the U. S. Census Bureau. According to these projections, Region I should have 5 per cent of the population in 1985, Region II 17 per cent, and the two other regions from III to IX should have 19, 16, 6, 9, 4, and 14 per cent respectively. If this distribution of population holds

\[ Q = A K^\beta L' \]

so that the percent time changes in real per capita income \( Q'/Q - L'/L = \beta (K'/K - L'/L) + A'/A \). The equilibrium path is that at which there is no net tendency for a priori savings to depart from a priori capital investment. With a fixed savings rate and distribution of income, this is equivalent to \( Q'/Q = K'/K \) which results in the equilibrium \( Q'/Q - L'/L = A'/A \).

With \( \beta \), the proportion of national income received in capital returns, roughly equal to .25, and the annual rate of technical progress \( A'/A \) not likely to be less than 2.25, the equilibrium growth path for per capita income should not be less than 3% per annum.

\[ 1 - \beta \]

\cf. \textit{United States Population Projections, Series B} (U. S. Bureau of the Census, July 1964). These projections are for "medium assumptions" as to economic and technical progress.

\(^{59}\text{cf. J. Kendrick and R. Sato, "Factor Prices, Productivity, and Growth" The American Economic Review (December, 1963) where the annual percent rate of change of real output is 3.15, the annual percent change in population is 0.76, so that the per capita real output increase is 2.39. This is not equal to maximum potential output, since these averages for the period 1919-1960 reflect the depression of the 1930's and subsequent smaller downturns in the late 1940's and 1950's. The equilibrium growth path may well follow the aggregate production function}^{60}\text{cf. United States Population Projections, Series B (U. S. Bureau of the Census, July 1964). These projections are for "medium assumptions" as to economic and technical progress.}
for 1985-2005, with total population increasing at the rates shown, then projections can be made of population increases in each power region through the year 2005. The projections imply modest increases in power demand -- since the regions likely to experience increased shares of total

Table 17: Five-Year Additions to Capacity in Nine Power Regions

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<td>38.2</td>
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<td>46.5</td>
<td>56.3</td>
<td>68.5</td>
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</table>

Source: \[ \log MW_t = 2.051 - 0.9836 \log(P \cdot 1000) + 0.9579 \log(Y/N) + 0.9039 \log(N) \]

for forecast values of P, Y/N, N in each of the nine power regions.
population are going to be highly populated and have the highest per capita incomes, and this technique of estimation from fixed relative population underestimates increases in these regions.

With lower electricity prices, higher personal incomes, and a larger population to spend the per capita incomes, there should be greater demand for additional generating capacity in the 1980's and the 1990's. The demand function, with these forecast values of the independent variables, shows substantial additions to capacity after 1985. Table 17 lists these forecast additions by power region. The low rate of population and income growth in New England, and continued high electricity price, can be expected to retard additions to capacity in Region I; but even here the amounts expected to be added are 50% greater in 1985-1989 than in 1968-1972. The extremely high rate of population and income growth in the north-central region should increase the annual addition to capacity there seven times over by the end of the century. The expected increases in the other seven regions are as substantial -- so that the total added capacity before the year 2000 is expected to be 900,000 thermal megawatts.

What proportion of these additions will be new nuclear reactor plants? This is a matter of the relative prices of nuclear and fossil-fuel-fired plants and of the relative fuel costs for these alternative facilities. Consider the last 15 years of history in nuclear projects to have set a pattern of reaction to relative prices, so that decisions to purchase or not to purchase more reactors in the 1980's and '90's will depend on percent changes in relative prices in the same fashion as at present. The history conforms to the relative share demand schedule

$$\log M_a/M = \alpha + \beta_1 \log(NFP) + \beta_2 \log(NKP) + \gamma_1 \log(FFP) + \gamma_2 \log(FKP).$$
Nuclear capacity was installed in four of the nine power regions in the period 1958-1962, six of the power regions in 1963-1967, and eight of the regions in 1968-1972. Capital and fuel prices for nuclear facilities (NKP and NFP) and for alternative fossil fuel facilities (FKP and FFP) were known before the installation of this new capacity -- the nuclear prices from the plants that were constructed, the fossil fuel prices from actual expenditures on the last fossil fuel plant built in each power region. The additions accounted for by new nuclear plants, and the equation explaining these additions as percents of total new capacity, are shown in Table 18. The explanation is much as expected from the purchase patterns of regulated companies. The higher the price of fossil fuel, and the lower the price of nuclear fuel, the greater the percent of capacity additions that are nuclear. But the important determinant of the nuclear percentage is the relative price of nuclear capital: the higher the price of nuclear capital the lower the demand; but the greater the margin of difference between nuclear and fossil capital prices, the larger the percent

---

61 That is, there are 18 observations of additional nuclear megawatts of capacity, expected prices for this new capacity, and the last experienced prices for fossil fuel capacity. Nuclear fuel costs and alternative fossil fuel costs were not usually specified at the time the contract for the new plant was signed. But calculations of assumed cost can be made from the technical characteristics and assumed rates of operation of the new plants. For the nuclear plants, fuel cycle costs were calculated in each case from \( C = \left( P_{i1}Q_i - P_{j}Q_j \right)/KWH \), where the \( i \)'s designate input quantities and prices for specified concentrations of Uranium 235, the \( j \)'s specify the salvagable present value of this substance in the final inventory, and KWH designates the present value of lifetime production in the plant with expected load factor and equipment durability. These calculations were made with fuel prices and load factors where these were specified; where all necessary parameters were shown, and there was also some discussion of expected fuel cycle costs, the calculations proved to be within 5 to 10% of the assumed fuel cycle costs.
of nuclear capital.\textsuperscript{62} The electricity companies' demands for input in the past not only show some favoritism towards higher priced nuclear capital -- as indicated by far lower elasticities of demand than compatible with perfect substitution between nuclear and fossil capital at identical prices -- but also by increasing favoritism as the gap between nuclear and fossil capital prices widens.

Table 18: Regional additions to Nuclear Capacity and the Explanatory Regression Equation, 1958-1972

<table>
<thead>
<tr>
<th>Region</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
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<td>8-62</td>
<td>482</td>
<td>609</td>
<td>940</td>
<td>73</td>
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<td>--</td>
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<td>3-67</td>
<td>1,473</td>
<td>116</td>
<td>410</td>
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<td>61</td>
<td>--</td>
<td>--</td>
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<td>5,582</td>
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<tr>
<td>8-72*</td>
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<td>13,325</td>
<td>10,413</td>
<td>4,929</td>
<td>15,586</td>
<td>6,286</td>
<td>--</td>
<td>835</td>
<td>6,616</td>
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\( M_n/M = \text{Nuclear MW}_t/\text{total MW}_t \); NFP = nuclear fuel cycle costs; NKP = nuclear capital price; FFP, FKP = fossil fuel and capital prices.

\[
\log(M_n/M) = \alpha + \beta_1 \log(\text{NFP}) + \beta_2 \log(\text{NKP}) + \log(\text{FFP}) + \log(\text{FKP})
\]

\[
\log(M_n/M) = 5.220 - 0.227 \log(\text{NFP}) - 1.347 \log(\text{NKP}) + 0.856 \log(\text{FFP}) - 0.694 \log(\text{FKP})
\]

\( R^2 = .462 \)

*forecast or ordered.

\textsuperscript{62} The percent elasticity with respect to nuclear capital prices is -1.347, so that any given percentage change in price will increase \( M_n/M \) by -1.3 times that percent change. The elasticity of \( M_n/M \) with respect to fossil capital price is -0.694, so that a decrease in fossil capital costs increases the nuclear share ceteris paribus; that is, if both nuclear and fossil capital were to cost 10% less than at present, then \( M_n/M \) would increase by 13.47% as a result of the change in NKP, and 6.9% as a result of the change in fossil capital price as well.
The demand for nuclear reactors in the 1980's and 1990's should not be different in kind from that in the last decade. Relative prices of capital and fuel should differ from those experienced, but the elasticities should not. The period of adjustment to new technology should not be any more complete during the last two decades of this century, because fast breeder reactors will be at the same stage of introduction into existing fossil and light water reactor systems as the light water reactors have been in recent years. The prices of nuclear and fossil capital, and the fuel cycle costs in the two systems, are expected to be the cause of rising nuclear shares. Both nuclear capital prices and fuel costs should fall, and the favoritism of nuclear should be even more evident in the strong reaction to a substantial decrease in the price of nuclear capital. This will be the case for nuclear versus fossil plant purchases and also for light water versus fast breeder installations: the relative prices should result in strong favoritism towards the new reactors. The forecast shares, however, follow only after specific forecasts have been made of prices for each reactor type.

