A Probability-Based Alerting Logic for Aircraft on Parallel Approach

by

Brenda Diane Carpenter

B.S. in Aerospace Engineering
University of Maryland at College Park, 1994

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of

Master of Science in Aeronautics and Astronautics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 1996

© Massachusetts Institute of Technology, 1996. All Rights Reserved.

Author ........................................................................................................................................

Department of Aeronautics and Astronautics
May 23, 1996

Certified by ...........................................................................................................................

James K. Kuchar
Assistant Professor of Aeronautics and Astronautics
Thesis Supervisor

Accepted by ............................................................................................................................

Professor Harold Y. Wachman
Chairman, Department Graduate Committee

Aero

MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

JUN 11 1996

LIBRARIES
A Probability-Based Alerting Logic for Aircraft on Parallel Approach

by

Brenda Diane Carpenter

Submitted to the Department of Aeronautics and Astronautics on May 23, 1996 in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics

Abstract

This thesis discusses the development and evaluation of an airborne collision alerting logic for aircraft on closely-spaced approaches to parallel runways. A novel methodology is used which links alerts to collision probabilities: alerting thresholds are set such that when the probability of a collision exceeds an acceptable hazard level an alert is issued.

The logic was designed to limit the hazard level to that estimated for the Precision Runway Monitor system: one accident in every one thousand blunders which trigger alerts. When the aircraft were constrained to be coaltitude, evaluations of a two-dimensional version of the alerting logic show that the achieved hazard level is approximately one accident in every 250 blunders. Problematic scenarios have been identified and corrections to the logic can be made.

The evaluations also show that over eighty percent of all unnecessary alerts were issued during scenarios in which the miss distance would have been less than 1000 ft, indicating that the alerts may have been justified. Also, no unnecessary alerts were generated during normal approaches.

Modifications to the two-dimensional logic were made in order to expand the situations under which the logic could operate effectively. Two altitude criteria were added to the logic, and the performance of these three-dimensional versions was compared to the two-dimensional version. Under three-dimensional parallel approaches, unnecessary alerts were reduced by almost forty percent, and the collision rate remained constant.

Finally, the effect of incorporating several avoidance maneuver options was investigated. When the logic could select one of two maneuvers, the performance remained virtually unchanged when compared to the single maneuver logic.

Thesis Supervisor: James K. Kuchar
Title: Assistant Professor of Aeronautics and Astronautics
Acknowledgements

This work could not have been completed without various contributions from a great many people. First, I must thank my parents for the love, support and understanding which has enabled me to complete nineteen long years of education. I will truly be a success if I can someday be as good a parent to my children as they were to me. Thanks also go to my advisor, Jim Kuchar, for his patience and guidance during our work together, and for his constructive criticism concerning this document. A whole slew of labmates prodded me along until the very last syllable was written: John-Paul Clarke, Rob Schwarz, and Sanjay Vakil (who receives special thanks for editing and distractions along the way). Finally, thanks to Jeremy Neilson who helped me keep my sanity as the work piled higher.

This work was supported by the National Aeronautics and Space Administration through MIT Lincoln Laboratory. I would like to thank the following individuals for their support of this project: Marvin Waller, NASA Langley; Steve Koczo, Rockwell-Collins; Trent Thresh, NASA Ames; Steve Campbell, MIT Lincoln Laboratory; and Professor R. John Hansman, Jr., MIT.
Table of Contents

Nomenclature ........................................................................................................ 13

1 Introduction .......................................................................................................... 15
  1.1 Parallel Approach ............................................................................... 16
  1.2 Alerting During Parallel Approach ....................................................... 18
  1.3 The Precision Runway Monitor ............................................................ 19
  1.4 Probability Thresholds versus PRM Thresholds .................................... 23
  1.5 Level of Safety ....................................................................................... 25
  1.6 Thesis Roadmap ..................................................................................... 27

2 Probability Threshold Design Methodology ...................................................... 29
  2.1 The Parallel Approach Situation ......................................................... 29
  2.2 Measurement Requirements and Accuracy Estimates ......................... 31
  2.3 Methodology ......................................................................................... 32
  2.4 Monte Carlo Simulations ..................................................................... 33
  2.5 Monte Carlo Results ............................................................................. 36
  2.6 Incorporation of Design Hazard Level ................................................ 38

3 Two-Dimensional Alerting Logic ...................................................................... 41
  3.1 State Measurements and Error Estimates ............................................. 41
  3.2 Monte Carlo Simulations and Nominal Intruder States ......................... 42
  3.3 Approximation of the Alerting Zone ..................................................... 45

4 Performance of the Two-Dimensional Alerting Logic .................................... 53
  4.1 Performance Evaluation Methodology ................................................ 53
  4.2 Performance Evaluation Results ......................................................... 60
  4.3 Resultant versus Design Hazard Level ................................................. 66

5 Three-Dimensional Alerting Logic .................................................................. 71
  5.1 Three-Dimensional Alerting Issues ..................................................... 71
  5.2 Altitude Criteria .................................................................................... 74
  5.3 Three-Dimensional Performance Evaluation Methodology .................. 76
  5.4 Performance of Alerting Logic under Altitude and Climb Rate Variations 76

6 Effect of Avoidance Maneuvers ..................................................................... 83
  6.1 Limitations of Single Maneuver .......................................................... 83
  6.2 Effects of Introducing a Level Turn Maneuver ...................................... 85
  6.3 Alerting Logic Variations ..................................................................... 89
  6.4 Evaluation of Alerting Logic Variations .............................................. 90

7 Conclusions ....................................................................................................... 93
  7.1 Summary of Significant Results ......................................................... 94
  7.2 Areas for Future Work ....................................................................... 96
List of Figures

Figure 1.1: Aircraft separations during parallel approach .......................................................... 16
Figure 1.2: Typical parallel approach scenario ............................................................................. 17
Figure 1.3: Example alerting threshold changing in response to state information ................ 19
Figure 1.4: PRM operating configuration ...................................................................................... 20
Figure 1.5: PRM alerting process .................................................................................................. 21
Figure 1.6: Example PRM threshold and probability threshold .................................................. 25
Figure 2.1: Intruder position defined relative to own aircraft ..................................................... 30
Figure 2.2: Schematic of alerting system operation ....................................................................... 34
Figure 2.3: Nominal versus actual intruder trajectories in Monte Carlo simulations ............ 35
Figure 2.4: Typical Monte Carlo results: P(C | maneuver).......................................................... 36
Figure 2.5: Typical Monte Carlo results: P(C | normal approach) ............................................. 37
Figure 2.6: Alerting threshold defined by hazard level ................................................................. 38
Figure 3.1: Approximate Monte Carlo results: Own aircraft performs an avoidance maneuver (N = 10,000) .................................................................................................................................................. 44
Figure 3.2: Approximate Monte Carlo results: Own aircraft follows normal approach course (N = 10,000) .................................................................................................................................................. 45
Figure 3.3: Collision curve ............................................................................................................. 47
Figure 3.4: Example location of collision curve with respect to Monte Carlo probability contours .................................................................................................................................................. 48
Figure 3.5: Simplified alerting threshold ......................................................................................... 49
Figure 3.6: Off-line calculations and real-time alerting ................................................................. 50
Figure 4.1: 5° bank angle blunder (145 kts, windy conditions) ..................................................... 53
Figure 4.2: 10° heading change blunder (145 kts, windy conditions) .......................................... 54
Figure 4.3: 15° heading change blunder (145 kts, windy conditions) .......................................... 54
Figure 4.4: 30° heading change blunder (145 kts, windy conditions) .......................................... 54
Figure 4.5: Fake blunder (145 kts, windy conditions) ................................................................... 55
Figure 4.6: Over-adjustment blunder (145 kts, windy conditions) .............................................. 55
Figure 4.7: Normal approach (145 kts, windy conditions) ........................................................... 55
Figure 4.8: Heading variations during normal approach ................................................................. 56
Figure 4.9: Bank angle variations during normal approach ........................................................... 56
Figure 4.10: Possible outcomes when no alert is issued ............................................................ 58
Figure 4.11: Possible outcomes when an alert is issued ............................................................ 59
Figure 4.12: Distribution of projected miss distances during unnecessary alert scenarios .......... 61
Figure 4.13: Projected miss distances during unnecessary alert scenarios as a function of runway separation ........................................................................................................... 63
Figure 4.14: Induced collision configuration .................................................................................. 65
Figure 5.1: Issuing an alert during a two-dimensional situation .................................................... 72
Figure 5.2: Issuing an alert during a three-dimensional situation .................................................... 73
Figure 5.3: Prediction of final relative altitude .............................................................................. 74
Figure 5.4: Unnecessary alert distribution: No Vertical Criterion (2D) ........................................ 78
Figure 5.5: Unnecessary alert distribution: Normal Approach Altitude Criterion (3D) ............... 79
Figure 5.6: Unnecessary alert distribution: Avoidance Maneuver Altitude Criterion (3D) ........... 79
Figure 5.7: Issuing an alert under Normal Approach Altitude Criterion ...................................... 81
Figure 6.1: Alerting zones under two-dimensional constraint ...................................................... 84
Figure 6.2: Alerting zones if intruder is climbing ......................................................................... 86
Figure 6.3: Representative alerting thresholds associated with different maneuvers ................. 90
List of Tables

Table 3.1: State Measurements and Standard Deviations ........................................ 42
Table 3.2: Range of Nominal Intruder State Measurements ........................................ 43
Table 3.3: Sample Lookup Table for Range Criterion ........................................... 51
Table 4.1: Possible Simulated Parallel Approach Outcomes .................................... 58
Table 4.2: Overall Performance of 2-D Alerting Logic ........................................... 60
Table 4.3: System Performance as a Function of Runway Separation ........................ 62
Table 4.4: System Performance as a Function of Intruder Airspeed .......................... 64
Table 4.5: Rate of Induced Collisions During Specific Blunders ............................... 65
Table 4.6: Resultant versus Design Hazard Level ................................................... 67
Table 4.7: Effect of Runway Separation on Hazard Level ......................................... 67
Table 4.8: Effect of Intruder Airspeed on Hazard Level .......................................... 68
Table 4.9: Hazard Level During Blunder Trajectories ............................................. 69
Table 5.1: Alerting Logic Performance Under 3-D Scenarios .................................. 77
Table 5.2: Alerting Logic Hazard Levels Under 3-D Approaches .............................. 82
Table 6.1: Performance of Alerting Logic with Two Maneuver Options ...................... 87
Table 6.2: Performance of Logic when Choosing Each Maneuver Option .................... 88
Table 6.3: Effect of Alerting Logic on Performance ............................................... 91
Table 6.4: Alerting Logic Hazard Levels ................................................................. 92
Nomenclature

\( g \): Gravitational constant.

\( h \): Altitude of the intruder relative to the own aircraft; a positive value indicates that the intruder is above the own aircraft.

\( h_{av} \): Predicted altitude separation assuming own aircraft initiates an avoidance maneuver.

\( h_{norm} \): Predicted altitude separation assuming own aircraft continues normal approach.

\( h_{int} \): Climb rate of the intruder; a positive value indicates that the intruder is climbing.

\( h_{own} \): Climb rate of the own aircraft; a positive value indicates that the own aircraft is climbing.

\( N \): Total number of simulations.

\( P(C \mid \text{trajectory}) \): Probability of a collision given that the own aircraft follows the flight path designated by \( \text{trajectory} \).

\( R \): Intruder’s turn radius; the quantity is always positive.

\( R \): Range limit; parameter calculated within the alerting logic which corresponds to the point along the collision curve at which \( P(C \mid \text{maneuver}) \) is equal to a threshold probability level.

\( t_c \): Time to collision parameter; the quantity is always positive.

\( V_{int} \): Airspeed of the intruder.

\( V_{own} \): Airspeed of the own aircraft.

\( x \): Lateral distance between the intruder and the own aircraft’s runway centerline; the quantity is positive when the intruder has not crossed the own aircraft’s runway centerline.

\( \bar{X} \): State vector: \( (x, y, h, \psi, \phi, V_{int}, h_{int}, V_{own}, h_{own}) \)

\( y \): Longitudinal position of the intruder relative to the own aircraft; a positive value indicates that the intruder is ahead of the own aircraft.

\( y_{\text{curve}} \): Longitudinal component of collision curve; calculated within the alerting logic and is the point on the collision curve corresponding to the intruder’s lateral position.
\( \phi \): Bank angle of the intruder; a positive value indicates a turn toward the own aircraft’s runway centerline.

\( \sigma \): Standard deviation of a Gaussian error distribution.

\( \psi \): Heading of the intruder relative to the runway heading; a positive value indicates that the intruder will intercept the own aircraft’s runway centerline.

\( \dot{\psi} \): Intruder’s rate of heading change; a positive value indicates that \( \psi \) will increase.
Chapter 1

Introduction

The goal of any alerting system is to provide protection from dangerous incidents. Ideally, an alerting system issues alerts only when such an incident is imminent, and will refrain from issuing unnecessary alerts in all other situations. Because of uncertainties within the physical system, it is not possible to eliminate unnecessary alerts. Therefore, the best possible alerting system optimizes the timing of alerts in order to maintain an acceptable level of safety while minimizing unnecessary alerts. Such a system issues alerts when just enough time remains to safely resolve a predicted conflict. Issuing alerts later decreases the amount of time available for the conflict to be avoided, and thus prevents the system from effectively providing the prescribed safety level. Issuing alerts sooner tends to increase the unnecessary alert rate.

The focus of this thesis is the development of an alerting logic which attempts to operate in this optimal fashion. The logic is specifically designed to operate during independent simultaneous parallel approaches to landing. It is based on probability concepts: alerts are issued only when the probability of a collision between the two aircraft involved reaches some threshold value.

This chapter describes the parallel approach situation and discusses the challenges of alerting under such conditions. One alerting system for parallel approach operations has already been designed and implemented. The operation of this system, the Precision Runway Monitor (PRM), is outlined, and some of its limitations are discussed.
1.1 Parallel Approach

Simultaneous parallel approaches involve two streams of aircraft approaching parallel runways. In visual meteorological conditions (VMC), the pilots may accept the responsibility of maintaining separation between their aircraft by visual means. For approaches conducted during instrument meteorological conditions (IMC), air traffic control personnel are responsible for separation between the aircraft [1].

The relative positions of the aircraft in parallel streams are either dependent or independent. In both cases in-trail separation is dictated by wake vortex considerations. During dependent approaches, aircraft are separated diagonally by at least 1.5 or 2.0 miles, depending on runway separation (Figure 1.1). During independent approaches, the diagonal separation restriction is lifted. However, during the turn onto the localizer, at least 3 miles radar separation or 1000 ft altitude separation must be maintained [1].

