Heat Transfer Effects on the Propellant Gauging of Three-Axis Stabilized Satellites

by

Sonia Ensenat

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Abstract

This document analyzes the effects of internal heat transfer on the propellant remaining predictions of a Hughes HS601 satellite. The HS601 is a three-axis stabilized geostationary satellite that utilizes a thermodynamic propellant gauging system (PGS). The PGS calculates the remaining propellant by injecting a known amount of pressurant gas into the propellant tanks and measuring the resulting pressure and temperature changes. PGS' remaining propellant prediction contains errors due to the fluctuating thermal environment of the propellant and pressurant tanks. As heat flows into the tanks, a time lag develops between the pressure and temperature readings due to the location of the sensors. A thermodynamic model of the pressurant tank is developed to estimate the tank temperature distribution as a function of orbital position. The pressurant tank model can be used to estimate the bias error introduced by the temperature sensor location and to assess the most effective location for the temperature sensor(s). Other issues, such as the best time of the year for gauging are discussed briefly.

Thesis Supervisor: R. J. Hansman, Jr.

Title: Assistant Professor
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Chapter 1. Introduction

The accurate determination of a satellite's remaining propellant has become more important as the aerospace industry struggles to reduce costs. Knowing a satellite's end of life to within a few months can minimize redundancy and lower replacement costs. Geostationary satellites receive an added advantage: the amount of fuel reserved for deorbit maneuvers can be greatly reduced. Propellant gauging can also be combined with repressurizations of the propellant tanks to control the mixture ratio of bipropellant propulsion systems. By actively controlling the mixture ratio, depletion of both fuel and oxidizer can be guaranteed to occur at nearly the same time, thus reducing propellant residuals.

The greatest challenge in the design of a low-gravity propellant gauging system comes from the difficulty in predicting the position and shape of the propellant in the tank. In sufficiently low gravity, surface tension forces become more significant than gravitational forces in determining the propellant's shape and position. Even when propellant management devices (PMDs) are used, the propellant may take on complex shapes. Spin-stabilized satellites avoid this problem since the dynamics of a spinning system provide for simple fuel measurements. A.D. Challoner [1] has developed a method for measuring the remaining propellant from a measurement of the propellant's slosh frequency. Another gauging method for spinning satellites uses the principle of conservation of angular momentum. Propellant is transferred between two propellant tanks and the corresponding change in spin speed is measured. By computing the corresponding change in the moment of inertia, the remaining propellant can be estimated. This method yields fairly accurate predictions without the need for a highly detailed model of the satellite's mass properties. Although propellant gauging of spin-stabilized satellites is fairly well understood, the growing demand for more powerful, three-axis stabilized satellites has fueled the development of new gauging techniques.

Hansman and Meserole [3] provide an overview of existing propellant gauging techniques, grouping them into four main categories: point sensors, line sensors, global systems and accounting systems. Both point and line sensors rely on direct measurement of fluid volume at given locations. Point sensors provide measurements of fluid volume at discrete points in the tank. Line sensors, such as ultrasonic or capacitative line sensors, measure fluid level along a line.
An algorithm based on the tank's geometry must be used to integrate measurements by point or line sensors at different locations and determine the overall amount of propellant left. Global systems determine the tank's entire fluid volume by measuring parameters like the fluid's pressure and temperature (thermodynamic methods) or its electromagnetic energy (electromagnetic absorption method). Finally, accounting (or bookkeeping) systems compute the amount of fuel removed from the tanks, through the use of flowmeters, or by simply keeping track of the thruster on-times and modeling the outflow of propellants from system pressures.

Choosing the appropriate gauging system, however, is not a clear-cut choice. As Hansman and Meserole point out, each method has its limitations. The applicability of point and line sensors decreases when the propellant's orientation is not favorable. Thermal gradients or bubbles trapped in the fluid change the effective density and lower the effectiveness of line, global and accounting methods. In addition, capillary forces can cause fluid to block sensors flush with the tank surface. Thus, the accuracy of each gauging method depends on factors like mission parameters and tank size.

Orazietti and Orton [5] conducted a comparative study of low-gravity propellant gauging systems for geostationary satellites. Several gauging concepts were evaluated and a few chosen for ground and on-orbit tests. They acknowledge the simplicity and low-cost of gas law (or thermodynamic) methods but add that they are "critically deficient" for bipropellant propulsion systems due to thruster mixture ratio differences, pressurant gas solubility and duty cycle variations. Also cited is the need for very high accuracy in predicting a satellite's end of life (1/2 to 1 month uncertainty), which may not be possible with gas law gauging methods. These deficiencies are relative, however. Some thermodynamic gauging methods do not rely on propellant flow models and are thus not affected by mixture ratio uncertainties and pulse firing of the engines. Besides their low cost and simplicity, thermodynamic gauging methods offer other advantages for the propellant gauging of three-axis stabilized geostationary satellites, as long as lifetime uncertainties on the order of 1-2 months are tolerable.

1.1 Thermodynamic Gauging Methods

The ideal propellant gauging system is accurate, light-weight, low-cost, simple to operate
and, if possible, does not interfere with or add much complexity to the propulsion subsystem. Thermodynamic gauging methods are low-cost and light-weight since the additional equipment needed consists of valves, extra propellant lines and temperature and pressure sensors. The weight of such a gauging system for a 6000-lb geostationary satellite can be less than 8 lbs [1]. In addition, operating the system is simple. Most thermodynamic gauging systems involve the transfer of some gas from the pressurant tank into the propellant tank while recording the change in the tanks' pressures and temperatures. A schematic of such a thermodynamic gauging system is shown in Figure 1.1.

![Figure 1.1 Schematic of the HS601 Propellant Gauging System](image)

The gauging maneuver only involves the opening and closing of one or two latch valves. As long as there is no chance of overpressurizing the propellant tank, the risk is minimal. The ability to transfer pressurant gas to the propellant tanks also offers the option of controlling the mixture ratio and thus prolonging the satellite's life. It is this simplicity and low-cost that has made thermodynamic techniques an attractive alternative for the gauging of geostationary satellites.

Several models have been developed for a thermodynamic gauging system that utilizes the transfer of pressurant gas. Torgovitsky's [6] analysis relies on knowing the mixture ratio, which adds considerable error. The pressurant and propellant in the propellant tanks are also assumed to be in thermal equilibrium. Orbital data confirms that this is not the case; thermal gradients
across the propellant tanks are normally present. Monti and Berry [4] developed and tested a thermodynamic gauging system based on an isothermal tank model. The resulting fill factor error, in spite of the temperature effects and pressure transducer inaccuracies, was on the order of 1.75%. However, the tests, conducted on the space shuttle, may not accurately reflect the geostationary thermal environment of the propellant tanks.

Figure 1.2 Hughes HS601 geostationary satellite

In developing a gauging system for the Hughes HS601 satellite (Figure 1.2), Chobotov and Purohit [2] recognize the need to take into account the orbital environment and the heat transfer in the propellant tanks. They model the propellant tank approaching an isothermal limit long after the gas injection, but never reaching it due to the changing solar flux into the satellite. They propose using the remaining propellant tank as a reference, to measure the effects of the changing thermal environment. This method has been attempted without success on the Hughes HS601 satellite. The propellant tanks, located on different sides of the satellite, do not experience exactly the same thermal environment. Furthermore, the transfer of pressurant gas into a propellant tank also disturbs the remaining tanks, eliminating the possibility of using them as
reference tanks. Analysis of orbital data shows other unexplained phenomena. A time lag between the pressure and temperature is present in both the pressurant and propellant tanks, possibly due to heat transfer within the tanks. Also visible in the propellant tanks is a recurring disturbance.

This study is restricted to the effects of the pressurant tank’s thermal environment on the accuracy of the propellant gauging system (PGS) of a Hughes HS601 satellite. Although analysis of the propellant tank is reserved for future studies, the thermodynamic effects present in the propellant tank and their impact on propellant gauging are discussed. A pressurant tank thermodynamic model is developed in Chapter 3. This model takes into account the heat transfer in the tank and seeks to explain the time lag between the tank’s pressure and temperature.
Chapter 2. Background

The design and testing of the HS601 propellant gauging system (PGS) is described by Chobotov and Purohit [2]. As the system has been put into practice, several operating difficulties have been encountered. These range from unforeseen effects of the orbital environment to finding a method to calculate the uncertainty in the propellant remaining estimate. As a result, several modifications were made to the gauging procedure and algorithm. However, even with these modifications a sufficiently accurate mass estimate is not obtained and some questions about the thermal environment of the pressurant and the propellant tanks remain unanswered.

