EXCHANGE RATE DYNAMICS WITH STICKY PRICES: 
THE DEUTSCHE MARK, 1974-1982 

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ABSTRACT

This paper estimates simultaneously dynamic equations for the Deutsche mark/dollar exchange rate and the German wholesale price index. These emerge from a model in which German prices are sticky. The stickiness is due to costs of adjusting prices of the form posited by Rotemberg (1982).

The main results of the empirical analysis are: first, the version of the model where prices are perfectly flexible is rejected. Second, in spite of the substantial price stickiness, we find that the nominal exchange rate undershoots in response to monetary innovations like those which appear to be typical in Germany.

KEY WORDS

Exchange rate, rational expectations, sticky prices.
1. INTRODUCTION

The idea that purchasing power parity holds with flexible exchange rates at every moment in time has been shown by the data since 1974 to be wrong. Real exchange rates, i.e. the ratios of aggregate price levels expressed in a common numeraire, are not constant, and, moreover, their changes tend to persist (see Frenkel (1981)). Furthermore, nominal exchange rates fluctuate more than price levels. One possible explanation for these phenomena, which is advanced by Dornbusch (1976), is that prices of goods produced for the domestic market change slowly. If asset markets are always in equilibrium, and goods prices move slowly, most of the volatility of the real exchange rate will be accounted for by movements of the nominal exchange rate. Our paper attempts to estimate a model of this type on German data since 1974. We too assume that prices move slowly. On the other hand, this paper differs in important respects both from Dornbusch's original formulation, and from its empirical implementation by Frankel (1979), Driskill and Sheffrin (1981), Backus (1984), Papell (1984) and Barr (1984).

First, we consider the impact of a relatively broad set of forcing variables on the paths of the price level and the exchange rate. In particular, we specify and estimate the effects of changes in prices of various imported goods and of changes in real wages. The effects of these latter variables on exchange rates has been the focus of theoretical work of Sachs (1980), among others.

Second, our price adjustment rule is not exogenously imposed. It arises instead from an explicit optimization problem followed by firms which, as in Rotemberg (1982a), are concerned that customers will desert them if they follow an erratic price behavior. This specification allows
for some contemporaneous reaction of the price level to current exogenous shocks, unlike the model estimated by Papell (1984) and Barr (1984), where quarterly price levels are predetermined.

Finally, we do not impose the assumption that forcing variables follow a random walk. Driskill and Sheffrin (1981) reject this hypothesis for the German money stock. Thus in our model the responses of exchange rates and prices to innovations in the forcing variables differ from those studied in the theoretical model of Dornbusch (1976), as well as from the simulation of Backus (1984). We let the data inform us as to the plausible stochastic processes followed by the forcing variables we consider. Then we assume that exchange rates and prices respond optimally in that private agents exploit their knowledge of these stochastic processes.

While we feel that these differences constitute important improvements over previous work, we must point out at the outset some important limitations. First, the period since 1974 is only partially a period of "flexible" exchange rates between Germany and the rest of the world which we aggregate into a dollar region. This is so because within Europe exchange rates are only allowed to move within bands. So, whether flexible exchange rates models can explain the DM exchange rate is an empirical question. Second, our model fails to incorporate the effects of government spending, the dynamics due to capital accumulation, and more generally the consequences of varying real interest rates. These simplifications allow us to derive closed-form expressions for endogenous variables, which we use to obtain maximum-likelihood estimates of the parameters and to perform simulations.
The structure of the paper is as follows. Section 3 estimates the model using German data, for the period from June 1974 to February 1983. Section 4 presents simulations of the responses of the endogenous variables to a variety of shocks and assesses the out-of-sample performance of our model. Finally, section 5 contains some concluding remarks. The appendices describe the solution of the model and the data used in the empirical section.

2. THE MODEL

Our economy is populated by a large number of monopolistically competitive firms, each producing a good that is differentiated from other domestic goods, and from foreign goods. Each firm faces the following demand curve for its product:

\[ Q_{it} = \left( \frac{P_{it}}{P_{dt}^\lambda (E_{t}P_{ct}^*)^{1-\lambda}} \right)^{-\gamma} \left( \frac{M_{t}}{P_{dt}^\lambda (E_{t}P_{ct}^*)^{1-\lambda}} \right) Q_t^* N_{lt} \]

where \( Q_{it} \) is demand for good \( i \) at time \( t \). The first term in the right hand side of (1) represents substitution between domestic and foreign goods, and captures substitution both by domestic residents and by foreigners. \( P_{dt} \) is the index of domestic goods prices, a geometrically weighted average of \( P_{it} \)'s. \( P_{ct}^* \) is the price of foreign consumption goods, taken a exogenous. This implies that although our country is "small," each domestic producer does not face a perfectly elastic demand schedule by foreign residents. The second term on the right hand side of (1) represents a real balance effect on domestic demand, \( Q_t^* \) stands for foreign activity, and \( N_{lt} \) is a random variable which affects demand for goods.
On the cost side, we assume that all domestic goods, together with imported intermediates and labor are used in the production of each good $i$. Ignoring constant terms, marginal costs of production of good $i$ is:

$$
\frac{\beta}{Q_{it}} \frac{(1-\alpha_1-\alpha_2)}{W_t} \left( E_t P^*_N \right)^{\alpha_2} N_{2t}^{\alpha_1}
$$

with $W_t$ the nominal wage rate, $P^*_N$ the foreign currency price of imported intermediate goods, and $N_{2t}$ is a random variable affecting productivity. When $\beta = 0$ we have the constant variable costs case.