The Economic Effects of Three Alternative Breeder Reactor Programs

The range of marginal costs for constructing and operating the three different types of fast breeder reactors, and the estimated demands for all types of nuclear reactors at various relative reactor prices, show any number of different effects from carrying out reactor research. The lowest marginal costs and the lowest reactor prices can produce buyers'
surplus from fast breeders that far outweighs the cost of carrying out the research. But higher marginal costs -- those shown in Table 14 for expected uranium and plutonium prices -- and prices the same for breeder as for the present light water reactors would result in negligible consumers' surplus from these new projects. The likely cost-price behavior, following from a continuation of recent output and price policies but with more sources of supply, results in surplus between the magnitudes associated with the two extreme cases.

At this stage of development, any one of three reactor projects can be considered likely candidates for a separate and exclusive program to terminate in a commercial demonstration plant in the middle 1980's. The Liquid Metal Fast Breeder, or the Steam Cooled Fast Breeder, or the Gas Cooled Fast Breeder could be funded for the costs of research necessary to terminate in a commercial operation (as shown in the preceding chapter). The single project would be carried out by a set of corporations not now involved in producing light water reactors. Then the company in the business of producing commercial copies of the demonstration reactor would be in "limited competition" with the two major producers of light water reactors and the companies providing fossil fuel electric generating equipment.

The experience with light water reactors sets the framework for "limited competition". The company responsible for designing and constructing the major prototype and demonstration breeder reactor is likely to have a commanding lead in constructing commercial copies. This company could form a collusive organization with those producing other types of reactors, or could go so far in the opposite direction as to reduce reactor prices
to the marginal costs of the lowest cost producer. But the pattern that has been established for price setting seems to come closest to Cournot-type behavior, where the results are positive price-marginal cost differences inversely related to the market demand elasticity times the number of firms.

Prices also depend upon the size of the pieces of equipment needed in each system at each point in time. The Federal Power Commission National Power Survey forecasted continued demands in the 1970's and early 1980's for small generating plants to meet peak loads and to provide safety margins against outage when 1,000 megawatt electric plants shut down for maintenance or repair. But the relative importance of small plants should decline so that by 1980 only 29 per cent of the generating capacity in the country will be provided by plants with ratings less than 500 megawatts thermal, and only 14 per cent will be provided by plants between 500 and 1000 MW_t. It is expected that plants with ratings between 1000 and 2000 megawatts thermal will provide 10 per cent of capacity, and that the largest plants will be the most important of all, with those with greater than 2000 MW_t rating accounting for 44 per cent of capacity at that time.\textsuperscript{63} If this size distribution of plants represents the results from minimizing expenditures on high-priced small plants -- subject to safety requirements -- then additions after 1980 should be made to maintain these proportions. The producer of new reactor equipment should offer prices for plants in each of these size classes.

Consider first the case in which the steam cooled breeder is the

research program. This is the simplest program with the earliest target date for completion of the demonstration plant. The company producing the first commercial steam breeders sets separate prices for each size of plant according to (1) the marginal costs and (2) the demand schedule for each size, and (3) the number and size of other firms offering comparable plants. The marginal costs are expected to be greater for large steam cooled plants than for small, as shown in Table 14. Demand schedules may very well differ from those for new additions to capacity in the 1950's and '60's, but there is no economic reason for this to be the case. The elasticities of \( m_{n}/M \) with respect to alternative capital and fuel prices are assumed to hold for every size of plant, given that decisions as to size are made on the basis of safety margin considerations alone. The number of firms offering reactor types should be equal to the two major producers of Light Water Reactor plus the major producer of the new Steam Cooled Fast Breeder. Then, with the profit margin above marginal costs determined by Cournot pricing conditions, the demand elasticities as in the log \( m_{n}/M \) function, and marginal costs, prices should vary from $28 per thermal kilowatt for the smallest reactor to $63 per thermal kilowatt for the full sized 1000 MWt (or 2500 MWe) reactor and the weighted average price for the expected distribution of reactor sizes should be roughly $37 per thermal kilowatt.

The extent of demands \( m_{n}/M = f(NKP, NFP, FKP, FPP) \) in each power region for this size distribution of plants depends upon the fuel cycle costs and relative prices of alternative plants. The expected advantage from fast breeder systems is relatively lower fuel cycle costs -- an advantage which would be realized if the expenditures on fuel in the marginal
costs of constructing and operating the fast breeder are as forecast. A "pass through" of these expenditures from the company constructing the breeder to the electricity generating company would assure realization of costs from slightly less than .5 to more than 1.2 mills per kilowatt hour of generation. These are the fuel cycle costs in the demand function equivalent to the factor expenditures \( P_F \cdot F \) in finding the marginal costs shown in Table 14 for building and operating the steam cooled breeder in various sizes.

The alternatives to the new steam reactors would include light water reactors at prices set by the two major firms now producing them. These established reactors will probably not be sold in the 1980's and 1990's at presently established prices. There is no reason to expect that there will be no further development of these systems -- that the present working plans and production costs will be frozen for the rest of the century. Rather, "improvements by repetition" in the production of components should guarantee a three per cent to five per cent reduction in costs each year, which should be reflected in at least a two per cent discount of present prices. The continuation of present ways of pricing and market sharing ought to result in reactors of less than 500 MWt being available at $60 per thermal kilowatt in the middle 1980's. Similarly, reactors from 500 to 1000 MWt should sell at $50 per thermal kilowatt, those with 1000 to

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These are also the "estimated equilibrium fuel cycle costs" of the operators of the generating facility faced with the same factor prices and the plant conditions posited by the limited nuclear corporation studies of small steam reactors and the Karlsruhe studies for large reactors. This follows from having derived the production function for Table 14 from these studies. Cf. footnotes 17 and 20 in this chapter for reference to the steam designs.
1750 MW_t capacity should center roughly on $40 per kilowatt, and the full scale 2500 MW_t reactor should be priced at $27 per thermal kilowatt. Twenty years of additional development experience -- as reflected in lower costs and thus lower prices -- makes the forecast average price for the size distribution of reactors equal to approximately $40 per thermal kilowatt. The fuel cycle costs for the light water plants should decline for the same reasons and to the same extent. The forecast fuel cycle costs for reactors to be completed between 1968 and 1972 range from 1.3 mills per kilowatt hour to 2.0 mills; improvements in fuel rod longevity from modest fuel cycle research programs -- either in place of the extensive programs in the fast breeder development projects or as small scale enlargements of these programs -- should establish costs no higher than 1.5 mills per kilowatt hour as the average for the reactors ranging from 250 MW_t to 2500 MW_t in a power region.

The other systems obviously able to provide alternatives to new breeders are steam boilers fired by coal, oil, or natural gas. There is some evidence that price reductions will occur for these non-reactor electricity generating plants, as well. There have been extensive rate reductions in the last decade on the transportation of coal in large volume,65 and -- since these still leave considerable disparity between rates and transport costs66 -- there should be another round of reductions of the same magnitude generated by the "initial competition" of the new reactors.


66Ibid. Cf., for example, Table 3, page 44, for marginal costs of $1.783 per ton and page 133 for a rate of $4.12 per ton on 1962 Trainload Transport from the Clemfield coal district to the generating plant at Eddystone, Pennsylvania.
The results should be fuel cycle cost reductions of .5 to 1.0 mills per kilowatt hour in all regions except VII and VIII.67

A round of capital price reductions cannot be expected to accompany the fuel cycle cost reductions. The largest and newest coal-fired plants now show costs per kilowatt of installed capacity slightly higher than those associated with the plants build in the early and middle 1960's. More pertinent, there is no incremental developmental program promising substantial productivity increases in the near future; only the entirely new technology in magnetohydrodynamics (MHD) "promises to open up new vistas for cost reduction and improved efficiency, and, hopefully, intensive, responsible efforts by the coal and conventional equipment manufacturers to regain an attractive price position"68 which is an uncertain and speculative venture indeed.

With this set of forecast capital and fuel prices, light water reactors could expect to capture some part of the demand for additional capacity after 1985. The choice between these plants of various sizes averaging out to $40 per thermal kilowatt, and fossil fuel plants at prices all somewhat above $30 per thermal kilowatt would favor the first; the lower fuel cycle costs of light water reactors -- with the difference approaching 1 mill per

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67Rate reductions which narrow profit margin for the railroads could be of almost any size. Assuming that the reductions are of the same order of magnitude as on the last round, however, profit margins would fall from roughly 100 per cent to roughly 50 per cent of calculated marginal costs on transport into the large generating stations on the Eastern seaboard. The same reductions elsewhere -- achieved on the first leg of delivery by barge or large volume ship transport -- would bring about cost savings from four cents to more than ten cents per million BTU which are equivalent to .5 mills per kilowatt hour at the least and more than 1.0 mills per kilowatt hour at the most.
kilowatt hour -- would add to the favoritism. The forecast additional nuclear capacity, found by multiplying total expected capacity M by $M_n/M$ for the relevant light water-fossil prices in each region, shows this favoritism. Towards the end of the century, reactors should make up one-third of the additional capacity in regions II and III, and should account for more than one-third of the additions made in regions V and IX. But nowhere can the light water plants alone expect to make up more than one-half the total new installations of plants of all sizes (as shown in Table 19).

