Effects of Compressor Face Boundary Conditions on the Dynamic Behavior of Supersonic Inlets

by

Jackkrit Thammavichai

B.S. Aeronautical Engineering and Engineering Sciences
United States Air Force Academy, 1994

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics

at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1996

©1996, Jackkrit Thammavichai. All rights reserved.
The author hereby grants to MIT permission to reproduce and to distribute publicly paper and electronic copies of this thesis document in whole or in part.

Author

Department of Aeronautics and Astronautics
May 24, 1996

Certified by

Dr. Choon S. Tan
Principal Research Engineer
Thesis Supervisor

Professor Edward M. Greitzer
H.N. Slater Professor of Aeronautics and Astronautics
Thesis Supervisor

Accepted by

Professor Harold Y. Wachman
Chairman, Department-Graduate Committee

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUN 11 1996
Effects of Compressor Face Boundary Conditions on the Dynamic Behavior of Supersonic Inlets

by

Jackkrit Thammavichai

Submitted to the Department of Aeronautics and Astronautics on May 24, 1996, in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics

Abstract

A computational study has been conducted to examine the effects of compressor face boundary conditions on the performance of supersonic inlets. A time-dependent compressor model was developed for use as an inlet exit boundary condition in the two-dimensional simulation of flow in a Mach 2.5 mixed-compression supersonic inlet. Computed results using the compressor model are compared with those using either uniform and fixed static pressure or uniform and fixed Mach number inlet exit boundary condition. The steady supersonic inlet performance metrics are total pressure recovery and stagnation pressure distortion at the compressor face. For unsteady flow comparisons, the figures of merit are transient movement of the terminal shock and static pressure history at the compressor face.

No significant difference was found in the computed steady performance characteristics using the three boundary conditions. The total pressure recoveries at the compressor face were all within 0.5% in mass-weighted-average dynamic head. However, the type of boundary condition imposed at the compressor face did have an impact on the unsteady performance of supersonic inlet. For an internal disturbance associated with a 4.0% decrease in the compressor face corrected mass flow, the terminal shock location, which is linked to the inlet unstart tolerance, using either uniform static pressure or uniform Mach number boundary condition differed by more than 25.0% compared to the results with compressor model. This trend also occurred when supersonic inlet was subjected to an internal disturbance associated with bleed operation or a freestream disturbance corresponding to a 5.0% increase in axial velocity.

Thesis Supervisor: Dr. Choon S. Tan
Title: Principal Research Engineer

Thesis Supervisor: Professor Edward M. Greitzer
Title: H.N. Slater Professor of Aeronautics and Astronautics
This thesis is dedicated to my family,
with all my love.
Acknowledgement

I would like to thank Dr. Choon S. Tan and Prof. Edward M. Greitzer for their help and support throughout the entire course of my study here at MIT. This thesis would not have been possible without their valuable advice, as well as their encouragement.

Thank all of my friends in GTL, especially in 31-255. Without you guys, life here in GTL can be very boring.

For all my friends at TSMIT and TSAB, especially P' Yoy, P' Joh, P' Gorn, P' Pep, P' Boy, P' Pui, P' Bun, P' It, Ging, Gon, Jor+, etc., I cannot find any words to describe how much you all mean to me. Thanks for great support and wonderful friendship.

My scholarship is sponsored entirely by the Royal Thai Air Force.

This achievement would not have been possible without my previous education at United States Air Force Academy, Virginia Military Institute, Royal Thai Air Force Academy, and Debsirin High School.

I cannot be where I am right now without support and encouragement from my wonderful family, especially my father who will always be there for me and my mother for whom I always have a place in my heart.

Finally, I wish to express my gratitude to his majesty King Bhumiphol, the great king of Thailand, for his majesty’s wonderful role model. His majesty is always there with the Thai people, sharing our sorrow and happiness. I am so proud to be Thai under his majesty’s royal guidance. Long Live The King.
Contents

1 Introduction  
1.1 Background ........................................ 12  
1.2 Problem Definition ................................... 13  
1.3 Objectives and Motivation ............................. 13  
1.4 Research Contribution ................................ 14  
1.5 Approach .............................................. 15  
1.6 Organization ........................................... 16

2 Literature Review on the Use of Inlet Exit Boundary Condition for Compressor Representation  
2.1 Introduction .......................................... 18  
2.2 Uniform Static Pressure Inlet Exit Boundary Condition ... 19  
2.3 Uniform Mach Number Inlet Exit Boundary Condition .... 20  
2.4 Supersonic Inlet as Part of Propulsion System .......... 21  
2.5 Experimental Supported Inlet Exit Boundary Condition ... 21  
2.6 Summary ................................................ 22

3 Governing Equations  
3.1 Introduction ........................................... 23  
3.2 Two-Dimensional Euler Equations ......................... 23  
3.3 Non-Dimensionalization ................................ 24  
3.4 Boundary Conditions ................................... 25

4 Numerical Algorithm ...................................... 28
4.1 Introduction ................................ 28
4.2 Spatial Discretization ........................... 28
4.3 Artificial Viscosity - Smoothing .................... 29
4.4 Time Integration ................................. 30
4.5 Boundary Condition Implementation .............. 32

5 Results and Discussions ............................ 35
5.1 Introduction ................................ 35
5.2 Steady State Flow Properties at the Compressor Face .... 35
  5.2.1 Results ........................................... 36
  5.2.2 Summary ...................................... 36
5.3 Supersonic Inlet Dynamic Behavior Due to Internal Disturbances .. 37
  5.3.1 Results for an Instantaneous Change in the Compressor Face
      Corrected Mass Flow ............................... 37
  5.3.2 Results for the Operation of Bleed System ............ 39
  5.3.3 Summary ...................................... 41
5.4 Supersonic Inlet Dynamic Behavior Due to Freestream Disturbances . 42
  5.4.1 Results for Freestream Disturbances with \( \beta \ll 1 \) .......... 43
  5.4.2 Results for Freestream Disturbances with \( \beta \gg 1 \) .......... 44
  5.4.3 Summary ...................................... 44
5.5 Results Summary .................................... 45

6 Conclusions and Recommendations for Future Research .................. 64
  6.1 Research Summary ................................ 64
  6.2 Conclusions ..................................... 64
  6.3 Recommendations for Future Research .................. 65

A Development of Compressor Model .................................. 66
List of Figures

1-1 Schematic diagram of the 120x20 structured grid for Mach 2.5 mixed-compression supersonic inlet ................................................................. 17

5-1 Static pressure distribution along the axial direction at the mid span for the terminal shock location of x = 3.6 ........................................ 47

5-2 Mach number distribution along the axial direction at the mid span for the terminal shock location of x = 3.6 ........................................ 48

5-3 Total pressure distribution along the axial direction at the mid span for the terminal shock location of x = 3.6 ........................................ 49

5-4 Distribution of flow properties along the axial direction at the mid span in the region close to the compressor for the terminal shock location of x = 3.6 ................................................................. 50

5-5 Total pressure distribution at the compressor face in terms of freestream total pressure for the terminal shock location of x = 3.6 ............... 51

5-6 Total pressure distribution at the compressor face in terms of total pressure difference with respect to average dynamic head for the terminal shock location of x = 3.6 ........................................ 52

5-7 Total pressure distribution at the compressor face in terms of freestream total pressure for the terminal shock location of x = 3.2 ............... 53

5-8 Total pressure distribution at the compressor face in terms of total pressure difference with respect to average dynamic head for the terminal shock location of x = 3.2 ........................................ 54
5-9 Transient movement of the terminal shock due to an instantaneous decrease in the compressor face corrected mass flow .......................... 55
5-10 Static pressure history at the mid span of the compressor face due to an instantaneous decrease in the compressor face corrected mass flow 56
5-11 Transient movement of the terminal shock due to the bleed operation 57
5-12 Static pressure history at the mid span of the compressor face due to the bleed operation .......................................................... 58
5-13 Temporal structures of freestream disturbances .......................... 59
5-14 Transient movement of the terminal shock due to freestream disturbances with $\beta \ll 1$ (quasi-steady) ................................. 60
5-15 Static pressure history at the mid span of the compressor face due to freestream disturbances with $\beta \ll 1$ (quasi-steady) ..................... 61
5-16 Transient movement of the terminal shock due to freestream disturbances with $\beta \gg 1$ (unsteady) ................................. 62
5-17 Static pressure history at the mid span of the compressor face due to freestream disturbances with $\beta \gg 1$ (unsteady) ..................... 63
List of Tables