In this chapter, the initial design of the PGS is briefly described. The improvements made to the gauging procedure and algorithm are then explained. Finally, the remaining problems with the gauging procedure and hypotheses as to their cause are presented.

2.1 Initial Design of the HS601 PGS

The HS601 satellite operates on a bipropellant propulsion system, shown in Figure 2.1. There are two pressurant tanks, one connected to the fuel tanks, the other connected to the oxidizer tanks. When the propellant gauging system is engaged, the propulsion system is operating in blowdown mode, which means the fuel and oxidizer tanks have been pressurized to a pre-determined amount and are depleted without re-pressurizing for the rest of the mission (except to perform propellant gaugings).

![Figure 2.1 Schematic of the HS601 Propellant Subsystem](image-url)
The design of Hughes' HS601 propellant gauging system (PGS) is described by Chobotov and Purohit. This design calls for the transfer of a given amount of pressurant gas from the pressurant tank into the propellant tank being gauged by opening one of the connecting latch valves (labeled LV 1 through LV 4 in Figure 2.1). Pressure and temperature readings are taken before and after the gauging from both the pressurant and propellant tanks.

Each propellant tank is equipped with a pressure transducer and two temperature sensors, as shown in Figure 2.2. The temperature sensor labeled $T_1$ measures $T_u$, or the ullage\(^1\) temperature, sensor $T_3$ measures the liquid propellant temperature, $T_L$. A third temperature sensor, $T_2$ is available in some satellites. It is located along the side of the tank, midway between $T_1$ and $T_3$. All temperature sensors are attached to the outside of the tank.

2.1.1 Propellant Remaining Calculation

The remaining propellant mass calculation as developed by Chobotov and Purohit is as follows. The pressurant mass removed from the pressurant tank is equated to the pressurant mass

\(^1\)The mixture of pressurant gas and propellant vapor in a propellant tank are referred to as ullage.
inserted into the propellant tank

\[-\Delta M_{Pr}^{\text{pressurant}} = \Delta M_{Pr}^{\text{propellant}}\]  \hfill (2.1)

where \(M_{Pr}\) = pressurant mass
and the subscripts indicate which tank is being referred to. Thus, the pressurant mass transferred out of the pressurant tank must appear as an increase in the ullage mass of the propellant tank being gauged. The negative sign accounts for the fact that the mass in the pressurant tank is decreasing, while the ullage mass in the propellant tank increases.

Using the ideal gas law, equation (2.1) can be written as

\[
\left[ \frac{PV}{T} \right]_i - \left[ \frac{PV}{T} \right]_f = \frac{PV_u}{T_u} \left[ \frac{PV_u}{T_u} \right]_i - \frac{PV_u}{T_u} \left[ \frac{PV_u}{T_u} \right]_i \]  \hfill (2.2)

where:
\(P\) = pressure
\(V\) = volume
\(P\) = pressurant
\(T\) = temperature
\(U\) = ullage

Subscripts:
i = initial
f = final

and the subscripts \(i\) and \(f\) denote initial and final values, or values before and after the pressurant transfer.

If this process can be assumed to be isothermal, then \((TP)_i = (TP)_f\) and similarly for the ullage temperature, \(T_{U}\). By making the substitutions

\[
\left[ P_p \right]_i - \left[ P_p \right]_f = \Delta P_p \]  \hfill (2.3a)

\[
\left[ P_u \right]_f - \left[ P_u \right]_i = \Delta P_u \]  \hfill (2.3b)

equation (2.1) can be re-written (for an isothermal process) as

\[
V_u = V_p \frac{T_u}{T_p} \left[ \frac{\Delta P_p}{\Delta P_u} \right] \]  \hfill (2.4)
From the ullage volume, the liquid propellant mass, \( m_L \), can be easily calculated

\[
  m_L = \rho_L V_L = \rho_L [V_T - V_u]
\]

where:
- \( V_T \) = tank volume
- \( \rho_L \) = liquid propellant density
- \( m_L \) = liquid propellant mass

The temperatures and pressure changes are measured, while the tank volume (as a function of pressure and temperature) can be calculated from acceptance test data. The propellant density (as a function of temperature) can also be calculated.

If the process is to be treated as isothermal, enough time must be allowed to pass after the pressurant transfer before 'final' measurements are taken. This allows transients to subside. However, the propellant tank's thermal environment is a function of the satellite's orbit and changes roughly every three hours.

The assumption of the pressurant transfer into a propellant tank as an isothermal process, as presented by Chobotov and Purohit, relies on the use of the remaining propellant tanks as reference tanks. The propellant tank being gauged experiences two perturbations: one is due to the insertion of pressurant gas, the other due to the changing thermal environment in the tank as a function of the satellite's orbit. By using the remaining propellant tanks to essentially subtract the perturbation due to the changing solar flux, only the perturbations due to the pressurant transfer will remain and an isothermal environment is re-created.

The reference tank method has proven impractical; the propellant tanks, due to their position on the satellite, do not have the same thermal environment. In addition, when pressurant is transferred into a propellant tank, the remaining propellant tanks experience a (smaller) perturbation. This has led to the abandonment of the calculation of remaining propellant based on an isothermal tank model.

Several other assumptions contained in the initial design of the PGS have been re-examined when developing the new gauging algorithm. The initial PGS design assumes that the pressurant gas behaves as an ideal gas. In reality, the gas deviates slightly from an ideal gas - its
compressibility factor is not 1. Although this effect is small, it is incorporated into the new gauging algorithm. The initial design does not account for the ullage gas dissolving into the liquid propellant or the vaporization of the liquid propellant. Along with the pressurant gas compressibility, these effects are included in the new gauging algorithm, developed chiefly by G.P. Purohit.

2.2 Improved Gauging Procedure and Algorithm

The new gauging algorithm still relies on the conservation of pressurant mass as expressed above in equation (2.1). However, when the effect of pressurant gas dissolving into the liquid propellant is taken into account, equation (2.1) becomes

\[-\Delta M_{p,\text{pressurant}}^{\text{tank}} = \Delta M_{p,\text{propellant}}^{\text{tank}} + \Delta M_{p,\text{sol., propellant}}^{\text{propellant}}\]  \hspace{1cm} (2.6)

where the last term indicates the pressurant mass dissolved in the liquid propellant. Re-writing equation (2.6) in terms of the ideal gas law, taking into account pressurant compressibility effects and including the effect of vapor pressure yields,

\[
\left[ \frac{P_p V_p}{Z_p R T_p} \right]_{\text{!initial}}^{\text{!final}} \text{Presstank} - \left[ \frac{P_p V_p}{Z_p R T_p} \right]_{\text{!initial}}^{\text{!final}} \text{Presstank} =
\]

\[
\left[ \frac{P_u - P_v}{Z_u R T_u} \cdot V_u \right]_{\text{!final}}^{\text{!initial}} \text{Proptank} - \left[ \frac{P_u - P_v}{Z_u R T_u} \cdot V_u \right]_{\text{!initial}}^{\text{!final}} \text{Proptank}
\]

\[\left[ M_{p,\text{sol., final}} - M_{p,\text{sol., initial}} \right] \]  \hspace{1cm} (2.7)

where \( P_v \) = vapor pressure

\( R \) = specific gas constant = ideal gas constant / molecular weight of gas

\( Z \) = compressibility coefficient

and the subscripts p, u and L have been used to indicate pressurant, ullage and liquid (propellant),
respectively.

Although equation (2.7) looks quite complex, most of the quantities are known. All the pressures and temperatures are measured. Expressions for the vapor pressure, compressibility, propellant density as a function of temperature, tank volume and dissolved pressurant mass can be found in Appendix A. They have not been incorporated into equation (2.7) for the sake of clarity. The ullage volume is the only unknown in equation (2.7).