**Figure 1.1:** Aircraft separations during parallel approach

A diagram of a typical parallel approach scenario is shown in Figure 1.2. After the turn onto the localizer, both aircraft maintain altitude until intercepting the glideslope, at which time they begin a 3° descent to the runway threshold. During the descent, the aircraft trajectories may lie exactly side-by-side, or may be offset in altitude somewhat due to the staggering of the runways.
The FAA allows independent parallel approaches to be carried out in VMC with a runway separation minimum of 700 ft. In IMC, independent approaches may be conducted to runways spaced at least 4300 ft apart. This minimum is reduced to 3400 ft if the airport is equipped with the Precision Runway Monitor (PRM) system [2].

A study performed by the Boeing Commercial Airplane Group predicts significant increases in the number of operations per hour if dependent approaches can be replaced by independent approaches [2]. At a single airport, a 27% increase in the number of operations per hour can be expected. Because of the capacity increases to be gained, it is desirable to reduce the minimum runway separation required for independent approaches.
1.2 Alerting During Parallel Approach

Aircraft are more closely spaced during parallel approach than during any other phase of flight. The potential exists for an aircraft in either stream to deviate off course toward another aircraft in the parallel stream. To increase safety, an alerting system is needed to warn flight crews of these blundering aircraft. The goal of the alerting system is to insure adequate separation between aircraft while allowing parallel approaches to be carried out.

The timing of alerts issued by the system is critical. For safety reasons, they should not be issued too late. If a late alert occurs, there may not be enough time for one aircraft to evade the other. On the other hand, early alerts increase the rate of unnecessary alerts. Unnecessary alerts during this phase of flight cause approaches to be aborted without justification. Aborted approaches disrupt the flow of traffic around the airport and potentially create hazardous situations during go-around procedures. Therefore, the alerting system should balance this trade-off and should ideally issue an alert with just enough time to safely avoid a collision.

To optimize the timing of alerts, the alerting thresholds within the system should be dependent on the particular situation. When the closure rate between two aircraft is high, the threshold should encompass enough time or space to permit the threatened aircraft to escape safely. If instead the closure rate is low but the threshold does not change, greater potential for an unnecessary alert exists. Therefore, the alerting threshold should be smaller, in a spatial sense, for low closure rates and larger for high rates.

For example, the scenario in Figure 1.3 shows one aircraft changing its heading toward another. In the left half of the figure, the circle surrounding aircraft B indicates a possible alerting threshold in this situation. Aircraft A is following the normal approach course with a minimal bank angle and a negligible heading deviation away from the run-
way heading. If A were to enter the circle in the same attitude, aircraft B would have just enough time to initiate a maneuver and avoid a collision.

![Diagram of aircraft A and B with alerting thresholds]

**Figure 1.3:** Example alerting threshold changing in response to state information

In the configuration on the right of Figure 1.3, A has altered its heading and bank angle. The threshold around B has grown larger in response to the change in the other aircraft’s attitude. The amount of time needed for B to successfully avoid aircraft A, with its new attitude, has increased, and thus the threshold has increased in size.

### 1.3 The Precision Runway Monitor

The Precision Runway Monitor (PRM) is a system designed to resolve dangerous situations which may occur between two aircraft on simultaneous parallel approaches. This system issues alerts when an aircraft deviates into a No Transgression Zone between the two runways.
Figure 1.4 shows the basic configuration of the PRM system. To perform simultaneous approaches in IMC using PRM, the runways must be spaced at least 3400 ft apart [2]. A 2000-ft wide No Transgression Zone (NTZ) is centered between the two runways. The alerting threshold of this system is the boundary of the NTZ. Penetration of this zone by one of the aircraft prompts a breakout instruction to the other aircraft from a ground controller [3].

![Figure 1.4: PRM operating configuration](image)

PRM uses a radar system with a 2.4-second update interval or better to allow ground controllers to monitor the positions of the two approaching aircraft with respect to the NTZ. The aircraft, NTZ, and runway centerlines are displayed on a high resolution color monitor. The lateral scale of the display may be adjusted to allow greater resolution of the aircrafts' lateral positions. Also, the display can be modified to include a predicted future trajectory ranging from two to ten seconds in length [3].
The system’s predictive capability is used to prepare controllers for possible alerting situations. The deviating aircraft’s icon changes from green to yellow and an aural ‘Caution!’ alert sounds when the system predicts NTZ penetration in ten seconds. Upon NTZ penetration, the blundering aircraft’s icon changes from yellow to red and an aural ‘Warning!’ alert sounds. These measures help controllers to detect blunders and issue alerts promptly [3].

Even with the predictive capabilities of the system, only the blundering aircraft’s lateral position is taken into account when an alert is issued to the pilot of the threatened aircraft. Other factors, including longitudinal separation of the two aircraft and the closure rate of the blundering aircraft, are not considered. Figure 1.5 illustrates the major components of the PRM alerting process.

Figure 1.5: PRM alerting process
The major limitation inherent in the operation of this system is the amount of time which might elapse between when the NTZ is penetrated and when the breakout maneuver is actually initiated. The controller must first detect that one of the aircraft has entered the NTZ. At this point the controller clears the local control frequency and issues a verbal breakout instruction to the endangered aircraft. The pilot then initiates the commanded avoidance maneuver. After the avoidance maneuver has begun, air traffic control resumes authority over subsequent traffic management.

Several experimental measurements of the controller’s response time delay have been made. One or two radar updates may occur before the controller decides that an alert is necessary. Analysis by Shank et al. indicates that the time delay between NTZ penetration and controller response is over five seconds in over twenty percent of the experimental situations involving the severest blunder trajectories [4]. When clearing the radio channel, the controller may be forced to wait until the pilot finishes a communication. Recordings made of terminal area communications indicate that the controller will be able to transmit without delay with a probability of over 0.93, but the maximum wait may be over eight seconds [4].

The pilot’s response time is difficult to determine experimentally. The pilot may already consider the parallel approach situation a dangerous one and may be ready to begin the avoidance maneuver almost immediately. However, it is possible that a pilot may receive only one breakout instruction in the course of his or her career and may take several seconds to begin the maneuver.

When the controller’s and pilot’s response delays are combined, the possible time delays involved in the alerting process place significant limitations on the runway separations for which PRM provides an adequate level of safety. The NTZ acts as a buffer zone which allows alerts to be issued with adequate time to resolve the conflict safely even
when time delays are present. If the size of the NTZ were to remain the same as runway separation is reduced, more unnecessary alerts would result as normal lateral deviations off the runway centerline place the aircraft within the NTZ. Alternatively, if the NTZ is reduced in size with runway separation, the time delays inherent in the alerting process will limit the system’s ability to provide adequate protection. Therefore, the runway separations for which PRM can provide a high level of safety are limited by the time delays associated with the alerting process [5].

To reduce the minimum runway separation below the PRM minimum, the ground-based alerting system should be replaced with an airborne one. The airborne system eliminates the controller and radio communications from the loop, reducing the time delays within the alerting process. To implement an airborne system, the two aircraft involved each broadcast their location and other state information. From this information, an airborne alerting system could determine when the other aircraft posed a threat and could issue an alert at that time. The only time delay remaining in the system involves the amount of time the pilot takes to react to the alert and the subsequent dynamic response of the aircraft.

1.4 Probability Thresholds versus PRM Thresholds

In the PRM system the alerting threshold is fixed in space and constant over time, regardless of the relative positions or closure rates of the two aircraft involved. A more effective alerting system would define alerting thresholds relative to the aircraft to be protected, and would alter the threshold geometry based on changes in the aircrafts’ states. Errors in the measurement of these states, as well as uncertainty about what each aircraft will do, make it difficult to simultaneously eliminate collisions and unnecessary alerts.
A probability-based logic allows the number of unnecessary alerts and collisions to be managed in terms of the probability that each will occur. The logic functions using a prediction of the intruder’s future trajectory. If a collision is predicted to occur with some level of probability, the system issues an alert. Therefore, the alerting thresholds can be designed by lowering the probability of a collision until the probability of an unnecessary alert rises to a level that is no longer acceptable. The trade-off between unnecessary alerts and system safety is discussed in work by Kuchar [6].

To determine whether a collision is likely, the system must have the ability to predict the future trajectories of the two aircraft involved. To be accurate, such a system requires more real-time state information from both aircraft than the single lateral deviation measurement required for the PRM system. Acquiring the measurements needed to predict future trajectories is feasible using a system such as Automatic Dependent Surveillance Broadcast (ADS-B) which broadcasts current state information and GPS position measurements.

PRM and a probability-based alerting system will generally issue alerts at different times (Figure 1.6). An intruder at location \( A \) warrants an alert under the PRM system even though the probability of a collision is very low. Because aircraft \( A \) is far ahead of the own aircraft, \( A \) must dramatically increase its bank angle or reduce airspeed in order to cause a collision. Even though the aircraft is blundering, the probability-based system does not issue an alert because the probability of a collision is not above the threshold.

Meanwhile, an intruder at \( B \) triggers an alert from the probability-based system before prompting an alert from the PRM system. The probability-based system predicts that aircraft \( B \) will cause a collision with some level of probability. The alert is issued from the probability-based system sooner than from PRM because the intruder’s high closure rate
is not taken into account within PRM. The design of such a probability-based alerting system is the focus of this thesis.

Figure 1.6: Example PRM threshold and probability threshold

1.5 Level of Safety

The safety level provided by the alerting system is defined to be the probability that an incident is successfully resolved during a parallel approach in which a blunder occurs. Hazard level is defined as the probability that an accident is left unresolved by the alerting system. By definition, the sum of safety level and hazard level is unity since all incidents are either resolved or unresolved.

A study by the Precision Runway Monitor Program Office assessed the safety level provided by the PRM system [3]. The report defines a worst-case blunder as a 30° heading change toward the other aircraft’s runway centerline. The study estimates that PRM allows only one accident in every 250 worst-case blunders, implying that PRM provides a
safety level of 249/250 or has a hazard level of 1/250 (0.004). As a baseline, it follows that other parallel approach alerting systems should provide a similar level of safety.

In the probability-based alerting logic, alerting thresholds are designed such that the system does not exceed a predefined hazard level. The alerting logic uses real-time state measurements to determine if the intruder is blundering and poses a threat. When those state measurements indicate that the probability of a collision exceeds an acceptable level, an alert is issued. This threshold collision probability corresponds to the hazard level defined above (unresolved incidents per intruder blunder). Therefore, the probability-based alerting logic is designed such that the hazard level (probability of a collision during a scenario in which a blunder occurs) approximates the PRM hazard level of 0.004.

It is important to realize that the hazard level associated with either PRM or the probability-based alerting logic is not the overall hazard level within the parallel approach situation. The overall hazard level of parallel approaches is defined as the probability of an accident occurring during a parallel approach situation. This statistic is related to the number of blunders which occur, as well as the ability of the alerting system to resolve those blunders. The following equation states this concept numerically:

\[
\text{Overall Hazard Level} = P(C) = P(C \mid B) \cdot P(B)
\]

where \(P(C)\) is the probability of a collision, \(P(B)\) is the probability of a blunder, and \(P(C \mid B)\) is the probability of a collision given that a blunder occurs. For instance, a study by the Precision Runway Monitor Program Office estimates that only one blunder will occur during every twenty-five million parallel approaches [3]. The overall hazard level is then given by:

\[
\left( \frac{1 \text{ accident}}{250 \text{ blunders}} \right) \cdot \left( \frac{1 \text{ blunder}}{25 \text{ million approaches}} \right) = \frac{1 \text{ accident}}{6.25 \text{ billion approaches}}
\]
This thesis focuses on designing and testing a probability-based alerting system based on the hazard level, defined by $P(C \mid B)$.

1.6 Thesis Roadmap

The remainder of this thesis discusses the methodology used to develop a probability-based alerting logic and the performance of several versions of the logic. Chapter 2 presents the basic methodology used to develop the first version of the logic. This first version is based on two-dimensional parallel approach geometries. The alerting logic which was produced using this methodology is presented in Chapter 3. An evaluation of the logic’s performance under two-dimensional approaches is presented in Chapter 4.

Next, the altitude restriction is lifted and modifications are made to the two-dimensional logic in an effort to improve the performance of the alerting logic under three-dimensional geometries. The development of the three-dimensional logic and the results of a series of performance evaluations appear in Chapter 5. Chapter 6 discusses general issues associated with alerting under three-dimensional parallel approach geometries. Chapter 7 summarizes the work and presents the significant conclusions.
Chapter 2

Probability Threshold Design Methodology

Alerting when the probability of a collision reaches some threshold value allows the system to be designed to provide a consistent level of safety. This chapter outlines the steps taken to develop the logic for such a system operating specifically under parallel approach conditions. First, a model of the parallel approach situation is developed which defines important variables and presents assumptions. Next, the required state measurements and associated measurement accuracies are outlined. The use of Monte Carlo simulation techniques to estimate the probability of a collision is explained in detail, and some representative results are included. This chapter also presents the process by which the alerting thresholds are defined based on the calculated values of collision probability.

2.1 The Parallel Approach Situation

To simplify the initial development, the focus is on the final approach segment; thus only the straight, descent portion of the approach is considered. Notably, runway stagger and the turn onto the localizer, including the required altitude separation, are ignored. Also, post-alert procedures to be managed by air traffic control are not considered in the development of the logic.

Throughout this thesis one aircraft is designated the threatened aircraft or “own aircraft”. This aircraft is equipped with the alerting system and performs an avoidance maneuver in response to an alert. The other aircraft is termed the intruder. It is assumed that the intruder does not respond to the threatened aircraft.
The intruder's position is defined relative to the origin of a set of axes placed on the own aircraft’s runway centerline (Figure 2.1). To derive this origin, the own aircraft’s longitudinal distance from the runway threshold is projected onto the runway centerline. It is assumed that the runway centerline is the own aircraft’s intended path and that any deviation from that path will be corrected. Figure 2.1 also defines other important variables. To simplify subsequent figures, the threatened aircraft always appears at the origin of the coordinate axes throughout this text.

Figure 2.1: Intruder position defined relative to own aircraft

In the figure above, the following six variables appear:

- Lateral distance from the intruder to the own aircraft’s runway centerline, \( x \)
- Longitudinal position of the intruder relative to the own aircraft, \( y \)
- Airspeed of the intruder, \( V_{\text{int}} \)
- Heading of the intruder relative to the runway heading, \( \psi \)
- Bank angle of the intruder, \( \phi \)
- Airspeed of the own aircraft, \( V_{\text{own}} \)

For simplicity, the following variables are not shown in the drawing but are important:
• Altitude of the intruder relative to the own aircraft, \( h \)
• Climb rate of the intruder, \( \dot{h}_{\text{int}} \)
• Climb rate of the own aircraft, \( \dot{h}_{\text{own}} \)

The combination of all these measurements describes the state vector:

\[
\bar{X} = (x, y, h, \psi, \phi, V_{\text{int}}, \dot{h}_{\text{int}}, V_{\text{own}}, \dot{h}_{\text{own}})
\]

Measurements of the variables which comprise \( \bar{X} \) are assumed to be available to the own aircraft through ADS-B or through measurement filtering techniques.

