AN INTEGRATED DESIGN, CONTROL, AND TRAJECTORY OPTIMIZATION ALGORITHM FOR FUTURE PLANETARY (MARTIAN) ENTRY/LANDER VEHICLES

by

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Thomas E. Bratkovich

Submitted to the Department of Aeronautics and Astronautics on May 13, 1993 in partial fulfillment of the requirements for the Degree of Master of Science

Abstract

Future planetary surface exploration missions will require entry/lander vehicles that are robust enough to provide pinpoint landing capabilities in the presence of atmospheric and aerodynamic uncertainties. An integrated design, control, and trajectory analysis methodology, based on the steepest descent algorithm, is generalized to the planetary entry/landing problem to provide a tool that permits early identification and solution of mission configuration and trajectory trades.

The analysis methodology is demonstrated on the Martian entry/landing problem. Applicable vehicle configuration and aerodynamics models are developed for a multiple stage vehicle, including hypersonic glide and parachute-aided descent flight phases. The vehicle configuration, control strategy, and flight trajectory are optimized with respect to system mass in the presence of dynamic pressure inequality constraints and average and extreme wind profiles. Significant design trade information is obtained, including proposals for increasing vehicle robustness in the uncertain Martian environment. Additionally, strategies to assure realizable solutions and numerical convergence of the analysis algorithm for the general planetary entry/landing problem are discussed and implemented.

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</tr>
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<td>$C_J$</td>
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D  Drag Force (N)

$D_o$  Nominal Parachute (Un-inflated) Diameter (m)

$D_o'$  Inflated Parachute Diameter (m)

da  Aeroshell Conical Base Diameter (m)

dl  Lander Vehicle Base Diameter (m)

$\frac{d C_{Np}}{d \alpha}$  Derivative of the Normal Force Coefficient With Respect to $\alpha$

$f$  Partial Derivative of the Vehicle State Vector with Respect to Time

$f^+_{s_i}$  Partial Derivative of the Vehicle State Vector with Respect to Time, Evaluated at the $i^{th}$ Switch Time with the Vehicle Dynamics Model Nearest the Trajectory "Landing" Condition

$f_{s_i}$  Partial Derivative of the Vehicle State Vector with Respect to Time, Evaluated at the $i^{th}$ Switch Time with the Vehicle Dynamics Model Nearest the Trajectory "Entrance" Condition

$f_p$  Partial Derivative of the Vehicle State Dynamics with Respect to the Design Parameter Vector

$f_u$  Partial Derivative of the Vehicle State Dynamics with Respect to the Control Vector

$f_x$  Partial Derivative of the Vehicle State Dynamics with Respect to the State Vector

$F_x$  Force Component in the Horizontal Direction (N)

$F_y$  Force Component in the Vertical Direction (N)

g  Vector Influence Function of the Design Parameter and Switch Time Perturbations
\( g_0 \) Martian Gravitational Constant (3.7176 m/s\(^2\))

\( H_p \) Partial Derivative of the Hamiltonian with Respect to the Design Parameter Vector

\( H_u \) Partial Derivative of the Hamiltonian with Respect to the Control Vector

\( I_{JJ} \) Scalar Influence Function of Cost

\( I_{ΨJ} \) Vector Influence Function of Constraints and Cost

\( I_{ΨΨ} \) Matrix Influence Function of Constraints

\( J(\tau) \) Mathematical Cost of the System

\( K_{CD} \) Parachute Drag Coefficient Correction Factor due to Effective Suspension Line Length

\( K_{le} \) Parachute Suspension Line Mass per Unit Length (kg/m)

\( K_p \) Aeroshell Structural Mass per Unit Area (kg/m\(^2\))

\( K_Q \) Weight on the Dynamic Pressure Inequality Constraint (Distributed Cost) for the Hypersonic Glide Phase

\( K_{Qp} \) Weight on the Dynamic Pressure Inequality Constraint (Distributed Cost) for the Parachute-Aided Descent Phase

\( K_{α} \) Weight on the Proposed Angle of Attack Inequality Constraint (Distributed Cost) for the Hypersonic Glide Phase

\( k \) Constant of Newtonian Flow, Assumed Equal to 2 for Classical Newtonian Flow

\( L \) Distributed Mathematical Cost

\( L_x \) Partial Derivative of the Distributed Mathematical Cost with Respect to the State Vector
\( L_p \)  Partial Derivative of the Distributed Mathematical Cost with Respect to the Design Parameter Vector

\( L_u \)  Partial Derivative of the Distributed Mathematical Cost with Respect to the Control Vector

\( L \)  Lift Force (N)

\( l_c \)  Parachute Suspension Line Length (m)

\( l_t \)  Parachute Trailing Distance (m)

\( M \)  Matrix Influence Function of the Design Parameter and Switch Time Perturbations

\( M_\infty \)  Mach Number of the Free Stream

\( m_a \)  Aeroshell Mass (kg)

\( m_c \)  Parachute Canopy Mass (kg)

\( m_l \)  Lander Mass (kg)

\( m_{le} \)  Suspension Line Mass (kg)

\( m_o \)  Parachute System Mass (kg)

\( m_{od} \)  Parachute Decelerator Mass (kg)

\( m_p \)  Payload Mass (kg)

\( m_v \)  Vehicle Mass (kg)

\( N \)  Number of Parachute Gores (Lines)

\( p \)  Vehicle Design Parameter Vector

\( p_a \)  Augmented Design Parameter Vector

\( p_{a_0} \)  Initial Augmented Design Parameter Vector

\( Q \)  Dynamic Pressure (N/m^2)

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\( Q_{\text{max}} \) Upper Bound on the Dynamic Pressure Inequality Constraint for the
Hypersonic Glide Phase (N/m²)

\( Q_{\text{maxp}} \) Upper Bound on the Dynamic Pressure Inequality Constraint for the
Parachute-Aided Descent Phase (N/m²)

\( R \) Gas Constant for Carbon Dioxide at Standard Atmosphere (189 \( \frac{\text{J}}{\text{kg} \cdot \text{K}} \),
35.1 \( \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot \text{K}} \))

\( T_{\infty} \) Temperature of the Free Stream (K)

\( t \) Time (s)

\( t_{si} \) \( i^{th} \) Switch Time

\( U \) Diagonal Matrix Containing Functions of Time to Weight the
Different Elements of the Control Vector

\( u \) Vehicle Control Vector

\( u_0 \) Step Function that has a Value of 1 When the Quantity in the
Parentheses is Positive, 0 When Negative

\( V \) Diagonal Matrix Containing Weights on the Design Parameter
Variations

\( V \) Vehicle Volume (m³)

\( V_{\infty} \) Velocity of the Free Stream (m/s)

\( v_x \) Horizontal Velocity Relative to the Fixed Inertial Frame (m/s)

\( v_{xl} \) Horizontal Velocity Relative to the Free Stream (m/s)

\( v_{xf} \) Horizontal Velocity at the Terminal Condition (m/s)

\( v_{xo} \) Horizontal Velocity at the Initial Condition (m/s)

\( v_y \) Vertical Velocity Relative to the Fixed Inertial Frame (m/s)

\( v_{yf} \) Vertical Velocity at the Terminal Condition (m/s)
\( \nu_f \) Vertical Velocity at the Initial Condition (m/s)

\( W \) Horizontal Wind Velocity (m/s)

\( x \) Vehicle State Vector

\( x_0 \) Initial Vehicle State Vector

\( x_f \) Terminal Vehicle State Vector

\( x_i \) \( i^{th} \) Element of the State Vector

\( x \) Horizontal Position (m)

\( x_f \) Horizontal Position at the Terminal Condition (m)

\( x_o \) Horizontal Position at the Initial Condition (m)

\( y \) Vertical Position (m)

\( y_f \) Vertical Position at the Terminal Condition (m)

\( y_o \) Vertical Position at the Initial Condition (m)

\( \alpha \) Angle of Attack, Measured Positive Counter-Clockwise from the Free Stream Velocity Vector to the Vehicle Centerline (°)

\( \alpha_{\text{max}} \) Proposed Upper Bound on the Absolute Value of the Angle of Attack Inequality Constraint for the Hypersonic Glide Phase (°)

\( \gamma \) Ratio of Specific Heats (carbon dioxide) = 1.297 (standard atmosphere)

\( \gamma_f \) Free Stream Relative Flight Path Angle, Measured Positive Counter-Clockwise from Horizontal to the Free Stream Velocity Vector (°)

\( \delta_c \) Cone Semivertex Angle (°)
\( \theta \) Pitch Angle, Measured Positive Counter-Clockwise from Horizontal to the Vehicle Centerline (°)

\( \rho_{\infty} \) Density of the Free Stream (kg/m³)

\( \lambda \) Costate Vector

\( \Lambda_1 \) Matrix that Includes State Dependent Influence Functions of Equality Constraints

\( \Lambda_2 \) Matrix that Includes Design Parameter Dependent Influence Functions of Equality Constraints

\( \tau \) Terminal Time of Trajectory -- Defined as "Entrance" to Martian Atmosphere (s)

\( \phi_1 \) Terminal Cost due to States

\( \phi_2 \) Terminal Cost due to Vehicle Design Parameters

\( \phi_{2,p} \) Partial Derivative of the Terminal Cost due to Vehicle Design Parameters with Respect to the Design Parameter Vector

\( \phi_3 \) Terminal Cost due to Elapsed Flight Time

\( \Psi \) Equality Constraint Vector

\( \Psi_p \) Partial Derivative of the Equality Constraint Vector with Respect to the Design Parameter Vector

\( \Psi_x \) Partial Derivative of the Equality Constraint Vector with Respect to the State Vector

\( \Omega \) Scalar Integration Cutoff Condition

\( \Omega_x \) Partial Derivative of the Integration Cutoff Condition with Respect to the State Vector
Chapter 1

Introduction

1.1 Research Motivation and Generalized Planetary Descent Requirements

Exploration of planetary bodies that have significant atmospheres will continue in the future. In the near term, envisioned surface missions to Venus, Mars, and Saturn's moon Titan all will require atmospheric entry/landing vehicles to transport important scientific packages. To this end, simulation tools are required that can analyze each phase of the planetary missions and identify trades on an integrated system basis. Correct development and utilization of these software tools will ensure that mission design options are thoroughly investigated, and the most effective and efficient alternatives are chosen. Early identification and correction of low performance or insufficiently robust mission configurations will ultimately lead to more capable vehicle designs with higher probability of mission success in uncertain planetary environments.
Atmospheric entry/landing profiles of general planetary bodies may contain several distinct flight phases. Initial atmospheric entry is accomplished by a direct hyperbolic intercept trajectory, or a planned de-orbit propulsive burn from a temporary parking orbit. The mission hardware is contained in an aerobrake or aeroshell configuration that may use ablative materials or an alternative thermal protection system. The high altitude flight is characterized by hypersonic flow regimes through a rarefied atmosphere. The aeroshell may provide forces both parallel and perpendicular to the free stream velocity vector, and some method of vehicle control may be available, such as angle of attack manipulation by attitude thrusters or aerodynamic moment producing devices. As the transonic flow regime is approached, the aeroshell may be jettisoned and a high drag decelerator may be deployed. Parachutes provide excellent drag characteristics at relatively low structure/mass penalties. The parachute-aided descent phase may be uncontrollable, but some mission specifications may require provisions for altering the direction and magnitude of the vehicle forces in this regime. After a large amount of system potential and kinetic energy has been dissipated, and parachute system may be separated, and in some instances an all-propulsive terminal landing phase may be initiated. A propulsive unit would have significant control authority to both provide safe landing velocities and perform final maneuvering to interesting touchdown locations. Depending on mission requirements, these phases may be employed separately or in conjunction to provide adequate descent performance.

The planetary entry vehicle design must be sufficiently versatile to accommodate significant mission constraints. Envisioned landings on a planetary surface will require pinpoint accuracy (within tens of meters) to facilitate
coordination with other missions, especially previously landed cargo flights and other scientific payloads. Alternatively, the vehicle entry and terminal descent profiles may not be exactly known during the vehicle design process, as mission requirements may call for an orbiting platform to locate and evaluate promising landing sites. The possible utilization of autonomous hazard avoidance systems adds further requirements to a precision landing capability. Such pinpoint capacity will place significant constraints on the vehicle configuration and guidance strategy, as a large amount of control authority is necessary to adequately assure mission success.

Planetary entry vehicle specifications are often cast as upper limits on various trajectory variables. The flight specific forces and loads must be constrained to ensure vehicle structural integrity. Additionally, manned vehicles will require more stringent loading limits to ensure the health of the occupants. Finally, structural component mass is often directly relatable to the maximum loads experienced during the entry trajectory. Lessening the structural mass of the vehicle design is advantageous, especially considering that mission launch costs are directly dependent on the system mass.

Thermodynamic heating experienced by the vehicle during different phases of the landing may also be a driving performance variable. Thermal considerations are crucial during the hypersonic glide phase of an trajectory, as aeroshell configurations may require considerable ablative material or a sophisticated thermal protection system. Additionally, advanced parachute designs allow decelerator deployment at supersonic Mach numbers, where thermodynamic heating becomes a significant driver of parachute system configuration and material selection. Finally, vehicle design requirements may be specified in terms of flight dynamic pressure. This
trajectory parameter provides a rough measure for monitoring both the thermodynamic heating and flight loads experienced by the vehicle.

A large amount of uncertainty about atmosphere properties may be present at entry. Planetary bodies have seasonal, diurnal, and equatorial variations in the atmospheric conditions, similar to the Earth. Even with extensive atmospheric research precursor missions, such as the Mars Observer, accurate modeling of these variations may not be possible. The effects of these atmospheric uncertainties may be mitigated by correct selection of the entrance time and location, but extensive vehicle design and control robustness must still be present in the mission architecture to handle such biases. Additionally, random disturbances in the atmosphere will undoubtedly be present. Temperature and density shears will result in wind fluctuations that may have significant impact over the lower altitudes of the trajectory. Finally, expected vehicle aerodynamic response will also be uncertain, as full scale testing in the environment will obviously be impossible. To the extent that atmospheric composition and condition variabilities can be modeled, these uncertainties should be anticipated in any mission configuration. Also, considering that lander/mission control communication links will be severed during the atmospheric entry, and long time delays will result from the planetary distances, the lander design must have enough control authority and versatility to operate in a robust and autonomous manner.
1.2 Research Objectives

The former National Space Council provided an incremental national plan [42] to land humans and scientific payloads on Mars during the second decade of the next century. A host of new technologies must be developed to make this journey feasible, advances that will span the disciplines of modern engineering and the biological sciences. The lengthy distances involved in such a journey may induce up to a 40 minute communication transmission time, requiring most performance capability be completely self-contained in the orbiting and landing vehicles. The entry/landing profile will contain all of the general planetary descent characteristics described above.

The integrated design, trajectory, and control methodology [18], [21] provides a method for investigating these issues. The integrated analysis algorithm specifically treats two-point boundary problems where state dynamics and system performance are highly coupled to the vehicle design configuration, flight controls, and trajectory path. The methodology has been previously applied to numerous analyses of advanced hypersonic air-breathing vehicles [2], [20], [21], [23], and is concurrently being implemented in an analysis of proposed National Launch System capabilities [22]. The Martian entry/landing problem is selected for assessment here to demonstrate the integrated analysis methodology's ability to analyze all aspects of generalized planetary descent missions.
Given this framework, the objectives of this research effort are as follows:

- Generalize the integrated design, trajectory, and control methodology to accommodate the Martian entry/landing vehicle analysis problem

- Explore analysis algorithm numerical stability and convergence issues, including development of peripheral gain controlling software

- Develop representative models of the vehicle configuration and mass, flight phase aerodynamics, atmospheric properties, and extreme wind profiles

- Demonstrate the integrated design, control, and trajectory analysis methodology on a multiple flight phase entry/landing profile, including consideration of vehicle performance and design robustness issues in the presence of atmospheric uncertainties

- Identify additional technical areas of future planetary and high speed atmospheric vehicle/mission designs that are addressable with this analysis methodology

1.3 Overview of Thesis Content

This report documents the research towards completion of the stated objectives. Chapter 2 provides a detailed examination of the capabilities of the integrated design, control, and trajectory analysis algorithm. The simulation computation process is described, a suitable iteration cutoff condition is offered, and strategies for effective gain term management are discussed. The simulation models developed over the
course of this research effort are presented in Chapter 3. Detailed accounts are given of the proposed vehicle geometry, mass, and aerodynamics characteristics. Martian environment models are developed, and a method for incorporating extreme wind uncertainties into the analysis is presented. Chapter 4 describes implementation issues for the Martian entry/landing problem, including formulation of a suitable performance index, state boundary conditions, equality and inequality constraint definition, and state dynamics models.

Demonstration results are first presented for a single flight phase hypersonic glide vehicle in Chapter 5. The effects of a dynamic pressure inequality constraint on the trajectory are explored, and vehicle performance when subjected to atmospheric wind conditions is evaluated. Chapter 6 presents a detailed vehicle performance analysis of the coupled hypersonic glide and parachute-aided descent flight phases. The vehicle is again subjected to dynamic pressure bounds and extreme atmospheric wind disturbances. A brief analysis highlights some of the vehicle capabilities.

A more detailed design sensitivity study is undertaken in Chapter 7. The vehicle design robustness in the presence of atmospheric winds is examined, and model improvements are suggested to limit solution variations. Design and trajectory solution sensitivity questions at the hypersonic glide/parachute-aided descent phase dynamics switch time are also explored. The aeroshell aerodynamics model is evaluated, and the implementation of inequality constraints across separate flight phases is discussed. Model fidelity issues are raised, and feasible improvements are suggested. Finally, a summary of the research results is included in Chapter 8, as well as a list of future research areas to expand the planetary entry/landing methodology capabilities.
Chapter 2

Analysis Algorithm Overview

2.1 Algorithm Features

The integrated design, trajectory, and control algorithm [21] is based on a first order gradient optimization methodology. The algorithm solves a two point boundary-value problem subject to equality and inequality constraints, while minimizing the performance criteria

\[ J(t) = \phi_1[x(t)] + 2(p) + 3(x) + \int L(x(t), u(t), p) \, dt \] (2.1)

with respect to design parameters, controls, and states.

The first term in Equation (2.1), \( \phi_1[x(t)] \), allows inclusion of functions that solely depend on the terminal states of the trajectory as performance criteria. Although not applicable to this effort, this term may be used to minimize the propellant mass of any single- or multi-stage vehicle [19], [21], [22]. The second term, \( \phi_2(p) \), facilitates the incorporation of functions dependent on vehicle design
parameters as performance measures. Expressly exploited in this study, this term provides a direct methodology for optimizing the vehicle configuration (and mass) in the presence of both equality and inequality state, control, and parameter constraints. The final term outside the integral, \( \phi_3(\tau) \) enables inclusion of functions dependent purely on the terminal time of the trajectory. Such functions might penalize any trajectory that is prolonged in duration. The term within the integral, \( L(x(t), u(t), p) \), operates as a distributed performance criteria over the length of the trajectory. The function becomes operational when a user specified performance boundary is violated, and inoperational at all other points in the trajectory. Examples include trajectory inequality constraints on the dynamic pressure, specific force, or thermodynamic heating. The functional implementation of \( L(x(t), u(t), p) \) should be smooth and monotonic with decreasing derivatives as the violation is lessened. Taken together, the terms of Equation (2.1) must define those aspects of the system performance that are substantive when conducting an encompassing vehicle design and control strategy investigation.

The algorithm also treats discrete changes in vehicle dynamics by including the logic to optimize the applicable switch times. These times identify points in the trajectory where different vehicle models, cost functional terms, and dynamics become effective. The times become optimization variables by incorporating each one as an additional term within the design parameter vector. Their values are perturbed on each analysis algorithm iteration using logic included to specifically compute switch time variations.
2.2 The Algorithm Computational Process

The trajectory is integrated in the "forward" direction from a given set of initial states using the vehicle dynamics equations, an initial guess of control and parameter values, and a fourth-order Runge-Kutta integration scheme. "Forward" integration occurs in negative time to simplify handling of some equality constraints. The integration continues until a previously specified monotonic cutoff condition is reached. A specified low altitude "landing" condition constitutes the initial state conditions at time zero; the targeted high altitude "entry" conditions are treated as terminal equality constraints, except for one that is implemented as the integration cutoff condition. Upon completion of the "forward" integration, costate and variational influence functions are integrated in the "backward" direction using the same integration routine. Results of the forwards/backwards computations are used to evaluate control and parameter perturbations prior to iterating the entire process.

The algorithm has been modified to separate the logic for reduction of performance index cost and constraint violations independently. Upon initial entrance to the algorithm, only equality constraint violations are lessened with each iteration. The user specifies an acceptable tolerance on each terminal constraint and its weighting relative to the others. At the end of each forward integration, the weighted root-sum-square of these violations is computed and compared to a user specified limit. Once the root-sum-squared constraint violation is within this limit, the algorithm initiates use of the cost reduction features. Any subsequent violations of the aggregate constraint boundary results in a temporary disabling of the cost reduction mechanisms until the constraint tolerances are again satisfied.
The optimization algorithm incorporates various features that simplify its use. The user may select an initial control history constructed from piecewise-linear segments or use a partly optimized history generated on a previous run. Piecewise-linear initial control histories may be finely or coarsely matched to a desired function, and are computed internally by using given values of estimated maximum trajectory "time-to-fly" as well as control values specified at various percentages of this maximum time. Because the effects of winds are an essential element of the research in this thesis, means were established for the user to change the wind profile, magnitude, and direction between each run, choosing from a previously determined list of profiles.