Table 19: Capacity Accounted for by Light Water Reactors, 1985-2004

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<td>23.3</td>
<td>28.2</td>
<td>34.2</td>
<td>41.7</td>
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Source: $M = f(P, Y/N, N)$ in Table 18 multiplied by

$M_n/M = f(NKP, NFP, FKP, FFP)$ for prices shown in the text.
The introduction of steam-cooled fast breeders should increase the relative sales of reactors. The lower fuel cycle costs provide an important and substantial basis for increased nuclear demand. The decrease in reactor prices from the entry and establishment of a third major producer would provide another, but not as substantial, reason for an increase in relative sales of nuclear-fueled plants. Fuel cycle costs close to 1 mill per kilowatt hour and capital prices of $37 per thermal kilowatt are expected to take the nuclear share closer to the total forecast additions to capacity after 1985. The additions accounted for by these new costs and prices, as shown in Table 20, could well be actual sales both of steam-cooled breeders and light water reactors with their prices reduced to the levels set by the breeders. The sum total of the two nuclear demands comes to more than three-fourths of the expected needs for capacity in these power regions in most of the five-year periods.

This increased demand shows one dimension of the substantial gains from this breeder for the electricity-generating companies and their consumers.

69 The effects of breeders include the increased capacity demanded of light water reactors, when this increased capacity follows from reduced prices after the entry of the new breeder manufacturer. No attempt is made to forecast the shares of light water and breeder sales following from the lower price, since both result from the "presence" of breeders.

70 In no case is the sum total greater than the five-year addition to capacity shown in Table 17. Two likely courses of action would follow if there were such a case. First, rather than a fixed relation between demands for capacity and electricity output as shown by the function \( M = f(P, Y/N, N) \) more capital and less of other inputs will be used when capital prices are as low as forecast for the steam-cooled breeder reactors. Second, the substantially lower capital prices associated with the new breeder allow reductions in unit cost of producing electricity which are reflected in final sales prices lower than those forecast. Given the present and forecast price circumstances, the relations between factor demand and final prices are fairly well reflected in the two-part demand function used here.
The full measure of gains is shown not by the difference between capacity additions $Q_0$ for the light water reactor systems and $Q_1^*$ for steam-cooled and light water systems together (in Figure 5) but by the area between the two demand curves for these two alternatives. The area under the first demand curve can be measured, assuming that such demand is $M = f(P, Y/N, N)$ times $M_n/M = f(NFP, NK, FFP, FKP)$ for the relevant prices, and that $Q_0$ equals the additions to capacity in each Power Region in Table 20 while $P_0$ equals the light water reactor average price of $40 per kilowatt for the assumed size distribution of plants. The same area can be measured for the additional

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<td>III</td>
<td>10.0</td>
<td>13.2</td>
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<td>IV</td>
<td>3.6</td>
<td>4.7</td>
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<td>9.7</td>
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<tr>
<td>V</td>
<td>10.0</td>
<td>13.1</td>
<td>17.1</td>
<td>23.4</td>
</tr>
<tr>
<td>VI</td>
<td>5.8</td>
<td>9.1</td>
<td>15.3</td>
<td>29.7</td>
</tr>
<tr>
<td>VII</td>
<td>5.6</td>
<td>4.4</td>
<td>10.2</td>
<td>14.2</td>
</tr>
<tr>
<td>VIII</td>
<td>3.6</td>
<td>4.6</td>
<td>5.9</td>
<td>7.6</td>
</tr>
<tr>
<td>IX</td>
<td>12.1</td>
<td>14.7</td>
<td>17.8</td>
<td>20.5</td>
</tr>
</tbody>
</table>

Source: $M = f(P, Y/N, N)$ in Table 18 multiplied by

$M_n/M = f(NFP, NK, FFP, FKP)$ for prices shown in the text. The total million megawatts are then reduced by the amounts shown in Table 19 for the L.W. reactor.
capacity accounted for by steam-cooled breeders by assuming the same demand curves and the appropriate breeder prices, with $Q^*_1$ as shown in Table 20 and $P^*_1$ the average steam cooled breeder price of $37 per kilowatt. The net gains are the differences between the two areas minus the marginal costs of providing the new breeder capacity.\footnote{These marginal costs are not for the given size distribution of plants but for total amounts determined by the regional capacities shown in Table 20. For power region I in 1985-89, the 7.8 thousand thermal megawatts are divided so that 29% of the generating capacity is in plants with less than 500 MW_t, 14% of capacity 500-800 MW_t, 11% of capacity in plants of 800-1000 MW_t, and 44% of capacity in plants with more than 1000 MW_t. Marginal cost curves for each such plant size are summed horizontally for the marginal cost of providing total amounts equal to these percentages times 7.8 thousand megawatts. The marginal costs of providing the entire 7.8 thousand megawatts are equal to the sum of these costs for each size class. This is the case for all other regional additions to capacity for the same size distribution of plants.}

The total of these gains in the 1980's and the 1990's is at least $23 billion dollars. The modest additions to demand in Power Regions I, IV, and VIII limit the gains to close to 500 million dollars in any five year period in each of these regions. But in the Middle Atlantic and far western states with high population and high fossil fuel plant prices, the gains from the new breeder system exceed $1.0 billion in most five year periods. The additions to population and per capita income forecast for the first five years of the 21st century add greatly to demand in that period; as a result, the capacity in nuclear generated by reactor fuel cycle costs and capital prices is much greater than in earlier years. The total net gains shown by the area between the demand functions for light water and breeder reactors should be more than $14 billion in that five-year period alone.\footnote{The calculations are made by integrating the demand curves $D_p$ for the light water reactor and $D_b$ for the breeder reactor between $Q_0$ and $Q_1^*$ (shown in Figure 5). These demand curves are the same simple two-part demand functions $M = f(P, Y/N, N)$ and $M_n/M = f(NFP, NKP, FFP, FKP)$ for the different fuel and capital prices NFP and NKP for non-breeder and breeder reactor types. (footnote continued)}
In some cases buyers' surplus would be generated by lower reactor costs as in Table 15 and, subsequently lower prices. Lower marginal costs from lower fuel and capital prices both add to the gains to the consumer by reducing the costs subtracted from the area under the demand curve and by reducing the level of prices so as to increase the capacity demanded. But in the case of the SCBR, lower marginal costs have no chance of realization -- at least no chance comparable to those for the other two reactor types. The gains shown in Table 21 are the probable economic effects of the SCBR. These effects are substantial. Even in the regions with the smallest population

Table 21: The Gains From Steam Cooled Breeder Reactor Prices and Costs, 1985-2004

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(millions of dollars)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>243</td>
<td>311</td>
<td>398</td>
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<tr>
<td>II</td>
<td>940</td>
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</tr>
<tr>
<td>III</td>
<td>904</td>
<td>1197</td>
<td>1591</td>
<td>2131</td>
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<tr>
<td>IV</td>
<td>328</td>
<td>439</td>
<td>588</td>
<td>795</td>
</tr>
<tr>
<td>V</td>
<td>912</td>
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<tr>
<td>VI</td>
<td>532</td>
<td>830</td>
<td>1344</td>
<td>2699</td>
</tr>
<tr>
<td>VII</td>
<td>507</td>
<td>686</td>
<td>935</td>
<td>1291</td>
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<tr>
<td>VIII</td>
<td>327</td>
<td>420</td>
<td>539</td>
<td>694</td>
</tr>
<tr>
<td>IX</td>
<td>1100</td>
<td>1335</td>
<td>1618</td>
<td>1967</td>
</tr>
</tbody>
</table>

Source: Estimates of the cross-hatched areas shown in Figures 4, 5, and 6.

The integral is net of the total costs associated with the assumed size distribution of plants. These costs are as calculated in the previous footnote, from the production function and factor prices for the "most likely" results from steam-cooled breeder research.

73The supply of plutonium, under fairly pessimistic expectations of new production, is forecast at 600 tons; but the demand consistent with steam (footnote continued)
and lowest per capita incomes, more than $1 billion of gains to equipment buyers follow from selling the assumed size distribution of plants over the period 1985-2004.

The second possibility is that the Gas-Cooled Breeder Reactor will be developed instead of the Steam-Cooled Breeder Reactor. The inauguration of a program for developing gas cooling could well begin within the next three years. The completion of the program cannot be promised to take place at the same time as that of the alternative program, however. The best estimate at this time is that the demonstration reactor based on a gas system would show commercial possibilities at the very end of the 1980's. Then the economic effects of installing gas breeders would be realized in the 1990's and the early years of the 21st century.