### 2.2 Measurement Requirements and Accuracy Estimates

The next step in the development of the alerting logic requires identifying the set of state measurements that are vital to the effective performance of the alerting system. Also, the development of such a system requires an estimate of the expected accuracy of state measurements.

Because alerts are issued when a collision is predicted to occur with some level of probability, the successful operation of a probability-based system relies on its ability to accurately predict future conditions. Therefore, it is important to obtain measurements of the initial conditions and formulate an accurate prediction based on this information. The nine variables defined above describe an initial state vector, \( \bar{X} \), from which the alerting logic extrapolates the future trajectories. The logic assumes that the own aircraft follows the runway centerline at a speed of \( V_{\text{own}} \) while the intruder performs a constant-rate turn defined by the measurements within the initial state vector.

The relative lateral and longitudinal positions, as well as the heading angle, provide a starting point for predicting the intruder’s turn. The intruder’s bank angle and airspeed imply a rate of heading change. Without a measurement of bank angle, it is more difficult to estimate turn rate. The own aircraft’s current speed enables the logic to predict its
approximate future location, assuming that the aircraft tracks the runway centerline at that speed. The own aircraft’s climb rate and the intruder’s relative altitude and climb rate allow the future relative altitude to be estimated.

As the alerting thresholds are developed, estimates must be made as to the accuracy of the state measurements by describing potential errors. Estimating error involves defining error distributions using probability density functions. A state error estimate is made based on the expected accuracy of the sensor and the noise in that state due to turbulence acting on the aircraft. Also, uncertainty about how future maneuvers may cause the actual trajectory to disagree with the predicted path are incorporated into the state measurement error estimate.

2.3 Methodology

Recall that the PRM hazard level was estimated to be one accident in every 250 blunders (0.004). As a baseline, the probability-based system is designed such that its hazard level is similar to PRM’s: one accident in every one thousand blunders (0.001). The alerting threshold is set at those points where the own aircraft will be unable to safely escape from the collision in a maximum of one case in every one thousand. Therefore, alerts are issued when the probability of a collision after an alert exceeds the maximum acceptable hazard level. The notation \( P(C \mid \text{maneuver}) \) corresponds to the probability of interest. Note that \( P(C \mid \text{maneuver}) \) is a function of the avoidance maneuver and is an implicit function of the situation defined by \( \bar{X} \).

The alerting thresholds are based on knowledge of \( P(C \mid \text{maneuver}) \). However, calculation of \( P(C \mid \text{maneuver}) \) cannot be done in real-time, so calculations must be made offline and stored for future use. Notably, \( P(C \mid \text{maneuver}) \) can be determined for every \( \bar{X} \)

1. “probability of a collision given a maneuver”
which is likely to occur during a parallel approach situation. Then, an estimate of $P(C \mid \text{maneuver})$ associated with that set of state measurements can be stored for future use, or a subset of state information which defines a physical alerting threshold can be stored. In this way, the alerting thresholds can be used while state measurements are taken in real-time.

2.4 Monte Carlo Simulations

Monte Carlo simulations can be used to estimate event probabilities for complex situations in which uncertainty exists [7]. In order to use Monte Carlo techniques, a set of initial information must be supplied, along with estimates about the accuracy of such information. Also required is a model of the situation dynamics which describes how the situation will change in the future.

For the development of the probability-based alerting logic, Monte Carlo simulations were used to calculate the value of $P(C \mid \text{maneuver})$ over a range of state measurements. The points at which $P(C \mid \text{maneuver})$ equals the maximum acceptable hazard level then define the alerting threshold for the system. Figure 2.2 depicts the development and ultimate operation of this system.
Monte Carlo techniques can be used to calculate the probability of a collision associated with a nominal situation defined by $\bar{X}$: the intruder’s and own aircraft’s initial states. Before a simulation begins, measurement errors are added to the nominal states, creating a set of actual states associated with the nominal states. Next, the own aircraft and intruder are simulated together starting from their actual initial states and using the constraints on situation dynamics over the course of the simulation. For the development of the alerting logic, the intruder is assumed to perform a constant-rate turn while the own aircraft performs a prescribed avoidance maneuver.

In order to derive $P(C \mid \text{maneuver})$ using this process, a large number of simulations are run with randomly generated errors introduced to the nominal intruder states. Because of the errors introduced at the beginning of each simulation, each actual intruder trajectory
differs somewhat from the nominal trajectory (Figure 2.3). This emulates the wide range of trajectories which are possible when uncertainties are present within the real system.

![Diagram](Image)

**Figure 2.3:** Nominal versus actual intruder trajectories in Monte Carlo simulations

By counting the number of collisions which occur during N simulations based on a set of nominal states, an estimate of collision probability associated with the nominal state measurements can be made. The following formula is used to calculate the probability of a collision assuming that the own aircraft follows an avoidance maneuver:

\[
P(C \mid \text{maneuver}) = \frac{\text{Number of collisions}}{N}
\]  

(2.1)

The magnitude of \(P(C \mid \text{maneuver})\) to be calculated and the desired accuracy of that calculation determine the number of simulations that need to be run. The following formula dictates the minimum number of simulations which must be run in order to calculate a certain level of probability, P, with a level of uncertainty defined by a standard deviation, \(\sigma\).
\[ N = \frac{P(1-P)}{\sigma^2} \] (2.2)

For example, if a probability of 0.001 is calculated over 10,000 simulations, then one standard deviation of error has a magnitude of 0.0003.

Instead of calculating \( P(C \mid \text{maneuver}) \) at each possible \( \bar{X} \), Monte Carlo techniques were used to estimate collision probabilities for a limited set of nominal parallel approach situations. By choosing a range of situations to comprise the set, the calculation of collision probability in an arbitrary situation can then be performed by interpolation.

### 2.5 Monte Carlo Results

The results of the Monte Carlo simulations can be best understood by creating contour plots of the resulting probability calculations. A representative plot is shown in Figure 2.4. Each plot combines probability information for the entire x-y plane and for one combination of intruder heading, bank angle and airspeed. The relative altitude of the intruder and the climb rates of both aircraft are assumed to be zero. Darker shades are associated with higher probabilities of collision, where a collision is defined to occur when the distance between the two aircraft falls below 500 ft.

![Figure 2.4: Typical Monte Carlo results: \( P(C \mid \text{maneuver}) \)](image)
The plot in Figure 2.4 depicts the results of Monte Carlo simulations assuming that the own aircraft performed an avoidance maneuver. Specifically, the own aircraft turns and climbs after a response time delay, and the intruder carries out a constant-rate turn at constant altitude as dictated by its initial states. As the intruder’s lateral distance from the own aircraft increases, the intruder is less likely to cause a collision because the own aircraft has more time to climb away from the intruder’s altitude. Also, if the intruder is far in front or far behind, regardless of lateral separation, a collision is less likely to occur without a significant change in the intruder’s airspeed.

To examine the need for an alert, it is of interest to run the same series of Monte Carlo simulations assuming that the own aircraft continues the normal approach instead of performing an avoidance maneuver. Again, probability contours over the entire x-y plane and for one combination of heading, bank angle and airspeed show dangerous regions (Figure 2.5).

![Figure 2.5: Typical Monte Carlo results: P(C | normal approach)](image)

In this case, the own aircraft remains coaltitude with the intruder throughout the intruder’s blunder. Even as the intruder’s lateral distance from the own aircraft increases, there is a longitudinal position from which the intruder is theoretically able to cause a col-
lision. Therefore, in this theoretical case, the probability contours extend laterally without limit.

2.6 Incorporation of Design Hazard Level

The maximum acceptable hazard level can be incorporated into the alerting system design using the Monte Carlo results. As mentioned in Section 1.5, the hazard level implies that at the time an alert is issued and the avoidance maneuver begins, the probability that an accident will occur is equal to that hazard level. Since \( P(C \mid \text{maneuver}) \) is the probability that a collision will occur, the alerting threshold should be set along the contour associated with \( P(C \mid \text{maneuver}) = \text{hazard level} \). Figure 2.6 shows an example alerting threshold established in this way.

![Figure 2.6: Alerting threshold defined by hazard level](image)

Alerts are issued only when \( P(C \mid \text{maneuver}) \) exceeds this hazard level. Thus, an intruder at \( A \) in Figure 2.6 does not warrant an alert even though the lateral separation between the two aircraft is low. \( A \) is far enough in front of the own aircraft such that a collision will only occur in what is assumed to be an unlikely event: \( A \)'s turn rate increases significantly. An intruder at \( B \), although projected to collide with the own aircraft if no
maneuver is undertaken, does not trigger an alert because an alert issued later still pro-
vides the target level of safety. Delaying the alert allows the system to collect more infor-
mation about the intruder’s projected path, and allows greater opportunity for the intruder
to resume its proper course. The intruder at C triggers an alert: the probability that a col-
losion will occur is greater than the maximum acceptable hazard level if the own aircraft’s
avoidance maneuver is initiated at that time.
Chapter 3

Two-Dimensional Alerting Logic

An alerting system was constructed using the methodology discussed in Chapter 2 and is described in this chapter. The relevant subset of state measurements and their assumed uncertainties are defined. The Monte Carlo simulations which were carried out are described, and representative results are shown. Alerting thresholds constructed based on the Monte Carlo evaluations are described in detail.

To begin investigating alerting issues for aircraft on parallel approach, altitude considerations were removed: the own aircraft and the intruder are co-altitude. The own aircraft’s avoidance maneuver initiates any altitude differences between the two aircraft. This restriction simplifies the visualization of alerting thresholds, as well as their storage and retrieval. Chapter 5 discusses a modification that extends the two-dimensional logic to the more complex three-dimensional case.

3.1 State Measurements and Error Estimates

For the two-dimensional alerting logic, the intruder is constrained to be co-altitude with the own aircraft as it follows the approach path. Vertical separation only takes place if and when the own aircraft begins an avoidance maneuver. Therefore, initial relative altitude and climb rates are excluded throughout the development of the two-dimensional alerting logic.

When constrained to a horizontal plane, the intruder’s future trajectory can be estimated if the following state measurements are made:

- Relative lateral position, x.
• Relative longitudinal position, y.
• Airspeed, \( V_{\text{int}} \).
• Heading, \( \psi \).
• Bank angle, \( \phi \).

If the own aircraft’s airspeed, \( V_{\text{own}} \), is included in the set of state measurements, relative horizontal separation can be predicted at points in the future.

To simplify the problem further, the speed of the own aircraft is assumed to be constant at 145 knots. Also the measurement of the intruder’s airspeed is assumed to be exact and thus will be assigned a deterministic value during each set of simulations. The remaining four state measurements each have a zero-centered Gaussian error distribution, defined by a standard deviation, \( \sigma \) (Table 3.1). The errors in lateral and longitudinal position correspond to measurements which could be attained using Differential GPS (DGPS). The values for \( \sigma_{\psi} \) and \( \sigma_{\phi} \) allow for aircraft attitude noise inherent in the approach, uncertainties concerning the intruder’s future turn rate, and sensor measurement errors. The final estimates of \( \sigma_{\psi} \) and \( \sigma_{\phi} \) were made based on an examination of aircraft state data from approach simulations recorded at Rockwell-Collins [8].

<table>
<thead>
<tr>
<th>State Measurement</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( \sigma_x = 35 ) ft</td>
</tr>
<tr>
<td>( y )</td>
<td>( \sigma_y = 35 ) ft</td>
</tr>
<tr>
<td>( \psi )</td>
<td>( \sigma_{\psi} = 2.5^\circ )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>( \sigma_{\phi} = 5.0^\circ )</td>
</tr>
</tbody>
</table>

### 3.2 Monte Carlo Simulations and Nominal Intruder States

To encompass a reasonable range of parallel approach geometries, the five intruder states were systematically varied as shown in Table 3.2. The lateral spacing limit of 4400 ft is based on the current runway separation minimum of 4300 ft for independent parallel
approaches in IMC. The longitudinal separation limit corresponds to 1.5 nmi—the range of longitudinal separations which could occur between the intruder and own aircraft based on minimum in-trail spacing limits of 3 nmi. The intruder airspeeds represent a reasonable sampling of final approach speeds for a range of aircraft, and the heading and bank angles are representative of a wide range of possible intruder attitudes. The result is that $P(C | \text{maneuver})$ must be calculated over $(12 \cdot 47 \cdot 4 \cdot 9 \cdot 7) = 142,128$ conditions.

**Table 3.2: Range of Nominal Intruder State Measurements**

<table>
<thead>
<tr>
<th>State</th>
<th>Range</th>
<th>Increments</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>0 ft - 4400 ft</td>
<td>400 ft</td>
<td>12</td>
</tr>
<tr>
<td>$y$</td>
<td>9200 ft behind - 9200 ft ahead</td>
<td>400 ft</td>
<td>47</td>
</tr>
<tr>
<td>$V_{\text{int}}$</td>
<td>120 knots - 180 knots</td>
<td>20 knots</td>
<td>4</td>
</tr>
<tr>
<td>$\psi$</td>
<td>40° away - 40° toward the own aircraft</td>
<td>10°</td>
<td>9</td>
</tr>
<tr>
<td>$\phi$</td>
<td>20° away - 40° toward the own aircraft</td>
<td>10°</td>
<td>7</td>
</tr>
</tbody>
</table>

Before the simulations are run, a collision must be defined. However, the definition of a collision is somewhat arbitrary. A miss distance of 10 feet between the two aircraft is technically not a collision since the miss distance is nonzero. However, the pilots involved may view the situation differently. For this work, a collision is said to occur if the distance between the centers of gravity of the two aircraft is 500 feet or less. This definition has been used in other related work [3].

Next, the own aircraft’s avoidance maneuver following the alert must be specified. For the two-dimensional version of the logic, the own aircraft is assumed to be limited to one maneuver [9]. In this case, the maneuver is defined as a combined turn and climb that begins after a response time delay of 2 seconds. The climb component entails a 0.25g pull-up until a climb rate of 2000 ft/min is reached. To perform the turn component, the aircraft rolls at a rate of 5°/s until reaching a 30° bank angle, rolling out at the same rate to
intercept a final heading of $45^\circ$ away from the intruder’s runway centerline. In addition, it is assumed that the own aircraft transitions to go-around thrust, resulting in an increase in airspeed of 15 kt at a rate of 1 kt/s.

Two sample contour plots of the Monte Carlo results are shown below. These particular plots were derived for the case in which $\psi = 20^\circ$ (toward the own aircraft) and $\phi = 20^\circ$ (toward the own aircraft). The results contained in Figure 3.1 are derived from simulations in which the own aircraft performs the avoidance maneuver described above. A single contour of $P(C \mid \text{turn/climb})$ is shown. Figure 3.2 shows the results for a case in which the own aircraft is assumed to follow the normal approach course. The graph shows a contour of $P(C \mid \text{normal approach})$. Note that the coarseness of the contours is due to the relatively large spread of conditions for which collision probability is calculated.