2.3 A Suitable Iteration Termination Condition

An optimization iteration end condition must be specified to signal achievement of a solution that is sufficiently close to optimal. Kirk [26] suggests a sufficiently small improvement to the cost function between succeeding iterations,

\[ J^{(i)} - J^{(i+1)} < \Delta J_c \]

where \( \Delta J_c \) is a preselected positive constant. Currently the algorithm employs a sufficiently small norm of Hamiltonian partial derivatives with respect to control and parameter variations, modified slightly from [11] to account for variations in design parameters:

\[ I_{JJ} - I_{\psi \psi}^T B C + g^T V g - g^T V M^T B C = 0 \quad (2.2) \]

where

\[ B = (I_{\psi \psi} + M V M^T)^{-1} \quad (2.3) \]
\[ C = (I \Psi J + M V g ) \]  

(2.4)

A partial clarification of these terms from [21] will illuminate the types of information present in the preceding equations.

\( I_{JJ}, I_{\Psi J}, I_{\Psi \Psi}, \) are control parameter influence functions of the form

\[ I_{JJ} = \int_{t}^{0} H_u U H_u^T dt \]  

(2.5)

\[ I_{\Psi J} = \int_{t}^{0} \Lambda_1^T f_u U H_u^T dt \]  

(2.6)

\[ I_{\Psi \Psi} = \int_{t}^{0} \Lambda_1^T f_u U f_u^T \Lambda_1 dt \]  

(2.7)

where \( U \) is a diagonal matrix containing functions of time to weight the different elements of the control variation vector, \( f_u \) is a vector containing the partial derivatives of the vehicle state dynamics with respect to the control variable vector, and \( H_u \) is the partial derivative of the Hamiltonian with respect to the control vector

\[ H_u = L_u + \lambda^T f_u \]  

(2.8)

Note that \( L_u \) is the partial derivative of the distributed mathematical cost term with respect to the control vector, and \( \lambda^T \) is the transpose of the costate vector. \( \Lambda_1 \) is a matrix that includes state dependent influence functions of the equality constraints, and can be defined by the following matrix differential equation and initial condition:

\[ \dot{\Lambda}_1 = - f_x^T \Lambda_1 \]  

(2.9)
\( \Lambda_2^T(\tau) = -\left( \Psi_x \frac{\Omega_x f}{\Omega_x f} \right) I_\tau \) \hspace{1cm} (2.10)

\( \Psi_x \) and \( \Omega_x \) are respectively the partial derivatives of the equality constraint vector and the state integration cutoff condition with respect to the state vector.

The design parameter influence functions \( V, g, \) and \( M \) also require additional specification. \( V \) is a diagonal matrix containing weights on the design parameter influence functions. \( M \) is a matrix

\[
M = \left[ \begin{array}{ccc}
\Lambda_2^T(0) & \Lambda_1^T(t_{s_1})(f_{s_1} - f_{s_1}) & \Lambda_1^T(t_{s_2})(f_{s_2}^+ - f_{s_2}) & \ldots
\end{array} \right]
\] \hspace{1cm} (2.11)

that contains the design parameter dependent influence functions of the equality constraints \( \Lambda_2 \), defined by

\[
\dot{\Lambda}_2 = -f_p^T \Lambda_2
\] \hspace{1cm} (2.12)

\[
\Lambda_2^T(\tau) = -\Psi_p
\] \hspace{1cm} (2.13)

and the switch time dependent influence functions containing the difference in the vehicle dynamics vector on either side of the \( i^{th} \) switch time \((f_{s_i}^+ - f_{s_i})\). The vector \( f_p \) is the partial derivative of the vehicle state dynamics with respect to the design parameters. The vector \( g \) is defined.
where $H_p$ is the partial derivative of the Hamiltonian with respect to design parameters

$$H_p = L_p + \lambda^T f_p$$ (2.15)

Finally, $\phi^T_{2p}$ is the transposed partial derivative of the previously defined design parameter dependent terminal cost term with respect to design parameters.

### 2.4 Algorithm Gain Term Management

Many gains and metrics are required to ensure stable convergence to the optimal solution. They trade the relative contributions to the performance index of the terminal and distributed cost terms. They also weight the relative importance of each individual constraint, and scale the relative effects of different controls and parameters. Finally, gains are used to manage the algorithm convergence step size. Some examples of the numerical effects of gain variations are provided in the following subsections.
2.4.1 Equality Constraint Emphasis ($C\psi$)

Lessening the equality constraint emphasis gains $C\psi$ slows algorithm convergence, but increases the stability by providing "awareness" of the terminal constraint boundary throughout the trajectory. An example of this effect is clearly seen in Figure 2-1, which plots angle of attack versus time from two optimization runs. The higher constraint gain case diverged toward a solution with a large angle of attack transient required to satisfy the equality constraints. The lower constraint gain case converges uniformly towards the correct solution.

![Figure 2-1. Effect of Lessening Equality Constraint Emphasis Gains $C\psi$](image)
2.4.2 Inequality Constraint Emphasis (K_Q)

Increasing the relative weighting of the inequality constraint violation cost (K_Q) relative to the terminal cost can retard algorithm convergence, but augments algorithm stability by providing "awareness" of the inequality constraint throughout the trajectory. Figure 2-2, which plots the dynamic pressure versus time for two optimization runs, illustrates the effect. Here the dynamic pressure inequality bound (400 N/m²) is moved backward in time with the reduced gain (and is also accompanied by a decrease in the magnitude of the angle of attack as seen in Figure 2-3). Manipulating the inequality constraint gain can also alter the optimal vehicle configuration.

![Figure 2-2. Effect of the Relative Weighting on the Dynamic Pressure Inequality Constraint (K_Q) on Vehicle Dynamic Pressure](image)
2.4.3 Requested Cost Improvement \((C_J)\) Selection

Another algorithm gain, the desired performance index cost improvement sought for each iteration \((C_J)\), must change according to the current position within the cost space. Large values of \(C_J\) are required when the algorithm is far from the solution and the cost \(J(\tau)\) is large. Significant reduction of this step size must occur as a minimum cost solution is approached or succeeding variations will initiate instability. To accomplish these reductions, a search routine has been added to the optimization algorithm that explores the effect of changes in \(C_J\) on cost improvement. In the routine, the user specifies both the number of different step size values to investigate and the strategy and range for the search. The routine computes the new
control history and design parameter variations for each $C_J$ value, and then integrates the states for each computed variational set to obtain improvement information. This mechanism currently eliminates a major source of numerical instability, and may be applied to other gains if necessary.

Two separate versions of this search routine are currently incorporated within the algorithm. The simplest mechanism operates when the inequality constraint is inactive (a distributed cost contribution of zero) and the cost reduction steps solely influence the terminal time, state, or design parameter dependent terms of the cost function. The step size that provides the greatest cost improvement within a user supplied constraint violation is chosen as the new $C_J$ for subsequent iterations. This strategy functions properly when 6-8 different step sizes are investigated over a narrow range both above and below the current step size. The routine is called once every 10 - 20 iterations to ensure stable convergence when both the equality constraint criterion are met and the inequality constraint is inactive. As the solution is approached, the searching routine is called even more often, preventing numerically unstable step sizes.

A more complicated version of this searcher operates when the inequality constraint is active (non-zero distributed cost contribution). When this term of the cost function is large relative to the terminal terms, logic similar to that described above is employed to gradually eliminate violations of the inequality constraint. However, when the inequality constraint violation is small, the algorithm becomes very sensitive to the step size $C_J$. The algorithm may assign design parameter and control variations in an entirely different manner depending on whether the inequality constraint is slightly violated or completely inactive. It is therefore advantageous to
eliminate slight violations of the inequality constraint as quickly as possible to ensure timely convergence. Assigning a large value to $C_J$ in this case will either completely eliminate the inequality constraint violation at the risk of algorithm instability or cause an increase in the cost associated with this term due to a "step-through" of the inequality constraint minimum. On the other hand, a very small $C_J$ in this region of the cost space provides stable but ineffectual algorithm convergence. An effective strategy would find a step size that is large enough to completely eliminate the inequality constraint violation but small enough to neither initiate a minimum "step-through" nor cause instability due to the computed perturbations.

The search routine employed for the inequality constraint cost reduction step size attempts to locate a $C_J$ that provides the greatest cost improvement within a user supplied equality constraint violation. If none of the searched values meets this criteria, the routine selects the step size that provides the greatest equality constraint violation below the user defined limit. The logic ensures that regions of increasing inequality constraint violation ("step through") are quickly exited and another attempt at complete elimination of the inequality constraint violation may be executed in a timely manner. This strategy functions properly when 3 - 5 different step sizes are investigated over a broad range (factor of 100) both above and below the current step size. The routine is called once every 1 - 3 iterations to ensure stable convergence when both the equality constraint criterion are met and the inequality constraint is active. When the inequality constraint violation is exceptionally small, the search routine must be called immediately and the base $C_J$ value around which the search is conducted must be reset with a smaller value computed relative to the violation.
2.4.4 Metrics

The current integrated analysis algorithm also allows the user to alter metrics that weight the relative importance of the variations in design parameters and the control history. The region of numerically stable convergence within the metric space can be different for each problem, and the metrics often require changes as the solution converges. A routine has been written that allows the user to compute distinct linear weightings in time on each control according to the current region within the cost space. A similar routine allows the user to separately specify the design parameter metrics depending on the proximity of the algorithm to an optimal solution. Utilization of these features requires significant user experience with the types of convergence paths that the algorithm can negotiate through the cost space. The step size searching routine described previously can also be automated to accommodate metric control. However, research for this thesis has not required the additional algorithm flexibility.
Chapter 3

Simulation Models

3.1 Introduction

Throughout the course of this research effort, the primary objective has been to demonstrate the practicality of utilizing the integrated design, control, and trajectory algorithm for the Martian entry/landing problem. In accordance with this guideline, highly accurate model information has been traded for representations that enable smooth encoding and uncomplicated analysis. Although this approach sacrifices some fidelity, advances are obtained in a broad exhibition of the algorithm's ability to deal with the important aspects of a general planetary entry descent with an atmosphere. The resulting analysis then does not provide detailed information for one specific mission, but rather a characterization of a class of vehicle designs across a range of entry profiles.

The simulation models are therefore as simple as possible while still retaining representative vehicle design and performance characteristics. Closed-form analytic
equations are used wherever available. Otherwise, individual sets of data are matched with empirical curve fits that yield analytic partial derivatives for implementation within the algorithm. Many of the models presented below have been updated based on simulation results that depicted glaring inaccuracies in their original assumptions. Recognizing that the modeling process is iterative in nature, significant leeway for improvements in the current models still exists. Some suggested model advancements may be found within Chapter 7.

3.2 Vehicle Geometry and Mass

3.2.1 Aeroshell Geometry

The integrated design, control and trajectory optimization algorithm, as applied to the Mars entry analysis problem, calls for parametric models of vehicle geometries that provide representative performance during the hypersonic glide and parachute-aided descent phases in the Martian environment. The hypersonic entry phase vehicle architecture is modeled as a constant volume cone with variable half angle $\delta_c$, as depicted in Figure 3-1. Designating a constant volume reflects an assumed requirement for the storage of the landing systems. The conical aeroshell encapsulates all useful mission equipment, including the lander (thrusters, fuel tanks, propellant, science package, communications, etc.), thermal protection system, and parachute deployment system. Utilization of this uncomplicated aeroshell configuration enables smooth implementation of the algorithm for a model of few variable parameters. Modeling of the vehicle forces is additionally simplified by the readily available aerodynamic data for supersonic flow around a cone.
3.2.2 Parachute/Lander Geometry

The parachute-aided descent phase initiates with the jettison of the conical aeroshell. Figure 3-2 portrays the simplified parachute/lander system with relevant dimensions. Initial literature searches suggest that most characteristics of the parachute can be inferred from the nominal un-inflated diameter, $D_0$.

The parachute deceleration phase is assumed to operate in the flight corridor between Mach $\sim 2.5 - \sim 0.2$. This wide range is selected with thought to the variable nature of the parachute deployment time within the optimization process. The selected parachute type is the disk-gap-band (12.5% geometric porosity), shown in Figure 3-3 from [6]. This parachute has been broadly investigated in both wind tunnels and flight tests, primarily as the design choice for the actual Viking [5], [16], [39], [41] and a contractor proposed Mars Rover Sample Return landers [33]. The disk-gap-band parachute incorporates structural strength at supersonic Mach numbers with adequate stability, aerodynamic, and thermodynamic characteristics.
Figure 3-2. Parachute/Lander Geometry
Disk-Gap-Band. The canopy is constructed as a flat circular disk and a cylindrical band separated vertically by an open space. A gore consists of a triangular top and rectangular bottom as illustrated. The disk, gap and band areas are 63 percent, 12 percent and 35 percent respectively of the total (nominal) area $S_0$. Data for a specific disk-gap-band parachute and load configuration are given below. Polyester materials were used for the Viking 63 ft diameter parachute to withstand the effects of heat sterilization and densely packed storage until deployed in the Martian atmosphere.

$$h_1 = \left[ \frac{S_0}{1.887 \frac{N}{\tan(180/N)}} \right]^N$$
$$h_2 = 0.113 \ h_1$$
$$h_3 = 0.33 \ h_1$$
$$e_s = 2h_1 \tan(180/N)$$

**Figure 3-3. The Disk-Gap-Band Parachute [6]**
3.2.3 The Aeroshell Mass

The vehicle mass at the atmospheric entrance condition can be defined as the sum of the fixed lander mass, the aeroshell mass, and the parachute system mass

\[ m_v = m_l + m_o + m_a \] (3.1)

The lander mass consists of a constant lump sum of the fuel tanks, propellant (needed in terminal propulsive descent), and expected useful landed mass

\[ m_l = 950 \text{ kg} \] (3.2)

During the parachute-aided descent phase the system mass is the sum of just the parachute system mass and the lander mass

\[ m_p = m_l + m_o \] (3.3)

The aeroshell mass is defined as

\[ m_a = \frac{\pi K_p d_a^2}{4} \left( 1 + \frac{1}{\sin \alpha} \right) \] (3.4)

where \( K_p \) is the structural mass defined in kilograms per square meter of aeroshell exterior surface area. \( K_p \) is in general a function of both the specific force and thermodynamic constraints imposed on the vehicle, but for simplicity in this analysis, \( K_p \) is established as a predetermined constant 10 kg/m\(^2\). The cone base diameter \( d_a \) can be written as

\[ d_a = \left[ \frac{24 (\tan \delta_c) V}{\pi} \right]^{\frac{1}{3}} \] (3.5)
where $V$ is the prescribed fixed volume of the vehicle, which is set at 200 m$^3$. The resulting aeroshell mass depends only on the cone half angle $\delta_c$, one of the design parameters that is allowed to vary in the analysis algorithm. As depicted in Figure 3-4, the aeroshell has a minimum mass of 2083.8 kg at a cone half-angle of 19.5°. This angle becomes an important benchmark for measuring the performance of a design, as optimized solutions tend towards the aeroshell mass minimum in the absence of constraints.

![Aeroshell Mass versus Cone Half Angle](image.png)

**Figure 3-4. Aeroshell Mass versus Cone Half Angle**

### 3.2.4 The Parachute Mass

The mass of the parachute system can be approximated by defining the following geometrical properties. Although the lander mass is constant, the lander diameter $d_l$ is assumed to be 80% of the aeroshell base diameter,
Parachute aerodynamic considerations (discussed in the next section) require the parachute trailing distance \( l_t \) to be defined

\[
  l_t = 8.5 \cdot d_l
\]

The simple geometric considerations of Figure 3-2 then automatically determine the suspension line length \( l_e \) as

\[
  l_e = \sqrt{\left(\frac{D_o'}{2}\right)^2 + l_t^2}
\]

The inflated disk-gap-band canopy diameter, \( D_o' \), is approximately 72\% of the nominal diameter [5]; using this information, the suspension line length becomes

\[
  l_e = \sqrt{\left(\frac{0.72 \cdot D_o}{2}\right)^2 + (8.5 \cdot d_l)^2}
\]

Pioneer Systems, Inc., outlines the basic structural components of a deployable disk-gap-band parachute system [33]. Tailoring their analysis to this report's study results in parachute system mass that may be approximated as a function of the canopy nominal diameter \( D_o \) and the lander diameter \( d_l \). The cloth canopy is constructed of one or more high strength, heat resistant cloths, usually Nylon, Dacron, or Nomex. The highest strength and temperature performance is obtained from Nomex, however the unit mass \( K_c \) of this cloth is greater than that of Nylon. The Viking landers used disk-gap-band parachutes constructed solely of Nylon [5], [16], [41], while a structural analysis for the Mars Rover Sample Return [33] dictates the
upper disk to be constructed of Nomex and the band of Nylon. Simplifying the model, the canopy is assumed to be 100% Nomex cloth \((K_c = 0.0983 \text{ kg/m}^2\), rated strength 213.6 N/cm\) for a conservative accommodation of parachute loads and heating. The mass of the canopy reflects the 12.5% geometric porosity of the disk-gap-band design

\[
m_c = 0.875 K_c A_p
\]  \hspace{1cm} (3.10)

where \(A_p\) is the parachute canopy area.

The number of gores, \(N\), is chosen to be 72, providing the maximum flexibility in rigging design. The material selected is 4.448 kN per line Kevlar, with a unit mass of \(K_{le} = 0.00397 \text{ kg/m}\). Again, this represents a conservative estimate of the required strength of each line. The mass of the suspension lines is

\[
m_{le} = N K_{le} l_e
\]  \hspace{1cm} (3.11)

The other component masses may be approximated simply as percentages of the sum of the canopy and line masses [14]. Add an additional 32% of this sum to account for radial and hem tapes, reinforcements, rigging aids, thread, ink, and break ties. The decelerator mass then becomes

\[
m_{od} = 1.32 (m_c + m_{le})
\]  \hspace{1cm} (3.12)

The auxiliary systems of the parachute are modeled as percentages of the decelerator mass. The deployment bag is allocated 3.5% of \(m_{od}\), the swivel, harness, and other structural connections are modeled as 20% of \(m_{od}\), and the deployment mortar is assumed 20% of \(m_{od}\). The final parachute system mass becomes
\[ m_0 = 1.435 \quad m_{od} = 1.8942 \ (m_c + m_{le}) \]  \hspace{1cm} (3.13)

Note that this equation reflects assumptions of the payload attitude during steady descent that are beyond the scope of this study.

### 3.3 Aerodynamic Models

#### 3.3.1 Aeroshell Aerodynamics

During the hypersonic glide phase, the drag and lift forces on the aeroshell are the only aerodynamic forces experienced by the vehicle

\[ L = \frac{1}{2} \rho_\infty V_\infty^2 C_{La} A_{po} \]  \hspace{1cm} (3.14a)

\[ D = \frac{1}{2} \rho_\infty V_\infty^2 C_{Da} A_{po} \]  \hspace{1cm} (3.14b)

The reference area \( A_{po} \) is the conical aeroshell base area.

An extensive literature search for aeroshell aerodynamic properties was conducted. Tabular wind tunnel data for lift and drag coefficients for a cone at an angle of attack for various Mach numbers and half angles is readily available [28], [29], [30]; coding these tables was deemed impractical for the scope of this study. Accordingly, a simple closed form approximation was sought for implementation in the algorithm. At sufficiently high Mach numbers, Newtonian flow theory has been shown to accurately predict the aerodynamic performance of a variety of mostly symmetrical bodies.

Newtonian flow theory idealizes the flow as a stream of molecules striking the surface of the vehicle, and no interaction with the shock wave occurs. The theory for
general bodies of revolution or symmetry has been thoroughly studied [24], [34] - [37]. For the cone at an angle of attack, Pike [34] derives the following expressions:

\[ C_{D_a} = C_{D_0} \left( \frac{5}{2} \cos^3 \alpha - \frac{3}{2} \cos \alpha \right) + \frac{3}{2} k \frac{A_{po}}{A_r} \cos \alpha \sin^2 \alpha \quad (3.15) \]

\[ C_{L_a} = C_{D_0} \left( \frac{5}{2} \cos^2 \alpha \sin \alpha + \frac{1}{2} \sin \alpha \right) + \frac{1}{2} k \frac{A_{po}}{A_r} \left( - \sin^3 \alpha + 2 \cos^2 \alpha \sin \alpha \right) \quad (3.16) \]

An equation for \( C_{D_0} \) using Newtonian flow is [31]

\[ C_{D_0} = \frac{2 (\gamma + 1) (\gamma + 7)}{(\gamma + 3)^2} \sin^2 \delta_c \quad (3.17) \]

Krasnov [31] suggests that a value of \( \gamma = 1.0 \) will best fit the exact Newtonian theory to experimental results in air. However, to simulate the real effect of the Martian atmosphere, \( \gamma = 1.3 \) is used in the algorithm. The difference in the \( \frac{2 (\gamma + 1) (\gamma + 7)}{(\gamma + 3)^2} \) term is then 3.25 % from the best experimental fit with air, and 0.85 % from the theoretical value for \( \gamma = 1.4 \).

Equation (3.17) operates under the assumption that the force coefficients and pressure distribution are independent of Mach number. In general, this is not a valid conjecture, especially at low supersonic speeds. Figure 3-5 from [31] portrays the dependence of the axial force coefficient on flow velocity at low Mach numbers; the coefficient dependence on Mach number is apparent. Figure 3-6, also from [31], depicts the slope of the normal force coefficient at 0° angle of attack for various cone angles and Mach numbers. This chart makes clear that the influence of Mach number diminishes as the cone angle increases, and the differences are negligible at high Mach numbers. Finally, Figure 3-7 from [24] shows that the Newtonian theory
equations match well with experimental data at high Mach numbers, and the drag coefficient is independent of Mach number for these large velocities.