The gas breeder would be the third independent source of supply of reactor technology, with the other two sources being accounted for by the two major light water reactor manufacturers. Setting prices for this breeder would be a matter of demand elasticities and the presence of two other sources of reactor supply (as well as the marginal costs of gas cooled plants of the desired sizes). The pricing policy of the manufacturer of the gas cooled plants, based on the existing supplies of the other manufacturers, may be expected to reduce prices for all sizes of plants so that the general level is close to $39-40 per thermal kilowatt; marginal costs vary from $55 per breeders providing the capacity shown above is forecast at 1100 tons because of the poor breeding performance of these plants and the larger number of such plants. As a result, the price of fuel probably will not fall to the minimal $1 per gram assumed for "low" fuel costs.

The fuel cycle costs on the demand side consistent with those shown above in the marginal production costs are somewhat less than .5 mills/kilowatt hour. This estimate is used as NFP; the fossil plant prices FFP and FKP are the same as used to estimate the gains from the steam cooled breeder reactor.
thermal kilowatt for the smallest plants at 156 MWt to $22 per thermal kilowatt at 2500 MWt, and profit margins above marginal cost are expected to be narrowly limited by the presence of the other manufacturers. The effects of these prices on the net gains of equipment buyers -- both for high and for marginal costs -- are shown in Table 22. There are no gains in the 1985-89

Table 22: The Gains From Gas-Cooled Breeder Reactor Prices and Costs, 1985-2004

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(millions of dollars)</td>
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<tr>
<td>I</td>
<td>*</td>
<td>297-323</td>
<td>382-414</td>
<td>491-533</td>
</tr>
<tr>
<td>II</td>
<td>*</td>
<td>1132-1224</td>
<td>1420-1537</td>
<td>1785-1932</td>
</tr>
<tr>
<td>III</td>
<td>*</td>
<td>1148-1241</td>
<td>1527-1653</td>
<td>2046-2214</td>
</tr>
<tr>
<td>IV</td>
<td>*</td>
<td>420-455</td>
<td>564-611</td>
<td>763-826</td>
</tr>
<tr>
<td>V</td>
<td>*</td>
<td>1140-1234</td>
<td>1490-1611</td>
<td>1957-2018</td>
</tr>
<tr>
<td>VI</td>
<td>*</td>
<td>797-863</td>
<td>1339-1448</td>
<td>2590-2803</td>
</tr>
<tr>
<td>VII</td>
<td>*</td>
<td>657-711</td>
<td>897-970</td>
<td>1238-1340</td>
</tr>
<tr>
<td>VIII</td>
<td>*</td>
<td>403-436</td>
<td>516-559</td>
<td>664-719</td>
</tr>
<tr>
<td>IX</td>
<td>*</td>
<td>1380-1385</td>
<td>1552-1679</td>
<td>1886-2042</td>
</tr>
</tbody>
</table>

*Forecast as negligible, since the demonstration plant is not expected to be completed before the end of 1987.

Source: As in Table 21; the first statistic is for forecast costs and prices and the second -- after the dash -- for "low" prices and costs.

---

75 Here as well as above the price-marginal cost difference is assumed to follow the Cournot pattern. Then \( P = MC(n/e) + 1 \) where \( n \) is the number of alternative, equal-size manufacturers and \( e \) is elasticity of demand with respect to capital prices. In this case, \( n = 3 \) and \( e = -1.5 \), the value indicated by the demand percentage \( M_p/M \) revised upwards for the effect of capital price on total capacity of all types.
period because of the late introduction of this type of reactor, but there are approximately $17 billion in buyers' returns in the 1990's and $13 billion in the first five years of the new century from the higher set of forecast prices and costs. The additional gains from the lower marginal costs and prices are not much greater; total gains come to $18 billion in the 1990's, rather than $17 billion, and to $14 billion rather than $13 billion in the 2000-04 period. 76

The effects of the gas breeder are not as substantial as those associated with the steam-cooled fast breeder reactor. The steam-cooled technology results in substantial buyers' gains in the 1985-89 period, while the gas breeder will still be in the process of development, and the gains resulting from the earlier introduction of the steam reactor are not cancelled by demands and buyers' surplus associated with the gas reactor later in this century. Approximately $7 billion of surplus are expected to accumulate from steam reactors in the 1985-89 period while more than $300 million on average in additional gains is realized in any five-year period later on from successful completion of the steam reactor rather than the gas reactor. The only factor to the contrary is the forecast of greater gains from gas cooling then those associated with steam cooling if "low" costs and prices are realized. Even here, $32 billion is forecast for the gas-cooled system under the lower cost conditions as compared to $36 billion for the steam-cooled system in the same period.

76 The small increase in gains from the lower cost obviously follow from the very limited reductions in cost associated with the gas-cooled fast breeder reactor. As was argued in the cost analysis above, the gas breeder is expected to have very low marginal cost for the larger plants as a result of extremely low fuel cycle costs associated with a high rate of breeding. But when fuel prices are reduced, so that net receipts from bred fuel are reduced, then revenue losses from high rates of breeding cancel in part the cost reductions on the fuel that is actually used up in the fission process. As a consequence, very little change in marginal costs of gas systems is expected as a result of lower fuel prices.
If the lower costs are realized, the steam breeder offers the larger total expected gains, and these gains in the earlier periods.

The third possibility, that of developing Liquid Metal Fast Breeder Reactors for all plant sizes, has forecast results remarkably different from those from either gas or steam-cooled breeder reactors. This reactor, ready for commercial adoption both a little sooner than the gas breeder and a little later than the steam breeder, would be sold at prices averaging $80 per thermal kilowatt given extremely low marginal costs at full capacity of 2500 MWt but given very high costs at 125 MWt, 250 MWt and 500 MWt. To meet the offerings of two other major sources of nuclear capacity -- the producers of light water reactors -- the LMFBR would have to be offered "across the board" as the plant to provide the entire size distribution of capacity. The (Cournot) prices consistent with this offering allow for slightly more than 20 per cent profit margins; a price of $62 per thermal kilowatt for the 1250 MWt reactor is greater than the marginal costs of $48 per kilowatt and in keeping with "sharing the sales" with the light water reactor manufacturers, while a price of $37 per kilowatt for the largest reactor is greater than marginal costs of $29 per thermal kilowatt by a similar appropriate amount. These price-cost disparities follow from the presence of other sources of supply -- they maximize profits subject to the supplies of other firms -- but they result in distinct disadvantages in shares for an independent third producer of reactor equipment. They result in relatively small buyers' gains, as well.

77Alternatively, the LMFBR program would be designed to provide only a few sizes in the desired plant-size distribution. The results would imply less than one additional competitor, and capacity adaptions less than half those shown here. Whether the higher price from less competition is cancelled by the lower costs of capacity will be discussed in the next chapter.
The gains generated by the liquid metal breeder are shown in Table 23. They are not comparable to those from the other reactor types. In Power

Table 23: Gains Resulting From Liquid Metal Fast
Breeder Reactor Prices and Costs, 1985-2004

<table>
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<tbody>
<tr>
<td></td>
<td>(Millions of Dollars)</td>
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<td></td>
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<td>I</td>
<td>30-84</td>
<td>38-110</td>
<td>48-141</td>
<td>64-180</td>
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<td>116-332</td>
<td>146-416</td>
<td>183-522</td>
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<td>112-319</td>
<td>148-422</td>
<td>196-562</td>
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</tr>
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<td>IV</td>
<td>40-115</td>
<td>54-154</td>
<td>72-207</td>
<td>99-280</td>
</tr>
<tr>
<td>V</td>
<td>113-320</td>
<td>146-418</td>
<td>192-547</td>
<td>252-720</td>
</tr>
<tr>
<td>VI</td>
<td>66-187</td>
<td>103-293</td>
<td>172-492</td>
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<td>VII</td>
<td>63-178</td>
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<td>40-115</td>
<td>52-148</td>
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<tr>
<td>IX</td>
<td>136-388</td>
<td>165-470</td>
<td>200-572</td>
<td>244-695</td>
</tr>
</tbody>
</table>

Source: As in Table 21; the number before the dash is for "high" costs, the number after the dash for "low" costs, as described in Tables 14 and 15.

Region I during the 1990-95 period, for example, only 3,000 megawatts of liquid metal capacity are expected to be purchased at the $80 per thermal kilowatt average price -- and that as a result of a fuel cycle cost advantage close to 1.6 mill- relative to the light water reactors. The area under the demand curve for the liquid metal reactors up to this level of capacity demand is equivalent to $1,074 million, but the surplus attributable to the light water reactors at their lower capital prices is $1,035 million, so that the gains are $97 million in this power region and time period.
Limited gains are also expected for the other power regions throughout the period 1985-2004—somewhat increased buyers' gains in regions with larger populations and incomes but the same where final demand for electricity is expected to stay close to 1985 levels (as shown in Table 23). Total gains are approximately $3 billion before the year 2000, and $1.7 billion in the first five years of the twenty-first century. The gains are enlarged if the lower costs and prices associated with excess supply in the fuels markets and with reduced capital costs are realized; the totals are $8 billion before the year 2000 and $5 billion for 2000-05. Then the range of buyers' gains from the LMFBR from 1985 to 2004 is from $3.4 billion to $13 billion.