As before, the propellant mass can be found from the ullage volume. Rewriting equation (2.4) gives

\[ V_u = V_T - V_L = V_T - \frac{m_L}{\rho_L} \]  

which can be substituted directly into equation (2.7).

Substituting equation (2.8) and the expressions in Appendix A into equation (2.7) yields a (long) equation that can be solved explicitly for the propellant mass. This equation constitutes the new gauging algorithm.

The experience of conducting several gaugings and of monitoring on-orbit tank pressures and temperatures has also helped improve the gauging procedure. Preliminary results indicate that the gauging must be performed at the time of year when the tank temperatures (and pressures) are changing the least.

![Figure 2.3 Spacecraft solar angle throughout orbit](image)

Figure 2.3 Spacecraft solar angle throughout orbit
The tank temperatures and pressures change the least near the solar solstices (in June and December). At the solstices, the sun makes a maximum angle (23.5°) with the Equator so that the satellite is always in view of the sun (Figure 2.3). Although the data gathered so far seems to indicate that a successful gauging must occur near the solar solstices, the effects of non-solstice gaugings have not been thoroughly investigated.

It was observed that the propellant tanks needed a long period to recover from a gauging. Once the pressurant was injected into the propellant tanks, several days were needed for the transients in the propellant tanks to dissipate. The "final" pressure and temperature data used in the gauging equation is data taken after the transients have elapsed.

The amount of gas injected into the propellant tank was found to affect the accuracy of the propellant mass estimate. The pressurant mass injected should be much larger than the uncertainty in the ullage mass calculation (due to sensor accuracy, etc.). The uncertainty in the propellant remaining is also largely affected by the poor resolution of the temperature sensors on older HS601 satellites. Figure 2.4 shows a propellant tank's ullage temperature over a period of two days. The step-like jumps in the temperature signal denote the limit in the accuracy of the temperature measurement due to the telemetry resolution. To reduce the temperature uncertainty, the temperature signal was filtered using a second order low-pass filter. The resulting temperature signal is also shown in Figure 2.4.

![Figure 2.4 Ullage temperature of a propellant tank on a typical HS601 satellite](image)

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2 Chapter 4 discusses the tank on-orbit temperatures and pressures in more detail.
2.3 Issues Remaining

The propellant gauging strategy described in the previous section is currently in place. However, certain gauging issues remain unresolved.

Orbital data shows a time lag between the pressure and temperature signals. Essentially, the peaks on the pressure and temperature signals are offset by a few minutes. The lag is present in all of the spacecraft that have been analyzed, in all tanks (both propellant and pressurant) and at different times of the year. The size of the pressure-temperature lag appears to depend mainly on the tank in question, although there is a small variation with the time of year. The size of the lag can be found numerically from the orbital data\textsuperscript{3}, but its cause remains yet to be identified.

The hypothesis of this study is that the lag is due to the transient thermal response of the pressurant and propellant tanks and the location of the temperature sensors. A simple case illustrates this hypothesis. Suppose a pressurant tank is being heated as shown in Figure 2.5, with the temperature sensor directly in the heat path.

\textbf{Figure 2.5 Schematic of pressure-temperature lag in a pressurant tank}

\textsuperscript{3} See Chapter 4 for details.
The temperature sensor at point A will record the temperature increase almost immediately. A finite time period, $t_1$, will elapse before the temperature of the gas on the opposite side of the tank (point B) rises in response. The tank pressure is a measure of the average tank temperature. A time lag, $t_2$, is then established between the temperature measured at point A and the tank pressure. In this case, the temperature signal will lead the pressure signal. If the tank is instead heated from point B, a similar argument can be made to show the existence of a time lag but now whether the pressure or temperature leads will depend on the main heat path in the tank. If the heat is mainly conducted along the tank wall, the temperature signal will still lead. Although a pressurant tank is used for illustrative purposes, the same mechanisms are believed to cause the pressure-temperature lag in the propellant tanks.

The locations of the temperature sensors also introduce a possible bias into the mass remaining calculations. The choice of the temperature sensor locations on the propellant tank is based on the assumption that the propellant is at the "bottom" of the tanks and around the walls, as shown in Figure 2.2. This is believed to be a good assumption due to the small amount of propellant in the tank and the use of propellant management devices (PMDs) which control the position of the liquid propellant. However, it is unclear whether $T_3$ alone is a good measure of the propellant temperature, $T_L$. For instance, is the propellant temperature better represented as a combination of $T_3$ and $T_2$?

A similar question arises regarding the ullage temperature, $T_U$ as it is measured by $T_1$. Since temperature gradients exist across the propellant tank even in steady-state conditions, $T_1$ may not be the best measure of the mass-averaged ullage temperature, $T_{UL}$. If this is the case, using $T_1$ as the ullage temperature in the mass remaining calculations would result in an erroneous mass prediction. Preliminary estimates show that this error could be as large as 13 kg for a combined bias of +3.0°C in $T_p$, -1.0°C in $T_L$ and +1.0°C in $T_U$ (the mass-averaged temperature of the pressurant gas in the pressurant tank). These temperature biases are representative of the tank thermal gradients predicted by a bulk thermal model of the spacecraft.\(^4\)

\(^4\)The bulk thermal model predicts tank temperatures based on the heat flux from and to other parts of the spacecraft but it makes large simplifications. For example, it models the propellant tanks as empty.

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This study seeks to better understand the temperature distribution of the pressurant tanks of the HS601 satellite. The hypothesis presented above is taken as a starting point. Secondary goals of this study are to estimate the bias error of the temperature measurement and evaluate the choice of the temperature sensor location. Analysis of the propellant tanks is reserved for later studies.
Chapter 3. Analysis

A thermodynamic analysis of the pressurant and propellant tanks is essential to understanding possible errors in the new mass prediction algorithm. Specifically, a model of the temperature distribution along the tank is desired along with an estimate of the bias errors due to sensor placement and nonuniform tank temperatures. An extensive thermodynamic model of pressurant and propellant tanks is complex. The heat flux into the tank is not simply from the outer walls facing the sun, but also from the radiator panels, electronics and attitude control mechanisms. Additionally, many heat transfer modes may be present and/or coupled: conduction, radiation, natural convection, combined heat and mass transfer. In the propellant tanks, additional effects may be present, for example: differences in propellant temperature result in surface tension gradients which can cause propellant motion (Maringoni effect). In modeling the pressure-temperature lags and the tanks' thermodynamic conditions (pressure and temperature profile), emphasis is placed on a qualitative understanding. Numerical solutions are obtained by making simplifying assumptions.

A thermodynamic model of the pressurant tank is developed in this chapter. It is assumed that the tank is not subjected to any sudden accelerations. The pressurant tank is treated as a closed, or constant-mass system. This constant-mass assumption limits the analysis to quiescent periods between propellant gaugings and maneuvers but the data used for propellant gauging calculations is only taken during such quiescent periods. All possible modes of heat transfer are compared and their relative importance determined. Based on this comparison, the dominant heat transfer modes are included in the tank model.

3.1 Pressurant Tank Description

A detailed schematic of an HS601 pressurant tank is shown in Figure 3.1. The tank is cylindrical with elliptical endcaps. The tank wall consists of a thin metal liner overwrapped with a composite layer. The open side of the tank connects to the propellant tanks via titanium lines. The connection between these lines and the pressurant tank is a metal structure, shown on the figure. The tank supports are made of composite materials and are not included in the schematic.
3.2 Dominant Heat Transfer Modes

Several heat transfer mechanisms can be present in the pressurant tank. Heat from the surroundings can be transferred to the tank by radiation and conduction. Within the tank, heat can be diffused by conduction through the tank wall as well as conduction through the gas. Heat is also radiated between different sections of the inner tank wall. Finally, although it is assumed that the gas in the tank does not undergo sudden accelerations, the net acceleration of the pressurant tank is not zero. The net accelerations on the pressurant tank are due to: 1) the oblateness of the earth, 2) the gravitational pull of the sun and moon 3) the difference between the gravitational pull of the earth and the centripetal acceleration of the orbit (since the pressurant tank is not at the spacecraft's center of gravity, these accelerations are not exactly balanced out), and 4) solar radiation pressure. Even small net accelerations can set up natural convection currents which can contribute significantly to the heat transfer in the tank.