Real wages relative to the CPI are given by:

$$
W_t = K_t P^*_t \left( E_t P^*_c \right)^{1-\lambda}
$$

where $K_t$ is the real consumption wage at time $t$.

Domestic producers are assumed to observe $M$, $P^*_C$, $P^*_N$, $Q^*$, $E$ and $K$ at the time of their pricing decision. In the absence of costs of changing prices, domestic producers would charge $\bar{P}_{it}$ at which marginal cost equals marginal revenue. In natural logarithms, it is equal to

$$
\frac{-1}{1+\beta^2} \left\{ [\beta \lambda (\gamma - d) + 1 - \alpha_1 - \alpha_2 - \lambda \alpha_1] P_{dt} + [\beta (1 - \lambda) (\gamma - d) + (1 - \lambda) \alpha_1] P_{ct}^* + a_2 P^*_N + \alpha_1 k_t + \beta d m_t + \beta f q_t^* + n_t \right\}
$$

where lowercase letters are the logs of the variables represented by the corresponding uppercase letters, and $n_t = n_{2t} + \beta n_{1t}$.

However, we follow Rotemberg (1982a) by assuming that monopolists also have convex costs of changing nominal prices. Once a second order
approximation to their profit function is used, the monopolists objective function is:

\[
(5) \quad \text{Min } E_t \sum_{j=0}^{\infty} \rho^j [ (\bar{p}_{it+j} - \bar{p}_{it+j})^2 + c(\bar{p}_{it+j} - p_{it+j-1})^2 ]
\]

where \( \rho \) stands for a constant discount factor, \( E_t \) is the expectations operator, conditional on information available at time \( t \), and \( c \) represents the cost of changing prices.

The first order conditions for this problem are:

\[
(6) \quad t^P_{it+1} - \left( \frac{1 + c + \rho c}{\rho c} \right) p_{it} + \frac{1}{\rho} p_{it-1} = - \frac{1}{\rho c} \bar{p}_{it}
\]

where for every variable \( x \), \( x_{t+j} \) indicates the expectation of \( x \) at time \( t+j \) conditional on information available at \( t \). The transversality condition is:

\[
(7) \quad \lim_{k \to \infty} (t^P_{it+k} - \bar{p}_{it+k}) + c(t^P_{it+k} - t^P_{it+k-1}) = 0
\]

Aggregating equation (6) one obtains:

\[
(8) \quad t^P_{dt+1} - \frac{1}{\rho \phi^3} p_{dt} + \frac{1}{\rho} p_{dt-1} = - \frac{1}{\rho \phi^3} (1 - (1 + \rho) \phi^3 - \phi^2) e_t + (1 - (1 + \rho) \phi^3 - \phi^2 - \frac{\alpha^2}{\phi^2}) P_{ct}^*
\]

\[
+ \frac{\alpha_1}{\phi^2} k_t + \frac{\alpha_2}{\phi^2} p_{Nt}^* + \phi_7 m_t + \phi_6 q_t^* + n_t
\]
where
\[ \phi_2 = (1 + \rho)c + \beta \gamma [(1 + \rho)c + (1 - \lambda)] + \lambda \beta \delta + (1 - \lambda)\alpha_1 + \alpha_2 \]
\[ \phi_3 = \frac{c(1 + \beta \gamma)}{\phi_2}, \quad \phi_6 = \frac{\beta \delta}{\phi_2}, \quad \phi_7 = \frac{\beta d}{\phi_2} \]

As shown below in equation (16), the solution to (8) which satisfies the aggregate version of the transversality condition (7) is that prices at \( t \) depend on prices at \( t-1 \) and on the current and expected future values of all the forcing variables in the model. If (and only if) all these variables obey a random walk the solution reduces to the partial adjustment formula of Dornbusch (1976), in which the rate of change of prices is proportional to the difference between \( \bar{p}_t \) and \( p_{t-1} \). It is this restrictiveness of the partial adjustment formula which makes its direct empirical application unappealing. While numerous theoretical alternatives to partial adjustment have been offered (see Obsloed and Rogoff (1984) for a survey), the empirical literature (including Frankel (1979), Driskill and Sheffrin (1981), Papell (1984), and Buiter and Miller (1982)) with the exception of Backus (1984), has focused on equations that do not flow from well posed individual optimization problems.