Figure 3-5. Axial Force Coefficient on a Cone at Angle of Attack [31]

Figure 3-6. Slope of the Normal Force Coefficient for Cones at Various Mach Numbers [31]
Although the preceding plots portray the Newtonian flow equations as imprecise over the glide phase analyzed for this effort, the relations do provide a closed form description of the vehicle aerodynamics that is sufficiently accurate to demonstrate the algorithm. Figures 3-8 and 3-9 respectively plot the drag and lift coefficients of Equations (3.15) and (3.16) as a function of angle of attack and various cone half angles. As discussed in Chapter 7, consideration should be given to improving the hypersonic glide phase aerodynamic model fidelity for future research.

Figure 3-7. Drag Coefficient -- Newtonian Flow versus Experiment [24]
Figure 3-8. Aeroshell Drag Coefficient Model for Several Cone Semivertex Angles

Figure 3-9. Aeroshell Lift Coefficient Model for Several Cone Semivertex Angles
3.3.2 Parachute Aerodynamics

At the commencement of the parachute aided descent phase, the aeroshell is jettisoned and a new set of aerodynamic coefficients are required. The drag force on the entire system may be written as a sum of the components on the parachute and lander

\[
D = \frac{1}{2} \rho_{\infty} V_{\infty}^2 (C_{D_i} A_i + C_{D_p} A_p)
\]  

(3.18b)

Most analyses within the literature use the parachute un-inflated area as the reference for drag coefficient calculations. Neither the disk-gap-band parachute nor the lander generate any forces normal to the free stream velocity vector, hence the system lift is assumed zero for this flight phase

\[
L = \frac{1}{2} \rho_{\infty} V_{\infty}^2 (C_{L_i} A_i + C_{L_p} A_p) = 0
\]  

(3.18a)

Drag forces on a general parachute in flight are dependent on many factors, including the dynamic pressure, the Mach number of the flow regime, the wake characteristics of the towing body, the parachute geometrical considerations, and the effective porosity of the canopy. Higher order effects are seen from the Reynolds number, the system relative elasticity, and the canopy stiffness. Although numerous investigators provide analytic treatment of the dynamics of the parachute/payload system, little effort has been dedicated to analytic prediction of the aerodynamic force coefficients because of the unsteady nature of the flow through and around the parachute canopy and the complexity associated with modeling such nonlinear phenomena. Instead, investigators have relied on wind-tunnel data and flight testing techniques that adequately evaluate parachute performance parameters for future
simulations. These methods have the advantage of highly reliable conclusions, yet lack the simplicity of a closed-form solution. Some useful formulas can be derived, however, from flight tests that were conducted with the Viking parachute system [6]. Drag coefficients were calculated for each test with respect to parachute/payload Mach number. Figure 3-10 shows the average drag coefficient of these tests, along with an empirical curve fit of the form

\[
\begin{align*}
C_{D_{\text{pa}}} &= 0.60355, \quad 0.1 < M_\infty \leq 0.80563 \\
C_{D_{\text{pa}}} &= -16.599 + 174.51 M_\infty - 633.3 M_\infty^2 + 1060.9 M_\infty^3 \\
&\quad - 837.26 M_\infty^4 + 252.28 M_\infty^5, \quad 0.80563 < M_\infty \leq 1.01123 \\
C_{D_{\text{pa}}} &= 0.79939 - 5.7367 M_\infty + 12.214 M_\infty^2 - 9.5059 M_\infty^3 \\
&\quad + 3.1789 M_\infty^4 - 0.3893 M_\infty^5, \quad 1.01123 < M_\infty \leq 2.6
\end{align*}
\]

Note that the first derivatives of the first two fits with respect to \(M_\infty\) are equal at the junction point \(M_\infty = 0.80563\). However, the first derivatives of the final two curves at the junction \(M_\infty = 1.01123\) are not equal, and must be carefully monitored to ensure that no optimization algorithm numerical problems occur due to the discontinuity in slope.

The impact of the lander wake on the parachute drag performance may be eliminated through the use of the non-dimensionalized trailing distance, \(l_f/d_f\) [1], [6], [39], [41]. The wake moves closer to the parachute as this parameter is lessened, decreasing the effective dynamic pressure experienced at the canopy relative to that seen at the lander. The result is a decrease in the drag force on the parachute, or a
corresponding drop in the drag coefficient. The effect is more pronounced in the low supersonic and transonic flow regimes, and therefore must be accommodated in this study. Because Equations (3.19a-c) are based on flight test data of the disk-gap-band parachute with a characteristic trailing distance of 8.5, they will lose accuracy unless the relative trailing distance is held constant at this value

$$\frac{L_t}{d_t} = 8.5$$  \hspace{1cm} (3.20)

The canopy inflates in such a way that the radial forces provided by parachute geometric considerations will determine the projected area normal to the flow. The geometric characteristic that is most representative of the projected area of inflation is
the effective line length, \( \frac{l_e}{D_o} \). As the effective length decreases, the projected diameter will also decrease, corresponding to a reduction in drag coefficient [6]. A rough method for incorporating this effect is multiplication of the average parachute drag coefficient \( C_{D_{pa}} \) by a correction factor \( K_{CD} \). Figure 3-11 (modified from [6] for this effort), shows the percentage reduction of drag coefficient for various effective suspension line lengths \( \frac{l_e}{D_o} \). The multiplicative factor 1.0 corresponds to the effective line length of the flight test drag coefficients shown in Figure 3-10 (\( \frac{l_e}{D_o} = 1.7 \)).

![Figure 3-11. Drag Coefficient Correction Factor \( K_{CD} \) due to Effective Suspension Line Length](image)

The curve can be represented by the functions

\[
K_{CD} = 0.54707 + 0.41093 \left( \frac{l_e}{D_o} \right) - 0.10055 \left( \frac{l_e}{D_o} \right)^2 + 0.00795 \left( \frac{l_e}{D_o} \right)^3,
\]

\[
0.4 \leq \frac{l_e}{D_o} \leq 4.95
\]

(3.21a)
\[ K_{CD} = 1.08168, \quad \frac{L}{D_o} \geq 4.95 \]  \hspace{1cm} (3.21b)

The final adjusted parachute drag coefficient to be implemented is

\[ C_{DP} = K_{CD} C_{Dpa} \]  \hspace{1cm} (3.22)

### 3.3.3 Lander Aerodynamics

Throughout the parachute deceleration phase, the lander is modeled as flat plate/disk of area \( A_1 \) and diameter \( d_1 \). The drag coefficient of flat plates/disks in high Reynolds number flows is [13]

\[ C_{D1} = 1.18 \]  \hspace{1cm} (3.23)

The lander drag coefficient is assumed constant over all Mach number regimes.

### 3.4 Martian Environment Models

#### 3.4.1 Density and Temperature

A simplified model for the Martian atmosphere has been adapted from [17]. The COSPAR Martian atmosphere consists of nominal, cold, and warm models for both the northern and southern hemispheres. The temperature, pressure, and density profiles from 0 to 100 kilometers are presented in tabular form at 2 kilometer increments, and nominal northern hemisphere models are used for all atmospheric variables. Figure 3-12 illustrates the temperature and density profiles plotted as a function of altitude from the reference ellipsoid.
Figure 3-12. Martian Temperature and Density Profiles [17]

Three separate exponential curve fits are used to provide a good match to the density data:

\[
\rho_\infty = 0.015569 \ e^{-0.088574 \times 10^{-3} \ y} \quad 0 \leq y \leq 22.75 \ \text{km} \tag{3.24a}
\]

\[
\rho_\infty = 0.022405 \ e^{-0.104572 \times 10^{-3} \ y} \quad 22.75 \leq y \leq 47.23 \ \text{km} \tag{3.24b}
\]

\[
\rho_\infty = 0.074733 \ e^{-0.130080 \times 10^{-3} \ y} \quad 47.23 \leq y \leq 100 \ \text{km} \tag{3.24c}
\]

Although the numerical values of these curve fits match at the junction points 22.75 km and 47.23 km, the value of the partial derivatives with respect to altitude are not equal, as illustrated in Table 3-1.
The curves are all well behaved and smooth with non-dimensionalized versions are implemented within the code. The gradient differences are acceptable for the purposes of this study.

Three empirical curve fits are again employed for the temperature profile:

\[ T_\infty = 214.0 - 0.05E-03 \, y - 0.025E-06 \, y^2 \quad 0 \leq y \leq 4 \text{ km} \]  
\[ (3.25a) \]

\[ T_\infty = 223.17 - 1.9094E-03 \, y - 9.1707E-09 \, y^2 + 1.3362E-14 \, y^3 \quad 4 \leq y \leq 70 \text{ km} \]  
\[ (3.25b) \]

\[ T_\infty = 139.0 \quad 70 \leq y \leq 100 \text{ km} \]  
\[ (3.25c) \]

Although the numerical values of these curve fits match at the junction points 4.0 km and 70.0 km, the value of the partial derivatives are not equal. Again, these gradient differences are acceptable for the purposes of this study.
3.4.2 Atmospheric Winds

A Martian wind model was also constructed to investigate the algorithm’s ability to simulate atmospheric variability. The MARS-GRAM atmospheric simulator [12] provides the code (FORTRAN) and documentation for a tool useful in this effort. Although the simulator can be attached to a state integrator to provide atmospheric data on each algorithm iteration, only extreme horizontal winds as a function of altitude were sought for this effort. A sample trajectory matching the flight corridor of interest was constructed for input into a stand-alone version of the MARS-GRAM. This trajectory was then run repeatedly on the simulator over a range of conditions, including Martian year and season, time of day and night, initial latitude and longitude, presence of dust storms and strength, and up to ± 3 standard deviations from the norm for density shears. The most extreme wind cases (both weak and mild) were selected to test the vehicle susceptibility to atmospheric uncertainty.

The selected wind profiles are shown in Figure 3-13. The strongest, most extreme case corresponds to a maximum intensity dust storm occurring during the northern hemisphere fall at about 60° latitude and 0° longitude. The weakest, least extreme case corresponds to profiles typically seen during the northern hemisphere summer solstice near the equator. The median oscillatory profile was added to further test the algorithm when subjected to additional variation.

These three profiles were fitted with empirical curves between altitudes of 0 and 35 kilometers:

Most Extreme: Northern Hemisphere Fall Dust Storm

\[
W = 58.875 + 8.284E-03 y - 0.06968E-06 y^2 \quad (3.26a)
\]
Least Extreme: Typical Northern Hemisphere Summer

\[ W = 17.365 + 1.9811\text{E-03} \, y - 0.08380\text{E-06} \, y^2 \]  
(3.26b)

Median Oscillatory Profile

\[ W = 40 + 30 \sin \left[ \frac{2 \pi}{30000} (y - 5000) \right] + 30000 \]  
(3.26c)

Finally, all profiles were coded with a feature allowing the winds to be scaled by any factor as warranted to get desired analysis results.

![Figure 3-13. Average and Extreme Martian Wind Test Profiles](image)

Figure 3-13. Average and Extreme Martian Wind Test Profiles
Chapter 4

Problem Implementation within the Analysis Algorithm

The integrated design, trajectory, and control analysis algorithm [21] requires definition of numerous relations specifically for the Martian entry/landing problem. Proper coordinate systems are allocated, and an appropriate gravitational model must be implemented. Vehicle dynamics states, design parameters, and time varying controls require definition. A pertinent performance index must be defined, boundary conditions for the states must be designated, an integration cutoff condition must be resolved, and the required vehicle dynamics model must be established. In addition, various first order partial derivatives of the dynamic model must be derived in a format applicable to the algorithm.
4.1 Required Frame and Vector Definitions

4.1.1 Coordinate Frames and Gravitational Model

This study aims to demonstrate the applicability of the integrated design, control, and trajectory optimization methodology to the Mars entry/landing vehicle analysis through the utilization of simplified but representative models; Cartesian coordinates are chosen as the defining axes for the problem. Neglecting the third spatial dimension sacrifices relevant information on vehicle crossrange performance, but greatly reduces the complexity of the equations required within the algorithm, especially the partial derivatives. Utilization of two dimensions also diminishes the computational time associated with the optimization runs. As depicted in Figure 4-1, the vertical axis is zeroed at the surface of the Martian reference "ellipsoid". The horizontal or downrange axis is not fixed to any specific Martian surface feature and merely represents the range flown by a vehicle configuration.

\[ g_o = 3.7176 \text{ m/s}^2 \] (4.1)

Figure 4-1. Coordinate Axes for the Martian Entry/Landing Problem

The Martian gravitational field is assumed constant for this analysis,

\[ g_o = 3.7176 \text{ m/s}^2 \] (4.1)
4.1.2 States, Design Parameters, and Controls

The vehicle state vector is defined within this framework for the hypersonic glide and parachute-aided descent phases as the horizontal and vertical vehicle positions and velocities

\[
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}
\] (4.2)

Section 3.2 of this thesis shows that all aspects of the vehicle configuration can be inferred by defining a design parameter vector containing the conical aeroshell semivertex angle and the nominal parachute (un-inflated) diameter

\[
p = \begin{bmatrix} \delta_c \\ D_0 \end{bmatrix}
\] (4.3)

The inclusion of switch times within the analysis methodology requires their treatment as variables in a manner similar to that of the design parameters. Accordingly, the design parameter vector may be augmented with the switch time between the hypersonic glide and parachute-aided descent phases of the trajectory

\[
p_a = \begin{bmatrix} \delta_c \\ D_0 \\ t_{s1} \end{bmatrix}
\] (4.4)

The proper selection of the vehicle control vector may be accomplished by examining the vehicle aerodynamics models of Section 3.3. The hypersonic glide phase aerodynamic coefficients have been cast as functions of the conical aeroshell half angle \( \delta_c \) (a design parameter) and the vehicle angle of attack \( \alpha \). Because \( \alpha \) is
measured relative to the free stream velocity vector, it can change rapidly due to fluctuations in the wind velocity. The control variable must therefore be directly relatable to the angle of attack. The vehicle pitch relative to the horizontal is chosen

\[ u = [\theta] \]  

(4.5)

where

\[ \theta = \alpha + \gamma_f \]  

(4.6)

The geometry of the disk-gap-band parachute is such that no forces are generated perpendicular to the free stream velocity vector. Realistically the parachute/lander configuration is free to rotate about the system center mass and separate local angles of attack may develop at the lander and the parachute. However, the added complications of these system dynamics have been excluded from this analysis. Accordingly, the angle of attack during the parachute-aided descent phase is zero, and the control "variable" \( \theta \) becomes dependent solely on the free stream relative flight path angle \( \gamma_f \), which is in turn dictated by the vehicle states. The result is a non-perturbable control during this portion of the flight phase.

4.2 A Suitable Performance Index

4.2.1 Definition

The general form of the performance index for a planetary entry/landing problem is
\[ J(t) = \phi_1[x(t)] + \phi_2(p) + \int_t^0 L(x(t), u(t), p) \, dt \] (4.7)

The time of flight provides no concrete estimate of performance for this problem, therefore \( \phi_3 \) has been excluded from Equation (2.1). Propellant consumption may be minimized by including \( \phi_1 \) in the performance index and the propellant mass as an element in the vehicle state vector. However, this thesis does not explicitly address the all-propulsive terminal phase assumed present as the final descent leg of most entry trajectories.

The applicable performance index therefore contains only the configuration terminal cost term and the distributed cost term,

\[ J(t) = \phi_2(p) + \int_t^0 L(x(t), u(t), p) \, dt \] (4.8)

The mass of the vehicle most accurately describes the vehicle performance characterized by the design dependent terminal term, because system mass may be directly related to mission launch costs. Therefore, assign

\[ \phi_2(p) = m_v = m_l + m_o + m_a \] (4.9)

The vehicle mass is the sum of the fixed lander mass, the aeroshell mass, and the parachute system mass.

The distributed performance index is used to incorporate an inequality constraint on the dynamic pressure, where two terms are utilized to reflect the possibility of different requirements on the hypersonic glide and the parachute-aided descent phases.
The variables $Q_{max}$ and $Q_{maxp}$ represent the user supplied upper boundaries on the dynamic pressure of the hypersonic glide and parachute-aided descent phases, respectively. Both $K_Q$ and $K_{Qp}$ are inequality constraint emphasis weights described previously in Section 2.4.2. These performance terms are triggered by the step function $u_0$ whenever the vehicle dynamic pressure violates the user defined upper boundary. Although no attempt has been made to individually model the effects of specific force or thermodynamic heating requirements within this thesis, dynamic pressure provides a rough approximation of the structural and thermodynamic limitations of the system, especially during the parachute-aided descent phase.

### 4.2.2 Required Partial Derivatives -- Terminal Term

The analysis algorithm requires significant partial derivatives of the performance index found in Equation (4.8). Because the terminal term is dependent only on the vehicle design parameters, we seek an expression for $\phi_{2p}$. Begin by noting that the three terms of Equation (3.1) are each individually differentiable [8]

$$
\phi_{2p} = \frac{\partial m_v}{\partial p} + \frac{\partial m_l}{\partial p} + \frac{\partial m_o}{\partial p} + \frac{\partial m_a}{\partial p}
$$

(4.11)

the lander mass partial simplifies to

$$
\frac{\partial m_l}{\partial p} = [0 \ 0]
$$

(4.12)

The parachute mass partial with respect to design parameters may be written
Finally, the partial derivative of the aeroshell mass with respect to the design parameter vector is

\[
\frac{\partial m_a}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial m_a}{\partial \delta_c} & 0 \end{bmatrix}
\]  

(4.21)

where

\[
\frac{\partial m_a}{\partial \delta_c} = \frac{\pi K_F d}{4 \sin \delta_c} \left[ 2 \frac{\partial d}{\partial \delta_c} \left( \sin \delta_c + 1 \right) - \frac{d}{\tan \delta_c} \right]
\]  

(4.22)
4.2.3 Required Partial Derivatives -- Distributed Term

The required partial derivatives of the distributed cost term are $L_x$, $L_p$, and $L_u$.

Using Equation (4.10), the partial derivative with respect to the state vector can be easily seen to be

$$L_x = 2 \left[ K_Q (Q - Q_{\text{max}}) u_d(Q - Q_{\text{max}}) + K_{Qp} (Q - Q_{\text{max}p}) u_d(Q - Q_{\text{max}p}) \right] \frac{\partial Q}{\partial x} \quad (4.23)$$

where

$$\frac{\partial Q}{\partial x} = \begin{bmatrix} 0 & \left( \frac{1}{2} \frac{\partial \rho}{\partial y} \mathbf{v}_\infty^2 - \rho \mathbf{v}_\infty \frac{\partial W}{\partial y} \right) & \rho \mathbf{v}_x & \rho \mathbf{v}_y \end{bmatrix} \quad (4.24)$$

Both $\rho$ and $W$ are simple differentiable polynomials found in Equations (3.24a-c) and (3.26a-c) respectively. The other two partial derivatives are zero for this analysis

$$L_p = 0 \quad (4.25)$$

$$L_u = 0 \quad (4.26)$$

4.3 Equality Constraints and Boundary Conditions

4.3.1 Definition

The generalized two point boundary problem requires two boundary conditions for each element defined in the state vector. For the Martian entry/landing problem, the "forward" integration occurs on a negative time scale, beginning with the initial state vector situated at the "landing" condition of the vehicle
The terminal states represent the selected "atmospheric entrance" condition of the mission

\[ x_0 = \begin{bmatrix} x_0 \\ y_0 \\ v_{xo} \\ v_{yo} \end{bmatrix} \quad (4.27) \]

Within the algorithm, the terminal boundary conditions may be implemented by first defining a "forward" integration scalar cutoff condition \( \Omega \) that is monotonic [19]. Although the vertical velocity might seem a useful candidate for this function, atmospheric entry profiles may contain regions where a "pull-up" maneuver is performed prior to reaching the landing state. The difference in the vehicle energy between the current state and final terminal state is more likely to be monotonic and is therefore chosen as the cutoff condition

\[ \Omega = \left( v_x^2 + v_y^2 + 2 g_o y \right) - \left( v_{xf}^2 + v_{yf}^2 + 2 g_o y_{tf} \right) \quad (4.29) \]

The final three required terminal boundary conditions are grouped as equality constraints in the vector

\[ \Psi = \begin{bmatrix} x - x_f \\ y - y_f \\ v_x - v_{xf} \end{bmatrix} \quad (4.30) \]
4.3.2 Required Partial Derivatives

The algorithm requires partial derivatives of the above terminal boundary conditions with respect to the state vector

$$\Omega_x = \begin{bmatrix} 0 & 2 g_0 & 2 v_x & 2 v_y \end{bmatrix}$$ (4.31)

and also the partial derivative of the equality constraint vector with respect to the design parameter vector

$$\Psi_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$ (4.33)

4.4 Vehicle State Dynamics

4.4.1 Definition

Some useful trajectory quantities that are purely dependent on the vehicle state vector may be initially defined. The free stream relative flight path angle $\gamma_f$, dynamic pressure $Q$, horizontal velocity relative to the free stream $v_{x1}$, free stream velocity $V_{\infty}$, and the flow Mach number $M_{\infty}$ are

$$\gamma_f = \tan^{-1} \left( \frac{v_y}{v_{x1}} \right)$$ (4.34)

$$Q = \frac{1}{2} \rho_{\infty} V_{\infty}^2$$ (4.35)

$$v_{x1} = v_x - W$$ (4.36)
\[ V_\infty = \sqrt{v_{x1}^2 + v_y^2} \]

\[ M_\infty = \frac{V_\infty}{\sqrt{\gamma R T_\infty}} \]

The two-dimensional state dynamics for the hypersonic glide phase can be written

\[
\frac{dx}{dt} = f = \begin{bmatrix}
v_x \\
v_y \\
\frac{F_x}{m_v} \\
\frac{F_y}{m_v}
\end{bmatrix}
\]

where

\[
\begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix}
-D \cos \gamma - L \sin \gamma \\
-D \sin \gamma + L \cos \gamma - m_v g_o
\end{pmatrix}
\]

and

\[ D = Q C_{Da} A_{po} \]

\[ L = Q C_{La} A_{po} \]

The two-dimensional state dynamics for the parachute-aided descent phase are only slightly different

\[
\frac{dx}{dt} = f = \begin{bmatrix}
v_x \\
v_y \\
\frac{F_x}{m_p} \\
\frac{F_y}{m_p}
\end{bmatrix}
\]
where

\[
\begin{pmatrix}
F_x \\
F_y
\end{pmatrix} = \begin{pmatrix}
-D \cos \gamma_f - L \sin \gamma_f \\
-D \sin \gamma_f + L \cos \gamma_f - m_p g_0
\end{pmatrix}
\] (4.44)

and

\[
D = Q \left( C_{Dl} A_l + C_{Dp} A_p \right)
\] (4.45)

\[
L = Q \left( C_{Ll} A_l + C_{Lp} A_p \right)
\] (4.46)

All aerodynamic coefficients have been previously defined in Section 3.3.