**Figure 3.1:** Approximate Monte Carlo results: Own aircraft performs an avoidance maneuver ($N = 10,000$) (contour corresponds to $P \geq 0.001$)
Figure 3.2: Approximate Monte Carlo results: Own aircraft follows normal approach course ($N = 10,000$) (contour corresponds to $P \geq 0.001$)

In the two-dimensional case, assuming that the own aircraft performs the climbing/turning avoidance maneuver when an alert is issued, $P(C \mid \text{turn/climb})$ is always less than or equal to $P(C \mid \text{normal approach})$. In other words, the single predefined avoidance maneuver is always as safe or safer than following the normal approach. This is not the case when altitude differences and climb rates are introduced, a situation that is discussed at length in Section 5.4.2.

3.3 Approximation of the Alerting Zone

As mentioned above, the Monte Carlo simulations produce values of $P(C \mid \text{turn/climb})$ for 142,128 combinations of $x$, $y$, $\psi$, $\phi$ and $V_{\text{int}}$, and the alerting thresholds are set at those combinations where $P(C \mid \text{turn/climb})$ is equal to the hazard level. In theory, the real-time determination of $P(C \mid \text{turn/climb})$ could be done by interpolation among all the states. However, a means of simplifying the alerting thresholds was developed through a set of
parameterized equations called the collision curve. The curve is used as a mechanism to more efficiently represent the alerting threshold.

The collision curve defines those locations in the horizontal plane from which the intruder will eventually be involved in a collision with the own aircraft, assuming that the own aircraft follows the normal approach and the intruder maintains a constant-rate turn. The collision curve can be conveniently expressed as two equations parameterized by the time to collision, t. These equations give lateral position, x, and longitudinal position, y, for the intruder relative to the own aircraft. Each combination of x and y represents a position in which an intruder is projected to collide with the own aircraft.

\[
x = R \{ \cos (\psi t + \psi) + \cos \psi \} \\
y = V_{\text{own}} t - R \{ \sin (\psi t + \psi) - \sin \psi \}
\]

where R is the intruder's turn radius:

\[
R = \frac{V_{\text{int}}^2}{g \tan \phi}
\]

where g is the gravitational constant. The time rate of heading change, \( \psi \), depends on the intruder’s bank angle and airspeed:

\[
\psi = \frac{g \tan \phi}{V_{\text{int}}}
\]

An example collision curve for a single combination of \( \psi \), \( \phi \), and \( V_{\text{int}} \) is shown in Figure 3.3.
Figure 3.3: Collision curve

The intruding aircraft at A will collide with the own aircraft on the normal approach path after ten seconds. Similarly, the intruder at B will collide with the own aircraft after fifteen seconds. The two intruder trajectories are arc segments of a circle whose radius is given by Equation 3.3.

The curve lies in the center of the contour associated with P(C | normal approach), but deviates from the center of the contour associated with P(C | turn/climb) as lateral separation increases (Figure 3.4). However, this deviation was generally found to be less than a few hundred feet for lateral separations less than 3000 ft. Also important is the fact that the longitudinal width of the contours is roughly constant (1600 ft) with lateral position. Given this information, the collision curve can be used as a tool to approximate the entire P(C | turn/climb) contour.
Recalling that alerts will be issued when $P(C \mid \text{turn/climb}) = 0.001$, approximation of the $P(C \mid \text{turn/climb})$ contour using the collision curve results in two alerting criteria:

1. The intruder’s longitudinal position must be within 800 ft of a collision curve defined by $V_{\text{int}}, \psi, \phi$ and $V_{\text{own}}$.
2. The intruder must be within a range defined by the $(x, y)$ point at which the collision curve crosses the $P(C \mid \text{turn/climb}) = 0.001$ contour, shown in Figure 3.4.

The first criterion relies on the fact that the longitudinal width of the contour is constant with lateral position and that the curve lies roughly in the center of the $P(C \mid \text{turn/climb}) = 0.001$ contour. Together, these two criteria define the alerting threshold shown in Figure 3.5.
Figure 3.5: Simplified alerting threshold

With these simplifications, an array indexed by $V_{int}$, $\psi$, $\phi$ and $V_{own}$ containing the horizontal range values associated with the point at which $P(C | \text{maneuver}) = 0.001$ is used to store the alerting thresholds for the 142,128 conditions mentioned earlier. With the calculation of the arrays containing range limit values performed off-line, real-time alerting can take place as shown in Figure 3.6. The specific process by which alerts are issued is described below.
Off-line Calculations

Finite set of Monte Carlo simulations over x-y plane

Range arrays indexed by \( \psi, \phi, V_{int} \)

Range limit found by interpolation among values in array

Intruder in range?

No

Yes

Using intruder's x, determine corresponding y on collision curve

y on curve within 800 ft of intruder's measured y?

No

Yes

Issue alert

Real-time measurements of \( x, y, \psi, \phi, V_{int}, V_{own} \)

Figure 3.6: Off-line calculations and real-time alerting

Based on the measured values of \( \psi, \phi \) and \( V_{int} \), a range limit is retrieved from the array using linear interpolation. If the intruder is within range, the time to collision, \( t_c \), is determined based on the time required for the intruder to reach the own aircraft’s runway centerline:

\[
    t_c = \frac{\cos^{-1} \left[ \cos (\psi) - \frac{x}{R} \right] - \psi}{\psi}
\]

Equation 3.5 was derived by manipulating the equation which defines the x-component of the collision curve (Equation 3.1). After this step, the longitudinal position component, y, along the curve corresponding to this point in time, \( t_c \), is determined using the second collision curve equation (Equation 3.2). This value for longitudinal position, designated
\( y_{\text{curve}} \) is compared to the intruder’s actual longitudinal position. If these two values of longitudinal position are within 800 ft of each other, the intruder has a probability of collision greater than or equal to 0.001 and an alert is issued.

Table 3.3 shows a sample array with heading angle listed in the left-hand column and bank angle across the top. Positive heading angles indicate that the intruder’s trajectory intercepts the own aircraft’s runway centerline. Positive bank angles increase the intruder’s heading angle toward the own aircraft. This array pertains to the case where \( V_{\text{int}} = 120 \) knots. The entire table for other values of \( V_{\text{int}} (140, 160 \text{ and } 180 \text{ knots}) \) is contained in Appendix A. The minimum value allowed for a table entry is 800 ft, which corresponds to the longitudinal width of the alerting zone. Given an arbitrary combination of \( \psi \) and \( \phi \), an appropriate range threshold value can be determined by interpolating among the values stored in the table.

**Table 3.3: Sample Lookup Table for Range Criterion**
(values in feet)

<table>
<thead>
<tr>
<th>( V_{\text{int}} = 120 \text{ kts} )</th>
<th>Intruder Bank Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -40^\circ )</td>
<td>-20°</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
</tr>
<tr>
<td>30°</td>
<td>923</td>
</tr>
<tr>
<td>40°</td>
<td>1256</td>
</tr>
</tbody>
</table>
For example, assume that the following state measurements are obtained: \( x = 1500 \text{ ft}, \ y = 700 \text{ ft}, \ V_{\text{int}} = 120 \text{ kts}, \ \psi = 20^\circ \) (toward the own aircraft), and \( \phi = 15^\circ \) (also toward the own aircraft). Also, the own aircraft’s airspeed is 145 kts. The range limit, \( R \), is retrieved from the lookup table by interpolation:

\[
R = 1917 + \left( \frac{15^\circ - 10^\circ}{10^\circ} \right) \cdot (2206 - 1917) = 2061.5 \text{ ft}
\]

This value is compared to the intruder’s range,

\[
\sqrt{x^2 + y^2} = \sqrt{1500^2 + 700^2} = 1655.3 \text{ ft}
\]

Since the intruder’s range is less than \( R \), the logic determines if the intruder is close enough to the collision curve to warrant an alert. For a lateral position of 1500 ft, the resulting \( y_{\text{curve}} \) is 962.4 ft. Since the intruder’s longitudinal position, \( y = 700 \text{ ft} \), is within 800 ft of the calculated value of \( y_{\text{curve}} \), an alert is issued. A version of this alerting algorithm written in pseudocode appears in Appendix B.

Recall that in Chapter 1, the advantages of designing thresholds which would adapt to the situation were discussed. For the simple situation described in that chapter, a change in heading of the blundering aircraft prompted an expansion of the alerting threshold. The table above exhibits a similar characteristic. As the heading angle of the intruder turns toward the own aircraft, the range limit increases and alerts are issued when there is a greater separation between the two aircraft. Similarly, as the bank angle increases, indicating a steeper turn toward the own aircraft, the range limit also increases. Close inspection of the remainder of the table in Appendix A shows that this behavior also holds for changes in airspeed: an increase in airspeed at the same heading and bank angle combination expands the alerting threshold by increasing the range limit.
Chapter 4

Performance of the Two-Dimensional Alerting Logic

An evaluation of the alerting logic was made to determine whether performance goals were being met. The methodology used to perform the evaluation is described in this chapter. Subsequent results of the alerting logic evaluation are included, and performance comparisons are made among the results.

4.1 Performance Evaluation Methodology

The performance of the two-dimensional alerting logic was evaluated using normal approach and blunder trajectories flown during a simulator study at Rockwell-Collins [8]. Aircraft state data were recorded at a rate of 2.2 samples per second during the simulations. Representative graphs of lateral versus longitudinal position for each of the seven trajectories are shown below in Figures 4.1 through 4.7. In all cases, the own aircraft’s runway is assumed to be located at a positive lateral distance from the intruder’s runway centerline.

Figure 4.1: 5° bank angle blunder (145 kts, windy conditions)
Figure 4.2: $10^\circ$ heading change blunder (145 kts, windy conditions)

Figure 4.3: $15^\circ$ heading change blunder (145 kts, windy conditions)

Figure 4.4: $30^\circ$ heading change blunder (145 kts, windy conditions)
Figure 4.5: Fake blunder (145 kts, windy conditions)

Figure 4.6: Over-adjustment blunder (145 kts, windy conditions)

Figure 4.7: Normal approach (145 kts, windy conditions)
Each of the seven trajectories was flown six times, once each at airspeeds of 130, 145, and 160 knots and once each in calm conditions and windy conditions. As noted, the example tracks shown above are trajectories flown at 145 knots in windy conditions. Figures 4.8 and 4.9 show the variation of heading and bank angle over the course of the 145-kt normal trajectory flown in windy conditions.

![Graph of heading variations during normal approach](image1)

**Figure 4.8:** Heading variations during normal approach

![Graph of bank angle variations during normal approach](image2)

**Figure 4.9:** Bank angle variations during normal approach

The own aircraft was assigned to follow one of two simulated normal approach trajectories at 145 knots. An intruder flew alongside this own aircraft at the same altitude,

---

1. The over-adjustment blunder trajectory was flown only under windy conditions, for a total of 39 trajectories (instead of $6.7 \times 6 = 42$ trajectories).
following one of the normal approach or blunder trajectories from the simulator study. The two aircraft were laterally spaced using runway separations of 1700, 2500, and 3400 ft. To emulate the range of possible longitudinal separations which could occur due to intrail spacing requirements, trajectories were offset longitudinally in 100 ft increments from -9100 ft to 9100 ft. Therefore, the total number of simulations to produce the data in this chapter is 42,822:

\[
(39 \text{ intruder trajectories}) \cdot (2 \text{ own aircraft trajectories}) \cdot (3 \text{ runway separations}) \cdot (183 \text{ longitudinal spacings}) = 42,822
\]

If an alert was issued during the course of the parallel approach, a second own aircraft was generated which broke away from the normal approach and followed the specified avoidance maneuver. Therefore in each simulation there were a maximum of three aircraft being simulated:

- Intruder
- Normal approach own aircraft
- Maneuvering own aircraft

Three-dimensional miss distances between the two simulated own aircraft and the intruder were recorded, as well as whether an alert was issued, in order to measure the performance of the alerting logic.

The outcome of each scenario is classified into one of six categories based on the miss distances and whether or not an alert was issued. The rate at which each outcome occurs over the course of N simulations is used to establish the performance of the alerting system. Each of the six outcomes is defined in Table 4.1.
Table 4.1: Possible Simulated Parallel Approach Outcomes

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Collision During Normal Approach?</th>
<th>Alert Issued?</th>
<th>Collision During Maneuver Trajectory?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct Rejection</td>
<td>No</td>
<td>No</td>
<td>N/A</td>
</tr>
<tr>
<td>Missed Detection</td>
<td>Yes</td>
<td>No</td>
<td>N/A</td>
</tr>
<tr>
<td>Unnecessary Alert</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Induced Collision</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Correct Detection</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Late Alert</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

No alerts are issued during those scenarios which are classified into the first two outcomes listed in the table. These two outcomes are shown in Figure 4.10. During a correct rejection scenario shown on the left, there is no need for an alert and one is never issued. During a missed detection shown on the right, no alert is ever issued even though a collision occurs at \( X \). In the current system, alerts are issued when the two aircraft close to within 800 feet of each other, regardless of the intruder’s attitude. Therefore, a collision (less than 500 feet miss distance) which occurs without an alert indicates a system malfunction or a severe state measurement error.

![Figure 4.10: Possible outcomes when no alert is issued](image)

58
When alerts are issued to the own aircraft during the approach, there are four possible outcomes. An example of each of these outcomes is diagrammed in Figure 4.11. During an unnecessary alert scenario pictured in the upper left corner, an alert is issued but no collision would have occurred if the own aircraft had followed the normal approach. An induced collision (upper right corner) is a special case of the unnecessary alert in that the avoidance maneuver prompted by the alert causes a collision which would not have occurred otherwise.

![Diagram](image)

**Figure 4.11:** Possible outcomes when an alert is issued

The lower two diagrams within Figure 4.11 are examples of scenarios in which a collision would have occurred over the course of the normal approach. If the alert was not issued early enough to insure adequate separation, it is termed a late alert. A late alert situation appears in the lower right corner. Otherwise, the alert enabled the own aircraft to successfully avoid a collision and the outcome is called a correct detection. Such a scenario appears in the lower left corner.
With an ideal system, only correct rejections and correct detections would occur. The alerting system would function such that safe situations do not trigger alerts while all situations in which collisions would occur prompt alerts which enable the conflict to be resolved safely. This ideal system requires the ability to predict the future trajectories of both aircraft without error. However, even with perfect sensors, the aircrafts’ future trajectories are uncertain because of the potential for either aircraft to initiate unexpected maneuvers. Therefore, designing the ideal alerting system is not possible and unnecessary alerts must be traded off against collisions.

### 4.2 Performance Evaluation Results

Table 4.2 presents the observed system performance of the two-dimensional alerting logic under purely two-dimensional parallel approach situations. The data encompass the results obtained over all trajectories, at all runway separations and at all intruder airspeeds, for a total of 42,822 simulations. Standard deviation estimates associated with the outcome probabilities were computed using Equation 2.2.