It is worth noting two thing at this point. First, while equation (8) is derived under the assumption that there are convex costs of changing prices, the same equation could have been derived from the model of Calvo (1983). This model assumes that producers who discount future profits by \( \rho \) are only capable of changing prices in any one period with probability \( \pi \). This equivalence in the aggregate implications of the two models is proved by Rotemberg (1987). Second, our focus here is on price rigidity only because, in Germany, overlapping wage contracts of the type observed in the US do not exist. (See Sachs (1979)).
The model is closed with the specification of asset markets equilibrium. The ex-ante interest rate differential is specified as follows:

\[ (9) \quad t^e_{t+1} - e_t = i_t - i^*_t + n_{3t} \]

where \( i_t \) and \( i^*_t \) are the domestic and foreign interest rate, respectively while \( n_{3t} \) is an exogenous risk premium. Finally, we have the money demand equation:

\[ (10) \quad m_t - \lambda p_{dt} - (1 - \lambda)(e_t + p^*_t) = aq_t - bi_t + n_{4t} \]

where \( n_{4t} \) is a random variable affecting velocity. In order to obtain the dynamics of the exchange rate, we need to specify the behavior of equilibrium output \( q_t \). We assume that domestic firms are never rationed in the amount of labor and intermediate goods they can buy. They supply whatever quantity of the good they produce is demanded. Then, equilibrium domestic output is given by aggregating (1):

\[ (11) \quad q_t = (e_t + p^*_t)(\gamma - d)(1 - \lambda) - [\gamma - (\gamma - d)\lambda]p_{dt} + f_{q} q^*_t + dm_t + n_{1t} \]

the dynamics of the exchange rate are obtained by substituting (11) into money demand, and using the relation (9):

\[ (12) \quad t^e_{t+1} = \phi_1 e_t - (1 - \phi_1 - \phi_4)p_{dt} = \phi_4 m_t + (\phi_1 - 1)p^*_t + \phi_5 q^*_t - i^*_t + n_{1t} \]
where \( \ddot{n}_t = a_1 n_{1t} / b + n_3 t + n_4 t / b \), \( \phi_4 = \frac{ad - 1}{b} \)

\[
\phi_1 = 1 + \left( \frac{1 - \lambda}{b} \right) (1 + a(\gamma - d)) \quad \phi_5 = \frac{af}{b}
\]

This completes the specification of the model which now consists of the two dynamic equation (8) and (12). For these equations to have a unique solution we need three boundary conditions. One is given by the predetermined value of \( p_{dt-1} \), while the other is given by the aggregation of (7) across firms. We obtain a third one by assuming that there are no exchange rate bubbles, (so that exchange rates are expected to converge to their long run value). Since these last two conditions imply only that the system converges over time, they are sufficient to generate a unique solution only if the characteristic equation of the system (8) and (12) has one stable root and two unstable roots. Then the solution can be written as:

\[
(13) \quad p_{dt} = \omega_0 p_{dt-1} - \left( \frac{1}{J_{21}} \right) \sum_{i=1}^{8} \omega_{i1} \left[ \sum_{j=0}^{\infty} \left( \frac{1}{J_{21}} \right) t^{z_{it+j}} \right]
\]

\[
- \left( \frac{1}{J_{22}} \right) \sum_{i=1}^{8} \omega_{i2} \left[ \sum_{j=0}^{\infty} \left( \frac{1}{J_{22}} \right) t^{z_{it+j}} \right]
\]

and

\[
(14) \quad e_t = \xi_0 p_{dt-1} - \left( \frac{1}{J_{22}} \right) \sum_{i=1}^{8} \xi_{i1} \left[ \sum_{j=0}^{\infty} \left( \frac{1}{J_{22}} \right) t^{z_{it+j}} \right]
\]

\[
- \left( \frac{1}{J_{22}} \right) \sum_{i=1}^{8} \xi_{i2} \left[ \sum_{j=0}^{\infty} \left( \frac{1}{J_{22}} \right) t^{z_{it+j}} \right]
\]
where the $w$'s, the $\xi$'s, $J_{21}$ and $J_{22}$ are functions of the parameters of (8) and (12) while the elements of the vector $z$ are $m$, $p^*_c$, $p^*_N$, $k$, $q^*$, $i^*$, $n$, and $\tilde{n}$.

While the model exhibits short run nominal rigidities in that prices and exchange rates depend on lagged prices, it is neutral in the long run with respect to once and for all changes in the stock of money. These eventually lead to equiproportionate increases of $p_d$ and $e$. However, the model is not superneutral: permanent changes in the rate of growth of nominal variables have real effects.

3. ESTIMATION

The estimation of the structural parameters (the $a$'s and the $\phi$'s) proceeds in two steps. We first obtain consistent (but inefficient) estimates by instrumental variables applied to (8) and (12). Then, we obtain efficient parameter estimates by maximum likelihood applied to (13) and (14). Maximum likelihood requires that these equations be estimated jointly with the stochastic process of the forcing variables. In practice, the results of these two procedures turn out to be similar.