4.4.2 Vehicle State Dynamics Partial Derivatives with Respect to States

The hypersonic glide phase partial derivatives with respect to the state vector are

\[
f_x = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\frac{1}{m_v} \frac{\partial F_x}{\partial x_1} & \frac{1}{m_v} \frac{\partial F_x}{\partial x_2} & \frac{1}{m_v} \frac{\partial F_x}{\partial x_3} & \frac{1}{m_v} \frac{\partial F_x}{\partial x_4} \\
\frac{1}{m_v} \frac{\partial F_y}{\partial x_1} & \frac{1}{m_v} \frac{\partial F_y}{\partial x_2} & \frac{1}{m_v} \frac{\partial F_y}{\partial x_3} & \frac{1}{m_v} \frac{\partial F_y}{\partial x_4}
\end{bmatrix}
\] (4.47)

where

\[
\frac{\partial F_x}{\partial x} = - \frac{\partial D}{\partial x} \cos \gamma_f - \frac{\partial L}{\partial x} \sin \gamma_f + (D \sin \gamma_f - L \cos \gamma_f) \frac{\partial \gamma_f}{\partial x}
\] (4.48)

\[
\frac{\partial F_y}{\partial x} = - \frac{\partial D}{\partial x} \sin \gamma_f + \frac{\partial L}{\partial x} \cos \gamma_f - (D \cos \gamma_f + L \sin \gamma_f) \frac{\partial \gamma_f}{\partial x}
\] (4.49)

The drag and lift force partials are

\[
\frac{\partial D}{\partial x} = \left( \frac{\partial Q}{\partial x} C_{Da} + \frac{\partial C_{Da}}{\partial x} \right) A_{po}
\] (4.50)
\[ \frac{\partial L}{\partial x} = \left( \frac{\partial Q}{\partial x} C_{L_a} + \frac{\partial C_{L_a}}{\partial x} Q \right) A_p \]  

(4.51)

while the flight path angle partial derivative has the form

\[ \frac{\partial \psi}{\partial x} = \begin{bmatrix} 0 & \frac{v_x}{v_y} \frac{\partial W}{\partial x} & -\frac{v_y}{v_y} \frac{\partial W}{\partial x} & \frac{v_x}{\sqrt{v_y}} \end{bmatrix} \]  

(4.52)

The aerodynamic coefficient partial derivatives are

\[ \frac{\partial C_{D_a}}{\partial x} = \frac{3}{2} \frac{\partial \alpha}{\partial x} \left[ C_{D_0} \left( -5 \sin \alpha \cos^2 \alpha + \sin \alpha \right) - 2 \sin^3 \alpha + 4 \sin \alpha \cos^2 \alpha \right] \]  

(4.53)

\[ \frac{\partial C_{L_a}}{\partial x} = \frac{\partial \alpha}{\partial x} \left[ C_{D_0} \left( 5 \sin^2 \alpha \cos \alpha - \frac{5}{2} \cos^3 \alpha + \frac{1}{2} \cos \alpha \right) - 7 \sin^2 \alpha \cos \alpha + 2 \cos^3 \alpha \right] \]  

(4.54)

where

\[ \frac{\partial \alpha}{\partial x} = -\frac{\partial \psi}{\partial x} \]  

(4.55)

The parachute-aided descent phase vehicle dynamics partial derivatives with respect to the state vector are

\[ f_x = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{m_p} \frac{\partial F_x}{\partial x_1} & \frac{1}{m_p} \frac{\partial F_x}{\partial x_2} & \frac{1}{m_p} \frac{\partial F_x}{\partial x_3} & \frac{1}{m_p} \frac{\partial F_x}{\partial x_4} \\ \frac{1}{m_p} \frac{\partial F_y}{\partial x_1} & \frac{1}{m_p} \frac{\partial F_y}{\partial x_2} & \frac{1}{m_p} \frac{\partial F_y}{\partial x_3} & \frac{1}{m_p} \frac{\partial F_y}{\partial x_4} \end{bmatrix} \]  

(4.56)

where Equations (4.48) and (4.49) still hold for the horizontal and vertical force partial derivatives. The lift and drag force partials are

\[ \frac{\partial D}{\partial x} = \frac{\partial Q}{\partial x} (C_{D_l} A_1 + C_{D_p} A_p) + \frac{\partial C_{D_p}}{\partial x} Q A_p \]  

(4.57)
The lift force partial derivative is zero because all lift coefficients and their derivatives are zero during this flight phase. The parachute drag coefficient partial has the form

\[
\frac{\partial C_{Dp}}{\partial x} = K_{CD} \frac{\partial C_{Dpa}}{\partial x}
\]  

(4.59)

where

\[
\frac{\partial C_{Dpa}}{\partial x} = 0, \quad 0.1 < M_\infty \leq 0.80563
\]  

(4.60)

\[
\frac{\partial C_{Dpa}}{\partial x} = \frac{\partial M_\infty}{\partial x} \left(174.51 - 1266.6 \ M_\infty + 3182.7 \ M_\infty^2 - 3349.04 \ M_\infty^3 + 1261.4 \ M_\infty^4\right),
\]  

\[
0.80563 < M_\infty \leq 1.01123
\]  

(4.61)

\[
\frac{\partial C_{Dpa}}{\partial x} = \frac{\partial M_\infty}{\partial x} \left(-5.7367 + 24.428 \ M_\infty - 28.5177 \ M_\infty^2 + 12.7156 \ M_\infty^3 - 1.9465 \ M_\infty^4\right),
\]  

\[
1.01123 < M_\infty \leq 2.6
\]  

(4.62)

and

\[
\frac{\partial M_\infty}{\partial x} = \begin{bmatrix}
0 & -M_\infty \left(\frac{v_{x}}{V_\infty^2} \ \frac{\partial W}{\partial y} + \frac{1}{2} \ \frac{\partial T_\infty}{\partial y}\right) & M_\infty \ \frac{v_{x}}{V_\infty^2} & M_\infty \ \frac{v_{y}}{V_\infty^2}
\end{bmatrix}
\]  

(4.63)

Again, \(\frac{\partial T_\infty}{\partial y}\) is a straight-forward polynomial function of the altitude \(y\).

### 4.4.3 Vehicle State Dynamics Partial Derivatives with Respect to Controls

The hypersonic glide phase dynamics partial derivatives with respect to the control vector are
\[
\mathbf{f}_u^T = \begin{bmatrix}
0 & 0 & \frac{1}{m_v} \frac{\partial F_x}{\partial u} & \frac{1}{m_v} \frac{\partial F_y}{\partial u}
\end{bmatrix}
\] (4.64)

where the horizontal and vertical force partial derivatives are
\[
\frac{\partial F_x}{\partial u} = - \frac{\partial D}{\partial u} \cos \gamma_f - \frac{\partial L}{\partial u} \sin \gamma_f
\] (4.65)
\[
\frac{\partial F_y}{\partial u} = - \frac{\partial D}{\partial u} \sin \gamma_f + \frac{\partial L}{\partial u} \cos \gamma_f
\] (4.66)

The lift and drag force partials may be written
\[
\frac{\partial D}{\partial u} = \frac{\partial C_{D_s}}{\partial u} Q A_{p_0}
\] (4.67)
\[
\frac{\partial L}{\partial u} = \frac{\partial C_{L_s}}{\partial u} Q A_{p_0}
\] (4.68)

with force coefficient partials
\[
\frac{\partial C_{D_s}}{\partial u} = \frac{3}{2} \frac{\partial \alpha_x}{\partial u} \left[ C_{D_0} \left( -5 \sin \alpha \cos^2 \alpha + \sin \alpha \right) - 2 \sin^3 \alpha + 4 \sin \alpha \cos^2 \alpha \right]
\] (4.69)
\[
\frac{\partial C_{L_s}}{\partial u} = \frac{\partial \alpha_x}{\partial u} \left[ C_{D_0} \left( 5 \sin^2 \alpha \cos \alpha - \frac{5}{2} \cos^3 \alpha + \frac{L}{2} \cos \alpha \right) - 7 \sin^2 \alpha \cos \alpha + 2 \cos^3 \alpha \right]
\] (4.70)
\[
\frac{\partial \alpha_x}{\partial u} = [1]
\] (4.71)

During the parachute phase, the hypersonic glide pitch control becomes equal to the free stream relative flight path angle \(\gamma_f\), which is a function of the states only (\(\frac{\partial \gamma_f}{\partial u} = 0\)). Also note that \(\frac{\partial Q}{\partial u}, \frac{\partial D}{\partial u}, \frac{\partial L}{\partial u}, \frac{\partial C_{D_p}}{\partial u}\) and \(\frac{\partial C_{L_p}}{\partial u}\) are all zero, therefore
\[
\mathbf{f}_u^T = \begin{bmatrix}
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (4.72)
which is consistent with the assumption of a non-perturbable control in the absence of an angle of attack.

4.4.4 Vehicle State Dynamics Partial Derivatives with Respect to Design Parameters

The hypersonic glide phase dynamics partial derivatives with respect to the design parameter vector are

$$\mathbf{f}_p = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_v} \frac{\partial F_x}{\partial p} - F_x \frac{\partial m_v}{\partial p} \\ \frac{1}{m_v} \frac{\partial F_y}{\partial p} - F_y \frac{\partial m_v}{\partial p} \end{bmatrix}$$

(4.73)

where

$$\frac{\partial F_x}{\partial p} = -\frac{\partial D}{\partial p} \cos \gamma - \frac{\partial L}{\partial p} \sin \gamma$$

(4.74)

$$\frac{\partial F_y}{\partial p} = -\frac{\partial D}{\partial p} \sin \gamma + \frac{\partial L}{\partial p} \cos \gamma - \frac{\partial m_v}{\partial p} g_o$$

(4.75)

The lift and drag force partials may be written

$$\frac{\partial D}{\partial p} = \left( \frac{\partial Q}{\partial p} C_{Da} + \frac{\partial C_{Da}}{\partial p} Q \right) A_{po}$$

(4.76)

$$\frac{\partial L}{\partial p} = \left( \frac{\partial Q}{\partial p} C_{La} + \frac{\partial C_{La}}{\partial p} Q \right) A_{po}$$

(4.77)

with force coefficient partials

$$\frac{\partial C_{Da}}{\partial p} = \frac{\partial C_{Da}}{\partial p} \left( \frac{5}{2} \cos^3 \alpha - \frac{5}{2} \cos \alpha \right)$$

(4.78)

$$\frac{\partial C_{La}}{\partial p} = \frac{\partial C_{La}}{\partial p} \left( -\frac{5}{2} \cos^2 \alpha \sin \alpha + \frac{1}{2} \sin \alpha \right)$$

(4.79)
and finally

\[
\frac{\partial A_{po}}{\partial p} = \left[ \left( \frac{8 \pi \gamma^2}{3 \sin \delta_c \cos^5 \delta_c} \right)^{\frac{1}{2}} \begin{array}{c} 0 \\ \end{array} \right] \quad (4.80)
\]

\[
\frac{\partial C_{Do}}{\partial p} = \left[ \frac{4 (\gamma + 1)(\gamma + 7)}{(\gamma + 3)^2} \begin{array}{c} \sin \delta_c \cos \delta_c \\ 0 \end{array} \right] \quad (4.81)
\]

The parachute-aided descent phase vehicle dynamics partial derivatives with respect to the design parameter vector are

\[
f_p = \left[ \begin{array}{ccc} 0 & 0 \\ \frac{1}{m_p} \frac{\partial F_x}{\partial \phi} - \frac{F_x}{m_p^2} \frac{\partial m_p}{\partial \phi} \\ \frac{1}{m_p} \frac{\partial F_y}{\partial \phi} - \frac{F_y}{m_p^2} \frac{\partial m_p}{\partial \phi} \end{array} \right] \quad (4.82)
\]

where the horizontal force partial derivative takes the form of Equation (4.74). The vertical force partial derivative with respect to the design parameter vector is

\[
\frac{\partial F_y}{\partial p} = -\frac{\partial D}{\partial p} \sin \gamma + \frac{\partial L}{\partial p} \cos \gamma - \frac{\partial m_p}{\partial p} g_0 
\]

Because \(\frac{\partial Q}{\partial p} = 0\) and \(\frac{\partial C_{Di}}{\partial p} = 0\), we may write the drag partial derivative as

\[
\frac{\partial D}{\partial p} = Q \left( C_{Di} \frac{\partial A_1}{\partial p} + C_{Dp} \frac{\partial A_p}{\partial p} + A_p \frac{\partial C_{Dp}}{\partial p} \right) 
\]

(4.84)

Recalling that the parachute/lander system has negligible lift coefficients and their derivatives, the partial derivative of the lift force with respect to design parameters is

\[
\frac{\partial L}{\partial p} = 0 
\]

(4.85)

The terms of Equation (4.84) can be expanded into
\[
\frac{\partial A_1}{\partial \mathbf{p}} = \begin{bmatrix}
\pi \frac{d_1}{2} & \frac{\partial d_1}{\partial \delta_c} & 0
\end{bmatrix}
\]

(4.86)

\[
\frac{\partial A_p}{\partial \mathbf{p}} = \begin{bmatrix}
0 & \frac{\partial A_p}{\partial D_o}
\end{bmatrix}
\]

(4.87)

\[
\frac{\partial C_{Dp}}{\partial \mathbf{p}} = C_{Dpa} \frac{\partial K_{CD}}{\partial \mathbf{p}}
\]

(4.88)

where

\[
\frac{\partial K_{CD}}{\partial \mathbf{p}} = \frac{\partial (\frac{l_c}{D_o})}{\partial \mathbf{p}} \left[ 0.41093 - 0.2011 \left( \frac{l_c}{D_o} \right) + 0.02385 \left( \frac{l_c}{D_o} \right)^2 \right]
\]

(4.89)

and

\[
\frac{\partial (\frac{l_c}{D_o})}{\partial \mathbf{p}} = \begin{bmatrix}
\frac{1}{D_o} \frac{\partial l_c}{\partial \delta_c} & \frac{1}{D_o} \left( \frac{\partial l_c}{\partial \delta_c} - \frac{l_c}{D_o} \right)
\end{bmatrix}
\]

(4.90)

Finally,

\[
\frac{\partial m_p}{\partial \mathbf{p}} = \frac{\partial m_o}{\partial \mathbf{p}} + \frac{\partial m_i}{\partial \mathbf{p}}
\]

(4.91)
Chapter 5

Initial Demonstration Results -- The Hypersonic Glide Phase

The integrated design, control, and trajectory optimization methodology was first verified on a single aspect of the Martian entry/landing problem, the hypersonic glide phase. The results displayed in this chapter demonstrate the types of information obtainable using the methodology when restricted to one flight phase (which avoids utilization of all switch time logic).

5.1 Problem Setup and Baseline Solution

5.1.1 Problem Setup

Minor modifications are required for a small subset of the implementation equations of the previous chapter. Although no vehicle state or control vector changes are necessary, the vehicle design parameter vector is reduced to reflect
problem termination without consideration of parachute operation (parachute sizing information is not required)

\[ p = \begin{bmatrix} \delta_c \end{bmatrix} \]  

(5.1)

The terminal performance index of Equation (4.8) also changes as no optimization is conducted on the parachute system configuration

\[ \phi_2(p) = m_v = m_a + m_p \]  

(5.2)

The vehicle mass is defined as the sum of the variable aeroshell mass and a constant payload mass that includes both the lander and parachute system mass

\[ m_p = 1000 \text{ kg} \]  

(5.3)

The boundary conditions for this problem are defined in the framework of Equations (4.27) and (4.28). The initial state vector is chosen as representative of the presumed hypersonic glide phase termination

\[ x_o = \begin{bmatrix} x_o \\ y_o \\ v_{x_o} \\ v_{y_o} \end{bmatrix} = \begin{bmatrix} 100000 \text{ m} \\ 5000 \text{ m} \\ 160 \text{ m/s} \\ -140 \text{ m/s} \end{bmatrix} \]  

(5.4)

The vehicle dynamics are integrated from this initial state to the required monotonic energy cutoff condition defined in Equation (4.29).

The terminal boundary states are identified using the piecewise linear initial control history guess feature in the algorithm. The initial piecewise control history is iteratively modified until the desired flight corridor is roughly simulated. The terminal state vector may then be slightly altered so that only small equality constraint
violations are encountered on subsequent runs, allowing the algorithm some freedom to adjust other vehicle parameters to find the optimal design solution. The terminal boundary states for the hypersonic "atmospheric entry" condition are

\[
\mathbf{x}_f = \begin{bmatrix} x_f \\ y_f \\ v_{xf} \\ v_{yf} \end{bmatrix} = \begin{bmatrix} 22622.0 \text{ m} \\ 33428.0 \text{ m} \\ 777.84 \text{ m/s} \\ -181.95 \text{ m/s} \end{bmatrix}
\]  

(5.5)

5.1.2 Baseline Solution

The initial test case analyzes vehicle performance when subjected to the above state boundary conditions in a windless atmosphere with no upper bound on the dynamic pressure inequality constraint. The algorithm successfully located a trajectory that satisfied the state boundary conditions of Equations (5.4) and (5.5) and converged the vehicle design to the minimum mass solution at \( \delta_c = 19.5^\circ \). Plots of the parameters of interest over this trial are depicted in Appendix Figure AI-1. Unless otherwise noted, all subsequent simulations use the results of this initial trial as the algorithm starting point.

5.2 The Dynamic Pressure Inequality Constraint

The effects of the dynamic pressure inequality constraint on the solution were investigated in some detail. Converged solutions were found for dynamic pressure bounds of 500, 450, 400, 350, and 300 N/m\(^2\) (Pascals). Table 5-1 shows the converged vehicle cone half angle as well as the maximum dynamic pressure attained during the trajectory.
Table 5-1. Effects of the Dynamic Pressure Inequality Constraint in a Windless Atmosphere (Hypersonic Glide Phase)

<table>
<thead>
<tr>
<th>Trial (Upper Q Bound) (Pa)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle $\delta_c$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No $Q_{\text{max}}$</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 500$</td>
<td>499</td>
<td>19.8</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 450$</td>
<td>450</td>
<td>20.3</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 400$</td>
<td>400</td>
<td>21.3</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 350$</td>
<td>377</td>
<td>23.8</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 300$</td>
<td>382</td>
<td>23.8</td>
</tr>
</tbody>
</table>

Plots comparing the most significant trajectory variables are shown in Figure AI-2. The trajectory altitude versus downrange displacement shows little variation and so has been omitted. Similarly, the free stream relative flight path angle histories ($\gamma_f$) do not differ significantly. According to Equation (4.6), the variation of the angle of attack along the trajectory closely mimics that of the pitch for constant $\gamma_f$, allowing the omission of pitch from the plots. Additionally, the trajectory solution for the 300 N/m$^2$ dynamic pressure limit closely resembles that of the 350 N/m$^2$ case for all parameters of interest.

The plots of Figure AI-2 differ significantly in the angle of attack history as the dynamic pressure inequality constraint is lowered. The region of the inequality constraint influence (approximately between -125 s and -80 s) shows increased angles of attack as the dynamic pressure is lowered. The vehicle flies at higher drag in this region in attempt to lessen the magnitude of the free-stream velocity vector and also the dynamic pressure.
The vehicle configuration also changes significantly as the dynamic pressure limit is reduced, until the minimum $Q$ is reached at approximately 377 Pa. The change in the cone half angle tapers as the minimum is approached, coinciding with uniformity of trajectory parameters between the 350 Pa and the 300 Pa cases. The evidence suggests the vehicle model cannot accommodate trajectories that fly at dynamic pressures lower than 377 N/m$^2$ when subjected to the state boundary conditions enumerated in Equations (5.4) and (5.5).