5.1 Steady state flow properties at the compressor face for the terminal shock location of $x = 3.6$ ........................................ 46
5.2 Steady state flow properties at the compressor face for the terminal shock location of $x = 3.2$ ........................................ 46
Nomenclature

A  area of each cell
b  intersecting point of compressor characteristic map
c  speed of sound
cp  specific heat at constant pressure
Cd  discharge coefficient
D  artificial viscosity
eC  compressor polytropic efficiency
E  internal energy per unit volume
fu,fv  external forces
F,G  fluxes in Cartesian coordinates
H  total enthalpy
L  length scale
m  slope of compressor characteristic map
ṁ  mass flow
M  Mach number
n̂  normal vector
P  pressure
q  dynamic head
Q  source term
r  radial direction
R  total flux
S  pressure switch
t  time
T  temperature
u,v  velocity components in Cartesian coordinates
U  conservative variables
Wf  work performed by external forces
x,y  Cartesian coordinates

Greek

α  angle of attack, constant in time integration
β  reduced frequency
Δ  change in quantity
Δt  explicit time step
γ  specific heat ratio
λ  CFL number
ν₂  second-difference smoothing coefficient
ν₄  fourth-difference smoothing coefficient
$\pi$ compressor total pressure ratio
$\rho$ density
$\sigma$ safety factor
$\tau$ compressor total temperature ratio

**Superscript**

$I$ non-dimensional quantity
$0,1,2,3,4$ stages in time integration
$n$ time index

**Subscript**

bleed bleed boundary
cf compressor face
corr corrected quantity
ff farfield boundary
inflow inflow boundary
$j,k$ cell index
max maximum quantity
min minimum quantity
$n$ normal direction, compressor stage index
off off-design condition
on on-design condition
outflow outflow boundary
s static condition
t tangential direction, total condition
wall wall boundary
x x component
y y component
$\infty$ freestream condition
Chapter 1

Introduction

1.1 Background

The development of propulsion systems for new supersonic aircrafts, such as high-speed civil transports and multi-role fighters, requires an understanding of the flow characteristics in the mixed-compression supersonic inlet. The primary function of a mixed-compression supersonic inlet is to capture and decelerate a supersonic freestream flow to a subsonic condition through several shock wave interactions which terminate in a normal shock. For the propulsion system to work efficiently, the inlet total pressure recovery should be as high as possible. The inlet should also be able to provide a uniform flow to the downstream compressor. Furthermore, the inlet internal aerodynamics must be such that the inlet shock system does not have a detrimental impact on inlet flowfield stability and inlet-engine flow matching. Poor inlet design can cause decreased engine performance, reduced stable flow range of the compressor, and increased risk of an inlet unstart, a condition for which the normal shock moves upstream of the inlet throat. Maximizing total pressure recovery, minimizing stagnation pressure distortion, and enhancing inlet unstart tolerance can thus be used as the figures of merit for characterizing supersonic inlet performance.

One design goal is for a mixed-compression supersonic inlet to stabilize the normal shock just downstream of the throat where the Mach number is slightly greater than
one. This condition minimizes the total pressure loss across the normal shock, but the risk of an inlet unstart due to a transient in the freestream flow or an engine transient increases. One approach to increase unstart tolerance is to stabilize the terminal shock further away from the inlet throat, resulting in higher loss across the shock. The requirements of high supersonic inlet performance and high unstart tolerance are thus in conflict. Inlet unstart tolerance can also be enhanced by increasing the bleed rate in the vicinity of inlet throat, but this approach still shares the same penalties on supersonic inlet performance as the previous one.

1.2 Problem Definition

The problem addressed in this research is that of the appropriate compressor face boundary condition for use in the analysis of the unsteady performance of supersonic inlet. Thus far, simple compressor face boundary conditions have been used to represent the inlet-compressor interface. Previous research has shown that the imposed inlet exit boundary condition had an impact on the dynamic behavior of supersonic inlet, but it did not clarify in specific terms the inadequacy associated with each boundary condition to represent the compressor influence. Additional work is thus needed to assess what constitutes a physically consistent inlet exit boundary condition for analyzing inlet-compressor transient interaction.

1.3 Objectives and Motivation

The main objectives of this research are:

1. To conduct a literature search and a critical review on the issue of the appropriate compressor face boundary condition for analyzing the unsteady performance of supersonic inlet.

2. To develop a simple compressor model, appearing as the axial forces per unit volume acting on fluid, for application to the unsteady two-dimensional simulation of flow in supersonic inlet.
3. To assess the computed steady and unsteady results obtained from the use of compressor model against those using uniform and fixed static pressure and uniform and fixed Mach number as inlet exit boundary conditions in terms of

(a) steady two-dimensional effects of the compressor model on the flow properties at the compressor face and,

(b) inlet-compressor transient interaction and quasi one-dimensional effects on the unsteady behavior of supersonic inlet in response to internal or freestream disturbances.

As alluded to in Section 1.4, accomplishing these objectives would serve to clarify what constitutes a physically realistic compressor face boundary condition for use in the unsteady supersonic inlet flow analysis.

1.4 Research Contribution

To analyze the unsteady behavior of supersonic inlet, one needs to formulate an inlet exit boundary condition which can realistically represent the dynamic response characteristics of compressor. Although some experimental work has been done on the transient propulsion system behavior, none provided data which could be used in the formulation of such an inlet exit boundary condition.

In the past, the assumptions of constant static pressure, constant mass flow, constant volumetric flow, and constant Mach number at the compressor face have been used for supersonic inlet flow computations. In this dissertation, the boundary conditions considered are uniform static pressure and uniform Mach number boundary conditions. The uniform static pressure boundary condition assumes that the compressor face static pressure is uniform and fixed in time. In contrast, the uniform Mach number boundary condition allows static pressure at the compressor face to adjust in a manner to maintain a uniform and fixed Mach number condition.
A goal of this research is to develop a simple compressor model that serves as a physically consistent inlet exit boundary condition for unsteady two-dimensional simulation of flow in supersonic inlet. The use of such a model relieves the need for any assumption to be made concerning the flow conditions at the compressor face. All flow properties at the compressor face are allowed to adjust in a physically consistent manner.

1.5 Approach

A simple compressor model is developed for use in the unsteady supersonic inlet flow analysis. The preliminary repeating row/repeating stage compressor design, described in Mattingly’s *Elements of Gas Turbine Propulsion* [10], is used for the compressor model development. The compressor characteristic used in this study is generic, and not based on any existing compressor. The detailed description of the compressor model development is presented in Appendix A.

The compressor model is used in the unsteady two-dimensional simulation of flow in a Mach 2.5 mixed-compression supersonic inlet. The geometry of the supersonic inlet used is also generic. Figure 1-1 shows the schematic diagram of the 120x20 structured grid for this generic Mach 2.5 mixed-compression supersonic inlet.

Unsteady simulations of flow in supersonic inlet are performed using a time-accurate two-dimensional Euler solver. Three different system configurations used in this research are an isolated inlet with uniform and fixed static pressure inlet exit boundary condition, an isolated inlet with uniform and fixed Mach number inlet exit boundary condition, and an integrated inlet-compressor system. The numerical simulations obtained from these configurations are used to determine the effects of inlet-compressor fluid dynamic coupling on steady and unsteady performance of supersonic inlet. The figures of merit used in the comparison of steady flow properties are total pressure recovery and average stagnation pressure distortion at the compressor face. For inlet dynamic behavior, transient movement of the terminal shock
and static pressure history at the compressor face are used as metrics.

1.6 Organization

There are six chapters in this thesis.

An introduction chapter begins with the description of technical background. It then states the problem definition, delineates the objectives and motivation, summarizes the research contribution, and finally presents the approach taken.

A literature search and a critical review on the subject of compressor face boundary condition for use in the analysis of the unsteady performance of supersonic inlet are presented in Chapter 2.

Chapter 3 describes the details of the governing equations and the associated boundary conditions used in the numerical simulation of flow in supersonic inlet.

In Chapter 4, the cell-centered finite volume scheme with a four-stage Runge-Kutta time integration algorithm for numerical solutions of Euler equations is presented.