<table>
<thead>
<tr>
<th>Probability σ</th>
<th>Correct Rejection</th>
<th>Missed Detection</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.3x10^{-1}</td>
<td>1.2x10^{-3}</td>
<td>0.0</td>
<td>3.3x10^{-2}</td>
<td>2.8x10^{-4}</td>
<td>3.4x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td>&lt;1x10^{-6}</td>
<td>8.7x10^{-4}</td>
<td>8.1x10^{-5}</td>
<td>8.8x10^{-4}</td>
<td>&lt;1x10^{-6}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of this evaluation process must be interpreted carefully. The probability values associated with each outcome are highly dependent on the parallel approach scenarios which were simulated. For this evaluation, most of the scenarios included a blunder situation. Also, the types of blunders which may occur during actual parallel approaches may differ significantly from those used for this evaluation. Therefore, the
probability values and corresponding standard deviations must be considered in the context of the type and number of blunders which were simulated.

Unnecessary alerts may not be undesirable when they occur during scenarios in which the miss distance between the intruder and the own aircraft would otherwise have been close to 500 ft. Therefore, miss distance establishes another measure of alerting logic performance. The graph in Figure 4.12 shows the cumulative probability distribution of miss distances over all blunder scenarios and at all intruder airspeeds. Over eighty percent of all unnecessary alerts occurred during scenarios in which the miss distances would have been less than 1000 ft had the alert not be issued. Thus, most unnecessary alerts may actually be considered appropriate.

![Cumulative Probability vs. Miss Distances](image)

**Figure 4.12:** Distribution of projected miss distances during unnecessary alert scenarios

Also important is the number of unnecessary alerts that occurred when the intruder flew a normal approach. An alerting logic that issues unnecessary alerts during normal approaches decreases the number of simultaneous parallel approaches that can be performed. Under two-dimensional conditions, the two-dimensional alerting logic was observed to not issue any unnecessary alerts during normal approaches at any runway separation or intruder airspeed. Unnecessary alerts only occurred during scenarios in which
the intruder blundered. Thus, it appears that the logic can successfully filter hazardous situations from normal localizer/glideslope tracking errors, indicating that parallel approaches to runways spaced 1700 ft apart may be feasible.

Parametric comparisons can be made by decomposing the performance data into subsets.

### 4.2.1 Effect of Runway Separation

As outlined in Section 3.2, it is desirable to safely reduce the runway separation minimum required for conducting independent parallel approaches. The PRM system has lowered this minimum from 4300 ft to 3400 ft. Comparing the performance of the probability-based alerting logic under different runway separations will provide insight as to whether the probability-based alerting system can safely lower the minimum still further. The data for comparison are listed in Table 4.3.

**Table 4.3: System Performance as a Function of Runway Separation**
*(all intruder airspeeds)*

<table>
<thead>
<tr>
<th>Runway Separation</th>
<th>Correct Rejection</th>
<th>Missed Detection</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700 ft</td>
<td>9.14x10^{-1}</td>
<td>0.0</td>
<td>4.72x10^{-2}</td>
<td>4.20x10^{-4}</td>
<td>3.79x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>2.3x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.8x10^{-3}</td>
<td>1.7x10^{-4}</td>
<td>1.6x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
</tr>
<tr>
<td>2500 ft</td>
<td>9.37x10^{-1}</td>
<td>0.0</td>
<td>3.00x10^{-2}</td>
<td>4.20x10^{-4}</td>
<td>3.24x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>2.0x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.4x10^{-3}</td>
<td>1.7x10^{-4}</td>
<td>1.5x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
</tr>
<tr>
<td>3400 ft</td>
<td>9.45x10^{-1}</td>
<td>0.0</td>
<td>2.30x10^{-2}</td>
<td>0.0</td>
<td>3.19x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>1.9x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.3x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.5x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
</tr>
</tbody>
</table>

These data show that as runway separation is increased, the number of unnecessary alerts declines. As runway separation decreases, more hazardous situations occur, prompting more unnecessary alerts. This trend can be seen in Figure 4.13. The graph shows the cumulative probability distribution of miss distances which would have
occurred had the unnecessary alerts not been issued. The results are graphed for each of the three runway separations tested. As mentioned before, the data are highly dependent on the number and type of blunders which were used in the evaluation process. The distribution of miss distances changes very little when the runway separation drops from 3400 ft to 2500 ft. However, a significant change occurs when the runway separation is decreased further to 1700 ft.

Figure 4.13: Projected miss distances during unnecessary alert scenarios as a function of runway separation

When the runway separation is reduced to 1700 ft, the intruder’s position triggers a greater number of unnecessary alerts. The majority of this difference takes place during the fake and over-adjustment blunder scenarios. During these scenarios, unnecessary alerts occurred when the runways were separated by 1700 ft but not at the two other separations. As can be seen in Figures 4.5 and 4.6, these two blunder trajectories have a maximum lateral deviation off the runway centerline of approximately 1000 ft. At larger runway separations, the alerting threshold is not crossed. At 1700 ft, however, the intruder closes to within 700 ft of the own aircraft’s runway centerline and prompts an alert in situations which would not have demanded an alert at the other runway separations. During
the other scenarios, the number of unnecessary alerts is roughly equivalent among the
three runway separations.

4.2.2 Effect of Intruder Airspeed

Similar comparisons of alerting logic performance are made for different intruder air-
speeds. Table 4.4 contains the results of this investigation.

Table 4.4: System Performance as a Function of Intruder Airspeed
(all runway spacings)

<table>
<thead>
<tr>
<th>Intruder Airspeed</th>
<th>Correct Rejection</th>
<th>Missed Detection</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>130 kt</td>
<td>9.33x10^{-1}</td>
<td>0.0</td>
<td>3.26x10^{-2}</td>
<td>0.0</td>
<td>3.41x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td>σ</td>
<td>2.1x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.5x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.5x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
</tr>
<tr>
<td>145 kt</td>
<td>9.30x10^{-1}</td>
<td>0.0</td>
<td>3.47x10^{-2}</td>
<td>5.60x10^{-4}</td>
<td>3.52x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td>σ</td>
<td>2.1x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.5x10^{-3}</td>
<td>2.0x10^{-4}</td>
<td>1.5x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
</tr>
<tr>
<td>160 kt</td>
<td>9.34x10^{-1}</td>
<td>0.0</td>
<td>3.29x10^{-2}</td>
<td>2.80x10^{-4}</td>
<td>3.29x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td>σ</td>
<td>2.1x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
<td>1.5x10^{-3}</td>
<td>1.4x10^{-4}</td>
<td>1.5x10^{-3}</td>
<td>&lt;1x10^{-6}</td>
</tr>
</tbody>
</table>

The performance of the alerting logic is reasonably consistent between each of the
three intruder airspeeds examined. However, induced collisions occur when the intruder’s
airspeed is 145 or 160 knots but none occur when the airspeed is 130 knots. The reason
for this phenomenon is related to the specific trajectories involved and is discussed in the
next section.

4.2.3 Problematic Trajectories

Only two of the blunder trajectory types caused collisions: the 5° bank angle blunder
and the 10° heading change blunder. No accidents occurred during the 15° and 30° head-
ing change blunders. Table 4.5 shows blunders during which collisions occurred.
Table 4.5: Rate of Induced Collisions During Specific Blunders

<table>
<thead>
<tr>
<th>Blunder Type</th>
<th>Induced Collisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>5° bank angle</td>
<td>16.7 %</td>
</tr>
<tr>
<td>10° heading change</td>
<td>83.3 %</td>
</tr>
<tr>
<td>15° heading change</td>
<td>0.0 %</td>
</tr>
<tr>
<td>30° heading change</td>
<td>0.0 %</td>
</tr>
</tbody>
</table>

All induced collisions occurred during trajectories in which the intruder was as fast or faster than the own aircraft and during trajectories in which the intruder’s nominal heading and bank angles were low. A further investigation as to the cause of these induced collisions points to the location of the intruder at the time the alert was issued. In 75% of the induced collisions recorded above, the intruder had already crossed the own aircraft’s runway centerline at the time of the alert. This leads to the geometry shown in Figure 4.14.

![Figure 4.14: Induced collision configuration](image)

Within the alerting logic, the range limit is always at least 800 ft. When the intruder crosses the own aircraft’s runway centerline, the intruder’s relative heading and bank angle switch from positive (toward the own aircraft) to negative (away from the own air-
craft). However, alerts may still be issued if the intruder crosses within 800 ft of the own aircraft. For the intruder at A, an alert is likely if the intruder’s airspeed is greater than the own aircraft’s airspeed. The intruder will overtake the own aircraft and enter the alerting zone from behind, especially in situations in which the intruder’s heading and bank angle are low. Once the alert is issued, the own aircraft turns toward the intruder, possibly causing a collision which would not have occurred otherwise. Based on these results, it may be appropriate to alter the alerting logic in order to inhibit alerts after the intruder crosses the own aircraft’s runway centerline. Further work needs to be done to determine whether such a change to the logic is appropriate.

4.3 Resultant versus Design Hazard Level

To determine whether the alerting logic achieves the design level of performance, the resulting hazard level was calculated from the scenario outcomes recorded above. The hazard level is defined as the probability that an accident will occur after an alert is issued during a scenario in which the intruder blunders. This statistic corresponds to the following combination of the outcomes defined above:

\[
\text{Hazard Level} = \frac{\text{Induced Collisions + Late Alerts + Missed Detections}}{\text{Unnecessary Alerts + Induced Collisions + Late Alerts + Correct Detections}}
\]  

(4.2)

The quantity in the numerator is the total number of collisions that were unresolved by the alerting system. The quantity in the denominator is the total number of alerts that were issued during blunder trajectories. The overall hazard level computed in this manner is shown in Table 4.6 and is compared to the maximum acceptable hazard level corresponding to the design goal. Ideally, the two quantities would be equivalent.
Table 4.6: Resultant versus Design Hazard Level

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Overall Result</th>
<th>Design Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hazard Level</td>
<td>0.0041</td>
<td>0.0010</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0003</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The discrepancy between these two values is due to a number of factors. The collision curve approximation to the actual alerting threshold established by contour plots of Monte Carlo results may not have been accurate enough, allowing a greater number of accidents than would have occurred if the simplifications had not been made. Also, prior to performing the Monte Carlo simulations, standard deviations of the error distributions were estimated. Specifically, the values chosen for $\sigma_w$ and $\sigma_\phi$ may not have been large enough to allow for unexpected future maneuvers by the intruding aircraft. In addition, the measurement of intruder airspeed was assumed to be deterministic and no error was included. A combination of all or some of these factors may have caused the discrepancy between the resultant and design hazard levels.

The hazard level was also calculated for the three different runway separations and for the three different intruder airspeeds (Tables 4.7 and 4.8).

Table 4.7: Effect of Runway Separation on Hazard Level

<table>
<thead>
<tr>
<th>Runway Separation</th>
<th>Observed Hazard Level</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700 ft</td>
<td>0.0049</td>
<td>0.0020</td>
</tr>
<tr>
<td>2500 ft</td>
<td>0.0067</td>
<td>0.0027</td>
</tr>
<tr>
<td>3400 ft</td>
<td>0.0000</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>
Table 4.8: Effect of Intruder Airspeed on Hazard Level

<table>
<thead>
<tr>
<th>Intruder Airspeed</th>
<th>Observed Hazard Level</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>130 knots</td>
<td>0.0000</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>145 knots</td>
<td>0.0080</td>
<td>0.0028</td>
</tr>
<tr>
<td>160 knots</td>
<td>0.0042</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

These data show that the resulting alerting logic achieves the design performance only when the intruder’s airspeed is 130 knots. This may be due to a decrease in the longitudinal width of the probability contour. Any variations in this width with respect to intruder airspeed were not taken into account when the alerting logic was developed. Therefore, the 1600 ft longitudinal width may be adequate for intruder airspeeds of 130 knots but too low for faster intruders. Possible sources of error in the calculation of the 1600-ft value were discussed above.

It should be noted that the observed hazard level is highly dependent on the number of unnecessary alerts which occur. As the number of unnecessary alerts increases, the hazard level decreases because the total number of alerts increases without a proportional increase in the number of collisions.

Finally, a hazard level associated with each blunder trajectory was calculated. The results appear in Table 4.9. The hazard levels associated with the 5° bank angle blunder and the 10° heading change blunder indicate that collisions are likely to occur during this type of blunder trajectory even when the alerting system is operating. Section 4.2.3 explained the cause of many of the collisions which occurred. Further work is needed to determine whether the logic can be altered such that the acceptable hazard level (0.001) is not exceeded when these types of blunders occur. Because no accidents occurred during the remaining four blunders, the hazard level associated with these blunders is zero.
### Table 4.9: Hazard Level During Blunder Trajectories

<table>
<thead>
<tr>
<th>Blunder Type</th>
<th>Observed Hazard Level</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5° bank angle blunder</td>
<td>0.0036</td>
<td>0.0025</td>
</tr>
<tr>
<td>10° heading change blunder</td>
<td>0.0136</td>
<td>0.0043</td>
</tr>
<tr>
<td>15° heading change blunder</td>
<td>0.0000</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>30° heading change blunder</td>
<td>0.0000</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Fake blunder</td>
<td>0.0000</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Over adjustment blunder</td>
<td>0.0000</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>
Chapter 5

Three-Dimensional Alerting Logic

Modifications to the baseline alerting logic were made in order to account for variations in the intruder’s altitude and climb rate. First, this chapter outlines the expected performance of the two-dimensional logic under three-dimensional parallel approach geometries and motivates the need for an enhancement to the alerting algorithm. To this end, two separate altitude criteria were incorporated into the logic, and a comparison of the system’s performance using each of the two criteria is made. As in the two-dimensional logic, only one avoidance maneuver is assumed: a combined turn with climb.

5.1 Three-Dimensional Alerting Issues

During two-dimensional parallel approaches, the alerting threshold lies completely within the probability contours associated with \( P(C \mid \text{normal approach}) \). In other words, \( P(C \mid \text{turn/climb}) \) is always less than or equal to \( P(C \mid \text{normal approach}) \). For example, consider the following two-dimensional scenario.

Scenario #1: An intruder is coaltitude with the own aircraft and slowly moves into a relative horizontal position such that there is an unacceptably high probability that a collision will occur during the normal approach. A short time later, the alerting threshold associated with the turn/climb maneuver is crossed and the own aircraft initiates that avoidance maneuver.

Recasting this scenario in terms of the quantities calculated with Monte Carlo simulations, \( P(C \mid \text{turn/climb}) \) increases from zero to 0.001 when the alerting threshold is crossed. Meanwhile, \( P(C \mid \text{normal approach}) \) increases from zero to a value greater than 0.001. Therefore, the probability of a collision along the normal approach is unacceptably high and the avoidance trajectory must be taken instead.
During a two-dimensional situation in which an alert is issued such as Scenario #1, there is only one possible sequence in which these increases in collision probability take place (Figure 5.1). In a situation denoted by A, the intruder is in a location from which a collision is unlikely along either the normal approach or avoidance trajectories. Therefore, both $P(C \mid \text{normal approach})$ and $P(C \mid \text{turn/climb})$ are less than 0.001. As the intruder’s location changes to one within region B, a collision is probable if the own aircraft remains on the normal approach. Thus, $P(C \mid \text{normal approach})$ exceeds 0.001. Finally, when the intruder penetrates region C, a collision becomes unacceptably likely along the avoidance maneuver trajectory and an alert is issued.

