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EVALUATING THE EFFECTS OF INCOMPLETE MARKETS ON RISK SHARING AND ASSET PRICING

John Heaton
Massachusetts Institute of Technology

Deborah Lucas
Northwestern University

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MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

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John Heaton*

Deborah Lucas**

*M.I.T. and NBER

**Northwestern University, and NBER

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1. Introduction

Incomplete markets in the form of an inability to borrow against risky future income has been proposed as an explanation for the poor predictive power of the standard consumption-based asset pricing model.[1] With complete markets, individuals fully insure against idiosyncratic income shocks, and individual consumption is proportional to aggregate consumption.[2] With limited insurance markets, however, individual consumption variability may exceed that of the aggregate, and the implied asset prices may differ significantly from those predicted by a representative consumer model. In this paper we study an economy in which agents cannot write contracts contingent on future labor income realizations. They face aggregate uncertainty in the form of dividend and systematic labor income risk, and also idiosyncratic labor income risk. Idiosyncratic income shocks can be buffered by trading in financial securities, but the extent of trade is limited by borrowing constraints, short sales constraints and transactions costs.

The motivation for considering the interaction between trading frictions and asset prices in this environment is best understood by reviewing the findings of a number of recent papers. Lucas (1991) and Telmer (1991) examine a similar model with transitory idiosyncratic shocks and without trading costs. Surprisingly, they find that even though agents cannot insure against idiosyncratic shocks, predicted asset prices are

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[1] For discussions of problems with the standard model, see for example, Hansen and Singleton (1983), Mehra and Prescott (1985) and Hansen and Jagannathan (1991).

[2] See R. Lucas (1978), for example.

similar to those with complete markets.[3] This occurs because when idiosyncratic shocks are transitory, consumption can be effectively smoothed by accumulating financial assets after good shocks and selling assets after bad shocks. Aiyagari and Gertler (1991) consider a model with no aggregate uncertainty and with transactions costs, in which agents trade to offset transitory idiosyncratic shocks. Differential transactions costs affect the relative returns on stocks and bonds, and reduce the total volume of trade. Finally, Constantinides and Duffie (1992) study the case of permanent idiosyncratic shocks. Although agents have unrestricted access to financial markets, no trade occurs. When the conditional variance of idiosyncratic shocks increases in a downturn the riskfree rate falls and the equity premium rises relative to the complete markets case.

These results suggest that the quantitative asset price predictions from this class of models will depend critically on several factors: (i) the extent of trading frictions in securities markets, (ii) the size and persistence of idiosyncratic shocks, and (iii) the correlation structure of idiosyncratic and aggregate shocks. To address (ii) and (iii), we develop an empirical model of individual income that captures both the size of the idiosyncratic shocks and the persistence of these shocks over time, based upon evidence from the Panel Study of Income Dynamics (PSID). The time series properties of aggregate income and dividends are estimated using the National Income and Product Accounts. This process is then used to calibrate the theoretical model.

Our theoretical model differs substantively from those discussed above

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[3] Using a more volatile aggregate income process, Marcet and Singleton (1991) also calibrate this model. The equity premium rises in the presence of short sales constraints.

by considering transactions costs in an environment with both aggregate and idiosyncratic shocks. Transactions costs play an important role because agents need to trade frequently in order to buffer shocks to their individual income. As a result, transactions costs can have two effects on asset prices.

First, (gross) rates of return on securities may be altered because agents require higher returns to compensate for transactions costs. This effect of transactions cost was emphasized by Aiyagari and Gertler (1991), Amihud and Mendelson (1986) and Vayanos and Vila (1992). In contrast, Constantinides (1986) argued that transactions costs should have only a small effect on asset returns. In his model in which there is no idiosyncratic risk and agents trade only to rebalance their portfolios, agents avoid most transactions costs by reducing the frequency of trades. As a result, asset returns are not much affected by the presence of transactions costs. However due to the idiosyncratic shocks that individuals face in our model, it is quite costly for individuals to change their asset trading patterns in response to trading costs.

A second, indirect, effect of transactions costs is that they limit the ability of agents to use asset markets to self-insure against transitory shocks, so that individual consumption does not move directly with the aggregate. The increased volatility in individual consumption reduces each individual's tolerance for the aggregate uncertainty reflected in dividends, for the utility specifications that we consider. The implied equity premium could rise in response to increases in transactions costs for this reason alone. This paper appears to be the first to evaluate the importance of this mechanism.

We calibrate the model under a variety of assumptions about the size and incidence of trading costs. When trading costs differ across markets, we find that agents readily substitute towards transacting in the lower cost market. For example, if transactions costs are only introduced in the stock market then agents trade primarily in the bond market and by this means effectively smooth transitory income shocks. In this case transactions costs have little affect on required rates of return. However if transactions costs are also introduced in the bond market in the form of a wedge between the borrowing and lending rate, then the bond return falls. With a binding borrowing constraint or a large wedge between the borrowing and lending rate, a small transactions costs in the stock market can produce an equity premium that is close to the observed value, and a low bond return that is close to the observed return on U.S. Treasury securities.

The remainder of the paper is organized as follows: In Section 2 we describe the model economy. Section 3 presents the empirical model of income and dividends. We also discuss the parameterizations for trading costs, borrowing constraints, and short sales constraints. Simulation results are reported in Section 4. Section 5 concludes.

2. Model

2.1 The Environment

The economy contains two (classes of) agents who are distinguished by their labor income realizations. At each time t, agent i receives stochastic labor income Y_t^i. By assumption, agents are not allowed to write contracts contingent upon future labor income.

Agents also receive income from investments in stocks and bonds. At time t, a share of stock, with price p_t^s, provides a claim to a flow of random dividends from time t+1 forward, $\{d_j\}_{j=t+1}^{\infty}$. The bond, with price p_t^b, provides a risk-free claim to one unit of consumption at time t+1. The agents trade these two securities to smooth their consumption over time. Trading is costly, with transactions cost function $\kappa(\cdot)$ in the stock market and $\omega(\cdot)$ in the bond market. Agents also face short sales and borrowing constraints.

At time t, each agent's preferences over consumption are given by:

$$(2.1) \qquad U^i \equiv E\left\{\sum_{\tau=0}^{\infty} \beta^{\tau} \frac{(c^i_{t+\tau})^{1-\gamma} - 1}{1-\gamma} \mid \mathcal{F}(t)\right\}, \ \gamma > 0 ,$$

where $\mathcal{F}(t)$ is the time t information set, which is common across agents. This information is generated by a state variable, Z_t, which is specified below. In principle, γ could be allowed to differ across agents. However, since we want to interpret the two groups as similar except for realizations of idiosyncratic shocks, it seems appropriate to equate γ across the two groups.[4]

At each date t, agent i maximizes (2.1) via the choice of consumption c^i_t, stock share holdings s^i_{t+1} and bond holdings b^i_{t+1} subject to the flow wealth constraint:

$$(2.2) \qquad c^i_t + p^s_t s^i_{t+1} + p^b_t b^i_{t+1} + \kappa(s^i_{t+1}, s^i_t; Z_t) + \omega(b^i_{t+1}; Z_t)$$
$$\le s^i_t(p^s_t + d_t) + b^i_t + Y^i_t$$

and short sales or borrowing constraints:

$$(2.3) \qquad s^i_t \ge K^s_t \qquad t = 0, 1, 2, \ldots$$

$$(2.4) \qquad b^i_t \ge K^b_t \qquad t = 0, 1, 2, \ldots$$

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[4]Dumas (1989) considers the implications of different risk aversion parameters in a complete markets setting.

The components of initial wealth d_0, s^i_0, b^i_0 and Y^i_0 and market prices are taken as given.

2.2 Trading Frictions

The extent to which individuals will use asset markets to buffer idiosyncratic income shocks depends on the size and incidence of trading costs, and the presence of borrowing and short sales constraints. Since the assumed form of these frictions qualitatively affect predicted asset prices, and since there is little agreement about the exact form of these costs, we consider several alternative cost structures.

Transactions costs in the stock market. Both buyers and sellers are assumed to face a quadratic transactions cost function:

$$(2.5) \qquad \kappa(s^i_{t+1}, s^i_t; p^s_t) = k_t\{(s^i_{t+1} - s^i_t)p^s_t\}^2$$

In the simulations, the parameter k_t is used to control the magnitude of the transactions cost. However, because the realized cost is endogenous, the range of attainable costs is bounded; an increase in the cost parameter eventually leads to an offsetting reduction in trade. Dividing the cost function by $(|s^i_{t+1}-s^i_t|)p^s_t$ gives the trading cost as a percent of the value of shares traded: $k_t|s^i_{t+1}-s^i_t|p^s_t$. In the simulation results we report the average of this percentage cost.

We use a quadratic cost function primarily for computational simplicity. However, it also captures the idea that as more assets are sold, agents must sell increasingly illiquid assets. The fact that many individuals hold no stock at all suggests that there may be significant fixed costs to entering this market.[5] To partially address this issue, we also estimate the income process conditioning on data from families who own non-negligible amounts of stock. Another possible objection to the quadratic form is that small changes in stock holdings may be at least as costly (proportionally) as large changes. In the discretized state space we will consider, however, infinitesimal shocks never occur so this limiting case is not relevant.

Bond market transactions costs. Bonds in this model represent private borrowing and lending. While it seems sensible to treat transactions costs symmetrically for sales and purchases of stock, this is less true for loans. Typically consumers pay a substantial spread over the lending rate to borrow. Part of the observed spread is a default premium which does not apply to the riskfree bonds of the model. However, a portion of the spread can be attributed to costs of financial intermediation or monitoring that must be incurred even if the debt is *ex post* riskfree.

To capture the asymmetry between effective borrowing and lending rates, the bond transactions cost function is assumed to have the form:

$$\omega(b^{i}_{t+1};Z_{t}) = \Omega_{t} min(0,b^{i}_{t+1}p^{b}_{t})^{2}. \tag{2.6}$$

The parameter Ω_t controls the magnitude of the cost. By convention borrowing at time t is represented by a negative value for b^{i}_{t+1}, so only the agent who borrows pays the transactions cost. As with stocks, the cost is reported as a percentage of the per capita amount transacted: $\Omega_t|b^{i}_{t+1}|p^{b}_{t}/2$.

We also will consider the implications of a symmetric quadratic cost

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[5] The effects of transactions costs of this form have been considered by Saito (1992).

function in the bond market of the form:

$$\omega(b^1_{t+1};Z_t) = \Omega_t(b^1_{t+1}p^b_t)^2. \tag{2.7}$$

As we shall see, the choice of (2.6) versus (2.7) will have a significant affect on the predicted equity premium.

To match the observed income and dividend process, the economy is assumed to be stationary in aggregate income growth rates. As a result, the price of the stock and the face value of the bond grow over time. To accommodate the growth in value, the borrowing constraint, K^b_t, is assumed to be linear in aggregate income, Y^a_t. Because the transactions costs are quadratic in the value of trade, to induce a constant average transactions cost to income ratio, we assume that $k_t = k/Y^a_t$ and $\Omega_t = \Omega/Y^a_t$ where k and Ω are constants.

Finally, we refer to the case where $\kappa(\cdot) \equiv 0$ and $\omega(\cdot) \equiv 0$ as the *frictionless model*. The frictionless model is similar to Lucas (1992) and Telmer (1992).

Borrowing and Short Sales Constraints. Consumption smoothing may also be curtailed by institutional limits on the amount of borrowing. This type of credit rationing is represented by (2.4). We will consider two scenarios. In the first, agents can borrow up to 10% of average per capita income. In the second, agents are precluded from any borrowing; only stock holdings can be used to buffer income shocks.

The choice of an appropriate upper bound on borrowing is not obvious. The value of household collateral, which is a plausible limit on debt, is not easily measured. Since agents rarely hit the assumed 10% upper bound for the income shocks considered, and since the agents always desire to do some borrowing, the chosen limits appear to bracket the relevant range.

No short sales are permitted in the stock market so that $K^s_t = 0$ in (2.3). This is motivated in part by the observation that it is costly for individuals to take short positions. Clearly, allowing short sales would increase the effective quantity of tradeable assets.

2.3 *Equilibrium*

At time t, aggregate output consists of the aggregate dividend, d_t and the sum of individuals' labor income $\sum_{i=1,2} Y^i_t$. Market clearing requires:

$$(2.8) \qquad b^1_t + b^2_t = 0 \qquad t = 0,\ 1,\ 2,\ \ldots$$

$$(2.9) \qquad s^1_t + s^2_t = 1 \qquad t = 0,\ 1,\ 2,\ \ldots$$

$$(2.10) \qquad \sum_{i=1,2} \{c^i_t + \kappa(s^i_{t+1}, s^i_t; Z_t) + \omega(b^i_{t+1}; Z_t)\} = d_t + y^1_t + y^2_t \qquad t=0,1,2,\ldots$$

Notice that in (2.8) we are assuming that bonds are in zero net supply.

When the short sales and borrowing constraints are not binding, the first order necessary conditions from the agent's optimization problem imply that for all i and t:

$$(2.11) \qquad [p^s_t + \kappa_1(s^i_{t+1}, s^i_t; Z_t)]u'(c^i_t) = \beta E\left\{u'(c^i_{t+1})[p^s_{t+1} + d_{t+1} + \kappa_2(s^i_{t+2}, s^i_{t+1}; Z_t)] \mid \mathcal{F}(t)\right\}.$$

and

$$(2.12) \qquad [p^b_t + \omega_1(b^i_{t+1}; Z_t)]u'(c^i_t) = \beta E\left\{ u'(c^i_{t+1}) \mid \mathcal{F}(t)\right\}.$$

If an agent is constrained by the short sale constraint (2.3), then (2.11) is replaced by:

$$(2.11') \quad s_t^i = K_t^s.$$

Similarly, if the agent is constrained by the borrowing constraint (2.4) then (2.12) is replaced by:

$$(2.12') \quad b_t^i = K_t^b \ .$$

At time t the unknowns are p_t^s; p_t^b; c_t^i, i=1,2; s_t^i,i=1,2; and b_t^i, i=1,2. The equations defining an equilibrium are the budget constraints (2.2), i=1, 2; the market clearing conditions (2.8), (2.9), and (2.10); and the asset pricing equations (2.11) or (2.11'), i=1,2, and (2.12) or (2.12'), i=1,2. By Walras's Law, one of the market clearing conditions or a budget constraint is redundant. We restrict our attention to stationary equilibria in which the consumption growth rate, portfolio rules, and equilibrium prices are functions of the time t state Z_t, which is described in detail in the next section.

3. State Variables

3.1 Empirical Model of Labor and Dividend Income

Our empirical model is designed to capture important features of the income process when labor income is uninsurable. For example, the model captures the correlation between aggregate and individual labor income growth, and the correlation in individual labor income shocks over time.

Since this income process serves as an input into an asset pricing model that must be solved numerically, we choose a relatively parsimonious specification.

The components of aggregate income include aggregate labor income, Y^{ℓ}_t, and aggregate dividends, D^a_t. The sum of Y^{ℓ}_t and D^a_t, aggregate income, is denoted by Y^a_t. The growth rate of Y^a_t and the logarithm of the share of D^a_t in Y^a_t is assumed to follow a bivariate autoregression. Letting $\gamma^a_t \equiv Y^a_t/Y^a_{t-1}$, $\delta_t \equiv D^a_t/Y^a_t$ and $X^a_t \equiv [log(\gamma^a_t)\ \ log(\delta_t)]'$, then X^a is assumed to be generated by:

$$(3.1) \qquad X^a_t = \mu^a + \Lambda^a X^a_{t-1} + \Theta^a \varepsilon^a_t, \ t=1, 2, 3, \ldots$$

where ε^a_t is a vector of white noise disturbances with covariance matrix I, the matrix Θ^a is assumed to be lower triangular, and μ^a is a vector of constants. We estimate these parameters using annual aggregate income and dividend data from the National Income and Product Accounts (NIPA).