Numerical results obtained from the use of three inlet exit boundary conditions and their physical interpretation are presented in Chapter 5. Specific aspects of supersonic inlet flowfield examined are: (i) steady state flow properties at the compressor face, (ii) inlet dynamic behavior due to a change in the compressor face corrected mass flow or the operation of bleed system, and (iii) effects of freestream disturbances on the transient behavior of supersonic inlet.

Chapter 6 summarizes the research and reports the main results. Recommendations for future research are suggested.

Finally, in Appendix A, the technique used to develop the compressor model for use in the unsteady supersonic inlet flow analysis is described in detail.
Figure 1-1: Schematic diagram of the 120x20 structured grid for Mach 2.5 mixed-compression supersonic inlet
Chapter 2

Literature Review on the Use of Inlet Exit Boundary Condition for Compressor Representation

2.1 Introduction

Although wind tunnel testing has been a primary tool in inlet designs, its high cost has led to the extreme use of computational fluid dynamics (CFD) to obtain critical flowfield information. In recent years, a number of CFD techniques have been used to examine this topic [2, 3, 4, 5, 6, 11, 12, 14]. There has been work related to improving the boundary condition associated with bleed operation and the inlet exit boundary condition for use in the unsteady flow analysis of mixed-compression supersonic inlet. This chapter reviews work on the effects of compressor face boundary conditions on the dynamic behavior of supersonic inlets.
2.2 Uniform Static Pressure Inlet Exit Boundary Condition

Benson and McRae [2] used the uniform and fixed static pressure boundary condition to analyze the unsteady behavior of supersonic inlet when subjected to freestream disturbances. They investigated how the type of freestream disturbances, which were caused by finite-rate changes in temperature and onset angle, affected the inlet transient behavior. For sub-critical disturbances, that did not induce an inlet unstart, the inlet transient responses were similar for both types of perturbations. In contrast, the dynamic responses of supersonic inlet were different depending on the perturbation type if freestream disturbances were super-critical, causing an inlet unstart.

For this inlet exit boundary condition, neither radial nor temporal variation of static pressure was allowed at the compressor face. Since static pressure at the compressor face could change radially and temporally in response to an engine transient or a transient in the freestream flow, this type of compressor face boundary condition was not a physically realistic representation of the transient interaction between inlet and compressor. Thus, it was not adequate for the unsteady supersonic inlet flow analysis.

To improve the analysis of the unsteady performance of supersonic inlet, Mayer and Paynter [11] imposed uniform static pressure, but allowed for temporal variation, as boundary condition at the compressor face. At each time step, the compressor face static pressure adjusted based on a mass-weighted-average total pressure and an input compressor face Mach number. The authors used this new compressor face boundary condition to investigate the inlet dynamic behavior due to an engine transient. Compared to the uniform and fixed static pressure compressor face boundary condition, this boundary condition was less reflective. Instead of being instantaneously changed to the new specified value, the compressor face static pressure gradually adjusted towards steady state condition.

Mayer and Paynter [12] also developed an Euler analysis procedure to simulate
the unsteady inlet flowfield caused by a transient in the freestream flow. This procedure could be used to predict the unstart tolerance of supersonic inlet when subjected to instantaneous freestream disturbances. However, since the inlet would experience temporally more gradual perturbations in practice, the procedure provided only qualitative prediction for the inlet unstart tolerance. The authors suggested that during a transient caused by short scale freestream disturbances, a constant volumetric flow at the compressor face was a physically realistic representation of the inlet-compressor interface. The inlet exit boundary condition previously developed was modified in such a way that the constant volumetric flow could be imposed at the compressor face boundary. The authors stated that perturbation due to an increase in freestream temperature was the most likely cause of inlet unstart.

Like its predecessor, however, the modified inlet exit boundary condition still assumed uniform static pressure at the compressor face. During transients, the inlet-compressor interaction may allow for radial variation of the compressor face static pressure. Assuming that static pressure was radially uniform at the compressor face, this boundary condition may be too specific to simulate generic inlet-compressor fluid dynamic coupling.

### 2.3 Uniform Mach Number Inlet Exit Boundary Condition

With experimental data showing a nearly uniform Mach number profile at the compressor face during transients, Chung and Cole [4] developed an inlet exit boundary condition based on the uniform and fixed Mach number condition. Imposing this boundary condition, the reflective nature of the transient response of supersonic inlet flow simulation decreased substantially. Disturbances traveling downstream were able to leave the computational domain at the compressor face boundary.

Although the authors claimed that computed results obtained with this boundary condition were in qualitative agreement with experiments, the data was based on an
engine with a relatively uniform Mach number profile at the compressor face. The boundary condition may thus be too specific to represent generic inlet-compressor interface. Furthermore, in an actual flow, all the compressor face flow properties adjusted during transients and it was unknown what effect this flow adjustment had. This question could be resolved by coupling an unsteady supersonic inlet flow simulation to a two-dimensional time dependent compressor model.

2.4 Supersonic Inlet as Part of Propulsion System

As shown in the work of Mayer and Paynter [11, 12] and Chung and Cole [4], the computed unsteady performance of a supersonic inlet was dependent on the type of boundary condition imposed at the compressor face. Clark [6] compared unsteady one-dimensional supersonic inlet flow simulations using several inlet exit boundary conditions to those using a model of an entire propulsion system. While subjected to the same type of freestream disturbances, each configuration generated different transient response. As a result of the lack of experimental data for comparison, it was not exactly known which configuration yielded the most physically realistic results.

2.5 Experimental Supported Inlet Exit Boundary Condition

Sajben and Freund [15] attempted to develop a new inlet exit boundary condition from experimental data. The authors expected the outcome of the ongoing research to be an experimentally supported boundary condition capable of predicting the transient response of compressor to short-time-scale acoustic disturbances. By integrating this short-time-scale compressor characteristics with the well understood long-time-scale compressor responses, all important aspects of the dynamic response characteristics of compressor could be described. This should result in a more realistic representa-
tion of the inlet-compressor transient interaction. While experimental work of this type would be useful for developing compressor face boundary condition on a sound physical basis, one should not obviate the usefulness of CFD tools to accomplish this exact same task.

2.6 Summary

Based on the literature search and critical review of the subject, currently used inlet exit boundary conditions are too specific to represent generic interaction between inlet and compressor during transients. Use of inlet exit boundary condition to simulate the compressor influence on inlet dynamic behavior may yield inadequate results if the assumption used in that boundary condition is violated. To formulate a generic and physically realistic boundary condition, inlet and compressor should be considered as an integrated system. During transients, inlet and compressor interact with each other. By considering inlet and compressor as individual component, the effects of the inlet-compressor dynamic flow coupling are neglected in the unsteady supersonic inlet flow analysis. Therefore, to obtain more physically realistic results, the unsteady numerical simulation of flow in supersonic inlet should incorporate the effects of the inlet-compressor transient interaction.
Chapter 3

Governing Equations

3.1 Introduction

This chapter describes the set of equations that govern the flow in a two-dimensional mixed-compression supersonic inlet and the associated boundary conditions. It also describes the non-dimensionalization of the flow variables and the governing equations.

3.2 Two-Dimensional Euler Equations

The unsteady two-dimensional Euler equations for an inviscid, compressible, and non-heat-conducting fluid, in conservative form and in the absolute frame of reference, can be written as

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{Q}. \tag{3.1}
\]

The conservative variables \( \mathbf{U} \) and the Cartesian fluxes \( \mathbf{F} \) and \( \mathbf{G} \) are given by

\[
\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho uH \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho vH \end{pmatrix}, \tag{3.2}
\]
where $\rho$ is density, $u$ and $v$ are velocity components in the x and y direction respectively, $P$ is static pressure, $E$ is internal energy per unit volume, and $H$ is total enthalpy.

The source term $Q$ is given by

$$Q = \begin{pmatrix} 0 \\ \rho f_u \\ \rho f_v \\ W_f \end{pmatrix},$$

(3.3)

where $f_u$ and $f_v$ are external forces in the x and y direction respectively, and $W_f$ is work performed by these external forces $W_f = \rho[(f_u u) + (f_v v)]$.

Assuming a perfect gas with constant specific heat ratio $\gamma$ (for air $\gamma = 1.4$), the equation of state can be used to relate static pressure $P$ to density $\rho$, velocity components $u$ and $v$, and internal energy $E$,

$$P = (\gamma - 1)[\rho E - \frac{1}{2} \rho (u^2 + v^2)].$$

(3.4)

This relation defines total enthalpy $H$ as

$$H = E + \frac{P}{\rho}.$$  

(3.5)

3.3 Non-Dimensionalization

The governing equations will be presented in dimensionless form as described in the following.