The three independent and separate breeder systems can be compared in terms of effects. The liquid metal system is ill-adapted to providing a range of plant sizes at prices dictated by the competition of light water and fossil fuel plants. The indications are clear that a single and exclusive reactor development program based on liquid metal technology will produce economic gains to the electricity generating companies and their customers, but the gains from the gas reactor in this century are expected to be $24 billion greater under the same market conditions, and the gains from the steam reactor are expected to be $8 billion greater still. The three separate and exclusive programs would seem to promise different results. The choice of one of these programs, or of combinations of them, depends on the balance of research costs against these promised economic effects.
6. THE ECONOMIC FAST BREEDER DEVELOPMENT PROGRAM

The research costs for developing various types of breeder reactors, and the subsequent economic effects from installing commercial copies of these reactors, determine what should be done in the next few years. When the forecast total of the costs exceeds the total of the gains from successful completion of the project, it is clear that the decision should be not to carry out this research. The value of the resources useful for other projects, as represented by total payments or expenditures on this research necessary to draw resources away from the other uses, is greater than the buyers' gains to be generated by this project. In other cases when the total of forecast expenditures does not exceed the gains from research then the choice among projects might be made on the basis of relative costs and gains. Those projects promising the greatest gains for given dollars of research expenditure -- that is, the highest internal rate of return -- are the better projects and should be chosen.

This economic strategy can be laid out for the first round of decisions as to the number and types of fast reactor projects. But comparing separate projects, and combinations of projects, is a matter of comparing

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1 This would seem a necessary, but not a sufficient, condition for considering a project worthy of support. The gains as measured here are social benefits if there are no further costs imposed on others by this activity, if the resulting distribution of benefits is optimal, and if the required rates of production here do not add to misallocation of resources in this industry and elsewhere. These conditions might hold for the forecast capacities shown in the last chapter -- at least there is a finite probability that they hold. But they cannot hold if the gains are less than the research costs.
the distributions of the internal rate of return generated by the research costs and gains in each case.

The initial view of the costs of research for each type of reactor arrived at the conclusion that "high" and "low" expenditures are equally probable but that the probability of "design" research costs occurring is roughly twice that of either "high" or "low" costs. The presence or absence of "research breakthroughs" at any of the separate stages of development could result in the occurrence or non-occurrence of "low" research costs at that stage without the same fortunate circumstances occurring at other stages; with three stages -- fuel development, core development and component development -- and three possible results at each stage, there are 27 probable levels of total research expense. The frequency distribution of costs of research can be expected to show 27 results for each project.

The gains forecast for each type of commercial fast reactor vary over a fairly wide range, as well. The marginal costs of constructing and operating the liquid metal fast reactor, for example, are expected to be "low" if there is reasonable technical progress in the fabrication of capital and if there are excess supplies of plutonium; these marginal costs imply reduced reactor prices which generate additional gains beyond those from the higher "forecast level" of costs. The probability of the lower costs is believed to be roughly equal to that of the higher costs -- of capital and fuel at $30 to $70 per thermal kilowatt -- in the cases of the gas-cooled and sodium-cooled breeder reactors. In the case of the steam-cooled breeder, extensive fuel requirements preclude any but zero probability of lower costs. Then two levels of gains are forecast with equal weight for the first two
reactor types and one value of gains is forecast for the third reactor type as a result of the levels of marginal production costs for reactor manufacturers and resulting capital and fuel prices for reactor buyers.

The research cost levels $C_t$ in year $t$ and later gains $G_t$ have results shown by the rate of return for that project $r$ for which

$$\sum_{t=1}^{\infty} C_t (1 + r)^{-t} = \sum_{t=1}^{\infty} G_t (1 + r)^{-t}.$$ Other levels of costs $C_t^*$ and gains $G_t^*$ generate different rates of return. The rates of return associated with all levels of research costs and gains make up a frequency distribution for a particular reactor project. There are distributions for the liquid, metal, steam, and gas fast reactor projects following the range of forecast research costs and gains for each.

The choice of the most economic projects in fast breeder reactor development comes to selecting those with the highest average and lowest variation in rates of return. Questions of choice are almost unlimited; but there are two that are most basic. If only one project is to be undertaken, then which type of reactor is most economic? If more than one project is conceivable, then how many, and how many competitive projects should be carried out at one time? An exposition of answers can be attempted on the basis of present knowledge and forecasts.

The Most Economic Single Reactor Project

The first application of economic strategy is to the choice of a long-term research project to build a demonstration plant of a single type
of fast breeder reactor. The candidates for exclusivity discussed here are the liquid metal fast breeder reactor, or the steam-cooled fast breeder reactor, or the gas-cooled breeder reactor. The one that has the most desirable forecast distribution of internal rates of return is "economic".²

This "economic" project is shown for the initial forecasts of costs and effects in the last two chapters. The costs of research for each project for 27 probable levels of success set against each of two possible levels for dollar gains³ from research imply the distribution of rates of return for the project. That is, the internal rates of return are calculated to set these research costs equal to the gains in present value terms. There are 27 probable internal rates of return for the steam-cooled breeder reactor -- since there is only a single probable level of gains -- and 54 probable rates of return for the liquid metal fast breeder or the gas-cooled fast breeder. These are summarized in Table 24 over probability intervals (where the probabilities are the products of those for a particular level of research cost and for a particular level of gains).⁴

²The choice is extremely limited, since it is constrained by the necessity to hold the number of projects to a single endeavor, not by the total amount of capital involved in research -- since these amounts differ from project to project -- or by the time period required to carry out the research. But the rationale is that anything less than a single project accomplishes nothing towards reducing nuclear fuel utilization in the last years of the twentieth century (That is, the projects of smaller size than full development of a fast breeder type must promise zero or negative economic returns). The rationale for more than one is explained in the next section.

³The totals for each five year period are assigned to separate years by a straight line interpolation between them for both the case of high prices based on cost as shown in Table 14 and low prices on costs in Table 15.

⁴The discussion in chapter 4 is to the point that the probabilities of "low" and "high" research costs together equal the probability of "design" research costs. The two levels of gains for the liquid metal fast breeder and the gas fast breeder seem equally probable. The single level of gains or surplus for the steam-cooled breeder reactor is the only level for which a probability is subjectively assigned.
Table 24:  The Forecast Rates of Return From Each of Three Reactor Projects

<table>
<thead>
<tr>
<th>Probability of $r &lt; r^*$</th>
<th>SCBR</th>
<th>GCBR</th>
<th>IMFBR</th>
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</thead>
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<tr>
<td>.01</td>
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<tr>
<td>.90</td>
<td>17.9</td>
<td>13.7</td>
<td>12.7</td>
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<tr>
<td>.99</td>
<td>18.3</td>
<td>14.1</td>
<td>13.1</td>
</tr>
</tbody>
</table>

source: calculated as outlined in the Text.

The three distributions of returns are revealing. The liquid metal fast breeder reactor quite evidently offers the lowest expected rate of return and the most variation in possible rates of return. The average or expected rate is 9.3 per cent, while the standard deviation of rates of return in this distribution is 3.1 per cent. 5 This again indicates that the liquid fast breeder reactor is the least efficient choice for capital investment, with the highest risk and variability in annual returns. For high costs and prices, the expected rate is 6.3 per cent, and for low costs and prices, it is 12.3 per cent. 5

5 Table 24 also shows that the distribution is bimodal -- centering on 6.3 per cent for "high" costs and prices and 12.3 per cent for "low" costs and prices. The sharp disparity of results is the combined effect of diseconomies in capital at small scale and of the changes expected in capital costs under "low" versus "high" cost conditions.
metal technology is ill-adapted for meeting the demands for a size distribution of plants that has a large percentage of small plants. But the distribution of rates of return also shows that the research costs of the liquid metal technology are relatively high and the gains are realized relatively late in this century.

The steam-cooled breeder reactor offers the opposite to the results from the liquid metal reactor. The expected rate of return is 17.5 per cent, and the standard deviation of these rates of return is 0.4 per cent. The average is high because the gains forecast from small steam breeders more than compensates for the cost disadvantage at the 2500 MWt level compared to the liquid metal reactor and because the relative simplicity of research implies low research expenditures. The standard deviation of rates is relatively low because "high" and "low" prices are not expected but rather a single "high" level is forecast: the "low" reactor production costs and prices assumed probable for the other two reactor types are not assumed possible in this case so that the variation in the distribution of rates of return is dependent on only the one forecast level of gains from high costs and prices. Then any realization of lower cost -- however improbable that may seem -- would only add to the variation in rates of return by adding to the higher end of the distribution.