Individual i's labor income as a fraction of aggregate labor income is given by η^i_t. In other words, $\eta^i_t \equiv Y^i_t/Y^{\ell}_t$ where Y^i_t is individual i's labor income at time t. $\{\eta^i_t\}_{t=1,\infty}$ is assumed to be a stationary process for each i. In the two person economy that we are considering, the law of motion for η^1_t implies a law of motion for η^2_t since $\eta^1_t + \eta^2_t = 1$. The basic specification for η^1_t that we consider is of the form:

$$(3.2) \qquad log(\eta^1_t) = \bar{\eta} + \rho log(\eta^1_{t-1}) + \varepsilon^1_t ,$$

where the $\{\varepsilon^1_t\}_{t=1,\infty}$ are individual shocks that have mean zero, that are independent over time and independent of ε^a_t for all t. The parameter ρ

captures persistence in each individual's income share.[6] Equation (3.2) implies that there is no correlation between the aggregate state and shocks to each individual's labor income share. In the appendix we present evidence that this is a reasonable assumption. Also in (3.2) we are assuming that the variance of ε_t^1 is not affected by the aggregate state. We discuss an alternative to this assumption below in Section 3.3.

The individual income process is estimated using annual household income data from the PSID. The Appendix has a detailed discussion of the selection criterion for families, and the construction of income. It also describes the estimation of equations (3.1) and (3.2), along with several alternative specifications.[7] For the aggregate dynamics we use the point estimates of the parameters of the vector autoregression, (3.1), as reported in Table A.1. For the individual labor income dynamics in the base case, we use the average of the cross-sectional estimates of (3.2) which are reported in Table A.3.

We have so far assumed that the only tradeable assets in positive net supply are claims to the dividend stream. Since dividends average only 3.9% of total income, this clearly understates the share of income from tradeable assets. Assets such as government securities, corporate bonds, etc., may also be sold or used as collateral for loans. In principle additional debt instruments could be incorporated, but doing so complicates

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[6] To represent labor income we are using a first-order autoregressive model. MaCurdy (1982) and Abowd and Card (1989) argue that the autocorrelation in individual income can be captured by a first-order moving average model for income growth. For our purposes, the first-order autoregressive model is advantageous since it provides a small state-space model that captures dependence in income growth.

[7] For instance, we estimate this law of motion conditional on income data from households that actually own stocks. The results are similar to those for the entire sample.

the analysis considerably.[8] Instead, we increase tradeable asset holdings to a more realistic level by grossing up the assumed fraction of dividend income so that tradeable income is 15% of total income on average. For comparison, capital's share of income in the NIPA averages about 30%.

3.2. *Markov Chain Approximations*

To use this estimated income process as an input for simulations of the Section 2 model, the VAR is approximated with a Markov chain using the method of Tauchen and Hussey (1991).[9] We focus on two specifications of the income process: the "Base Case" and the "Cyclical Distribution Case" models.

The Base Case model closely approximates the estimated process in equations 3.1 and 3.2. We use a state vector with eight dimensions, and a corresponding matrix of transition probabilities. Table 3.1 gives the values of the state variables in the different states along with the transition probability matrix. Notice that the individual share of labor income takes on two values, with a value in the good state that is 1.65 times as large as the value in the bad state. The persistence of these individual shocks is captured in the transition probability matrix.

In the Base Case model, idiosyncratic labor income shocks are assumed to be independent of the aggregate growth rate and dividend realization

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[8] Including additional debt instruments is problematic due to nonstationarities in the data. For example, net interest payments from the corporate sector to the household sector (as measured by "net interest" from the NIPA) account for 1.4% of total income in 1946, and rise to 12.1% of total income in 1990. Computationally, adding assets has the disadvantage that it greatly increases size of the state space.

[9] The GAUSS code to implement these procedures was kindly provided to us by George Tauchen.

because, as discussed below, there is little evidence of significant correlation between individual and aggregate shocks in the PSID data. However, Mankiw (1986), and Constantinides and Duffie (1992) show that the distribution of labor income over the business cycle can play an important role when individuals cannot trade claims to this income. In particular, they show that if the distribution of labor income widens in a downturn then individuals may demand a large equity premium to hold stocks. To examine this potential effect, we consider the Cyclical Distribution Case (C.D.C.) model.

In the C.D.C. model, we set $\eta^1 = 0.5$ (no idiosyncratic shock) when the growth rate of aggregate income, γ^a_t, is high. We then choose two values of η^1 in the low state for γ^a_t such that we maintain the unconditional variance of η. The result is the set of states given in Table 3.2, with the transition matrix as in Table 3.1. This concentrates the idiosyncratic shocks into low growth states for the aggregate economy.

There is in fact some evidence in the PSID that the distribution of income widens during downturns. Letting $\hat{\sigma}^\eta_t$ be the estimated cross-sectional standard deviation of $log(\eta^1_t)$ for each t, the estimated correlation between σ^η_t and $log(Y^\ell_{t+1}/Y^\ell_t)$ is -0.06. However, when this correlation is incorporated into the Markov chain model, it results in a very small departure from the model of Table 3.1.

3.3 Summary of State Variables

The exogenous state variables include γ^a_t, δ_t and η^1_t. These evolve according to the Markov process specified above. An endogenous component of the state is portfolio composition. In our two person economy, this is summarized by agent 1's holdings of stocks and bonds, since agent 2's holdings can be derived using the market clearing conditions (2.8) and

(2.9). We define the state vector of the economy by $Z_t = \{\gamma^a_t, \delta_t, \eta^1_t, s^1_t, b^1_t\}$. Asset prices, consumption policies, and trading policies are found as a function of Z_t.

4. Simulation Results

In this section we report the results of Monte Carlo simulations of the economy of Section 2, using the exogenous driving processes described in Section 3. We solve for the equilibrium numerically using a modified version of the "auctioneer algorithm" described in Lucas (1991).

A number of summary statistics are reported. First, we consider the volatility of individual consumption implied by the model and compare this to the volatility of aggregate consumption. This statistic is of interest because risk sharing is not complete and, as a result, there is no direct link between individual and aggregate consumption. The extent to which individual consumption behaves like the aggregate indicates the degree to which risk sharing is being accomplished by trading securities. We also report the mean and standard deviation of implied equilibrium asset prices. These results allow us to evaluate whether the model with incomplete markets and transactions costs can explain the equity premium puzzle of Mehra and Prescott (1985). Finally, we report the mean and standard deviation of trading volume, in order to gauge the sensitivity of trade to the level of transactions costs.

To compute these statistics, we assume that initially each agent holds half the shares of stock and no debt. The economy evolves for 1000 years, driven by realizations of the exogenous income process. Asset returns, trades and consumption growth between years 999 and 1000 are recorded for each history. The reported statistics are based on averages across 1500 of these experiments[10].

4.1. *Representative Agent Baselines*

Before turning to the simulations of the model with incomplete insurance markets, we briefly examine the implications of the complete markets version of the model. This experiment is very similar to the one undertaken by Mehra and Prescott (1985), except that we assume a different consumption and dividend process. Mehra and Prescott equate dividends and consumption, while we treat consumption as the sum of dividends and labor income.

Table 4.1 reports the results of the complete markets version of our model for $\gamma = 1.5$ and $\beta = 0.95$, and each of the specifications of individual labor income. The column labeled "Aggregate" is for the complete markets case in which the representative agent is assumed to consume aggregate income. The table also reports, in the column labeled "Data", the sample moments from the data. The moments of stock returns were calculated using annual returns from the S&P 500 from 1947 through 1990 converted to real returns using the CPI. The moments of the bond returns were calculated using annual Treasury bill returns from 1947 through 1990 also converted to real returns using the CPI. Although the consumption process here differs from those used in previous studies, the implied asset prices are similar.[11] For example, the predicted average return on risk-free bonds is high and close to the stock return in each case whereas the observed average bond return is quite low. Further, the standard deviation of the stock return is

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[10] The long time horizon was chosen to ensure that the effect of initial conditions was eliminated.

[11] See, for example, Mehra and Prescott (1985).

low relative to historical levels. Notice that these result are independent of the model of idiosyncratic income since this risk is completely shared.

To assess whether the presence of uninsurable labor income shocks has the *potential* to explain the poor performance of the complete markets, representative agent model, consider the columns in Table 4.1, labeled "Individual B.C." for the Base Case model and "Individual C.D.C." for the Cyclical Distribution (C.D.C.) model. These statistics were calculated in a representative agent model using one of the agent's labor income dynamics *as if* they were the aggregate labor income dynamics. These results show that if the representative consumer is forced to consume his idiosyncratic labor income process, then the predicted equity premium becomes much larger. For example, in the Base Case model the premium is predicted to be 6.8%. The relatively high level of both the bond and stock returns could be corrected by using a larger value of β. The standard deviation of the stock return is also predicted to be 38.6% so that the stock return is quite volatile, which is consistent with the data. Notice that in the C.D.C. model, the equity premium is even higher because the idiosyncratic shocks are concentrated in periods of low aggregate growth.

In general the results in Table 4.1 indicate that the model with uninsurable labor income has the potential to explain some of the observed moments of asset returns. Of interest, however, is whether these results change once trading is allowed in the stock and bond markets.

4.2 Frictionless Trading

Table 4.2 summarizes the case in which individuals can trade costlessly in both the stock and bond market, subject to the restriction that stocks cannot be sold short, and that individuals can borrow only up to 10% of per capita income. In the Table, the rows labeled "Avg Consump Growth" and "Std

Dev Consump Growth" give the average and standard deviation of consumption growth for agent 1. Notice that individual consumption is only slightly more volatile than aggregate consumption, (see the "Aggregate" column in Table 4.1) and is much less volatile than individual income (see the "Individual" columns in Table 4.1).

The result that idiosyncratic shocks are offset by asset trades when trading is costless strengthens the findings of Lucas (1991) and Telmer (1991), who perform a similar experiment under the assumption that labor income shocks are independent over time. In our model, the labor income shares are correlated over time and hence the idiosyncratic shocks are relatively persistent.[12] This persistence should, via a wealth effect, make it more difficult to self-insure through the asset markets. A striking example of this is given by Constantinides and Duffie (1992), who construct cases in which labor income shocks follow a random walk and no smoothing occurs. However, the results of Table 4.2 indicate that with the persistence that we consider (which matches the PSID observations of labor income) agents can still effectively smooth their idiosyncratic income shocks. As a result, equilibrium asset prices are virtually identical to the complete markets case reported in Table 4.1.

Given the parameterization used here, an increase in predicted consumption volatility appears to require the introduction of some form of trading friction. This observation motivates the experiments with transactions costs and borrowing constraints that follow.

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[12] The estimated autocorrelation coefficient is approximately 0.5. We also experimented with income processes exhibiting higher persistence in the income shocks, allowing autocorrelation coefficients as high as 0.7. This had little affect on the results reported in Table 4.2.

4.3. *Transaction Costs in Both Markets, Asymmetric Bond Market Costs*

We now consider the effects of transactions costs on equilibrium prices and consumption. We will focus on specifications in which agents face transactions costs in both markets *simultaneously*. In simulations not reported here, we find that imposing a friction in one market alone has a negligible impact on asset prices. Because portfolio balance is a second order consideration for these agents relative to intertemporal consumption smoothing, they tend to substitute towards trading in the lower cost market, and equilibrium returns are largely unaffected. This is related to the finding of Constantinides (1986) that transactions costs cause agents to delay their portfolio rebalancing and therefore have only a second order effect on asset returns.

In the first specification that we consider, the cost structure in the bond market is such that only the borrower pays the transactions cost. Recall that in this case the transactions cost functions are given by (2.5) for the stock market and (2.6) for the bond market. We allow the parameter Ω to vary from 0 to 2.0. To balance the average costs in the two markets we set $k = \Omega/2$. The results[13] of these specifications are given in Figures 4.1a-d for the Base Case model and 4.2a-d for the C.D.C. model. In each of the figures the x-axis gives the value of Ω.

First consider Figures 4.1a and 4.2a which give the average stock and bond returns, along with the equity premium, as a function of Ω for each of the two models. Due to the cost of borrowing there are two different rates of return in the bond market: a lending rate and a borrowing rate. In the

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[13] In constructing these results a grid of values for Ω between 0 and 2.0 are used. To smooth across these grid points we fit a third order polynomial through the points. This curve is reported in the figures. This smoothing of the results only serves to remove some very small variability in the figures.

figures we plot the average return to lending, since the equity premium is typically measured using the average return on a treasury security, and individuals cannot borrow at this rate. Notice that as Ω increases the average bond return falls and the equity premium widens. The average transactions costs paid by agent 1, as a percentage of the value of trade, are reported in figures 4.1b and 4.2b. As Ω increases the average costs paid increase. For example in the Base Case model the average cost paid in the stock market rises from 0 to 4.1% of the value of trade. Figures 4.1c and 4.2c report the average level of trading in the two markets as a percentage of per capita income. As the transactions costs increase the level of trading drops off. This is reflected in a higher level of consumption volatility which can be seen in Figures 4.1d and 4.2d, which give the standard deviation of consumption growth for agent 1. Notice that when Ω = 2.0 individual consumption volatility is close to the volatility of individual income.

In this model an increase in transactions costs results in a decrease in the average bond return but the average stock return remains about the same at all levels of the transactions costs. This result is best understood by considering the direct and indirect effects of transactions costs.

The Direct Effect of Transactions Costs.

With moderate transactions costs in each market, agents trade every period to buffer their idiosyncratic income shocks. The individual facing the adverse idiosyncratic shock borrows in the bond market and sells stock in the stock market to partially offset the shock. The agent receiving the favorable shock buys both bonds and stocks to create a buffer against future adverse shocks. The borrower is willing to pay a relatively high borrowing rate including transactions costs. For example, consider Figure 4.1a and

Ω=2. The average return on the bond is about 0.3%, but the borrower faces an average transactions cost of 3.3%. Since the cost function is quadratic, the marginal transaction cost that the borrower faces is 6.6% which implies a net marginal borrowing rate of 6.9%. The lender, however, only receives a 0.3% rate of return on average. The lender is satisfied with this average rate of return because the high consumption variability when Ω = 2 (see Figure 4.1d) creates a strong precautionary demand for bonds.

Notice that the stock return remains at a little over 8% for all levels of Ω in each model. The direct effect of transactions costs on the equilibrium stock return is difficult to predict since the buyer of stock demands a lower price to compensate for transactions costs whereas the seller demands a higher price. Note that the net return from buying stock is lower than 8% due to transactions costs, although in equilibrium the observed return is not greatly affected. The lower net return also reflects a precautionary demand for assets due to higher consumption volatility.

In sum, for this cost specification there is a direct effect of transactions costs that depresses the observed bond return and hence increases the observed equity premium. Both stocks and bonds have a lower net return due to a precautionary demand induced by higher consumption variability.

A similar effect of differential stock and bond market transactions costs on relative observed returns is found by Aiyagari and Gertler (1991) and Vayanos and Vila (1992). However in those models there is no aggregate risk so that differences in rates of return are generated solely by the differences in transaction costs across asset markets. In our model there is a further indirect effect of transactions costs which is due to an increase in the covariance between stock returns and consumption.

The Indirect Effect of Transactions Costs

To the extent that the increase in consumption volatility increases the covariance between individual consumption and the net returns on stock, the net-of-transactions-costs equity premium that agents demand widens. To assess this effect we need to construct a net-of-transactions-costs equity premium. There are several ways to do this construction. First consider the payoff to an individual investor from investing in a unit of the stock at time t to be liquidated at time t+1. A transactions cost is paid in the current period and at liquidation, but the size of this cost depends on the portfolio position today and the expected portfolio position tomorrow. The marginal transactions cost that individual i pays at time t in purchasing a unit of the stock is: $2k_t(s^i_{t+1}-s^i_t)(p^s_t)^2$ (see (2.5)). If that position is liquidated at time t+1 the marginal transactions cost is $-2k_{t+1}(s^i_{t+2}-s^i_{t+1})(p^s_{t+1})^2$. As a result, the net (of transactions costs) one period rate of return from a unit of investment in the stock on the part of individual i is given by:

$$(4.1) \qquad r^{s,net}_{t,t+1} \equiv \frac{p^s_{t+1} + d_{t+1} - 2k_t(s^i_{t+2}-s^i_{t+1})(p^s_{t+1})^2}{p^s_t + 2k(s^i_{t+1}-s^i_t)(p^s_t)^2} - 1 .$$

Notice that this rate of return satisfies

$$(4.2) \qquad E\left\{\beta \frac{u'(c^i_{t+1})}{u'(c^i_t)} (1 + r^{s,net}_{t,t+1}) \mid \mathcal{F}(t)\right\} = 1,$$

which is just the Euler equation for individual i (see (2.11)).