There are two differences between (8) and (12) on the one hand and the equations we actually estimate by instrumental variables on the other. First, we multiply equation (8) by $\phi_3$. This change in normalization facilitates testing for the absence of costs of changing prices: in that case $\phi_3 = 0$. Second, we replace expected values at $t$ ($t^p_{t+1}$ and $t^e_{t+1}$) by the corresponding realized values. This leads to composite disturbance terms equal to $\xi_{1t}$ for (8) and $\xi_{2t}$ for (12):

$$\xi_{1t} = \epsilon_{1t+1} + \eta_t,$$
$$\xi_{2t} = \epsilon_{2t+1} + \tilde{n}_t$$
Here $\epsilon_{1t+1} = p_{dt+1} \cdot t^p_{dt+1}$ and $\epsilon_{2t+1} = \epsilon_{t+1} - t^{e_{t+1}}$. As Cumby, Huizinga and Obstfeld (1983) point out, $z_{1t}$ and $z_{2t}$ are likely to be serially correlated. The structural disturbances $n_t$ and $\tilde{n}_t$ affect $p_{dt}$ and $e_t$ unexpectedly, thus changing $\epsilon_{1t}$ and $\epsilon_{2t}$. Therefore, even when $n_t$ and $\tilde{n}_t$ are not serially correlated, $\epsilon_{1t}$ and $\epsilon_{2t}$ are first order moving average processes. The estimates of the parameters are obtained taking this into account, since we follow the generalized instrumental variables procedure of Hansen (1982), Hansen-Singleton (1982), and Cumby-Huizinga-Obstfeld 1983). The results are presented in Table 1.

Column I contains estimates for our original specification. However, since $\alpha_1$ and $\alpha_2$ pertain to production functions estimated more precisely elsewhere, we reestimate our model equating them to 0.25 and 0.11, respectively. These are the values obtained by Dramais (1980). In addition to identifying $\phi_2$, this imposes an additional constraint. Column II presents the resulting estimates. The estimates of column I and II are obtained allowing the disturbance terms to be conditionally heteroskedastic. The instruments are $p_{d_t}$, $e$, $p_c^*$, $p_n^*$, $q^*$, $\Delta t^*$, $k$ and $\Delta m$ at $t-1$. These are valid instruments as long as they are not correlated with either $n_t$ or $\tilde{n}_t$ since, by the rational expectations assumption, they cannot be correlated with $\epsilon_{1t+1}$ or $\epsilon_{2t+1}$. We use rates of change rather than levels of $m$ and $i^*$ because, as argued below, the estimated stochastic processes of these two variables appear to be stationary only in the first differences.

Columns I and II show evidence of misspecification. Hansen's (1982) test of overidentifying restrictions which is reported in the last row
resoundingly rejects these restrictions. Durbin-Watson statistic of the exchange rate equation indicates that the estimated first order autocorrelation of the residuals is fairly high. This is not inconsistent with the theoretical model because, as argued above, we expect the composite disturbances to follow a first order MA process. However, a high autocorrelation of $\xi_{1t}$ and $\xi_{2t}$ might also be due to autocorrelation in the structural disturbances $n_t$ and $\tilde{n}_t$.

We thus also consider the hypothesis that the structural disturbances follow an autoregressive process:

$$(15) \quad n_t = \rho_1 n_{t-1} + u_{1t}, \quad \tilde{n}_t = \rho_2 \tilde{n}_{t-1} + u_{2t}$$

By quasi first-differencing equations (8) and (12) we obtain estimating equations whose residuals are given by:

$$\eta_{1t} = \epsilon_{1t+1} - \rho_1 \epsilon_{1t} + u_{1t}, \quad \eta_{2t} = \epsilon_{2t+1} - \rho_2 \epsilon_{2t} + u_{2t}$$

These quasi first-differenced equations are estimated using the same instrumental variables procedure, with instruments dated at $t-2$. Once again the errors have an MA component. The results are reported in columns III and IV of Table 1. The estimated autocorrelation coefficient is positive and significant in the exchange rate equation, but insignificant in the price equation. Moreover, the J statistic, which is distributed as a chi-square with 6 degrees of freedom in column III and 7 degrees of freedom in column IV, is well within the acceptance range. The table also shows that the estimates are not substantially affected either by quasi first-differencing, or by constraining the $a$'s.
In order to obtain maximum likelihood estimates we need to express the right-hand sides of equations (13) and (14) in terms of contemporaneous and past forcing variables. Exchange rates and prices depend on expected future values of such variables. The maximum likelihood procedure imposes the constraint that the stochastic process followed by the forcing variables is known to the public, and is used to form the expectations in (13) and (14). As shown by Hansen and Sargent (1980), the sums of discounted values of the forcing variables can be reduced to a function of present and past values of these variables. The complexity of this function, however, depends on the complexity of the stochastic process followed by the forcing variables. Therefore, we seek to model this stochastic process parsimoniously.

Hansen and Sargent's (1980) formulae require that past values of $p_d$ and $e$ do not help predict the forcing variables. Granger causality tests reported in Table 2 are consistent with this requirement. This table is obtained by first differencing $m$ and $i^*$ (as the coefficient estimates of a nondifferenced system suggested) and by using six lags for all variables. Lack of Granger causality is related to identification. If past structural disturbances in the price and exchange rate equations affect arbitrarily future values of the forcing variables, then it is impossible to recover the independent effect of structural disturbances on prices and exchange rates.