The gas-cooled breeder reactor promises results between those of the steam-cooled breeder on the high side and the liquid metal breeder on the low side. The average rate of return is 13.0 per cent, while the standard deviation of rates is 0.5 per cent. The average is less than the steam-cooled breeder, because the gas technology arrives on the scene some
five years later and results in higher costs on smaller reactors not altogether compensated by lower costs on the 2500 megawatt thermal reactor size. The costs of research are somewhat greater as well, and have greater variation; but this variation in expenditures to realize certain results is compensated for by low variation in gains from commercial adoption of gas reactors. The standard deviation of rates, while reflecting somewhat greater risk in research, reflects greater stability in final buyers' gains under the conceivable range of prices and costs.

The most economic project would seem to be the steam-cooled breeder reactor program. The commercial steam reactors of various sizes should meet the demands at the lowest possible cost level only for the smallest sizes, but should be generally close to low costs for all sizes. The research costs for achieving commercial performance are the lowest of all breeder reactor types. As a consequence, the distribution of rates of return forecast for the steam-cooled breeder is higher, and has less variation, than those for the other two potential breeder reactors.

The Economic Combination of Projects

The fast breeder reactor development program could be limited to a single set of research activities designed to produce one type of commercial demonstration reactor. But it is conceivable that history could repeat itself, with the Atomic Energy Commission underwriting initial research once again on a number of separate types of nuclear reactors. The liquid metal fast breeder reactor development program underway since the early 1960's could be continued at the same or faster pace; steam-cooled and gas-cooled reactors
could become major new projects by escalating the private company programs now under way to status as large-scale Federal research programs. Other technologies not discussed here, or not yet beyond sets of equations, could provide plans for building commercial demonstration reactors in the middle 1980's or later. All of these could be undertaken at once, and further decisions made to continue them in the tradition of thermal reactor development: start as many projects as can provide plans, and continue them until the difference between forecast and realized research costs of one or two projects are significantly less than for the others; then delete all the other research projects.

The questions of strategy raised by these possibilities are sharply delineated. The technologies of liquid metal, steam, and gas fast reactors show possibilities for rapid and safe development -- at costs which vary within ranges shown in chapter 4. There is some indication that not only can these reactors be developed, but "consolidated" large scale testing and construction programs can promise the required results at lower costs than separate and independent projects, one for each reactor type, at smaller scale. But the gains from research in the adoption of commercial fast breeders follow in good part from capital and fuel price reductions associated with new technology and with a larger number of alternative sources of supply of reactors. The new technologies promise lower fuel cycle costs for the buyers of generating equipment -- from "passing through" the cost savings in intense utilization of the initial inventory and the bred inventory of fuel -- while the addition of independent sources of reactors reduces the margin between capital prices charged to the buyers and the capital costs of
the reactor manufacturers. The forecast costs indicate advantages for "consolidated" research, while the forecast gains for commercial breeders are expected to be greater when there are "competitive" research projects. The choice requires favoring one of two Atomic Energy Commission goals: projects put together to reduce research expenses, or projects parceled out among commercial companies so as to maximize competition in the final market for reactors.

Choosing the number of reactor projects, and "consolidation" or "competition", can be put in terms of economic effects. The number of projects, and their arrangement, resulting in the most favorable forecast distribution of rates of return, is most likely three projects parceled out to add to competition. This can be seen from the contrasts between rates of return on projects that maximize competition and those that provide the greatest consolidation.

Competitive Fast Breeder Development Projects

One, two, or three new types of fast breeder reactor could be added to the alternatives available to the electricity generating companies in the 1980's and 1990's. The first could come from that company favored with contracts to develop the technology for steam-cooled fast breeder plants, on the presumption that the contractee learns the technology by doing while other companies do not (in the tradition of the development of the two light water thermal reactors: the company building the experimental and demonstration reactors in a particular technology is the one that sells all or almost all of that type of reactor). After the steam-cooled demonstration plant has
been built and a number of copies have been sold, either a liquid metal or a gas-cooled reactor demonstration plant could appear. It is expected that the liquid metal demonstration plant could be constructed successfully in the middle 1980's, and the gas-cooled plant in the late 1980's. Either might be built by a separate and independent reactor manufacturer -- companies other than the two large light water manufacturers -- now carrying out research in these two technologies, and contracts could be let to give these companies a strong lead in further development. This would maximize the number of competitors.

The economic effects from three separate and competitive research projects differ from those expected from single and exclusive projects. The costs of research on a reactor type are the same when that type comprises the only research program (There may be small cost savings in cross reference of results from separate core performance research projects, but these are negligible). But the gains from commercial adoptions following research are different for the second and third projects because reactor sales prices are different. Prices can follow any number of patterns when there are four or five reactor manufacturers each offering plant types for all sizes in each Power Region. Prices might be maintained as if there were still only two firms, given the loyalty of the third, fourth and fifth firms to the policies of the established manufacturers; in this case, the only gains from breeders are found in increased buyers' demands from lower fuel cycle costs. To the other extreme, the presence of five firms might break the Cournot pattern and establish approximately perfect competition. Then prices would approach marginal costs and the buyers' gains would be a maximum (for those marginal
costs and for the costs of research resulting from this scheme of separate projects). Neither extreme seems as likely as continuation of past history. Conformity to a pattern of equal shares, with the shares declining from 50 per cent for each to 25 per cent for each when there are four reactor types, would result in substantial price reductions. Cournot prices would decline from 37 to 32 dollars per thermal kilowatt with the entry of liquid metal breeders as the fourth source of nuclear plants, or from 37 to 30 dollars per thermal kilowatt for the gas-cooled breeder as the fourth source of nuclear capacity for generating companies.

Consider the liquid metal fast breeder program as an addition to a steam-cooled program to reap the benefits from more competition. The steam-cooled program is chronologically "first", appearing to have the capability to construct a working demonstration reactor in the early 1980's, and is also "first" in terms of forecast internal rates of return. The liquid metal project might follow the steam project with a demonstration plant showing good commercial possibilities in the middle 1980's, and with commercial copies available at prices reduced to make sales in the presence of two established light water reactors and the newly established steam-cooled breeder reactor. The commercial copies would be available in sizes greater than 1250 MW only, because smaller LMFBR's could not be priced equal to or below SCBR's and cover the marginal costs of building them. The size distribution of plants would be sold at lower prices, given the prices of the SCBR's for small plants but lower prices for the LMFBR larger plants.6

6 The level of LMFBR prices for plants greater than 1200 MW is forecast on Cournot principles, depending on the elasticity of reactor demand and the presence of four firms selling plants of that size or larger.
The gains from doing so would be equal to the additional buyers' surplus shown by the increased demand generated by lower fuel cycle costs and lower capital prices. These gains, as the integral of the demand function \( Q(N/Q) = f(p_{nf}^*, p_{nk}^*) \) for lower fuel and capital prices \( p_{nf}^*, p_{nk}^* \) minus the integral of the same function for prices for the steam-cooled fast breeder reactor \( p_{nf}, p_{nk} \), are as follows:

<table>
<thead>
<tr>
<th>Power Region</th>
<th>Buyer's Gains, 1985-2004 (millions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>581 to 1212</td>
</tr>
<tr>
<td>II</td>
<td>2166 to 4513</td>
</tr>
<tr>
<td>III</td>
<td>2311 to 4815</td>
</tr>
<tr>
<td>IV</td>
<td>853 to 1777</td>
</tr>
<tr>
<td>V</td>
<td>2259 to 4705</td>
</tr>
<tr>
<td>VI</td>
<td>2165 to 4510</td>
</tr>
<tr>
<td>VII</td>
<td>1357 to 2826</td>
</tr>
<tr>
<td>VIII</td>
<td>786 to 1637</td>
</tr>
<tr>
<td>IX</td>
<td>2389 to 4976</td>
</tr>
</tbody>
</table>

The gains are much greater than those associated with the liquid metal breeder as a single project. It would seem that reduced liquid metal reactor prices are more than compensated for by increased demands, particularly when accompanied by specialization in the range of reactor sizes.

---

\(^7\) They are shown for both "low" and "high" marginal plant production costs and associated plant prices.
where this reactor type has cost advantages. These estimates produce relatively high rates of return. Balancing "high" and "low" gains against the costs of liquid metal research shown in chapter 4 produces rates of return as in Table 25. These forecast marginal rates of return for adding a competitive liquid metal project to an on-going steam-cooled project are on average 15.4 per cent, with a standard deviation of 2.4 per cent. The average is 6.1 points greater, and the standard deviation 0.6 points less, than when the LMFBR is the only breeder reactor project.