The net (of transactions costs) rate of return from a unit of investment in the bond is given by:

$$(4.3) \qquad r^{b,net}_{t,t+1} = \begin{cases} 1/p^b_t & \text{if } b^1_{t+1} \geq 0 \\ 1/[p^b_t + 2\Omega_t b^1_{t+1} (p^b_t)^2] & \text{if } b^1_{t+1} < 0 \end{cases} .$$

The indirect effect of transactions costs occurs to the extent that the covariance between individual consumption and the net return on the stock changes. To assess this effect we plot the "net premium" in Figures 4.1a and 4.2a. This measure of the equity premium is given by: $E\left\{r^{s,net}_{t,t+1} - r^{b,net}_{t,t+1}\right\}$ which we calculate for individual 1. The "net premium" reflects the changing conditional covariance between $r^{s,net}_{t,t+1}$ and consumption growth as transactions costs increase. In the Base Case model when Ω=2 the equity premium is 7.8% but the net premium is only 1.3%. However in the C.D.C. model when Ω=2, the equity premium is 7% the net premium is 2%. As a result the indirect effect does affect the observed equity premium, although the direct effect on the bond return dominates.

Another measure of the net-of-transactions-costs premium is to look at the difference between average stock and bond returns, subtracting out the average realized transactions costs in each market. Notice that a borrower must pay transactions costs on the entire amount borrowed each period. However in selling stock the transactions cost is small as a percentage of the total return on stock holdings, since only a portion of the stock portfolio typically is bought or sold each period. The average transactions cost in the stock market as a percentage of the value of the stock portfolio are small, rising to a maximum of 0.46% when Ω = 2 (k=1) in the C.D.C. model. As a result the average net return on an individual's portfolio of stocks is about 7.5% for Ω = 2. Since the average return to a lender for Ω = 2 is about 1%, this implies a sizeable net-of-transactions-costs equity premium of 6%.

Which of these two measures is a more appropriate measure of the net-of-transactions-costs premium depends on the question being asked. The first measure directly reflects the relation between risk and return in the model, while the second is of no obvious theoretical interest. However, in comparing the predictions of the model to data, the second measure can be calculated using only information on returns and average transactions costs, while the former would also require detailed information on consumption.

The results of this section indicate that for relatively low transactions costs the model does predict a substantial equity premium and a low risk-free rate of return. For example in the B.C. model when $\Omega = 1$, the average transactions costs are 1.7% in the bond market and 2.3% in the stock market and the equity premium is 4.1%. The net premium is 0.5% in this case so that there is some effect of increased consumption volatility on individuals attitude towards aggregate risk. However the largest effect is the drop in the required rate of return on the bond due to a precautionary demand for savings.

4.4. *Transactions Costs in Both Market, Symmetric Bond Market Costs*

To further investigate the effect of the form of transactions costs on the predicted equity premium, we examine the case in which the borrowing costs are shared equally by the lender and the borrower; the bond cost function (2.6) is replaced with (2.7). In this case any difference between the average bond and stock return should reflect differences in risk between the two markets.

The results of this specification of costs are reported in Figures 4.3a-d for the Base Case model and 4.4a-d for the C.D.C. model. Since transactions costs are paid by both the borrower and the lender in the bond market, we limit Ω to the range [0,1], and set k equal to Ω. This produces average transactions costs that are similar to those in Figures 4.1 and 4.2. so that the results of the two cost structures can be easily compared.

Notice that in Figures 4.3a and 4.4a the equity premium rises only slightly as a result of the increase in consumption volatility. In the C.D.C. model the premium widens more than in the Base Case model, which is to be expected given the fact that the individual income shocks are concentrated during economic downturns. Because transactions costs are paid by both the lender and the borrower, the drop in the bond return is not due to differences in the transactions costs structure in the two markets, but instead to an increased precautionary demand for savings.

The effect of precautionary demand can be seen most clearly by considering the net-of-transactions-costs bond return. For example in the C.D.C. model with Ω = 1 the average bond return is 6.3% and the average bond market transactions costs is 2.7%. As a result the marginal return from investing in the bond market is 0.9%. This is not reflected in the observed bond return because of the form of the cost function. Since the transactions costs are balanced in each market the equity premium in this case reflects the net premium as measured by $E\left\{r^{s,net}_{t,t+1} - r^{b,net}_{t,t+1}\right\}$, where $r^{s,net}_{t,t+1}$ is given by (5.1) and

$$(4.4) \qquad r^{b,net}_{t,t+1} = E\{1/[p^b_t + 2\Omega_t b^i_{t+1}(p^b_t)^2]\} \ .$$

Notice that the net premium and the measured equity premium track each other closely in this case. In this specification of transactions costs there is some effect on the observed equity premium but it is less dramatic than in the asymmetric bond market costs case examined in Section 4.4.

4.5. Transactions Costs in Stocks, No Borrowing

Here we consider a market structure that permits costly trading in stocks but precludes borrowing (i.e., the borrowing constraint is at 0). In this case, the (shadow) price of bonds is calculated using the marginal rate of substitution of the agent with a good idiosyncratic shock. A second way to close the bond market would be to impose a very high cost on the borrower in the bond market. This is equivalent to the case where borrowing is not allowed.[14] Hence, the results of this section can be interpreted as approximating a situation where there is extreme credit rationing or where there is a very large wedge between the borrowing and lending rates.

Figures 4.5a-d report results for the Base Case model without borrowing, in which the only cost parameter that varies is k. For any level of the average costs the effect of stock market transactions costs on the predicted bond return and the equity premium is slightly larger than in the case where costly borrowing is allowed (see Figures 4.1a-d). This occurs because agents cannot substitute towards the bond market to avoid transactions costs.

The results for the C.D.C. model without borrowing are reported in Figures 4.6a-d. These results are similar to Figures 4.5a-d except that the equity premium is larger for any level of average transactions costs. As before this occurs because shocks to individual labor income only occur during economic downturns. Notice however that the additional equity premium predicted by this effect is not large.

[14] See Heaton and Lucas (1992) for a discussion of this issue.

5. Concluding Remarks

In this paper we have examined asset prices and consumption patterns in a model in which agents face both aggregate and idiosyncratic income shocks, and insurance markets are incomplete. Agents reduce consumption variability by trading in a stock and bond market to offset idiosyncratic shocks, but transactions costs in both markets limit the extent of trade. To calibrate the theoretical model, we estimated an empirical model of labor and dividend income, using data from the PSID and the NIPA.

Although the agents in the model are not very risk averse, the model predicts a sizable equity premium and a low riskfree rate. By simultaneously considering aggregate and idiosyncratic shocks, we can decompose this effect of transactions costs on the equity premium into two components. The direct effect is due to the fact that individuals equate net-of-cost margins, so an asset with lower associated transactions cost will have a lower market rate of return.

The size of the direct effect varies widely with the structure of transactions costs. When the cost structure in the stock and bond market are assumed to be similar, the direct effect is negligible. However, when we assume that the transactions cost in the bond market creates a wedge between the borrowing and lending rate, or that there is a binding borrowing constraint, the direct effect can account for over three quarters of the total premium. This is related to the findings of Aiyagari and Gertler (1991), who report that realistic transactions costs may account for about 50% of the observed equity premium.

A second, indirect effect occurs because transactions costs result in individual consumption that more closely tracks individual income than aggregate consumption. The higher variability of individual consumption increases the covariance between consumption and the dividend process, and hence increases the systematic risk of the stock. While the size of the indirect effect increases with the assumed level of transactions costs, it is relatively insensitive to the structure of transactions costs. In the base case analysis, the indirect effect accounts for about 20% of the premium.

While the model can explain the observed first moments of stocks and bonds, it does not explain observed second moment differentials. It shares the feature of many consumption-based models, that an increase in the equity premium predicted by the model is not accompanied by a substantial change in the relative volatility of bond and stock returns. Typically in models that fit the equity premium, the volatility of the bond return is too high. However in the model considered in this paper an increase in transactions costs implies a widening in the equity premium, but the volatility of *observed* returns does not greatly increase. In particular the volatility of the bond return remains fairly low as transactions costs increase. The volatility of the net-of-transactions-costs bond and stock returns do rise in response to transactions costs due to the increase in the variability of the marginal rate of substitution. It remains an open question whether there is a realistic assumption about transactions costs that can simultaneously explain the low volatility of short bond rates and the high volatility of stock returns.

Several extensions of the analysis are left to future research. Our two-person economy is the simplest case of heterogeneity, and intuitively we expect most of the results to carry over to the more general case. Conceptually the model could easily be extended to the case of n agents. Because of the practical difficulty of increasing n using current computational techniques, an interesting question is whether it would be possible to exploit a different modeling strategy to develop a tractable way to allow for a large number of individuals. This would allow for a more detailed description of the income distribution and its evolution over time. One potentially important factor that could be studied in this context would be the impact of a small probability of a very bad idiosyncratic shock. The model also could be extended to include a production or storage technology, and a labor/leisure decision.

Appendix: Data Description and Estimation Results Using the PSID

In Section 3.1 we briefly described the empirical model of aggregate and individual labor income used in the calibrations. In this appendix we describe the estimation in more detail, and describe a more general model of labor income that was also investigated.

A.1 Further Specification of the Model of Individual Labor Income

As in Section 3, individual i's labor income as a fraction of aggregate labor income is given by η^i_t, where $\{\eta^i_t\}_{t=1,\infty}$ is assumed to be a stationary process for each i. The model for $\{\eta^i_t\}_{t=1,\infty}$ must account for correlation over time in η^i_t and correlation between shocks to η^i_t and the aggregate state. The most general specification of η^i_t that we consider is of the form:

$$\text{(A.1)} \qquad log(\eta^i_t) = \bar{\eta}^i + \zeta^{i\prime}\varepsilon^a_t + \rho^i log(\eta^i_{t-1}) + \varepsilon^i_t,$$

where the $\{\varepsilon^i_t\}^{i=1,n}_{t=1,\infty}$ are individual shocks that have mean zero, that are independent over time and across individuals and independent of the aggregate shock ε^a_t for all t. $E\{(\varepsilon^i_t)^2\}^{1/2} \equiv \sigma^i$. $\{\bar{\eta}^i\}^{i=1,n}$, $\{\zeta^i\}^{i=1,n}$ and $\{\rho^i\}^{i=1,n}$ are parameters.

The parameters $\{\bar{\eta}^i\}^{i=1,n}$ capture permanent differences in relative labor income. The parameter vector $\{\zeta^i\}$ captures the degree to which shocks to individual i's relative income are correlated with the aggregate shocks. Differences in ζ^i across individuals allow the aggregate shock to differentially affect individuals. The parameter ρ^i captures the persistence in shocks to individual i's labor income.

As it stands, the specification in (A.1) does not use all of the cross-sectional information. For example, the parameter $\bar{\eta}^i$ is free to vary across individuals in the model. It may be possible to link variation in $\bar{\eta}^i$ to observable family characteristics. However, unless variation in $\bar{\eta}^i$ were linked to just two distinct subgroups of the population, it is not clear how this information could be used in our investigation of the implications of the two person economy of Section 2.

A.2. Data

The PSID provides a panel of annual observations of individual and family income, consumption of food, and other variables. In using the PSID, we took family income per family member as of a measure of Y^i for our model. Because of the changing character of many of the families, we took a subsample from the PSID consisting of those families for which the head of the household was male and for which neither the head nor his spouse changed over the sample. This selection was used so that we did not have to keep track of new families, family split-offs, and other dramatic changes in the family. The complete PSID over-samples poorer members of the U.S. population due to a sample of poor individuals from the Survey of Economic Opportunity. To make the sample closer to a random sample of the U.S. population, those families who were originally part of the Survey of Economic Opportunity were excluded from the sample.

Total labor income of the head of the household and his wife along with total transfers to the family was used for total family income. The transfers included unemployment compensation, worker's compensation, pension income, welfare, child support and so on. If a family had zero income from all of these sources in any year, it was excluded from the sample. At this point we had a sample of 860 families with income data spanning the years 1969 to 1984. Because families differ in size, family income per family member was created by dividing each family's income by the total number of family members in each year[15]. This measure of nominal family labor income was weighted by the Consumer Price Index (CPI) to obtain a measure of real labor income per family member.

The model of individual income dynamics was estimated using this full sample, and also using a subsample restricted to households owning stock. The sample was split because a large segment of the population does not hold financial assets. In the asset pricing model that we are examining, it is clearly more appropriate to consider the income dynamics of those individuals who participate in securities markets. Following Mankiw and Zeldes (1990), the sample was split based upon individual holdings of securities using questions about stock holdings in the 1984 PSID. If a family reported some holdings of stocks in 1984, it was included in the group called *stockholders*.

For both the complete sample and the stockholder sample, η_t^i was then constructed as $Y_t^i/\left\{\sum_{i=1}^{n} Y_t^i\right\}$ where n is 860 for the complete sample and 327 for the stockholders sample.

Along with observations of individual labor income from the PSID, measures of annual aggregate labor income and dividends were taken from the NIPA for the years 1947 through 1989, obtained from CITIBASE. For labor income we used "total compensation of employees". The aggregate series were weighted by the total U.S. population and the CPI in each year to obtain real per capita labor income and dividends.

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[15] We also conducted the analysis where family income was weighted only by the number of adults in the family. The results were similar and hence are not reported.

III.3. Empirical Results

As described above, our measure of aggregate labor income was taken from the NIPA. A natural alternative would have been to use total income from the PSID. However because of the limited time dimension in the PSID, the aggregate dynamics could not be estimated with much precision. Although the NIPA measure of labor income does not exactly correspond to the measure of labor income obtained from the PSID, the two measures of aggregate income are similar. For example, Figure A.1 gives a plot of the CITIBASE measure of aggregate labor income growth $[log(Y^a_t/Y^a_{t-1})]$ along with the corresponding measure constructed from the PSID. The PSID measure of aggregate labor income per capita was constructed by summing household labor income across households in our sample and then weighting by the total number of people in our sample for each year. The correlation between the two series is 0.9. As a result, measuring aggregate labor income shocks from the NIPA series should do a reasonable job in capturing aggregate labor income dynamics.

Table A.1 reports ordinary least squares estimates of the law of motion for aggregate labor income and aggregate dividends (see equation (3.1)). Using the fitted residuals from this estimation, the model given by (A.2) was estimated individual by individual using ordinary least squares.

All Individuals. The estimation was first performed for our largest subsample of households from the PSID. A summary of these findings is given in Table A.2 which reports sample averages of the parameter estimates along with sample standard errors. The estimates reported in Table A.2 are consistent with results reported in MaCurdy (1982) and Abowd and Card (1989), for example. In particular, the estimated parameter values imply that individual income growth is negatively correlated over time. Abowd and Card (1989) also argue that aggregate variation in income has little affect on the autocorrelation structure of individual income. In our sample from the PSID, on average only 16% of individual income variation is captured by the aggregate shocks. For this reason, we also estimated the law of motion for the η^i's with $\zeta^i \equiv 0$. The results of this estimation are given in Table A.3 which reports the cross section average of the parameters and cross sectional standard errors. Notice that the average autoregressive parameter is slightly larger as is the estimated variance of ε_t^i.

Stockholders. Tables A.4 and A.5 present results of estimating (A.2) for the *stockholder* subsample. In comparing these results to tables A.2 and A.3, notice that the average autoregressive parameters are smaller for the stockholders than in the regressions for the entire sample. Also the variance of the idiosyncratic shock, σ^i, for the stockholders is slightly larger. However, the results are generally consistent with a model in which η_t^i is correlated over time and in which there is a very important role for idiosyncratic shocks. On average 12% of the variation in η_t^i over time is explained by the aggregate shocks.

Based upon these results we use the estimated parameters reported in Table A.3 as our base case in Section 3. In Section 3 we also discuss a case that captures variation in the cross-sectional distribution of labor income over the business cycle.