![Figure 5.1: Issuing an alert during a two-dimensional situation](image)

When the approach geometries are three-dimensional, this sequence does not necessarily hold. Consider the following scenario in which the same two-dimensional alerting logic is used to issue alerts to the own aircraft during a three-dimensional approach situation:

Scenario #2: An intruder is several hundred feet above the own aircraft. Slowly, the intruder moves into the relative lateral and longitudinal positions from Scenario #1 but maintains the initial vertical separation. After a
short time, the intruder crosses the alerting threshold and the own aircraft initiates the turn-climb maneuver.

In terms of probability values, \( P(C|\text{turn/climb}) \) in Scenario #2 is greater than \( P(C|\text{normal approach}) \) since the normal approach involves a shallow descent away from the intruder while the turn/climb maneuver will increase the own aircraft’s altitude toward that of the intruder. Therefore, an induced collision may result. However, because the alerting thresholds only consider horizontal position, the alert is still issued. Figure 5.2 shows this process.

\[
\begin{array}{c|c|c}
\text{P(Collision|Normal Approach)} & < 1/1000 & > 1/1000 \\
\hline
\text{Both approach and maneuver are safe} & \text{Approach is unsafe but alert can be delayed} \\
\text{Alert issued} & \text{Alert issued} \\
\end{array}
\]

**Figure 5.2:** Issuing an alert during a three-dimensional situation

Path A is identical to the alerting process which occurs during a two-dimensional situation. Path B, which can occur if the intruder is above the own aircraft or is climbing, is problematic. Because the probability of a collision along the turn/climb trajectory exceeds the threshold value, an alert is issued even though the normal approach is actually a safer option. Therefore, the two-dimensional logic operating under three-dimensional conditions is likely to issue more unnecessary alerts. Also, since the avoidance maneuver is actually less safe than continuing the approach, induced collisions may occur. In order to
reduce the number of unnecessary alerts which may be issued and which may potentially result in induced collisions, the two-dimensional alerting logic should be augmented in a way that accounts for the vertical separation and climb rates of the two aircraft.

5.2 Altitude Criteria

To improve the performance of the two-dimensional alerting logic under three-dimensional conditions, two preliminary altitude criteria were added separately to the two-dimensional algorithm. Both criteria use the time-to-collision parameter, $t_c$, calculated to determine whether the intruder’s longitudinal position places it within the alerting zone (Equation 3.5). Figure 5.3 shows the state measurements and calculations involved.

Figure 5.3: Prediction of final relative altitude

The first criterion, called the Normal Approach Altitude Criterion, predicts the vertical separation, labelled $h_{\text{norm}}$ in Figure 5.3, at the closest point of approach between the intruder and the own aircraft which follows the normal approach. This calculation is based on the aircraft climb rates and their relative altitude, along with the time-to-collision parameter. If this final relative altitude is less than some threshold distance, and if the
intruder’s lateral and longitudinal position would otherwise prompt an alert, an alert is issued.

Similarly, the second criterion, called the Avoidance Maneuver Altitude Criterion, predicts vertical separation, labelled $h_{av}$, at the closest point of approach which would occur if the own aircraft performed an avoidance maneuver. The definition of the own aircraft’s avoidance maneuver, along with the intruder’s initial relative altitude and climb rate, is used to calculate $h_{av}$. Again, if the final relative altitude is less than some threshold distance, an alert is issued.

In both cases, the vertical threshold distance is associated with the height of the alerting zone in the vertical plane. This zone was established in the same manner that the longitudinal width of the alerting zone in the horizontal plane was established. In determining the longitudinal width, Monte Carlo simulations were run at a longitudinal increments across a range of lateral separations. From this process, it was determined that the width of the $P(C \mid maneuver) = 0.001$ contour was roughly constant (1600 ft) with lateral position and with initial heading and bank angle.

To determine the height of the same contour in three-dimensional space, Monte Carlo simulations were run at a variety of altitudes across a range of lateral separations. This same process was performed at a series of longitudinal positions. From these calculations, the height of the alerting zone was determined to be approximately 1600 ft. This value held reasonably well across lateral separations and the longitudinal positions which were investigated. Since the alerting zone was found to have a height of approximately 1600 ft, an alert is issued if the predicted vertical separation is less than 800 ft.
5.3 Three-Dimensional Performance Evaluation Methodology

The three-dimensional performance evaluation methodology is similar to the methodology used to evaluate two-dimensional performance described in Section 4.1. Each intruder trajectory, including blunders and normal approaches, was run alongside one of two normal approaches of the own aircraft. A third aircraft was simulated which performed the avoidance maneuver if an alert was issued.

To simulate three-dimensional parallel approach geometries, the intruder was permitted to deviate vertically from the own aircraft’s altitude. During each of the scenarios, the intruder was assigned a vertical offset of -1000, -500, 0, 500, or 1000 ft from the own aircraft’s altitude. Also, a climb rate of -1000, -500, 0, 500, or 1000 ft/min was assigned to begin as soon as the intruder deviated off course and began to blunder. With these additional scenario variations, the total number of simulations run to produce the data in this chapter is 1,070,550:

\[
(39 \text{ intruder trajectories}) \cdot (2 \text{ own aircraft trajectories}) \cdot (3 \text{ runway separations}) \cdot \\
(183 \text{ longitudinal spacings}) \cdot (5 \text{ altitude offsets}) \cdot (5 \text{ climb rates}) = 1,070,550
\]

5.4 Performance of Alerting Logic under Altitude and Climb Rate Variations

In order to evaluate and compare the effectiveness of the two altitude criteria, the evaluation process was performed twice, once with each criterion. Also, the performance of the two-dimensional logic under three-dimensional parallel approach scenarios was assessed for comparison. The results are listed in Table 5.1. These data encompass all intruder trajectories at all airspeeds and at all runway separations.
Table 5.1: Alerting Logic Performance Under 3-D Scenarios

<table>
<thead>
<tr>
<th>Logic</th>
<th>Correct Rejection</th>
<th>Missed Detection</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Vertical Criterion (2D)</td>
<td>9.3x10^-1</td>
<td>0.0</td>
<td>5.8x10^-2</td>
<td>2.0x10^-4</td>
<td>9.3x10^-3</td>
<td>3.5x10^-5</td>
</tr>
<tr>
<td>σ</td>
<td>2.4x10^-4</td>
<td>&lt;1x10^-6</td>
<td>2.3x10^-4</td>
<td>1.4x10^-5</td>
<td>9.3x10^-5</td>
<td>5.7x10^-6</td>
</tr>
<tr>
<td>Normal Approach (3D)</td>
<td>9.6x10^-1</td>
<td>0.0</td>
<td>3.1x10^-2</td>
<td>2.1x10^-4</td>
<td>9.3x10^-3</td>
<td>3.5x10^-5</td>
</tr>
<tr>
<td>σ</td>
<td>1.9x10^-4</td>
<td>&lt;1x10^-6</td>
<td>1.7x10^-4</td>
<td>1.4x10^-5</td>
<td>9.3x10^-5</td>
<td>5.7x10^-6</td>
</tr>
<tr>
<td>Avoidance Maneuver (3D)</td>
<td>9.5x10^-1</td>
<td>0.0</td>
<td>4.0x10^-2</td>
<td>1.8x10^-4</td>
<td>9.2x10^-3</td>
<td>5.2x10^-5</td>
</tr>
<tr>
<td>σ</td>
<td>2.1x10^-4</td>
<td>&lt;1x10^-6</td>
<td>1.9x10^-4</td>
<td>1.3x10^-5</td>
<td>9.3x10^-5</td>
<td>7.0x10^-6</td>
</tr>
</tbody>
</table>

Each row refers to the performance of a different version of the alerting logic. The No Vertical Criterion row refers to the performance of the two-dimensional alerting logic under three-dimensional approach geometries. The Normal Approach and Avoidance Maneuver rows list the performance of the two versions of the alerting logic with those altitude criteria.

5.4.1 Unnecessary Alerts

The unnecessary alert rate has been significantly reduced with the addition of either of the altitude criteria. During three-dimensional approach geometries, collisions are unlikely to occur in two situations, even if a blunder takes place:

1. The intruder is above the own aircraft and climbing.
2. The intruder is below the own aircraft and descending.

If the alerting logic disregards altitude and climb rate information when issuing alerts, an intruder in either of these situations may prompt an alert based solely on lateral and longitudinal position. The two-dimensional logic discounts all altitude information and issues a high number of unnecessary alerts during the two situations listed above. Figure 5.4
shows the rough distribution of unnecessary alerts issued by the two-dimensional alerting logic under three-dimensional approaches as a function of intruder’s altitude and climb rate. This is a coarse contour plot based on datapoints at the intersection of gridlines. For example, 4-6% of all unnecessary alerts occurred when the intruder was 500 ft above the own aircraft and climbing at a rate of 500 ft/min.

![Figure 5.4: Unnecessary alert distribution: No Vertical Criterion (2D)](image)

When the Normal Approach three-dimensional alerting logic is used instead, the total number of unnecessary alerts issued under the two conditions listed above is reduced. The logic predicts future vertical separation based on the initial altitude separation of the two aircraft involved and both aircrafts’ climb rates. Since the own aircraft is in a very shallow descent, alerts are issued most often when the intruder is coaltitude, above and descending, or below and climbing. Therefore the majority of unnecessary alerts associated with an intruder that is above and climbing or below and descending are eliminated. Figure 5.5 shows the distribution of unnecessary alerts using the Normal Approach Altitude Criterion.
Figure 5.5: Unnecessary alert distribution: Normal Approach Altitude Criterion (3D)

The distribution of unnecessary alerts changes again when the Avoidance Maneuver Altitude Criterion is considered. This version of the logic alerts in situations where the intruder will be co-altitude with the avoidance maneuver trajectory at the projected time of collision. Since all alerts are issued in this manner, most unnecessary alerts are issued in situations in which the intruder is above the own aircraft or ascending. This trend can be seen in Figure 5.6.

Figure 5.6: Unnecessary alert distribution: Avoidance Maneuver Altitude Criterion (3D)
5.4.2 Induced Collisions and Late Alerts

As shown in Table 5.1, the number of induced collisions and late alerts that occurred with the three alerting logic versions tested under three-dimensional parallel approach geometries remains roughly constant. However, slight differences in the number of induced collisions and late alerts arise because of differences in the alerting process.

When the Normal Approach Altitude Criterion is added to the two-dimensional logic, the induced collision rate increases. This increase may be due to the fact that the alerts are issued slightly later because both horizontal and vertical position requirements must be met before the alert is issued. However, more analysis needs to be performed to better understand the reason for this increased induced collision rate.

Also, a subtle change in the alerting process may have contributed to the increase in induced collisions when the Normal Approach Altitude Criterion is used. Section 5.1 contrasted the processes by which alerts were issued by the two-dimensional and three-dimensional alerting logics. However, when the Normal Approach criterion was used, a step was added to this process which assures that the normal approach trajectory has become dangerous before issuing an alert (Figure 5.7).
In case B, the potential exists for alerts to be issued when \( P(C \mid \text{turn/climb}) \) is much greater than \( P(C \mid \text{normal approach}) \). Therefore, it is likely that induced collisions will occur. This is seen when comparing the performance of the two-dimensional logic to that of the Normal Approach logic.

When the Normal Approach criterion is replaced with the Avoidance Maneuver criterion, the rate of induced collisions declines. The criterion within the logic checks to see if the probability of a collision along the avoidance trajectory has exceeded the maximum acceptable limit before an alert is issued, thus better assuring the safety of the avoidance maneuver.

When the Avoidance Maneuver criterion is used, a greater number of late alerts occur. The Avoidance Maneuver criterion places a different requirement on the intruder’s altitude which may allow late alerts to occur more often. More work needs to be done in order to fully explain the increase in late alert rate.
5.4.3 Hazard Levels

The hazard levels for each version of the alerting logic were calculated and are included in Table 5.2. The increase in calculated hazard level between the two-dimensional logic and both versions of the three-dimensional logic is not due to the increase in the number of collisions which occurred, which varied by about 3%. Instead, the increase is due to the significant reduction in the number of unnecessary alerts. Because hazard level is defined as unresolved blunders over total number of alerts (Equation 4.2), fewer alerts implies that the hazard level increases, even though the performance may have actually improved slightly.

<table>
<thead>
<tr>
<th>Alerting Logic</th>
<th>Hazard Level</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Dimensional Logic</td>
<td>0.0035</td>
<td>0.0002</td>
</tr>
<tr>
<td>Normal Approach Altitude Criterion</td>
<td>0.0060</td>
<td>0.0004</td>
</tr>
<tr>
<td>Avoidance Maneuver Altitude Criterion</td>
<td>0.0048</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Chapter 6

Effect of Avoidance Maneuvers

This chapter describes the possible advantages of incorporating additional avoidance maneuvers into the existing alerting logic. First, the performance limitations due to having only a single avoidance maneuver available are discussed, as well as the expected benefits of introducing an additional maneuver. Next, a preliminary investigation was made in which the logic selects one of two avoidance maneuvers. The results from this investigation are presented.

When the alerting logic is designed to choose among several maneuvers, additional alerting options arise. The nature of the situation may dictate that an alert should be issued when one maneuver becomes unsafe, or when all but one maneuver are unsafe. The consequences of these designs are discussed and some preliminary evaluations are made.

6.1 Limitations of Single Maneuver

In the most general three-dimensional case, each possible own aircraft trajectory has associated with it a semi-infinite alerting zone. This zone consists of all intruder locations in three-dimensional space which will eventually result in a collision with the own aircraft. These zones were discussed in Section 3.3 and formed the basis for the alerting logic. In two-dimensional situations, the alerting zone has a finite size for all own aircraft trajectories except the normal approach trajectory. This collapse of a semi-infinite alerting zone down to a finite one enabled the logic developed earlier to function effectively.
In the logic discussed throughout the previous chapters, alerts were issued when the intruder entered the zone associated with the turn/climb avoidance maneuver. In the two-dimensional parallel approach situation, this zone is contained entirely within the normal approach alerting zone (Figure 6.1). The solid line $A$ shows the intruder locations which will cause a collision between the intruder and the own aircraft following the turn/climb avoidance maneuver. It is representative of the center of the probability contour corresponding to $P(C \mid \text{turn/climb}) = \text{hazard level}$. Similarly, solid line $B$ shows locations from which an intruder would collide with the own aircraft on normal approach. This curve is the collision curve developed in Section 3.3.