Order of magnitude calculations were used to determine the relative importance of these heat transfer mechanisms. The closed end of the pressurant tank is supported by a metal bracket that connects to one of the spacecraft's main structures. This support is modeled and included in the tank model. The heat conducted via the gas lines was also found negligible. Using data from Hughes
Thermophysics' spacecraft bulk thermal model and estimating the conductive resistance along the gas line as 1137.0 K/W, the heat transfer by conduction along the gas line was found to be 0.0025 W (winter solstice, second year of life, spacecraft hour = 0, north pressurant tank, HTK1)\(^1\) while the total heat transferred to the pressurant tank from all external nodes was around 16.06 W. The conductive heat transfer accounts for only 0.015\% of the total heat transfer to the pressurant tank. If, instead, one compares the heat transferred by conduction along the gas line to the heat transferred by radiation between the two nodes joined by the gas line (0.82 W), the heat transferred by conduction is still only responsible for a small percentage of the total heat transferred between the two nodes (0.30 \%). Although a thorough investigation of the importance of conduction under different circumstances was not undertaken, these results were taken to be representative, since conductive heat transfer will be proportional to the temperature difference between the nodes and radiative heat transfer is proportional to the difference of the fourth power of the temperatures. Heat transferred by conduction into the pressurant tank is therefore neglected in the tank model.

The Rayleigh number was used to estimate whether natural convection is an important mode of heat transfer. The Rayleigh number is a non-dimensional parameter that indicates the importance of natural convection relative to conduction. It is defined as,

\[
Ra = \frac{\beta \Delta T g L^3}{\nu \alpha}
\]

where

\(\beta =\) volumetric coefficient of thermal expansion (for an ideal gas, it is equal to \(1/T\), where \(T\) = temperature) \\
\(\Delta T =\) temperature gradient which creates density difference in gas

\(^1\) For the south pressurant tank (HTK2), using data from winter solstice, second year of life, spacecraft hour = 0, the conduction heat transfer is 0.0045 W, the total heat transfer is 13.93 W.
\[ g = \text{net acceleration of the pressurant tanks} \]
\[ L = \text{characteristic length scale - tank diameter was used} \]
\[ \nu = \text{kinematic viscosity} \]
\[ \alpha = \text{thermal diffusivity} \]

When calculating the net acceleration on the pressurant tank, only the magnitude of the acceleration was considered and the largest acceleration was used to calculate the Rayleigh number. The acceleration due to solar radiation pressure is approximately \(1.145 \times 10^{-7}\) m/s\(^2\) at beginning of life - this is based on calculations by Hughes' Orbital Operations. The net acceleration due to the difference in the centripetal and earth's gravitational acceleration is \(1.26 \times 10^{-8}\). The accelerations due to the solar and lunar gravity are \(5.01 \times 10^{-6}\) m/s\(^2\) and \(1.09 \times 10^{-5}\) m/s\(^2\). Finally, the acceleration due to the earth's oblateness is on the order of \(10^{-11}\) m/s\(^2\). The net acceleration on the pressurant tank was taken to be due to lunar gravity, since this effect was the most significant.

Using this acceleration, the Rayleigh number for the pressurant tank was calculated to vary from 3.4 to about 1500 over a wide range of operating conditions\(^2\). Usually, a Rayleigh number of less than \(10^3\) is indicative that natural convection can be neglected. The pressurant tanks' Rayleigh number was only above 1000 for the cases of high tank pressure and high tank temperature gradients. For temperature gradients of 5 degrees C or less, the Rayleigh number had a maximum value of 756.8. As the temperature gradients across the tank are not expected to be greater than 5 degrees C, natural convection was neglected and heat transfer through the pressurant gas was taken to be mainly by conduction.

The main heat transfer modes captured in the model are then conduction through the tank wall and the pressurant gas, and radiation both from other parts of the spacecraft and within the pressurant tank.

### 3.3 Model

---

\(^2\) Tank pressure ranging from 150 to 700 psi, tank temperature ranging from 4 to 60 degrees C and temperature difference across the tank ranging from 1 to 15 degrees C.
The energy conservation equation for the pressurant tank, written in cylindrical coordinates is (assuming constant thermal conductivity),

$$mCp \frac{\partial T}{\partial t} = \frac{k}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial T}{\partial r} \right] + \frac{k}{r^2} \frac{\partial^2 T}{\partial \phi^2} + k \frac{\partial^2 T}{\partial z^2} + Q_{rad}$$  \hspace{1cm} (3.2)

where:

- \( T \) = temperature
- \( m \) = mass
- \( Cp \) = specific heat coefficient
- \( k \) = thermal conductivity
- \( Q_{rad} \) = radiative heat transfer [W]

Equation (3.2) can be written for each of the systems in the pressurant tank: the composite part of the wall, the metal wall liner and the pressurant gas. These three equations, together with the equation of state for an ideal gas (modified for compressibility) form a system of non-linear partial differential equations to be solved for the pressure of the gas and the temperature distribution. Some simplification of the equations is possible, for example, the metal layer of the tank wall is very thin and the temperature drop across it in the radial direction can be neglected. However, the equations remain difficult to solve due to their non-linearity, the complexity of the boundary conditions (partly due to the presence of the endcaps) and the temperature and spatially varying thermal properties (the composite's thermal conductivity varies spatially, the pressurant gas' thermal conductivity and thermal capacitance are functions of temperature and pressure.

To simplify the problem, the tank was divided into nodes of constant volume and constant temperature and a numerical approach was used to calculate the heat transfer between connecting nodes. The different heat transfer mechanisms can be "turned on and off" to examine their relative importance on the tank thermal response.

3.3.1 Assumptions

Several assumptions were made in the modeling process. The difference between the
temperature of the temperature sensor and the tank wall (due to the resistance provided by the bonding materials) was assumed negligible. The time lag between the tank wall and sensor temperatures was estimated to be negligible\(^3\). The volume change of the pressurant tank is quite small and the effect of work done on the gas by the expansion or contraction of the tank was neglected. Since the tank nodes were defined as constant-volume nodes, no work is done on individual nodes by expansion of nearby nodes. For the gas nodes, the effect of changing density of the nodes is seen as a change in the node mass.

The heat transfer between the nodes is modeled as linear and instantaneous. Conduction between two nodes is then assumed to occur immediately, rather than through the formation of a boundary layer and a time-dependent penetration. This assumption was made for simplicity and its effect needs to be evaluated.

Radiation into the tank from other parts of the spacecraft is obtained from predictions made by Hughes Thermophysics' spacecraft bulk\(^4\) thermal model. The bulk model takes into account the orbital position and the time of year and predicts the temperature of the different spacecraft components over a period of time. The heat flow from other parts of the spacecraft into the pressurant tank was updated every hour, which should be sufficient since the spacecraft's thermal environment does not change much over any three-hour period.

### 3.3.2 Numerical Statement of Problem

The pressurant tank is broken up into nodes as shown in Appendix B. For each node, \(i\), the following energy conservation equation can be written,

\[^3\text{A measure of a material's thermal time lag is } d^2/\alpha, \text{ where } d \text{ is the distance across which heat travels and } \alpha \text{ the material's thermal diffusivity. For the bonding material, } d^2/\alpha \sim 0.5 \text{ seconds, much faster than the typical sampling rate of 1 reading every 30 seconds.}\]

\[^4\text{The bulk spacecraft thermal model uses the NEVADA software to calculate the radiative exchange between surfaces. The total heat exchange is then calculated using the CINDA software.}\]
\[ m_i C_p_i \frac{dT_i}{dt} = \sum_{j}^{N} \frac{T_j - T_i}{R_{ij}} + Q_{i,\text{int}} + Q_{i,\text{ext}} \tag{3.3} \]

where:

- \( m_i \) = mass of node i
- \( C_p_i \) = specific heat coefficient of node i
- \( T_i \) = temperature of node i
- \( T_j \) = temperature of adjacent node, j
- \( R_{ij} \) = effective resistance between nodes i and j
- \( Q_{i,\text{int}} \) = heat into node i from other parts of the tank
- \( Q_{i,\text{ext}} \) = heat into node i from outside the tank

and the last two terms of equation (3.3) represent heat transfer that cannot be modeled by a linear relationship, namely radiation heat transfer. The radiative and conductive heat transfer due to external nodes has been calculated using the spacecraft bulk model from Hughes' Thermophysics Group. The bulk model takes into account the change in the spacecraft's thermal environment due to its orbital position and models the heat transfer among the larger spacecraft components. The spacecraft bulk model was modified to subdivide the pressurant tank wall into the same nodes as used in our detailed model. The corresponding heat transfer into each node from other spacecraft components is updated every hour. Since the satellite's solar angle changes significantly once every three hours, an update time of once every hour was deemed sufficient.