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Table 3.1

Markov Chain Model for Exogenous State Variables

Base Case

State Number	States γ^a	δ	η^1
1	0.991	0.1522	0.3772
2	1.047	0.1517	0.3772
3	0.991	0.1483	0.3772
4	1.047	0.1478	0.3772
5	0.991	0.1522	0.6228
6	1.047	0.1517	0.6228
7	0.991	0.1483	0.6228
8	1.047	0.1478	0.6228

Transition Probability Matrix

$[p_{ij}]$ where $p_{ij} = Pr\{\text{state } j \text{ at time } t+1 \mid \text{state } i \text{ at time } t\}$

$$\begin{bmatrix}
0.3562 & 0.2430 & 0.08506 & 0.05803 & 0.1237 & 0.08436 & 0.02953 & 0.02015 \\
0.3392 & 0.3360 & 0.03370 & 0.03338 & 0.1178 & 0.1166 & 0.01170 & 0.01159 \\
0.03338 & 0.03370 & 0.3360 & 0.3392 & 0.01159 & 0.01170 & 0.1166 & 0.1178 \\
0.05803 & 0.08506 & 0.2430 & 0.3562 & 0.02015 & 0.02953 & 0.08436 & 0.1237 \\
0.1237 & 0.08436 & 0.02953 & 0.02015 & 0.3562 & 0.2430 & 0.08506 & 0.05803 \\
0.1178 & 0.1166 & 0.01170 & 0.01159 & 0.3392 & 0.3360 & 0.03370 & 0.03338 \\
0.01159 & 0.01170 & 0.1166 & 0.1178 & 0.03338 & 0.03370 & 0.3360 & 0.3392 \\
0.02015 & 0.02953 & 0.08436 & 0.1237 & 0.05803 & 0.08506 & 0.2430 & 0.3562
\end{bmatrix}$$

Table 3.2

Markov Chain Model for Exogenous State Variables

Cyclical Distribution Case

	States		
State Number	γ^a	δ	η^1
1	0.991	0.1522	0.3263
2	1.047	0.1517	0.5
3	0.991	0.1483	0.3263
4	1.047	0.1478	0.5
5	0.991	0.1522	0.6737
6	1.047	0.1517	0.5
7	0.991	0.1483	0.6737
8	1.047	0.1478	0.5

Transition Probability Matrix

As in Table 3.1

Table 4.1

Moments Implied by the Complete Markets Cases

Moment	Data	Aggregate	Individiual B.C.	Individual C.D.C.
Avg Consump Growth	0.021	0.019	0.019	0.019
Std Dev Consump Growth	0.029	0.028	0.109	0.166
Avg Bond Return	0.008	0.082	0.055	0.060
Std Dev Bond Return	0.026	0.006	0.175	0.266
Avg Stock Return	0.089	0.083	0.138	0.167
Std Dev Stock Return	0.173	0.030	0.386	0.465

Table 4.2

Momemts Implied by the Frictionless Model

Moment	Base Case	Cyclical Distribution
Avg Consump Growth	1.019	1.019
Std Dev Consump Growth	0.037	0.036
Avg Bond Return	0.081	0.081
Std Dev Bond Return	0.034	0.035
Avg Stock Return	0.081	0.083
Std Dev Stock Return	0.047	0.049
Avg Bond Trades (% of Consumption)	0.070	0.062
Std Dev Bond Trades	0.058	0.059
Avg Stock Trades (% of Consumption)	0.121	0.113
Std Dev Bond Trades	0.057	0.090

Table A.1

Aggregate Dynamics

$$\gamma_t^a \equiv Y_t^a / Y_{t-1}^a, \quad \delta_t \equiv D_t^a / Y_t^a \text{ and } X_t^a \equiv [\log(\gamma_t^a) \ \ \log(\delta_t)]'$$

$$X_t^a = \Lambda^a X_{t-1}^a + \Theta^a \varepsilon_t^a + \mu^a$$

Ordinary Least Squares Estimates[16]

$$\Lambda^a = \begin{bmatrix} \begin{matrix} 0.0805 \\ (0.1673) \end{matrix} & \begin{matrix} 0.0626 \\ (0.0344) \end{matrix} \\ \\ \begin{matrix} -0.5166 \\ (0.2329) \end{matrix} & \begin{matrix} 0.9480 \\ (0.0706) \end{matrix} \end{bmatrix} \qquad \Theta^a = \begin{bmatrix} 0.0276 & 0 \\ \\ 0.0056 & 0.0432 \end{bmatrix}$$

$$\mu^a = \begin{bmatrix} \begin{matrix} 0.2249 \\ (0.1141) \end{matrix} \\ \\ \begin{matrix} -0.1630 \\ (0.2343) \end{matrix} \end{bmatrix}$$

[16] Standard errors in parentheses.

Table A.2

Individual Income Dynamics with Aggregates

All Individuals

Cross sectional means and standard deviations of coefficient estimates.

$$\eta^i_t \equiv Y^i_t / Y^\ell_t$$

$$log(\eta^i_t) = \bar{\eta}^i + \rho^i log(\eta^i_{t-1}) + \zeta^{i\prime} \varepsilon^a_t + \varepsilon^i_t,$$

$$\sigma^i = \{E(\varepsilon^i_t)^2\}^{1/2}$$

Coefficient	Cross-Sectional Mean	Standard Deviation
$\bar{\eta}^i$	-3.5284	2.6094
ζ^i_1	0.0036	0.3600
ζ^i_2	-0.0071	0.0722
ρ^i	0.5032	0.1268
σ^i	0.2292	0.1226

Table A.3

Individual Income Dynamics without Aggregates

All Individuals

Cross sectional means and standard deviations of coefficient estimates.

$$\eta^1_t \equiv Y^1_t / Y^\ell_t$$

$$log(\eta^1_t) = \bar{\eta}^1 + \rho^1 log(\eta^1_{t-1}) + \varepsilon^1_t,$$

$$\sigma^1 = \{E(\varepsilon^1_t)^2\}^{1/2}$$

Coefficient	Cross-Sectional Mean	Standard Deviation
$\bar{\eta}^1$	-3.3499	2.4134
ρ^1	0.5290	0.3320
σ^1	0.2508	0.1309

Table A.4

Individual Income Dynamics with Aggregates

Stockholders

Cross sectional means and standard deviations of coefficient estimates.

$$\eta^i_t \equiv Y^i_t / Y^\ell_t$$

$$log(\eta^i_t) = \bar{\eta}^i + \rho^i log(\eta^i_{t-1}) + \zeta^{i\prime}\varepsilon^a_t + \varepsilon^i_t,$$

$$\sigma^i = \{E(\varepsilon^i_t)^2\}^{1/2}$$

Coefficient	Cross-Sectional Mean	Standard Deviation
$\bar{\eta}^i$	-4.6673	2.1603
ζ^i_1	0.0113	0.0930
ζ^i_2	-0.0441	0.1676
ρ^i	0.2308	0.3479
σ^i	0.3587	0.1494

Table A.5

Individual Income Dynamics without Aggregates

Stockholders

Cross sectional means and standard deviations of coefficient estimates.

$$\eta^i_t \equiv Y^i_t / Y^\ell_t$$

$$log(\eta^i_t) = \bar{\eta}^i + \rho^i log(\eta^i_{t-1}) + \varepsilon^i_t,$$

$$\sigma^i = \{E(\varepsilon^i_t)^2\}^{1/2}$$

Coefficient	Cross-Sectional Mean	Standard Deviation
$\bar{\eta}^i$	-4.2409	1.8692
ρ^i	0.2987	0.3039
σ^i	0.3826	0.1589