A physical explanation of this phenomena may be found by referring to the aeroshell aerodynamic model depicted in Figures 3-8 and 3-9. As the angle of attack is increased above $\sim 35^\circ$, the aeroshell experiences some growth in drag coefficient, however a drastic decrease in lift coefficient occurs. Notice also that increases in cone half angle tend to attenuate rather than augment the vehicle drag coefficient in this angle of attack region. Therefore, any further enlargement in the drag characteristics of the aeroshell to meet the required dynamic pressure inequality constraint will either lessen the cone half angle or increase the angle of attack. On one hand, lessening the cone half angle amplifies the lift and drag characteristics (also decrementing the aeroshell mass) in the region of dynamic pressure bound influence, but as a result drag force losses will occur during the critical low altitude/high density region of the trajectory. On the other hand, increasing the angle of attack in this region incurs a penalty in trajectory lift performance. The analysis algorithm finds the solution that best keeps both the vehicle mass and dynamic pressure violations to a minimum, but further optimization of the performance index results in a vehicle design and control history that are not robust enough to simultaneously meet the state boundary conditions for the requested dynamic pressure boundary.
5.3 Effects of Head Winds

The initial demonstration of the integrated design, control, and trajectory optimization methodology includes testing the vehicle performance criteria in the presence of the wind profiles of Figure 3-13. Analysis of optimization runs with an atmospheric bias towards head winds proved less complicated than for tail winds; therefore these cases are discussed first. The starting trajectory for these cases is always the converged baseline solution with no upper bound on the dynamic pressure constraint that is outlined briefly in Section 5.1.2.

5.3.1 Case 1 -- Typical Low Latitude Northern Hemisphere Summer Solstice

Analysis of the head wind profiles begins with the implementation of the least extreme model typical of the northern hemisphere summer solstice near the equator. Different wind profile magnitudes of 50% and 100% are shown to illustrate the development of the solution in the presence of an upper bound on the vehicle dynamic pressure. A summary of the vehicle design parameter and the maximum dynamic pressure attained during the trajectory is shown in Table 5-2. Results of the windless converged solutions are also presented for comparison.

Plots of the relevant comparison parameters for the optimization runs without an upper limit on the dynamic pressure inequality constraint are illustrated in Figure A1-3. The vehicle is successfully able to fly between the two sets of boundary states at the vehicle design condition that yields the minimum possible mass (the only applied performance criterion).
Table 5-2. Effects of Head Wind Case 1

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle $\delta_c$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No $Q_{\text{max}}$, No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>No $Q_{\text{max}}$, Wind 100%</td>
<td>586</td>
<td>19.5</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 400$, No Wind</td>
<td>400</td>
<td>21.3</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 400$, Wind 50%</td>
<td>400</td>
<td>20.1</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 400$, Wind 100%</td>
<td>450</td>
<td>20.4</td>
</tr>
</tbody>
</table>

The plots of the angle of attack profile show the pronounced effects of the head wind in region of greatest sensitivity, the higher atmospheric density altitudes near the vehicle "landing" condition, near $t = 0$ seconds. The head wind tends to increase the free stream velocity vector and dynamic pressure experienced by the vehicle, corresponding to augmented lift and drag forces over what is necessary to reach the boundary states. The algorithm has therefore altered the vehicle control history such that the angle of attack in this region is diminished, reducing both the lift and drag coefficients.

Addition of a sample dynamic pressure bound of 400 N/m² results in vehicle trajectory characteristics that show less variation due to the applied wind magnitude. As depicted in Figure AI-4, only the angle of attack and the dynamic pressure histories show dissimilarity for the different wind magnitudes. The peaks present in the angle of attack histories of Figure AI-4 occur precisely at the time (-114 and -118 seconds) that the dynamic pressure meets the upper bound. Discontinuities in the slope of state or control trajectories are often encountered at the entrance or exit of inequality constraints; this characteristic is inherent to the steepest descent
methodology. The presence of the head wind affects the vehicle design by allowing a slightly smaller cone half angle, as less drag is required to satisfy the boundary conditions.

5.3.2 Case 2 -- Median Oscillatory Profile

Input of the median oscillatory wind again resulted in converged solutions that satisfied both the inequality and equality constraints. A summary of the trajectory maximum dynamic pressure and the vehicle half angle is displayed below in Table 5-3.

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle $\delta_c$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No $Q_{max}$, No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>No $Q_{max}$, Wind 100%</td>
<td>692</td>
<td>19.5</td>
</tr>
<tr>
<td>$Q_{max} = 400$, No Wind</td>
<td>400</td>
<td>21.3</td>
</tr>
<tr>
<td>$Q_{max} = 400$, Wind 100%</td>
<td>404</td>
<td>22.9</td>
</tr>
</tbody>
</table>

Table 5-3. Effects of Head Wind Case 2

Figures AI-5 and AI-6 show the important trajectory values for both the trials with dynamic pressure bound and those with a $Q_{max} = 400$ N/m$^2$. Large deviations are seen for all parameters of interest, especially with respect to the angle of attack histories. Although the optimal aeroshell configuration of 19.5° is reached without the inequality constraint, application of an upper bound on the dynamic pressure results in degraded vehicle performance in the presence of the wind. The head wind serves to increase the amount of dynamic pressure that must be "eliminated" from the
trajectory in order to meet the constraint. The vehicle cone half angle is therefore enlarged, increasing the configuration mass but providing the required drag characteristics that are necessary to meet the inequality constraint.

The small violation of the dynamic pressure bound for the 100% magnitude wind case illustrates another feature of the algorithm convergence properties. The relative weighting of the inequality constraint \( K_Q \) is set low enough that small violations of the upper bound on the inequality constraint are allowed if the vehicle configuration mass may still be decreased. The algorithm in effect trades the required dynamic pressure performance for a decrease in the system mass.

5.3.3 Case 3 -- Most Extreme High Latitude Dust Storm

Table 5-4 shows the important variables associated with the application of the most extreme wind profile.

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle ( \delta_c ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No ( Q_{\text{max}} ), No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>No ( Q_{\text{max}} ), Wind 100%</td>
<td>912</td>
<td>19.5</td>
</tr>
<tr>
<td>( Q_{\text{max}} = 400 ), No Wind</td>
<td>400</td>
<td>21.3</td>
</tr>
<tr>
<td>( Q_{\text{max}} = 400 ), Wind 100%</td>
<td>480</td>
<td>18.3</td>
</tr>
</tbody>
</table>

Table 5-4. Effects of Head Wind Case 3

In the absence of a dynamic pressure upper bound, the vehicle design parameter remains at the configuration minimum mass, but Figure AI-6 illustrates that the trajectories are highly altered. The large dynamic pressures seen with this wind case
are due to the relatively extreme head winds and correspondingly high Mach numbers, especially at the lower altitudes.

Figure AI-7 shows the effect of an added dynamic pressure upper bound of 400 N/m$^2$. Most of the optimal trajectory falls above this bound with extreme head wind active, and again the other trajectory parameters differ notably. In both cases, the angle of attack history is consistently lowered due to the strong head wind, as an equivalent amount of lift and drag is generated due to higher relative velocities at lower angles of attack.

### 5.4 Effects of Tail Winds

The three wind test profiles of Figure 3-13 were also examined with the integrated design, trajectory, and control methodology as tail winds. The vehicle performance contrasts distinctly from all of the previous trials, inducing a different method of investigation. The flown trajectory does not reach the required state boundary conditions of Equations (5.4) and (5.5) within the requested tolerance in any of the tail wind cases, often resulting in algorithm instability. Because of this, no effort was made to study the effects of an upper bound on the dynamic pressure inequality constraint, as the algorithm does not attempt to optimize the performance index until after the equality constraint boundary has been satisfied.

Efforts instead concentrated on locating the *fraction* of the tail wind that the simulation could successfully negotiate for each of the sample wind profiles. It was possible to increase this fraction using an incremental solution approach, whereby previous weak wind trajectories and vehicle optimal design solutions became the
algorithm starting point for a new stronger wind trial. Tail wind magnitudes are increased by small percentages between trials. Using this method, only the first simulation uses the optimized solution of the baseline trial (Figure AI-1) as the algorithm starting point. The incremental solution approach ensures stable algorithm convergence in the presence of slightly greater tail winds.

5.4.1 Case 1 -- Typical Low Latitude Northern Hemisphere Summer Solstice

Table 5-5 shows the important parameters of the converged solutions for the trials implementing the incremental wind profile technique on the least extreme wind profile associated with the northern hemisphere summer solstice. The original, windless results are also shown for comparison. The equality constraints cannot be satisfied for tail wind profiles that are above 60.8% magnitude of the original least extreme case.

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle $\delta_c$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>Wind 25%</td>
<td>715</td>
<td>20.0</td>
</tr>
<tr>
<td>Wind 40%</td>
<td>677</td>
<td>21.3</td>
</tr>
<tr>
<td>Wind 50%</td>
<td>641</td>
<td>22.6</td>
</tr>
<tr>
<td>Wind 60%</td>
<td>576</td>
<td>24.6</td>
</tr>
<tr>
<td>Wind 60.5%</td>
<td>568</td>
<td>24.8</td>
</tr>
<tr>
<td>Wind 60.8%</td>
<td>563</td>
<td>25.0</td>
</tr>
</tbody>
</table>

Table 5-5. Effects of Tail Wind Case 1
A comparison of the final 60.8% magnitude tail wind and the windless case baseline trajectory is depicted in Figure AI-9. The plots are dominated by the effect of the decrease in the free stream velocity vector \( V_\infty \) due to the presence of the tail wind. The vehicle inertial state velocities \( v_x \) and \( v_y \) stay greater in magnitude for a longer period of time due to decreases in lift and drag force experienced from diminished \( \alpha \) and \( V_\infty \). The trend in the dynamic pressure peaks seen in Table 5-5 can be roughly explained by increases in inertial velocity that more than offset the decrease in relative velocity due to the tail wind.

The cone half angle increases (along with the vehicle mass) significantly as the wind magnitude grows. This allows the vehicle to dissipate more excess kinetic energy at lower angles of attack by increasing the drag coefficient. A detrimental penalty in lift coefficient eventually results in a vehicle configuration that cannot reach the terminal constraints for greater tail winds.

Examination of the tail wind control history and the aeroshell lift and drag coefficients illustrated in Figures AI-9, 3-8, and 3-9 also yields information on the vehicle performance. At the "atmospheric entry" condition (large negative time), the angle of attack is such that the drag coefficient is small while the lift coefficient is negative and relatively large. The lift and drag are then slowly increased as altitude is lost until the peak drag is reached in the region where the wind has the greatest influence. This physically translates to a vehicle that seeks to quickly dive into denser regions of the atmosphere, where the excess energy provided to the system by the tail wind may be more easily dissipated by augmented vehicle forces.
5.4.2 Case 2 -- Median Oscillatory Profile

The incremental approach was again applied to the median oscillatory profile implemented as a tail wind. Algorithm instability occurs as the vehicle mass is decreased and the angle of attack exceeds 90°, prompting the use of the trajectory flown at the onset of this instability. Special note should therefore be made that the solutions obtained were not optimal as defined by the iteration end condition of Equation (2.2). Important results for this case are shown in Table 5-6, while Figure AI-10 depicts the trajectory parameters of merit compared to the original windless solution.

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle δc (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>Wind 21%</td>
<td>660</td>
<td>21.3</td>
</tr>
<tr>
<td>Wind 25%</td>
<td>619</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Table 5-6. Effects of Tail Wind Case 2

The results are similar to that of the Case 1 tail wind trials, except for the unstable performance near the terminal state boundary constraints (between -140 and -160 seconds). Use of any tail wind of greater magnitude than 25% of the median oscillatory profile results in unstable trajectories and a vehicle design that can not satisfy the equality constraints.

5.4.3 Case 3 -- Most Extreme High Latitude Dust Storm

Once again the incremental approach is used to slowly build up the wind magnitude present in the simulation. The algorithm responds to slightly increased
wind profiles by first eliminating violations of the equality constraints, and then optimizing with regard to the defined performance index. In the course of this optimization, algorithm instability results whenever angle of attack values exceeds 90°. Table 5-7 and Figure AI-11 show the parameters of merit compared to the converged windless solution.

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle δc (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>Wind 3%</td>
<td>617</td>
<td>19.7</td>
</tr>
<tr>
<td>Wind 6.6%</td>
<td>704</td>
<td>20.0</td>
</tr>
<tr>
<td>Wind 6.7%</td>
<td>706</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Table 5-7. Effects of Tail Wind Case 3

Any wind magnitude greater than 6.7% of this extreme wind profile results in instability and trajectories that can not adequately satisfy the equality constraints. The vehicle performance attributes shown above may all be described using the same arguments presented for the other two tail wind cases.

5.5 Analysis of Results

The demonstration results provide significant information for planetary vehicle systems analyses. Although the head wind cases all resulted in satisfactory converged solutions, the tail wind cases highlight a fundamental inadequacy in the vehicle configuration assumptions for this simulation. The equality constraints could not be satisfied for tail winds over 60.8% magnitude for the least extreme low latitude
The decrease in the relative free stream velocity vector $V_\infty$ translates directly into the loss of lift and drag forces over the altitudes of greatest sensitivity. Two options are available to increase the robustness of the vehicle design configuration.
The first, increasing the lift characteristics of the vehicle aeroshell model, can result in better performance over a wider range of tail wind cases. This effect is illustrated in an optimization run where the aeroshell lift coefficient is increased by 10% and all other vehicle parameters held constant. The "high-lift" configuration is tested against the least extreme tail wind profile, that typical to the northern hemisphere summer solstice near the equator. The incremental method is applied, and important results are shown in Table 5-8 and Figure AI-12 with comparisons to the optimal windless solution and the maximum tail wind "regular lift" solution.

<table>
<thead>
<tr>
<th>Trial (% Magnitude)</th>
<th>Resulting Maximum Q (Pa)</th>
<th>Cone Half Angle δc (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Original Lift Model</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Wind</td>
<td>527</td>
<td>19.5</td>
</tr>
<tr>
<td>Wind 60.8%</td>
<td>563</td>
<td>25.0</td>
</tr>
<tr>
<td><em>+10% Lift Model</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind 67.5%</td>
<td>674</td>
<td>22.0</td>
</tr>
<tr>
<td>Wind 75.0%</td>
<td>640</td>
<td>23.1</td>
</tr>
<tr>
<td>Wind 80.0%</td>
<td>602</td>
<td>24.3</td>
</tr>
<tr>
<td>Wind 81.5%</td>
<td>584</td>
<td>24.8</td>
</tr>
</tbody>
</table>

Table 5-8. Least Extreme Tail Wind Case 1 with +10% Lift Model

The trends in the maximum dynamic pressure and the conical aeroshell half angle are consistent with those of the other tail wind trials. The final trajectory flown (wind 81.5% magnitude) closely resembles that of the 60.8% tail wind magnitude trial, but the vehicle is now successfully able to negotiate 81.5% of the tail wind profile. These results justify the assumptions that a vehicle aeroshell possessing
greater lift characteristics would prove more robust in the uncertain Martian atmosphere.

The other option for alleviating the vehicle susceptibility to tail winds is increasing the drag capability of the vehicle to facilitate the dissipation of energy. The use of a parachute phase during landing after an initial glide phase will provide augmented configuration drag capabilities [19], [33], [14] at a relatively small mass penalty. This information provides the basis for the implementation and analysis of the parachute-aided descent phase presented in the next chapter of this thesis.
Chapter 6

The Coupled Hypersonic Glide and Parachute-Aided Descent Phases

This chapter describes the application of the integrated design, control, and trajectory optimization methodology to the coupled hypersonic glide and parachute-aided descent phases of the Martian entry/landing problem. The applicable vehicle dynamics, a suitable performance index, and required partial derivatives are outlined in depth in Chapter 4. The hypersonic glide phase terminates at the variable switch time $t_{s_i}$, activating the parachute-aided descent phase dynamics.

6.1 Problem Setup and Baseline Solution

6.1.1 Problem Setup

The boundary conditions for this problem are again defined in the framework of Equations (4.27) and (4.28). In an effort to keep this analysis as compatible with the
initial hypersonic glide phase demonstration as possible, the terminal state boundary condition is chosen as the previous "atmospheric entry" condition.

\[
\mathbf{x}_f = \begin{bmatrix}
    x_f \\
    y_f \\
    v_{xf} \\
    v_{yf}
\end{bmatrix} = \begin{bmatrix}
    22622.0 \text{ m} \\
    33428.0 \text{ m} \\
    777.84 \text{ m/s} \\
    -181.95 \text{ m/s}
\end{bmatrix}
\]  

(6.1)

The initial state vector is chosen to represent a typical vehicle state at the termination of the parachute-aided descent phase.

\[
\mathbf{x}_0 = \begin{bmatrix}
    x_0 \\
    y_0 \\
    v_{xo} \\
    v_{yo}
\end{bmatrix} = \begin{bmatrix}
    100000 \text{ m} \\
    3000 \text{ m} \\
    25 \text{ m/s} \\
    -60 \text{ m/s}
\end{bmatrix}
\]  

(6.2)

Identification of this state was made with reference to mission profiles [33], [5] that are representative of past and expected future Martian lander trajectories. At the initial state boundary condition, the vehicle is assumed to commence an all-propulsive landing phase, although these vehicle dynamics are not simulated in this study.

6.1.2 Baseline Solution

In a general case, the integrated analysis algorithm locates an optimal solution according to the selected gains described in Section 2.4. For this analysis, the gains are chosen such that the different performance index terms are optimized in a specific order. For the baseline optimization run, equality constraints are first met, then the design parameter dependent terminal term of the performance index is optimized. Because most of the vehicle mass (~2100 kg) is concentrated in the aeroshell, the
algorithm then seeks to shift the cone half angle to 19.5°, the angle of minimum mass. Finally, the parachute system mass (~60 kg) is minimized until the vehicle is unable to satisfy the state boundary conditions with any further alteration of the configuration.

The baseline glide-parachute solution is located by an optimization run with the dynamic pressure inequality constraint and all atmospheric winds inactive. An initial control history and design parameter vector are selected with a desire to approximately imitate the previously demonstrated hypersonic glide phase trajectories. Experimentation yields the following initial augmented design parameter vector

\[ \mathbf{p}_{a0} = \begin{bmatrix} \delta_c \\ D_0 \\ t_{s1} \end{bmatrix} = \begin{bmatrix} 23.0° \\ 17.0 \text{ m} \\ 17.7 \text{ s} \end{bmatrix} \quad (6.3) \]

and a control history that closely resembles the glide phase demonstration baseline solution depicted in Figure AI-1. The algorithm successfully locates a solution that satisfies the equality constraints and minimizes the configuration mass. The important aspects of the trajectory are portrayed in Figure AII-1, while Table 6-1 presents some parameters of interest. The resulting discontinuities in the angle of attack and pitch histories occur due to the change in vehicle dynamics at the switch time.
Table 6-1. Initial Baseline Trajectory for the Coupled Hypersonic Glide and Parachute-Aided Descent Phases

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>ts (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline, δc = 23°</td>
<td>1350</td>
<td>3087.7</td>
<td>13.7</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
</tbody>
</table>

The baseline solution dynamic pressure builds up in the glide phase only after the vehicle aeroshell mass has been optimized and the analysis algorithm begins to minimize the parachute diameter. This effect is shown explicitly in Figure AII-2, where the final optimized baseline solution is compared to the intermediate solution characterized by minimum aeroshell mass only. Table 6-2 presents a summary of the valid performance parameters.

Table 6-2. Performance Parameter Comparison for the Baseline Solution and a Partially Converged Solution

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>ts (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>Partially Converged Baseline</td>
<td>842</td>
<td>3100.7</td>
<td>19.4</td>
<td>17.0</td>
<td>-17.1</td>
</tr>
</tbody>
</table>

The major difference between the two solutions detailed in Table 6-2 is that the baseline parachute diameter is over 3 meters less than the partially converged solution, at a mass savings of 13 kg. The trimmed decelerator area translates into a loss of vehicle kinetic energy dissipation capability during the parachute-aided descent phase. The fully minimized solution therefore relies on a lengthened
parachute descent time, an increased decelerator drag coefficient due to the high Mach number of deployment, and a hypersonic trajectory that "digs" deeper into the atmosphere to augment drag forces before the parachute-aided descent phase. The higher atmospheric density and the increased hypersonic glide phase free stream velocity result in the steep peak seen in the baseline solution dynamic pressure history.

6.2 The Effects of a Dynamic Pressure Inequality Constraint

6.2.1 Approach

The effects of imposing a dynamic pressure inequality constraint are examined. The large peak in the dynamic pressure trajectory displayed in the baseline solution of Figure AII-1 occurs during the vehicle hypersonic glide phase; effort concentrated on the analysis algorithm's ability to reduce this peak.

Two different solutions are available for the algorithm starting point in the presence of an upper bound on the dynamic pressure. Choosing the completely optimized initial baseline trajectory allows the algorithm to meet modest dynamic pressure bounds, but no further performance improvement is possible as the upper boundary is lowered below ~800 N/m². This phenomenon is attributable to the analysis algorithm converging to a local minimum in this region. The fully optimized baseline solution appears highly sensitive to the low requested upper bounds on the dynamic pressure, as the individual terms of the performance index are in conflict. To decrease dynamic pressure bound violations, more drag is needed, implying an increased parachute diameter or conical aeroshell half angle. Either of these changes
will add to the configuration mass, increasing cost due to the design parameter dependent terminal term of the performance index.

Utilization of the partially converged intermediate solution solely possessing optimized aeroshell results in convergence to a different local minimum that allows trajectories that satisfy more restrictive inequality constraint limits. The initial partially converged solution possesses enough decelerator area that the configuration drag is adequate and system mass improvements are still possible. Because of the better characteristics of the local minimum derived from this starting history, this solution becomes the analysis algorithm initialization point for the trials that investigate the effects of the dynamic pressure inequality constraint.

The weight on the dynamic pressure inequality constraint $K_Q$ is set at 100 for all trials included in this section. This value of $K_Q$ results in a distributed performance index term that is initially three to ten orders of magnitude greater than the terminal vehicle configuration cost. With such a large emphasis on the inequality constraint violations, the distributed cost term is quickly eliminated, and final converged solutions stay very near or below the upper bound.