Use of freestream static density $\rho_\infty$, speed of sound $c_\infty$, and length scale $L$ allows
one to write the flow quantities in dimensionless form as follows:

\[
\begin{align*}
    x' &= \frac{x}{L} \quad y' = \frac{y}{L} \quad t' = \frac{t}{L/c_\infty} \\
    u' &= \frac{u}{c_\infty} \quad v' = \frac{v}{c_\infty} \quad \rho' = \frac{\rho}{\rho_\infty} \\
    P' &= \frac{P}{\rho_\infty c_\infty^2} \quad E' = \frac{E}{\rho_\infty c_\infty^2} \quad H' = \frac{H}{\rho_\infty c_\infty^2}.
\end{align*}
\] (3.6)

The freestream conditions are now given as:

\[
\begin{align*}
    \rho'_\infty &= 1 \quad P'_\infty = \frac{1}{\gamma} \\
    u'_\infty &= M_\infty \cos \alpha \quad v'_\infty = M_\infty \sin \alpha \quad c'_\infty = 1 \\
    E'_\infty &= \frac{1}{\gamma(\gamma-1)} + \frac{M_\infty^2}{2} \quad H'_\infty &= \frac{1}{\gamma-1} + \frac{M_\infty^2}{2},
\end{align*}
\] (3.7)

where \( M_\infty \) is freestream Mach number and \( \alpha \) is angle of attack.

For convenience, the primes are dropped and all flow quantities are taken to be dimensionless.

### 3.4 Boundary Conditions

To obtain a solution for the Euler equations, proper boundary conditions must be imposed. The boundary conditions to be imposed in this study are at inflow boundary, farfield boundary, wall boundary, bleed boundary, and outflow (compressor face) boundary.

**Inflow Boundary**

Since the incoming flowfield is supersonic, all flow properties at the inflow boundary will be extrapolated from the freestream values.

**Farfield Boundary**

Due to the supersonic outflow condition, all flow variables at the farfield boundary can be extrapolated from the computational domain. A zero gradient extrapolation is used for this process.
Wall Boundary

For solid walls, there must be no flux through the wall surface. As the inlet flow is assumed inviscid, the non-slip boundary condition is not applied.

Bleed Boundary

The flow through the bleed opening is assumed to be choked. The type of bleed boundary condition used in this study, taken from Chyu et al. [5], models the normal velocity at the bleed opening as

\[ v_n = C_d \sqrt{\frac{\gamma P}{\rho}}, \]  

(3.8)

where \( C_d \) is discharge coefficient and taken to be 0.07 for critical flow.

Outflow (Compressor Face) Boundary

Two types of the outflow boundary conditions, uniform static pressure and uniform Mach number, are used in this study. Characteristic analysis of the one-dimensional Euler equations normal to the boundary, as described in detail in References [1] and [8], shows that only one boundary condition must be specified at the outflow boundary because of the subsonic outflow condition.

For the uniform static pressure boundary condition, only static pressure \( P \) at the outflow boundary is specified, while the other three variables \((u,v,\rho)\) are extrapolated from the computational domain.

On the other hand, for the uniform Mach number boundary condition which allows for radial variation of static pressure, the procedures of imposing the required boundary condition are as follow:

1. Calculate the local total pressure \( P_t(\tau) \) from the upstream point value.

2. Extrapolate \( u,v,\rho \) from the computational domain.
3. Calculate the local static pressure $P_s(r)$ from the isentropic relation

$$
P_s(r) = \frac{P_i(r)}{(1 + \frac{\gamma-1}{2} M_{cf}^2)^{\frac{\gamma}{\gamma-1}}},$$  \hspace{1cm} (3.9)

where $M_{cf}$ is the specified compressor face Mach number.
Chapter 4

Numerical Algorithm

4.1 Introduction

The numerical technique used to develop the Euler solver is a cell-centered finite volume scheme with a four-stage Runge-Kutta time integration. This algorithm was developed by Jameson, Schmidt, and Turkel [9]. To perform the integration of the Euler equations, the method first discretizes the spatial derivatives to obtain a system of coupled ordinary differential equations. Then, time integration is performed on these equations using a linear multistage scheme. In addition to spatial discretization and time integration, this chapter also describes the use of artificial viscosity and the implementation of boundary conditions.

4.2 Spatial Discretization

The governing equations are discretized in space using the cell-centered finite volume approach. For the cell-centered algorithm, the area of interest is discretized into non-overlapping cells, each of which stores the conservative variables \((\rho, \rho u, \rho v, \rho E)\) at its center. The semi-discrete Euler equations, for any cell \((j,k)\), can be written as

\[
A_{j,k} \frac{d}{dt} U_{j,k} + R_{j,k} = A_{j,k} Q_{j,k},
\]

(4.1)
where $A$ is the area of each cell and

$$R_{j,k} = \sum_{edges} (F \Delta y - G \Delta x)_{j,k}. \quad (4.2)$$

To obtain the correct shock jump relation and shock position, the spatial discretization scheme must be conservative. For a Cartesian grid, this type of spatial discretization is equivalent to the second order central differencing.

### 4.3 Artificial Viscosity - Smoothing

Many discrete approximations to the Euler equations have problem concerning odd-even oscillations. Artificial viscosity or smoothing in the form of fourth order difference can be added to suppress this type of oscillation.

In addition, a second order artificial viscosity is added to prevent the Gibbs phenomenon associated with presence of shock waves. However, second order smoothing introduces first order errors. Away from shock waves, therefore, the second order artificial viscosity is turned off by a pressure switch, which is in the form of second order difference of pressure.

With the artificial viscosity $D_{j,k}$, the semi-discrete Euler equations can be written as

$$A_{j,k} \frac{d}{dt} U_{j,k} + R_{j,k} = A_{j,k} Q_{j,k} + D_{j,k}, \quad (4.3)$$

where

$$D_{j,k} = \nu_2 [\delta_x (S_x \frac{A}{\Delta t} \delta_x U_{j,k}) + \delta_y (S_y \frac{A}{\Delta t} \delta_y U_{j,k})] - \nu_4 [\delta_x (\frac{A}{\Delta t} \delta_x^2 U_{j,k}) + \delta_y (\frac{A}{\Delta t} \delta_y^2 U_{j,k})]. \quad (4.4)$$

In the above equation, $S_x$ and $S_y$ are pressure switches in the $x$ and $y$ direction respectively, $\nu_2$ and $\nu_4$ are non-dimensional constants, whose values depend on the form of pressure switch $S$, and $\Delta t$ is time step size. In this research, the values of $\nu_2$ and $\nu_4$ are set to be 1.5 and 0.025 respectively. The factor $A/\Delta t$ is needed because
the artificial viscosity has its inverse as a multiplication factor in the linear multistage

time integration.

The pressure switches $S_z$ and $S_y$ are given by

$$S_z = \left| \frac{\delta^2 P}{\partial x^2} \right| = \left| \frac{P_{j+1,k} - 2P_{j,k} + P_{j-1,k}}{P_{j+1,k} + 2P_{j,k} + P_{j-1,k}} \right|, \quad S_y = \left| \frac{\delta^2 P}{\partial y^2} \right| = \left| \frac{P_{j,k+1} - 2P_{j,k} + P_{j,k-1}}{P_{j,k+1} + 2P_{j,k} + P_{j,k-1}} \right|. \quad (4.5)$$

To maintain conservative spatial discretization, the pressure switch $S$, the cell area $A$, and the time step size $\Delta t$ used in the calculation of the artificial viscosity $D_{j,k}$ must be calculated at the faces. The average values of these quantities obtained from those of the two cells sharing the face will be used. This technique of calculating the artificial viscosity is valid for all faces, except those on the edge of the computational domain and those next to the domain edge. For the faces on the edge of the computational domain, the artificial viscosity is set equal to 0.0. To determine the artificial viscosity of the faces located next to the edge of the computational domain, a second order extrapolation is used to obtain the required information.