Figure 4.1a: B.C., Returns

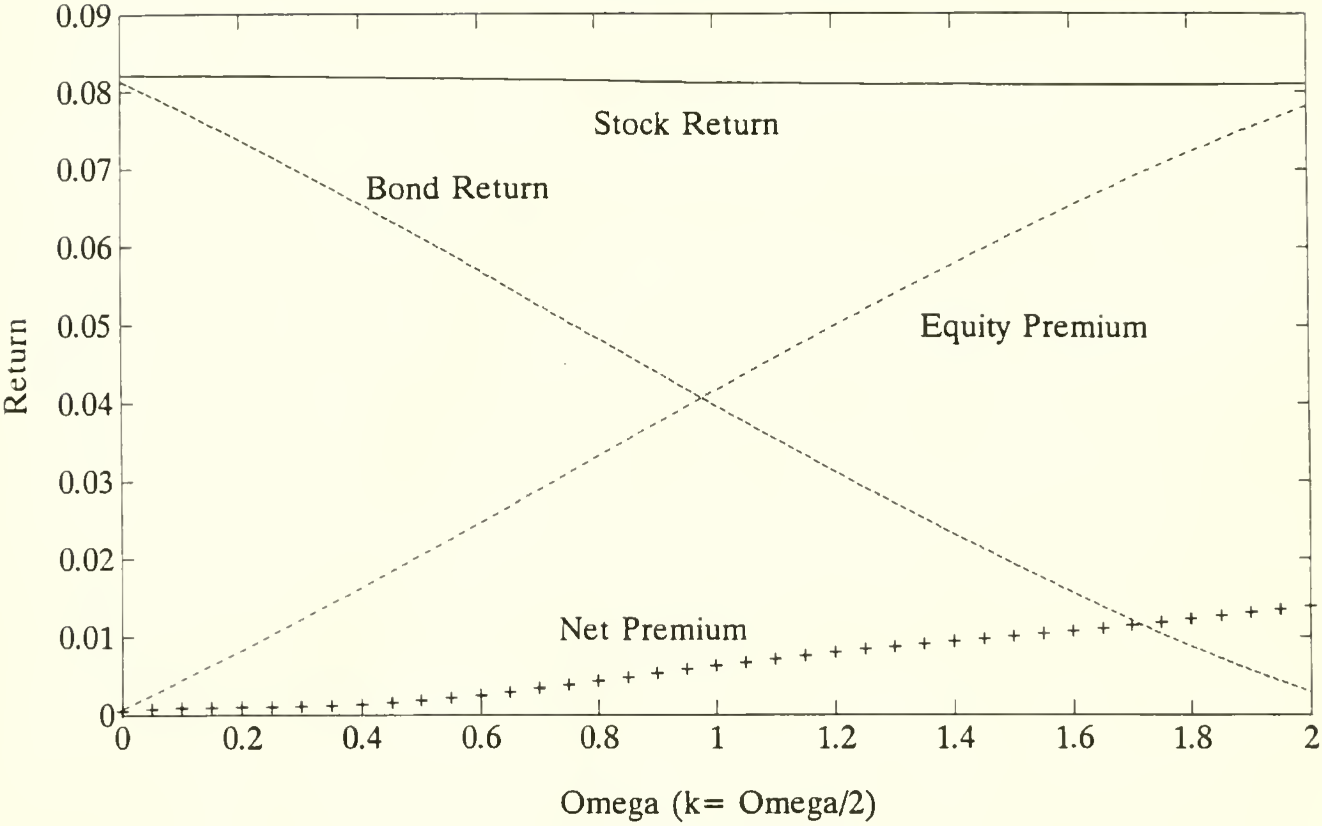


Figure 4.1b: B.C., Avg. Costs

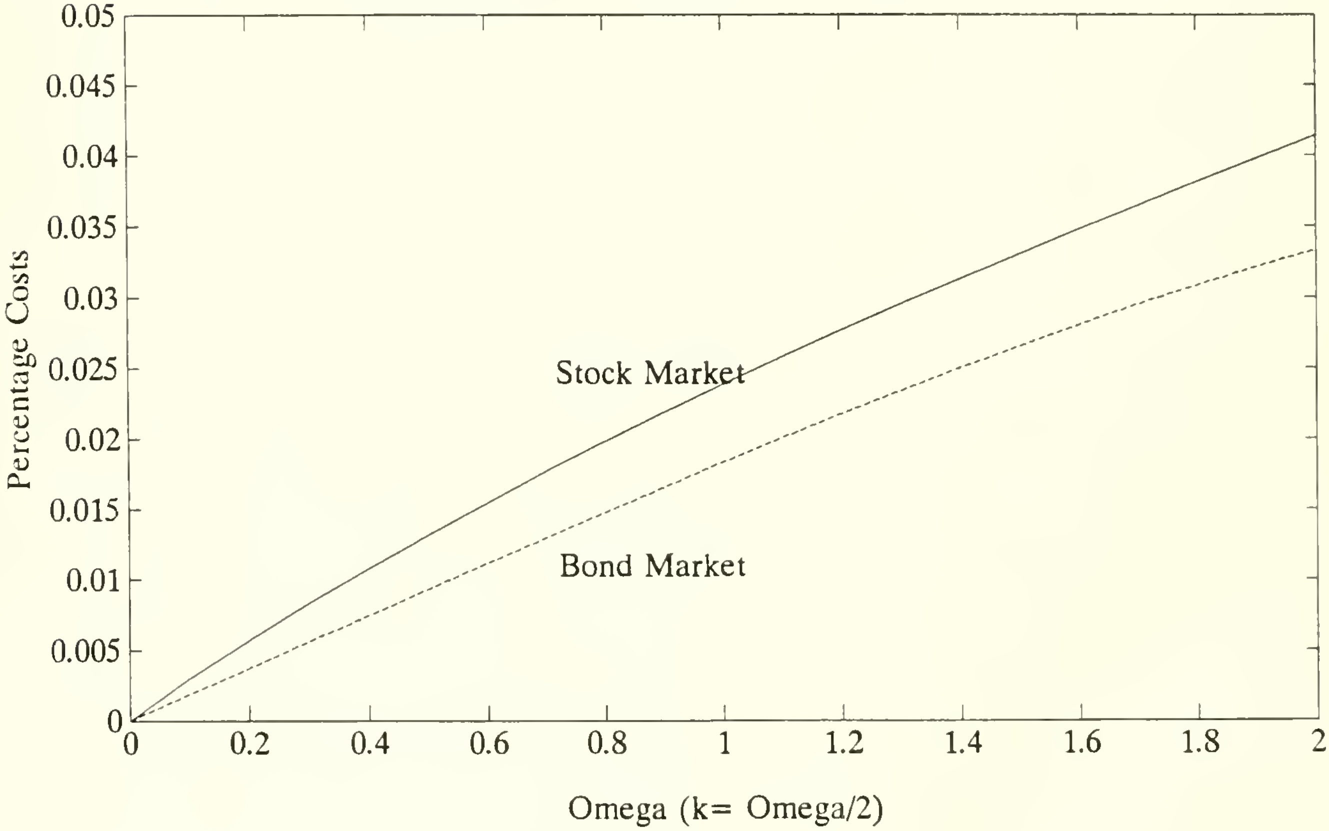


Figure 4.1c: B.C., Average Trading

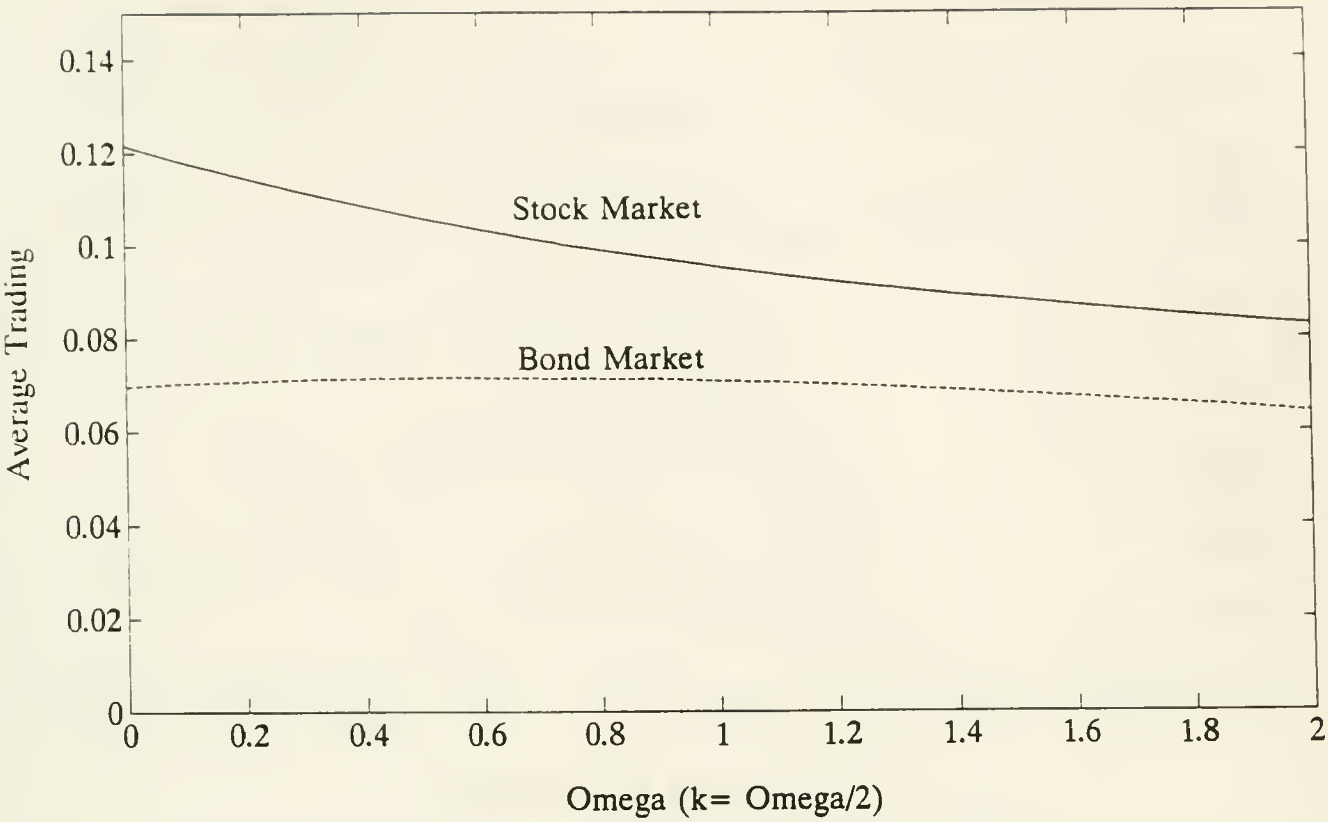


Figure 4.1d: B.C., STD of Consumption

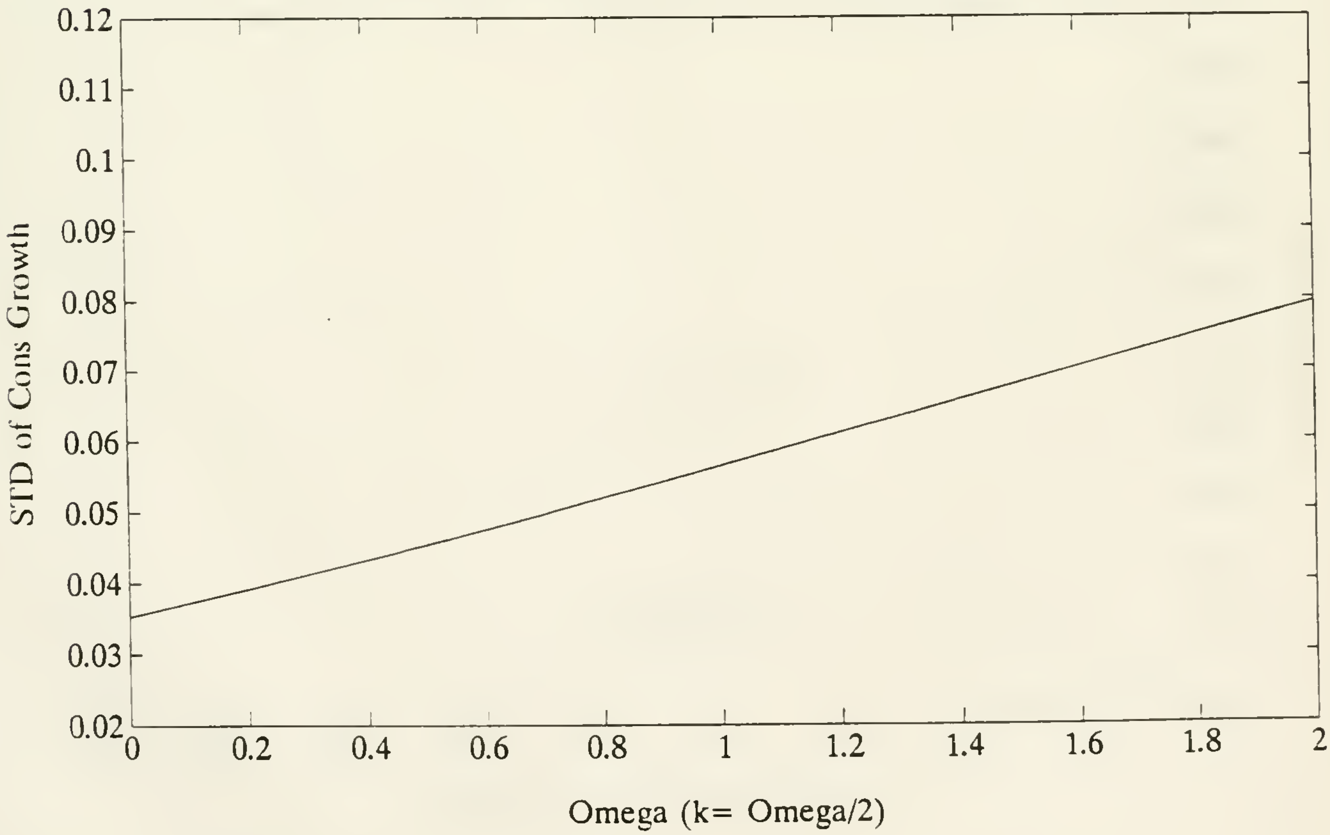


Figure 4.2a: C.D.C., Returns

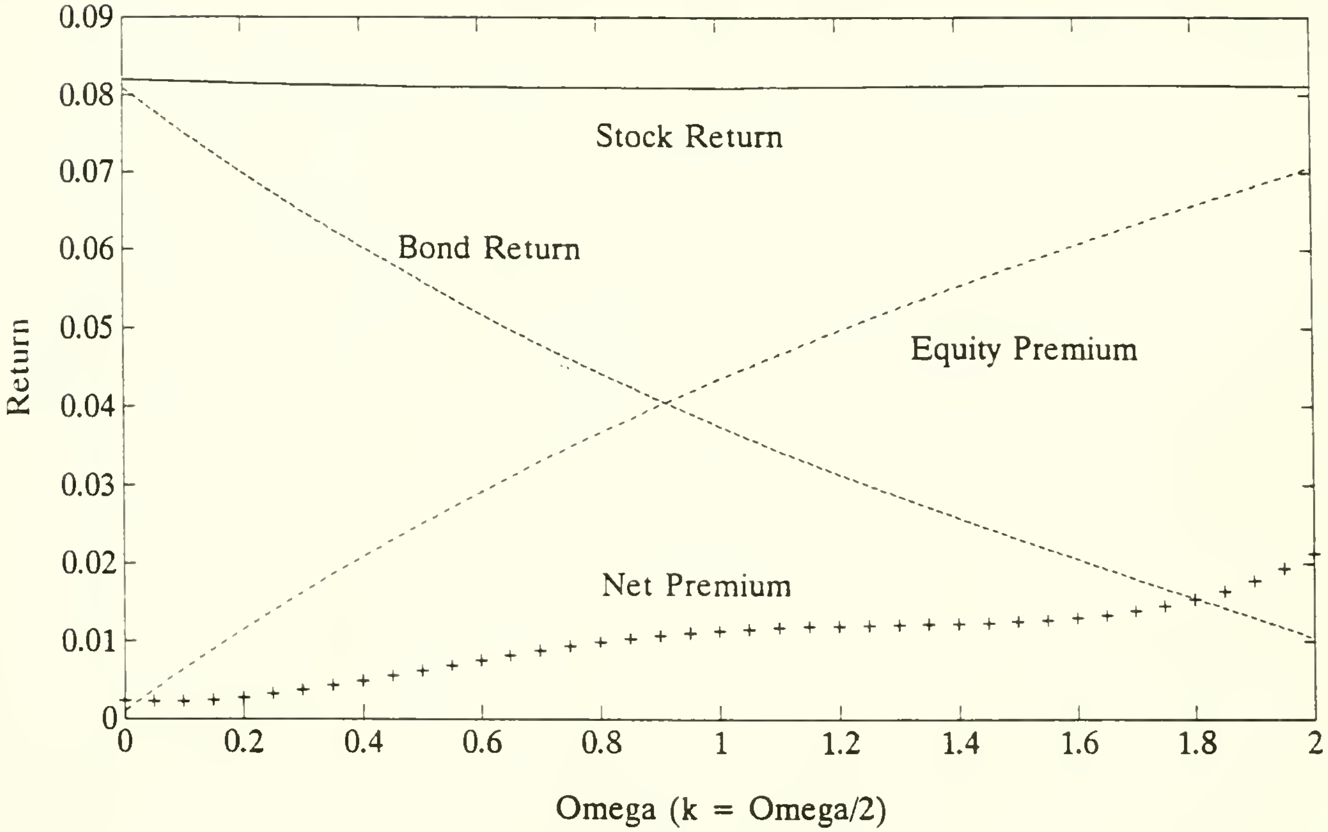


Figure 4.2b: C.D.C., Avg. Costs

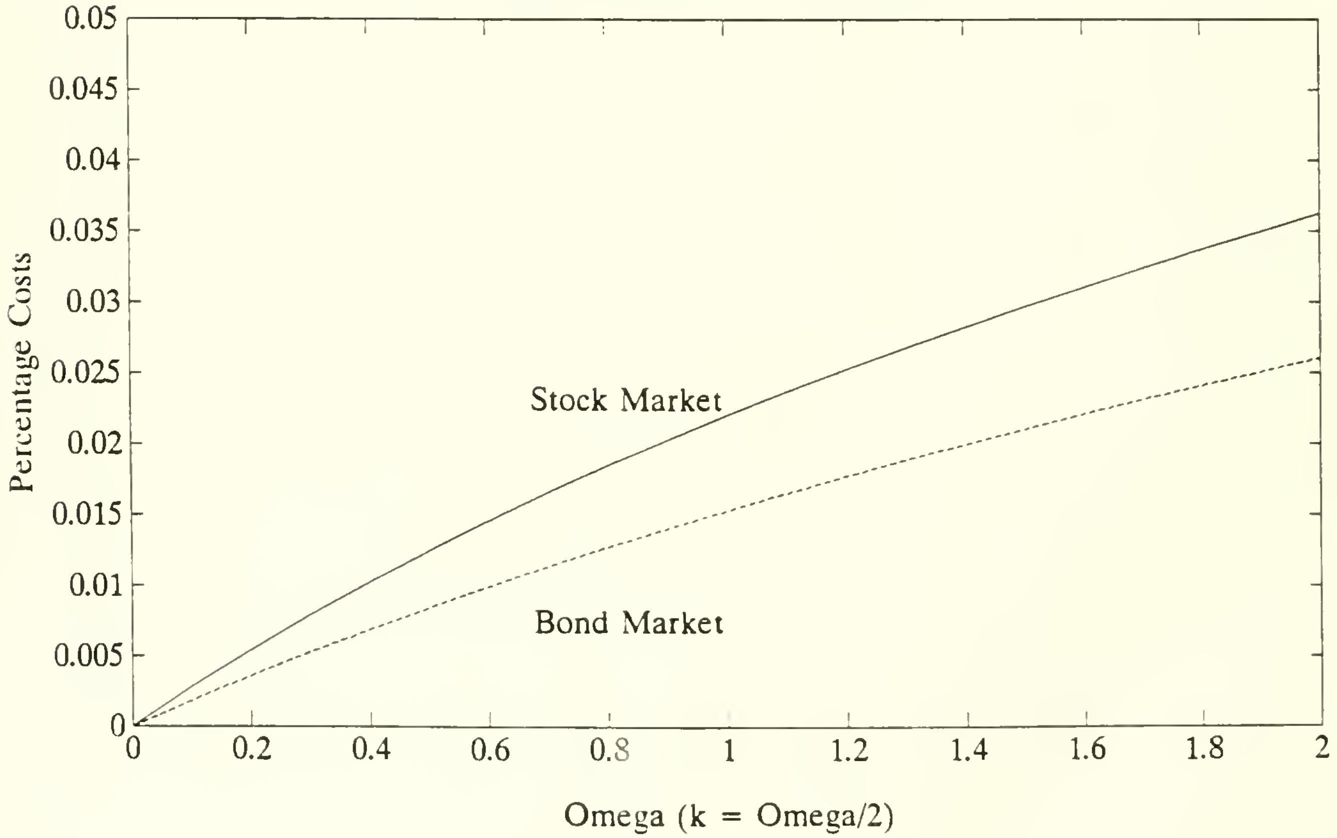
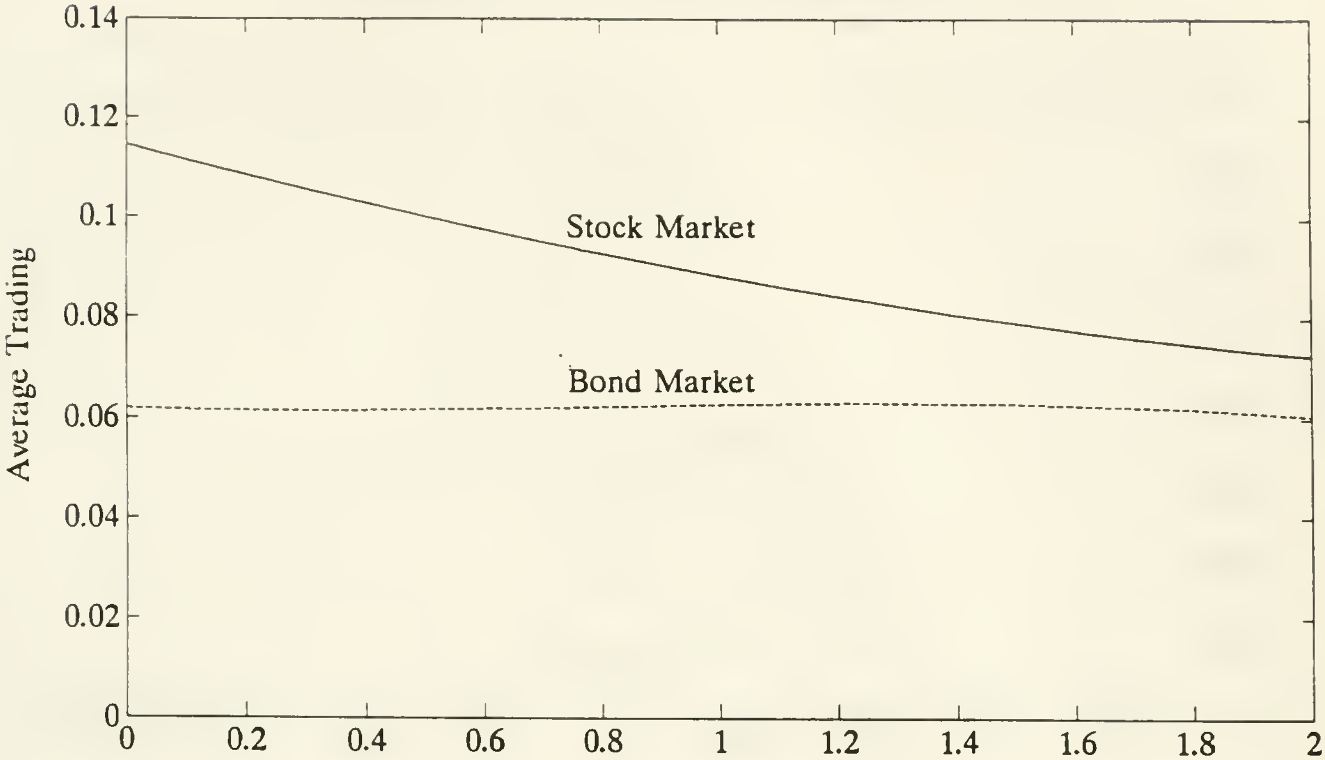
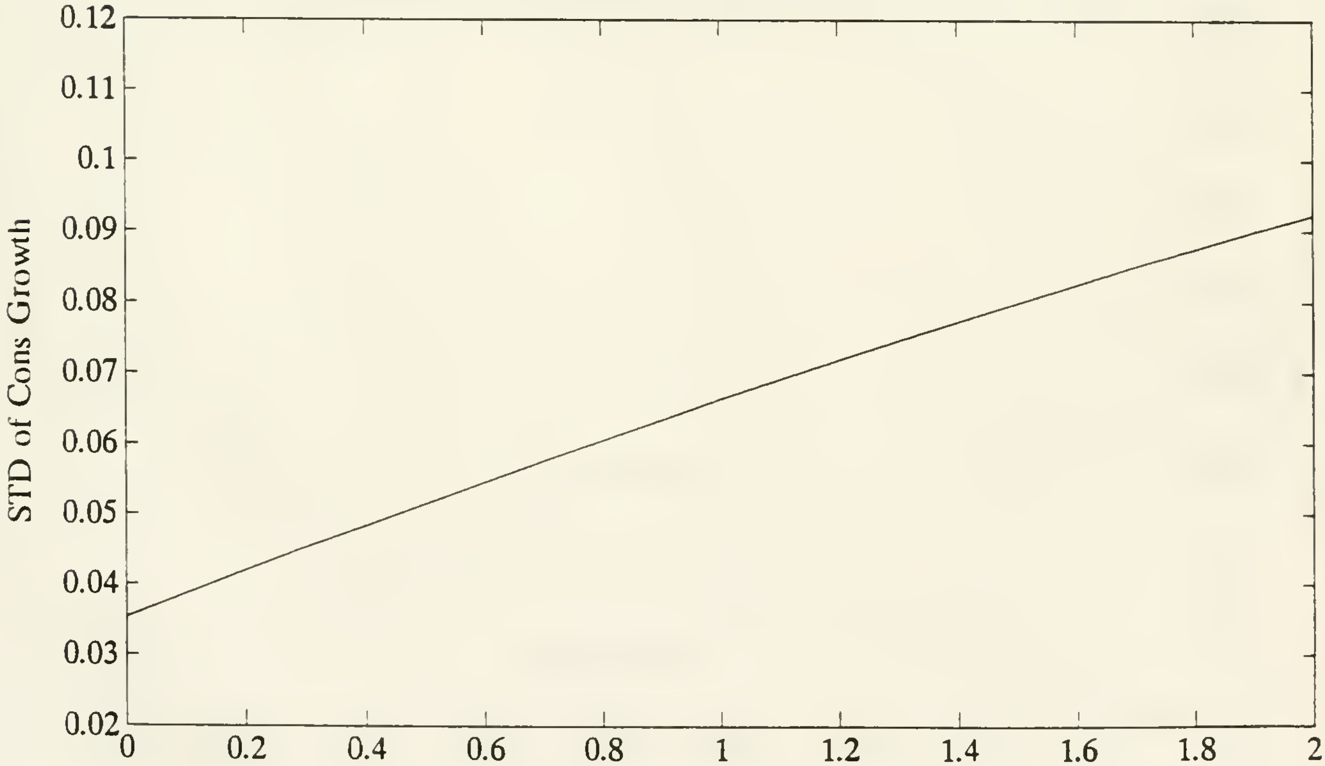