Table 25: The Forecast Rates of Return From Competitive Reactor Projects

<table>
<thead>
<tr>
<th>Probability of $r &lt; r^*$</th>
<th>LMFBR (second fast reactor)</th>
<th>GCBR (second fast reactor)</th>
<th>LMFBR and GCBR (second and third fast reactors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>11.8</td>
<td>8.3</td>
<td>9.6</td>
</tr>
<tr>
<td>.10</td>
<td>12.4</td>
<td>8.6</td>
<td>10.2</td>
</tr>
<tr>
<td>.20</td>
<td>12.8</td>
<td>8.9</td>
<td>10.6</td>
</tr>
<tr>
<td>.35</td>
<td>13.2</td>
<td>9.3</td>
<td>11.0</td>
</tr>
<tr>
<td>.50</td>
<td>13.8</td>
<td>10.2</td>
<td>11.7</td>
</tr>
<tr>
<td>.65</td>
<td>17.5</td>
<td>11.8</td>
<td>15.2</td>
</tr>
<tr>
<td>.80</td>
<td>17.8</td>
<td>12.1</td>
<td>15.5</td>
</tr>
<tr>
<td>.90</td>
<td>18.1</td>
<td>12.6</td>
<td>15.7</td>
</tr>
<tr>
<td>.99</td>
<td>18.4</td>
<td>13.0</td>
<td>16.3</td>
</tr>
</tbody>
</table>

Source: calculated as outlined in the Text.
The alternative to the liquid metal fast breeder as the second competitive project is the gas-cooled fast breeder reactor. This alternative is almost as promising. Fast reactor prices set by options to buy gas-cooled plants are forecast to be a bit lower than the combination of steam-cooled and liquid metal prices, with the capital prices an average of 30 dollars per thermal kilowatt and fuel cycle costs as low as .5 mills per kilowatt hour. The economies of scale associated with the larger gas-cooled breeder allows substantial price reductions on large plants in the size distribution of buyers' facilities on the entry of this fourth source of supply; and the lack of substantial dis-economies of small scale -- in comparison with those in the liquid metal reactor -- allow the gas breeder to offer comparable prices for middle-sized reactors in the required distribution of plants. Only the very late entry of this technology reduces the buyers' gains below those in the liquid metal system. The buyers' gains associated with the gas reactor prices, net of those already identified with the steam-cooled breeder entry some years earlier, are as follows:

<table>
<thead>
<tr>
<th>Power Region</th>
<th>Buyers' Gains, 1990-2004 (millions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>516 to 989</td>
</tr>
<tr>
<td>II</td>
<td>1910 to 3656</td>
</tr>
<tr>
<td>III</td>
<td>2079 to 3981</td>
</tr>
<tr>
<td>IV</td>
<td>769 to 1474</td>
</tr>
<tr>
<td>V</td>
<td>2021 to 3869</td>
</tr>
<tr>
<td>VI</td>
<td>2081 to 3985</td>
</tr>
<tr>
<td>VII</td>
<td>1230 to 2356</td>
</tr>
<tr>
<td>VIII</td>
<td>698 to 1337</td>
</tr>
<tr>
<td>IX</td>
<td>2079 to 3981</td>
</tr>
</tbody>
</table>
The gains are somewhat greater when "low" marginal costs for gas reactors bring about somewhat lower reactor prices; shown as the second figure on each line above, the higher estimates of gains are again as much as the lower estimates because the slightly lower fuel costs both reduce marginal costs and substantially increase demand. Balancing all probable levels of research costs against the two probable levels of buyers' gains results in the distribution of internal rates of return shown in Table 25 for this type of reactor. The distribution is lower than for the liquid metal reactor but has less variance; the average rate of return is 10.6 per cent and the standard deviation is 1.5 per cent. The first is two thirds as much, while the second is only six tenths as much, as in the case of the marginal liquid metal project.

Policies for the establishment of more reactor manufacturers might well lead to three competitive programs. But what if there can be only two? This would be to require the choice between liquid metal and gas as a second fast breeder project. Economic strategy within this limited frame of reference does not clearly call for favoring either the liquid metal or the gas fast breeder reactor. The expected or average economic effect, as measured by the average internal rate of return on research, is greater for the liquid metal project than for the other program but the variation around the average is greater for the liquid metal project as well, so that more is expected at greater risk that it will not be realized. Policies for adding to competitors, but strongly against taking "chances" on poor results, would favor the gas program while those for taking "chances" in expectation of much stronger competitors and higher gains particularly on large reactors would favor the liquid metal program.
What are the results from three "competitive" fast reactor programs? Given ambivalence in choice of the second program, the third could be either the liquid metal or gas reactor. Consider, then, no choice at all but rather adding both the liquid metal fast breeder reactor and the gas fast breeder projects to the precedent steam-cooled breeder reactor program. The marginal benefits from two more projects would derive from the increased demand generated by lower prices from adding two more reactor manufacturers each with separate technologies. The producer of LMFBR's would almost certainly be the firm carrying out the last stages of research in liquid sodium technology. This would be a company separate and independent from the two light water manufacturers as well as from the steam-cooled reactor manufacturer. The same would be true of the producer of high temperature gas fast reactors -- this company would specialize in this technology in order to make the number of competitors as large as possible. Five manufacturers following the Cournot pattern for setting prices end up with profit margins of all firms reduced below those for four manufacturers and with more or less equal division five ways of all sales. The large number of alternatives would result in "passing through" the lowest fuel cycle costs -- less than one-half mill per kilowatt hour under the low-cost conditions associated with excess supplies of plutonium -- and offering the buyer the chance to select two or more fast reactor types best able to provide the plant size needed to make up his required size distribution of plants. Capital prices on the small scale steam cooled plants or medium scale gas plants would be expected to preclude contracts

8. "More or less" equal division in this instance follows from expecting that some types will specialize in providing only small or large reactors, so that only firm alternatives are available at any particular size. The sizes in most demand in any five year period will then determine the division of shares of new capacity among the five companies.
for small liquid sodium plants altogether, and the prices on the middle-sized gas plants and the largest liquid sodium plants would be "competitive" in the sense that they would be so close to marginal costs of steam reactors for these plants as to preclude sales of large steam reactors.

The average capital price for the size distribution of plants associated with five alternative reactor sources of supply is forecast at $30 per thermal kilowatt. These lower prices should increase the demand for nuclear reactors, when met in kind by other producers or made equivalent by lower capital prices at their (higher) fuel prices. The increases in demand generate additional buyers' gains when measured against the demands and gains already forecast to follow from the steam cooled nuclear reactor as the initial fast breeder type. The net gains from the LMFBR and GCBR over those of the SCBR are as follows:

<table>
<thead>
<tr>
<th>Power Region</th>
<th>Buyers' Gains, 1985-2004 (millions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>613 to 1247</td>
</tr>
<tr>
<td>II</td>
<td>2282 to 4648</td>
</tr>
<tr>
<td>III</td>
<td>2434 to 4959</td>
</tr>
<tr>
<td>IV</td>
<td>899 to 1830</td>
</tr>
<tr>
<td>V</td>
<td>2379 to 4847</td>
</tr>
<tr>
<td>VI</td>
<td>2281 to 4646</td>
</tr>
<tr>
<td>VII</td>
<td>1429 to 2911</td>
</tr>
<tr>
<td>VIII</td>
<td>827 to 1686</td>
</tr>
<tr>
<td>IX</td>
<td>2516 to 5126</td>
</tr>
</tbody>
</table>

The total is forecast to lie in the range from $15 billion to more than $32 billion. The distribution of internal rates of return setting these gains equal to the sum total costs of separate liquid metal and gas cooled fast
reactor research projects has a fairly low average and relatively wide standard deviation. The average of the returns, shown in the last column of Table 25, is 13.1 per cent and the standard deviation is 2.3 per cent. These are the base dimensions of expected results from adding both the gas and liquid metal project so as to provide three fast breeder reactor types in the 1990's. The dimensions are greater on average -- 13.1 per cent as compared to 9.3 per cent -- and less in standard deviation -- 2.3 per cent compared to 3.1 per cent -- than for the single and exclusive LMFBR program. They show that more is to be gained, in terms of rate of return on a dollar of research expenditure, from the development of the LMFBR as one of three fast reactor projects than from that project by itself.

A Consolidated Research Program for Developing Breeder Reactors

Rather than funding a number of separate attempts to build unique reactor technologies, the Atomic Energy Commission could merge projects so that there was in effect a single development program. The merged program would develop multiple-purpose fuel rods -- those designed for high core performance levels in a number of different coolant environments. Core performance levels for the fuel rods under conditions of either liquid metal cooling, steam cooling, or gas cooling, could be achieved by experimentation in a large scale fast environment test facility. In the last stage, particular components for one or the other of the fast reactor types could be developed on order. The costs of such research are forecast to be equal to the costs of the liquid metal fast breeder reactor program along with "marginal additions" for adding on steam and gas-cooled fast core performance experimentation and component development (as shown by the "marginal
additions" in the summary tables in chapter 4).