The resistances are calculated by the model based on physical parameters (i.e. distance...)

---

5 The radiative heat transfer calculation follows from a model developed in Mills, pp. 571-573 (Reference 11).
between nodes) and the materials' thermal conductivities. A distinction is made between the thermal conductivity of the composite in the direction parallel to the fibers and in the direction perpendicular to the fibers. The thermal conductivity of the pressurant gas nodes is updated periodically since it depends on the temperature of the gas. This update time is also an input into the model. The specific heat of the pressurant gas varies little with temperature and was taken to be constant.

The system of equations that are contained in equation (3.3) are solved by creating a linear system of equations and numerically integrating it by marching forward in time to yield the node temperatures as a function of time. The next section explains this solution process.

3.3.3 Matrix Representation

For N nodes, equation (3.3) can be expanded into a system of N equations, which can be written in matrix form:

\[ P \frac{\partial T}{\partial t} = A^T CAT + Q \quad (3.4 \text{a}) \]

or

\[ P \frac{\partial T}{\partial t} = GT + Q \quad (3.4 \text{b}) \]

P is a diagonal matrix containing the products of the nodes' masses and specific heats, T and Q are column vectors with the temperature of each node and heat into each node, respectively. Both P and Q have N - 1 rows. The original system has N equations, but one of the nodes must be grounded, which removes one equation. The solution of the linear system is a set of N - 1 temperatures which are expressed with respect to the grounded node (if \( T(2) = -3.0 \), this means that the temperature of node 2 is 3 degrees lower than the temperature of the grounded node).

The matrix G is the conductivity matrix and is composed of the resistance terms from equation (3.3). As shown in equation (3.4 a), the matrix G is composed by the combination of matrices A and C, which are easier to generate. A is called the connectivity matrix and is constructed by noting how
the nodes in the system are connected. It has nconn rows and N - 1 columns, where nconn is the number of connections between nodes. Each row of A is composed of a -1 in the column corresponding to the node from which the connection leaves and a 1 in the column corresponding to the node which it enters. For example, connector 2 "leaving" node 3 and "entering" node 5 would have a second row of A that looks like: \[ 0 0 -1 0 1 0 0 \ldots \]. The direction of the connection flows (which node is designated as -1) is arbitrary as long as it is consistent, it only indicates in which direction the heat is flowing. The matrix C is a diagonal matrix of size nconn. The ith component of C is 1/resistance of the ith connector. The conductivity matrix G is formed by multiplying the transpose of A, C and A

\[ Q \] is a column vector of size N - 1. It contains the sum of the external and internal heat flows. As explained above, this vector is updated periodically, but treated as constant between updates.

Multiplying equation (3.4b) by the inverse of matrix P gives a linear system of equations of the form

\[ \frac{\partial T}{\partial t} = AT + B \]

(3.5)

where A is a matrix and B an N - 1 column vector. Equation (3.5) has solutions of the form

\[ T_j = \sum_{i}^{N-1} V_{ij} e^{-\lambda_i t} \]

(3.6)

where:

\[ \lambda_i = \text{eigenvalues of matrix A} \]

\[ V_i = \text{a function of initial conditions, A and B} \]

Therefore, the smallest eigenvalue of matrix A is an indicator of the time it takes the pressurant tank to reach thermal equilibrium. It is only an indicator because the magnitude of the exponential, \( V_{ij} \), controls the magnitude of each exponential decay term. The value \( 1/\lambda_{\text{min}} \) is referred to as the tank's

\[ \text{---} \]

\[ ^6 \text{The numerical algebra analysis briefly described here is taken from Strang, pp 110 - 120 (Ref. 19)} \]
thermal time constant and it is an indicator of the time required for the temperature transients to subside to 0.37 of their initial value.

Equation (3.5) is discretized in time using the bilinear transform and the node temperatures are calculated at discrete points in time. If the integration is performed over a long enough time period (greater than $3/\lambda_{\text{min}}$) the initial transients decay (to 0.05 of their initial value) and the steady state solution is obtained. This is the equivalent of setting $\delta T/\delta t = 0$ in equation (3.5) and solving for $T$.

For the case of the HS601 pressurant tank, although $\lambda_{\text{min}}$ is on the order of 10 minutes, the settling time is limited by the magnitude of other exponential terms and the tank temperatures do not reach steady state conditions until approximately 3 hours after an initial disturbance. Since the tank thermal environment is updated every hour, the tank never reaches thermal equilibrium - the thermal environment changes before the tank can completely adjust to it.

From the node temperatures and the tank pressure, the pressurant mass in the tank and the effective tank temperature can be calculated. This effective temperature is the temperature the tank would have if it was isothermal but otherwise under the same conditions (at the same pressure and containing the same pressurant gas mass). The pressurant gas mass is calculated first,

$$m_{\text{tot}} = \frac{P}{R} \sum_{i}^{N} \frac{V_i}{Z_i T_i} \quad (3.7)$$

and it is then used to calculate the effective gas temperature and effective gas compressibility (which is a function of both $P$ and $T_{\text{eff}}$, and must be solved for iteratively).

$$T_{\text{eff}} = \frac{PV_{\text{tot}}}{m_{\text{tot}} R Z_{ \text{eff} }} \quad (3.8)$$

where:

- $P$ = gas pressure
- $R$ = ideal gas constant/molar mass
- $T_{\text{eff}}$ = effective tank temperature
\[ T_i = \text{temperature of node } i \]
\[ V_i = \text{volume of node } i \]
\[ V_{\text{tot}} = \text{total tank volume} \]
\[ Z_{\text{eff}} = \text{effective pressurant gas compressibility} \]
\[ m_{\text{tot}} = \text{total mass of pressurant gas} \]

The quantities in equations (3.7) and (3.8) are evaluated based on the equilibrium (steady-state) temperatures of the tank.

The temperature gradients given by the model and the difference between the effective tank temperature and the sensor temperature are indicators of the bias errors incurred by the current propellant gauging system. Not included in this estimate of the bias errors are errors due to sensor calibration or lags due to sensor response times. Also, the assumption of negligible temperature drop between the tank wall and the sensor needs to be checked more carefully. Although the distance between the wall and the sensor is very small, the cross sectional area is also small and the conductive resistance could be significant.

The accuracy of the pressurant tank model developed here needs to be validated. Since the tank is equipped with only one sensor, validation is not possible based on available orbital data. Particular attention should be paid to the assumption that conductive heat transfer between nodes is instantaneous.

A sample calculation of the tank temperature distribution is presented in Chapter 4.
Chapter 4. Data Analysis

The model developed in Chapter 3 can be used to gain insight into the behavior of the pressurant tanks, which in turn leads to more accurate propellant gauging procedures. In this chapter, trends in tank pressures and temperatures are first described (for both pressurant and propellant tanks). The long-term pressure and temperature behavior of the tanks is not easily determined from the model due to the large number of calculations that would be required. Instead, trends in orbital tank pressures and temperatures are reviewed for their impact on gauging procedure such as the scheduling of gaugings.

The statistical calculation of pressure-temperature lags from orbital data is then explained. Statistically calculated lags for several cases are presented.

Finally, a sample calculation of the tank temperature distribution is performed using the tank model.