![Diagram of alerting zones under two-dimensional constraint](image)

**Figure 6.1**: Alerting zones under two-dimensional constraint

Because the intruder is constrained to be co-altitude with the own aircraft in the two-dimensional case, the probability contour (represented by curve $A$) associated with the avoidance maneuver is limited in size. As the lateral separation between the intruder and own aircraft increases, the avoidance trajectory will provide more vertical separation
between the two aircraft after the alert. At some point in time, this vertical separation will exceed 500 ft in which case a collision is impossible. Meanwhile, if the own aircraft were to have followed the normal approach, no vertical separation would have taken place and collisions can occur over a wider range of initial intruder lateral positions. Therefore, curve A is significantly shorter than curve B.

When the intruder’s altitude restriction is lifted, A and B are both likely to change in size. If the intruder matches the avoidance maneuver climb rate and altitude, A becomes larger than B. In this situation, the size of curve B decreases: the altitude separation limits the initial horizontal positions from which the intruder can cause an accident. The size of A increases since altitude separation between the two aircraft disappears when the own aircraft climbs along the avoidance path.

This effect reduces the ability of the current alerting logic to function well under three-dimensional conditions if only one maneuver is allowed. The two-dimensional logic only determines whether the intruder is in a horizontal position which will cause an accident. If the intruder matches the avoidance maneuver climb rate, the maneuver may be more dangerous than the normal approach course but the alert may be issued anyway.

**6.2 Effects of Introducing a Level Turn Maneuver**

It is proposed that introducing additional maneuver options into the alerting logic can reduce the effects described above. Specifically, a constant-altitude turn maneuver could be incorporated into the logic along with the climbing maneuver which is already present. The result of this addition can be seen in Figure 6.2 using three curves similar to the ones shown in Figure 6.1. However the curves will shift depending on the intruder’s altitude or climb rate. Figure 6.2 shows the three curves if the intruder climbs at the same rate required for the turn/climb avoidance maneuver.
In this case, curve A corresponds to the locus of initial intruder positions that will cause a collision between the intruder and an own aircraft following the climbing turn trajectory. The curve lies in the center of the $P(C \mid \text{turn/climb}) = 0.001$ probability contour. Curve B is the locus of points which will cause an accident if the own aircraft follows the normal approach (i.e. the center of the $P(C \mid \text{normal approach}) = 0.001$ contour). Finally, curve C corresponds to a new probability contour: $P(C \mid \text{turn}) = 0.001$.

Curve C is the same size as curve A from Figure 6.1. The altitude separation between a climbing intruder and level own aircraft limits the number of lateral intruder positions which can cause an accident. Also, the heading change associated with the maneuver increases the separation between the two aircraft. Curve B is similar in size because the same altitude separation may occur, although the lack of a heading change means that this curve is somewhat larger than curve C. Again, curve A is the largest of the three because less altitude separation occurs if both the intruder and the own aircraft climb.
A preliminary evaluation of an alerting logic which chooses between the level turn or climbing turn has been implemented. The climbing turn is the same maneuver which has been used throughout this thesis. The level turn entails the same turn but the aircraft performs a 0.25g pull-up from the glideslope descent rate until the aircraft is maintaining altitude--the pull-up is not continued into a climb.

The logic first performs the horizontal position test outlined in Section 3.3. This test involves the intruder’s lateral and longitudinal position and the predicted time to collision. Next, the Normal Approach Criterion from Section 5.1 is applied. If the predicted vertical separation between the intruder and the own aircraft following the normal approach path is less than 800 ft, an alert is issued.

At this point, a choice is made between the two maneuvers. Vertical separation is predicted between the intruder and two own aircraft: one which performs the level turn, and another which performs the climbing turn. The maneuver that is predicted to result in the maximum vertical separation between the two aircraft is performed. The results of this preliminary evaluation are listed in Table 6.1, with the Normal Approach Altitude Criterion evaluation results included for comparison.

**Table 6.1: Performance of Alerting Logic with Two Maneuver Options**

<table>
<thead>
<tr>
<th>Logic</th>
<th>Correct Rejection</th>
<th>Missed Detection</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Maneuver Options</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ</td>
<td>9.59x10⁻¹</td>
<td>0.0</td>
<td>3.14x10⁻²</td>
<td>2.08x10⁻⁴</td>
<td>9.25x10⁻³</td>
<td>4.95x10⁻⁵</td>
</tr>
<tr>
<td></td>
<td>1.9x10⁻⁴</td>
<td>&lt;1x10⁻⁶</td>
<td>1.7x10⁻⁴</td>
<td>1.4x10⁻⁵</td>
<td>9.3x10⁻⁵</td>
<td>6.8x10⁻⁶</td>
</tr>
<tr>
<td>Normal Approach (3D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ</td>
<td>9.59x10⁻¹</td>
<td>0.0</td>
<td>3.14x10⁻²</td>
<td>2.12x10⁻⁴</td>
<td>9.26x10⁻³</td>
<td>3.46x10⁻⁵</td>
</tr>
<tr>
<td></td>
<td>1.9x10⁻⁴</td>
<td>&lt;1x10⁻⁶</td>
<td>1.7x10⁻⁴</td>
<td>1.4x10⁻⁵</td>
<td>9.3x10⁻⁵</td>
<td>5.7x10⁻⁶</td>
</tr>
</tbody>
</table>

The performance of the alerting logic which chooses between the level turn and climbing turn maneuver is similar to the performance of the three-dimensional logic using the
Normal Approach Altitude Criterion logic. Induced collisions were reduced while the rate of late alerts increased. The unnecessary alert rate and hazard level are approximately equal between the two systems.

The outcomes resulting during scenarios in which the turn/climb maneuver was chosen and when the level turn maneuver was chosen are separated in Table 6.2. The Correct Rejection and Missed Detection outcomes are not included in the table because no alert is issued during these scenarios, and thus no maneuver is selected. During the 43,837 simulations in which alerts were issued, the turn/climb maneuver was chosen 40,109 times, and the level turn maneuver was chosen during the remaining 3,728 scenarios.

**Table 6.2: Performance of Logic when Choosing Each Maneuver Option**

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turn/Climb</td>
<td>7.53x10⁻¹</td>
<td>1.97x10⁻³</td>
<td>2.44x10⁻¹</td>
<td>1.32x10⁻³</td>
</tr>
<tr>
<td></td>
<td>2.2x10⁻³</td>
<td>2.2x10⁻⁴</td>
<td>2.1x10⁻³</td>
<td>1.8x10⁻⁴</td>
</tr>
<tr>
<td>Level Turn</td>
<td>9.28x10⁻¹</td>
<td>3.86x10⁻²</td>
<td>3.30x10⁻²</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>4.2x10⁻³</td>
<td>3.2x10⁻³</td>
<td>2.9x10⁻³</td>
<td>&lt;1x10⁻⁶</td>
</tr>
</tbody>
</table>

The most significant result shown above is the increase in induced collision rate associated with the level turn maneuver. Based on the argument outlined in Section 4.2.3, it is possible that a greater number of unnecessary alerts became induced collisions than was expected because of the own aircraft's constant altitude flight. When the intruder crosses the own aircraft's runway centerline, the own aircraft remains at constant altitude instead of climbing. Therefore, what might have been an unnecessary alert becomes an induced collision because no altitude separation takes place.
6.3 Alerting Logic Variations

The introduction of additional avoidance maneuvers into the alerting logic allows the logic to function in a number of different ways. These logic variations arise because each maneuver has associated with it a distinct alerting threshold. The thresholds are set around the points in space where $P(C \mid \text{maneuver})$ is greater than the hazard level goal. Therefore, for each avoidance maneuver, a different threshold defined by $P(C \mid \text{maneuver})$ appears.

A conservative or a critical system may be designed based on the number of maneuver thresholds which must be crossed before an alert is issued (Figure 6.3). The logic may be designed to issue alerts when one of the thresholds associated with a particular maneuver is crossed. In this case, the probability of a collision over the course of that maneuver is greater than the threshold level. For the other maneuvers, the probability of a collision is much lower than the threshold level and the logic chooses the safest of the remaining maneuvers. This is a conservative system.

On the other hand, the logic may be designed to issue an alert only when all of the thresholds have been crossed. For example, when this variation of the logic is operating with three maneuver options available, the intruding aircraft only prompts an alert when all three of the thresholds have been crossed. Two of the maneuvers have associated with them a probability of collision which is greater than the threshold level. The remaining maneuver has a probability of collision equal to the threshold level and is deemed to be the safest of the three. The alert is issued and this maneuver is performed as it is the last option available. This is a critical system.

Figure 6.3 depicts some representative thresholds associated with particular avoidance maneuvers and depicts the alerting thresholds within the critical and conservative systems. The alerting threshold of the conservative system is the union of all of the probability con-
tours associated with all of the possible maneuvers and is shown in the figure with a heavy, dashed line. In the critical system, the alerting threshold is the intersection of all of these contours, denoted by the heavy, solid line.

**Figure 6.3:** Representative alerting thresholds associated with different maneuvers

The conservative system would be expected to reduce the number of collisions that occur, but would also increase the number of unnecessary alerts. In contrast, the critical system would lower the unnecessary alert rate but may allow more accidents to occur.

### 6.4 Evaluation of Alerting Logic Variations

The concept described above was implemented in a preliminary fashion to assess the predicted performance variations. To do this, alerting thresholds were established for three different avoidance maneuvers:

1. Turn and climb maneuver (described earlier).
2. Level turn maneuver (described earlier).
3. Straight-ahead climb maneuver (0.25g pull-up to a 2000 ft/min climb rate--no heading change).
Each maneuver probability contour was assumed to lie centered on the collision curve and was assumed to be 1600 ft wide in the longitudinal direction. For each maneuver, a new range limit array was developed. These are included in Appendix A.

For the purpose of the evaluations, three systems were investigated: a conservative system, a semi-critical system, and a critical system. For the conservative system, alerts are issued when any one of the three thresholds is crossed. For the semi-critical system, two thresholds are crossed before the alert is issued. In the critical system, all of the thresholds must be crossed before the alert is finally issued.

To simplify the development of the logic and the choice of avoidance maneuver, the evaluations were run under two-dimensional conditions. The selected avoidance maneuver was chosen to have the smallest range limit value. The results of this evaluation are included in Table 6.3. As in the two-dimensional performance evaluation, the total number of simulations is 42,822.

<table>
<thead>
<tr>
<th>System</th>
<th>Correct Rejection</th>
<th>Missed Detection</th>
<th>Unnecessary Alert</th>
<th>Induced Collision</th>
<th>Correct Detection</th>
<th>Late Alert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservative</td>
<td>8.9x10^{-1}</td>
<td>0.0</td>
<td>7.7x10^{-2}</td>
<td>9.3x10^{-5}</td>
<td>3.4x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td>Semi-Critical</td>
<td>9.2x10^{-1}</td>
<td>0.0</td>
<td>4.4x10^{-2}</td>
<td>1.6x10^{-4}</td>
<td>3.4x10^{-2}</td>
<td>0.0</td>
</tr>
<tr>
<td>Critical</td>
<td>9.3x10^{-1}</td>
<td>0.0</td>
<td>3.4x10^{-2}</td>
<td>7.0x10^{-5}</td>
<td>3.4x10^{-2}</td>
<td>0.0</td>
</tr>
</tbody>
</table>

As predicted, the unnecessary alert rate declines as more thresholds must be crossed before an alert is issued. In fact, the conservative system issues alerts during normal approaches because the alerting threshold had expanded. The overall rate of unnecessary alerts during normal approaches was 2.4x10^{-2}. Ninety-three percent of these unnecessary alerts occurred when the runways were spaced by only 1700 ft, with the remainder occurring when the runways were spaced by 2500 ft. This indicates that the conservative sys-
tem would not be appropriate for a runway spacing of 1700 ft, and may not even be suitable for a runway spacing of 2500 ft.

The rates of late alerts are zero for each logic evaluated, and the rates of induced collisions are similar.

The hazard level present in each version of the alerting logic was calculated from the performance data above. These results are included in Table 6.4.

**Table 6.4: Alerting Logic Hazard Levels**

<table>
<thead>
<tr>
<th>Alerting Logic</th>
<th>Hazard Level</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservative</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
<tr>
<td>Semi-Critical</td>
<td>0.0021</td>
<td>0.0008</td>
</tr>
<tr>
<td>Critical</td>
<td>0.0010</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

The hazard level associated with the conservative alerting logic is lower than for the other two because of the high number of unnecessary alerts which were issued. The significantly higher hazard level associated with the semi-critical system is due to a small increase in the number of induced collisions combined with a decrease in the number of unnecessary alerts. Further work is needed to determine why a greater number of induced collisions occurred under the semi-critical alerting system.
Chapter 7

Conclusions

A probability-based alerting logic for aircraft on parallel approach was designed and tested. Monte Carlo simulations were used to calculate the probability of a collision over an extensive range of possible parallel approach geometries. Based on the results, alerting thresholds were established such that alerts are issued when the probability of a collision exceeds a maximum acceptable hazard level.

Direct incorporation of probability into the alerting logic is a novel approach to alerting system development. It allows explicit trade-offs to be made between safety and utility, and it eliminates the guesswork involved in defining an alerting threshold based solely on fixed physical parameters. It enables the logic to be designed to provide a consistent level of safety over a range of possible situations.

Several modifications were made to a baseline two-dimensional version of the alerting logic. First, altitude considerations were added in order to expand the situations under which the logic could operate effectively. Also, the logic was redesigned to select one of several avoidance maneuver options. Performance evaluations were made of all versions of the logic. The evaluation procedure used pilot-flown blunder trajectories recorded during a simulator study at Rockwell-Collins. When appropriate, the logic issued alerts based on ‘real-time’ state measurements and an avoidance maneuver was initiated in response to those alerts. The eventual outcome of each approach was then categorized to allow an analysis of performance.
7.1 Summary of Significant Results

Two-Dimensional Logic

The two-dimensional logic consists of two alerting criteria: 1) the intruder must be within a range limit which is dependent on the intruder’s state measurements, and 2) the intruder must be in a position from which a collision is likely if a constant-rate turn trajectory is followed. No allowance is made for climb rates or altitude separation.

The overall performance delivered by the two-dimensional logic under two-dimensional approaches (aircraft are coaltitude) is approximately equal to the level of performance that the system was designed to achieve. The hazard level goal was set at 0.001 and the system exhibited an overall hazard level of 0.004, which is similar to the PRM level. This hazard level implies that approximately one in 250 blunders will not be resolved by the alerting system.

The performance of the alerting logic indicates that it may be successful in providing protection for parallel approaches to runways spaced less than 3400 ft apart. When the intruder followed a normal approach course at runway separations ranging from 1700 ft to 3400 ft, no alerts were ever issued. This may indicate that the alerting thresholds can be expanded, providing additional safety while maintaining an acceptable rate of unnecessary alerts.