Finally, $D_{j,k}$ is brought over to the other side of the equation and added to the total flux terms to be summed over all faces. The modified semi-discrete equations can then be written as

$$A_{j,k} \frac{d}{dt} U_{j,k} + R'_{j,k} = A_{j,k} Q_{j,k}, \quad (4.6)$$

where

$$R'_{j,k} = \sum_{edges} (F \Delta y - G \Delta x - D)_{j,k}. \quad (4.7)$$

### 4.4 Time Integration

Time integration is implemented through the use of the four-stage Runge-Kutta time-stepping scheme. After each stage, the sum of the modified fluxes around the faces of each cell must be updated. However, the artificial viscosity $D$ and the source term $Q$ are evaluated only at the beginning of each time integration so as to reduce the required computations.
Given the solutions at time step \( n \), the new solutions at time step \( n+1 \) are obtained using the four-stage Runge-Kutta time integration as follows:

\[
\begin{align*}
U^0 &= U^n \\
U^1 &= U^0 - \alpha_1 \frac{\Delta t}{A}[R'(U^0)] + \alpha_1 \Delta tQ \\
U^2 &= U^0 - \alpha_2 \frac{\Delta t}{A}[R'(U^1)] + \alpha_2 \Delta tQ \\
U^3 &= U^0 - \alpha_3 \frac{\Delta t}{A}[R'(U^2)] + \alpha_3 \Delta tQ \\
U^4 &= U^0 - \alpha_4 \frac{\Delta t}{A}[R'(U^3)] + \alpha_4 \Delta tQ \\
U^{n+1} &= U^4
\end{align*}
\]  

(4.8)

where \( \alpha_1, \alpha_2, \alpha_3, \) and \( \alpha_4 \) for the four-stage Runge-Kutta scheme are \( \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \) and 1 respectively.

A cell's local time step \( \Delta t \) is given by

\[
\Delta t = \frac{\sigma \lambda \Delta x \Delta y}{|u|\Delta y + |v|\Delta x + c\sqrt{\Delta x^2 + \Delta y^2}}
\]

(4.9)

where \( c \) is local speed of sound, \( u \) and \( v \) are local velocity components in the \( x \) and \( y \) direction respectively, \( \Delta x \) and \( \Delta y \) are cell dimensions, \( \sigma \) is a safety factor, set equal to 0.9, and \( \lambda \) is the CFL number which is less than or equal to \( 2\sqrt{2} \) for the four-stage Runge-Kutta scheme.

For steady state solutions, a maximum local time step for each cell can be used. To obtain time accurate solutions, the same time step must be used for all cells in the computational domain. To maintain the stability of the scheme, the smallest local time step in the computational domain is used as the time step for all cells. For this study, the explicit time step used for all unsteady supersonic inlet flow simulations is set to a non-dimensional time of 0.01.
4.5 Boundary Condition Implementation

To calculate the Cartesian fluxes $F$ and $G$ and the total flux $(F \Delta y - G \Delta z)$ at the faces on the edge of the computational domain, the boundary conditions described in the previous chapter must be enforced.

**Inflow Boundary**

Due to the supersonic flow at the inflow boundary, the variables at the inflow faces of the computational domain are set equal to the freestream values:

\[
\begin{align*}
  u_{\text{inflow}} &= u_\infty \\
  v_{\text{inflow}} &= v_\infty \\
  \rho_{\text{inflow}} &= \rho_\infty \\
  P_{\text{inflow}} &= P_\infty.
\end{align*}
\]

(4.10)

With this information, the fluxes at the faces on the inflow edge of the computational domain can be calculated from Equations (3.2), (3.3), and (3.5). The total flux is given by $F \Delta y$ because $\Delta z$ for each face on the inflow edge is equal to zero.

**Farfield Boundary**

At the faces on farfield boundary, the dummy cell technique is used to implement the farfield boundary condition. Using a zero gradient extrapolation, the flow quantities at the dummy cells are set equal to those of the first cell inside the computational domain:

\[
\begin{align*}
  u_{ff} &= u_k \\
  v_{ff} &= v_k \\
  \rho_{ff} &= \rho_k \\
  P_{ff} &= P_k,
\end{align*}
\]

(4.11)

where $k$ is 1 for bottom boundary and $k_{\text{max}}$ for top boundary.

Knowing flow variables at the dummy cells, the fluxes for those cells can be calculated. To calculate the total flux at the farfield edge, the average of the fluxes from the dummy cells and their corresponding first cells inside the domain is used.
Wall Boundary

There must be no flux through the solid wall surface. For inviscid flow, the non-slip boundary condition is not applied so that one can use the reflection/dummy cell technique to impose this wall boundary condition. The flow variables at the dummy cells are given by:

\[
\begin{align*}
(\vec{u} \cdot \vec{n})_{wall} &= (\vec{u} \cdot \vec{n})_k \\
(\vec{u} \cdot \vec{t})_{wall} &= (\vec{u} \cdot \vec{t})_k \\
\rho_{wall} &= \rho_k \\
P_{wall} &= P_k,
\end{align*}
\]

(4.12)

where \(\vec{n}\) and \(\vec{t}\) are normal and tangential vectors to the wall respectively, and \(k\) is defined the same way as that in the farfield boundary section.

The fluxes and total flux at the wall boundary can be calculated using the same procedure described in the farfield boundary condition implementation.

Bleed Boundary

At the bleed opening, the flow is allowed to leave the computational domain. The technique used to impose the bleed boundary condition is taken from the work of Chyu et al. [5]. The flow is assumed to be choked through the bleed opening. Like the previous two cases, the dummy cell technique is used to impose this boundary condition, defining the flow variables at the dummy cells as:

\[
\begin{align*}
(\vec{u} \cdot \vec{n})_{bleed} &= C_d \sqrt{\frac{T_k}{\rho_k}} \\
(\vec{u} \cdot \vec{t})_{bleed} &= (\vec{u} \cdot \vec{t})_k \\
\rho_{bleed} &= \rho_k \\
P_{bleed} &= P_k - \rho_k (\vec{u} \cdot \vec{n})_{bleed} [(\vec{u} \cdot \vec{n})_{bleed} - (\vec{u} \cdot \vec{n})_k],
\end{align*}
\]

(4.13)

where \(\vec{n}\), \(\vec{t}\), and \(k\) are defined the same way as that in the wall boundary section, and \(C_d\) is a discharge coefficient.

To determine the fluxes and total flux at the bleed boundary, the procedure described in the implementation of farfield and wall boundary conditions is used.

Outflow (Compressor Face) Boundary

Only one boundary condition must be specified at the subsonic outflow boundary. The other three variables can be extrapolated from the computational domain.
For the uniform static pressure boundary condition, only static pressure at the outflow boundary is specified. The flow quantities at the outflow edge are:

\[ u_{\text{outflow}} = u_{j_{\text{max}}} \quad v_{\text{outflow}} = v_{j_{\text{max}}} \]
\[ \rho_{\text{outflow}} = \rho_{j_{\text{max}}} \quad P_{\text{outflow}} = P_{\text{specified}}. \] (4.14)

For the uniform Mach number boundary condition, the procedure used to impose the required boundary condition is given in Section 3.3. The flow variables at the outflow edge can be defined as:

\[ u_{\text{outflow}} = u_{j_{\text{max}}} \quad v_{\text{outflow}} = v_{j_{\text{max}}} \]
\[ \rho_{\text{outflow}} = \rho_{j_{\text{max}}} \quad P_{\text{outflow}} = P_{j_{\text{max}}} / \left(1 + \frac{\gamma - 1}{2} M_{\text{specified}}^2 \right)^{\frac{\gamma}{\gamma - 1}}. \] (4.15)

The procedure used in the inflow boundary condition implementation is used to determine the fluxes and total flux at the faces on the outflow edge of the computational domain.
Chapter 5

Results and Discussions

5.1 Introduction

Effects of compressor face boundary conditions on the steady and unsteady performance of supersonic inlets are discussed in this chapter. The results obtained from numerical simulation of flow in supersonic inlet using a simple compressor model are assessed against those using either uniform and fixed static pressure or uniform and fixed Mach number inlet exit boundary condition. The supersonic inlet performance used for assessment are steady state flow properties at the compressor face, supersonic inlet dynamic behavior due to internal disturbances, and supersonic inlet dynamic behavior due to freestream disturbances.