The merged program could be run by a single manufacturing company. This company, particularly if it has had a wealth of experience in developing reactors, not only could integrate the work in general fuel research with the separate steps lending to the three new technologies, but also prevent duplication of exactly the same experiments under the guise of "liquid metal research" as compared to "steam research". The single enterprise can be conceived of as producing three types of fast reactors to prevent duplication in research and in adoption of fast breeder plants of different types at the same size, as well. The costs of production of steam-cooled breeders of very small sizes are expected to be considerably lower than those for the other reactor types, so that the only small fast reactors offered would be based on steam-cooled technology. The cost of gas-cooled breeders of medium scale are forecast to be lower than those of other reactor types, so that all plants of 1,000 MW_t to 1,500 MW_t offered for commercial adoption would be gas-cooled. The economies of scale favor the liquid metal fast breeder at full scale; all fast reactor plants of size greater than 2500 MW_t would be copies of the most successful liquid metal fast breeder demonstration design. As a consequence, there would be no competition among fast reactor types nor -- if the company carrying out the final stages of research is one of the two experienced reactor producers -- would there be competition between thermal and fast breeder reactors.

The research costs and gains following from carrying through this most extreme plan for organizing research are impressive. The research costs are forecast to be much less than the sum of the cost for three
separate and independent fast reactor research projects; the sum total of the IMFBR research outlays along with marginal additions for the other two programs are forecast as $2.8 billion when all design targets are realized, while the sum total of costs for three separate programs are $4.3 billion under the same conditions. The gains from adoption of one or the other of the three "exclusive-size" reactors are substantial, but they are also less than the gains under competition because profit margins -- prices minus marginal costs -- are higher for only two firms rather than five. Based on marginal costs for three types of reactors at three separate sizes ranging from $16 per thermal kilowatt at 156 MWt (for an SCBR) to $29 per thermal kilowatt at 2500 MWt (for the IMFBR of that size only), prices, even under extreme Cournot conditions for only two sources of supply, are forecast to range from $23 per thermal kilowatt for the small reactor to $43 for the largest reactor. They are two to six dollars higher than when there are five independent sources of reactor capacity, and average $35 rather than $29 per thermal kilowatt. The gains forecast for buyers of generating stations in the nine Power Regions over the period 1985-2004 are as follows:

<table>
<thead>
<tr>
<th>Power Region</th>
<th>Buyers' Gains, 1985-2004 (millions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>995 to 1572</td>
</tr>
<tr>
<td>II</td>
<td>3710 to 5857</td>
</tr>
<tr>
<td>III</td>
<td>3950 to 6148</td>
</tr>
<tr>
<td>IV</td>
<td>1457 to 2306</td>
</tr>
<tr>
<td>V</td>
<td>3860 to 6107</td>
</tr>
<tr>
<td>VI</td>
<td>3710 to 5851</td>
</tr>
<tr>
<td>VII</td>
<td>2319 to 3666</td>
</tr>
<tr>
<td>VIII</td>
<td>1340 to 2123</td>
</tr>
<tr>
<td>IX</td>
<td>3978 to 6460</td>
</tr>
</tbody>
</table>

9 The first forecast is the sum of the figures in parentheses in the Chapter 4 Summary Tables while the second is the sum of the figures for the separate programs not shown in parentheses.
These gains are large for a single project -- as indicated particularly by the second number on each line. The occurrence of lower nuclear fuel costs, as a result of bred fuel from high-productive gas cooled reactors being used in the low-productive steam reactors, reduces marginal production costs and prices, so that the "high" gains total more than $40 billion. The distribution of buyers' gains between these low and high values, when measured against all probable levels of costs of research in the consolidated program, result in internal rates of return higher than those shown above for an exclusive liquid metal fast breeder project but lower than for other projects.

Table 26: Forecast Rates of Return on a Single Research Program to Develop Three Fast Reactors

<table>
<thead>
<tr>
<th>Probability of ( r )</th>
<th>( r^* )</th>
<th>Internal rate of return ( r^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td></td>
<td>8.4</td>
</tr>
<tr>
<td>.10</td>
<td></td>
<td>9.0</td>
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<tr>
<td>.20</td>
<td></td>
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<tr>
<td>.35</td>
<td></td>
<td>9.8</td>
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<tr>
<td>.50</td>
<td></td>
<td>10.6</td>
</tr>
<tr>
<td>.65</td>
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<td>11.4</td>
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<td>.80</td>
<td></td>
<td>11.9</td>
</tr>
<tr>
<td>.90</td>
<td></td>
<td>12.2</td>
</tr>
<tr>
<td>.99</td>
<td></td>
<td>12.7</td>
</tr>
</tbody>
</table>

source: as in Tables 24 and 25
The range of internal rates of return is shown in Table 26. All of these rates of return exceed those from the base or exclusive LMFBR program, but fall short of the marginal rates of return for an independent steam-cooled breeder program and even for a second independent liquid metal or gas cooled reactor program. The variation of rates of return also falls short of the experience in the separate, independent programs because of the relatively smaller gains made in reducing prices when there are "low" marginal costs -- relatively smaller because the size of the price industries associated with one reactor manufacturer are relatively smaller to start with. That is, the range of probable rates of return has a relatively low average and low variation because of the higher price-cost margin that follows from a single fast reactor manufacturer.

The policy to foster research "in series" without creating any new suppliers of reactors is extreme. As an example case, examination of the policy indicates that buyers' gains are less from moving in this direction than from adding to competition among reactor types. Gains accrue in research cost savings from a highly integrated reactor research program, and from producing specific types of new breeder reactors only for particular sizes of plants. But greater gains accrue from the opposite policy -- from separate programs with higher research costs which result in independent producers of distinct types of breeders -- because the reductions expected to take place in margins between reactor prices and costs result in lower prices even with higher research costs.

Decisions for the Next Decade Given the Expected Gains from Research

The Atomic Energy Commission has assigned the highest priority to "finding and executing the research and development program required for
mastering the technically difficult and challenging sodium-cooled fast breeder reactor concept in a timely manner."10 The application of economic strategy suggests a reappraisal of this priority and a considerable extension of the program beyond its scope.

The economics stress both cost and gains from developing new reactors. The costs of research of the liquid metal program are somewhat greater under most conditions than those expected for the steam-cooled breeder program, and roughly comparable to those from a gas fast breeder program scaled up to promise comparable results. If costs are dominant, so that any single program is limited to the expenditure level forecast for the LMFBR, then the liquid metal project offers the smallest forecast gains from these expenditures. The range of probable rates of return from spending the required amount for a single steam-cooled project is much higher than for the liquid metal project. Development of wholly new gas-cooled fast breeder reactors also promises more than the liquid metal project -- when promise is measured in terms of rates of return implicit in the gains of buyers of electricity generating equipment.

Where the costs are not all-determining, but rather rates of return per dollar of research expenditure can be compared, then taking on more than a single reactor project has strongly positive economic effects. The economic strategy not only requires priority to steam-cooled breeder research, so as to develop an appropriate size distribution of fast breeder plants, but also doing the two other projects "in parallel" so as to add competitors. The frequency distribution of rates of return forecast for the gas-cooled fast breeder as a second, independent fast reactor project

has strong advantages over that for the liquid metal fast breeder reactor as a single exclusive project. The distribution of rates of return on a liquid metal development program as a second, competitive program is better than that from the same technology as a single, exclusive program. If a case can be made for one program and for the LMFBR as that program, then two independent programs present a better case. Three independent programs also present a better case. The two gas and liquid metal programs in parallel following after a steam cooled program offer a higher average and lower variance in forecast rates of return than the single, exclusive liquid metal fast reactor project. More "competitors" from additional reactor projects reduce prices so as to add more to buyers' gains than the additional costs of research in taking on another project or two. Most important, it is not the number of projects but the number of competitors that is most critical. The gains for buyers of generating equipment from independent reactor manufacturers is greater than from a large scale single manufacturer offering types of fast breeders in appropriate sizes even though there are cost savings in research and in production of fast breeder reactors in the single consolidated research program. Lower costs are more than lost in higher profit margins for the single manufacturer, as shown by the lower distribution of rates of return in Table 26.

Economic strategy calls for favoring research on a much larger scale than is now being carried on. Programs should be underway to develop liquid metal fast breeder reactors of large size, gas fast breeders of large and medium size, and steam-cooled fast breeder reactors designed for small plants. Strategy points to funding large scale research and experimentation facilities
to carry out the work on the three reactor types separately, since competitive development projects would appear to promise more for the buyers of equipment in the long run. The three reactors, offered for overlapping size classes of plants, promise marginal rates of return up to three times that of the sodium-cooled project now assigned "the highest priority". For these gains to be realized, it is time to expand the size, the number of reactor types, and the number of firms in the reactor research program. There should be a return to the policies of project selection set in the thermal reactor development program -- to historically proven patterns of selection -- for economic reasons.