Figure 4.2c: C.D.C., Average Trading



Omega (k = Omega/2)

Figure 4.2d: C.D.C., STD of Consumption



Omega (k = Omega/2)

Figure 4.3a: B.C. Symmetric Costs, Returns

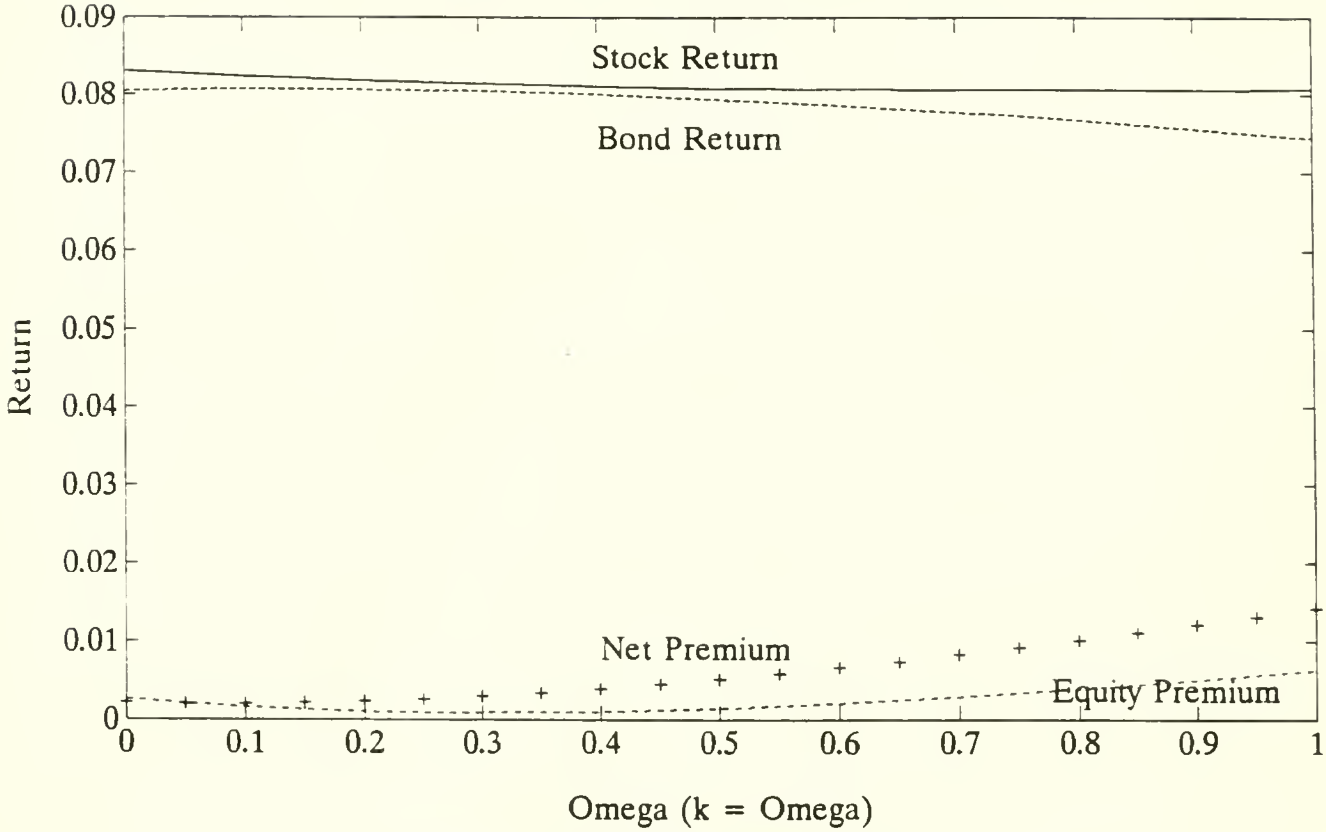


Figure 4.3b: B.C. Symmetric Costs, Costs

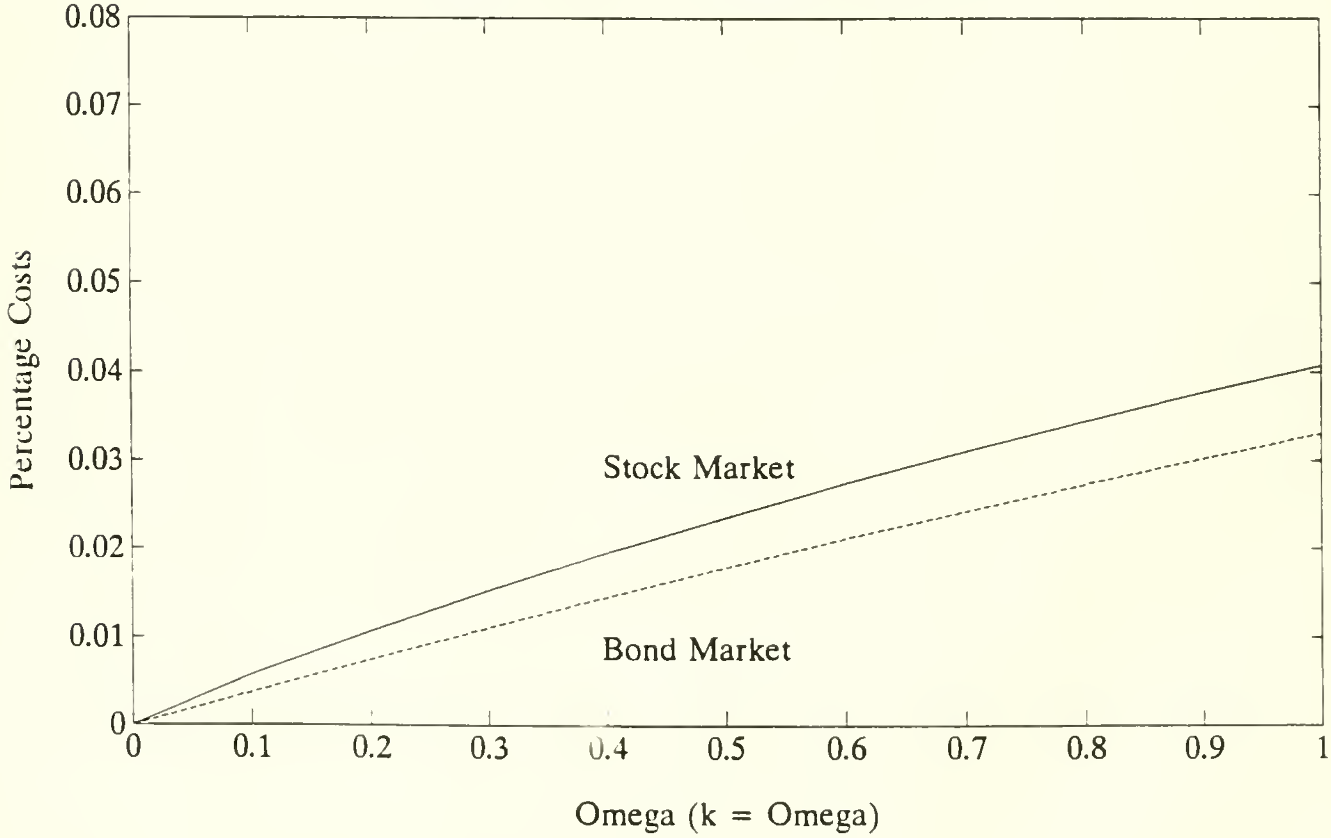


Figure 4.3c: B.C. Symmetric Costs, Average Trading

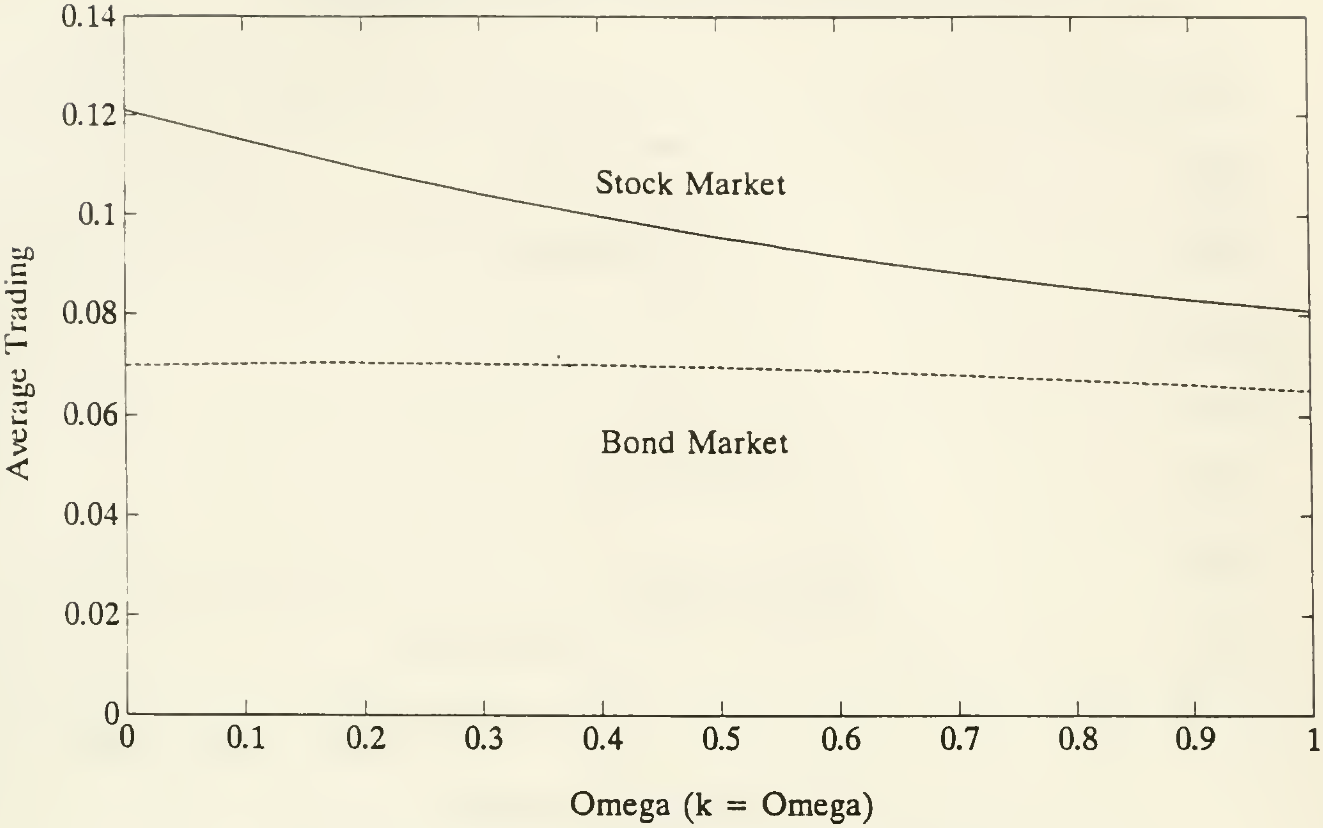


Figure 4.3d: B.C. Symmetric Costs, STD of Consumption