5.2 Steady State Flow Properties at the Compressor Face

The first issue considered is the steady state flow properties at the compressor face. For all three system configurations, simulations are performed to place the terminal shock downstream of the inlet throat at \( x = 3.6 \). This location is chosen because it is at the midway between the on- and off-design terminal shock locations of \( x = 3.2 \) and \( x = 4.0 \), which are the assumptions used in the compressor model development.
Since the upstream influence of perturbation generated at the compressor face decays over a length scale of compressor annulus height, the type of inlet exit boundary condition used affects the steady inlet flowfield only in the region close to the compressor. Figures 5-1, 5-2, and 5-3 respectively show the distribution of static pressure, Mach number, and total pressure along the axial direction at the mid span for the three system configurations. Only inlet flowfield in the region close to the compressor is affected by the boundary condition imposed at the compressor face. This can be seen in Figure 5-4, which shows the distribution of flow properties along the axial direction at the mid span in the region close to the compressor face.

5.2.1 Results

The steady flow properties at the compressor face for all three system configurations are shown in Table 5.1. The total pressure distribution at the compressor face is presented in Figure 5-5 in terms of freestream total pressure, and in Figure 5-6 in terms of total pressure difference with respect to mass-weighted-average dynamic head. These results show that with the terminal shock at the same location, the corrected mass flow and the steady flow properties at the compressor face for the three system configuration are approximately the same. The difference in the compressor face total pressure recoveries are within 0.5 % in average compressor face dynamic head. Although the average stagnation pressure distortion coefficient at the compressor face obtained from the configuration using uniform Mach number boundary condition is higher than those of the other two cases, this difference is not significant.

5.2.2 Summary

The computed results show that differences in the steady flow properties at the compressor face for the three system configurations are not significant. All types of compressor face boundary conditions examined here can be used in the analysis of the steady performance of supersonic inlet.
5.3 Supersonic Inlet Dynamic Behavior Due to Internal Disturbances

During flight, supersonic inlet internal disturbances can be generated, for example, due to a change in the corrected mass flow at the compressor face and due to the operation of bleed system. In response to these disturbances, the supersonic inlet has to adjust through the movement of the terminal shock. If the disturbance amplitude is large enough, an inlet unstart may occur.

In this section, the initial conditions are the steady state inlet flowfield obtained from the previous section, in which the terminal shocks are located at $x = 3.6$ for all three system configurations. With the same initial inlet flowfield, it can be seen how the imposed compressor face boundary condition affects the inlet dynamic behavior.

5.3.1 Results for an Instantaneous Change in the Compressor Face Corrected Mass Flow

The first case discussed is the dynamic behavior of supersonic inlet caused by an instantaneous decrease in the compressor face corrected mass flow. This type of engine transient causes a forward movement of the terminal shock. To decrease the corrected mass flow at the compressor face requires an increase in the compressor face static pressure, corresponding to a decrease in the compressor face Mach number. For the compressor model, the operating point must shift to the left, resulting in a higher pressure ratio across the compressor. In the computation, this is implemented by specifying a higher exit static pressure.

Based on the results obtained in the previous section, if the compressor face corrected mass flow for each system configuration is changed by the same amount, the inlet flowfield will adjust to reach approximately the same steady state condition. Since the corrected mass flow at the compressor face and the location of the terminal shock are linked, the control variable used is the movement of the terminal shock.
For each system configuration, the terminal shock is moved from the initial location of \( x = 3.6 \) to a final steady location of \( x = 3.2 \), corresponding to a 4.0 % decrease in the compressor face corrected mass flow. This range of terminal shock movement is chosen such that the assumption used in the compressor model development is not violated. The steady flow properties at the compressor face for the terminal shock location of \( x = 3.2 \) are shown in Table 5.2. The total pressure distribution across the compressor face in terms of freestream total pressure and in terms of total pressure difference with respect to average compressor face dynamic head are presented in Figures 5-7 and 5-8 respectively. As expected, no significant difference exists in the results obtained from the three system configurations.

Results for the supersonic inlet dynamic behavior due to an instantaneous decrease in the corrected mass flow at the compressor face are shown in Figures 5-9 and 5-10. These figures respectively give the transient movement of the terminal shock and the static pressure history at the mid span of the compressor face for all three system configurations. Disturbances generated by this type of engine transient initially cause the static pressure at the compressor face to rise sharply, with the magnitude of this pressure jump depending on the type of boundary condition imposed.

For the uniform and fixed static pressure boundary condition, the solution appears oscillatory. A damped oscillation exists in both the transient movement of the terminal shock and the static pressure history at the compressor face. The oscillation in the transient movement of the terminal shock could cause a false inlet unstart if the magnitude of the change in the compressor face corrected mass flow is large enough, or the final location of the terminal shock is close to the inlet throat.

For the uniform and fixed Mach number boundary condition, the inlet dynamic response shows no oscillation. As shown in Figure 5-10, this type of inlet exit boundary condition allows for a gradual adjustment of static pressure at the compressor face.

For the compressor model, the inlet dynamic behavior shows no overshoot in the
transient movement of the terminal shock. The initial oscillation in static pressure at the compressor face is followed by a gradual adjustment towards steady state condition. This indicates that after an initial oscillatory response caused by an engine transient, the compressor model allows disturbances to be transmitted through and reflected at the compressor.

For the configuration with uniform static pressure boundary condition, the terminal shock reaches the peak overshoot value of 6.9 % after approximately 2.5 flow-through time. At this flow-through time, supersonic inlet is most likely to become unstart. By comparing the terminal shock locations for the three configurations at that critical time, one can determine an inaccuracy associated with the unstart tolerance estimation for the configurations using simple inlet exit boundary conditions. Compared to the system configuration using compressor model, the terminal shock location, which is linked to the inlet unstart tolerance, differs by approximately 26.0 % for the configuration using either uniform static pressure or uniform Mach number boundary condition.

5.3.2 Results for the Operation of Bleed System

The second case considered in the dynamic behavior of supersonic inlet caused by internal disturbances is associated with bleed operation. If properly located between the inlet throat and the terminal shock, the operation of bleed system can be useful for increasing unstart tolerance. In this study, the bleed openings are modeled as single slots, located just downstream of the inlet throat on both the cowl and the centerbody at \( x = 3.025 \) and \( x = 3.125 \). The bleed rate is not specified, but the flow through the opening is assumed to be choked. Like the previous case, the initial condition used for each configuration is the steady state inlet flowfield obtained from the previous section with the terminal shock at \( x = 3.6 \).

Figures 5-11 and 5-12 give the transient movement of the terminal shock and the static pressure history at the mid span of the compressor face for the three system configurations. For each configuration, the amount of mass flow bled out of the system
is approximately the same, about 2.0% of the total incoming flow. The transient responses of the three systems are quite different. Figure 5-11 shows that the bleed operation causes the terminal shock to move forward in the case of uniform and fixed static pressure boundary condition and to move backward in the case of uniform and fixed Mach number boundary condition. For the compressor model, the terminal shock moves a little bit forward, and its steady state location is in between those of the two inlet exit boundary conditions.

This condition can be explained using simple one-dimensional flow principles. When the bleed system is in operation, the area at the bleed opening increases. In supersonic flow, an area increase causes local axial velocity and Mach number to increase, and local static pressure and density to decrease. If the terminal shock remains at the same location, the flow properties both at the terminal shock and the compressor face have to adjust. An increase in Mach number at the bleed opening causes higher terminal shock Mach number, higher loss across the terminal shock, and lower Mach number at the compressor face. A decrease in static pressure at the bleed opening causes lower static pressure at the terminal shock, resulting in lower compressor face static pressure. Finally, since loss across the terminal shock increases due to an increase in the terminal shock Mach number, the total pressure at the compressor face will be lower than the initial value.

Because the flow has to meet the specified requirement at the compressor face, the flow properties in that region have to adjust to overcome the initial changes associated with bleed operation. The only adjustment the flowfield can make is through forward or backward movement of the terminal shock. As shown in Figure 5-12, for all three system configurations, static pressure at the compressor face initially drops as a result of bleed operation. For the uniform static pressure inlet exit boundary condition, the boundary condition specified forces the compressor face static pressure to increase back to its undisturbed value. Thus, the terminal shock has to move forward, resulting in higher terminal shock static pressure and higher static pressure at the compressor face. Also, the static pressure at the compressor face oscillates during transients. This
damped oscillation in the compressor face static pressure indicates that disturbances generated by the bleed operation are reflected at the compressor face and propagate back upstream.

With uniform Mach number at the compressor face, the terminal shock must move backward to decrease the compressor face stagnation pressure, keeping the ratio between total pressure and static pressure constant. Because this boundary condition is not reflective, disturbances generated by the bleed operation leave the computational domain at the compressor face boundary. This can be seen in Figure 5-12, in which static pressure at the compressor face gradually adjusts towards new steady state condition.