4.1 Tank Pressure and Temperature Trends

The pressures and temperatures of the propulsion system tanks at any point in the mission depend on factors such as the propellant and pressurant masses in the tanks and the heat flow into and out of the tanks. In the case of the HS601 satellite, all stationkeeping maneuvers are performed in blowdown mode with the pressurant tanks isolated from the propellant tanks. Except during propellant gaugings, no pressurant gas is removed from the pressurant tanks. Thus, the pressurant tanks' pressure will only change due to heating of the tanks or to propellant gaugings. On the other hand, the pressure in the propellant tanks will decrease throughout the mission as propellant is used and will increase when gaugings are performed. The change in tank pressures and temperatures due to mass introduced or removed from the tank will be ignored since it does not apply to the propellant remaining calculation\(^1\). The rest of the chapter will only deal with tank pressures and temperatures for this constant mass case, where the only changes in pressures and temperatures are due to

\(^1\)Although propellant gauging involves the transfer of pressurant from pressurant to propellant tanks, the data used in calculating the propellant mass is taken before and after this transfer. During these periods of data gathering the tanks are isolated and can be considered closed systems.
thermodynamic effects.

Aside from a dependance on the propellant and pressurant masses, the tanks' pressures and temperatures are a measure of the surrounding thermal environment. The variation in tank temperatures due to the changing thermal environment is best described as three effects acting in parallel. The tank temperatures show a daily cycle, an annual cycle and an overall increase over the life of the spacecraft. These temperature changes cause a corresponding pressure change.

The diurnal cycle in the tank pressures and temperatures is due to the changing solar radiation that the satellite is exposed to as it rotates around the sun. Figure 4.1 correlates the satellite position over a day to a pressurant tank’s pressure and temperature. Both pressurant and propellant tanks are located near the side of the satellite that faces away from the earth at all times (labeled side A).

![Figure 4.1 Filtered pressure and temperature of an HS601 pressurant tank](image-url)
The time reference frame illustrated in Figure 4.1 is called the satellite's time reference frame. From this point on, data will usually be compared on this time scale, to allow comparison of satellites at different longitudes. The pressure and temperature profiles shown in Figure 4.1 correspond to a period near one of the solar solstices, when the satellite is always in view of the sun. Thus, even between the hours of 18:00 and 6:00 the satellite receives direct sunlight - the orbital plane is tilted out of the paper. At certain times of the year (eclipse season), the satellite can be in the earth's shadow during part of its orbit - the orbital plane is the plane of the paper and at 0 hrs, the earth shields the satellite from the sun.

The tank pressures and temperatures also follow an annual cycle as the satellite orbits around the sun. Figure 4.2 shows the satellite's orbit around the sun and the corresponding daily-averaged temperatures in two propellant tanks: a north-facing tank and a south-facing tank. During the period shown the tanks are not entirely undisturbed, a gauging is performed in each of the tanks and some propellant is used up. However, the temperature curves are not very sensitive to the changes in the propellant and pressurant gas masses inside the tanks. The temperature trends shown in Figure 4.2 are the same as those seen in undisturbed tanks.

Both south and north-facing tanks show average temperatures peaking around the solar solstices (June 21 and December 22), which nearly coincide with aphelion (July 4) and perihelion (January 3). The average tank temperatures then decrease as the satellite enters eclipse season and are lowest near the equinoxes (March 21 and September 22). Note that the propellant tank temperatures change the least near the solar solstices and equinoxes. However, the gaugings are currently performed only near solstices because the period of near-constant tank temperatures is longer then. The periods around the equinoxes when the temperature is nearly constant are not long enough for transients due to the injection of pressurant to settle out. The south-facing tank is hotter during the winter solstice since it is slightly closer to the sun (due to the angle of the orbit). By the same reason, the north-facing tank is hotter during the summer solstice. Although gaugings were performed on the tanks shown, the tank temperatures are not as sensitive as the tank pressures so the trend remains the same.

Finally, there is a general temperature increase with the life of the satellite. This is due to the
degradation of the radiators' emissivity over the life of the satellite. Figure 4.3 shows the predicted

Figure 4.2 Solar orbit for an geosynchronous satellite and corresponding (daily-averaged) ullage temperatures in a north-facing propellant tank and a south-facing propellant tank of an HS601 satellite.
ullage temperature increase of an HS601 propellant tank over the life of the spacecraft. The nominal spacecraft lifetime is 15 years. Although HS601 orbital temperature data has only been continuously collected for a maximum of three years on any given spacecraft, the available data shows the same trends as the predictions of Figure 4.3.

Figure 4.3 Predicted ullage temperature of an HS601 fuel tank over S/C lifetime

Seasonal variations have been omitted for clarity

\[ Tu = \text{ullage temperature} \quad \text{To} = \text{ullage temperature, first year of life} \]

4.2 Pressure -Temperature Lag

The existence of a lag between the propulsion system tanks' pressures and temperatures does not present a problem to the gauging calculations if the lag can be accurately calculated and removed. The study of the pressure-temperature lag is limited to how it affects the calculation of remaining propellant. The calculations of remaining propellant use data collected over a 2-3 day period during which no propellant is used and no pressurant gas transferred (both pressurant and propellant tanks are closed systems). The lags referred to in this section correspond to similar data periods.

4.2.1 Lag Calculation

The pressure-temperature lag was statistically calculated by maximizing the pressure-temperature cross-correlation function. In the time domain, the cross correlation of two functions measures how "events" in one function at a given time, \( t \), relate to events in the second function at
a different time, \( t + \tau \). If the two functions describe processes that affect each other (like heat flux and temperature), the cross correlation function shows how quickly one function reacts to a change in the other (the time lag or response time).

Y. W. Lee [15] defines the cross-correlation function between two functions \( f(t) \) and \( g(t) \) as

\[
\phi_{fg}(\tau) = \frac{1}{T} \int f(t)g(t + \tau) \, dt
\]  

where:

- \( \phi_{fg} \) = cross-correlation function
- \( \tau \) = time lag/lead (chosen arbitrarily)
- \( T \) = total time interval
- \( t \) = time

or, for a set of \( N \) discrete data points,

\[
\phi_{fg} = \frac{1}{N \Delta t} \sum_{n=1}^{N} f(n \Delta t)g(n \Delta t + \tau) \Delta t
\]

where:

- \( N \) = total number of data points
- \( \Delta t \) = time interval between data samples (\( N \Delta t = T \))
- \( n \) = sample number
- \( \tau \) = time lag/lead (arbitrarily chosen)

The lag is found by calculating the value of the cross-correlation function, \( \phi \), for several (arbitrary) values of the time lag, \( \tau \). If \( f(t) \) and \( g(t) \) are non-periodic functions, the value of \( \tau \) for which the cross-correlation function is a maximum yields the time lag between the two functions. If no lag is present, the cross-correlation function will be a maximum for \( \tau = 0 \). Figure 4.4 shows a graphical example of the lag calculation for two non-periodic functions with a time lag, \( \tau \). Figure 4.4a shows the two functions, \( f \) and \( g \) as they would be measured in real time. The maximum of the \( f(t) \) occurs at \( t = 26 \), while the maximum of \( g(t) \) occurs at \( t = 37 \). The cross-correlation of the two functions is plotted on Figure 4.4b. The cross-correlation function has maximum value at
approximately $\tau = 11$, indicating that the functions $f$ and $g$ follow a similar behavior but $g(t)$ lags $f(t)$ by 11 minutes.
The pressures and temperatures of the pressurant and propellant tank are periodic functions, with a period of (approximately) 24-hrs. This means that the cross-correlation function can be maximized for several values of the time lag, \( \tau \). Figure 4.5 shows a plot of the pressure-temperature cross-correlation function of a typical HS601 propellant tank.

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**Figure 4.5a Pressure and temperature of HS 601 propellant tank**

![Graph showing pressure and temperature over time]

**Figure 4.5b Cross-correlation function**

![Graph showing cross-correlation function over time lag]
Since both the pressure and the temperature have the same period, the cross-correlation function's maxima will repeat with the same periodicity of the pressure and temperature functions. Thus, the maximum shown in Figure 4.5 at $\tau = -51$, will repeat at nearly 24 hours apart ($\tau = 3549$, etc.). But which maximum gives the pressure-temperature time lag? This depends on whether it is the pressure or the temperature that leads, which is a function of the direction of heat flow and the sensor position (Figure 2.5). Since this cannot be determined from orbital pressure and temperature alone, the lag with the smallest absolute value was taken as the statistically calculated lag. This lag value can be easily corrected based on the physical model because the subsequent maxima are approximately 24 hrs. apart.