6.2.2 Results

Converged solutions were found for dynamic pressure bounds of 1000, 800, 600, and 400 N/m$^2$. The important time dependent trajectory parameters are portrayed in Figure AII-3 for the 800, 600, and 400 Pascal dynamic pressure upper bound cases; the baseline solution is offered for comparison.

The apparent discontinuity in the angle of attack history for the $Q_{\text{max}} = 400$ Pa trial is due to a deficiency in the aeroshell aerodynamics model that was not apparent.
in the initial demonstration of the decoupled hypersonic phase. This phenomena is explained in detail in Section 7.3, and similar α histories appear in many of the subsequent optimization runs of this chapter. Although it may be possible to improve the model to eliminate the discontinuity, this research effort does not address the problem further. All analysis presented in this thesis treats the aeroshell aerodynamics as a closed form analytic model, and as such its merits may be evaluated within framework of the integrated design, control, and trajectory methodology.

Table 6-3 shows the important performance variables of these trials, along with the initial baseline solution for comparison.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>ts (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline, No Qmax</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>Qmax = 1000</td>
<td>1000</td>
<td>3088.2</td>
<td>19.4</td>
<td>13.8</td>
<td>-33.0</td>
</tr>
<tr>
<td>Qmax = 800</td>
<td>799</td>
<td>3088.8</td>
<td>19.4</td>
<td>14.0</td>
<td>-31.9</td>
</tr>
<tr>
<td>Qmax = 600</td>
<td>600</td>
<td>3089.7</td>
<td>19.4</td>
<td>14.2</td>
<td>-30.4</td>
</tr>
<tr>
<td>Qmax = 400</td>
<td>400</td>
<td>3091.9</td>
<td>19.5</td>
<td>14.8</td>
<td>-26.8</td>
</tr>
</tbody>
</table>

Table 6-3. Effects of the Dynamic Pressure Inequality Constraint in a Windless Atmosphere

6.2.3 Analysis

The vehicle configuration formulation has significant authority to decrease the dynamic pressure experienced throughout the hypersonic glide phase. The angle of attack histories in Figure AII-3 show a trend towards increased aeroshell drag coefficient as the dynamic pressure upper bound is diminished. In this way, the
vehicle sheds excess velocity during the hypersonic glide phase. Additionally, the parachute diameter enlarges, providing the extra deceleration required to satisfy the state "landing" condition. Although this tends to slightly increase the vehicle mass, the drag force is augmented and a large amount of energy is dissipated at a relatively small cost. On the other hand, a similar reduction in energy can be accomplished in the hypersonic glide phase by bolstering the aeroshell drag (increased conical half angle), resulting in a severe mass penalty. The analysis algorithm successfully negotiates this tradeoff, yielding specific vehicle performance sensitivity information.

As illustrated in Figure AII-3, analysis of any upper dynamic pressure bound below ~400 Pa will activate the separate distributed performance index term on the parachute-aided descent phase dynamic pressure. The current choice of the initial solution guess causes convergence to a local minimum from which the analysis algorithm is unable to lessen the dynamic pressure during the parachute phase. This characteristic is explored more fully in Chapter 7 of this thesis, and several suggestions for alleviating the problem are presented. Consequently, the analysis of this chapter concentrates on the effects of the dynamic pressure inequality constraint on the hypersonic glide phase only.

6.3 Effects of Head Winds

The integrated design, control, and trajectory optimization methodology was applied to the coupled hypersonic glide and parachute-aided descent phases which included the atmospheric wind profiles presented in Figure 3-13. Winds alter the performance of the parachute-aided descent phase, as the atmospheric disturbances
have the greatest influence in the denser low altitude section of the trajectory. The decelerator drag coefficient is also highly dependent on the flow Mach number, and the current implementation of the parachute phase provides no direct means of vehicle control.

Numerical sensitivities in the analysis algorithm require the user to slowly increase the wind magnitude to values of interest between analysis runs. This incremental approach begins with an initial guess that closely mimics the hypersonic glide phase demonstration baseline. A small head wind magnitude is injected into the simulation, and the algorithm perturbs the vehicle design and control history until the state boundary conditions are satisfied. This solution then becomes the initial guess for a simulation in which the wind magnitude has been increased. The initial guess design parameters may also be altered to slightly improve algorithm convergence in the presence of augmented wind magnitudes. This method insures quick and stable algorithm performance when addressing the effects of the most extreme atmospheric wind profiles.

6.3.1 Case 1 -- Typical Low Latitude Northern Hemisphere Summer Solstice

The analysis of vehicle performance in the presence of the head winds begins with the least extreme profile, that typical of the low latitude northern hemisphere summer solstice. Table 6-4 shows the most important performance variables for optimization runs conducted with the maximum magnitude of this wind case. The most important trajectory parameters for the trials with and without a dynamic pressure upper bound are plotted in Figures AII-4 and AII-5. The windless optimized baseline solutions are presented for comparison. The dynamic pressure upper bound
of 600 N/m² is chosen as a moderate constraint that influences only the hypersonic glide phase.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>ts, (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Q_max, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>No Q_max, Wind 100%</td>
<td>1466</td>
<td>3078.0</td>
<td>19.4</td>
<td>10.6</td>
<td>-25.2</td>
</tr>
<tr>
<td>Q_max = 600, No Wind</td>
<td>600</td>
<td>3089.7</td>
<td>19.4</td>
<td>14.2</td>
<td>-30.4</td>
</tr>
<tr>
<td>Q_max = 600, Wind 100%</td>
<td>600</td>
<td>3082.2</td>
<td>19.4</td>
<td>12.0</td>
<td>-21.5</td>
</tr>
</tbody>
</table>

Table 6-4. Effects of Head Wind Case 1

The head wind causes the free stream velocity vector to increase during the dense low altitude region of the descent, providing greater drag forces for similar vehicle configurations and control histories. The result is a less stringent drag requirement for the parachute-aided descent phase, and a decrease in both parachute size and deployment time. The maximum wind velocity of ~29 m/s occurs at an altitude of ~11.5 kilometers.

For the trial without an upper bound on the dynamic pressure, the head wind allows the analysis algorithm to locate a "low-drag" vehicle configuration that has a parachute diameter approximately 3 meters less than the windless design; the parachute is also deployed nearly 10 seconds later in the trajectory. These parachute-aided descent characteristics are attained without altering the aeroshell cone half angle, but the trajectory is flown at a slightly lower altitude than in the windless case, resulting in augmented aeroshell forces. This configuration shaves nearly 10 kg from the total vehicle mass required to satisfy the state boundary constraints in the windless atmosphere. Additionally, the dynamic pressure peak increases due to the larger V∞.
experienced by the vehicle due to the head wind, and inertial velocities are diminished notably in the denser regions of the trajectory.

The application of both a 600 N/m² upper bound on the dynamic pressure and the maximum wind magnitude results in similar trends in the vehicle performance. The parachute diameter and deployment are decreased by ~2 meters and ~9 seconds respectively from the windless case, at a mass savings of 5.5 kilograms. All trajectory time histories plotted in Figure AII-5 display matching characteristics, except for the high altitude angle of attack performance. A closer examination of the specific lift and drag force histories for these trials show that the different angles of attack result in similar vehicle forces. This result is a direct consequence of the aeroshell lift and drag coefficient model deficiencies that were previously noted, and is not an outcome of the injected wind.

6.3.2 Case 2 -- Median Oscillatory Profile

Incorporation of the median oscillatory profile also resulted in successfully optimized solutions that satisfy the state boundary constraints and minimize the specified performance index. Table 6-5 presents the important parameters for these trials, along with baseline solutions for comparison. Once again, the results are dominated by the increased free stream velocity vector and the associated augmented vehicle forces. The maximum wind velocity of 70 m/s occurs at ~12.5 kilometers altitude. Figures AII-6 and AII-7 illustrate the important trajectory variables for the trials with and without the dynamic pressure inequality constraint, respectively.
<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δ_c (°)</th>
<th>D_o (m)</th>
<th>t_s (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Q_max, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>No Q_max, Wind 100%</td>
<td>1550</td>
<td>3075.8</td>
<td>19.4</td>
<td>9.7</td>
<td>-23.5</td>
</tr>
<tr>
<td>Q_max = 600, No Wind</td>
<td>600</td>
<td>3089.7</td>
<td>19.4</td>
<td>14.2</td>
<td>-30.4</td>
</tr>
<tr>
<td>Q_max = 600, Wind 100%</td>
<td>600</td>
<td>3082.4</td>
<td>19.4</td>
<td>12.1</td>
<td>-17.6</td>
</tr>
</tbody>
</table>

Table 6-5. Effects of Head Wind Case 2

The solution for the trial without an upper bound on the dynamic pressure has an optimized configuration characterized by a parachute diameter that is 4 meters less than the windless case, and the decelerator deployment time is diminished by 10 seconds. This trims almost 12 kilograms from the vehicle design, a significant decrease in the desired performance criteria. The important trajectory variables plotted in Figure AII-6 display almost identical performance as those for the least extreme head wind of Case 1. The similarity can be understood when the two wind profiles plotted in Figure 3-13 are compared at the low altitudes typical for the parachute-aided descent phase (3 - ~5 km). The two different wind cases have similar profiles in these altitudes, where the vehicle is most sensitive to the wind disturbances. The median oscillatory solution displays a greater hypersonic glide phase dynamic pressure, and also a diminished parachute diameter. These trends are attributable to the augmented vehicle forces experienced during hypersonic glide phase due to the higher head winds present in the median oscillatory profile.

Adding an upper bound on the dynamic pressure of 600 N/m² also results in performance that is similar to that of the low latitude northern summer solstice wind profile of Case 1, as portrayed in Figure AII-7. The increased free stream velocity
vector allows dissipation of the vehicle energy with a parachute diameter over 2 meters less than the windless configuration. The decelerator deployment time is also decremented by nearly 13 seconds, delaying the onset of the high drag parachute-aided descent phase. The requested dynamic pressure bound is satisfied with no increase in the aeroshell half angle.

6.3.3 Case 3 -- Most Extreme High Latitude Dust Storm

The final head wind case implemented in the analysis algorithm is the most extreme profile representative of a high latitude Martian dust storm. The analysis algorithm was unable to satisfy a requested upper bound on the dynamic pressure of 600 N/m², a larger bound of 800 N/m² is therefore presented in this analysis. Table 6-6 presents the important parameters for these optimization runs. In all trials furnished below, the solutions satisfy the state boundary conditions and minimize the previously defined performance criteria.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>D₀ (m)</th>
<th>tₛ₁ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Qₘₐₓ, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>No Qₘₐₓ, Wind 100%</td>
<td>1621</td>
<td>3064.5</td>
<td>19.4</td>
<td>2.5</td>
<td>-19.1</td>
</tr>
<tr>
<td>Qₘₐₓ = 800, No Wind</td>
<td>799</td>
<td>3088.8</td>
<td>19.4</td>
<td>14.0</td>
<td>-31.9</td>
</tr>
<tr>
<td>Qₘₐₓ = 800, Wind 100%</td>
<td>800</td>
<td>3073.7</td>
<td>19.4</td>
<td>8.8</td>
<td>-10.1</td>
</tr>
</tbody>
</table>

Table 6-6. Effects of Head Wind Case 3

The large wind magnitudes of this profile initiate significant alterations in the vehicle configuration and trajectory. The maximum wind magnitude of 258 m/s
occurs at 35 kilometers altitude, and the "landing" condition magnitude is 83 m/s at 3 kilometers. Figures AII-8 and AII-9 depict the relevant variables over the trajectory.

The trial without an upper bound on the dynamic pressure shows a striking reduction in the parachute diameter of over 11 meters relative to the optimized windless case. This reduction is accompanied by ~15 second decrease in the parachute deployment time. Both results are again explainable by the augmented drag forces available to the vehicle due to the increased free stream velocity vector. This vehicle design is over 13 kilograms less massive than the windless optimal configuration. The low altitude wind magnitude for this case is nearly four times the comparable magnitudes of the previous wind cases; the parachute diameter decreases by a proportional amount.

The trajectory parameters plotted in Figure AII-8 show the large dynamic pressure peak encountered over the entire entry profile and the large decrease in the inertial horizontal velocity. Near the beginning of the hypersonic glide phase, the windless angle of attack history is small and negative, resulting in negative lift forces that steepen the vehicle flight path, as illustrated by the altitude versus range plot. With the extreme wind added, the vehicle does not display this "dig down" characteristic, as vehicle forces are sufficiently large to eliminate the need for quick increases in the atmospheric density.

Addition of an upper bound on the dynamic pressure illuminates a deficiency in the selected vehicle configuration capabilities when subjected to head wind disturbances. The analysis algorithm could not accommodate dynamic pressure bounds lower than ~800 N/m² in the presence of the most extreme head wind. Below this bound, the large relative weight on the distributed cost term (K_Q) causes the
algorithm to accept significant mass penalties to accommodate very small decrements in the trajectory dynamic pressure. The result is an conical aeroshell half angle that diverges quickly from the minimum mass solution at 19.5° (~1° for each 1 Pa lessened from the dynamic pressure). The trial presented above represents the realizable dynamic pressure limit for acceptable vehicle configurations.

Figure AII-9 plots the important trajectory parameters for this trial against the windless \( Q_{\text{max}} = 800 \text{ Pa} \) optimization run. The windless case must again "dig down" into the atmosphere to provide the necessary vehicle forces, while the augmented free stream velocity provides the required forces when the wind is active. The inertial velocities are also diminished in the presence of the head wind.

### 6.4 Effects of Tail Winds

The integrated design, control, and trajectory optimization methodology was also applied to the coupled hypersonic glide and parachute-aided descent phases subjected to tail winds. The vehicle performance is independently evaluated for each of the three wind profiles of Figure 3-13 in the following sections. The previously described incremental approach is again utilized to build up the wind magnitudes to values of interest. As in the hypersonic glide phase demonstration of Chapter 5, the vehicle configuration sensitivities permit only a certain magnitude of each wind to be implemented while still satisfying the state boundary conditions. Accordingly, the tables and figures below present the winds as a percentage of the maximum wind profile magnitude. Plots of the actual tail winds experienced by the vehicle over the trajectory time may be found in Section 6.5.
6.4.1 Case 1 -- Typical Low Latitude Northern Hemisphere Summer Solstice

The analysis of vehicle performance in the presence of the tail winds begins with the least extreme profile, that typical of the low latitude northern hemisphere summer solstice. The vehicle configuration is successfully able to satisfy the state boundary conditions in the presence of 91% of this wind profile. Any additional increase in this wind magnitude results in conditions for which the algorithm is unable to satisfy the trajectory solution requirements.

The most important performance variables for optimization runs are presented in Table 6-7, and the windless optimized baseline solutions are presented for comparison. The most important trajectory parameters for the trials with and without a dynamic pressure upper bound are plotted in Figures AII-10 and AII-11. The dynamic pressure upper bound of 400 N/m² is chosen as the lowest hypersonic glide phase limit obtainable before the overlap with the parachute inequality constraint.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>t_{s1} (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Q_{max}, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>No Q_{max}, Wind 91%</td>
<td>1107</td>
<td>3098.9</td>
<td>19.4</td>
<td>16.5</td>
<td>-65.0</td>
</tr>
<tr>
<td>Q_{max} = 400, No Wind</td>
<td>400</td>
<td>3091.9</td>
<td>19.5</td>
<td>14.8</td>
<td>-26.8</td>
</tr>
<tr>
<td>Q_{max} = 400, Wind 91%</td>
<td>399</td>
<td>3099.0</td>
<td>19.5</td>
<td>16.6</td>
<td>-60.9</td>
</tr>
</tbody>
</table>

Table 6-7. Effects of Tail Wind Case 1

This wind profile reduces the free stream velocity in the low altitude regions of the trajectory while increasing it at high altitudes. The result is reduced vehicle forces at the altitudes of greatest sensitivity, the high atmospheric density region of the
parachute-aided descent phase. The tail winds "push" the vehicle in the horizontal direction, effectively adding energy to the system. The configuration is altered such that this extra energy is dissipated in the most efficient manner (lowest vehicle mass) by increasing both the parachute nominal diameter and deployment time. The tail wind trajectories also last a longer time, allowing the vehicle forces to dissipate additional energy.

For the trial without an upper bound on the dynamic pressure, the tail wind allows the analysis algorithm to locate a "high-drag" vehicle configuration that has a parachute diameter almost 3 meters greater than the windless design; the parachute is also deployed nearly 31 seconds earlier in the trajectory. Additionally, the tail wind solution possesses a parachute deployment state that is nearly 2 kilometers in altitude and 0.25 Mach number greater than that of the windless trajectory. All of these factors contribute to increased drag forces and energy dissipation during the parachute-aided descent phase. The conical aeroshell half angle remains constant at 19.4°, only a tenth of a degree less than the aeroshell mass minimum. The resulting vehicle mass is over 11 kilograms greater than the windless vehicle configuration. The analysis algorithm successfully recognizes that large vehicle drag forces may be obtained at the least mass penalty by increasing the area of the relatively lightweight parachute canopy.

The dynamic pressure profile plotted in Figure AII-10 illustrates the significant decrease in the free stream velocity, as the tail wind trajectory solution has a peak that is almost 250 Pa less than that experienced in the windless trajectory. The angle of attack histories display nearly identical behavior in the low altitude region of the trajectory, except for the displacement caused by the difference in parachute
deployment time. However, the tail wind configuration angle of attack tends towards ninety degrees near the "atmospheric entry" state boundary condition. Consulting the lift coefficient plot of Figure 3-9, this corresponds to a negative lift "dig" down maneuver at high altitudes. The result is vehicle trajectory that at a slightly lower altitude between approximately 30 and 25 kilometers than the windless case. The higher altitude of the tail wind vehicle trajectory near the "landing" condition is consistent with the augmented drag performance required by the parachute phase, as the high Mach number, altitude, and time of deployment all serve to increase the energy dissipation capabilities of the "terminal" flight phase.

The application of both a 400 N/m² upper bound on the dynamic pressure and 91% of the tail wind magnitude results in similar trends in the vehicle performance. The parachute diameter and deployment time are both greater by nearly 2 meters and 34 seconds, respectively. The conical aeroshell half angle again stays constant at the minimum mass solution. The result is a vehicle configuration that has augmented energy dissipation capabilities with an increase in mass of just over 7 kilograms. The important trajectory parameters illustrated in Figure AII-11 show nearly identical behavior with the windless trial, again except for the time displacement.

6.4.2 Case 2 -- Median Oscillatory Profile

The vehicle configuration is evaluated during an optimization run with the median oscillatory tail wind profile. The analysis algorithm successfully locates a vehicle design and control history that satisfy the state boundary conditions for 53% magnitude of this wind. The addition of even slightly more wind again results in conditions for which the algorithm is unable to satisfy trajectory requirements. Table
6-8 presents the important parameters for cases with and without an upper bound on the dynamic pressure constraint, along with the baseline solutions for comparison.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>$\delta_c$ (°)</th>
<th>$D_o$ (m)</th>
<th>$t_{s_i}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No $Q_{max}$, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>No $Q_{max}$, Wind 53%</td>
<td>1072</td>
<td>3096.9</td>
<td>19.4</td>
<td>16.1</td>
<td>-50.3</td>
</tr>
<tr>
<td>$Q_{max} = 400$, No Wind</td>
<td>400</td>
<td>3091.9</td>
<td>19.5</td>
<td>14.8</td>
<td>-26.8</td>
</tr>
<tr>
<td>$Q_{max} = 400$, Wind 53%</td>
<td>400</td>
<td>3097.4</td>
<td>19.4</td>
<td>16.2</td>
<td>-46.0</td>
</tr>
</tbody>
</table>

Table 6-8. Effects of Tail Wind Case 2

The trends in the optimal vehicle configuration are similar to those seen in the previous low latitude wind case. Once again, the results are dominated by the increased free stream velocity vector and the associated diminished vehicle forces. Figures AII-12 and AII-13 illustrate the important trajectory variables for the trials with and without the dynamic pressure inequality constraint, respectively.

Without a constraint on the trajectory dynamic pressure, the vehicle configuration is altered to provide greater energy dissipation characteristics in the presence of the tail wind. The conical aeroshell half angle remains constant at 19.4°, but the parachute nominal diameter and deployment time are increased by ~2.5 meters and ~15 seconds, respectively, from that seen in the windless trial. The resulting optimal vehicle configuration is over 9 kilograms more massive than in the windless case.

Figure AII-12 portrays plots of the most important trajectory variables for this case; the trends are similar to the that of the previous tail wind trial. The dynamic pressure peak is over 275 N/m² less than that occurring during the windless trajectory. The angle of attack histories are similar at low altitudes, albeit displaced...
by the additional parachute deployment time. The large peak in $\alpha$ at the high altitude "entrance" condition represents a vehicle attempt attain sufficient negative lift to "dig down" into the denser regions of the atmosphere. Any further reductions in the vehicle mass and parachute diameter cause the analysis algorithm to perturb the angle of attack above $90^\circ$, causing unstable performance. It is interesting to note the delay of this "dig down" maneuver in the median oscillatory profile from that seen in the analysis of the Case 1 tail wind. This may be explained by the wind sign change seen at high altitudes in the profile typical to the low latitude regions of the Martian atmosphere. The median oscillatory profile lacks this high altitude head wind bias; the "dig down" maneuver is therefore delayed until the atmosphere is dense enough to have significant effect on the vehicle forces.