For the compressor model, during the operation of bleed system, the compressor face corrected mass flow decreases, causing the compressor operating point to shift to the left. This results in a higher overall compressor pressure ratio. The transient movement of the terminal shock is such as to be consistent with constant static pressure at the compressor exit. This causes the terminal shock to move forward.

To estimate inaccuracy associated with the analysis of supersonic inlet unstart tolerance of each system configuration, the steady state locations of the terminal shock obtained from the three configurations are used. By using the system configuration with compressor model as comparison standard, the terminal shock location, associated with the inlet unstart tolerance, for the configuration using either one of the two simple inlet exit boundary conditions is off by approximately 14.0 %.

5.3.3 Summary

The results show that the type of boundary condition imposed at the compressor face has an impact on the dynamic behavior of supersonic inlet due to internal disturbances in that it can cause an inaccurate estimation of the inlet unstart tolerance. With the same internal disturbances, the configuration with uniform and fixed static pressure boundary condition has the highest risk of experiencing an inlet unstart,
and the simulation using this boundary condition underestimates the unstart tolerance of supersonic inlet. With uniform Mach number boundary condition, the unstart tolerance is overestimated.

5.4 Supersonic Inlet Dynamic Behavior Due to Freestream Disturbances

During flight, supersonic inlet experiences a variety of freestream disturbances, which can be classified into acoustic, entropy, and vorticity modes [15]. The disturbances seen by the compressor are the combination of these three modes. A freestream disturbance associated with an axial velocity fluctuation is used in this study because it is the type of perturbation that is most likely to be encountered by supersonic inlet in actual practice. Effects of the change in axial velocity on freestream Mach number and stagnation flow properties can be determined using the relationships derived by Mayer and Paynter in Reference [12].

The initial inlet flowfield for each configuration is the steady state condition obtained when the terminal shock is located at \( x = 3.2 \). To assess the effects of compressor face boundary conditions on the inlet dynamic behavior when subjected to freestream disturbances, the background inlet flowfield will be kept constant. After the passage of freestream disturbances, the inlet flowfield returns to its steady undisturbed condition. The temporal variation of imposed freestream perturbation is presented by a half sinusoidal wave as shown in Figure 5-13. The wavelength used in this study is chosen so the reduced frequencies \( \beta \), which determine the flow steadiness, have values of 0.1 and 5.0.

To limit the movement of the terminal shock to within the range of shock location assumed in the compressor model development, the amplitude of freestream disturbance employed is small. Initially, the perturbation used is a 5.0 % decrease in the mean axial velocity. However, this disturbance amplitude causes the throat Mach number to fall below 1.0, resulting in an inlet unstart so the computed results are
meaningless for examining the effects of inlet exit boundary conditions. This occurs even with an axial velocity decrease of 1.0 %. The freestream disturbance used in this study is thus a 5.0 % increase in the mean axial velocity. Although this perturbation causes the terminal shock to move away from the inlet throat, the effects of compressor face boundary conditions on supersonic inlet dynamic behavior can be examined.

5.4.1 Results for Freestream Disturbances with $\beta \ll 1$

Results for the quasi-steady case of $\beta \ll 1$ are shown in Figures 5-14 and 5-15. These figures respectively give the transient movement of the terminal shock and the static pressure history at the mid span of the compressor face caused by freestream disturbances.

For the uniform and fixed static pressure boundary condition, the constraint at the compressor face causes an overshoot in the transient movement of the terminal shock. This overshoot can cause a false inlet unstart if the undisturbed location of the terminal shock is near the inlet throat, or the disturbance amplitude is large enough.

For the uniform and fixed Mach number boundary condition, the compressor face static pressure adjusts to maintain a constant ratio of total pressure to static pressure. The transient movement of the terminal shock for this compressor face boundary condition shows no oscillation and the inlet flowfield gradually adjusts back to its steady state condition. However, it takes more than 13.0 flow-through time for the system to return back to its original condition. Also, compared to the other two boundary conditions, the terminal shock moves furthest away from its undisturbed location.

For the compressor model, it takes approximately the same amount of time as the configuration with uniform static pressure boundary condition to adjust back to its original condition. However, the terminal shock does not overshoot its undisturbed location.
Since freestream perturbation of interest has an effect of moving the terminal shock closer to the inlet throat, one does not have to take into account the overshoot appearing in the terminal shock movement for analyzing supersonic inlet unstart tolerance. Instead, to estimate inlet unstart tolerance, the peak location of the terminal shock obtained from each system configuration is used. Compared to the compressor model, the use of uniform Mach number boundary condition causes a 27.0% difference in the terminal shock location, which is linked to the inlet unstart tolerance. On the other hand, inlet unstart tolerance for the configuration using uniform static pressure boundary condition is approximately the same as that with compressor model. However, the terminal shock for the configuration using compressor model takes 0.5 flow-through time longer to reach the peak value.

5.4.2 Results for Freestream Disturbances with $\beta \gg 1$

For the unsteady case of $\beta \gg 1$, the transient movement of the terminal shock and the static pressure history at the mid span of the compressor face are shown in Figures 5-16 and 5-17 respectively. Although unsteady freestream disturbances cause the compressor face static pressure to adjust, they seem to have little impact on the transient movement of the terminal shock. The terminal shock for each system configuration barely moves from its undisturbed location. The implication is that if freestream disturbances are highly unsteady, supersonic inlet will hardly experience their effects. As before, an overshoot exists only in the transient movement of the terminal shock for the configuration with uniform static pressure boundary condition.

5.4.3 Summary

For quasi-steady freestream disturbances ($\beta \ll 1$), the type of inlet exit boundary condition imposed has an impact on the dynamic behavior of supersonic inlet. However, for highly unsteady flow ($\beta \gg 1$), the differences are small and can be neglected. In addition, the configuration using uniform Mach number boundary condition is sensitive to this freestream perturbation in that its terminal shock moves furthest away.
from the undisturbed location. If imposed freestream disturbances have an effect of moving the terminal shock closer to the inlet throat, the system configuration using uniform Mach number boundary condition will have the highest risk of an inlet unstart.

5.5 Results Summary

The computed results indicate that the effects of compressor face boundary conditions on the steady flowfield within supersonic inlet have minimal impact on the inlet steady performance. However, the type of boundary condition imposed at the compressor face does have an impact on the dynamic behavior of supersonic inlet in that it can cause an inaccurate estimation of the inlet unstart tolerance. For the uniform and fixed static pressure boundary condition, the inlet unstart tolerance is underestimated when subjected to both types of internal disturbances discussed in this study. When subjected to quasi-steady freestream disturbance, the results obtained from the use of uniform and fixed Mach number boundary condition underestimate the unstart tolerance of supersonic inlet. Compared to the configuration using compressor model, inaccuracy in this unstart tolerance estimation can be as high as 27.0 %, as shown in the case of quasi-steady freestream disturbances.
Table 5.1: Steady state flow properties at the compressor face for the terminal shock location of $x = 3.6$

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$x_{\text{shock}}$</th>
<th>$P_t$</th>
<th>$T_t$</th>
<th>$\dot{n}_{\text{corr}}$</th>
<th>$q$</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform P</td>
<td>3.6013</td>
<td>10.5552</td>
<td>5.9554</td>
<td>0.7114</td>
<td>0.7794</td>
<td>0.6317</td>
</tr>
<tr>
<td>Uniform M</td>
<td>3.5994</td>
<td>10.5625</td>
<td>5.9515</td>
<td>0.7107</td>
<td>0.7709</td>
<td>0.7258</td>
</tr>
<tr>
<td>Comp Model</td>
<td>3.6008</td>
<td>10.5953</td>
<td>5.9608</td>
<td>0.7090</td>
<td>0.7878</td>
<td>0.6353</td>
</tr>
</tbody>
</table>

Table 5.2: Steady state flow properties at the compressor face for the terminal shock location of $x = 3.2$