Equation 4.1 and the previous discussion are only valid for a stationary, zero-mean function. A stationary function is one whose average does not change with time. As seen in Figure 4.2, the tank pressures and temperatures are not stationary, their diurnal average changes over a year, especially during eclipse season. However, as gaugings take place near solar solstices where the slope of the pressure and temperature curves is near zero (Figure 4.2), the pressure and temperature curves can be assumed stationary. The decreasing value of the successive maxima in Figure 4.5 is partly due to the non-stationary nature of the pressure and temperature functions (it is also due to noise in the signals).

4.2.2 The Effects of Filtering Pressure and Temperature Data on Lag Calculations

Raw pressure and temperature orbital data is not suitable for gauging calculations. Some preprocessing is required to minimize the error introduced. The signals are first interpolated, then filtered. The effect of this pre-processing on the computed lag is discussed here.

The raw pressure and temperature signals are not evenly spaced in time. The signals must first be interpolated since both the filter and lag calculation processes - equation (4.1) - require evenly spaced data points. The pressures and temperatures are sampled at a rate of one reading every 30-60 seconds. This rate is believed to be fast enough to avoid aliasing of the signal. The raw data is then linearly interpolated to yield evenly spaced data points.

After being interpolated, the signal is filtered. The pressure and temperature signals are filtered to achieve different effects. The propellant tank pressures are filtered to remove transients
seen as spikes in Figure 4.6.

Figure 4.6 Unfiltered pressure of an HS601 propellant tank

The pressurant tank pressure signal does not contain such transients and is therefore not filtered. The pressurant and propellant tank temperatures are filtered, as shown in Figure 2.4, to reduce the effects of quantization. Quantization results when the analog temperature signal is converted to a digital signal - the step size is proportional to the range of the temperature sensor and decreases as the number of bits available for transmission increases. A low pass filter is effective in removing the effects of quantization (with little error) when the step size is much smaller than the range of variation of the signal [16]. This is not the case for many of the propellant tank temperatures examined here (i.e. data shown on Figure 2.4) due to the coarseness of the temperature sensors used. Although the filter error is not negligible in this case, filtering is still done since the filter can reduce the signal uncertainty by up to 40%.

It is after this processing of the data (interpolating and filtering) that the lag and the remaining propellant are calculated. Although the interpolating is required for lag calculations, the lag is a weak function of the quantization and the lag of the unfiltered data can be calculated instead [this is done to simplify the data reduction].

Table 4.1 compares the lags calculated from interpolated pressure and temperature data to those calculated after interpolation and filtering of the data. Filter A is representative of the optimal filter chosen for this particular temperature sensor. Filter B yielded a filtered signal with larger errors but even with this unsuitable filter the lag remains fairly constant.
Table 4.1 Pressure-temperature time lag (in minutes) of a typical HS601 satellite calculated with unfiltered and filtered data. [T1, T2 and T3 indicate the temperature sensor positions shown in Figure 2.22]

<table>
<thead>
<tr>
<th>Tank</th>
<th>f-g</th>
<th>Lag (minutes)</th>
<th>Lag (minutes)</th>
<th>Lag (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressurant</td>
<td>P-T</td>
<td>12</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Oxidizer</td>
<td>P-T1</td>
<td>94</td>
<td>106</td>
<td>100</td>
</tr>
<tr>
<td>Oxidizer</td>
<td>P-T2</td>
<td>-165</td>
<td>-162</td>
<td>-159</td>
</tr>
<tr>
<td>Oxidizer</td>
<td>P-T3</td>
<td>-369</td>
<td>-362</td>
<td>-368</td>
</tr>
</tbody>
</table>

Note: A positive lag value indicates that the temperature signal leads. A negative lag value indicates that the pressure signal leads.

Since the pressure-temperature lag does not change much with the filtering of the pressure and temperature signals, the lags calculated in this section are those of the unfiltered signals. The pressure and temperature signals are still filtered before propellant remaining calculations are performed.

4.3 Calculated Values of the Pressure-Temperature Lag

Given the tank temperature variations described in section 4.1, the pressure-temperature lag can be expected to vary greatly from season to season and show a lesser variation from year to year. Also, the lags should differ within the propellant tanks since their position in the spacecraft gives them slightly different thermal environments. Finally, the lag of the pressurant tanks will differ from that of the propellant tanks not only due to the different thermal environment of the tanks, but to the different thermal capacities and resistances of the tanks.

The pressure-temperature lag of propellant and pressurant tanks was statistically calculated for different cases:

\[ T_1 \text{ sensor is closest to pressurant gas, } T_3 \text{ is closest to liquid propellant, } T_2 \text{ is roughly halfway between } T_1 \text{ and } T_3. \]
### Table 4.2 Pressure-temperature time lag (in minutes) of HS601 pressurant tanks

<table>
<thead>
<tr>
<th>Spacecraft</th>
<th>Year of Life</th>
<th>Time of Year</th>
<th>Pressurant 1</th>
<th>Pressurant 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>summer solstice</td>
<td>-7</td>
<td>-12</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>summer solstice</td>
<td>27</td>
<td>-29</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>winter solstice</td>
<td>-10</td>
<td>-21</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>fall</td>
<td>-17</td>
<td>-8</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>spring</td>
<td>10</td>
<td>-13</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>summer</td>
<td>-26</td>
<td>-44</td>
</tr>
</tbody>
</table>

### Table 4.3 Pressure-temperature time lag (in minutes) of fuel tanks for HS601 spacecraft A

<table>
<thead>
<tr>
<th>Year of Life</th>
<th>Time of Year</th>
<th>Fuel 1 Press - T1</th>
<th>Fuel 1 Press - T3</th>
<th>Fuel 2 Press - T1</th>
<th>Fuel 2 Press - T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>summer solstice</td>
<td>-79</td>
<td>-42</td>
<td>not found</td>
<td>not found</td>
</tr>
<tr>
<td>2</td>
<td>fall</td>
<td>-65</td>
<td>-115</td>
<td>-185</td>
<td>-83</td>
</tr>
<tr>
<td>2</td>
<td>spring</td>
<td>-81</td>
<td>1</td>
<td>-145</td>
<td>-281</td>
</tr>
<tr>
<td>1</td>
<td>summer</td>
<td>110</td>
<td>98</td>
<td>118</td>
<td>616</td>
</tr>
</tbody>
</table>

### Table 4.3 Pressure-temperature time lag (in minutes) of oxidizer tanks for HS601 spacecraft A

<table>
<thead>
<tr>
<th>Year of Life</th>
<th>Time of Year</th>
<th>Oxidizer 1 Press - T1</th>
<th>Oxidizer 1 Press - T3</th>
<th>Oxidizer 2 Press - T1</th>
<th>Oxidizer 2 Press - T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>summer solstice</td>
<td>-55</td>
<td>not found</td>
<td>-141</td>
<td>not found</td>
</tr>
<tr>
<td>2</td>
<td>fall</td>
<td>-51</td>
<td>not found</td>
<td>-29</td>
<td>not found</td>
</tr>
<tr>
<td>2</td>
<td>spring</td>
<td>-44</td>
<td>-332</td>
<td>-79</td>
<td>-319</td>
</tr>
<tr>
<td>1</td>
<td>summer</td>
<td>70</td>
<td>612</td>
<td>98</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: A positive lag value indicates that the temperature signal leads. A negative lag value indicates that the pressure signal leads.
The data of Tables 4.1 through 4.3 show that pressure-temperature lags were observed on several spacecraft, at different times of the year and different periods in the spacecraft's lifetime. However, the data shows no obvious correlation between the time of year or the tank type and the pressure-temperature lag. The tank thermal environment and the propellant masses in the tanks may be the leading factors determining the pressure-temperature lag size.