Table 2 is also consistent with univariate representations for the processes followed by $p_c^*$, $q^*$, $k$ and $i^*$. For these variables we specify univariate processes. We also use a univariate model for $p_N^*$, thereby treating the correlation between $p_N^*$ and lags of $m$ as an ex-post statistical
regularity that agents would not employ in forming forecasts of $p_N^*$. On the other hand, we specify agents' forecasts of future changes in $m$ as depending also on US interest rate changes, as the date would seem to suggest. Table 3 contains the estimates, obtained by ordinary least squares, of our favored parsimonious representation (found by Box-Jenkins type specification searches) of the stochastic process followed by the forcing variables.

The residuals of the maximum likelihood estimating equations (13) and (14) consist only of current and predicted future values of both structural disturbances $n$ and $\tilde{n}$. Again, we consider both the case in which these disturbances are white noise, and the case in which they have first order serial correlation.

If $n$ and $\tilde{n}$ are white noise, their expected future values, conditional on currently available information, are zero. Under the normality assumption, maximum likelihood involves the minimization of the covariance matrix of the disturbances. These disturbances include those of the equations describing the forcing variables, as well as $n$ and $\tilde{n}$.

The actual estimation is done, as described in Berndt, Hall, Hall and Hausman (1974) by taking a single Newton step from the consistent estimates presented in Tables 1 and 3. To carry out this step, we numerically compute, for each set of parameters the values of $J_{21}$, $J_{22}$, the $\omega$'s and $\xi$'s to obtain the residuals of (13) and (14), taking into account the prediction formulae of Hansen and Sargent (1980). Maximum likelihood estimates and standard errors for the case in which $n$ and $\tilde{n}$ are serially uncorrelated are presented in the first two columns of Table 4.
In the case in which \( n \) and \( \bar{n} \) have first order serial correlation, maximum likelihood involves the minimization of the determinant of the covariance matrix which includes \( u_1 \) and \( u_2 \) (as defined in equation (15)) instead of \( n \) and \( \bar{n} \). Computation of \( u_1 \) and \( u_2 \) for the purpose of the minimization requires further transforming of the residuals of (13) and (14). The resulting estimates are reported in the second two columns of Table 4.

As can readily be seen, the estimates of Table 4 are very similar to those reported in Table 1. The estimates of the stochastic processes of the forcing variables, which are not reported, are virtually unchanged from Table 3.

The bottom two rows present likelihood ratio tests for the validity of the restrictions imposed by the model. The first, \( 2(L_1 - L_0) \), gives twice the difference of the log-likelihood of our constrained estimates \( (L_0) \) and those of a model that includes the same equations for the forcing variables but which leaves the price and exchange rate equations unconstrained. It allows prices and exchange rates to be explained by all the current and lagged values of the forcing variables that enter in our restricted model. Under the null hypothesis that the constraints derived from (13) and (14) are valid, this test statistic is distributed as a chi-square with degrees if freedom equal to the difference between the number of parameters in the two models. The second test, \( 2(L_2 - L_0) \) also considers the possibility that the forcing variables follow a different stochastic process. It thus contrasts the likelihood of our model with that of an unconstrained vector autoregression which includes six lags of all variables. All test statistics lead us to reject the constraints imposed by (13) and (14).
Further evidence of misspecification is revealed by the D.W. statistics in columns III and IV, which indicate that the ex-post $u_1$'s have serial correlation equal to 0.7.

We now turn to the discussion of the parameter estimates. The values of $\phi_1$ (ranging from 1.63 to 1.90 in Table 4) are significantly greater than one, thus indicating that the effect of the exchange rate on the German price deflator is significantly different from zero. The coefficient is an increasing function of the income elasticity of money demand, and of the terms of trade effect on aggregate demand.

The value and significance of $\phi_3$ permits us to strongly reject the version of the model with perfectly flexible prices. This is particularly interesting, since Hoffman and Schlagenhauf (1983) and Woo (1984) are unable to reject versions of the flexible-prices monetarist model which are very similar in all other respects to the one considered here. The increased ability of our test to reject the hypothesis of perfect price flexibility may be due to the use of a specific alternative hypothesis.

With $b > 0$, $\phi_4$ is negative if $ad < 1$, where $a$ is the income elasticity of money demand, and $d$ is the elasticity of aggregate demand to an increase in real balances. $\phi_4 < 0$ is thus consistent with the U.S. evidence of Goldfeld (1973) and Rotemberg (1982b) who have found values of $a$ and $d$ below 1.

The estimates of $\phi_6$ are of the wrong sign, and insignificant in the version of the model with autocorrelated structural disturbances. $\phi_7$, which should be positive, is indeed positive in columns III and IV of Table 4, but is negative and insignificant in columns I and II. $\alpha_1/\phi_2$ and $\alpha_2/\phi_2$
are unprecisely estimated in columns I and III, thus validating the constraints we impose in columns II and IV.

Finally, Table 1 and Table 4 report the eigenvalues of the dynamic system consisting of the equations (8) and (12). These eigenvalues are computed from the parameter estimates. All estimates of Table 1 and the estimates of columns II, III and IV of Table 4 produce, as is required for our solution, two eigenvalues which are bigger than 1, while the other is smaller than one.