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$x_{\text{shock}}$</th>
<th>$P_t$</th>
<th>$T_t$</th>
<th>$\dot{n}_{\text{corr}}$</th>
<th>$q$</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform P</td>
<td>3.1992</td>
<td>10.9824</td>
<td>5.9306</td>
<td>0.6823</td>
<td>0.7526</td>
<td>0.7906</td>
</tr>
<tr>
<td>Uniform M</td>
<td>3.2009</td>
<td>10.9865</td>
<td>5.9256</td>
<td>0.6817</td>
<td>0.7409</td>
<td>0.9044</td>
</tr>
<tr>
<td>Comp Model</td>
<td>3.2004</td>
<td>11.0217</td>
<td>5.9349</td>
<td>0.6801</td>
<td>0.7599</td>
<td>0.7938</td>
</tr>
</tbody>
</table>
Figure 5-1: Static pressure distribution along the axial direction at the mid span for the terminal shock location of $x = 3.6$
Figure 5-2: Mach number distribution along the axial direction at the mid span for the terminal shock location of $x = 3.6$
Figure 5-3: Total pressure distribution along the axial direction at the mid span for the terminal shock location of $x = 3.6$. 
Figure 5-4: Distribution of flow properties along the axial direction at the mid span in the region close to the compressor for the terminal shock location of $x = 3.6$
Figure 5-5: Total pressure distribution at the compressor face in terms of freestream total pressure for the terminal shock location of $x = 3.6$
Figure 5-6: Total pressure distribution at the compressor face in terms of total pressure difference with respect to average dynamic head for the terminal shock location of \( x = 3.6 \)
Figure 5-7: Total pressure distribution at the compressor face in terms of freestream total pressure for the terminal shock location of $x = 3.2$
Figure 5-8: Total pressure distribution at the compressor face in terms of total pressure difference with respect to average dynamic head for the terminal shock location of $x = 3.2$
Figure 5-9: Transient movement of the terminal shock due to an instantaneous decrease in the compressor face corrected mass flow
Figure 5-10: Static pressure history at the mid span of the compressor face due to an instantaneous decrease in the compressor face corrected mass flow.
Figure 5-11: Transient movement of the terminal shock due to the bleed operation
Figure 5-12: Static pressure history at the mid span of the compressor face due to the bleed operation
Figure 5-13: Temporal structures of freestream disturbances
Figure 5-14: Transient movement of the terminal shock due to freestream disturbances with $\beta \ll 1$ (quasi-steady)
Figure 5-15: Static pressure history at the mid span of the compressor face due to freestream disturbances with $\beta \ll 1$ (quasi-steady)
Figure 5-16: Transient movement of the terminal shock due to freestream disturbances with $\beta \gg 1$ (unsteady)
Figure 5-17: Static pressure history at the mid span of the compressor face due to freestream disturbances with $\beta \gg 1$ (unsteady)
Chapter 6

Conclusions and Recommendations for Future Research

6.1 Research Summary

The problem addressed is the appropriate compressor face boundary condition for analysis of the unsteady performance of supersonic inlet. To examine this, a simple compressor model is developed and applied as an inlet exit boundary condition to the unsteady two-dimensional numerical simulation of flow in supersonic inlet. The results obtained from the system configuration using the compressor model are compared with those using either uniform and fixed static pressure or uniform and fixed Mach number compressor face boundary condition. The supersonic inlet performance used for comparison are steady flow properties at the compressor face, supersonic inlet dynamic behavior due to internal disturbances, and supersonic inlet dynamic behavior due to freestream disturbances.

6.2 Conclusions

From the computed results, the following conclusions are deduced:
1. With approximately the same corrected mass flow at the compressor face, corresponding to approximately the same terminal shock location, no significant difference exists in the steady performance of supersonic inlet. The differences in the total pressure recoveries at the compressor face obtained from the three system configurations are within 0.5 % in mass-weighted-average dynamic head.

2. The boundary condition imposed at the compressor face does have an impact on the dynamic behavior of supersonic inlet in that it can cause an inaccurate estimation of the inlet unstart tolerance.

   (a) Compared to the compressor model, the use of simple inlet exit boundary condition to represent compressor influence can cause as high as 27.0 % difference in the terminal shock location, which is linked to the inlet unstart tolerance, at the critical time when supersonic inlet is most likely to become unstart.

   (b) With the compressor model, no constraint is specified at the compressor face. Therefore, the flow properties in the region close to the compressor can adjust in a physically consistent manner, giving a more realistic representation of the inlet-compressor transient interaction.

### 6.3 Recommendations for Future Research

The following is the list of recommended work for improving the unsteady analysis of supersonic inlet flowfield:

1. Development of a three-dimensional analysis to allow for the investigation of supersonic inlet flowfield parameters, such as angle of attack and circumferential distortion.

2. Use of Navier-Stokes equations to examine the effects of throat blockage and boundary layer behavior on the dynamic response of supersonic inlet.
Appendix A

Development of Compressor Model

The preliminary repeating row/repeating stage compressor design, described in Mattingly's *Elements of Gas Turbine Propulsion* [10] is used in the development of the compressor model. For simplicity, the compressor is assumed to be a straight duct located right after the diffusing part of the inlet and consists of 30 grid columns. In this research, the operating range of the compressor model is between the compressor total pressure ratio of 8.0 and 10.0. For this operating range, the compressor model does not encounter surge or stall during transients. The total pressure ratio across each compressor stage varies linearly with the corrected mass flow at that stage. The polytropic efficiency $e_c$ of the compressor is assumed to be 0.95.

The following are the procedures used in the compressor model development:

1. Perform the numerical simulation of supersonic inlet using uniform static pressure inlet exit boundary condition to place the terminal shock at the assumed on-design location of $x = 3.2$ and obtain on-design average total pressure $P_t$, and total temperature $T_t$, at the compressor face.

2. Assume that the on-design compressor pressure ratio $\pi_{on}$ is 10.0.

3. Assuming that the change in $T_t$ across each compressor stage is constant, obtain
\( \Delta T_t \) across each stage from

\[
\Delta T_t = T_t \left( \frac{n}{n_{neq}} - 1 \right). \tag{A.1}
\]

4. Calculate \( T_t \) at each stage from

\[
T_{tn} = T_{tn-1} + (n - 1) \Delta T_t \tag{A.2}
\]

where \( n = 2, \ldots, 31 \).

5. Calculate \( \tau \) and \( \pi \) across each stage from

\[
\begin{align*}
\tau_n &= \frac{T_{tn+1}}{T_{tn}} \\
\pi_n &= \frac{T_{tn}}{T_{n-1}} \tag{A.3}
\end{align*}
\]

where \( n = 1, \ldots, 30 \).

6. Calculate \( P_t \) at each stage from

\[
P_{tn} = P_{tn-1} \pi_{n-1} \tag{A.4}
\]

where \( n = 2, \ldots, 31 \).

7. Calculate corrected mass flow \( m_{corr} \) for each stage from

\[
m_{corr} = \frac{\dot{m} \sqrt{T_{tn}}}{P_{tn} A} \tag{A.5}
\]

where \( n = 1, \ldots, 30 \).

8. Assuming that the compressor total pressure ratio at the off-design condition \( \pi_{off} \) is 8.0, repeat Steps 1-7 to calculate required properties for the off-design condition, where the location of the terminal shock is assumed to be at \( x = 4.0 \).

9. Calculate slope \( m \) and intersecting point \( b \) for the characteristic map of each
compressor stage from

\[ m_n = \frac{\pi_{on_n} - \pi_{off_n}}{\tilde{m}_{corr_{on_n}} - \tilde{m}_{corr_{off_n}}} \]

\[ b_n = \pi_{on_n} - (m_n \tilde{m}_{corr_{on_n}}) \]  

(A.6)

where \( n = 1, \ldots, 30 \).

The Euler-solver computer program reads in the values of \( m \) and \( b \), obtained from Step 9, for each compressor stage (grid column). The source term \( Q \) for each local cell inside the compressor region is calculated from the following set of equations:

\[ \dot{m}_{corr} = \frac{\dot{m}_v T_i}{P_{iA}} \]

\[ \pi = m(\dot{m}_{corr}) + b \]

\[ \tau = \pi^{\frac{\gamma - 1}{\gamma + \beta}} \]

\[ T_{tw} = \frac{\tau T_i}{\tau + 1} \]

\[ T_{iB} = T_{iw} \tau \]

\[ f_u = \frac{\gamma (\tau - \tau_{tw})}{\Delta x} \]

\[ Q = \begin{pmatrix} 0 \\ \rho f_u \\ 0 \\ \rho f_u u \end{pmatrix} \]  

(A.7)

For cells outside the compressor region, all components of the source term are set equal to zero.
References