Two types of blunders (the 5° bank angle blunder and the 10° heading change blunder) were found to cause collisions under two-dimensional approaches. A modification to the logic is proposed which should greatly improve performance under these blunder scenarios.
Three-Dimensional Logic

Two separate enhancements were made to the two-dimensional logic in order to improve performance under three-dimensional approach geometries. One version of the three-dimensional logic issues alerts based on predicted altitude separation if the own aircraft follows the normal approach path (Normal Approach criterion). The other version issues alerts based on predicted altitude separation if the own aircraft initiates an avoidance maneuver (Avoidance Maneuver criterion).

Performance evaluations were made of the two-dimensional logic and both versions of three-dimensional logic under three-dimensional parallel approach geometries. The Normal Approach criterion lowered the unnecessary alert rate by forty-six percent but allowed a slight increase in the rate of induced collisions. The Avoidance Maneuver criterion issued a slightly higher rate of unnecessary alerts than the Normal Approach criterion but lowered the induced collision rate below the level allowed by the two-dimensional logic.

Performance evaluations were made of an alerting logic which chooses among several maneuvers. First, a level turn option was added to the alerting logic, for a total of two maneuver options. Expected performance improvements were not realized, largely because of the induced collisions which occurred when the level turn maneuver was selected. Second, additional maneuver options allow the development of several alerting systems which range from conservative (alert when one avoidance maneuver becomes unacceptably risky) to critical (alert when all but one maneuver becomes unacceptably risky). Evaluations of such a series of alerting systems shows that the conservative system issues almost twice as many unnecessary alerts without a commensurate increase in safety.
7.2 Areas for Future Work

A number of issues arose which demand further investigation but were beyond the scope of this initial study.

Error Estimates

The statistics which establish error estimates are crucial to the successful attainment of a performance goal. Alerting thresholds were defined based on probability values calculated using Monte Carlo simulations. The validity of such calculations, and thus the accuracy of the alerting thresholds, depends on accurate state measurement error distributions. This version of the logic estimated \( \sigma_x \) and \( \sigma_y \) based on DGPS position measurements. Estimates of \( \sigma_{\psi} \) and \( \sigma_{\phi} \) were designed to encompass several different uncertainties within the system: measurement error, variations caused by disturbances such as wind conditions during the approach, and the ability of the intruder to perform a maneuver which differs from the assumed constant-rate turn. Better estimates of uncertainty will enable the resulting performance calculations to more closely conform to the design goals. Data being collected by MIT Lincoln Laboratory at Memphis International Airport may help define more realistic probability density functions [10].

A few significant error measurements were excluded from the development of this logic due to the difficulty in estimating their magnitudes. Variations in the pilot’s response time were ignored, as well as any variation in the pilot’s aggressiveness while performing a prescribed avoidance maneuver. Both of these parameters are important, but experimental measurements are difficult because of possible learning effects when subjects are exposed to more than one alert.
Definition of a Collision

The development and predicted performance of the alerting logic depends greatly on the definition of a collision which is used. During both the development of the alerting logic and during subsequent performance evaluations, a collision was defined as a miss distance of 500 ft or less.

It may be useful to design the alerting logic to provide a minimum separation of 500 ft but classify scenario outcomes into several categories. For instance, miss distances less than 500 ft would still be collisions. Scenarios in which an alert was issued but the minimum miss distance was between 500 ft and 1000 ft would be termed near misses. Unnecessary alerts would comprise scenarios in which an alert was issued but the miss distance was greater than 1000 ft. Of course, these numerical definitions could be changed. Creating this additional category in which to place a scenario’s outcome would allow more accurate analysis of the alerting system’s performance.

Induced Collisions

Induced collisions accounted for a significant portion of the accidents allowed by the alerting logic. A significant fraction of these occurred during scenarios in which the alert was issued after the intruder had already crossed the own aircraft’s runway centerline. A number of options exist to eliminate these accidents: the alerting logic can be modified or additional maneuvers can be permitted.

First, the alerting logic can be modified such that two separate components exist. One component would operate under normal situations; the other component would become active when the intruder crossed the own aircraft’s runway centerline. Both components would be developed using the same probability-based method outlined in this thesis.
A second option involves using the same logic but allowing a different maneuver when the intruder crosses the own aircraft’s runway centerline. The induced collision is a direct result of the own aircraft turning toward the intruder. If the direction of turn were reversed, the collision would not occur. However, the own aircraft would be turning toward the parallel stream of aircraft, creating other potential conflicts not considered by the alerting logic.
References


Appendix A

Range Limit Arrays

The alerting logic relies on the arrays included in this appendix for a physical parameter which defines the alerting threshold. This parameter changes as the situation changes and as the planned avoidance maneuver changes. Therefore, each of the following tables contains range limits for various intruder headings, bank angles, and airspeeds. Also, each possible avoidance maneuver discussed in the text has a corresponding array.

Note that the heading associated with each range limit value is listed to the left of each row. Also, the bank angle is listed across the top and the intruder’s airspeed is noted prior to each nine-by-seven array.

Table 1: Range Limit Array for Turn and Climb Maneuver

<table>
<thead>
<tr>
<th>Intruder Airspeed: 120 knots</th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1302</td>
<td>1775</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1076</td>
<td>1447</td>
<td>1842</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1196</td>
<td>1637</td>
<td>2015</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1081</td>
<td>1435</td>
<td>1822</td>
<td>2159</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1341</td>
<td>1707</td>
<td>2029</td>
<td>2273</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>1344</td>
<td>1644</td>
<td>1943</td>
<td>2217</td>
<td>2488</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>1095</td>
<td>1687</td>
<td>1917</td>
<td>2206</td>
<td>2481</td>
<td>2815</td>
</tr>
<tr>
<td>30°</td>
<td>923</td>
<td>1555</td>
<td>1919</td>
<td>2169</td>
<td>2424</td>
<td>2728</td>
<td>3013</td>
</tr>
<tr>
<td>40°</td>
<td>1256</td>
<td>2040</td>
<td>2196</td>
<td>2421</td>
<td>2695</td>
<td>2938</td>
<td>3330</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intruder Airspeed: 140 knots</th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1341</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>956</td>
<td>1545</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1230</td>
<td>1724</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1103</td>
<td>1550</td>
<td>1956</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1095</td>
<td>1462</td>
<td>1802</td>
<td>2186</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>1137</td>
<td>1464</td>
<td>1801</td>
<td>2069</td>
<td>2482</td>
</tr>
<tr>
<td>Angle</td>
<td>20°</td>
<td>30°</td>
<td>40°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>800</td>
<td>1512</td>
<td>1802</td>
<td>2110</td>
<td>2410</td>
<td>2757</td>
</tr>
<tr>
<td>30°</td>
<td>800</td>
<td>800</td>
<td>1873</td>
<td>2156</td>
<td>2417</td>
<td>2711</td>
<td>3106</td>
</tr>
<tr>
<td>40°</td>
<td>800</td>
<td>1927</td>
<td>2212</td>
<td>2470</td>
<td>2738</td>
<td>3090</td>
<td>3357</td>
</tr>
</tbody>
</table>

Intruder Airspeed: 160 knots

<table>
<thead>
<tr>
<th>Angle</th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>877</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1189</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1002</td>
<td>1504</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>965</td>
<td>1403</td>
<td>1785</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1017</td>
<td>1379</td>
<td>1719</td>
<td>2076</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>1080</td>
<td>1433</td>
<td>1774</td>
<td>2119</td>
<td>2421</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>800</td>
<td>1502</td>
<td>1849</td>
<td>2168</td>
<td>2414</td>
<td>2797</td>
</tr>
<tr>
<td>30°</td>
<td>846</td>
<td>1571</td>
<td>1913</td>
<td>2257</td>
<td>2510</td>
<td>2853</td>
<td>3187</td>
</tr>
<tr>
<td>40°</td>
<td>1455</td>
<td>2026</td>
<td>2332</td>
<td>2604</td>
<td>2923</td>
<td>3214</td>
<td>3610</td>
</tr>
</tbody>
</table>

Intruder Airspeed: 180 knots

<table>
<thead>
<tr>
<th>Angle</th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1040</td>
<td>1192</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1192</td>
<td>1437</td>
<td>1777</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1192</td>
<td>1437</td>
<td>1777</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1271</td>
<td>1635</td>
<td>2059</td>
<td>2472</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>1066</td>
<td>1363</td>
<td>1635</td>
<td>1931</td>
<td>2207</td>
<td>2583</td>
</tr>
<tr>
<td>20°</td>
<td>955</td>
<td>1410</td>
<td>1730</td>
<td>2059</td>
<td>2321</td>
<td>2635</td>
<td>2954</td>
</tr>
<tr>
<td>30°</td>
<td>1255</td>
<td>1831</td>
<td>2165</td>
<td>2472</td>
<td>2758</td>
<td>3019</td>
<td>3436</td>
</tr>
<tr>
<td>40°</td>
<td>1887</td>
<td>2280</td>
<td>2632</td>
<td>2917</td>
<td>3151</td>
<td>3515</td>
<td>3851</td>
</tr>
</tbody>
</table>

Table 2: Range Limit Array for Climb Maneuver

<table>
<thead>
<tr>
<th>Angle</th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>2483</td>
<td>2885</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1722</td>
<td>2650</td>
<td>3006</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>2359</td>
<td>2837</td>
<td>3234</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1856</td>
<td>2608</td>
<td>2985</td>
<td>3415</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>2482</td>
<td>2808</td>
<td>3234</td>
<td>3673</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>1679</td>
<td>2746</td>
<td>3061</td>
<td>3462</td>
<td>3880</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>1132</td>
<td>2776</td>
<td>2963</td>
<td>3286</td>
<td>3746</td>
<td>4106</td>
</tr>
<tr>
<td>30°</td>
<td>923</td>
<td>1518</td>
<td>2923</td>
<td>3215</td>
<td>3600</td>
<td>3936</td>
<td>4289</td>
</tr>
<tr>
<td>40°</td>
<td>1342</td>
<td>2335</td>
<td>3134</td>
<td>3483</td>
<td>3912</td>
<td>4192</td>
<td>4326</td>
</tr>
</tbody>
</table>
### Table 3: Range Limit Array for Level Turn Maneuver

<table>
<thead>
<tr>
<th>Intruder Airspeed: 140 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intruder Airspeed: 160 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intruder Airspeed: 180 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>1250</td>
</tr>
<tr>
<td>1968</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intruder Airspeed: 120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>-40°</td>
</tr>
<tr>
<td>-30°</td>
</tr>
<tr>
<td>-20°</td>
</tr>
<tr>
<td>-10°</td>
</tr>
<tr>
<td>0°</td>
</tr>
<tr>
<td>10°</td>
</tr>
</tbody>
</table>

**Intruder Airspeed: 140 knots**

<table>
<thead>
<tr>
<th></th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1310</td>
<td>1718</td>
<td>2150</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1189</td>
<td>1628</td>
<td>2036</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1189</td>
<td>1628</td>
<td>2036</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1189</td>
<td>1628</td>
<td>2036</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>1241</td>
<td>1586</td>
<td>1987</td>
<td>2332</td>
<td>2736</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>1634</td>
<td>1969</td>
<td>2282</td>
<td>2696</td>
<td>3117</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>829</td>
<td>2018</td>
<td>2360</td>
<td>2675</td>
<td>3033</td>
<td>3548</td>
</tr>
<tr>
<td>30°</td>
<td>800</td>
<td>2043</td>
<td>2379</td>
<td>2701</td>
<td>3031</td>
<td>3481</td>
<td>4140</td>
</tr>
<tr>
<td>40°</td>
<td>800</td>
<td>2043</td>
<td>2379</td>
<td>2701</td>
<td>3031</td>
<td>3481</td>
<td>4140</td>
</tr>
</tbody>
</table>

**Intruder Airspeed: 160 knots**

<table>
<thead>
<tr>
<th></th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>810</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>810</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>810</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>810</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>1169</td>
<td>1559</td>
<td>1934</td>
<td>2361</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>1149</td>
<td>1601</td>
<td>1975</td>
<td>2357</td>
<td>2750</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>800</td>
<td>1642</td>
<td>2041</td>
<td>2418</td>
<td>2823</td>
<td>3267</td>
</tr>
<tr>
<td>30°</td>
<td>849</td>
<td>1645</td>
<td>2131</td>
<td>2524</td>
<td>2886</td>
<td>3362</td>
<td>3924</td>
</tr>
<tr>
<td>40°</td>
<td>1455</td>
<td>2177</td>
<td>2660</td>
<td>3227</td>
<td>3601</td>
<td>3972</td>
<td>4624</td>
</tr>
</tbody>
</table>

**Intruder Airspeed: 180 knots**

<table>
<thead>
<tr>
<th></th>
<th>-20°</th>
<th>-10°</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>-30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>-20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>-10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>0°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>10°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>20°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>30°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>40°</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
</tbody>
</table>
Appendix B

Alerting Logic in Pseudocode

The alerting logic was originally written in C code. What follows is a generalized version of that original code with comments signifying the processing which is occurring. The variables which appear as text are defined similarly to those which appear in the body of this thesis.

```c
if (x > 0.0) { /* intruder between runways? */
    psi = psi;
    phi = phi;
}
else { /* Otherwise, change signs */
    psi = (-1)*psi;
    phi = (-1)*phi;
}
t = 0;
y_curve = 0;
rangelimit = find_max_range(); /* find range limit from table */
range = sqrt(x*x+y*y); /* compute intruder's actual range */
if (range < rangelimit) { /* within range? */
    if (fabs(phi) < 0.001) { /* for turn rates close to zero */
        if (psi < 0.0) { /* and small heading deviation */
            hit = FALSE; /* set hit flag to false */
        }
        t = x/(v*sin(psi)); /* compute t_c */
        y_curve = vown*t - v*cos(psi)*t; /* determine y_curve */
    }
    else { /* for non-zero turn rates */
        r = (v*v/(32.2*tan(phi)))); /* compute turn radius */
        if (phi > 0.0 && x > (r + r*cos(psi))) { /* small radius? */
            hit = FALSE; /* set hit flag to false */
        }
        else if (phi < 0.0 && x > (-r + r*cos(psi))) { /* banked away? */
            hit = FALSE; /* then set hit flag to false */
        }
    }
else { /* Otherwise */
    if ((psi>0.0) || (phi>0.0)) { /* If heading or bank positive */
        psidot = v/r; /* compute rate of heading change */
        t = (acos(-x/r + cos(psi)) - psi)/psidot; /* compute t_c */
        y_curve = vown*t - r*(sin(psidot*t + psi) - sin(psi));
    }
else /* Otherwise, attitude is away from own aircraft */
    */
```
hit = FALSE; /* set hit flag to false */

} 

if (fabs(y - y_curve) <= 800 ft) /* check distance to */
    hit = TRUE; /* collision curve */
} else
    hit = FALSE;

if (hit == TRUE) { /* if two conditions are satisfied, issue alert */