4. SIMULATIONS

In this section we use (13) and (14) to simulate the responses of the nominal exchange rate and the price level to a number of exogenous shocks. One important empirical question we can analyze with simulations of our model is whether the Deutsche mark exchange rate overshoots its steady state response to innovations in the German money stock. This type of question has received considerable attention in the theoretical and empirical literature since the seminal work of Dornbusch (1976), because these overreactions have the potential of explaining the volatility of exchange rates. Exchange-rate overshooting tends to be mitigated by two effects. The first is the immediate increase in the deflator for real money balances due to the exchange rate depreciation, the second is the increase in income and money demand associated with the worsening of the terms of trade. Note that, while the contemporaneous response of the price level reinforce the first effect, it decreases the importance of the second effect. By these two channels, the impact of the increase in money supply
on the domestic interest rate is dampened, thus reducing the need for large exchange rate adjustments [see Dornbusch (1976) and Mussa (1982)].

Figure 1 shows the responses to an innovation in the process followed by the German money stock. The simulation is carried out as follows. Starting at a steady state (where all variables are normalized to zero), we produce a unit increase in the disturbance of the \( m_t \) auto-regression at month 10. The price level and the exchange rate respond to such innovation according to our maximum likelihood estimates of column IV in Table 4. The money stock responds following the same maximum likelihood estimates. The figure shows that the money stock does increase further after the innovation, and eventually reaches a permanently higher level.

Figure 1 also shows that the exchange rate rises more on impact than the price level, while it "undershoots" its new steady state level. This undershooting results from the fact that money demand increases by more than money supply at an unchanged interest rate (so nominal interest rates must rise). Since the response of the deflator to an increase in money seems modest we ascribe the relatively large response of money demand to a large effect of the terms of trade on output. This is consistent with the relatively large estimate of \( \phi_1 \) we obtain. While we obtain undershooting, it is important to stress that the exchange rises much more than prices, the real exchange rate depreciates on impact (by 0.35 for a one percent increase in the money stock). Because of long-run neutrality it gradually returns to its previous steady state level.

Figure 2 shows the effects of a unity money innovation under the assumption that the money stock follows a random walk. This simulation is interesting because, as pointed out in the work of Currie (1984) and Meese
and Singleton (1983) among others, the nature of the stochastic process followed by the money stock may have an important impact on the degree of overshooting. The figure shows that the lack of exchange rate overshooting in Figure 1 is not due to the specific stochastic process followed by the money stock in Germany. Instead, it appears that the size of the coefficient $\phi_1$ best explains the results of our simulations. A high value of $\phi_1$ implies that exchange rate depreciations, given the nominal interest rate, tend to reduce real money balances relative to money demand. They do this by raising both output and the consumption deflator. As we argued above, both these factors prevent overshooting.

Figures 3 and 4 illustrate the effects of an increase in the U.S. Treasury Bill rate. Given $p_c^*$, this amounts to a real interest shock from the U.S. As shown in Figure 3, the estimated reaction of $m$ implies that an increase in U.S. interest rates is followed by a large monetary contraction in Germany. In the model, changes in $i^*$ and $m$ have contrasting effects on the real exchange rate. As discussed above, falls in $m$ tend to produce an immediate real appreciation, followed by a progressive depreciation. On the other hand, an increase in $i^*$ tends to depreciate the real exchange rate on impact. The latter effect dominates. In addition, the sharp monetary contraction generates a price deflation in the long run. As in the case of an innovation in $m$, the estimated process for $i^*$ shows that interest rate shocks have a permanent component. A permanent increase in the foreign real interest rate brings about a permanent real exchange rate depreciation because at full employment higher interest rates are associated with lower real balances and therefore lower domestic demand.
The real exchange rate depreciation crowds in net foreign demand for domestic output.

Figures 3 and 4 also suggest that the sizeable real depreciation of the DM/dollar rate observed from 1980 to 1985 cannot be ascribed to higher U.S. real interest rates alone: even the steady state response of the real exchange rate to a 1 percent real interest rate shock is only 1.92 percent. This suggests that other variables, like monetary shocks in Germany and various supply shocks might also have played a role in the recent rate experience. To illustrate the ability of all the variables in our model to track the recent real depreciation of the DM/dollar rate we compare predictions (for our detrended data) generated by the estimates of column IV in Table 4 to the actual experience from June 1980 to July 1985. Note that our parameters are estimated using data only through January 1983.

Figure 5 plots actual and simulated values of $p^* + e - p_d$. The figure shows that model traces out many of the turning points of the DM/dollar real exchange rate from 1980 to 1985. However, it is unable to reproduce the trend depreciation: only half of the 30 percent cumulated real depreciation of the DM from June 1980 to July 1985 is reproduced by the model. In addition, the sizeable swing from June 1984 to July 1985 is also significantly underestimated.

5. CONCLUDING REMARKS

This paper specifies and estimates a rational expectations model of sticky prices in an open economy. The results are mixed. On the one hand, the model is not rejected when estimated by instrumental variables and it produces reasonable parameter estimates. On the other hand, the cross
equation restrictions that require that agents, when they are forecasting the future values of various forcing variables, use the estimated stochastic processes for these variables are rejected. These apparently inconsistent results might be due to the different power of the two tests, to the failure of the structural disturbances to follow an unconditional normal distribution, and to errors in the specification of either the structural equations or the forecasting equation for the forcing variables. Thus there is unfortunately little we can conclude from this negative test result.