Adding an upper bound on the dynamic pressure of 400 N/m$^2$ also results in performance that is similar to that of the low latitude wind profile of Case 1, as portrayed in Figure AII-13. The vehicle configuration allows higher drag dissipation by utilizing a parachute with an increased canopy diameter of nearly of 1.5 meters and a deployment time lengthened by over 19 seconds. These differences translate to a vehicle that is 5.5 kilograms more massive than the optimal windless design. The relatively stringent dynamic pressure bound may be met by the configuration, and all trajectory plots show nearly similar behavior to both the windless trial and the Case 1 tail wind solution.

6.4.3 Case 3 -- Most Extreme High Latitude Dust Storm

The final tail wind case implemented in the Martian entry/landing simulation is that typical of the most extreme high latitude dust storm. The vehicle is successfully
able to negotiate up to 19% of this wind profile magnitude while still satisfying trajectory solution requirements. Table 6-9 shows the important performance parameters for trials both with and without the 400 N/m² dynamic pressure bound. The baseline converged solutions are again presented for comparison. Figures AII-14 and AII-15 depict the relevant variables over the trajectory.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>t₀ₕ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Qₘₐₓ, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>No Qₘₐₓ, Wind 19%</td>
<td>1105</td>
<td>3097.1</td>
<td>19.4</td>
<td>16.1</td>
<td>-50.0</td>
</tr>
<tr>
<td>Qₘₐₓ = 400, No Wind</td>
<td>400</td>
<td>3091.9</td>
<td>19.5</td>
<td>14.8</td>
<td>-26.8</td>
</tr>
<tr>
<td>Qₘₐₓ = 400, Wind 19%</td>
<td>400</td>
<td>3097.6</td>
<td>19.4</td>
<td>16.2</td>
<td>-45.7</td>
</tr>
</tbody>
</table>

Table 6-9. Effects of Tail Wind Case 3

The vehicle performance in the absence of an upper bound on the dynamic pressure is once again dominated by configuration alterations that offset the decrease in the relative free stream velocity vector. The decelerator nominal diameter is increased by almost 2.5 meters, and the parachute deployment time is increased by nearly 16 seconds. The resulting configuration is nearly 9.5 kilograms more massive than its windless counterpart. Figure AII-14 shows that the dynamic pressure peak is decreased by nearly 250 Pa due to the diminished free stream velocity vector and higher altitude of the trajectory. The angle of attack performance is nearly identical to that seen in the previous median oscillatory tail wind trials and may be explained with similar arguments.

Finally, the addition of the 400 N/m² dynamic pressure limit results in performance that is similar to the other two tail wind profiles. The energy dissipation
characteristics of the vehicle are augmented by an increase in the parachute diameter of nearly 1.5 meters, and the parachute is also deployed for almost 19 seconds longer than the windless trial. These alterations yield a vehicle that is nearly 6 kilograms more massive than the optimal windless solution. Figure AII-15 shows that the dynamic pressure peak in hypersonic glide phase is completely eliminated, and all other trajectory variables display behavior very similar to the baseline solution.

### 6.5 Analysis of Results

The results of the coupled hypersonic glide and parachute-aided phases yield specific information regarding the system performance in the presence of a variety of atmospheric winds and dynamic pressure inequality constraints. Tables 6-10 and 6-11 present a summary of the system performance parameters for all the wind cases investigated in the previous sections.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum $Q$ (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>$\delta_c$ ($^\circ$)</th>
<th>$D_0$ (m)</th>
<th>$t_{sl}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline, No Wind</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>Case 1 Head Wind 100%</td>
<td>1466</td>
<td>3078.0</td>
<td>19.4</td>
<td>10.6</td>
<td>-25.2</td>
</tr>
<tr>
<td>Case 2 Head Wind 100%</td>
<td>1550</td>
<td>3075.8</td>
<td>19.4</td>
<td>9.7</td>
<td>-23.5</td>
</tr>
<tr>
<td>Case 3 Head Wind 100%</td>
<td>1621</td>
<td>3064.5</td>
<td>19.4</td>
<td>2.5</td>
<td>-19.1</td>
</tr>
<tr>
<td>Case 1 Tail Wind 91%</td>
<td>1107</td>
<td>3098.9</td>
<td>19.4</td>
<td>16.5</td>
<td>-65.0</td>
</tr>
<tr>
<td>Case 1 Tail Wind 53%</td>
<td>1072</td>
<td>3096.9</td>
<td>19.4</td>
<td>16.1</td>
<td>-50.3</td>
</tr>
<tr>
<td>Case 1 Tail Wind 19%</td>
<td>1105</td>
<td>3097.1</td>
<td>19.4</td>
<td>16.1</td>
<td>-50.0</td>
</tr>
</tbody>
</table>

Table 6-10. System Performance Parameters for Trials Without the Dynamic Pressure Inequality Constraint
Table 6-11. System Performance Parameters for Trials with the Dynamic Pressure Inequality Constraint

Both data sets displayed above show the flexibility the vehicle configuration has in satisfying the state boundary conditions over a wide range of wind conditions and dynamic pressure limits. The trends in vehicle configuration show definitively that the addition of the parachute-aided descent phase has desensitized the aeroshell design to the different trajectory requirements of this research effort. Nearly all of the
vehicle design variation occurs in the parachute canopy nominal diameter and
decelerator deployment time, between 2.5 and 16.5 meters and -19.1 and -65.0
seconds respectively. This translates to a system mass range of only 34.4 kilograms,
all of this fluctuation occurring in the parachute system mass.

The head wind trials show that the large increase in the free stream velocity
vector allows the vehicle configurations to successfully negotiate the most extreme
anticipated Martian winds. Unfortunately, the increased free stream velocity vector
renders some proposed dynamic pressure constraints unattainable for the hypersonic
glide phase. Local minima are present just below 600 N/m² for the typical low
latitude and the median oscillatory wind profiles, and also just below 800 Pa for the
most extreme dust storm profile. These minima restrict any further dynamic pressure
performance gains by the vehicle configuration as more constrained solutions become
unrealizable.

As in the hypersonic glide phase demonstration, the tail wind trials show a
definite magnitude limit past which the algorithm cannot successfully locate a
solution that satisfies the state boundary constraints. This solution limit threshold is
evident in Figure 5-1, where the three tail wind profiles are plotted versus the
trajectory time for the optimized solutions without the dynamic pressure bound. The
wind velocities differ by only a few meters per second during the parachute aided
descent phases, between 0 and ~65 seconds. The low latitude wind of Case 1 is
slightly greater in the parachute phase, but the high altitude head wind bias provides
the vehicle with extra time and energy dissipation capabilities in the hypersonic glide
phase.
An optimization run of the Case 1 low latitude tail wind converged to a different local minimum than that illustrated in the previous analysis. As shown in Table 6-12, this alternate solution has nearly identical vehicle performance parameters. However, a comparison of the trajectory variables plotted in Figure AII-16 shows the great disparity in the optimized solution. The alternate trial shows an impressive vehicle pull up maneuver after a low altitude energy dissipation region of the trajectory. This is accompanied by a discontinuous angle of attack history reflective of the aeroshell aerodynamics model. The presence of multiple local optima in the solution space will be explored in depth in the next chapter. Special note should be made that alternate solutions may be present for the other tail wind cases examined in this study, and that
these solutions may yield enhanced vehicle performance in the presence of increased tail winds.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>δc (°)</th>
<th>Do (m)</th>
<th>tsi (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Solution</td>
<td>1107</td>
<td>3098.9</td>
<td>19.4</td>
<td>16.5</td>
<td>-65.0</td>
</tr>
<tr>
<td>Alternate Solution</td>
<td>977</td>
<td>3098.7</td>
<td>19.4</td>
<td>16.5</td>
<td>-66.0</td>
</tr>
</tbody>
</table>

Table 6-12. Comparison of Local Minimum Solutions for Case 1 Tail Wind

The information gleaned from these results yield insight into vehicle design robustness. The vehicle model must be desensitized to the tail wind magnitudes, and the wide variation of optimal parachute sizes requires some reduction. The analysis algorithm also may require additional features to enable its handling of a broader set of trajectory constraints. The next chapter discusses a few of these issues in detail, and provides some suggestions for future study.
Chapter 7

Proposed Resolution of Robustness, Numerical, and Fidelity Issues

7.1 Improving Vehicle Robustness to Atmospheric Winds

7.1.1 Modeling Issue Definition

Atmospheric wind disturbances are manifested in the vehicle forces through the squared free stream velocity magnitude present in the dynamic pressure, and also in any atmospheric dependence of the aerodynamic coefficients. The hypersonic glide phase is flown in the low density, high altitude regions of atmosphere, significantly reducing the influence of the atmospheric winds. The aeroshell forces are highly nonlinear functions of the free stream velocity, but there is enough control authority is provided to mitigate these effects. When coupled with the high energy dissipation of the parachute-aided descent phase, these model characteristics tend to desensitize the aeroshell design to the injected disturbances. This analysis is supported by the small
variation in the aeroshell half angle illustrated in the coupled hypersonic glide/parachute-aided descent analysis presented in Tables 6-10 and 6-11.

A closer examination of the parachute-aided descent phase implementation will explain the sensitivity of the parachute size and deployment time to the wind disturbances. The parachute/lander system lacks lift force capability and is uncontrolled. The design therefore is unable to rotate the aerodynamic force directions relative to the system centerline. Coupled with the pinpoint terminal "landing" conditions of the vehicle, this system has little authority to compensate for disturbances in the atmosphere which alter the vehicle course.

Many aspects of the parachute-aided flight phase dynamics model tend to intensify the poor disturbance rejection qualities of the design. The atmosphere is denser at the low altitudes of parachute operation, heightening wind forces experienced by the parachute/lander system. The system mass is also lessened as the aeroshell is jettisoned, causing the wind disturbances to have even greater influence on the vehicle state dynamics.

Additionally, the parachute drag coefficient dependence on the Mach number causes the parachute dynamics to be very sensitive to the wind. Table 7-1 displays the Mach number, altitude of decelerator deployment, and vehicle design parameters for the baseline and tail wind trials. Typical windless solutions have a parachute deployment Mach number of ~1.3, which yields the highest decelerator drag coefficient (~0.75), as shown in Figure 3-10. The most sensitive tail wind solutions, however, are characterized by high Mach numbers of parachute deployment, near 1.5. The parachute drag coefficient is still quite high in this flow regime (~0.7) and nearly 50% greater than that for typical head wind trials.
### Table 7-1. Energy Dissipation Transfer from the Hypersonic Glide Phase to the Parachute-Aided Descent Phase

<table>
<thead>
<tr>
<th>Trial</th>
<th>Deployment Mach Number</th>
<th>Deployment Altitude (m)</th>
<th>$\delta_c$ (°)</th>
<th>$D_o$ (m)</th>
<th>$t_{s_1}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline, No Wind</td>
<td>1.3</td>
<td>4470</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>Case 1 Tail Wind 91%</td>
<td>1.5</td>
<td>6410</td>
<td>19.4</td>
<td>16.5</td>
<td>-65.0</td>
</tr>
<tr>
<td>Case 1 Tail Wind 53%</td>
<td>1.5</td>
<td>5500</td>
<td>19.4</td>
<td>16.1</td>
<td>-50.3</td>
</tr>
<tr>
<td>Case 1 Tail Wind 19%</td>
<td>1.5</td>
<td>5500</td>
<td>19.4</td>
<td>16.1</td>
<td>-50.0</td>
</tr>
</tbody>
</table>

7.1.2 Thought Experiment to Understand the Vehicle Sensitivity to Tail Winds

A hypothetical example is useful to demonstrate this sensitivity described above. The methodology operates in backwards time, such that the initial state boundary "landing" conditions are fixed at the proposed parachute phase termination. An optimized solution is first constructed for a trial with a relatively large tail wind magnitude, such as one seen in Figure 6-1. The addition of a small amount of wind magnitude to this converged solution (the incremental approach) results in decreased free stream velocities, Mach numbers, and drag forces during the parachute phase. Because the parachute/lander system has low mass, a prolonged flight time, and minimal control characteristics, the small wind increment induces severe violations of the initial guess of the hypersonic glide phase termination point. The initial assumed aeroshell control history has no compensation information that will bring the vehicle to the terminal state "atmospheric entry" boundary condition, resulting in large equality constraint violations.
The results from application of the analysis algorithm counteract these violations by stretching the deployed parachute phase and slightly increasing the decelerator diameter. Because the tail wind parachute diameters are already large (~16 meters), immense aerodynamic forces are encountered near the switch time. Even small (< 1 second) increases in the decelerator deployment time will therefore greatly increase the energy dissipated during the entire flight phase. The analysis algorithm successfully recognizes that the parachute-aided descent phase is more efficient at dissipating excess energy; the kinetic and potential energies of the vehicle state are therefore increased at parachute deployment time. Evidence of this phenomenon is presented in Table 7-1.

For tail winds beyond the solution threshold presented in the above table, several of the energy dissipation characteristics of the parachute/lander system come into conflict. The tail winds tend to decrease the free stream velocity, yielding a sharp decrease in the parachute dynamic pressure and drag coefficient. Small tail wind increments may also cause the analysis algorithm to seek large increases in the switch time to mitigate these effects, as the extra energy imparted to the system may be dissipated by longer decelerator deployment times. The analysis algorithm is unsuccessful at negotiating the nonlinearities associated with the large required variations in the parachute deployment time and Mach number. This leads to a circumstance where the model precludes a realizable solution as the decelerator quickly exceeds the assumed deployment Mach limit of 2.6 due to large increases in the switch time.

Preliminary trial runs bolster this explanation. Increasing the deployed parachute phase duration guess by 6 - 10 seconds will allow the analysis algorithm to
satisfy the state boundary conditions for extra wind increments of only +0.1% of the solution threshold magnitudes. Even a small increase in system energy cannot be satisfied while still meeting the pinpoint state "landing" conditions. Any small increase in tail wind will therefore significantly alter the decelerator performance. Tail winds both increase the energy dissipation requirement and decrease the Mach number over the entire parachute regime. The Mach number loss tends to decrease both the drag coefficient and the flight dynamic pressure, severely restricting the energy dissipation capabilities over the long deployment times seen in these cases, and eventually reaching conditions which preclude realizable solution. The following section provides some suggestions for reducing this sensitivity.

7.1.3 Proposed Problem Resolution

The vehicle sensitivity to tail wind disturbances may be alleviated by the inclusion of an all propulsive landing phase [14], [15], [19], [33] at the termination of the parachute-aided descent phase. A mono- or bi-propellant propulsion package will provide the vehicle with significant control authority in the high density region of the atmosphere, where winds will have the greatest impact. For the two dimensional case, this control may be provided by a throttable engine with a selectable maximum thrust design parameter. The vehicle should also have the ability to alter the thrust direction, either through lander pitch control or a gimbaled rocket engine.

Implementing this flight phase will require additional analysis algorithm logic. New vehicle state, control, and design parameter vectors must be defined, following the framework of [19]. Vehicle mass and aerodynamics models must be revised to a form that is suitable for an all-propulsive descent. An additional dynamics switch
time must be introduced into the vehicle design parameter vector, and analysis algorithm stability issues at the new state dynamics discontinuity must be investigated. Finally, suitable performance index and equality constraints must be defined to optimize the amount of propellant burned and match its volume to that of the fuel tanks. The added flight phase models should be kept as simple as possible to facilitate future analysis.

The state initial "landing" condition may now be fixed to the actual surface of the planet, freeing the vehicle state at the termination of the parachute phase. The parachute/lander system will still display high sensitivity to atmospheric winds, but the uncontrollable nature of this descent phase will now impose minimal constraints on the vehicle performance. Any vehicle state errors occurring in this flight regime may be compensated by the control authority present in the other two flight phases. The result will be a vehicle design that is more robust to the wind disturbances present in Martian environment.

7.2 Alleviating Design/Trajectory Solution Sensitivity to the Dynamics Discontinuity at the Switch Time

7.2.1 Modeling Issue Definition

The analysis of the coupled hypersonic glide and parachute-aided descent phases revealed that the current vehicle configuration is unable to satisfy the state boundary conditions for certain imposed inequality constraints and atmospheric wind conditions. Although the vehicle is successfully able to satisfy the state boundary conditions for all of the head wind profiles of interest, the dynamic pressure performance is degraded as the free stream velocity vector is increased. The current
vehicle configuration also cannot handle tail winds in excess of those depicted in Figure 6-1. Wind magnitude increases result in numerical analysis algorithm convergence problems of various forms. Even those optimization runs that result in fully converged solutions for tail winds display a wide range of vehicle configurations, primarily in the parachute nominal diameter and the decelerator deployment time.

One aspect of this solution character sensitivity is caused by the current planetary entry/landing problem implementation. In general, the addition of a state dynamics discontinuity adds a significant amount of nonlinearity to the optimization problem. Extra logic is required to treat this effect, and additional terms are present in the control and design parameter perturbation equations. Additionally, gains must be specified to weight the switch time dependent terms of these equations relative to the vehicle configuration parameters. The computational process is more complicated, and numerical errors are more likely to occur.

The sensitivity due to the dynamics discontinuity is reinforced by the contrasting dynamics of the two vehicle flight phases. For the hypersonic glide phase, aerodynamic forces are available both parallel and perpendicular to the free stream velocity vector. The aeroshell lift and drag force coefficients depend on the conical half angle and angle of attack, which is in turn a function of both the free stream relative flight path angle and the control, the vehicle pitch angle. The vehicle control is crucial to mitigating the nonlinear terms of the free stream velocity in both the flight path angle and dynamic pressure, found in Equations (4.34) and (4.35).

However, the parachute-aided descent dynamics model differs significantly from that of the hypersonic glide phase. Most notable is the discontinuity in the
vehicle mass at the switch time as the aeroshell is jettisoned. The instantaneous loss of over 2/3 of the system mass introduces an important nonlinearity to the switch time dynamics. Also, no aerodynamic forces are produced perpendicular to the free stream direction, as the parachute and lander are assumed to produce drag only. Additionally, the parachute drag coefficient model has direct nonlinear dependence on the flight Mach number, which is in turn directly proportional to the free stream velocity. This results in a third order dependence of the decelerator drag force on the free stream velocity, and no vehicle control is available to regulate the force magnitudes. These differences from the hypersonic glide dynamics are manifested in the high degree of sensitivity of the switch time in the solution space.

7.2.2 Proposed Problem Resolution

Three approaches are proposed to lessen the effects of the vehicle dynamics differences at parachute deployment and thereby decrease the analysis algorithm sensitivity at the switch time. The first is to assume that the aeroshell is jettisoned at the end of the parachute-aided descent phase rather than at the beginning. Use of this maneuver has been previously documented [33] as a method for reducing problems associated with the separation dynamics at high Mach numbers. Retaining the aeroshell will eliminate the nonlinearity due to the mass change at the switch time, and will additionally provide the vehicle with lift force and control (angle of attack) capabilities. Although retaining this mass increases the potential energy dissipation requirements of the vehicle configuration, the result will be a smoother dynamics transition, and decreased trajectory solution sensitivity to the hypersonic glide/parachute-aided descent phase switch time.
Another strategy is to provide the vehicle with parachute reefing capability [33]. The parachute inflated diameter $D_0'$ may be lessened by providing a line system to restrict the skirt perimeter during a certain flight phase. Explosive charges are then used to sever the reef line at the appropriate time in the trajectory, allowing the parachute to inflate to its full decelerator area. If the aeroshell is retained, the end of the reeled phase may also provide a natural jettison point at a suitable subsonic Mach number. Finally, parachute reefing will lessen the sharp drag force spike (snatch load) experience at the parachute deployment time.

Although inclusion of the reeled parachute phase would require extra analysis algorithm logic and an additional dynamics switch time, the transition between the aeroshell and the high area decelerator dynamics will be smoothed by the presence of an intermediate energy dissipation phase. The reefing system may provide vehicle robustness to both head and tail winds and an array of dynamic pressure bounds. The parachute diameter and system mass may also be desensitized to the high and low energy requirements of the uncertain atmospheric conditions. This will provide the parameterized design model with the flexibility to alter the operation times of the intermediate drag reeled system and the high drag maximum area parachute.

A final approach to reducing design and trajectory solution sensitivities at parachute deployment is to provide the vehicle with a lifting decelerator that provides greater vehicle control. Although this option has not been explored thoroughly in the literature, it may become a viable system component for future missions. Each of the above proposed approaches may be found to be of less significance when the effects of the previously described all-propulsive landing phase are also included in the
overall trajectory/design analysis. However, future research efforts should explore these options.

7.3 Required Improvements in the Aeroshell Aerodynamics Model

7.3.1 Modeling Issue Definition

The angle of attack history plotted in Figure A1-2 for $Q_{\text{max}} = 400$ Pa displayed nearly discontinuous behavior, which prompted further investigation of the aeroshell lift and drag models. Figure 7-1 depicts the specific lift and drag force ($L/m_v$, $D/m_v$) histories plotted versus the trajectory time for this case.

![Figure 7-1. Specific Forces for an Optimized Solution with Nearly Discontinuous Angle of Attack](image)
The discontinuity in the angle of attack occurs near -150 seconds, at the precise location of the steep valleys in the specific forces experienced by the vehicle. Despite these sharp dips in the vehicle forces, lift and drag values remain nearly continuous on either side, even though the angle of attack values are nearly 105° different. A reference to the aeroshell aerodynamic properties illustrated in Figure 3-8 and 3-9 shows that this is indeed possible at the angles in question (~40° and ~65°).

This property of the aeroshell aerodynamics model is undesirable because solutions including this control region may require control actuation that provides unattainable vehicle pitch rates. Furthermore, this model characteristic introduces the possibility that some test cases may have more than one locally optimal solution for the control vector and state trajectory. This phenomenon was encountered often in the course of this research, especially when two radically different initial guesses are employed for a trial with the same performance criteria. Table 7-2 presents the relevant parameters for two such cases; the notes in parentheses show the difference in the initial guess.