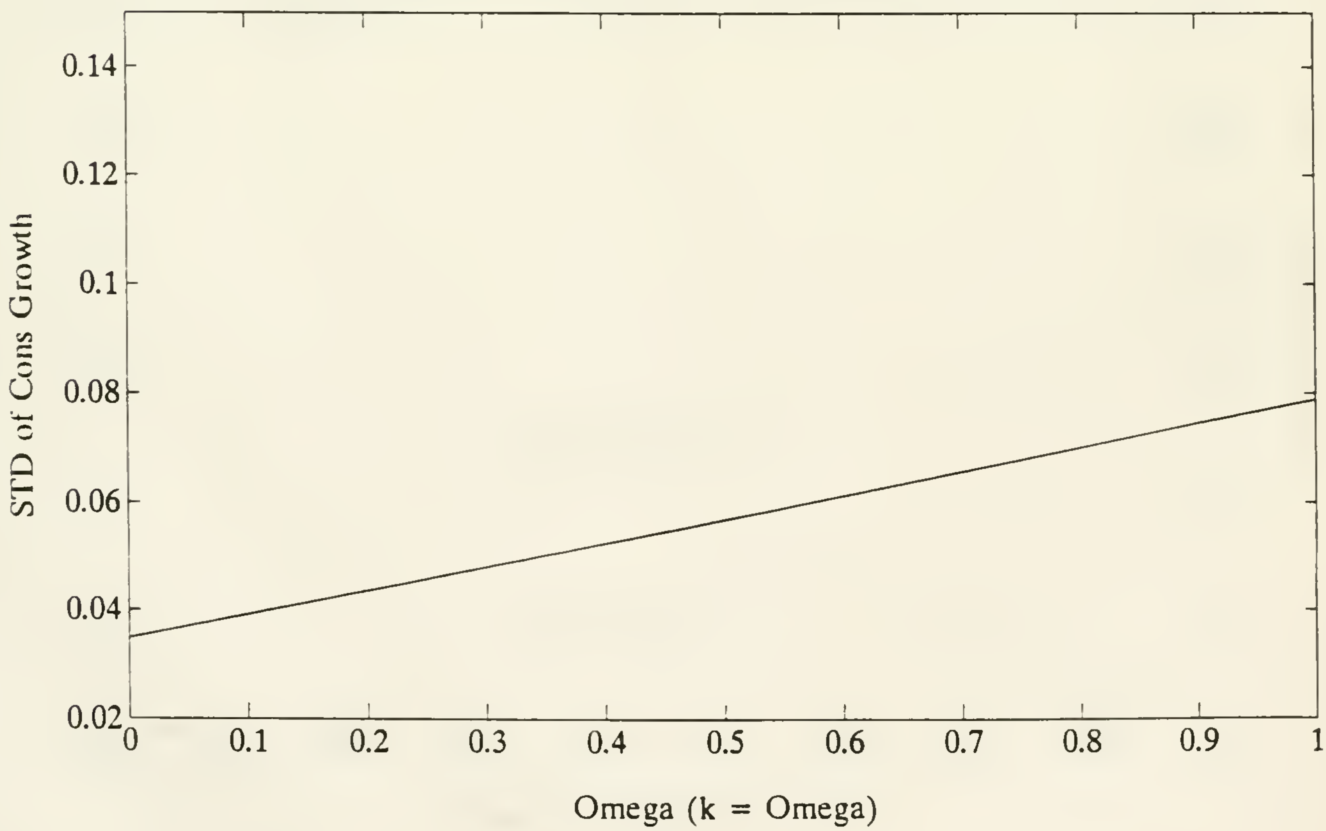


Figure 4.4a: C.D.C. Symmetric Costs, Returns

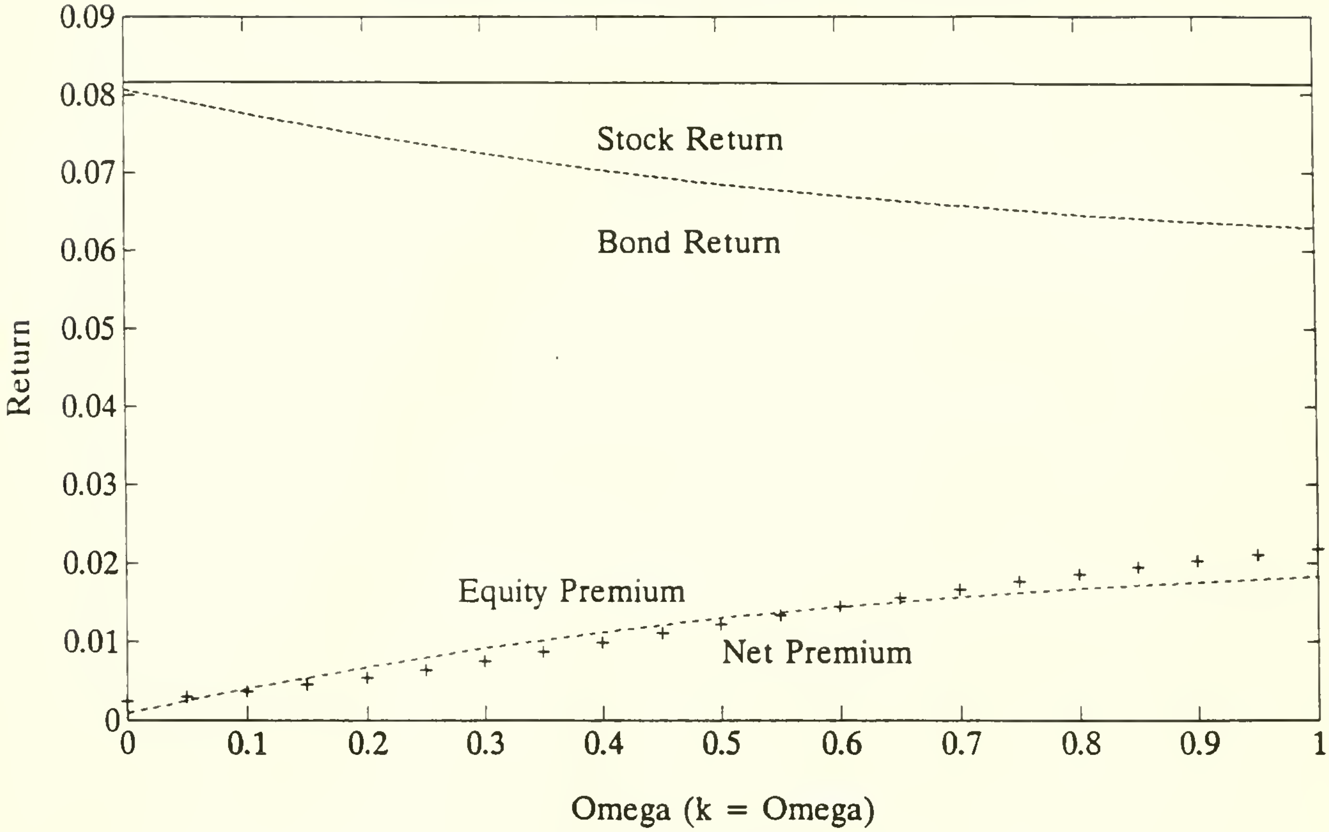


Figure 4.4b: C.D.C. Symmetric Costs, Costs

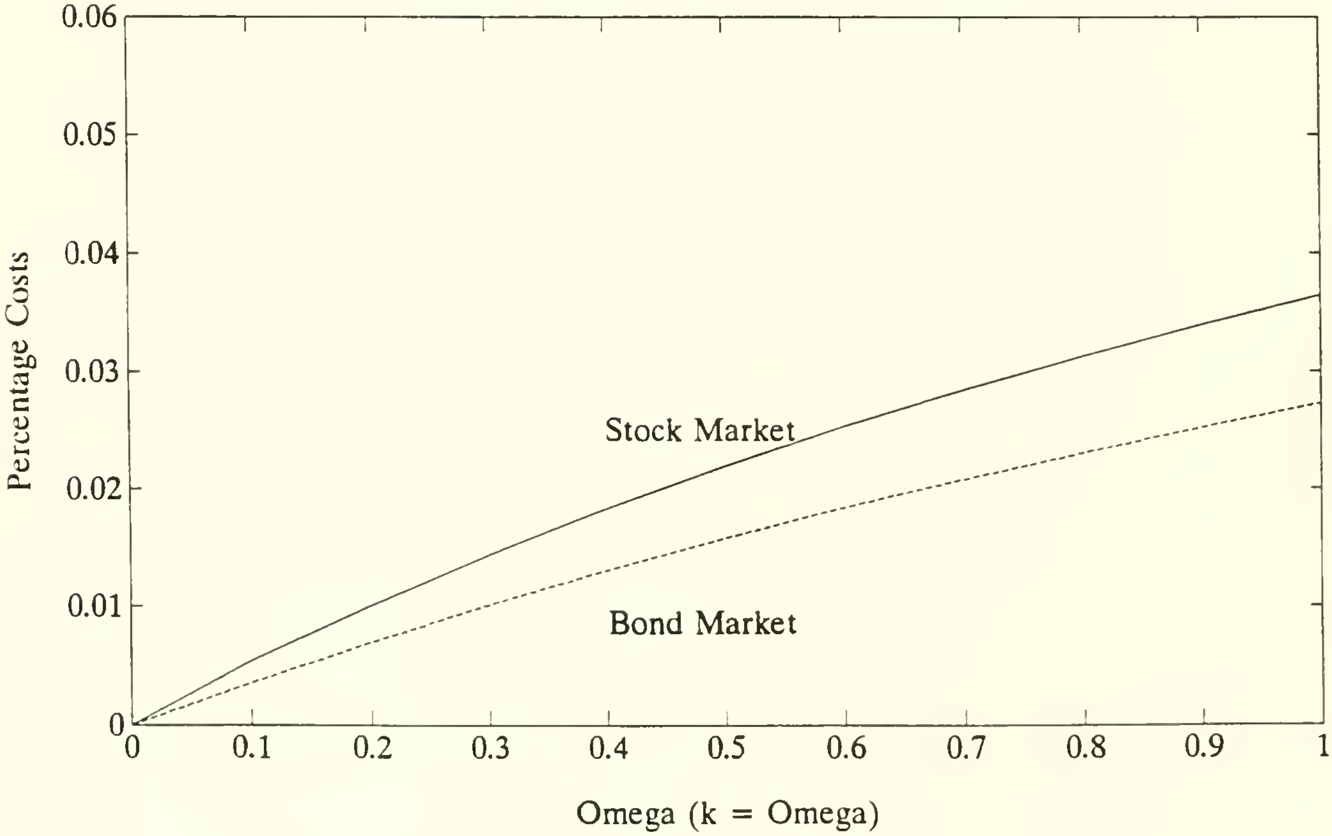


Figure 4.4c: C.D.C. Symmetric Costs, Average Trading

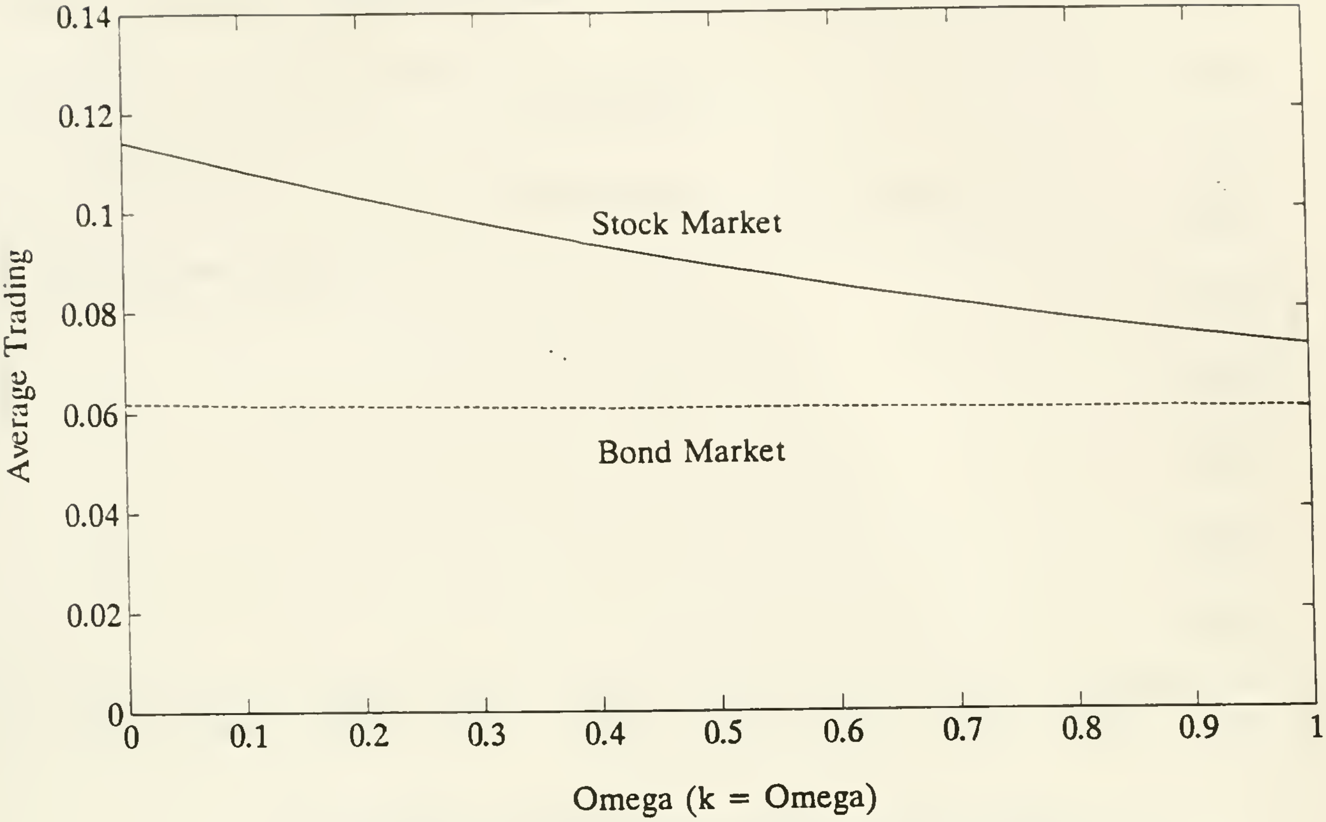


Figure 4.4d: C.D.C. Symmetric Costs, STD of Consumption

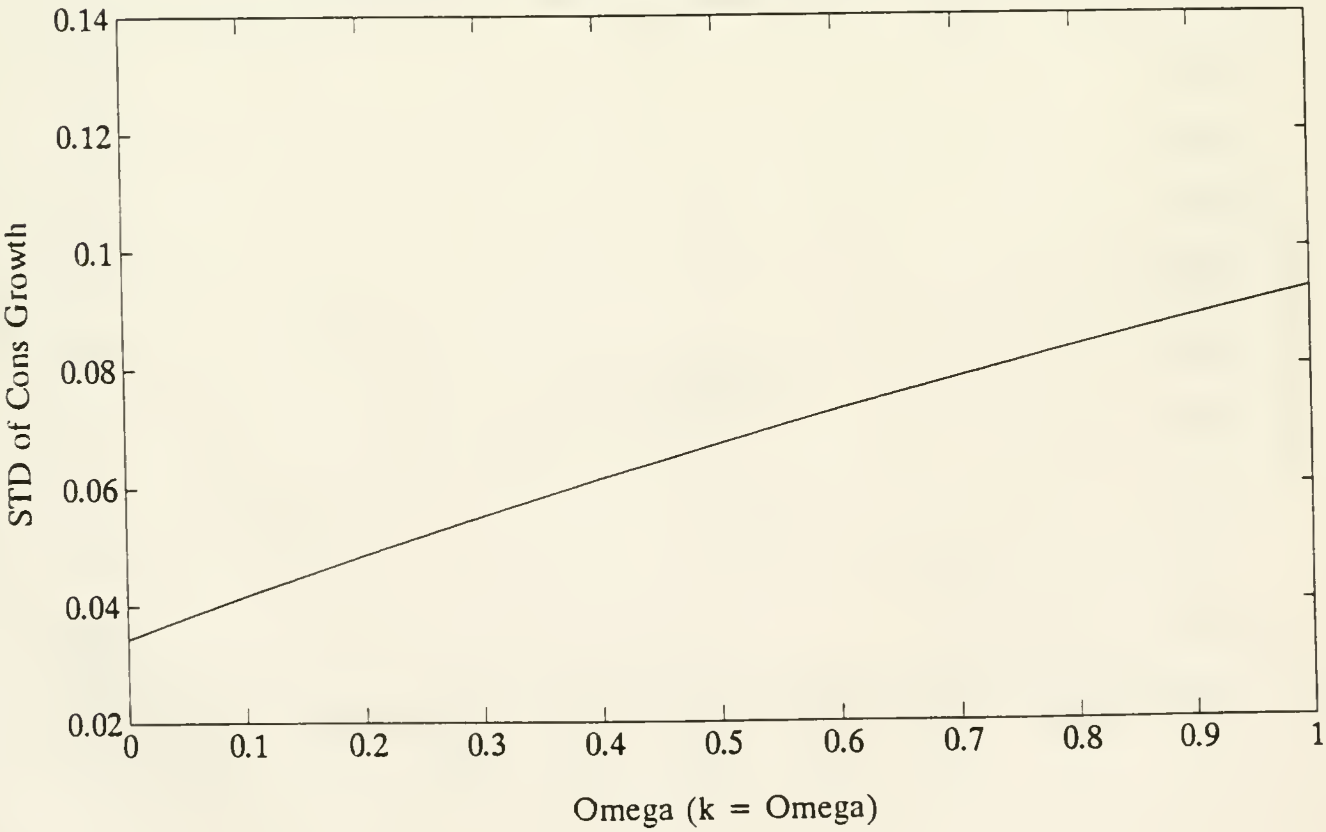


Figure 4.5a: B.C. No Bonds, Returns