Since at least in some respects our model performs well, we believe that empirical research on well specified structural exchange rate models is worth pursuing further. Thus we do not feel that this style of research is necessarily doomed by the findings of Meese and Rogoff (1983) that simple structural exchange rate models are worse at predicting exchange rates than random walks. For instance, while our model does not predict the level of the real exchange rate particularly well, it appears able to explain several of its turning points. This suggests that simple comparisons of root mean square prediction errors may be an incomplete method for gauging a model's usefulness. It would seem that more research on methods for validating the out-of-sample forecasting ability of different models as well as research on improvements in structural models is needed before one can decide whether the ideas of current exchange rate theories are useful for understanding the behavior of actual exchange rates.

Improvements along these lines could include endogenizing some of the forcing variables, like the real wage rate, and, possibly, money supply. Also, the explicit inclusion of real interest rate in aggregate demand
might improve the fit of the model. These extensions, however, considerably increase the size and the complexity of the model, and would make maximum likelihood estimation quite expensive.
ACKNOWLEDGMENT

Robert Cumby and John Huizinga kindly provided us with software for generalized instrumental variables estimation. Financial support from the John M. Olin Foundation, the Fondazione Einandi, and the National Science Foundation is gratefully acknowledged.
APPENDIX: THE DATA

This paper presents an empirical analysis of the dollar mark exchange rate, but unlike earlier work, aggregates the rest of the world (for Germany) as a single dollar area.

Data for $P_c^*$, $Q^*$ is computed using geometrically weighted averages of the individual series form the countries appearing below.

<table>
<thead>
<tr>
<th>Country</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
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</tr>
<tr>
<td>France</td>
<td>.275</td>
</tr>
<tr>
<td>Italy</td>
<td>.184</td>
</tr>
<tr>
<td>Netherlands</td>
<td>.252</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>.132</td>
</tr>
</tbody>
</table>

The weights are the average from 1974 to 1981 of the ratio of the value of imports plus exports of each country with Germany over the total trade of Germany with these countries. In 1981 the 5 countries in Table A2.1 represented 45 percent of the total trade of Germany. Individual countries' indices for $P_c$ and $q$ are aggregated using the respective exchange rates vis a vis the U.S. dollar.

Most of the data is obtained from the International Financial Statistics (IFS) tape. The index of wages in Germany and the index of intermediate inputs prices are computed using data from the Deutsche Bundesbank, Monthly Report, various statistical supplements.

The exchange rate is the price of a U.S. dollar in Deutsche marks, line rf in IFS (average over the month).
The domestic price level is line 63 of IFS, the index of wholesale prices.

\( P^*_c \) is computed aggregating the various countries indices of consumer prices, line 64 in IFS.

\( P^*_n \) is a weighted average of the IFS index of all commodities prices excluding oil, line 76a, and the index of the dollar price of oil for Saudi Arabia, line 76aa.

The weights are chosen from the geographical composition of Germany's imports, using data from the Deutsche Bundesbank Monthly Report. From the period 1974-1982 we computed the average share of imports from OPEC, in total imports less finished goods. The average value share of imports from OPEC is 22 percent.

\( i^* \) is the U.S. treasury bill rate, line 60c in IFS.

\( Q^* \) is the weighted average of industrial production indices, line 66c, expressed in dollar terms, by dividing each country's index by the real dollar exchange rate.

\( M \) is line 34 in IFS.

\( K \) is the index of wage costs in industry, per man/hour, divided by the CPI. The former is from Bundesbank Monthly Bulletin, the latter is IFS line 64.

Theory requires that all series be realizations of stationary stochastic processes. The mean and trend of all series have been removed by regressing the log of each against a constant, time and time squared. For the seasonally unadjusted data 11 monthly dummies were also included in the regression.
References


<table>
<thead>
<tr>
<th></th>
<th>I</th>
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<th>III</th>
<th>IV</th>
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<td>(.0021)</td>
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<td>(.004)</td>
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<td>2.26</td>
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<td>6.260</td>
<td>6.21</td>
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Columns I and II contain the DW statistics for the estimates of $\zeta_{1t}^t$ and $\zeta_{2t}^t$. Columns III and IV contain the DW statistics for the estimates of $\eta_{1t}^t$ and $\eta_{2t}^t$.
Table 2. Significance Levels for the Absence of Causality From X to Y in Partially First Differenced Vector Autoregressions

<table>
<thead>
<tr>
<th>Variable X</th>
<th>$p_N^*$</th>
<th>$p_C^*$</th>
<th>$q^*$</th>
<th>$\Delta i^*$</th>
<th>$\Delta m$</th>
<th>$k$</th>
</tr>
</thead>
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<td>$p_N^*$</td>
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<td>0.19</td>
<td>0.74</td>
<td>0.46</td>
<td>0.15</td>
<td>0.58</td>
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<td>$p_c^*$</td>
<td>0.5</td>
<td>0.54</td>
<td>0.92</td>
<td>0.004</td>
<td>0.004</td>
<td>0.995</td>
</tr>
<tr>
<td>$q^*$</td>
<td>0.47</td>
<td>0.73</td>
<td>0.88</td>
<td>0.65</td>
<td>0.15</td>
<td>0.59</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>0.31</td>
<td>0.99</td>
<td>0.80</td>
<td>0.47</td>
<td>0.72</td>
<td>0.76</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>0.69</td>
<td>0.80</td>
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<td>0.053</td>
<td>0.36</td>
<td>0.67</td>
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<td>$p_d$</td>
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<td>0.64</td>
<td>0.65</td>
<td>0.95</td>
<td>0.47</td>
<td>0.55</td>
</tr>
<tr>
<td>$e$</td>
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<td>0.68</td>
<td>0.65</td>
<td>0.95</td>
<td>0.47</td>
<td>0.55</td>
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Table 3. Estimated Stochastic Process of Forcing Variables

<table>
<thead>
<tr>
<th>Variable X</th>
<th>$P_{ct}$</th>
<th>$P_{nt}$</th>
<th>$k_t$</th>
<th>$q_t$</th>
<th>$(m_t - m_{t-1})$</th>
<th>$({i_t}^<em>_t - {i_t}^</em>_t-1)$</th>
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</thead>
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<tr>
<td>$X_{t-1}$</td>
<td>1.247</td>
<td>1.175</td>
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<td>.948</td>
<td>.064</td>
<td>.325</td>
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<tr>
<td></td>
<td>(.093)</td>
<td>(.095)</td>
<td>(.096)</td>
<td>(.028)</td>
<td>(.093)</td>
<td>(.094)</td>
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<tr>
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<td>.194</td>
<td>--</td>
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<tr>
<td></td>
<td>(.093)</td>
<td>(.095)</td>
<td>(.097)</td>
<td></td>
<td>(.097)</td>
<td>(.094)</td>
</tr>
<tr>
<td>$X_{t-3}$</td>
<td>--</td>
<td>--</td>
<td>.164</td>
<td>--</td>
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<td>--</td>
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<td></td>
<td></td>
<td></td>
<td>(.092)</td>
<td></td>
<td>(.096)</td>
<td></td>
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</table>

Coefficients of:

- $(i_{t-1}^* - i_{t-2}^*)$  | -- | -- | -- | -- | .068 | -- | (.113)

- $(i_{t-2}^* - i_{t-3}^*)$  | -- | -- | -- | -- | -- | -.238 | -- | (.118)

- $(i_{t-3}^* - i_{t-4}^*)$  | -- | -- | -- | -- | -- | .023 | -- | (.118)

- $(i_{t-4}^* - i_{t-5}^*)$  | -- | -- | -- | -- | -- | .268 | -- | (.113)

D.W.  | 2.03   | 2.08   | 2.06  | 1.85  | 2.00 | 2.00 |

$r^2$  | .95    | .93    | .27   | .91   | .21  | .12  |
Table 4. Maximum Likelihood Estimates of the Price Level and Exchange Rate Euler Equations

<table>
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<tr>
<td>( \phi_1 )</td>
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<td>1.737</td>
<td>1.903</td>
<td>1.806</td>
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<tr>
<td></td>
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<td>(.061)</td>
<td>(.135)</td>
<td>(.116)</td>
</tr>
<tr>
<td>( \phi_3 )</td>
<td>.501</td>
<td>.500</td>
<td>.501</td>
<td>.501</td>
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<tr>
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<td>(.002)</td>
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<td>(.001)</td>
</tr>
<tr>
<td>( \phi_4 )</td>
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<td>-.129</td>
<td>-.271</td>
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<tr>
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<td>(.060)</td>
<td>(.060)</td>
<td>(.091)</td>
<td>(.095)</td>
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<td>( \phi_5 )</td>
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<td>.416</td>
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<tr>
<td></td>
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<td>(.059)</td>
</tr>
<tr>
<td>( \phi_6 )</td>
<td>-.004</td>
<td>-.003</td>
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<tr>
<td></td>
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<td>(.001)</td>
<td>(.001)</td>
<td>(.002)</td>
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<td>( \phi_7 )</td>
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<tr>
<td>( \frac{a_1}{\phi_2} )</td>
<td>.011</td>
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</tr>
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<td>(.001)</td>
<td></td>
</tr>
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<td></td>
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<td>(.090)</td>
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<td>( \rho_2 )</td>
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<td></td>
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<td>( \Delta w ): ( n_{t} u_{2t} )</td>
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<td>(.056)</td>
<td>(.057)</td>
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</table>


\(^a\) Columns I and II contain the DW statistics for the estimates of \( n_t \) and \( \bar{n}_t \). Columns III and IV contain the DW statistics for the estimates of \( u_{1t} \) and \( u_{2t} \).
Fig. 1: Money Innovation

Percent Deviations from Steady State

Time (months)
The Random Walk Case

FIG. 2: Money Inflation

Percent Deviations from Steady State
Fig. 3: US T-Bill Rate Innovation

US TBill Rate and German Money Stock
Percent Deviations from Steady State

Time (months)
Fig. 5: The DM Real Exchange Rate

Simulation of the 1980-1985 Episode
Date Due

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