<table>
<thead>
<tr>
<th>Trial (Initial Guess)</th>
<th>Maximum Q (Pa)</th>
<th>Vehicle Mass (kg)</th>
<th>$\delta_c$ (°)</th>
<th>$D_0$ (m)</th>
<th>$t_{s_1}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline ($\delta_c = 23^\circ$)</td>
<td>1350</td>
<td>3087.7</td>
<td>19.4</td>
<td>13.7</td>
<td>-34.2</td>
</tr>
<tr>
<td>Alternate Baseline ($\delta_c = 19^\circ$)</td>
<td>1379</td>
<td>3087.6</td>
<td>19.4</td>
<td>13.6</td>
<td>-34.2</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 800$ (Baseline)</td>
<td>799</td>
<td>3088.8</td>
<td>19.4</td>
<td>14.0</td>
<td>-31.9</td>
</tr>
<tr>
<td>$Q_{\text{max}} = 800$ (Alternate Baseline)</td>
<td>799</td>
<td>3088.8</td>
<td>19.4</td>
<td>14.0</td>
<td>-31.8</td>
</tr>
</tbody>
</table>

Table 7-2. Important Performance Parameters for Similar Trials with Different Initial Guesses
Although the above performance variables show remarkable uniformity regardless of the initial guess, Figure 7-2 shows that the control histories for the baseline trial with different initial aeroshell cone half angle guesses display the discontinuous behavior inherent in the aeroshell aerodynamics model. A similar result is observed for the $Q_{\text{max}} = 800$ Pa trial presented above.

![Figure 7-2. Angle of Attack for Baseline Trials with Different Initial Conical Aeroshell Half Angle Guesses](image)

During a typical analysis case, the vehicle configuration is optimized such that the boundary conditions are satisfied and the applicable performance index is minimized. For the hypersonic glide phase, this means the aeroshell mass is minimized to the relatively low drag, high lift half angle of 19.5°. Decrementing the parachute system mass also severely reduces the vehicle drag forces during the low
altitude flight phase. Minimization of the performance index therefore often results in diminished drag forces required to meet the terminal state boundary conditions. The analysis algorithm counteracts this phenomenon by perturbing the vehicle control history such that the aerodynamic coefficients provide the necessary force augmentation.

7.3.2 Thought Experiment to Understand the Model Discontinuity Effect

The analysis algorithm perturbs control, design, and transition time variables such that the vehicles forces are augmented in the face of an evermore lightweight, lower drag vehicle configuration. This usually means alteration of the hypersonic glide phase pitch and angle of attack histories in such a way that more drag or lift force is generated at the required time in the trajectory. Because the steepest descent method relies on first order partial derivatives to achieve these perturbations, the slope of the lift and drag coefficient curves of Figure 3-8 and 3-9 often dominate the other terms of the control history perturbations.

In a hypothetical case, assume that in a specific region of the optimization space and a given region of the trajectory more lift force is required to satisfy one of the problem constraints. Consulting Figure 3-9, any current angle of attack in the trajectory time history greater than $-30^\circ$ will be perturbed in the positive direction, increasing the aeroshell lift coefficient. However, any angle of attack less than $-30^\circ$ will be perturbed in the negative direction, also yielding an augmented lift coefficient. As the required lift force increases, so does the separation of the control values over a small amount of time steps. Eventually, both control values deliver the required
vehicle force, and the lift and drag force profiles become nearly continuous, as shown in Figure 7-1.

In an extreme case, so much lift force must be generated that the control is altered such that the large lift coefficients near -90° are attained, and an analysis algorithm numerical instability may occur due to trigonometric discontinuities at ±90°. Any angle of attack currently on the central maxima of the lift coefficient curve cannot provide any lift coefficient greater than that at ±30°, and a discontinuity will occur in the angle of attack and pitch histories between two adjacent time steps. Without additional model features to provide an unambiguous control strategy in the problem region, this aeroshell aerodynamics representation will often yield degraded analysis algorithm performance.

7.3.3 Proposed Problem Resolution

Although the thought experiment over simplifies the analysis algorithm perturbation method, it demonstrates the deficiency of the current aerodynamics model when using the steepest descent optimization methodology. Two model improvements are recommended for consideration in future implementation of the Newtonian flow equations.

The least difficult and expedient solution is to change the lift and drag coefficient Equations (3.15) and (3.16) so that their values become constant when the absolute value of the angle of attack exceeds a preselected value. The Newtonian flow approximations are only accurate below $\alpha = 130^\circ$ [34]; this limit provides a natural value at which constant coefficient equations may be patched to Equations (3.15) and (3.16). Care should be taken to nearly equate the slopes across the two
different relations, as analysis algorithm instabilities may occur if slope discontinuities are large.

A more complicated solution involves using the specified performance index logic to penalize excessive angle of attack values at any point in the trajectory. This methodology will place a detrimental cost on operations in "undesired" control regimes. An additional distributed performance index term may be added to Equation (4.8) in the same format as the dynamic pressure inequality constraint of Equation (4.10)

\[ L = K_\alpha (\alpha - \alpha_{\text{max}})^2 u_0(\alpha | - \alpha_{\text{max}}) \]  

(7.1)

This term reflects the assumed even or odd functional behavior of the coefficient equations. Although use of the proposed inequality constraint provides the user with greater flexibility, extra complexity will be added into the methodology. An appropriate weighting \( (K_\alpha) \) relative to the other performance index components must be empirically determined, and extra required partial derivatives must be computed. Because the angle of attack depends on both the pitch angle and the free stream relative flight path angle, derivatives with respect to both the state and control vector must be implemented in the analysis algorithm. Additionally, more factors must be addressed to assure analysis algorithm stability and convergence. Future applications of this methodology to the planetary atmosphere entry/landing problem may warrant this effort.
7.4 Allowing Different Inequality Constraint Limits on Separate Vehicle Flight Phases

7.4.1 Modeling Issue Definition

Application of the integrated design, control, and trajectory optimization methodology to the planetary atmospheric entry/landing problem has highlighted an area for analysis algorithm improvement. The proposed distributed cost function for the coupled hypersonic glide and parachute-aided descent phases,

\[ L = K_Q (Q - Q_{\text{max}})^2 u_0 (Q - Q_{\text{max}}) + K_{Qp} (Q - Q_{\text{max}_p})^2 u_0 (Q - Q_{\text{max}_p}) \]  (7.2)

allows the user to define dissimilar upper bounds on the trajectory dynamic pressure during different vehicle flight phases. The anticipated implementation of these inequality constraints is shown for a general flight trajectory in Figure 7-3.
For the planetary entry/landing descent problem, the first term of Equation (7.2) is active only when the upper bound on the dynamic pressure is violated during the hypersonic glide phase, while the second term is activated when violations occur on the separate bound imposed during the parachute-aided descent phase. A typical dynamic pressure history for the coupled hypersonic glide and parachute-aided descent phases is illustrated in Figure 7-4.

![Figure 7-4. Typical Dynamic Pressure Inequality Constraint Implementation for the Planetary Entry/Landing Problem](image)

Functional attempts to set separate bounds on the hypersonic glide and parachute-aided descent phases, however, result in numerical instability of the analysis algorithm. In a typical optimization run with both terms of the distributed performance index operational, the constraint violations tend to diverge in both flight phases. In addition to the modeling nonlinearities of the parachute phase described previously, the current analysis algorithm formulation logic is unable to successfully handle the separate constraint limits portrayed in Figure 7-4.
The inequality constraint functions defined in Equation (7.2) depend only on the vehicle state vector. Although the vehicle dynamics are discontinuous across the switch time marking the parachute deployment, the state vector remains continuous at this event. The two-sided nature of the inequality constraint formulation presents a problem when a violation is encountered specifically at the switch time. In such cases, the disparate user defined upper limits may activate only one of the two distributed constraint terms. The gradient information calculated by the analysis algorithm for the control history and design parameter perturbations will contain information based only on one set of vehicle dynamics, although both sets actually apply at the switch time. The result is an attempt to alter the state history such that the dynamic pressure constraint is satisfied on only one side of the switch time. Unfortunately, the state vector must remain continuous, and the control perturbations for the other flight phase lack the necessary information to meet this requirement.

7.4.2 Proposed Problem Resolution

The integrated design, control, and trajectory optimization methodology must be expanded beyond the current formulation to incorporate the treatment of separate state and control dependent inequality constraints during different vehicle flight phases. The previous discussion highlights a requirement for a constraint on the vehicle state vector at an interior point in the trajectory, namely at the switch time. This constraint may take the form [11]

\[ N \left[ x \left( t_{s_i} \right), t_{s_i} \right] = 0 \] (7.3)
Adjoining this function to the performance index and taking the first variation will yield the required necessary conditions for an optimal solution, which then may be incorporated into the framework of the integrated analysis algorithm. Although treatment of an intermediate state dependent point conditions in the steepest descent method is well documented, specifying the required time of constraint activation as the free dynamics switch time $t_s$, may require significant future research effort.

This formulation may also be applied to any general problem with multiple state dynamics phases and separate state dependent inequality constraints. The logic may extended to include intermediate point constraints on control vector dependent functions. Such capability will permit the location of a locally optimal solution for cases similar to those depicted in Figures 7-3 and 7-4. However, certain problem formulations will not lend themselves easily to this additional logic.

Figure 7-5. Problem Formulation with Unreachable Dynamic Pressure Inequality Constraint Limits
An example of this is shown in Figure 7-5, where the dynamic pressure point constraint cannot be satisfied simultaneously on either side of the parachute deployment time without grossly altering the parachute performance. Care must be taken that the nonlinearities associated with the switch times do not preclude solutions that satisfy the requested inequality constraint bounds.

### 7.5 Additional Suggested Model Fidelity Improvements

#### 7.5.1 Aeroshell Configuration and Fixed Lander Mass Improvements

The current conical aeroshell configuration tends toward designs that are not practical for the envisioned class of planetary missions. The optimal aeroshell design has a half angle of $19.5^\circ$, which translates to a vehicle length of 11.5 meters and diameter of 8.1 meters for a 200 m$^3$ volume requirement. Unfortunately, this design will not meet general launch vehicle diameter requirements, such as the $\sim$4.5 meter limit of the Titan IV shroud [33]. The aeroshell configuration also possesses a large amount of surface area and its associated structural mass.

A biconic aeroshell design may be a feasible alternative to the cone utilized in this research effort. With a constant volume requirement, the variable biconic design will utilize less aeroshell structural material and provides flexibility for geometric constraints. Additionally, biconic aerodynamic information is readily available [31], and the nose cone angle dominates the aerodynamic properties of the design. This attribute would help to decouple the vehicle aerodynamic forces from the fixed volume requirement.
Finally, the assumed lander mass for this study was 950 kg. When added to the parachute system mass of ~50 kilograms, this translates into a packed payload density of 5 kg/m\(^3\) for a 200 m\(^3\) aeroshell volume. For the general class of missions envisioned here, a more realistic packing density of 50 kg/m\(^3\) [33] yields a lander mass of ~10000 kilograms, nearly 10 times the current value. Reducing the volume requirement to 100 m\(^3\) would translate into ~5000 kilogram fixed lander mass, which would bring the lander design into a class expected for future Martian missions.

7.5.2 Parachute-Aided Descent Phase Model Improvements

In order to eliminate the lander wake effects from the parachute aerodynamics model, the parachute trailing distance has been fixed by Equation (3.7). At the minimum aeroshell mass half angle 19.5°, the conical base diameter is 8.1 meters, which results in parachute trailing distance of 55 meters. Using Equation (3.9), Table 7-3 shows the parachute suspension line length for various parachute nominal diameters at the optimal aeroshell base diameter of 8.1 meters.

<table>
<thead>
<tr>
<th>Parachute Diameter, (D_0) (m)</th>
<th>Suspension Line Length, (l_e) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>55.4</td>
</tr>
<tr>
<td>6</td>
<td>55.4</td>
</tr>
<tr>
<td>10</td>
<td>55.5</td>
</tr>
<tr>
<td>14</td>
<td>55.6</td>
</tr>
<tr>
<td>18</td>
<td>55.8</td>
</tr>
</tbody>
</table>

Table 7-3. Parachute Suspension Line Length for Various Nominal Diameters at the Optimal Aeroshell Base Diameter of 8.1 m
This table makes plain the relative insensitivity of the suspension line length over the range of parachute diameters encountered in this study. In cases where the parachute diameter is small, the lines account for the overwhelming majority of the parachute system mass. The geometric model may require revision to negate these effects, possibly by decoupling the line length from the aeroshell base diameter and providing a correction factor for the wake effects.

The effective line length $l_e/D_0$ also varies drastically (between 27.7 and 3.1) over this range of parachute diameters. The parachute drag coefficient correction factor $K_{CD}$ is directly dependent of the value of the effective line length, as shown in Figure 3-11. The assumptions inherent to this model are that the effective line length will stay near $\sim 1.7$ to accurately match the disk-gap-band flight test data. Unfortunately, the vehicle aerodynamic and geometric requirements do not allow this, and exceedingly large line lengths are encountered for small parachute diameters. The drag coefficient factor is decidedly a second order effect, as maximum drag coefficient variations due to $l_e/D_0$ are less than 10%. The above suggested geometrical and aeroshell revisions may desensitize the effective line length over the range of vehicle designs encountered. If not, the correction factor may be eliminated with only a small loss in the aerodynamic fidelity of the parachute-aided descent phase.

In addition to dynamic pressure limits, the investigation of both thermal and loading forces is recommended. Thermal considerations are critical for parachutes deployed at high Mach numbers, as significant heat damage has been encountered in disk-gap-band parachute flight tests [16]. High snatch forces are also experienced in the canopy, gores, and payload as the parachute is deployed. These loads will
determine the necessary line and canopy strengths, and also their masses per unit of geometric measurement. An approximate method of calculation is straightforward [6], [33], and could lead to reductions in the system mass or identify the need for bolstered structural components. Inequality constraints on both the thermodynamic heating and the vehicle loads may be implemented as distributed cost performance terms in the integrated analysis methodology.

Some additional notes on the parachute-aided descent phase model fidelity are in order. The assumption that the vehicle angle of attack is zero is a first order approximation that is not complete. In general, the dynamics of the parachute/lander system will display oscillatory behavior about the system center of mass, producing effective angles of attack at the canopy. This behavior has been displayed in numerous research efforts and is treated analytically in the literature. Use of a mortar for parachute deployment from the vehicle is the baseline for this study, primarily because of the tested reliability of such devices. Choosing a tractor rocket or deployment bag only will provide mass savings that may be traded against performance [6], [33].

All of the suggested improvements in this chapter should be cautiously implemented if the all-propulsive landing phase is incorporated into the methodology. With this additional flight phase, any deviation from the previously studied vehicle configuration will complicate future analyses, and the results presented in this thesis will lose their comparative value.
Chapter 8

Conclusion

8.1 Summary of Results

The integrated design, control, and trajectory analysis methodology has been successfully generalized to accommodate the planetary vehicle entry/landing problem. The analysis algorithm has been demonstrated to be a worthwhile tool for system and mission design studies at the early stages of concept generation. The methodology allows timely identification of system configuration, control, and trajectory trades in a smooth simulation environment. Significant performance information may be obtained, facilitating the implementation of more robust vehicle configurations and flight strategies that are better able to handle the wide range of uncertainties associated with planetary atmospheric entry and landing.
The integrated analysis methodology was demonstrated on the multistage Martian entry/landing problem. A significant amount of knowledge has been gained on design robustness issues for a class of vehicle configurations. The initial demonstration was sufficient to provide relevant vehicle design trade information, including results that highlighted the need for a high drag parachute-aided descent phase to increase vehicle robustness to tail winds.

With the addition of a parachute-aided descent flight phase, the aeroshell conical design has been desensitized to the various dynamic pressure requirements and atmospheric conditions, including tail winds. The aeroshell configuration mass is kept at the minimum possible, resulting in a design that has relatively high lift and low drag capabilities. Because of the lower atmospheric density of the hypersonic glide phase corridor, this desensitivity has a direct physical explanation, and the analysis algorithm successfully optimizes the system performance.

However, the parachute design is very sensitive to these same effects. The analysis algorithm is able to recognize that a wide variety of atmospheric conditions may be negotiated at a relatively small mass fluctuation by sharply altering the parachute diameter and changing the energy dissipation characteristics of the design. The vehicle configuration was also shown to have degraded performance in the presence of tail wind profiles that are typical to the Martian environment. This information elicits a search for strategies to increase vehicle robustness and desensitize the design. Several possible strategies that should be investigated are summarized in the next section.

Ultimately, the use of the analysis methodology will result in higher performance and more robust planetary entry/landing vehicle configurations. With
such a tool, many aspects of the iterative preliminary design process may be streamlined, providing a more effective utilization of engineering resources.
8.2 Suggested Future Research

Several proposals for future research have been discussed throughout this thesis. These improvements will allow greater user flexibility in studying the general planetary entry/landing problem. Additionally, model improvements will increase the analysis fidelity and provide more robust vehicle designs. A summary of these suggestions is provided below.

Include an All-Propulsive Landing Phase in the Analysis

Inclusion of an all-propulsive landing phase will increase the vehicle robustness to average and extreme wind conditions by providing a high degree of control authority in dense regions of atmosphere. Vehicle simulation models should be kept as simple and representative as possible to facilitate the analysis and keep computational time to a minimum.

Allow Aeroshell Jettison at the End of the Parachute-Aided Descent Phase

Delaying the aeroshell jettison maneuver until the end of the parachute flight phase will reduce the sensitivity of design solutions to the vehicle dynamics discontinuity at parachute deployment. The increased energy dissipation requirement associated with this maneuver may adversely affect vehicle performance; a trade study may be conducted to determine the best jettison strategy.
Provide Parachute Reefing Capability

Reefed parachute capability will add increased flexibility in satisfying mission objectives. An investigation should be conducted to determine if vehicle designs are also desensitized to atmospheric disturbances and mid-trajectory dynamic pressure performance.

Investigate Lifting Decelerators

Introducing lift into the parachute-aided descent phase may provide the vehicle with additional control authority and a robust design. Care should be taken to minimize the modeling uncertainties associated with these mainly experimental devices.

Resolve Poor Aerodynamic Performance Model Deficiencies for the Aeroshell

The conical aeroshell aerodynamics model is deficient at high angles of attack, which has an adverse effect when applying the steepest descent analysis methodology. Improvement of the lift and drag coefficient equations at high angles of attack or incorporation of an inequality constraint on $\alpha$ is advised.

Study Biconic Aeroshell Configurations

The use of biconic aeroshell design will partially decouple the hypersonic glide phase aerodynamics from the fixed volume requirement. Payload masses and packing densities should more accurately reflect the envisioned Martian mission configurations simulated in this thesis.
Alter the Parachute Drag Coefficient Correction Factor Model

The current correction factor used to approximate the parachute geometric effects on the decelerator drag is not fully representative. The model may be dropped if second order effects are not desired.

Expand Integrated Analysis Algorithm to Account for Intermediate Point Constraints

The integrated design, control, and trajectory analysis methodology cannot currently manage different inequality constraints during separate vehicle flight phases. The required logic should be derived and added to provide flexibility in modeling and analysis.

Incorporate Thermodynamic Heating and Vehicle Load Limits into the Analysis

Although addition of these inequality constraints will complicate analysis, many vehicle structural parameters are directly relatable to these limits. Thermal and structural considerations are critical to the hypersonic glide and parachute-aided descent phases, and may provide a more accurate performance measure than the trajectory dynamic pressure.
Appendix I

Decoupled Hypersonic Glide Phase Trajectory Plots
Figure AI-1. Parameters of Interest for the Initial Baseline Trajectory
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Figure AI-3. Effects of Head Wind Case 1 Without a Dynamic Pressure Bound
Figure AI-4. Effects of Head Wind Case 1, Dynamic Pressure Bound $Q_{\text{max}} = 400$ Pa
Figure AI-5. Effects of Head Wind Case 2 Without a Dynamic Pressure Bound
Figure AI-6. Effects of Head Wind Case 2, Dynamic Pressure Bound $Q_{\text{max}} = 400$ Pa
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Appendix II

Coupled Hypersonic Glide and Parachute-Aided Descent Phases Trajectory Plots
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Figure AII-2. Comparison of the Initial Baseline Trajectory and a Partially Converged Solution
Figure AII-3. Effects of Different Dynamic Pressure Inequality Constraints
Figure AII-4. Effects of Head Wind Case 1 Without a Dynamic Pressure Bound
Figure AII-5. Effects of Head Wind Case 1, Dynamic Pressure Bound $Q_{\max} = 600$ Pa
Figure AII-6. Effects of Head Wind Case 2 Without a Dynamic Pressure Bound
Figure AII-7. Effects of Head Wind Case 2, Dynamic Pressure Bound $Q_{\text{max}} = 600$ Pa
Figure AII-8. Effects of Head Wind Case 3 Without a Dynamic Pressure Bound
Figure AII-9. Effects of Head Wind Case 3, Dynamic Pressure Bound $Q_{\text{max}} = 800$ Pa
Figure AII-10. Effects of Tail Wind Case 1 Without a Dynamic Pressure Bound
Figure AII-11. Effects of Tail Wind Case 1, Dynamic Pressure Bound $Q_{\text{max}} = 400$ Pa
Figure AII-12. Effects of Tail Wind Case 2 Without a Dynamic Pressure Bound
Figure AII-13. Effects of Tail Wind Case 2, Dynamic Pressure Bound $Q_{\text{max}} = 400$ Pa
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Figure AII-16. Comparison of Local Minima for the Case 1 Tail Wind Trials Without a Dynamic Pressure Bound
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