4.4 Sample Tank Temperature Calculation

The temperature response of an HS601 pressurant tank to a step impulse was calculated. Figures 4.7a and 4.7b show the resulting temperature of the tank wall nodes, labeled by tank sections\(^3\). The temperatures are to an arbitrary reference temperature, T\(_{\text{ref}}\). The prescribed heat flow is not representative of the thermal environment of the tank but is meant as a simple example. The heat flows into the tank through the two nodes on the aft-facing part of cylinder 2 and flows out through the two earth-facing nodes of cylinder 2. Thus it is not surprising that the nodes on cylinder 2 show the highest temperature differential. The maximum temperature gradient across the tank is 5 - 6 degrees K.

The temperature of the pressurant gas nodes are shown in Figure 4.8a. The temperature of each gas node closely follows the temperature of the nearest wall section. This may be an artificial effect, arising from the assumption that all heat flow between nodes takes place instantaneously.

Figure 4.8b compares the effective gas temperature, T\(_{\text{eff}}\) and the measured sensor temperature, T\(_{\text{sensor}}\). The effective gas temperature should follow the gas pressure exactly, so the lag between T\(_{\text{sensor}}\) and T\(_{\text{eff}}\) should be the same as the pressure-temperature lag. In this case, the orbital pressure and temperature showed no lag. The approximately 5 degrees K bias between T\(_{\text{sensor}}\) and T\(_{\text{eff}}\) would translate into an error in the gauging calculation. The size of the bias will change based on the tank's thermal environment. To establish if the temperature sensor location is appropriate for propellant gauging, orbital data from different gauging periods should be examined to find the maximum bias that can occur between the measured and effective gas temperatures.

\(^3\) Appendix B contains a diagram of the tank nodes and sections
Figure 4.7a Temperature of pressurant tank nodes

Figure 4.7b Temperatures of pressurant tank nodes
Figure 4.8a Temperature of pressurant gas

- gas temperature
- effective gas temperature

Figure 4.8b Temperatures of pressurant gas & sensor

- effective gas temperature
- sensor temperature
Chapter 5. Conclusions

A thermodynamic model of the HS601 pressurant tank was developed to estimate the temperature distribution within the tank. The tank was divided into constant volume nodes and the conductive and radiative heat transfer between the nodes was modeled. Natural convection was shown to be a negligible heat transfer mechanism and was therefore not modeled. The thermodynamic model includes the effects of gas compressibility and the dependence of gas properties (such as thermal conductivity) on temperature and pressure. The tank expansion is not modeled as its effects were shown to be negligible.

The pressurant tank model can be used to estimate the errors introduced into the gauging by the temperature sensor location and to determine the most effective location for the sensor. Given the pressurant tank's temperature and pressure sensor readings and a model of the heat transfer into the tank from other sections of the spacecraft, the effective gas temperature can be estimated. This effective gas temperature is the temperature the tank would have if it were isothermal but otherwise under the same conditions (thermal environment and under the same pressure and containing the same pressurant mass). The effective temperature, not the sensor temperature should be used in the propellant gauging equations, thus reducing propellant gauging errors.

Several assumptions made in developing the pressurant tank model may limit its applicability. The heat transfer between the tank nodes was assumed to be linear and instantaneous. The result is that the temperature of each gas node closely follows the temperature of the nearest wall section. The thermal resistance of the sensor was assumed to be negligible; in other words, the temperature sensor was assumed to read the temperature of the wall that it is attached to. Although the resistance of the bonding material is very small, the sensor cross sectional area is also small and the conductive resistance could be significant. Finally, the model does not account for sensor calibration errors or lags due to sensor response times. The effect of these assumptions on the temperature distribution given by the model needs to be checked in more detail and the tank model should be validated against orbital data.

The existence of a pressure-temperature lag in both propellant and pressurant tanks was established. However, the cause of the lag and the mechanisms that control it require further investigation.
Bibliography

I. Low-Gravity Propellant Gauging


II. Heat Transfer


12. G. P. Purohit, Propulsion System Model [of the HS 601 spacecraft]


IV. Signal Processing


V. Reference


Appendix A. Correction Factors to Ideal Gauging Equations

This appendix lists the equations used to correct for non-ideal effects in the gauging equations described in Section 2.2. The constants used are specific to the propellants and pressurant used by the HS601 satellite - monomethyl hydrazine (MMH) and nitrogen tetraoxide (NTO) as propellants, helium gas (He) as pressurant. The correlations were obtained from Air Force Technical Report AFRPL-TR-69-149.

Compressibility (He):

\[
Z = \left[ a_1 + a_2 P \right]^{a_3} \\
\]

\[
a_1 = 1.000933 \\
a_2 = 3.73505 \times 10^{-5} \left[ \frac{1}{\text{psi}} \right] \\
a_3 = 460 [R]
\]

\[ P = \text{gas pressure (psia)} \]
\[ T = \text{gas temperature (degrees Rankine)} \]

Vapor Pressure:

\[
\log P_v = b_1 + \frac{b_2}{T} + \frac{b_3}{T^2}
\]

\[
\begin{align*}
\text{MMH} & : \\
b_1 &= 5.39797 \\
b_2 &= -1988.299 [R] \\
b_3 &= -493216.9 [R^2]
\end{align*}
\]

\[
\begin{align*}
\text{NTO} & : \\
b_1 &= 6.6571 \\
b_2 &= -2524.3 [R] \\
b_3 &= -183600 [R^2]
\end{align*}
\]

\[ P_v = \text{vapor pressure (psia)} \]
\[ T = \text{liquid propellant temperature (degrees Rankine)} \]

Mass of dissolved pressurant:

\[
\log K_p = c_1 \cdot T^{c_2} + c_3
\]

\[
K_p = \frac{\text{mass}_{\text{He}}}{\text{mass}_{\text{propellant}}} \cdot \frac{1}{\text{pressure}_{\text{He}}} \left[ \frac{1}{\text{psi}} \right]
\]
$MMH:$
\[ c_1 = 2.90683 \cdot 10^{-3} \left[ \frac{1}{K} \right] \]
\[ c_2 = 1 \]
\[ c_3 = -8.38480 \]

$NTO:$
\[ c_1 = -609.335 \left[ \frac{R}{K} \right] \]
\[ c_2 = -1 \]
\[ c_3 = -5.47913 \]

$T =$ propellant temperature (degrees Rankine)

**Propellant Density:**

\[ \rho = d_1 + d_2 T + d_3 T^2 \]

$MMH:$
\[ d_1 = 71.8132 \left[ \frac{lbm}{ft^3} \right] \]
\[ d_2 = -0.032584 \left[ \frac{lbm}{ft^3 \cdot R} \right] \]
\[ d_3 = 0 \]

$NTO:$
\[ d_1 = 129.850 \left[ \frac{lbm}{ft^3} \right] \]
\[ d_2 = -7.175 \cdot 10^{-2} \left[ \frac{lbm}{ft^3 \cdot R} \right] \]
\[ d_3 = -6.3276 \cdot 10^{-6} \left[ \frac{lbm}{ft^3 \cdot R^2} \right] \]

$\rho =$ propellant density (lbm/ft$^3$)

$T =$ propellant temperature (degrees Rankine)
Appendix B. Pressurant Tank Nodes

The pressurant tank wall was divided into 14 nodes. Preliminary calculations indicated that most of the heat transfer in the wall was in the circumferential direction and the radial temperature drop in the wall was small. Thus, a single radial node is used to represent the wall - the specific heat of the wall and the resistances between nodes take into account the contributions from the metal and composite parts of the wall. The figure below shows the division of the pressurant tank wall into constant-volume, constant-temperature nodes.

Nodes 1 and 2 make up the boss. The pressurant gas nodes inside the boss are as shown below. Shaded nodes indicate wall nodes.
Nodes 3, 4, 13 and 14 make up the tank's elliptical endcaps. To distinguish them, the endcap closest to the boss (made up of nodes 3 and 4) is called endcap 1 and the endcap farthest from the boss is called endcap 2. Each endcap contains two pressurant gas nodes:

The tank's main body is made up of two cylindrical sections divided into nodes 5 through 8 and 9 through 12. Again, the section closest to the boss (nodes 5 through 8) is called cylinder 1 and the cylindrical section farthest from the boss is called cylinder 2. Unlike the two endcaps, these two cylindrical sections are physically identical. The pressurant gas inside the cylindrical sections is broken up into nodes as follows: