Power Electronics Models for
Integrated Design of High Speed Electric Motor Drives

by

KEITH D. SZOLUSHA

Submitted to the Department of Electrical Engineering and
Computer Science in partial fulfillment of the requirements for the
degree of

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in
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February 4, 1998

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Chairman, Department Committee on Graduate Theses

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ABSTRACT

This thesis explores a method to simulate three phase currents in a three-phase wye configured electric motor drive system. The Current Simulation Method calculates and stores the three phases of simulated current over a specified number of fundamental electrical motor rotation cycles within a constant interval time-stepping loop. At each new time step in the loop, the new values of current are calculated according to the time of the last excitation switch, the current time, current, and the motor parameters.

The simulation program developed is integrated into a main optimal motor design program which examines thousands upon thousands of motor designs for optimal performance criteria. The simulation program serves as an addition to the existing main program by examining the efficiency and total harmonic distortion effects of the integration of power electronics and control (pulse width modulation) technology.

The Matlab simulation program developed is tested for its performance as a half-cycle steady state three-phase current simulator. The success of the simulations in this thesis depend on the ability for the simulation to accurately calculate real-time phase currents and to predict their amplitudes and initial values for steady state operation.

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# Power Electronics Models for Integrated Design of High Speed Electric Motor Drives

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Chapter 1

Introduction

This thesis investigates the development of power electronics models for the integrated design of high speed electric motor drives. These power electronic models, currently developed within the Matlab environment, are integrated with electromagnetic motor models to obtain a useful tool for the design and performance prediction of high speed electric motor drives. A conventional approach adopted in the design of a high speed electric motor drive system is to optimize the power electronics and motor designs separately in terms of calculating losses and other performance parameters. Since the development of motors and controllers are aimed at system integration for the complete electric motor drive system, the ‘overall system’ efficiency and performance optimization is more important to the customer than the optimization of the individual components such as the electric motor and the controls. Although the conventional approach mentioned may optimize the power electronics and motor designs separately, it may not optimize the overall system design as well as the integrated method due to its many nonlinearities.

1.1 Problem Statement and Definition [2]

The proliferation of faster computers, new design tools, and new materials for electric motor drives provide an opportunity for more efficient and powerful motor drives, superior to existing or past electric motor drive systems and economically competitive to hydraulic and mechanical systems in a large number of applications and markets. Significant development activity is currently improving semiconductors to achieve higher motor drive operating voltages and temperatures, increased power and current densities, and higher switching speeds [2]. Digital signal processing (DSP) tools are integral to the formulation of different pulse width modulation (PWM) motor control and drive technologies to create higher drive efficiencies and smoother (without harmonics) inverter switching schemes for drive operation. Today’s permanent magnet materials offer high magnetic energies, allowing for miniaturization and more compact machine designs. Motors and generators, such as the variable-reluctance type [1], offer wide speed ranges, high speeds, high starting torques, and unparalleled robustness, even in harsh, high-temperature, high-vibration environments. Emphasis is also placed upon effective cooling schemes, packaging, scaleability, modularity, reliability and cost. New manufacturing techniques, such as electrospray [3] and homopolar pulsed consolidation [4], are also being introduced as a mean towards more efficient, less expensive motor drives.
Such advances in components and subsystems have expanded the capabilities and applicability of electric motor drives. Electric motors come in a variety of topologies, which when coupled with appropriate electronics and effective cooling schemes can define drives at higher operational speeds, higher efficiencies, or higher torques, that can be interconnected and centrally controlled through fast communication networks. This fusion of fast growing engineering disciplines has created and imposed the need for a systems approach to the development of innovative electric motor drive products. More now than ever, there is a strong urgency for effective and fast integration, that allows better system understanding, leading to more robust, more efficient and cost-effective designs.

The range of technical possibilities available today, coupled to the growing need for high efficiency, lightweight, and compact electric power systems in a number of diverse markets, such as aircraft, electric vehicles, and air-conditioning, has created a need for fast and reliable tradeoff studies. From an engineering perspective, the cooling approach, the motor type, and the adopted power electronics topology determine and directly affect application attributes such as size, weight, efficiency, and cost. There is a need in today’s technology organization for the means to perform reliable tradeoff studies that are capable of effectively integrating all complex, engineering interactions that take place in an electric motor drive. Before embarking on detailed design and prototype development, all the tradeoffs should be well understood and the consequences of choice of a particular technology should be identified and quantified within the context of the engineering framework. [5]

1.2 Research Challenges and Objectives

Currently, electric motor drive system design is neither easy nor inexpensive to achieve. A number of technical challenges arise in the system design stage.

The first challenge is that of creating accurate models for the separate aspects of the electric motor drive system, namely models that predict the electromagnetic performance of a motor topology, the power electronics efficiency and loss characteristics, and the controller’s pwm waveform characteristics. The concentration in this thesis is placed on the power electronics models. It is essential to ensure that each model is accurate to a given degree, since the overall design specifications will be only as accurate as the models used to develop them.

The power-electronics configuration in this thesis is three phases of IGBT (integrated gate bipolar transistor) module technology. Each IGBT module is a switch containing a transistor and diode in parallel but oriented in opposite directions. Ideally, this switch pair operates without loss or delay. However, the realistic and accurate representation of these IGBT modules in electric motor drive system modeling must account for the losses which arise from each switch execution and the transistors’, diodes’, and switch recovery characteristics. Switching losses are crucial in the calculation of the overall efficiency of the power electronics since they are the largest source of energy loss and efficiency degradation. There is some energy lost during
ambient, non-switching, conduction times in which the IGBT module is turned-on and
providing a current path from the power source to the motor, but the saturation voltage is
very small (close to zero) during these times.

Figure 1.1 Switching waveforms (turn-on and turn-off) of a typical 150A, 600V Powerex IGBT module. Powerex half-bridge mode
(p. 1-21). Another on page A-35. \( V_{CE} \) (collector-emitter) is the IGBT module voltage and \( I_C \) is the current.

In Figure 1.1, the switching waveforms for both turn-on and turn-off possess non-
ideal characteristics. The switch from high to low takes a small amount of time, causing
a crossover of sloping current and voltage. This crossover, as seen in the power
characteristics of the switch, results in significant energy losses. Quicker switches,
however, can lower the amount of energy lost.

The power electronics models should accurately estimate the amount of energy
lost in the switching process. The exact model of each switch turn-on and turn-off
involves too much detail and would take too much time for a motor design program to
run through thousands and thousands of designs, each of which have hundreds of
switches and calculations for each fundamental cycle. Therefore, the sloped switching
waveforms are simplified by modelling them as square-trapezoid waveforms (see Figures
3.4 and 3.5) whose energy loss content is approximately the same as the actual energy
lost in the sloped switching pattern.

The second challenge is that of system integration. Each of the system’s
component models must be tied together to create system variables such as ‘system
efficiency’ and ‘total power loss’. Another aspect that system integration amplifies is the
issue of harmonics. The harmonics created by the controller, and applied to the power
electronics during switching will also affect the power supplied to and lost in the motor.

Previously the motor topology Monte Carlo analysis program, described below,
performed a number of calculations based on each successfully generated motor design.
The next step was evaluation of the newly generated motor design against the others
followed by the synthesis of a new design. The addition of the power electronics models
must fit into this existing circular format. Therefore, the power electronics time-based
three-phase current simulation, and subsequent energy loss analysis is placed in the loop.
The execution of the new Monte Carlo system optimization program steps out of the
main Monte Carlo analysis for each design and performs the time-based power
electronics simulation and analysis. This is a time consuming simulation, but is currently
the only forseeable method to evaluate the nonlinear effects of the power electronics.
Figure 1.2 demonstrates the flow of the system optimization program.
The challenge of **evaluation speed versus accuracy** is important in the design of the system models. Although accuracy is extremely important in choosing the appropriate design parameters for an optimal electric motor drive system, speed of evaluation is traded off to achieve this. Optimization, discussed in detail later, involves running thousands upon thousands of model simulations and calculations over a rather short period of time (up to a day or a week) to calculate the most probable optimal design. In a time-based simulation, increased accuracy decreases the time step and increases the number of steps in a fundamental cycle (in other words, increased accuracy increases model and simulation resolution as well as run-time). The most accurate computer models in the world could possibly take years to reach an optimal electric motor drive system design if they were programmed to extraordinarily accurate measures with magnificent detail and resolution. The designer must choose an appropriate balance of design accuracy and simulation speed.

Decreasing the time-step to extreme accuracy is not completely necessary since the generated list of possibly optimal designs is going to be reviewed and analyzed further by field experts. Although it may be a future project goal, the current design optimization program serves as a first-run tool for generating design possibilities and pinpointing optimal design ranges.

The final technical challenge is **model evaluation**. Computer simulations and models should be able to predict motor drive performance within acceptable confidence intervals. The square-trapezoid switch waveform model in this thesis should have similar overall energy loss content to the actual switching waveforms in the lab. Performance predictions of electric motor drive parameters should match (or come close to) in-lab measurements and calculations on these systems. Oftentimes models represent not only the amount of understanding that the designer has about the actual characteristics and physics of the system, but also the amount of misunderstanding that the designer possesses. Aspects of a model that have been overlooked may be discovered in experimental verification. Alterations based on the experiments will result in more accurate system design tools.

Other technical challenges arise when determining a superior drive design. Choosing a superior design is not always easy because of the tradeoff nature of these challenges. Although one design parameter has been optimized, another has been
sacrificed at its expense. Somewhere in between, the designer must choose where the superior tradeoff lies. The following is a list of such tradeoffs.

- The first challenge is that of maintaining rotor integrity over the drive’s speed range. For each motor topology, it is essential to ensure that mechanical strength and material retention are guaranteed at all times.
- Increasing the motor speed, by increasing the driving frequency, increases the motor losses and can reduce efficiency considerably. To first order, the hysteresis losses on the stator are proportional to the frequency and the eddy current losses are proportional to the frequency squared.
- Higher speed also implies a large number of switching operations in the power electronics section of the system. This in turn results in additional power loss, further degrading the efficiency of the motor drive.
- Higher speeds and more switching raise a number of thermal management issues for both the power electronics and the motor. The selection of an appropriate cooling scheme is critical for successful operation.
- Industry desires low-cost, easily-manufactured components, but physics provide higher performance and high efficiency specialized components that are both expensive and difficult to manufacture.

These technical challenges are reflected in all aspects of the motor drive system, from motor topology to power electronics and controls. Past methods such as spreadsheet analysis and gradient search algorithms have been employed to study the results of varying one of the above technical aspects at a time in order to reach an optimum system configuration. The Monte Carlo approach used in this thesis integrates all of the above challenges to enable tradeoff studies at the system level. Trends and effects of raising system power losses and lowering system harmonics can be determined using this approach.

By creating the power electronics and controls simulation software and integrating it into the Monte Carlo design approach in conjunction with an existing motor topology program, designers can reach a more desirable system design before construction and testing takes place. The goal of this thesis is the development of power electronics and controls models for the integrated design of high speed electric motor drive systems. It will also involve performing the integration within the Monte Carlo framework. Whereas past design results were sufficient in merely satisfying requirements and constraints, within a margin of simulation error, the new ‘system’ Monte Carlo method will establish a more accurate way to reach an overall optimal systems design. The process will not only be simplified by making the motor topology and power electronics (with controls) selection and sizing become one step instead of two, but the results will be more accurate, incorporating harmonics and overall system losses and efficiency. These results will target the customer’s real focus better than motor efficiency and power electronics would separately.

1.3 Thesis Scope
This thesis addresses several major issues concerning the development of power electronics models for integrated design of high speed electric motor drives. The first issue is the relative importance of the Monte Carlo design tradeoff approach as a system design tool. As stated above, past techniques for tradeoff studies have been gradient-search methods and spreadsheet analysis for separate components of the electric motor drive system. The Monte Carlo approach can cover a greater design space, and when used appropriately, can find optimal or superior designs. However, engineering biases may limit the design space by choosing the lower and upper bounds of the design attributes. Selection of these bounds should only be performed by someone that is knowledgeable in all aspects of electric motor drive system design.

The second major issue in this thesis is the accurate and fast prediction of harmonic creation and resulting power electronics losses. The power electronics model employed in this thesis is a time-based, step-through simulation of electrical calculation in the motor and the power electronics, based on the motor parameters and the switching in the power electronics as defined by the controller. Issues such as the power calculations (losses) in the inverter switches during switching and during saturation are important in the development of the models. Reverse recovery characteristics of the diodes in the switches affect the power loss and efficiency calculations as well.

Other issues involve the type of controller technology used. Different pwm controller schemes such as asynchronous sinusoidal and center-aligned will affect not only the number of switches in the inverter, but the timing of those switches which, in turn, affect the harmonics of the current waveforms in the three phases of the motor. Although some pwm schemes, matched with certain switches, will result in very small low-order harmonics, their numerous pulses may also result in high power loss. Other schemes with less pulses may result in less power loss, but higher low-order harmonics. These schemes are considered in this thesis.

1.4 Thesis Outline

The outline for the remainder of this thesis is as follows. Chapter 2 contains the background necessary to understand the Monte Carlo design tradeoff approach, the motor drive 'system', several common controller schemes, and the inverter topology. Chapter 3 presents the technical approach used to determine the phase currents and energy losses during the time-loop current simulation (see Current Simulation Method) as well as other model output parameter (thd). Chapter 4 describes the Matlab software developed to run the current simulation and power electronics efficiency program in conjunction with the existing Monte Carlo framework. Chapter 5 describes the numerical results from running the different pwm technologies in the power electronics current simulation program. Chapter 6 gives the summary of what was done in the thesis. Chapter 7 presents conclusions on what knowledge was gained from the models and simulations in this thesis. Finally, Chapter 8 presents recommendations for future research. Advanced study in this field should take the content of Chapter 8 into consideration. An appendix of Matlab programs is listed in Chapter 9.
Chapter 2

Background

2.1 Introduction

In order to establish a foundation for examining the issues addressed in this thesis, this chapter discusses the Monte Carlo design tradeoff approach and the electric motor drive system to which it is being applied. The first section describes the synthesis, analysis, and evaluation steps of the Monte Carlo process. The second, third, and fourth sections give theoretical, topological, and numerical backgrounds on the electric motor drive system and its components -- the motor, inverter, and controls respectively.

2.2 The Monte Carlo Design Tradeoff Approach

Figure 2.1 is a block diagram of the Monte-Carlo-based integrated design tradeoff approach.

![Block diagram of Monte Carlo approach](image)

The first step in the approach (and one of great importance) is the formulation of three classes of goals which describe the performance of the electric motor drive. These classes are the application-specific requirements, constraints, and attributes briefly described below.

- The requirements are defined as the set of design specifications that must be met with equality. Output power, operating voltage, and speed are typical input requirements for an electric motor drive system.
The constraints are defined as a set of design specifications that must be met with inequality. Target machine envelope and efficiency are typically defined as constraints that must not be violated. All feasible designs must conform to these constraints.

The attributes are defined as a set of design specifications that must be optimized (maximized or minimized) over the degrees of freedom which remain after the design requirements and constraints have been met. This set is strongly dependent on the application. For example, typical attributes for electric motor drive systems are efficiency, weight (or volume), and cost. However, for a submarine application, a typical attribute is noise.

The next step in the process is that of Monte Carlo synthesis. During synthesis, a drive design is synthesized by randomly selecting motor, power electronic, and controller (and possibly thermal) topologies and their design variables within their permissible limits or within the design space. Other methods are available and could have been adopted such as grid-gradient searches and synthesis based on expert rules [11], [13], [14]. However, such methods can become trapped on local maxima or minima and, as a result, the complete exploration of the design space is often not guaranteed. Design creativity can therefore be limited. Furthermore, the calculation of gradients requires that all variables be continuous. Some discrete (integral) variables, such as the number of turns in a motor and number of poles, and some choice variables such as pwm technique and IGBT module are not continuous. In addition, synthesis based on expert rules can also exhibit limited creativity because the rules reflect the prejudices of the experts.

Monte Carlo methods [10], on the other hand, facilitate design creativity. The independent variables can assume any values within permissible limits set by the designer based upon his or her understanding of the design constraints. These limits are chosen wide enough so as not to inhibit creativity, but may still reflect prejudices of the designer. Monte Carlo methods are easy to formulate and can be easily applied to complicated systems. No information on the gradients of any function with respect to the individual design variables is required, and minimal knowledge of prior electric motor drive design is required for its implementation, but familiarity and understanding of motor drive design is needed to choose proper constraint limits and to determine which designs are optimal and feasible. Changes to the design problem such as the introduction of new, more complicated models and topologies, inclusion of additional constraints and design variables, both discrete and continuous, do not affect the synthesis process significantly.

A disadvantage of the Monte Carlo process is that it can be more time consuming than other approaches because a large number of designs must be synthesized and analyzed to adequately span the permissible design space. However, with the advent of faster computers, this problem assumes less significance. In addition, adaptive procedures can be generated to reduce the design space [18], but this may limit the design space based once again on the prejudices of the experts which inhibit creativity.

Each candidate electric motor drive system, output from the Monte Carlo synthesis stage, is input to the analysis stage. During analysis, a candidate electric motor drive system’s performance is determined based upon appropriate models for the system,
its components, and its interconnections. Based upon these models, the program may determine that the design requirements and/or constraints cannot be met. In this case, the candidate design is discarded and a new design is generated by the synthesis stage. The user-defined design requirements and constraints, if too strict, may result in a large percent of the synthesized designs to be discarded during analysis. If the percent is too high, time may be wasted during analysis of inferior designs. Analysis of the combined mechanical, electromagnetic, electrical, (and possibly thermal) topologies ('overall system' synthesis), may be the most time consuming stage of the Monte Carlo approach, depending on the complexity of the topologies. During analysis, the independent design variables are used to calculate many more complex design specifications and parameters according to the system models. The analysis stage also performs limitation checks and calculations to ensure that boundary conditions are not violated. However, synthesis simply generates random design values, according to the appropriate distribution, and compares them to user-defined limits.

Accurate models determine the success of the tradeoff analysis effort; the results of integrated designs are only as accurate as the models used to generate those designs. The assumptions and approximations are critical, and the detail and degree of sophistication is tightly coupled to the information desired and the accuracy required. Two conflicting issues arise when examining the applicability and effectiveness of a model: its degree of accuracy, and the computational time and effort required. The issue of computational time and effort is amplified even more when viewed within the context of an iterative approach such as the one adopted here; the models have to be accurate enough to provide meaningful results, yet fast enough to allow for a large number of candidate designs to be evaluated within a small time frame. The adopted models must be detailed enough to ensure accurate tradeoff studies. However, it is important to note that detailed analysis and performance characterization of the system can only be accomplished through a more extensive analysis of the mechanical, electromagnetic, electronic, and thermal subsystems, accomplished through numerical techniques such as finite element analysis.

The final stage in the process is that of evaluation. During evaluation, the attributes of each candidate drive system design are compared to those which have come before it, based upon rules of multi-attribute dominance. If all attributes of at least one old design are better than the attributes of the new design, the new design is discarded. Otherwise, the new design is retained in a database of dominant designs that form the frontier of optimal performance. If all attributes of the new design dominate the attributes of any of the designs existing in the database, then those existing designs are all replaced by the new one. Tradeoff analysis and optimization by multi-attribute dominance offers major advantages over optimization based on a scalar objective function. First, it is very difficult to assign weights to the various performance variables that appear in a scalar cost function. In the case of multi-attribute dominance, weightings are not necessary. Second, the optimization of a scalar objective function results in a single drive design, while optimization based on multi-attribute dominance results in a multi-dimensional surface that defines the frontier of optimal performance. The surface allows the designer to visualize the tradeoffs of emphasizing one attribute over another.
More detailed models and experimental data should be used to fine-tune the performance estimates and validate the designs of interest.

2.3 The Electric Motor Drive System

Figure 2.2 illustrates the key components of a typical electric motor drive system. The integrated design and development of such an electric motor drive system is interdisciplinary, since the machine exhibits complex interactions during the conversion of electrical to mechanical energy. Optimal systems supply high power and constant torque to the motor with minimal power losses and minimal harmonics. System design and tradeoff analysis requires knowledge of each of the components of the electric motor drive system. The following sections will give a description of the components of the electric motor drive system and their interactions.

![Figure 2.2 - Key Components of an Electric Motor Drive System](image)

The typical electric motor modeled in this thesis is a three phase machine. The three phases of the motor are metal windings that carry nearly sinusoidal current from the inverter through the stator of the motor. Flux excitation in the air gap of the motor results in rotational torque, driving the rotor and supplying mechanical energy to a load. The inputs to the electric motor are the three phases of sinusoidal voltage. The output of the electric motor is the motor positioning data provided to the controller by an incremental encoder or positioning resolver that is enclosed in the case of the motor. The phase current data is also fed back to the controller by current sensors placed on at least two of the phases of the motor.

The inverter contains three sets of switches for the three phases of the electric motor. Each switch set places the voltage excitation of its respective motor phase high or low with signals from the controller. The purpose of having an inverter with binary voltage levels which supplies three phases of sinusoidal current to the motor is its power efficiency. Common inverter technology provides greater than ninety percent power efficiency to the motor, and although the switching losses in the inverter are significant, they are superior to alternative AC power supply techniques such as AC sources with amplification technology of approximately fifty percent power efficiency. The inputs to the inverter are the high and low DC voltage supply lines and the three controller signals which set the switches high or low. The output of the inverter is the three phases of high or low voltage to the motor.
The controller is very sophisticated and expensive technology such as a digital signal processor which is programmed to perform complex operations based on the feedback from the motor and different pulse width modulation techniques. Using control-loop computations based on the motor position and phase current data, the controller determines when each phase switches high or low in an electric motor drive system. The inputs to the controller are the motor position data from the incremental encoder or position resolver of the motor and at least two of the three phases of current data from current sensors placed on the phases of the motor. The output of the controller is the inverter switching commands.

The DC voltage source simply supplies voltage and power to the motor drive system through the inverter. The output of the DC voltage source is the DC voltage supplied to the inverter which also provides the power to the motor. For inverter component protection, there is usually an overcurrent setting on the source which limits the current and therefore the power flow into the inverter.

2.3.1 Electric Motor Topology

The motor used in this thesis is a high speed permanent magnet, three-phase, wye configured electric motor. Figure 2.3 gives the electrical model of such a motor.

![Electrical model of a 3-phase, wye configured motor.](image)

Fig. 2.3 Electrical model of a 3-phase, wye configured motor.
Forming the wye ('Y') interconnection, the three phases of the motor windings join at a floating neutral point, not tied to ground. Electrically, each phase possesses both resistance (R), from permanent magnet and metal winding power dissipation characteristics, and inductance (L), representative of the coil-like windings of N-turns, in series. In this model, each phase resistance is assumed to be identical in ohmic value, as is each phase inductance. In addition, the inductance may have a functional dependency on the magnitude of phase current, but its differentiation with respect to the current is set to zero here, and any functionality is ignored for purposes of simplicity and single magnitude performance modeling (at top speed). R is constant.

Another component of the electrical model which must not be ignored is the sinusoidal back-emf (electro motor force or voltage). Back-emf is a voltage that arises on each phase from the spinning characteristics of the motor. Depending on the topology of the motor, the back-emf may have a square-pulse or sinusoidal characteristic relative to the phase, speed, position of the motor, and strength (K) of the permanent magnets in the motor. The high speed permanent magnet motors in this thesis have a sinusoidal back-emf to because of the availability of motors with sinusoidal back-emf to gain experimental data from.

Alternating voltage applied at the phase excites the motor. The three phase leads are connected to the switches of the inverter and excited with voltage pulses. The pulses, depending on the pwm technique, create three phases of (hopefully) sinusoidal current in the motor. Most motors are designed assuming ideal sinusoidal inputs at the three leads, since the result is optimal continuous torque. Optimal motor models therefore contain perfectly sinusoidal current and back-emf waveforms, convenient for simple calculations. Actual motor currents, ensuing from inverter pwm waveforms, are not ideal sinusoids, and contain a variety of harmonics that promote loss in the motor.

2.3.2 Inverter Topology (Switching)

The electrical topology of the inverter is demonstrated in Figure 2.4. For a three-phase system, the inverter includes three sets of transistor/diode pairs. One transistor/diode in each pair connects the corresponding motor phase to high voltage (Vbus) of the DC source, while the other transistor/diode creates a path to ground of the DC source. The excitation voltage on each motor phase is determined by the signals from the controller, turning either the upper transistor on and the lower transistor off, or the lower transistor on and the upper transistor off at all times. In each phase, the upper and lower switches always possess opposite states (on/off) so that the power source is not shorted and the motor current has a continuous flow path. However, due to the switching waveforms of the transistor, adjustments in switch timing may be made so that the time delay in switching from on to off or vice versa does not affect the switch states or power losses significantly.
The switches and diodes are biased in one direction only, due to their physical structures. Thus, the positioning of the transistors and diodes only allows current to flow in one direction relative to the position of the component. Figure 2.5 contains a diagram of the direction of current flows based on which state the transistors are set by the controller (hi - upper transistor on and lower off, or lo - upper transistor off and lower on). The arrows on the components in both Figure 2.4 and 2.5 represent the direction that current can possibly flow through each component.

Since each phase of the motor has inductance (coils), the phase current must be continuous. Therefore, the unidirectional nature of the transistors requires that the phase currents need two alternative paths of unidirectional current flow to compensate for the reverse directional current for each transistor. Not only do the diodes provide a reverse path for reverse directional current flow, but they provide reverse recovery paths for current during switching transitions in which neither switch is operational or current-carrying.

Although the diodes and transistors are not ideal switches or ideal unidirectional current paths (ideal short), they provide the cheapest, quickest, most efficient solution to the inverter requirement. Other sinusoidal sources require amplifiers to boost the voltage and current up to the high motor drive levels. These amplifiers operate at approximately 50% efficiency, not nearly enough for a motor drive system. The diodes and transistors each have a slight reverse bias voltage in series with a resistance when activated. Therefore, the terminal voltage, seen by the motor and generated by the inverter, is only approximately Vbus or Ground. The terminal voltage at each phase of the motor is somewhat lower or higher than Vbus (or Ground) due to the voltage drop of VceSat and the drop across the internal switch resistance.
2.4 Several Pwm Techniques

Pulse width modulation (pwm) is an integral aspect of three-phase electric motor drive system control and drive. Pulse width modulation is a series of pulses (of varying widths) that can simulate virtually any waveform by substituting the amplitude data of a waveform with pulses of proportionate widths. Since there are many different techniques of calculating pulse width modulation timing and three-phase balancing, several of those techniques are employed in this thesis.

The comparator, synchronous sinusoidal, asynchronous sinusoidal, six-step with harmonics elimination, and center-aligned techniques are all described below. Each of these pwm techniques is aimed at forcing three phases of sinusoidal current in the motor by driving sinusoidal pwm voltage waveforms at the inverter. These three phases each have a one hundred twenty degree phase shift with respect to each other for optimal three-phase motor drive.

2.4.1 Comparator

For pulse width modulation, a current comparator compares the actual current in each phase of a motor to an ideal sinusoidal current which is computed according to the speed and torque of the spinning motor. When the difference between the two waveforms exceed the threshold (delta) the comparator signals for a switch to occur to drive the actual current back within threshold difference of the ideal current source.
2.4.2 Asynchronous Sinusoidal

Asynchronous sinusoidal pwm generates pulses by comparing a triangular carrier wave with three sinusoidal reference waveforms that are proportionate to the motor position. The amplitude of the carrier wave and the amplitude of the three sinusoidal waveforms may vary by a proportionality factor (M) for supplying the motor with different amounts of power and torque. The comparison of the carrier and reference waveforms yields three phases of pulsed width logic, high values corresponding to times when the reference waveform is greater than the carrier wave, and low values corresponding to all other times. Fig. 2.7 is a block diagram that describes this logic.
Fig. 2.7 (A)Synchronous sinusoidal technique.

Fig. 2.8 (a) Three sinusoidal reference phases and triangular carrier wave. Asynchronous sinusoidal. M=1 (M=amplitude of sinusoids divided by amplitude of triangle waveforms). (b) Three-phase modulation arising from reference and triangle wave comparison.

Figure 2.8a is a detailed plot of three sinusoidal motor position waveforms (reference) and the triangular carrier comparison waveform. The magnitude proportionality factor (M) is one, for optimal torque. Note that in Figure 2.8a the three waveforms are in sync.
with the carrier waveform. Phases A, B, and C can be characterized by a frequency that is an integral dividend of the carrier wave frequency; the frequency of A, B, and C is \(~1/15\) of the frequency of the carrier waveform. Therefore, the carrier waveform and reference waveforms are synchronized, and if the motor slows, and the frequency of the reference waveforms reduce, then the frequency of the carrier wave will reduce proportionally. The modulation waveforms will always be similar to each other, with the number of switches per cycle remaining constant.

Asynchronous sinusoidal PWM is slightly different. The triangular carrier wave always maintains the same frequency, whether or not the motor reference waveforms speed up or slow down. Therefore, as the speed of the motor changes and the frequency of the reference waveforms alters, the number of switches per period of motor position changes as an indirect proportion. If the motor speed halves, the motor frequency halves, but the switches per motor period double. [16]

The three-phase modulation arising from reference and triangle wave comparison is also the switching waveforms which control the inverter switch timing. When the modulation is high, the switch turns on, or goes to high bus voltage, and when the modulation is low, the switch turns off, or connects to ground. Figure 2.8b is a representation of the modulation and switching waveforms arising from Figure 2.8a.

### 2.4.3 Six-Step with Harmonics Elimination

Maheshwari’s technique for harmonics elimination using six-step PWM with notching is described below.

Six-step PWM requires a unique method of modeling a cosinusoidal waveform to produce its result. The waveform, F(x)=\(\cos(x)\) is divided into six sixty degree segments, describing the first segment from 0 to 60 degrees as \(f(x)=\cos(x)\). Each sixty degree segment is therefore equated using a form of this \(f(x)\) such as \(f(-x)\) for -60 to 0 degrees. \(f(-x+360)\) for 300 to 360 degrees. Between 60 and 120 degrees and between 240 and 300 degrees, the cosinusoidal wave is a combination of alterations of \(f(x)\). Table 2.1 lists each sixty degree representation of \(F(x)\).

<table>
<thead>
<tr>
<th>(F(x)) = (\cos(x)) (from 0 to 360 degrees)</th>
<th>(\cos(x))</th>
<th>0 to 60 degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\cos(-x+120)+\cos(x-60))</td>
<td>60 to 120 degrees</td>
<td></td>
</tr>
<tr>
<td>(-\cos(-x+180))</td>
<td>120 to 180 degrees</td>
<td></td>
</tr>
<tr>
<td>(-\cos(x-180))</td>
<td>180 to 240 degrees</td>
<td></td>
</tr>
<tr>
<td>(-\cos(x-240)+\cos(-x+300))</td>
<td>240 to 300 degrees</td>
<td></td>
</tr>
<tr>
<td>(\cos(-x+360))</td>
<td>300 to 360 degrees</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Representing a cosinusoidal wave by using forms of \(f(x)\) from 0 to 60 degrees.

In six-step PWM, similar to the method in Table 2.1, the line-line voltages from the switches' values are also alterations of a 60 degree segment, initially a single pulse of width equal to sixty degrees. The fundamental segment from 0 to 60 degrees in line-line voltage is \(g(x)\). The entire waveform (line-line) is \(g(x), -g(-x+120)+g(x-60), -g(-x+180),\) etc. as in Table 2.1.
The most interesting results arise when harmonics are removed from the switching function using Maheshwari’s [16] method. Notches cut in the fundamental sixty degree pulse are also seen in the rest of the line-line switching waveform as the different alterations of \(g(x)\). These notches are specifically calculated to remove specific harmonics.

Every modulation (or switching) waveform contains undesired harmonics that reach the motor and cause thermal and electrical losses. Since the systems in this thesis are three-phase, any third harmonic, or multiples thereof, are seen on all three phases of the motor, and when shifted by zero, one hundred twenty, and two hundred forty degrees in the three phases, they cancel themselves out and are not seen as losses. Even (multiples of two) harmonics have the same effect because of their multiplicity properties. The harmonics of concern are thus the 5th, 7th, 11th, 13th, 17th, 19th, 23rd, etc. (all \(6k \pm 1\) s.t. \(k\) is an element of all positive integers). Maheshwari’s method of harmonic elimination begins with the 5th, and then subtracts as many additional harmonics as desired.

To calculate where the notches should be cut in the fundamental \(g(x)\) to remove specified harmonics, the locations of notch cuts, alphas, are calculated using the following format. It should be noted that the number of alphas for a desired set of harmonics eliminated is always the number of harmonics plus one.

<table>
<thead>
<tr>
<th>Harmonics Eliminated</th>
<th>(a(1))</th>
<th>(a(2))</th>
<th>(a(3))</th>
<th>(a(4))</th>
<th>(a(5))</th>
<th>(a(6))</th>
<th>(a(7))</th>
<th>(a(8))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>32.93</td>
<td>39.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, 7</td>
<td>25.83</td>
<td>28.82</td>
<td>49.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, 7, 11</td>
<td>21.79</td>
<td>23.51</td>
<td>41.69</td>
<td>47.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, 7, 11, 13</td>
<td>18.28</td>
<td>19.27</td>
<td>35.54</td>
<td>39.32</td>
<td>51.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, 7, 11, 13, 17</td>
<td>15.16</td>
<td>16.79</td>
<td>31.47</td>
<td>34.02</td>
<td>46.07</td>
<td>51.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, 7, 11, 13, 17, 19</td>
<td>14.08</td>
<td>14.52</td>
<td>27.69</td>
<td>29.41</td>
<td>40.86</td>
<td>44.57</td>
<td>53.67</td>
<td></td>
</tr>
<tr>
<td>5, 7, 11, 13, 17, 19, 23</td>
<td>12.76</td>
<td>13.08</td>
<td>25.14</td>
<td>26.40</td>
<td>37.12</td>
<td>39.85</td>
<td>48.73</td>
<td>53.31</td>
</tr>
</tbody>
</table>

Table 2.2

### 2.4.4 Center-Aligned

Center-aligned PWM generates pulses on each phase according to electrical motor position. This PWM technique has a predetermined pulse generation frequency \(f_{\text{pulse}}\) about which the pulses are centered. At the beginning of each \(\Delta t\) interval, the electrical motor position, \(\theta_E\), is measured by the encoder or resolver on the motor.

\[
\Delta t = \frac{1}{f_{\text{pulse}}}
\]

The three-phase pulse widths, \(\delta_A\), \(\delta_B\), \(\delta_C\), for that \(\Delta t\) is then calculated with a duty cycle ratio, \(\beta\) and centered about the \(\Delta t\) interval.

\[
\delta_A = \beta \frac{\cos(\theta_E) + 1}{2} \Delta t \tag{2.1}
\]

\[
\delta_B = \beta \frac{\cos(\theta_E - 120) + 1}{2} \Delta t \tag{2.2}
\]
\[ \delta_C = \beta \frac{\cos(\theta_E - 240) + 1}{2} \Delta t \]  

Figure 2.9 Center-aligned pwm. The width of each frequency centered pulse is determined by the measurement of the electrical motor position at the beginning of each \( \Delta t \) interval.
Chapter 3

Technical Approach

3.1 Introduction

This chapter describes the technical approach of this thesis to model an electric motor drive system, its losses and efficiencies, and different mathematical principals involved. The technical approach includes a description of the power electronics motor model developed in Matlab in the Monte Carlo format and a numerical analysis of some of the calculations involved in this thesis such as energy loss and efficiency calculations.

3.2 Matlab Model Development

The Matlab power electronics motor drive model Isim.m (see Appendix) provides necessary numerical data to the Monte Carlo motor design program to which it is appended. For each qualifying motor design, the Matlab power electronics model is called to generate the following analysis:

- One half cycle of all three phases of constant speed phase currents ($I_A$, $I_B$, $I_C$).
- Energy losses in the power electronics and the motor.
- Total harmonic distortion (thd) in the current waveforms.
- Power electronics weight and cost.
- Maximum current supplied to the motor from the power electronics.

3.2.1 Phase Currents

Current Simulation Method

In order to calculate the time-based three-phase motor currents, an understanding of the components of the three-phase wye configured electric motor and their electrical equations are necessary. The three-phase wye-configured motor model in Figure 3.1 is the basis for the mathematical calculation of the currents in this thesis. Amps and volts are the respective units of measurements for all currents and voltages.
Figure 3.1 Three-phase wye-configured motor model. $U_a$ is the terminal voltage reference to ground at phase A, $I_a$ is the phase current, Backemf$_a$ is the sinusoidal emf resulting from the rotor rotation, $V_e$ is the difference between the terminal and neutral voltage, and $V_n$ is the neutral voltage of the motor referenced to ground.

The terminal-to-neutral voltages ($V_A$, $V_B$, $V_C$) are the sums of voltage across the phase resistance ($R$) and the voltage induced by the flux linked ($\lambda_A$, $\lambda_B$, $\lambda_C$) in the corresponding phases of the motor (Equation 3.1). The flux linked in the motor has two components, one from the windings and one from the magnets of the motor. This is indicated in Equation 3.2 below. The voltage from the winding inductance is the time derivative of the flux in the windings ($\lambda_A$, $\lambda_B$, $\lambda_C$); this is the voltage on the inductor in Figure 3.1. The sinusoidal voltage source in each phase in the figure is the back emf ($\text{BEMF}_A$, $\text{BEMF}_B$, $\text{BEMF}_C$) of the motor. The BEMF is the time derivative of the flux from the magnets.

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix}
= R \begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix}
+ \frac{d}{dt} \begin{bmatrix}
\lambda_A \\
\lambda_B \\
\lambda_C
\end{bmatrix}
\]

(3.1)
The voltage from the winding inductance is the time derivative of the flux by the windings. The relative position of the windings creates mutual inductance values \( M \) in addition to the self inductance \( L \) of each winding such that

\[
\begin{bmatrix}
\lambda_A \\
\lambda_B \\
\lambda_C
\end{bmatrix} =
\begin{bmatrix}
L & -M & -M \\
-M & L & -M \\
-M & -M & L
\end{bmatrix}
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} - K_1 \begin{bmatrix}
\cos(p\omega t) \\
\cos(p\omega t - 120) \\
\cos(p\omega t - 240)
\end{bmatrix}
\]  

(3.2)

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix} =
R
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} +
\frac{d}{dt}
\begin{bmatrix}
\lambda_A \\
\lambda_B \\
\lambda_C
\end{bmatrix} +
\begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix}
\]  

(3.3)

The back emf (BEMF) of each phase is a sinusoidal voltage arising from the magnets according to Equation 3.5. The magnitude of the back emf is proportional to the speed of the motor. The frequency is the same as the electrical frequency of the motor. The phase shift of each phase of the back emf with relation to the phase A electrical motor position is -120 and -240 for phase B and C respectively. The back emf motor constant \( K \) does not vary with motor speed. However, when multiplied by the (actual) motor speed, it will give the magnitude of the back emf waveforms.

\[
\begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix} = K\omega
\begin{bmatrix}
\sin(p\omega t) \\
\sin(p\omega t - 120) \\
\sin(p\omega t - 240)
\end{bmatrix}, \text{ where } K = K_1\omega
\]  

(3.5)

The terminal voltages, \( U_A, U_B, \) and \( U_C \), usually have two values, high or low. A high value corresponds to the top switch at a particular phase being ‘On’. The ‘On’ value is supplied by a power supply connected to the power electronics. The low value, resulting from the bottom switch at being ‘On’, is zero, the power supply ground, in this thesis.

The neutral voltage \( V_N \), is the potential, in relation to the power supply ground, at the node where the three phase windings are connected. As shown below, \( V_N \) is the average of the three terminal voltages. The voltage difference between the terminal and neutral voltages \( V_A, V_B, V_C \) is significant for determining how much current is in each phase of the motor. However, the neutral voltage can be eliminated from the calculations, making the current in each phase a function of the changing terminal voltages alone.
From Figure 3.1, 
\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix} = \begin{bmatrix}
U_A - V_N \\
U_B - V_N \\
U_C - V_N
\end{bmatrix}
\] 
(3.6)

By substituting Equation 3.6, Equation 3.3 can be rewritten as
\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} = \begin{bmatrix}
U_A - V_N \\
U_B - V_N \\
U_C - V_N
\end{bmatrix} - \frac{d}{dt} \begin{bmatrix}
\Lambda_A \\
\Lambda_B \\
\Lambda_C
\end{bmatrix} - \begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix}
\] 
(3.7)

\(I_A, I_B,\) and \(I_C\) sum to zero (Equation 3.8) by simple KCL at the neutral node. Equation 3.9 follows directly from Equation 3.8. (Note: since these currents always sum to zero, it is necessary to only know two of their values in order to calculate the third, Equation 3.12)

\[I_A + I_B + I_C = 0 \] 
(3.8)

\[R(I_A + I_B + I_C) = 0 \] 
(3.9)

Below, a combination of Equation 3.7 and Equation 3.9 contains the terminal voltages, the neutral voltage, and all three phases of the flux-linked voltage (the inductive produced and back emf) (Equation 3.10). By proving that the sum of the three inductive values and the sum of the three back emf values equal zero, the neutral voltage is described by the terminal voltages alone.

\[0 = U_A + U_B + U_C - 3V_N - \frac{d}{dt}(\Lambda_A + \Lambda_B + \Lambda_C) - (BEMF_A + BEMF_B + BEMF_C) \] 
(3.10)

\[\frac{d}{dt} \begin{bmatrix}
\Lambda_A \\
\Lambda_B \\
\Lambda_C
\end{bmatrix} = \frac{d}{dt} \begin{bmatrix}
LI_A - MI_B - MI_C \\
- MI_A + LI_B - MI_C \\
- MI_A - MI_B + LI_C
\end{bmatrix} \] 
(3.11)

In Equation 3.12 it is clear that the sum of the windings terms (\(\Lambda\)'s) of the flux-linked values (Equation 3.11) sum to zero, and thus the voltages about the windings sum to zero. Equation 3.8 is used to not only to group the (L+M) terms to create Equation 3.12, but to eliminate the three (L+M) terms in Equation 3.12 by showing that they sum to zero.

The sum of the three equations in Equation 3.11 gives
\[
\frac{d}{dt}(I_A + I_B + I_C) = \frac{d}{dt}((L+M)I_A + (L+M)I_B + (L+M)I_C) = 0 \quad (3.12)
\]

Summing three sinusoids of equal magnitude and frequency which are each 120 degrees out of phase from each other gives zero at all times (Equation 3.13). Therefore, since the three BEMFs are each 120 degrees out of phase from each other, the BEMF portion of Equation 3.10 sums to zero (Equation 3.14).

\[
\sin(x) + \sin(x-120) + \sin(x-240) = 0 \quad (3.13)
\]

\[
BEMF_A + BEMF_B + BEMF_C = 0 \quad (3.14)
\]

It is concluded that the sum of the terminal voltages and three times the neutral voltage is zero. According to Equations 3.10, 3.12, and 3.14, the neutral voltage (\(V_N\)) is the average of the three terminal voltages (Equation 3.15).

\[
\therefore V_N = \frac{U_A + U_B + U_C}{3} \quad (3.15)
\]

To solve for the phase currents, Equation 3.3 is rewritten, making \(I_{\text{phase}}\) the primary differential variable. Each phase current must be expressed as a function of only itself, and of neither of the other two phase currents for simple differential solution methods to be applicable. Each equation in Equation 3.16 has all three.

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} + \frac{1}{R} \frac{d}{dt} \begin{bmatrix}
L & -M & -M \\
-M & L & -M \\
-M & -M & L
\end{bmatrix} = \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
U_A \\
U_B \\
U_C
\end{bmatrix} - \frac{1}{R} \begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix} \quad (3.16)
\]

Equation 3.16 is rewritten as 3.17. In Equation 3.17, each equation is independent (relatively), of the other two, containing only one current phase.

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} + \frac{(L+M)}{R} \frac{d}{dt} \begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} = \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
U_A \\
U_B \\
U_C
\end{bmatrix} - \frac{1}{R} \begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix} \quad (3.17)
\]

This format enables the first order differential equations to be solved using simple differential equation methods. The solution for each phase current has two parts, a particular solution and a homogenous solution. The homogenous solution for each phase current is the part of each phase current which solves the differential equation (such as Equation 3.17) with the right hand side set to zero and it is used to match initial conditions. The particular solution further satisfies the equation by making the full
differential equation (Eq. 3.17) a true statement. Each particular solution in Equation 3.17 has two distinct parts, a switch value (U) part and a BEMF part (Equation 3.18).

\[
I_p, A = I_{p, A} + I_{h, A} = I_{p, A, U} + I_{p, A, BEMF} + I_{h, A}
\]  

(3.18)

The homogenous solution is the solution to the differential equation in Equation 3.19. The homogenous solution (Eq. 3.20) contains the time-transient of initial conditions of the current values. The initial conditions of the phase currents are the values of the phase currents at the time when a switch occurs, in the immediately following equations, the initial conditions occur at time equals zero. Part of the homogenous solution (H) cannot be calculated until after the particular solution is found. Once the particular solutions are known, the initial conditions are used to determine H.

\[
\begin{bmatrix}
I_{h, A} \\
I_{h, B} \\
I_{h, C}
\end{bmatrix} + \frac{(L + M)}{R} \frac{d}{dt} \begin{bmatrix}
I_{h, A} \\
I_{h, B} \\
I_{h, C}
\end{bmatrix} = 0
\]

(3.19)

\[
\begin{bmatrix}
I_{h, A} \\
I_{h, B} \\
I_{h, C}
\end{bmatrix} = \begin{bmatrix}
H_A \\
H_B \\
H_C
\end{bmatrix} e^{-\frac{R}{L+M}t} \quad H_A, H_B, H_C \text{ based on initial values.}
\]

(3.20)

The particular solution is the solution to the differential which ignores the initial conditions of the system. It is also considered to be the steady-state solution, as time approaches infinity, assuming that the transient decays over time. In the three phases of the motor, the particular solution to the currents is broken down to two components. The switch values (U) and the BEMF values account for the two parts of the particular solution. Since the particular solution for the switch values for each non-switching period is constant, the time-derivative of the switch value part of the particular solution is zero. This simple part of the particular solution is shown in Equation 3.21. The switch values are constant for a period of time, beginning with time zero, so \( U_A, U_B, \) and \( U_C \) in the following examples are considered constant until the next switch occurs and new reference values are stored.

\[
\begin{bmatrix}
I_{p, A, U} \\
I_{p, B, U} \\
I_{p, C, U}
\end{bmatrix} = \frac{1}{3R} \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
U_A \\
U_B \\
U_C
\end{bmatrix}
\]

(3.21)

The particular solution for the back emf portion solves Equation 3.22. The switch values are excluded and the time-derivative is still important since the form of \( I_{p, \text{phase, BEMF}} \) contains sinusoids.
Equation 3.5 is necessary to solve for this part of the particular solution. After some simple math, the solution for the back emf part is shown in equation 3.23.

\[
\begin{align*}
\begin{bmatrix}
I_{p,A,BEMF} \\
I_{p,B,BEMF} \\
I_{p,C,BEMF}
\end{bmatrix} + \frac{(L+M)}{R} \frac{d}{dt} \begin{bmatrix}
I_{p,A,BEMF} \\
I_{p,B,BEMF} \\
I_{p,C,BEMF}
\end{bmatrix} = -\frac{1}{R} \begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix}
\end{align*}
\]

(3.22)

The unknown part of the homogenous solution (H in 3.20) is found by setting the sum of the homogenous and both parts of the particular solution to the initial condition of the phase current at time equal to zero (or the time of the most recent switch). For now, the initial condition is \(I_{iA}, I_{iB}, I_{iC}\) and the initial time is \(t=0\). Equation 3.24 gives the value of H, the final part of the solution of the phase currents, when solved.

\[
\begin{align*}
\begin{bmatrix}
I_{iA} \\
I_{iB} \\
I_{iC}
\end{bmatrix} &= \begin{bmatrix}
H_A \\
H_B \\
H_C
\end{bmatrix} \cdot e^{-\frac{R}{L+M} t} + \begin{bmatrix}
I_{p,A,U} \\
I_{p,B,U} \\
I_{p,C,U}
\end{bmatrix} + \begin{bmatrix}
I_{p,A,BEMF} \\
I_{p,B,BEMF} \\
I_{p,C,BEMF}
\end{bmatrix} \\text{at } t = 0
\end{align*}
\]

(3.24)

It follows that the solution for the homogenous coefficients is

\[
\begin{align*}
\begin{bmatrix}
H_A \\
H_B \\
H_C
\end{bmatrix} &= \begin{bmatrix}
I_{iA} - I_{p,A,U}(0) - I_{p,A,BEMF}(0) \\
I_{iB} - I_{p,B,U}(0) - I_{p,B,BEMF}(0) \\
I_{iC} - I_{p,C,U}(0) - I_{p,C,BEMF}(0)
\end{bmatrix}
\end{align*}
\]

(3.25)

Assuming the initial conditions of the phase currents are \(I_{iA}, I_{iB}, I_{iC}\) and the initial time is \(t=0\), the solution to phase A current, and similarly the other phase currents is shown in Equation 3.26. The homogenous and both parts of the particular solution are included. At time equals zero, \(I_A(t) = I_{iA}\). As time approaches infinity, the switch values and back emf values remain, but the initial condition \(I_{iA}\) decays to zero.
The phase currents are driven by power electronics whose switches are frequently turning the terminal voltages high and low. The format of Equation 3.26 is proper for describing a three-phase motor whose terminal values (switch values) are set at time equals zero, and do not change for the rest of time. However, the switches repeatedly change after time equals zero. Each time one of these switches occurs, each phase of the phase current takes on new reference current and time values, changing the slope of the continuous phase current. The slope of the phase current is not continuous due to the switching nature of the power electronics. The phase currents are more properly described by equations 3.27 and 3.28. The reference time (t_ref) is the time that the last switch occurred (on any of the phases). The initial condition of the current changes each time a switch occurs and remains in the equation as I_A(t_ref) until another switch occurs and a new initial condition is stored. When t = t_ref, the phase current is in a new initial condition.

Ideally, the switches have binary discrete switch states. They are either on or off (high or low), and change only at certain times (determined by the controls), remaining constant (as a discrete-time variable) while the phase current transitions and until the next switch occurs.

Equations 3.27, 3.28 for phase currents I_A, I_B respectively.

Equations 3.27 and 3.28 are used in a time-loop model of phase current simulation. After each switch occurs, the reference time and phase currents are reset. Next, at each small time step before the next switch, the new phase currents are calculated, advancing t, but holding t_ref and the reference currents constant. Once another switch occurs, the new terminal voltages are used, as well as the new reference times and phase currents. The three-phase motor current simulation model continues to calculate the phase currents through half of one fundamental electrical motor cycle on these principles.

One point of inquiry about the phase current description is when the switches take place. This section speculates that when a switch happens, the terminal voltages are set and the transition occurs until the next switch happens, but it is not clear here when these switches work. The switch times are determined mainly by the controls of the system. Different pwm control schemes are utilized in this thesis to control the switch activity.
Pulse width modulation generates a method of varying the pulse widths on each phase to stimulate three phases of sinusoidal currents on the motor phases. The type of pulse width modulation chosen and implemented by the controller determines when the switchings occur. The power electronics time loop model calculates what the current is on each phase of the motor at small incremental time steps according to the equations above. Inside the power electronics model there are choices for different pulse width modulation techniques which simulate the controller's ability to generate switch times.

In order to keep each execution of the power electronics model to minimal time, subsequently keeping the total time of the Monte Carlo motor-drive design program minimal, the time-stepping for-loop (see Figure 1.2) only covers one half cycle of phase current during a steady state speed. The anti-symmetrical nature of the two half cycles of a sine wave allows the entire sine wave to be described by either half cycle alone. The losses in three phases of one half cycle will generally be exactly repeated over the other half cycle, and the efficiency can be determined from one half cycle alone. The total harmonic distortion of a half wave is going to be the same for an entire wave with repeated harmonics.

The calculated half cycle in each design choice has to be steady state, without transitional components. To accomplish this, the initial current values chosen by the power electronics model must be three phase values which are already in steady state and sum to zero. The initial currents (see Initial Currents below), correspond to the initial electrical motor position which is $\theta_0 = 0$ at time equals zero. Since the motor inductance makes the phase currents continuous in time and since the R-L motor component causes a transitional phase current time decay to steady state when there is an offset or no initial value at all, the initial currents in the three phases cannot be ignored. Regardless of what current is initially given to the current phases at time zero, they will eventually reach three-phase steady state in which the magnitude of each phase is exactly the magnitude of the others and there are no dc offsets. However, this may take ten fundamental cycles or more, depending on the R-L relation of the motor. The power electronics model's time-minimization attribute is reason to target the three phase values in the correct phase with the pwm signals to calculate the initial three-phase current values, skipping any transitional time periods and generating one half cycle of steady state (constant speed) phase currents.

Minimally, an entire sine wave can be described by alterations of a sixty degree segment. However, this model requires that each phase reaches a peak maximum or minimum current value which gives its magnitude or offset in relation to the other current phases. If the maximum values of the absolute value of each phase current half-cycle waveform are within a tolerable percentage (>95%), the currents are considered to be in steady state, and any transitional current offsets have been dampened out. Otherwise, if the largest and smallest maximum are not within that tolerable percentage, the initial phase current values are not correct for steady state, and transitional current offsets are present, creating non-steady state loss data, giving an incorrect energy efficiency. In this case, the power electronics model tells the main program that the results are invalid and must be disregarded. In the future, there could be better methods of controlling the
maximum values or choosing the initial values that would not eliminate some of the possibilities because they are not in steady state during the half-cycle generated.

3.2.2 Energy Losses

Energy losses in the power electronics and motor are calculated as the power electronics program steps through the half-cycle for-loop. After each time step (increment) a new calculation of the three-phase currents (see Section 3.2.1) occurs and the energy losses at that time are calculated. There are several consumers of energy in the motor drive system.

- **Switching losses** are calculated only when switches occur.
- **Conduction losses** in the power electronics occur during non-switching times.
- **Ohmic losses** in each phase of the motor are continuous with phase current.
- **Energy delivered** to the system is useful energy that powers the motor.

Although they are generally small, the **switching losses** in the power electronics are usually the main source of inefficiency in the inverter. Inverter efficiencies are approximately ninety five percent or greater, and most of that five percent is lost during switching. When a switch occurs, one phase of the inverter closes a path for the current to flow through either the positive or ground of the power source and opens a path for it to flow through the other line of the power source. Delay timing in the controls eliminates the risk of shorting the power source but will not be discussed in this thesis. Switching this path also switches the terminal voltage at the node connecting the phase of the inverter and the phase of the motor. The pwm voltage signals on the terminals of the motor are created in this switching method.

When a switch signal is given to a phase of the inverter, switching does not occur instantaneously. The physics of the transistor and diode components in the inverter make these transitions less than ideal. Figure 3.2 shows typical non-ideal switching waveforms in the inverter which give rise to the switching energy losses.
Figure 3.2 Typical voltage and current switching waveforms giving rise to the main source of energy loss in the power electronics.

\[
P(t) = V(t) \cdot I(t)
\]

\[
E = \int_{t_i}^{t_f} P(t) \, dt
\]

Power is the product of instantaneous voltage and current. Energy is the integral, with respect to time, of the instantaneous power. Energy is always consumed (lost) in each IGBT as well. By summing all of the switching and conduction losses in the upper and lower IGBT’s in all three phases of the motor drive, the total inverter energy loss for a period of time (half-wave of phase current) is calculated.

Figure 3.2 shows how non-uniform the switching slopes and timing between the current and voltage can be. The slopes and delays in the waveforms are not predictable with the limited information provided in manufacturer databooks such as Powerex [22]. Therefore, the energy loss calculations are not predictable either. Due to the complexity of the switching waveforms and the in-depth calculations that would be necessary for precise energy loss data, this thesis uses a square-triangle method for approximating the switching voltage and current waveforms as well as the energy losses.

Below, the square-trapezoid switch waveform approximation method will be described for each of the four switching scenerios for each phase of the motor. Each scenerio involves a transition (switch) in both the upper and lower IGBT modules. One turns off and the other turns on. These actions lead to energy losses in both the Upper and Lower IGBT modules during every switch. Table 3.1 lists the four scenerios and relates them to the proper switch transition.

Figure 3.3 gives the IGBT modules’ current and voltage conventions used in this thesis. For example, instantaneous power lost in the Upper and Lower IGBT modules are \(V_{hi} \cdot I_{hi}\) and \(V_{lo} \cdot I_{lo}\) respectively. Figure 3.3 also shows that the IGBT module current...
paths of $I_{hi}$ and $I_{lo}$ are determined by the sign of the phase current, $I_{phase}$; they can either pass through the diode or transistor, causing the energy to dissipate in that part of the module alone. The transistor/diode paths are discussed in greater detail in Section 2.

<table>
<thead>
<tr>
<th>$I_{phase} &gt; 0$</th>
<th>Upper</th>
<th>Lower</th>
<th>$I_{phase} &lt; 0$</th>
<th>Upper</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>switch Hi</td>
<td>turn ON</td>
<td>turn OFF</td>
<td>switch Hi</td>
<td>turn ON</td>
<td>turn OFF</td>
</tr>
<tr>
<td>switch Low</td>
<td>turn OFF</td>
<td>turn ON</td>
<td>switch Low</td>
<td>turn OFF</td>
<td>turn ON</td>
</tr>
</tbody>
</table>

Table 3.1 Four switch transitions. Switching a phase hi (on) and low (off) when the phase current is both positive and negative.

Fig. 3.3 Single phase inverter IGBT modules. (a) Current flows through either the upper transistor (hi) or the lower diode (lo) when the phase current is positive or leaving the inverter. (b) Current flows through either the upper diode (hi) or lower transistor (lo) when the phase current is negative, or entering the inverter from the motor.

Figures 3.4 and 3.5 show the square-trapezoid method of approximation for both the Upper and Lower IGBT modules during turn-on and turn-off. Notice that $I_{hi}$ and $-I_{lo}$ always sum to $I_{phase}$. The switch transitions in each case take place during the rise and fall times, $t_r$ and $t_f$ respectively. Once again, note that the delay timing from the gating signal to the beginning of the switch transition is not included in this thesis because it does not significantly affect the power loss or efficiency calculation.
The approximated switching waveforms in the square-trapezoid method adopted in this thesis model the voltage transitions in the IGBT modules as ideal (square) switches. The current waveforms transition from hi to low or low to hi over a tiny fall or rise time with constant slope. The reason for such a simple model is to create simple energy loss calculations which depend on the given IGBT module parameters \( t_n, t_f, V_{diode}, V_{ceSat} \) and the design/model parameters \( I_{phase} \) and \( V_{bus} \). The equations for energy loss during each
switch in Equations 3.31-3.34, are simple area-under trapezoid calculations. The simplicity keeps the power electronics energy loss calculations from being time-exhaustive inside the time-loop.

\[ I_{\text{phase}} > 0 \]

\[
E_{\text{sw(on)Upper}} = I_{\text{phase}} \left( \frac{V_{\text{bus}} + V_{\text{diode}}}{2} \right) t_r, \quad E_{\text{sw(off)Lower}} = I_{\text{phase}} \left( \frac{V_{\text{diode}}}{2} \right) t_f \quad (3.31)
\]

\[
E_{\text{sw(off)Upper}} = I_{\text{phase}} \left( \frac{V_{\text{bus}} + V_{\text{diode}}}{2} \right) t_f, \quad E_{\text{sw(on)Lower}} = I_{\text{phase}} \left( \frac{V_{\text{diode}}}{2} \right) t_f \quad (3.32)
\]

\[ I_{\text{phase}} < 0 \]

\[
E_{\text{sw(on)Upper}} = I_{\text{phase}} \left( \frac{V_{\text{diode}}}{2} \right) t_f, \quad E_{\text{sw(off)Lower}} = I_{\text{phase}} \left( \frac{V_{\text{bus}} + V_{\text{diode}}}{2} \right) t_f \quad (3.33)
\]

\[
E_{\text{sw(off)Upper}} = I_{\text{phase}} \left( \frac{V_{\text{diode}}}{2} \right) t_f, \quad E_{\text{sw(on)Lower}} = I_{\text{phase}} \left( \frac{V_{\text{bus}} + V_{\text{diode}}}{2} \right) t_f \quad (3.34)
\]

The total switch energy loss in each case above is the sum of the energy lost in the Upper and Lower IGBT module.

\[
E_{\text{switch(Hi)}} = E_{\text{sw(on)Upper}} + E_{\text{sw(off)Lower}} \quad (3.35)
\]

\[
E_{\text{switch(Lo)}} = E_{\text{sw(off)Upper}} + E_{\text{sw(on)Lower}} \quad (3.36)
\]

During non-switching times, there are still very small energy losses in the IGBT modules. Figures 3.4 and 3.5 show that some non-switching times have both non-zero current and voltage, resulting in power dissipation in the IGBT module. The physical nature of the transistors and diodes contain both offset voltages and small resistive elements which unfortunately still dissipate power. Although \( V_{\text{diode}} \) and \( V_{\text{cesat}} \) both have some functional dependency on current, this thesis considers them constant to avoid time-consuming calculations. The 'conduction' (non-switching) energy losses are thus simple \( V*I*t \) equations specific to the path of the current which has been shown to be determined by the sign of the phase current and the state of the switch (Hi or Low). Equations 3.37-3.40 cover all conduction losses in each phase.
\( I_{phase} > 0 \)

\[ E_{\text{cond}(Hi)} = I_{phase} \cdot V_{ceSat} \cdot t \] (3.37)

\[ E_{\text{cond}(Lo)} = I_{phase} \cdot V_{diode} \cdot t \] (3.38)

\( I_{phase} < 0 \)

\[ E_{\text{cond}(Hi)} = I_{phase} \cdot V_{diode} \cdot t \] (3.39)

\[ E_{\text{cond}(Lo)} = I_{phase} \cdot V_{ceSat} \cdot t \] (3.40)

The ohmic losses in the motor are \( I^2R \) per phase power losses. The energy lost in the ohmic resistance is simply the time integral of these power losses once again. In the wye configured three-phase motor model (Figure 2.3), each phase has a basic series ohmic resistance associated with it. This resistance exists in the copper windings which are long conductors with significant enough cross-sectional area to include the ohmic losses when more than a few amps of sinusoidal current are run through them. This thesis assumes that the phase resistance of the motor is a known quantity and determination of that quantity is out of the scope of this thesis.

The energy delivered to the motor is the useful energy that powers the motor to drive whatever load to which it is attached. The energy delivered to the motor is the time integral of the instantaneous power. The power delivered at a particular time is calculated by multiplying the instantaneous phase current by the instantaneous BEMF (Back EMF) voltage of that phase.

Phase shifts between the Back EMF and the phase current reduce the maximum power delivered to the motor. Without a phase shift, the maximum power delivered to the motor is |BEMF||phase| (VI*). In order to maintain the maximum power transfer, or something close to it, the controller has to have a phase shift adjustment to excite a correctly shifted current. The power electronics model creates maximum power delivery by making the Back EMF and phase currents properly aligned.

Inside the time-loop of the power electronics model, at each time interval and in each phase, the energy delivered to the motor is computed by multiplying the instantaneous BEMF voltage, the instantaneous current, and the time-step interval size.

\[ \eta_{\text{system}} = \frac{E_{\text{deliv}}}{E_{\text{deliv}} + E_{\text{cond}} + E_{\text{ohm}} + E_{\text{switch}}} \] (3.41)
The efficiency of the system \( (\eta_{\text{system}}) \) is given in Equation 3.41. Without any ohmic, conduction, or switching losses, the system would be one hundred percent efficient. However, the losses can not be ignored and must be properly accounted for. The system efficiency gives an insightful view of not only the effectiveness of the particular devices used in the inverter or the topology of the motor, but also the switching technique (pwm) employed. Some switching techniques which eliminate harmonics but have more switches in each fundamental cycle drive the switching losses up and reduce the efficiency of the motor. The tradeoffs are observable and can be implemented in the main Monte Carlo program to decide tradeoff criteria for superior design selection.

### 3.2.3 Other Important Values

The power electronics model returns a number of values that the Monte Carlo main program considers attributes. The weight and cost of the IGBT modules are returned to the main program along with the efficiency. In research and development, these attributes are a factor affecting the feasibility of producing a given design. A superior motor drive system may be much more expensive or heavy, and without considering the cost and weight, there will be problems down the line in the production stage.

The maximum current generated in the motor by the power electronics and controller techniques is also returned to the main Monte Carlo program as a system attribute. Maximum current may be an inequality constraint if the designer chooses to set a limit in the main program. Since each phase reaches a positive or negative current peak in the half-cycle that is calculated, the maximum absolute value in each phase is the magnitude of the sinusoidal current waveforms. As previously mentioned, if the maximum currents are not within a certain percent of each other, there exists a transitional offset that has not dampened out and the efficiency data is not accurate.

### 3.2.4 Total Harmonic Distortion

The total harmonic distortion of a waveform is defined by Schlect, Verghese, and Kassakian in ...[8]. Equation 3.42a demonstrates the calculation of the total harmonic distortion in a waveform as a percent. Total harmonic distortion is a simple way to measure how much ripple is in a non-ideal sinusoidal waveform. Ideal sinusoids have zero percent total harmonic distortion because there are no harmonics present. However, the three-phase current waveforms created from pwm switching techniques have ripple that adds harmonics to the generally sinusoidal waveform. The amount of ripple tells how big these harmonics are getting without going into detail about which particular subharmonic is exerting itself.

\[
\text{thd}_{\text{rms}} = \sqrt{\frac{I_{\text{rms}}^2 - I_{\text{1,rms}}^2}{I_{\text{1,rms}}^2}}
\]  

(3.42a)
In Equation 3.42a, the difference between the fundamental rms component \( I_{1,\text{rms}} \) of the waveform and the overall rms component \( I_{A,\text{rms}} \) creates the value of total harmonic distortion. The fundamental and overall rms components are calculated using equations 3.43a and 3.44a respectively below when one half cycle of the waveform in question, \( I(\theta_e) = I \sin(\theta_e) \) is known. If the waveform was a full cycle from zero to two pi, the scaling factor \( (\pi) \) in the denominator of Equation 3.43a would be square root of two times pi and both integrals would be from zero to two pi according to [8].

\[
I_{1,\text{rms}} = \frac{\pi \int_0^\pi I(\theta_e) \sin(\theta_e) d\theta_e}{\pi} \quad (3.43a)
\]

\[
I_{A,\text{rms}} = \frac{\pi \int_0^{2\pi} I^2(\theta_e) d\theta_e}{2\pi} \quad (3.44a)
\]

Since the three-phase currents have phase shifts of zero, negative one hundred twenty, and negative two hundred forty for phases A, B, and C respectively, the phase A current waveform is used to determine total harmonic distortion. The software function THD.m developed (see Section 5.6 and Appendix) requires a waveform with zero phase shift for calculations. The current in phase A is aligned with the BEMF of phase A which never has a phase shift due to the nature of the phase alignment.

### 3.2.5 Phase Alignment and Initial Currents

When the semi-square terminal waveforms \((U_A, U_B, U_C)\) and the previous (or initial) phase currents are provided, the Current Simulation Method for phase current calculation in a three-phase electric motor is used to determine the resulting phase current waveforms. However, the semi-square terminal pwm voltage waveforms must contain a phase shift with the electrical motor position \( (\theta_e) \), and subsequently with the Back EMF which is aligned with the electrical motor position. This phase shift is necessary, and possibly made more clear in the mathematics below, because the complex impedance of the motor phase provides a phase shift in the phase current as compared to the voltage excitation. It is efficient to align this phase-shifted current with the corresponding Back EMF to provide most efficient power out of the mechanical end of the motor. Since the Back EMF is phase-aligned with the electrical motor position, the phase current, when aligned with the BEMF, will also be phase aligned with the electrical motor position. Therefore, the pwm voltage waveforms at the power electronics terminals must contain a phase shift compared to the electrical motor position to maintain the phase currents' alignment with the Back EMFs.

The value of the initial phase currents is also important to the success of the power electronics model. As mentioned above, if the initial phase currents have transitional offsets which prevent the steady state operation of the system during the first
and only half cycle of simulation, the loss and efficiency calculations are not reliable sources of system parameters. Therefore, the initial phase currents have to be phase-aligned with the Back EMF and electrical motor position. The magnitude of the phase currents is the next issue.

The following method is used to compute the phase current magnitude (I) and the terminal voltage pwm waveform phase shift (\(\phi\)) if the duty cycle value of the pwm waveforms (M) is known, or to compute the pwm phase shift and the duty cycle value if the magnitude of the phase currents is known. The values of I, M, and \(\phi\) are therefore generalized below and Equations 3.49 and 3.50 are used to solve for the unknowns.

It is a convention of this thesis that the electrical motor position (\(\theta_e\)) is in phase with the Back EMF of phase A (BEMF\(_A\)). Equation 3.42 demonstrates \(\theta_e\)'s numerical representation which is recognized in each of the sinusoidal motor equations in this thesis. Equation 3.43, used in this section of the thesis, is a form of Equation 3.5 which is more useful for this section.

\[
\theta_e = p \omega t \quad (3.42)
\]

\[
\begin{bmatrix}
BEMF_A \\
BEMF_B \\
BEMF_C
\end{bmatrix} = K \sin(\theta_e)
\begin{bmatrix}
\sin(\theta_e - 120) \\
\sin(\theta_e - 240)
\end{bmatrix} \quad (3.43)
\]

Maximum power is delivered to the motor when the three phase currents are phase-aligned with their respective Back EMFs. For that particular reason, the phase currents generated in the power electronics models are also phase-aligned with their respective Back EMFs. The form of the three phase currents is therefore generally the form of Equation 3.44, disregarding the harmonics. The amplitude (I) of the current waveform is not significant yet. The phase current harmonics other than the fundamental are not important here as long as the fundamental dominates the current and the power delivered to the motor is maximized when the fundamental is phase-aligned with the Back EMF. In situations where this is not the case, the harmonic content of the phase currents is too high, and the architecture or control scheme is not considered as an optimal design.

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} = \begin{bmatrix}
I \sin(\theta_e) \\
I \sin(\theta_e - 120) \\
I \sin(\theta_e - 240)
\end{bmatrix} \quad (3.44)
\]

From 3.44, it is noted that the initial currents for each phase are chosen at \(\theta_e = \) zero which is also time = zero. The guesses for the initial currents in the power electronics program are simple to choose once the value of I is determined.
The terminal-neutral voltage ($V_A$, $V_B$, $V_C$ in Equation 3.3) is rewritten in Equation 3.45 and simplified in Equation 3.46 to help determine the duty cycle ($M$) of the pwm waveforms with a given magnitude of current or to determine the current waveform amplitude given a duty cycle. In both cases the power supply voltage remains fixed at $V_{bus}$, but the changing of the duty cycle varies not the pwm pulse height, but the pwm pulse widths.

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix}
= R_I \begin{bmatrix}
\sin(\theta_e) \\
\sin(\theta_e - 120) \\
\sin(\theta_e - 240)
\end{bmatrix}
+ K_{p\omega} \begin{bmatrix}
\sin(\theta_e) \\
\sin(\theta_e - 120) \\
\sin(\theta_e - 240)
\end{bmatrix}
+ (L + M) p\omega \begin{bmatrix}
\cos(\theta_e) \\
\cos(\theta_e - 120) \\
\cos(\theta_e - 240)
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix}
= \sqrt{(R_I + K_I p\omega)^2 + ((L + M) p\omega)^2} \begin{bmatrix}
\sin(\theta_e + \tan^{-1}\left(\frac{(L + M) p\omega}{R_I + K_I p\omega}\right)) \\
\sin(\theta_e - 120 + \tan^{-1}\left(\frac{(L + M) p\omega}{R_I + K_I p\omega}\right)) \\
\sin(\theta_e - 240 + \tan^{-1}\left(\frac{(L + M) p\omega}{R_I + K_I p\omega}\right))
\end{bmatrix}
\]
In Equation 3.6, the terminal voltages \((U_A, U_B, U_C)\) are the sum of the respective terminal-neutral voltages \((V_A, V_B, V_C)\) and the neutral voltage \((V_N)\) which is the average of the three terminal-neutral voltages. In order to determine either the duty cycle of the terminal voltages or the magnitude of the phase current waveforms, only the fundamental element of the terminal voltage square waveforms is used in this thesis. Although the sub-harmonics of square waveforms may be relatively high in magnitude in comparison to the fundamental, the mathematics involved in calculating all of the harmonics and using them to determine the duty cycle or current magnitude would be too time-consuming and equation-exhausting to be beneficial for the very slight increase in accuracy that the resulting duty cycle or current magnitude would gain. \(V_N\) is therefore the sum of three fundamental sinusoidal waveforms (each centered about \(V_{bus}/2\)), shifted by 120 degrees with respect to each other. The neutral voltage is therefore \(V_{bus}/2\) (seen by averaging the three terminal voltage values for all time in Equation 3.47). The pwm waveforms (terminal voltages) fundamental components are shown in Equation 3.47.

\[
\begin{bmatrix}
U_A \\
U_B \\
U_C
\end{bmatrix} = M_{bus} \begin{bmatrix}
\sin(\theta_e + \phi) \\
\sin(\theta_e -120+\phi) \\
\sin(\theta_e -240+\phi)
\end{bmatrix} + \frac{V_{bus}}{2}
\] (3.47)

By substitution, Equation 3.48 enables one to find solutions for \(I\), \(M\), or \(\phi\) depending on the givens and unknowns mentioned above.

\[
M_{bus} \begin{bmatrix}
\sin(\theta_e + \phi) \\
\sin(\theta_e -120+\phi) \\
\sin(\theta_e -240+\phi)
\end{bmatrix} + \frac{V_{bus}}{2} = \sqrt{\left(\frac{RI + K_I p\omega}{L + M p\omega}\right)^2 + \left(\frac{(L + M p\omega)^2}{L + K_I p\omega}\right)^2}
\]

(3.48)

From 3.48, Equations 3.49 and 3.50 are easily distinguished. Either \(M\) or \(I\) can be calculated in 3.49 as long as the other is known. When the magnitude of the current waveform is given or predetermined, the duty cycle and the pwm phase shift are simple to determine. However, when the duty cycle value is known and the current magnitude is unknown, a quadratic solution is necessary to find the two possible values of \(I\). Then the phase shift value can be calculated.

\[
M_{bus} = \sqrt{\left(\frac{RI + K_I p\omega}{L + M p\omega}\right)^2 + \left(\frac{(L + M p\omega)^2}{L + K_I p\omega}\right)^2}
\] (3.49)
\[ \phi = \tan^{-1} \left( \frac{(I + M)p\omega l}{RI + K_I p\omega} \right) \]  

(3.50)

In this thesis, I is not given. I is a result of the pwm waveforms operating on a motor system which has series inductance and resistance in each phase as well as a Back EMF which is a function of the operating speed of the motor. If one wanted to use the other calculation method and determine the duty cycle of the pwm technique (see *Recommendations*), the power electronics program would have to be slightly altered, and the power delivered to the motor would be known at every speed.
Chapter 4

Software

The following Matlab programs make up the power electronics/three-phase current simulation models that are the main focus of this thesis. The code for each program is listed under its own heading in the Appendix. However, each Matlab program is described in Chapter 5 to provide an understanding of how to use or alter the program.

- Isim.m
- sw_setup.m
- setbac.m
- setup_triwave.m
- centpwm.m
- thd.m

4.1 Isim.m

Figure 4.1 Logic flow in Isim.m — the current simulation program (also called the power electronics model). A function call is made to Isim and the five steps are performed as shown. Then the output is returned to the main program.

The current simulator, Isim.m is the main power electronics model. Figure 4.1 demonstrates the flow of Isim from the function call in the main Monte Carlo program to
the return to the main program with the output values. Isim is called from the main Monte Carlo program for each successful motor design and it performs a three-phase current simulation as well as an energy loss and efficiency analysis. The following paragraphs describe the inputs, outputs, and flow of the current simulator and its sub-functions listed above which perform small tasks for the main simulator.

The inputs to the current simulator are basically the electrical motor and power electronics parameters and the testing characteristics as well as a few random choice selections determined in the Monte Carlo program, outside of the current simulator. The following is a list of inputs to the current simulator:

- Motor parameters - R, L, M, p, K
- Power electronics - \( V_{\text{cesat}}, V_{\text{diode}}, t_r, t_f, V_{\text{bus}} \)
- Controls - \( f_{\text{carrier}}, \text{sixstV}_{a,b,c}, \text{helim}, \text{pwm choice} \)
- Testing conditions - \( \omega_m \)

The motor parameters from the electrical model of a three-phase, wye configured motor are easily recognized in Figure 2.3. For a further description of these parameters, see Section 2.3.1. In this thesis and in the models developed herein, each phase of the motor has exactly the same armature resistance (R) and inductance (L) as the other two. In real life this is a slight approximation since the manufacturing precision can be excellent, but will never be perfect. There may be imperfections in the wire material or the magnets, but if they exist, they are insignificant in a well-built machine. The magnet strength motor constant (K) which, as a product of motor speed, determines how large the BEMF of each phase gets, is also characteristic of the particular motor topology chosen. The number of pole pairs (p) is selected by the random integer generator in the Monte Carlo main program and usually lies between one and four, but occasionally five pole pair machines are examined. K is a product of a motor constant \( K_1 \), and the number of pole pairs. Therefore, for the same type of motor design and magnet type, two motors with different pole pairs will have the same \( K_1 \) value, but different K values.

The power electronics parameters are chosen from a database of existing power electronics IGBT modules. However, the current power electronics models haven't been appended with a database so the values for \( V_{\text{cesat}}, V_{\text{diode}}, t_r, \text{ and } t_f \) have to be preselected by the programmer in the main program before the call to Isim is made. Figures 3.4 and 3.5 demonstrate the function of each of the power electronics parameters in Isim. The energy loss equations 3.31 to 3.40 show the calculations used to approximate the energy losses in the current simulation program. The power source high voltage \( V_{\text{bus}} \) is the high voltage that the power electronics switch onto the motor phases. Slight deviations from \( V_{\text{bus}} \) (the diode and transistor offsets) are used to calculate the energy loss approximations as mentioned above, but are not used to simulate the current waveforms in the motor because of the overall insignificant effects that they would have on the bus-voltage dominated phase current values.

The controls inputs exist to assist in the construction of the specific pwm waveforms employed on each motor drive design choice. The pwm technique choice (\textit{scheme} in Isim) is an integral random selection determined in the main Monte Carlo
design synthesis program. There are choices for a comparator scheme, asynchronous sinusoidal, six-step with harmonics elimination, and center-aligned pwm. The random choice is displayed in the dominant design file to further understand which pwm techniques promote desired and undesired results. The pwm carrier frequency $f_{\text{carrier}}$ (carrfreq in Isim) is the pwm frequency and the number of pulses per half electrical fundamental motor wave is easily calculated with the knowledge of the motor speed. The other three inputs ($\text{sixst}V_a, \text{sixst}V_b, \text{sixst}V_c$) contain the harmonics elimination logic in the six-step pwm technique described in Section 2.4.4. The logic is seven sets of fundamental line-ground six-step waveforms to be turned into switching modulation waveforms in Isim. Each additional set contains the necessary notches cut out for an additional non-even, non-triplen harmonic to be suppressed. The choice for which set to use is contained in $h_{\text{aim}}$, a random integral number which is also determined in the Monte Carlo design synthesis stage.

The motor speed input $\omega_m$ is the mechanical rotational speed of the three-phase motor in radians per second. The mechanics of the motor topology determine the desired motor speed and input it to the current simulator at the Isim call. The desired speed of the motor is very important because it is how fast the mechanical system attached to it will turn. If the desired speed is in revolutions per minute, simple calculations can make the adjustments, but radians per second is easier to use in sinusoidal calculations. The electrical speed of the machine $\omega_e$ is simply the mechanical speed times the number of pole pairs in the machine. The calculations in Isim operate on the electrical speed of the motor because the current waveforms are repeated over the number of pole pairs, and no new knowledge is gained by observing multiple cycles of the same current waveforms.

The outputs of the current simulator are the parameters that are returned upon completion of simulating one half of a fundamental electrical current cycle and are either used for further calculation or more importantly used for dominance and superior design examination as well as representative data stored in the database of dominant designs for further verification outside of the program limits. The following are the outputs of Isim.

- system efficiency
- weight and cost of power electronics
- g (good-or-not)
- current amplitude percent
- total harmonic distortion

The system efficiency (syseff in Isim) in Equation 3.41 is calculated after all of the energy losses over the half-wave have been determined. The energy delivered to the motor and the conductin, switching, and ohmic losses are all necessary for the calculation of the system efficiency.

The weight and cost of the power electronics is an option that is not necessarily exercised for the completion of the efficiency and simulation calculations. From a business perspective, these parameters may be very important in choosing an economically superior design as opposed to the very expensive, and very heavy dominant design from the engineers point of view. For the purposes of this thesis though, the
weight and cost of the power electronics is not discussed in-depth and does not give rise to any truly insightful results.

The g (good-or-not) value is part of the main Monte Carlo framework and tells the program if a design is no longer valid for dominance tests. If the three phase current exceeds the maximum limit or if the current amplitudes are not within a certain tolerance (percent) of each other and there remains a transitional current component that is throwing off energy loss results, then the good-or-not variable is set to zero and the design is thrown out. There may be other tests implemented in the Isim program that determine that the particular design is no longer valid, setting g to zero and telling the program that it is time to move on and synthesize another drive design.

The minimum current waveform percent (min_per in Isim) is a steady state testing value that is returned to the main program for good-or-not (g) determination. Since there are three sinusoidal current waveforms which sum to zero by definition of KCL (joined at the neutral point), the amplitudes of the three waveforms should be equal to within a tiny percent tolerance due to the harmonics of the waveform. If there are any transitional current offsets from incorrectly choosing the initial current values (see Equation 3.44) or the pwm phase shift (\( \phi \)) is incorrect, the maximum amplitudes of the three resulting waveforms are not going to be equal or close to equal. Therefore, by examining the minimum percentage of three amplitude comparisons, the accuracy of the pwm phase shift and the initial current values is determined.

Total harmonic distortion (thd in Isim) is an output due to its usefulness. Section 3.2.4 discusses the mathematical determination of total harmonic distortion. The half cycle of current waveforms contain many non-triplen, non-even harmonics which are not only sources of electrical losses in the motor, but can cause thermal difficulties inside the motor and its magnets, changing different aspects of the motor parameters. Phase resistance, BEMF, \( K \), and other parameters can go into saturated values, or become unstable under certain thermal and mechanical situations involving harmonic distortion. These are not covered by the scope of this thesis though, so the main goal is to return the amount of total harmonic distortion in the current waveform to the main program and let other professionals interpret the results later from the file of dominant designs.

### 4.2 Sw_setup.m

Sw_setup.m is the function call to the switch initialization Matlab program. The current simulator and power electronics model requires arrays that can maintain the switch states in all three phases of the power electronics during the half-wave time loop. The switch states are initialized to zero state for convenience. Also, the switch setup program creates the time array \((T)\) which contains the time data matching each of the \(x/2\) or \(x/2+1\) data points in the current waveforms. According to the speed of the motor, and the number of data points \((x)\) required for one full cycle of electrical motor rotation, the time interval between adjacent data points is calculated using Equation 4.1.

\[
\text{Interval} = \frac{2\pi \times \text{omm} \times \text{p}}{x}; \quad (4.1)
\]

where \( x \) is twice the length of the half cycle current array, \( \text{omm} \times \text{p} \) is \( \omega_c \).
This interval is used not only to increment the time (t) during the steps of the current simulation time loop, but to integrate the power lost to find the conduction energy lost during non-switching time steps in Equations 3.37 to 3.40. The variable containing the number of switches taking place in each phase for the half-cycle is initialized to zero as well. After these values are calculated and initialized, they are output to the Isim program.

4.3 Setbac.m

Setbac.m is called to setup and initialize the BEMF components of the half-cycle, the current amplitude (refamp), and the initial phase current states at time equals zero. The BEMF ($V_{backABC}$ in Isim) is computed before the time loop so that one line of calculation is created instead of x/2 (order of 1000) lines of calculation to save time. The BEMF is independent of the current in the phases; it is only determined by the speed and position of the motor and the motor topology. However, the initial currents and current amplitude additionally require knowledge of the power supply voltage.

The inputs to the setbac function are the following motor drive parameters:

- $R$, $L$, $M$, $K_1$, $\omega_m$, $p$, $V_{bus}$, and $T$

The BEMF portion of the current from Equation 3.23 in Szolusha’s Method ($I_{p,phase,BEMF}$) is calculated from the above inputs ($BackABC$ in Isim) also to eliminate unnecessary time by including it in the time-loop. The phase current amplitude $I$ (refamp in Isim) in Equation 3.49 is solved by using quadratic roots method. The positive root is the solution in amps. The duty cycle $D$ in Equation 3.49 is set to 100% in this thesis, but it can be altered to fit the needs of the user. If the BEMF amplitude $K_1\omega_0$ is greater than $D*V_{bus}/2$ (the modulation fundamental component amplitude), some problems arise and the solution to $I$ becomes complex and eliminates the motor design as a possibly dominant choice.

4.4 Setup_triwave.m

Section 2.4.2 describes the asynchronous sinusoidal pwm method. Setup_triwave uses the motor speed, time interval, and carrier frequency $f_{carrier}$ ($carrfreq$ in Isim) data to develop a triangle waveform to compare to the reference waveforms in Isim. To create this triangle waveform, the number of triangular cycles in a half wave of fundamental electrical motor rotation is computed using Equation 4.2.

$$\text{number} = 2\pi p_i * carrfreq / \omega_m \quad (4.2)$$

where $\omega_m$ is $\omega_q$.

The triangle comparison wave is always started at zero radians for simplicity. The asynchronous characteristic comes from the shifting of the reference waveforms that it is compared to by angle $\phi$ (phi in Isim).
4.5 Centpwm.m

Center-aligned pwm is described in Section 2.4.4. When the center-aligned pwm scheme is selected, the program makes a function call to centpwm.m to generate the pwm (switching) waveforms for the half-cycle of current excitation. The phase angle \( \varphi \), the number of pulses per half-cycle, and the number of data points \( x \) in a half-cycle are the only inputs required to generate the pwm waveforms. Once the three-phase mod waveforms are returned to Isim, they are multiplied by the bus voltage \( V_{\text{bus}} \) (\( V_{dc} \) in Isim) to become the terminal voltage pwm waveforms. The widths \( \delta_{A,B,C} \) (length in centpwm) of each pulse are calculated by using Equations 2.1 to 2.3.

4.5 THD.m

After the three-half-cycle phases of approximately sinusoidal current waveforms are generated, the final output parameter in Isim is calculated, the total harmonic distortion. Section 3.2.4 describes the method used to calculate the amount of ripple in the current waveforms as a percent of the fundamental.

THD.m, listed in the Appendix, only has one output, the percent of total harmonic distortion which is given to the main program as a drive system attribute. Equation 3.42a shows how thd is calculated. The inputs to THD are the phase A current waveform \( I_{ABC}(1,:) \) in Isim) the radial step size as opposed to the time interval, the set of radial steps, and the number of fundamental cycles in the waveform which is one half in this thesis.

The calculations in THD.m require an integration of the multiplication of sinusoidal waveforms performed in Equations 3.43a and 3.44a. The function i_grate.m performs this task by doing a step-averaging integration method (see Appendix).
Chapter 5

Numerical Results

The power electronics and current simulation Matlab program Isim.m is run with each of the four pwm techniques described in the thesis (Comparator, Asynchronous Sinusoidal, Six-Step with Harmonics Elimination, and Center-Aligned Pwm). The half-wave results listed in the Monte Carlo Half-Wave Format section below are compared to some adjusted waveform’s results in the next section (Adjusted Results) of this chapter. The Monte Carlo half-wave program Isim and the calculations that it performs are not always accurate. The Adjusted Results section discusses this issue and demonstrates possible solutions to ensure accurate calculations in future work.

Some typical motor and power electronics parameters are chosen for each simulation for proper comparison of results. These baseline values are listed below.

- \( R = 2.3 \) phase resistance (ohms)
- \( L = 0.14 \) phase self inductance (henrys)
- \( M = 0.00100 \) mutual inductance between phases (henrys)
- \( K = 0.0067 \) magnet strength constant (volts*seconds/radian)
- \( p = 2 \) number of pole pairs
- \( \omega_m = 1000 \) mechanical motor speed (rad/s)
- \( f_{carrier} = 15000 \) modulation carrier frequency (hz) asynchronous and center aligned
- \( V_{bus} = 100 \) power supply voltage (volts)

The results from each of the pwm techniques are described below. Each simulation develops the three-phase current waveforms which are graphically shown in the figures below. From the half-wave three-phase current waveforms, one can see the slight inaccuracies in phase shift and initial current values that cause non-ideal three-phase waveforms. This effect also distorts the total harmonic distortion calculation by not correctly aligning the phase A current waveform and the zero-shifted BEMF\(_A\) waveform. After allowing the simulation to run for a number of cycles (this number varies depending on the initial error and the phase shift), the above mentioned inaccuracies are dampened out and the total harmonic distortion and energy loss equations are possibly more accurate.

### 5.1 Isim Results - Monte Carlo Half-Wave Format

In each of the pwm techniques below, the resulting three-phase current waveforms are shown as well as the three-phase BEMFs, the terminal voltage wave at phase A (\( U_A \)), and the neutral voltage (\( V_n \)). The values for the total harmonic distortion, the system energy efficiency, the energy delivered, the conduction losses, the ohmic losses, and the switching losses are given.
5.1.1 Hysteresis comparator half-wave results

The comparator is set to 5% hysteresis tolerance in the results below. See Section 2.4.1 for details about the operation of the hysteresis comparator. Figure 5.1, the three-phase current waveforms during the half-cycle have some significant ripple. The 5% tolerance setting can be reduced to decrease the amount of ripple in the waveform but increase the amount of switching that takes place, therefore increasing the switching losses.

![Figure 5.1 Three-phase current waveforms during the half-cycle of fundamental electrical rotation.](image)

In the comparator switching scheme the current waveforms are phase-aligned with the respective BEMF components on each phase as seen in Figure 5.2(a). The small number of switches that arise from such a loose 5% tolerance rating are easily countable in the phase A terminal voltage waveform (U_A) in Figure 5.2(b). It is interesting to observe the phase differences that are necessary in the BEMF and U of each phase in order for the difference of the BEMF and the sum of V_n (Figure 5.2(c)) and U to excite the three-phase current waveforms which are phase-aligned with the BEMF components.

![Figure 5.2 Half-wave of the hysteresis comparator significant wave formulations. (a) Three-phase BEMF. (b) U_A. (c) V_n](image)
The following are the numerical results from the half-wave hysteresis comparator current simulation.

- total harmonic distortion (thd) = \(3.84\%\)
- system energy efficiency (eleceff) = \(70.16\%\)
- energy delivered to motor (Edeliv) = \(0.028\)J
- switch energy losses (Eigbt) = \(3.0894\times10^{-5}\)J
- ohmic energy losses (Eohm) = \(1.7138\times10^{-4}\)J
- conduction energy losses (Econd) = \(9.8452\times10^{-4}\)J

5.1.2 Asynchronous sinusoidal pwm half-wave results

The asynchronous sinusoidal pwm method has much smaller ripples but has much higher thd than the hysteresis comparator in the three-phase current waveforms (Figure 5.3). The thd calculation in Section 5.1 is not completely accurate and it is improved in Section 5.2 below. Although the ripple is small, it is more frequent as a result of the higher number of switching transitions in 5.4(b) which is determined by \(f_{\text{carrier}}=15000\). It can also be noted that the current waveforms in the asynchronous simulation are not phase aligned with the BEMF in 5.4(a). Additionally, the peak values of the three phases appear to be growing apart, containing some type of transitional offset. Refer to Section 2.4.2 for details about the implementation of the asynchronous sinusoidal pwm method.

![Asynchronous Sinusoidal PWM Waveforms](image)

Figure 5.3 Three-phase current waveforms during the half-cycle of fundamental electrical rotation.

The following are the numerical results from the half-wave asynchronous sinusoidal pwm current simulation.

- total harmonic distortion (thd) = \(5.74\%\)
- system energy efficiency (eleceff) = \(67.87\%\)
- energy delivered to motor (Edeliv) = \(0.024\)J
- switch energy losses (Eigbt) = \(1.1164\times10^{-4}\)J
- ohmic energy losses (Eohm) = \(1.3413\times10^{-4}\)J
conduction energy losses (Econd) = $8.7951e^{-4}J$

The energy delivered to the motor in the two cases above should be equal since the target amplitude and phase-alignments of the current waveforms are equal. However, the comparator and asynchronous energy delivered quantities are not equal, indicating a slight problem in the amplitude or phase of the asynchronous method.

When $f_{\text{carrier}}$ is increased, the switching losses in the IGBT module also increase. To examine this effect, the same simulation above is run using $f_{\text{carrier}}=60000$ Hz. Figure 5.4.5 shows the smoother waveform (less harmonics) from a higher switching rate of the carrier frequency.

The numerical results from the 60,000 Hz simulation are listed here.

- system energy efficiency (eleceff) = 62.32%
- energy delivered to motor (Edeliv) = .0024J
- switch energy losses (Eigbt) = 4.1866e-4J
- ohmic energy losses (Eohm) = 1.3396e-4J
- conduction energy losses (Econd) = 8.8165e-4J
As can be expected, the energy lost in the switches (E_{igbt}) increased by a factor of four when the carrier frequency, and hence the switching frequency, increased by a factor of four. If the pulse width modulation frequency gets high enough, the switching losses (E_{igbt}) become the dominant consumer of system energy.

5.1.3 Six-step pwm with harmonics eliminated half-wave results

The 5th, 7th, and 11th harmonics are suppressed in this execution of the six-step pwm with harmonics eliminated technique. The number of switches (17 per 1/2 cycle in one phase (Figure 5.6(b))) is very low for the small amount of ripple in the current waveforms (Figure 5.5). The current waveforms are supposed to be phase-aligned with the respective BEMF components on each phase as seen in Figure 5.6(a). However, just as the asynchronous sinusoidal method had some transitional offsets, so do the currents in Figure 5.5. This gives support to the theory that the phase shift-and-reference amplitude calculations in 3.45 and 3.46 are inaccurate without full consideration of all harmonics. The inaccuracies are discussed later in *Conclusions*. See Section 2.4.3 for details about the implementation of six-step pwm with harmonics elimination.

![Figure 5.5 Three-phase current waveforms during the half-cycle of fundamental electrical rotation.](image)

The following are the numerical results from the half-wave six-step pwm with harmonics elimination current simulation.

- total harmonic distortion (thd) = 37.01%
- system energy efficiency (eleceff) = 68.02%
- energy delivered to motor (Edeliv) = 0.0027J
- switch energy losses (E_{igbt}) = 4.1970e-5J
- ohmic energy losses (E_{ohm}) = 1.8050e-4J
- conduction energy losses (E_{cond}) = 0.0010J

The dominating energy consumer here is the conduction losses. The conduction loss results are discussed in *Conclusions*. 55
5.1.4 Center-aligned pwm half-wave results

The center-aligned pwm frequency is set to 15000 hz in the results below. The efficiency of the small ripples in the current waveforms in Figure 5.7 is overshadowed by the inaccuracy of the phase and/or reference amplitude or the current waveforms. The large number of switches that helped create the low-ripple three-phase current waveforms are seen in Figure 5.8(b). See Section 2.4.4 for details about the operation of the hysteresis comparator.

The following are the numerical results from the half-wave center-aligned current simulation.

- total harmonic distortion (thd) = 36.72%
- system energy efficiency (eleceff) = 67.90%
- energy delivered to motor (Edeliv) = 0.0024J
- switch energy losses (Eight) = $1.1000e^{-4}$J
- ohmic energy losses (Eohm) = $1.3277e^{-4}$J
- conduction energy losses (Econd) = $8.7434e^{-4}$J

Figure 5.8  Half-wave of the center-aligned pwm significant wave formulations. (a) Three-phase BEMF. (b) $U_A$. (c) $V_n$

The differences in the results in this section are discussed in Conclusions. Further discussion in that section relates the above half-wave Isim results in the Monte Carlo format to the adjusted waveforms’ results below.

5.2 Adjusted Steady State Waveform Results

The results in the half-wave Monte Carlo Isim format above are not completely accurate. It can be seen in the non-comparator Figures (5.3, 5.5, 5.7) that the three-phase current waveforms ($I_A$, $I_B$, $I_C$) are not in steady state. By definition in this thesis, the current waveforms in a three-phase wye configured electric motor are in steady state operation when the motor is running at constant speed and the current waveforms are fundamentally sinusoidal with zero offsets and non-changing frequency or phase-alignment. Careful inspection of the above mentioned Figures reveals that there are offsets in each of the three phases of the current in each of the Figures (5.3, 5.5, 5.7).

The effect of non-steady state waveforms is two-fold. One problem with non-steady state current waveforms for the present analysis is that the total harmonic distortion of the waveforms is incorrectly calculated, with the fundamental not aligning itself properly with the matching zero-phase sinusoidal wave generated for multiplication in THD.m to assist in the calculation of Equation 3.43a, the fundamental component’s

\[
 I_{1, \text{rms}} = \sqrt{\frac{\int_0^\pi \sin^2(\theta) d\theta}{\pi}} \quad (3.43a)
\]

The following numeric results are gathered using alterations of the Isim program. First, the asynchronous sinusoidal pwm generated current waveforms in Figure 5.3 are further generated, past one half cycle, to three complete fundamental electrical cycles to be able to see more clearly the non-steady state properties of the generated waveforms.
Next, one full cycle of the asynchronous sinusoidal pwm generated waveforms are offset and phase adjusted to compare energy loss and total harmonic distortion results. Then, the six-step method is similarly adjusted for offsets and phase shifts to further verify or disprove the seemingly excessive total harmonic distortion result in Section 5.1.3.

The results in this section are tied together and further analyzed in Conclusions. The purpose of this section is to explain the tests performed using the Isim software, or alterations thereof, and to provide the results of those tests.

5.2.1 Three Cycles of Comparator and Asynchronous

In this section, three cycles of the comparator and the asynchronous sinusoidal pwm generated current waveforms are presented. The Isim model is adjusted to continue computing phase currents for three full fundamental electrical cycles of the motor. The first half-cycle in Figures 5.9 and 5.10 are replicas of Figures 5.1 and 5.3. The second half-cycle in Figure 5.10 gives some interesting insight to the nature of the steady state properties of the currents. As mentioned above, this insight is discussed in Conclusions.

![Figure 5.9](image)

Figure 5.9  Three electrical cycles of hysteresis comparator generated phase currents using the Isim simulator without holding the half-wave restriction.

The hysteresis comparator three-cycles display the uniformness of the waveform generated from a closed-loop system. The losses in the system over the three fundamental cycles are given below. The total harmonic distortion of these waveforms is not examined here.

- system energy efficiency (eleceff) = 70.05%
- energy delivered to motor (Edeliv) = .0167 J
- switch energy losses (Eigbt) = 1.7543e-4 J
- ohmic energy losses (Eohm) = .0010 J
- conduction energy losses (Econd) = .0060 J
The losses in the system over the three fundamental cycles are given below. The total harmonic distortion of these waveforms is not examined here.

- system energy-efficiency (eleceff) = 67.63%
- energy delivered to motor (Edeliv) = 0.0165J
- switch energy losses (Eigbt) = 7.7482e-4J
- ohmic energy losses (Eohm) = 0.0011J
- conduction energy losses (Econd) = 0.0060J

5.2.2 Change of Reference Amplitudes and Initial Values in Asynchronous

One may assume that the numeric results obtained in Section 5.1.2 are inaccurate after observing the three cycle waveforms in Figure 5.10. The question arises as to how the numeric results would change if a steady state current waveform was simulated for the asynchronous sinusoidal pwm technique.

In order to answer this question, the offsets in the current simulation that are present in Figures 5.3 and 5.10 can be compensated for when choosing the initial values in the Isim program. Figure 5.11 shows the resulting waveform when \( I_A(1), I_B(1), \) and \( I_C(1) \) are initialized according to the waveform offsets and the amplitude of the current waveforms that are predicted (set in Isim) properly. From Figure 5.10 it is learned that the amplitude is actually 0.1815 amps as opposed to the calculated \( \text{refamp} \) in Isim.m (I in Equation 3.49) of 0.1531. The difference in calculated and actual amplitudes is discussed in Conclusions. Also from Figure 5.3 it is learned that the phase angles of the three phases is not 0, \(-2\pi/3\), and \(-4\pi/3\) degrees for A, B, and C respectively. If this was the case, the initial value of \( I_A \), zero, (corresponding to zero radians) would be equal to the final value of \( I_A \) in the Figure (corresponding to \( \pi \) radians). The difference of -0.0759 shows that there is not only an offset, but a phase shift. This phase shift is calculated by first removing the offset, then by discovering the inverse sine of the initial value (with offset removed) of \( I_A \). Assuming that the waveform still represents exactly \( \pi \) radians, and using the law that \( \sin(\phi) = -\sin(\phi + \pi) \), the offset must be \((I_A(0) + I_A(\pi))/2 = -0.0379\).
Compensation makes $I_A(0)=.0379$. The other two phases are also centered, removing the offsets. Figure 5.11 is the simulated (Isim) full cycle of phase currents with the adjusted amplitude and initial values calculated above.

The following Isim output parameters resulted from changing the initial currents and reference amplitude to the values discussed above. Total harmonic distortion of this waveform is discussed in the next section.

- system energy efficiency (eleceff) = \(67.86\%\)
- energy delivered to motor (Edeliv) = \(0020\) (units?? joules??)
- switch energy losses (Eigbt) = \(2.5833e-4\)
- ohmic energy losses (Eohm) = \(3.4686e-4\)
- conduction energy losses (Econd) = \(0.0020\)

### 5.2.3 Phase Shift for Total Harmonic Distortion

Total harmonic distortion calculation methods (see Section 3.2.4) in this thesis require a sinusoidal waveform with zero phase shift. The adjusted waveform in Figure 5.11 has a phase shift, similar to the waveforms generated in the Isim simulations (Figures 5.3, 5.5 5.7). The THD calculation in the numeric results above therefore might not be accurate, and a phase shifted asynchronous sinusoidal pwm generated waveform is going to be examined for THD to understand this further.

The three-phase current waveforms in Figure 5.11 have a negative phase angle of $\arcsin(I_A(0)/\text{max}(I_A))$ in relation to 0, $-2\pi/3$, $-4\pi/3$ radians. Figure 5.12 shows the waveforms shifted by $-2.085$ radians.

The total harmonic distortion of the asynchronous sinusoidal pwm method with initial current offsets and phase shifts is listed below. The usefulness of this value is also discussed in-depth in Conclusions.

- total harmonic distortion (thd) = \(2.17\%\)
In Section 5.1.3, the calculated total harmonic distortion of the six-step pwm with harmonics elimination method is almost 40%. For the current waveforms in Figure 5.5, this seems to be very high in comparison with the above calculated thd for a similarly distorted looking waveform. The method of removing the current offset and the phase shift is applied to the current waveforms generated in Section 5.1.3 to recalculate the total harmonic distortion and further understand the performance of the Isim program. The adjusted current amplitude of 0.2121 amps, the removal of the current offsets, and the shifting of the phase angle of the current waveforms by -0.2117 radians gives the steady state zero-phase-shift six-step current waveforms in Figure 5.13 which is used to calculate the actual thd of the waveform in Figure 5.5.

![Graph](image-url)

Figure 5.13 Half cycle of six-step pwm with harmonics elimination generated phase currents from Figure 5.5 shifted by -0.2117 radians for proper calculation of total harmonic distortion after initial current values are set to remove offsets.

- total harmonic distortion (thd) = 2.01%

This thd value is much different than the one in Section 5.1.3. Its significance is discussed further in Conclusions.
Chapter 6

Conclusions

This chapter discusses the implications of the numerical results in Chapter 5 which give insight to the usefulness of the Isim power electronics and current simulation model both in the Monte Carlo framework and as a simulation and waveform generation tool.

The following topics of discussion on the results of Chapter 5 include the following.

- Switching methods comparison
- Total Harmonic Distortion
- Dependability of Isim
- Isim as a full wave generator
- Improvements

The usefulness of the following results is not simply in learning about the operation of a three-phase electric machine, but in learning about the performance of the motor drive simulation software. The error in some results gives more insight into the possibilities and future research on the simulation software than into the acceptance of other results as verifiable numbers.

6.1 The Switching Methods

In this section, the similarities and differences in the numeric results of the four switching schemes are discussed. Topics of interest include the system energy losses, the total harmonic distortion, the offset currents and phase shifts. The differences in the pwm technologies make some methods more efficient in specific performance aspects and other methods in different aspects. The goal of the Monte Carlo analysis is not only to find a single superior design of optimality with a specific pwm scheme and a specific sized motor, but also to better understand trends in design and performance criteria. Within a given design space there may be not only points of optimality, but regions of optimality as well. By examining many different motor models and creating a database of optimal designs, there may be trends in the design space which magnify or stress a certain parameter of interest, leading one to understand and predict that trend in future motor design. The trends in the four switching schemes in this thesis are discussed in this section as they relate to the output of the power electronics models. System efficiency, the output parameter of main focus in this section, has four components - the ohmic, conduction, and switching losses as well as the energy that is actually delivered to and used by the machine. The numerical results of Chapter 5 are used to examine the trends in these switching schemes and to further understand how the choice of switching, or pwm, affects the prediction of efficiency and energy losses in the system.
6.1.1 Switch Energy Losses (Eibgt)

The energy losses in the IGBT modules as defined in Section 3.2.2 (Eibgt) in Chapter 5 are the switching losses of the power electronics. Since the energy loss equations in Chapter 3 directly demonstrate how the IGBT module parameters such as $V_{cesat}$ affect the calculation of the switching losses, that will not be discussed here. The following paragraphs will discuss the relation of the switch loss values to the different pwm techniques and the parameters associated with them.

In the four switching schemes and their given inputs used in Chapter 5, the technique resulting in the highest switch energy loss is also the technique which performs the most switches in the simulated half-cycle. Both asynchronous sinusoidal pwm and center-aligned pwm methods are set to $f_{carrier}=15,000$ Hz for the half-wave simulation which results in approximately forty five switches over the half-cycle. These two switching schemes also have approximately the same IGBT module energy losses. Table 6.1 lists the switching losses for the numeric results in Section 5.1 and their corresponding number of switches.

<table>
<thead>
<tr>
<th></th>
<th># switches in half-cycle</th>
<th>IGBT module switch energy loss (Watt*s)</th>
<th>Loss per switch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteresis Comparator</td>
<td>10</td>
<td>3.09E-05</td>
<td>3.089E-06</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 15KHz</td>
<td>45</td>
<td>1.12E-04</td>
<td>2.481E-06</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 60KHz</td>
<td>180</td>
<td>4.19E-04</td>
<td>2.326E-06</td>
</tr>
<tr>
<td>Six-Step Pwm w/ Harm. Elimin.</td>
<td>17</td>
<td>4.20E-05</td>
<td>2.469E-06</td>
</tr>
<tr>
<td>Center-Aligned Pwm</td>
<td>45</td>
<td>1.10E-04</td>
<td>2.444E-06</td>
</tr>
</tbody>
</table>

Table 6.1 Section 5.1 results of switching losses and the related number of switches.

For three of the switching schemes, the loss per switch is approximately equal to the others. This indicates that the switching losses in the power electronics are proportional to the number of switches per cycle as long as the amplitude of the current waveforms remains fixed. In the asynchronous sinusoidal case where the carrier frequency is increase by a factor of four (from 15KHz to 60KHz) the switching losses are also just about multiplied by four. The loss per switch is slightly decreased, but for generalization sake, the loss per switch can be considered equal in each of the 15KHz and 60KHz case. It is concluded here, to a small approximation, that the switching losses in any switching scheme are proportional to the number of switches that occur.

Therefore, the switching schemes with the highest number of switches in a fundamental cycle are going to have the highest switching losses as well. Increased switches in a pwm scheme drives down the harmonic content of both the switching terminal voltage waveform and the three-phase current waveforms in the motor. It is easily noticed how much smoother the current waveforms in Figure 5.4.5 are than the current waveforms in Figure 5.3 - the same switching technique, but four times faster carrier wave and four times the switch executions. The value of total harmonic distortion should be decreased with an increase in the number of switches within the same pwm technique. However, this generalization cannot be applied to different switching schemes.
since switch timing as well as the number of switches plays a key role in waveform harmonics.

As mentioned earlier, but not directly apparent here, the switching losses are a major concern of the system designer because they are usually so large compared to the conduction losses in the power electronics. The major source of loss in the power electronics is considered to be the switching losses over the conduction losses. For more powerful modulation techniques, with hundreds of switches in the modulation waveform per fundamental cycle, the switching losses are driven very high up, causing a source of significant loss in the power electronics which is usually greater than 95% efficient.

However, it appears that the conduction losses (Econd) in the power electronics, for the four switching techniques in Section 5.1 are greater than the switching losses. The next section discusses the conduction losses and examines why the previous statement is true here.

6.1.2 Conduction Losses (Econd) and Ohmic Losses (Eohm)

The conduction losses in the power electronics are the major source of non-useful energy lost in the drive system, specifically the power electronics. The conduction losses in each of the four switching schemes in Section 5.1 are approximately equal. However, the conduction losses should be more than just approximately equal since the current waveforms have the same amplitude in each case and the current is being excited on the same IGBT. The conduction losses, regardless of the number of switches, are a function of the magnitude of the current waveform. Different switching techniques with different switch placement and with far different number of switches in one half-cycle generate the same basic energy loss in the conduction process of the power electronics. Table 6.2 demonstrates the previous statement.

<table>
<thead>
<tr>
<th></th>
<th># switches in half-cycle</th>
<th>IGBT module conduction energy loss (Watt*s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteresis Comparator</td>
<td>10</td>
<td>9.85E-04</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 15KHz</td>
<td>45</td>
<td>8.80E-04</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 60KHz</td>
<td>180</td>
<td>8.82E-04</td>
</tr>
<tr>
<td>Six-Step Pwm w/ Harm. Elimin.</td>
<td>17</td>
<td>1.00E-03</td>
</tr>
<tr>
<td>Center-Aligned Pwm</td>
<td>45</td>
<td>8.74E-04</td>
</tr>
</tbody>
</table>

Table 6.2 Conduction energy losses in one half-cycle for different switching schemes in Section 5.1.

The difference in the conduction losses listed in Table 6.2 for the different pwm techniques could be due to the following observation. The shift of the current waveforms resulted in some of the waveform being negative, switching the device that produces the conduction loss such as the upper diode to the lower transistor.

The conduction losses are clearly independent of the number of switches in the switching scheme. The conduction losses are generally based on the amplitude of the current waveforms (I). There is a slight part of the conduction losses that isn’t captured in this thesis and the models within. The conduction losses occur only during the non-switching times. In the power electronics models Isim, the conduction losses are
calculated at all times during the cycle, including the switching times for simplicity of programming. There may therefore be a small error which makes the conduction losses too large and drives the system efficiency down slightly. However, this error is most likely amplified more and more as the number of switches goes up. The 60KHz asynchronous sinusoidal pwm method executed four times the number of switches that the 15KHz asynchronous sinusoidal method executed, causing approximately four times the error in conduction loss overshoot. With ultra-fast pwm methods, this model could become more and more erroneous. However, for the small number of switches in Section 5.1, the conduction losses are not extremely erroneous.

The ohmic losses ($E_{ohm}$) are similar to the conductin losses in their functional dependency only on the amplitude of the current and the resistive value of the motor phases. Table 6.3 displays the ohmic energy losses for the different pwm simulations mentioned above. The ohmic losses cannot be completely controlled by the power electronics switching scheme or the type of power electronics used in the drive system. The ohmic losses occur in the motor phases. To decrease or control these losses, one needs to obtain a motor with specified phase resistance. Decreasing the harmonic content of the pwm schemes can slightly decrease the harmonic $I^2R$ losses in the motor, but not significantly enough to have an effect on the motor efficiency through the improvement of ohmic losses.

<table>
<thead>
<tr>
<th></th>
<th>IGBT module ohmic energy loss (Watt's)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteresis Comparator</td>
<td>1.71E-04</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 15KHz</td>
<td>1.34E-04</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 60KHz</td>
<td>1.34E-04</td>
</tr>
<tr>
<td>Six-Step Pwm w/ Harm. Elimin.</td>
<td>1.81E-04</td>
</tr>
<tr>
<td>Center-Aligned Pwm</td>
<td>1.33E-04</td>
</tr>
</tbody>
</table>

Table 6.3 Ohmic energy losses in one half-cycle for different switching schemes in Section 5.1.

Refer to Section 3.2.3 for further explanation of the energy loss calculations used in this thesis and the power electronics and current simulation model Isim.

6.1.3 Energy Delivered to the Motor (Edeliv)

The energy delivered to the motor, as described in Section 3.2.3, is the time integrated power into the BEMF waveform. The phase current of each phase multiplied by the BEMF is the power that is being delivered to the motor at each time during the half or full-cycle of electrical motor rotation. Table 6.4 lists the energy delivered to the motor during the half-wave simulations in Section 5.1.
Table 6.4 Energy delivered to the motor in one half-cycle for different switching schemes in Section 5.1.

<table>
<thead>
<tr>
<th>Switching Scheme</th>
<th>Energy Delivered to Motor (Watt*s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteresis Comparator</td>
<td>0.0028</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 15KHz</td>
<td>0.0024</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 60KHz</td>
<td>0.0024</td>
</tr>
<tr>
<td>Six-Step Pwm w/ Harm. Elimin.</td>
<td>0.0027</td>
</tr>
<tr>
<td>Center-Aligned Pwm</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

The energy delivered to the motor in each case should be equal. However, the phase shifts in the current waveforms and the offsets prevent the maximum efficiency in energy delivery to be achieved. Three of the methods contain similar looking waveforms, and similar phase shifts and offsets. Each of these is associated with the same amount of energy delivered to the motor (.0024 Joules).

The energy delivered to the motor is a function of not only the BEMF amplitude and the phase current amplitude, but also the phase alignment of the BEMF and the phase current. The energy delivered to the motor is maximized when the two waveforms involved, the BEMF and current of each phase, are phase-aligned. As mentioned earlier in Section 3.2.5, it is a convention of this thesis that the BEMF of each phase, as seen in Figures 5.2, 5.4, 5.6, and 5.8 is phase-aligned with the electrical motor position, starting with BEMF_A equals zero at time zero. The phase currents therefore must contain the same phase as their respective BEMFs. In Section 5.2, it is mentioned that the phase currents developed in Section 5.1 using the Isim half-wave simulator experienced dc offsets and phase shifts even though the initial values at time equals zero of phase A is set to zero. Taking away the dc offsets in Section 5.2 one can quickly understand why the energy delivered to the system in the pwm schemes in Section 5.1 is not maximized. Figures 5.3 and 5.5 have phase shifts of -.2085 radians and -.2117 radians respectively as calculated in Section 5.2.3.

The comparator scheme has the most energy delivered to the motor out of the switching techniques. The waveform isn’t the smoothest, nor is it the least lossy. It is simply phase-aligned with the BEMF. The comparator doesn’t require a phase angle shift of the terminal waveform to be known before it excites the current. The comparator logic feedback determines when to switch according to the values of the phase currents, allowing the phase of the terminal voltages to naturally fall into place, creating phase currents in phase with BEMFs, delivering maximum energy to the motor.

6.1.4 System Efficiency (Eleceff)

Equation 3.41, \( \eta_{system} = \frac{E_{deliv}}{E_{deliv} + E_{cond} + E_{ohm} + E_{switch}} \) (E_switch=Eight), gives the system energy efficiency. The efficiency values for the switching schemes in Section 5.1 are as correct as the components discussed above. For example, if the energy delivered to the system is not maximized because of a phase-shift in the current waveform, then the maximum power is not going to be delivered to the system and the overall efficiency of
the system is going to go down. The efficiencies in table 6.5 are therefore not completely representative of the switching scheme alone, but of the success of the method of aligning the phase currents and the BEMF. Since these methods which are used in Isim are not entirely accurate, the switching schemes’ system efficiencies may not accurately be captured during one half-cycle of non-steady state current waveforms.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>System Energy Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteresis Comparator</td>
<td>0.7016</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 15KHz</td>
<td>0.6787</td>
</tr>
<tr>
<td>Asynchronous Sinusoidal Pwm 60KHz</td>
<td>0.6232</td>
</tr>
<tr>
<td>Six-Step Pwm w/ Harm. Elim.</td>
<td>0.6802</td>
</tr>
<tr>
<td>Center-Aligned Pwm</td>
<td>0.6790</td>
</tr>
</tbody>
</table>

Table 6.5 System energy efficiency during one half-cycle for different switching schemes in Section 5.1.

The system efficiencies are approximately 70% for each switching scheme and the given inputs to the Isim simulator. In order to better calculate system efficiencies, more work must be done to ensure that the phase currents are in steady state. The current amplitude calculation and modulation waveform phase shift calculations in Equation 3.46 and 3.50 must be improved. See Improvements below.

### 6.2 Total Harmonic Distortion

The second output variable of importance to the Monte Carlo framework is the total harmonic distortion (thd). The total harmonic distortion calculation method developed in this thesis and implemented in THD.m (see Section 4.6 and 3.3) works only on steady state (no dc offset) sinusoidal waveforms with zero phase shift. The current waveforms produced in Section 5.1 do not possess these criteria and have gained bogus thd values in the numerical results section. However, Section 5.2.3 describes and implements a solution to the phase shift and dc offset bugs to calculate the actual total harmonic distortion of the shifted waveforms created in Isim.

The phase-aligned comparator current waveforms returned 3.84% thd, a very low number. However, the non-aligned six-step method returned a 37% thd value even though the waveform appears much smoother than the comparator’s. The problem as evidenced in the results in Section 5.2.3 is the phase shift and offsets. When these non-idealities are removed the total harmonic distortion of the six-step generated waveform is only 2.01%, much nicer than the 37% value generated before. The asynchronous sinusoidal value is also refigured with offsets removed in this section to be 2.17%.

This method of adjusting the phase angle and adjusting the offsets, although frustrating and tedious, proves that the actual total harmonic distortion and the steady state waveforms are possible to simulate, even though the calculations within Isim are not currently 100% accurate. With some slight tweaking, the waveforms can be captured in steady state, with maximum power delivered to the motor and a proper calculation of total harmonic distortion each and every time. However, until this tweaking occurs, these adjusted values show that the Isim current simulator cannot function properly without the
correct initial phase currents and the correct calculation of phase current amplitude. The further improvements are discussed below in the section title Improvements.

6.3 Dependability of Present Isim

Isim has much potential to be completely dependable as a steady state current simulation and system efficiency tool. Isim possesses the capability to incorporate almost any kind of pwm technology to control the current waveforms in a three-phase machine. The methods used in this thesis, the hysteresis comparator, asynchronous sinusoidal pwm, six-step pwm with harmonics elimination, and center-aligned pwm are just a few of the many methods of generating phase currents in a three-phase electric motor drive system. More methods can be easily added using the same general format as the others, or even added in a new language if the software is converted to something nicer such as C++.

Isim also possesses the capability to compute the total harmonic distortion of the three phase current waveforms which it generates, whether they are one-half cycle or numerous cycles as long as there is no phase shift beyond the understood 0, -2\pi/3, -4\pi/3 three-phase phase shifts. Isim has the capability to be used as a quick-time half wave generator as is useful in the Monte Carlo framework for spitting out test after test after test as quick as possible. For analysis on specific designs, as was shown in Section 5.2, the half-wave generator can be turned into a full-wave or beyond generator and more results can be gathered, including just eyeballing the figures for major problems as the offsets in Figure 5.10. Numbers of cycles can be useful to ensure that transitional components settle out and steady state is reached eventually. This option can be both time and memory consuming and is not suggested to be relied upon.

With future improvements on the Isim program, more and more accurate results for specific power electronics devices and their diode and transistor functional dependencies can be incorporated therein. Higher resolution can generate more accurate results as well, and more computer memory and higher speed shall allow these higher resolutions to be easily incorporated into the existing system.

6.4 Isim as a Full-Wave Generator

This thesis and the technical methods in this thesis have aimed at creating one half-wave of fundamental electrical motor rotation for the Monte Carlo framework. This has been shown to be possible, both with correct phase angles and current amplitudes as in the case of the hysteresis current comparator and the shifted waveforms, and with slightly shifted current waveforms and imprecise calculations. However, there exists the option as discussed in Section 5.2.1 to generate any number of cycles for a given three-phase current system. Figures 5.9 and 5.10 show three complete cycles of current waveforms which reveal more than the simple half-waves did since they were not in steady state. The notion that the half-wave possesses all of the information necessary to describe the entire rest of the waveform is true only if the wave is in steady state with zero dc offsets. For plotting and descriptive purposes as some of the waveforms in this thesis, the option to print one full wave instead of one half is very useful.
6.5 Improvements

Isim is not a completely reliable tool which has many possible improvements, some which are not straightforward or obvious how to achieve. This section describes some specific improvements that must be addressed in order for the Isim power electronics and efficiency prediction model to be fully dependable and incorporated into the Monte Carlo framework for superior motor drive design. The improvements that are covered in this section are listed below.

- Proper calculation of phase current amplitude $I_{\text{refamp}}$ in Isim.
- Phase shifted total harmonic distortion calculation.
- Including torque in the calculations.
- Switching losses.

The main improvement, as mentioned earlier in this chapter, to the Isim program that needs to be made is the adoption of a better current amplitude or phase calculation. The solution to Equation 3.49 is not completely accurate because only the fundamental terminal voltage $(U_A, U_B, U_C)$ waveforms are used and the harmonics which are integral in defining the square-wave nature of the terminal voltages are ignored completely in the calculation of the phase current amplitude. For a square wave, these non-fundamental harmonics are key components whose amplitudes may be more than 10% of the fundamental.

The phase currents, although ideally phase-locked with the BEMF for maximum power delivered, will not always possess the desired phase shift of zero. There may be times when there are great design reviews with a good profile of energy delivered to the system despite a slight phase shift. When these times come along, the total harmonic distortion method (THD.m) needs to possess the ability to compute total harmonic distortion with phase-shifted waveforms. This can easily be achieved by matching the phase shift in the sinusoidal waveform in the fundamental components rms calculation to the phase of the shifted current waveform.

Many motor designers aim at a goal of providing a specified amount of torque at a certain required speed. The calculations in Current Simulation Method and this thesis in general do not include torque to determine the amount of current. Instead, the amount of current is calculated from a predetermined, set motor speed and a given power supply voltage, without an imposed restriction on power supply current. The torque resulting from such a motor can be calculated from the phase current and BEMF waveforms generated in the Isim program. This is the torque that is delivered to the system, but it is determined by the current and it doesn’t determine the current. Some engineers would prefer to give a torque requirement, and learn about how much current that would require. Without some improvements to Isim, this isn’t possible currently.

The simple switching loss calculations in this thesis are very rough, square-trapezoid approximations of slightly unpredictable switching waveforms. The given charts and graphs and rise-time, fall-time, delay-time values in the available data books is
useful to a certain point. There is one slight problem with the data obtained. The timing or phase difference ($t_{\text{wait}}$ in Figure 6.1) between the rising edge of either the current or voltage and the falling edge of the other is not defined. This is a crucial value in the calculation of energy losses and without it, one cannot accurately predict the switching energy losses for a given IGBT module at a given current rating.

![Figure 6.1](image)

Figure 6.1 Unknown wait time ($t_{\text{wait}}$) between ramp-up and ramp-down of the switching waveforms.

The knowledge of these timing issues could give a better idea on how to mathematically give a simple, but more complex than in this thesis, model and predict the switching energy losses in the three phase electric machine.
Chapter 7

Summary

This thesis aims to develop power electronics switching efficiency and energy loss prediction models by simulating three phases of sinusoidal current in an electric wye configured machine. In Chapter 2, the necessary background is given for understanding the Monte Carlo optimization process in which the power electronics models in this thesis are aimed to be incorporated. The structure of the electric motor drive system is simply described with details provided about the motor topology, the power electronics' (IGBT modules') structure, and some background on different pwm and switching schemes used to spin the motor in this thesis.

Chapter 3 is the heart of the technical description of this thesis. First, the Current Simulation Method of phase current calculation delivers a mathematical lesson on calculating the three phases of phase current arising from power electronics' switching waveforms and BEMF components. The three equations (3.27, 3.28) are used to analyze the current at all times given the switching waveforms and motor constants.

\[ I_A(t) = \left[ I_A(\omega t) \frac{2U_A}{3R} \frac{v_A - v_d - v_c}{3R} \right] + \frac{K_1 R}{R^2 + (\omega R + L + M)^2} \frac{2U_A}{3R} \frac{v_A - v_d - v_c}{3R} \frac{K_1 R}{R^2 + (\omega R + L + M)^2} \]

\[ I_B(t) = \left[ I_B(\omega t) \frac{2U_B}{3R} \frac{v_A - v_d - v_c}{3R} \right] + \frac{K_1 R}{R^2 + (\omega R + L + M)^2} \frac{2U_B}{3R} \frac{v_A - v_d - v_c}{3R} \frac{K_1 R}{R^2 + (\omega R + L + M)^2} \]

Equations 3.27, 3.28 for phase currents \( I_A, I_B \), respectively.

Calculations used in the Isim program involving energy loss and switching waveforms are described in their respective sections. The switching waveform approximations are graphically as well as mathematically represented in these sections as well as all the other energy loss and delivery equations. The total harmonic distortion method used by Kassakian, Schlecht, and Verghese in [8] is briefly described.

Chapter 4 describes the separate Matlab programs that makeup the software implementation of this thesis. The main program Isim.m is the current simulator time-loop which steps through the half-wave current simulation and power electronics energy loss predictions. The other programs such as Setbac.m are small functions used in the main program which perform tasks described in this chapter. A full description of the total harmonic distortion software (THD.m) also exists in Chapter 4.

Chapter 5 contains the numerical results from the current simulation Isim being executed first with four different switching schemes over one half cycle of fundamental electric motor rotation, then with alterations of the norm. Multiple cycles are created and
phase shifts and offsets are removed to perform further and more accurate THD and energy
loss calculations on the waveforms generated in Isim. The results to a few different
methods are used in comparisons and analysis in the next chapter.

Chapter 6 discusses the results in Chapter 5 and interprets what the results mean.
In Conclusions, these results are analyzed further and suggestions for future
improvements and observations about current positive and negative system qualities are
made. Chapter 6 ties together the goals and the results of this thesis. Certain aspects of
the current simulation program Isim, need to be improved to function as a powerful tool.
The current amplitude prediction, the initial current values and the total harmonic
distortion phase alignment are three areas that require some attention in future studies.
Chapter 8

Recommendations

This thesis has focused on the development of accurate power electronics and efficiency predictions for electric motor drive systems. However, there is much more to be accomplished. Future research and development in this field should consider the recommendations below.

8.1 Code Language

The power electronics models of this thesis were developed within the Matlab framework. Matlab was chosen for its simple and straightforward programming framework, computational and plotting abilities, and familiarity within the engineering background of the designers. However, the drawbacks of Matlab as a programming base are not prevalent when the Monte Carlo motor design program operates alone, without the inclusion of the power electronics portion. The most obvious drawback of the Monte Carlo motor design program before the power electronics are included is the time of execution. Motor design simulations with calculations and evaluations take only 1 second each to complete. To generate one hundred superior motor designs, the program may take up twelve hours or more. A faster language can significantly decrease the operation time.

With the power electronics program included, the run-time is severely longer. Each power electronics simulation and evaluation takes up to forty seconds, and slightly longer when appended to the motor design calculations. Faster languages, with quicker ‘for-loop’ and ‘if-else’ operations, or more object-oriented functionality could result in drastic reductions of the run-time of the program.

The computational limitations of memory and speed may also be a result of the computer or operating system of choice. Matlab may run faster on different computers, and there may be less memory limitations on computers with excess RAM, allowing the Matlab design program to run quicker.

The ability of languages such as C and C++ to hold structural variable types would be a major advantage in programming simplicity for the power electronics models. The functional call for the power electronics program uses a long list of variables such as phase resistance, inductance, mutual inductance, maximum speed, maximum current, switching scheme choice, frequency choice, and more. If all of these variables were combined into one structure, not only would the function call be shorter and simpler, but the power electronics program would be able to be compacted significantly without having to isolate separate switching schemes and therefore repeat lots of code currently shared by all switching schemes.

Libraries of different IGBTs and their specifications spark some more ideas for future development in this field. Expandable libraries would enhance and simplify the
comparison of different inverter technologies. Using a language with more power and memory capabilities than Matlab, it is possible to include the functional dependencies and nonlinearities of parameters such as rise and fall time. These library entries could be altered or available for examination at the click of a button with the addition of a graphical user interface (GUI).

### 8.2 Switching models

The switching models used in the power electronics program will be more accurate if the sloping waveforms are included in the switching losses and calculations. The square-trapezoid waveforms used in this program (Figure 8.2), and thus the simple loss calculations in Equations 3.31 to 3.40 are good approximations which can be sufficient for optimization purposes but not entirely accurate or reliable. A firm understanding of switching waveforms and their functionality depending on the type of device used, the magnitude of the current, and the voltage used will be necessary to perform extensive and specific prediction of the power electronics switching losses, resulting in more accurate system performance predictions.

![Figure 8.1 Switching waveforms from Powerex Data book.](image)

![Figure 8.2 Square-wave switching approximations in this thesis.](image)

The switching waveforms have different sloping and recovery characteristics which may be characteristic of the type of load and the magnitude of the power. The Powerex IGBT Module data book [22] indicates that there are different switching waveforms for half-bridge, resistive load, and short-circuit devices. For the electric motor drive application, there needs to be a switching waveform model for a three-phase inductive and resistive load and functionality of the parameters of this waveform based on the current, the power, and the load. The Powerex data books are a good place to start,
but they are not specifically targeted to three-phase inductive loads. The experimental data in this thesis is a starting point.

Switching parameters also have some functional dependencies. Rise and fall times for switch transitions \((t_r\) and \(t_f\)) are functions of the current. It makes sense that a constant rate of change takes longer to change more. Higher magnitudes of current, therefore, have higher switch transition times, causing more breakdown in an ideal power electronics pulse-generation model. Reverse recovery characteristics such as time \((t_{rr})\) and magnitude \((I_{rr})\) of reverse recovery current increase with an increasing emitter current. None of these functional dependencies are included in the models in this thesis. Future improvements to the power electronics program could include these functional dependencies as a library of look-up tables for each power electronics device. The changing values of \(t_r, t_f, V_{ceSat, t}, I_{rr}\) and others will be active in the power electronics simulation and this feature will create more accurate power electronics and efficiency predictions.

8.3 Pwm Techniques

There are several pwm techniques that are not implemented in this thesis. For example, space-vector pwm is not included as an option for a power electronics control technique. Other more advanced pwm techniques such as adding third harmonics and other combinations of several third order harmonics to decrease one phase of switching and increase maximum power output are also not employed in the power electronics model prototype. With limited time to complete this thesis, the final decision was to include certain pwm techniques and leave others for future research since it would be too time consuming to find and include every pwm technique available. A more extensive development would include more pwm techniques hence being able to better optimize the electric motor drive designs.

Future studies might also explore the role of harmonics in the different aspects of pwm technologies. Certain techniques, such as the six-step with harmonics elimination, base the elimination of different harmonics on very specific calculations and placements of pulses. These values are ideal and do not consider rise and fall times, or even signal propagation delays within the IGBTs. Every IGBT has a finite rise and fall time and a finite turn on and off delay that would slightly skew the actual placement of the notches in the terminal voltages. Also, since the pwm operates off of a dsp chip, and the timing of the signals are digital, with specific timing intervals, the resolution of the notching pulse times may be defined by the timing resolution of the dsp chip. These slight changes in notch-edge placements will change the harmonic content of the current waveforms, and could possibly result in having much worse harmonics than predicted. It would be interesting to perform further non-idealistic tests on these highly mathematical and timing-specific pwm techniques.

8.4 Controller Theory
In addition to the power electronics and motor topology sections of this thesis, there is also a space reserved for the controller theory implementation. The controller technology is mentioned minimally in this thesis due to the concentration on the power electronics. The controller’s filter and gain values and the control algorithm can affect the operation of the pwm techniques. Further studies in the relation of not only the types of power electronics used, but also in the types of controller algorithms included in the pwm technique (which is a controller-defined parameter) will give an even more in-depth understanding of optimal electric motor designs, leaving very little motor drive technology out of the design stage. Although some independent motor, power electronics, or controls developers might desire to optimize their particular fields separate of their interfacing technology, such as the adjoining controls, power electronics, or motor, this is not a proper approach to optimize the entire system.

By implementing the controls, the turn-on and turn-off transitions of electric motor drives could be explored in more depth, as well as the high-end speeds and torques defined additionally by the feedback and control loops and not simply the power source and pwm technique. Speed and stability of switch transition and stability of steady state is an important issue that can also be examined with the addition of controls.

From a systems integration perspective, each part of the electric motor drive system and how it interconnects with the others is important to understand and control. System integration, and more importantly, system optimization can only be achieved when the motor drive and all of its components are designed and tested as a single unit.

### 8.5 Extreme Values

Extremely high speeds (above fifty or sixty thousand rpm) can have unpredictable effects on power electronics’ performance. Assuming that some power electronics have been designed for such high-end applications and are supplied with specifications of values at these data points, it is necessary to examine the changing effects of high speeds on the switching times, recovery magnitudes, and propagation delays of the power electronics in order to verify that the high speed designs are not linear (first order) estimations based on middle speed data values. At very high speeds, high currents, and high power, the power electronics’ characteristics display nonlinear effects which should be accounted for in any high speed design models.

Low speeds are also subject to nonlinearities. Motor drives do not always operate continuously at maximum speed and torque, and are never designed and tested for that speed range alone. During periods of low demand, or idle time, the motor drive design should not break down or stall. Just as important as the high end capabilities are the middle and low end operating qualities. Since the low speed and low torque range of motor operation may exhibit nonlinearities, the design and verification of the motor drive’s performance must consider this.

Further studies might consider examining the problems in designing a very, very high speed motor drive (possibly above one hundred thousand rpm) using linear (first order) estimations based on middle speed (ten to thirty thousand rpm) motor drive characteristics. Finding out which linear models break down more (the motor, the power
electronics, the controller) might give direction on which portion of the electric motor drive is the most constraining. Improvements targeted in one area could improve the motor drive performance more than other areas.

8.6 Experimental Data Points

Many experimental data points must be gathered to verify that a model is accurate throughout its range of parameters (minimum to maximum speed, torque, etc.). In the design stage of electric motor drive systems, several types of modeling software are used to predict the performance of different model specifications. These software packages include finite element analysis programs for mechanical and thermal devices, magnetic flux simulators for electric motor designs, and other electrical engineering software such as PSpice or the power electronics program in this thesis. However, the software and the models are only as dependable as the experimental data that supports them. The commonly used simulation software has been extensively tested on many real-world devices to examine the simulation accuracy. Different devices in different combinations at different levels of power have been tested for model verification. Due to the nonlinear effects of devices mentioned above, the simulations would not be accurate if they weren’t first tested and modified to fit the actual experimental data.

The power electronics software developed in this thesis is backed by some experimental data from a real electric motor drive system. The switching waveforms and pwm signals from the actual motor drive and from the power electronics models are compared in this thesis. However, these data points are minimal in number; they reflect tests performed on a single inverter module, a single motor, and a single pwm scheme. With a more lucrative budget, many different motors, inverters, and control technologies from different companies could be brought together and tested for switching waveforms, propagation rise time and fall time delays, efficiency and harmonics content. Verification of the motor drive technology models in this thesis might alone take another year to accomplish if an exhaustive number of data points were to be collected.

8.7 Thermal Calculations

There are still pieces of the electric motor drive system not yet designed which will improve the accuracy of the system optimization program. Thermal calculations are crucial to the design and verification of both the motor and power electronics components. Without these calculations, the results of the Monte Carlo design program have to be analyzed individually by a thermal expert. A lot of time and frustration will be eliminated from the design process with these calculations included in the program. The thermal calculations are similar to the motor topology calculations in the way that there is no time loop involved in finalizing the thermal results. The time that the thermal results program would take for each generated design would similarly be less than one second, not significantly affecting the total time for each design to be analyzed as a complete system.
8.8 Fast Fourier Transform

A fast fourier transform operation performed on the current waveforms generated in each of the simulation schemes would give the list of harmonics and their amplitude. The fft could be performed on not only the motor currents, but on the switching waveform of the inverter. The harmonics created, or present can be an indication of the feasibility of a design.

8.9 Monte Carlo Optimization Process

The Monte Carlo Optimization process generates new random number choices for each parameter in each design. These random numbers are generally constrained to a range of acceptable values which itself may limit the design space of the program to the creativity of the user (programmer) as mentioned previously. However, one of the fundamental ideas behind the superiority of this Monte Carlo process of optimization may also be a reason for a major refinement.

The Monte Carlo design optimization process is implemented first in Satcon’s motor design and now electric motor drive system design program. Ideologically this process is superior to a grid-gradient process because of it’s ability to expand the design space of the design generation program. No longer is the design space limited to search specific integrals of predetermined size for each design variable. By using grid-gradient methods, large regions of design space would never be reached.

For example, a design variable A has a minimum and maximum value of 10 and 20 respectively. If the grid searched every .01 interval between 10 and 20, there would be 2001 total possibilities for A. It is possible that there are more optimal choices of A that lie between specific choices of A which cannot be reached without more resolution. What if 18.02 and 18.03 were the optimal values of A using the grid-gradient method with .01 as an interval, but 18.025 was far superior to each of the previous choices? There could possibly be an entire region or regions of superior choices that lie between the chosen intervals of A. No matter how small you make the interval, there will be more and more spaces of superior values that need higher resolution to find. The Monte Carlo method used in this program generates random numbers in Matlab using the randn command. There is more (possibly much more) than fifty decimal places in each randn choice. With the Monte Carlo method, there are no longer possible design choices which are not capable of being generated by the program and the design space has expanded immensely.

Unfortunately, the random numbers generated by Matlab using the randn command have over fifty decimal places, possibly much more. Although the design space has now expanded immensely, it is too large! The concept of finding more optimal values between grid-gradient intervals has been eliminated, but not without creating the possibility of finding such a small section of superior values with resolution that cannot be met in the manufacturing of this optimal design. Motor manufacturers have only so much resolution to tolerate due first to mechanical design tool limitations, and second to
material limitations and molecular dimensions. The motor parameters generated have no particular resolution limit.

Therefore, the numbers that are delivered to the file or screen and printed out are in the format of four decimal places. However, the actual calculations performed on these variables were using the entire fifty+ decimal place value for each and every non-integral design variable. The provided four decimal place numbers are possibly not part of one of the optimal areas, and the performance of a motor with the four decimal parameters could be greatly different than with the fifty+ decimal places. If the program is altered to provide the fifty+ decimal places, the manufacturers would have extreme difficulty providing this accuracy.

The design space, therefore should be limited to the resolution of the manufacturers precision, and possibly include any manufacturer’s tolerance to determine weather the design can still be generally optimal if the values are-slightly not as precise as the manufacturers’ provided values. There could possibly be a small region of extreme optimality with an extremely high surrounding gradient whose slight shift (within manufacturer’s tolerance) would drastically alter the motor’s performance.
Chapter 9

APPENDIX

9.1 References

[3] Center for Electromechanics, The University of Texas at Austin.

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