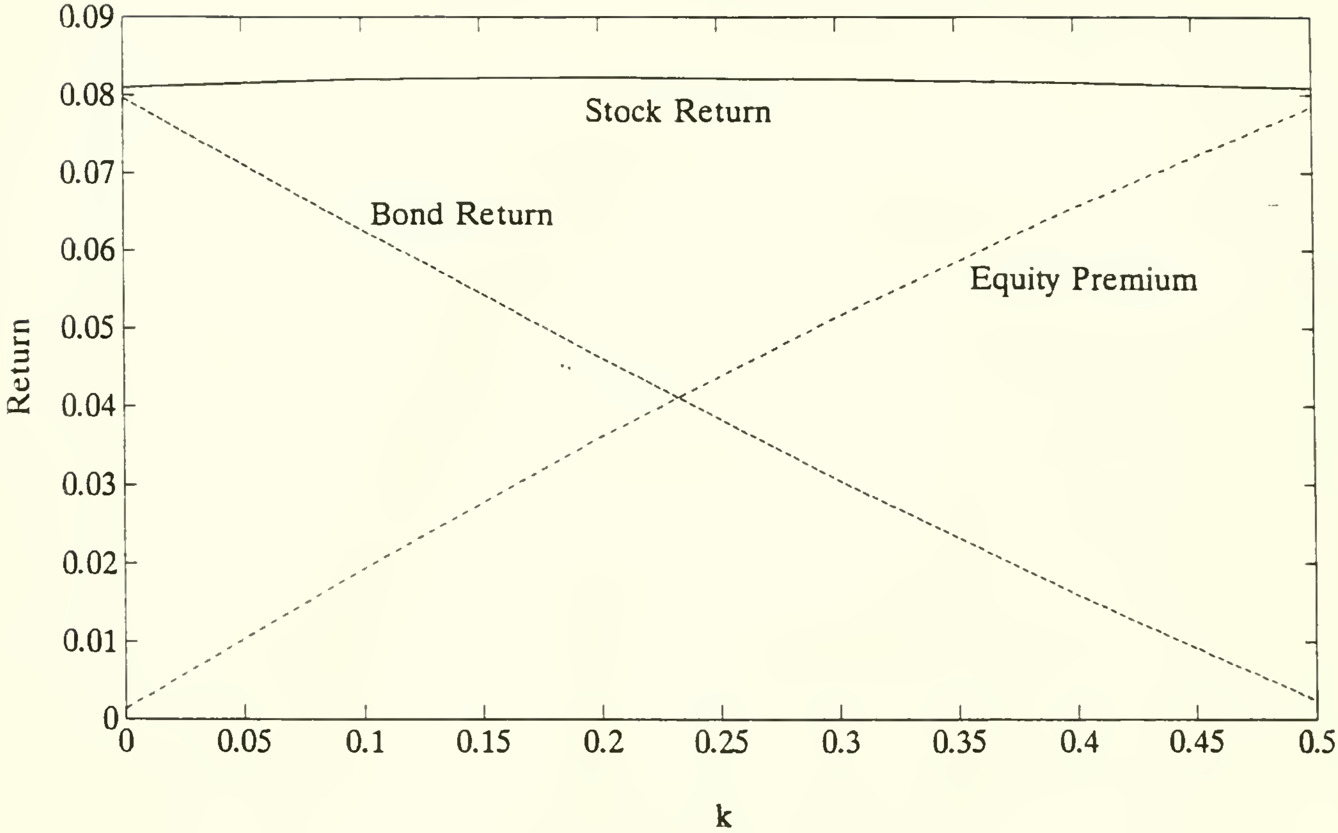


Figure 4.5b: B.C. No Bonds, Avg. Costs

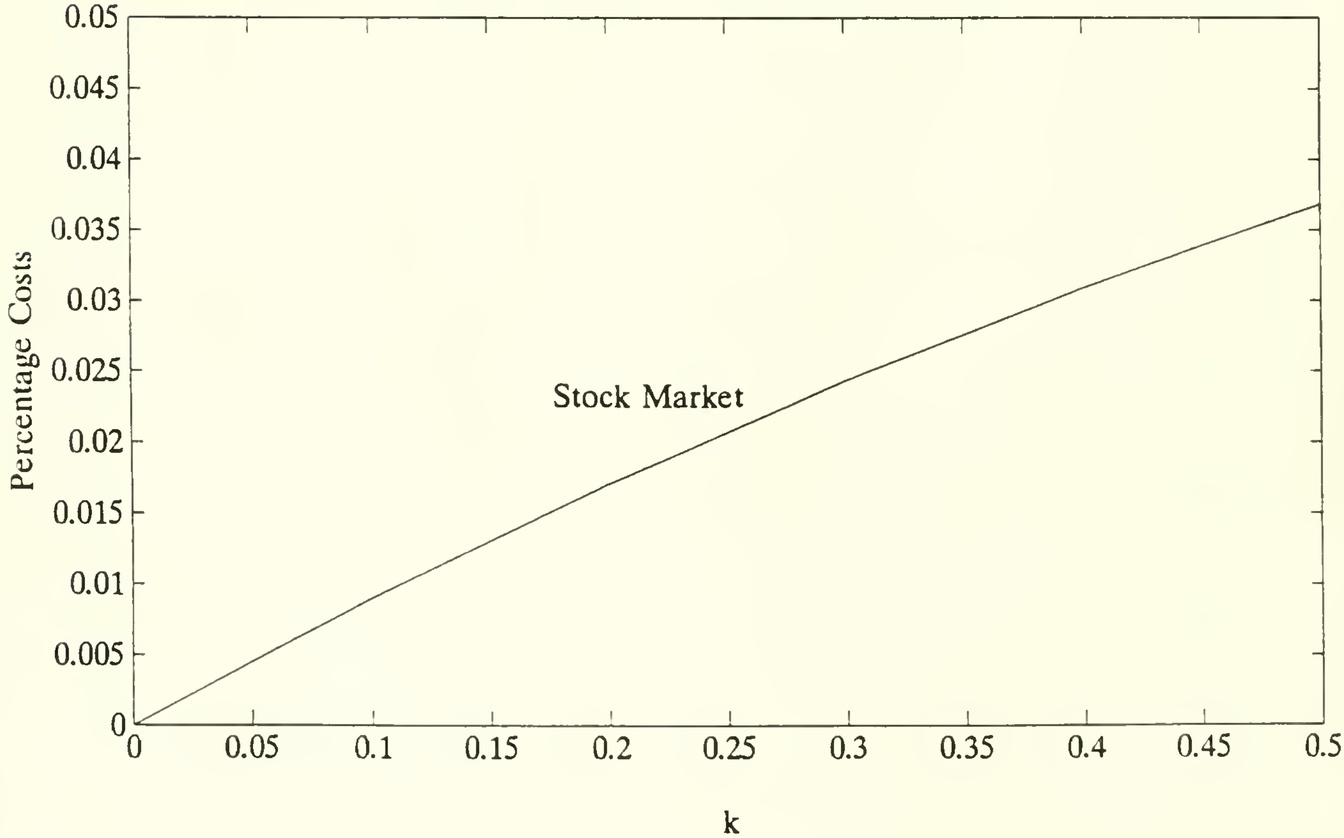


Figure 4.5c: B.C. No Bonds, Average Trading

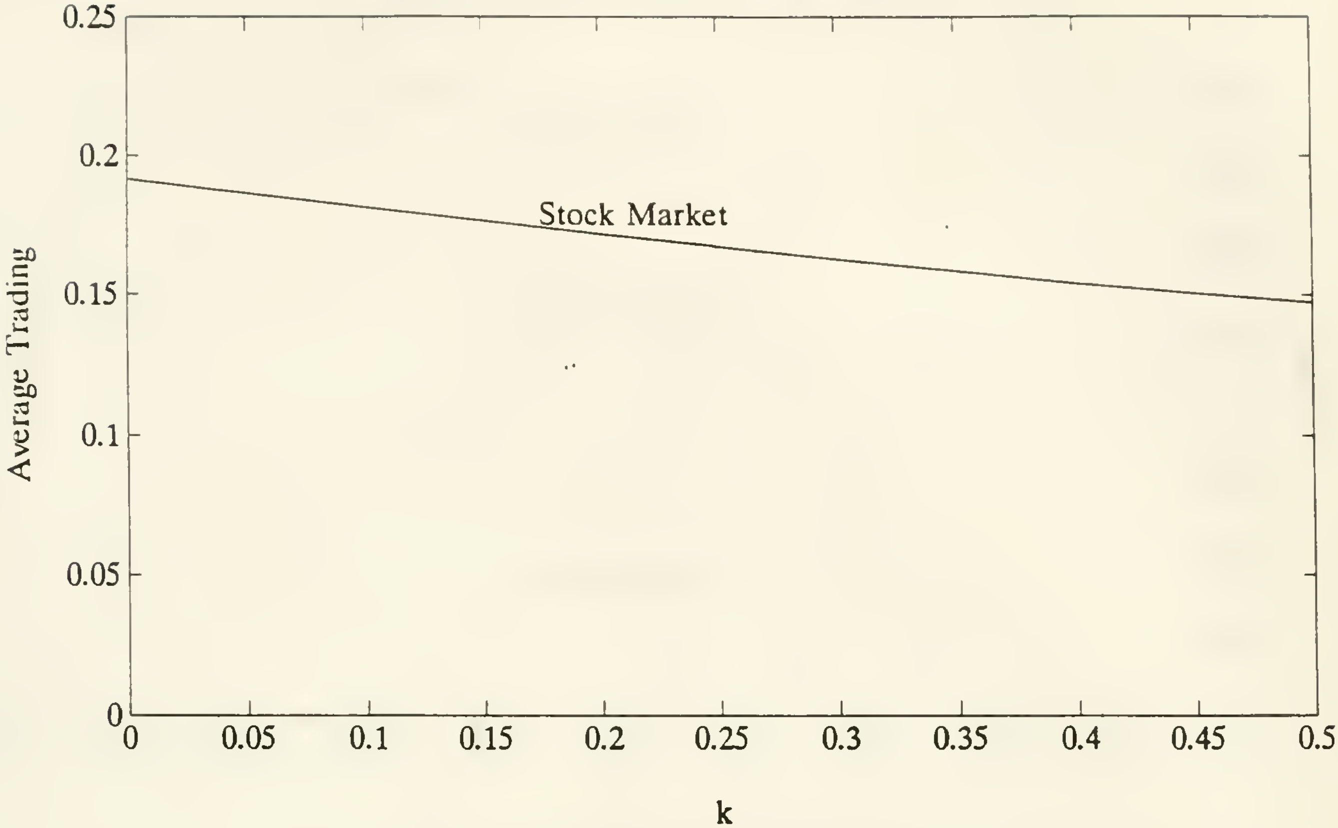


Figure 4.5d: B.C. No Bonds, STD of Consumption

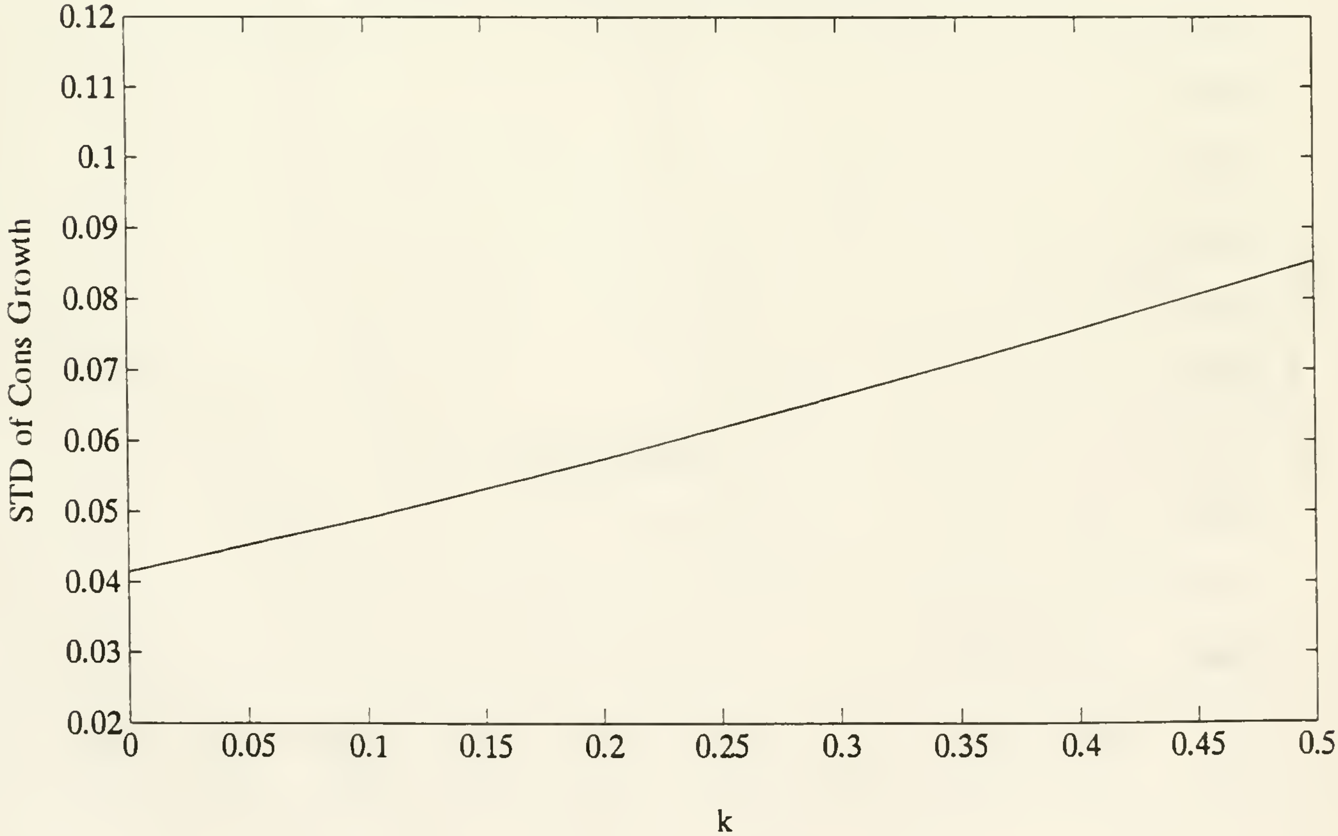


Figure 4.6a: C.D.C. No Bonds, Returns

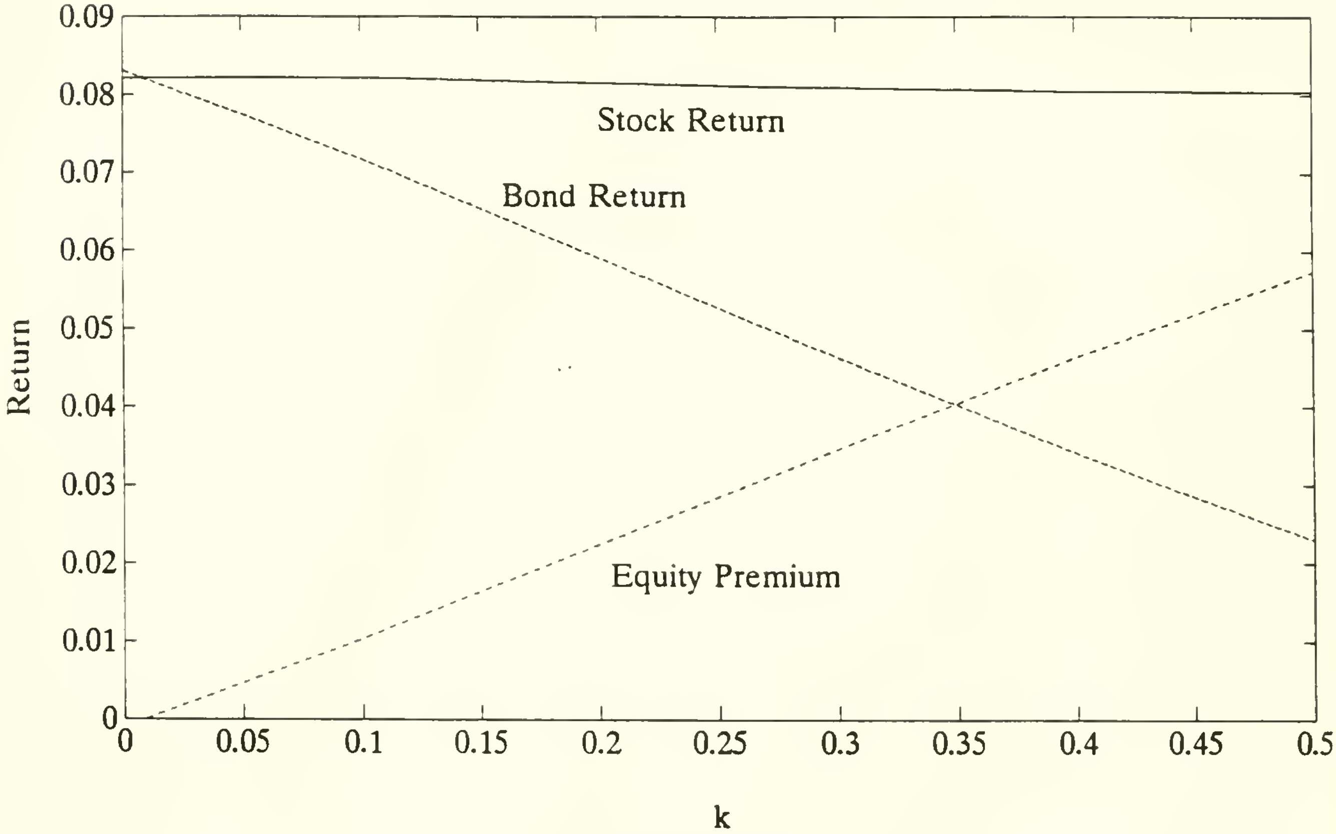


Figure 4.6b: C.D.C. No Bonds, Avg. Costs

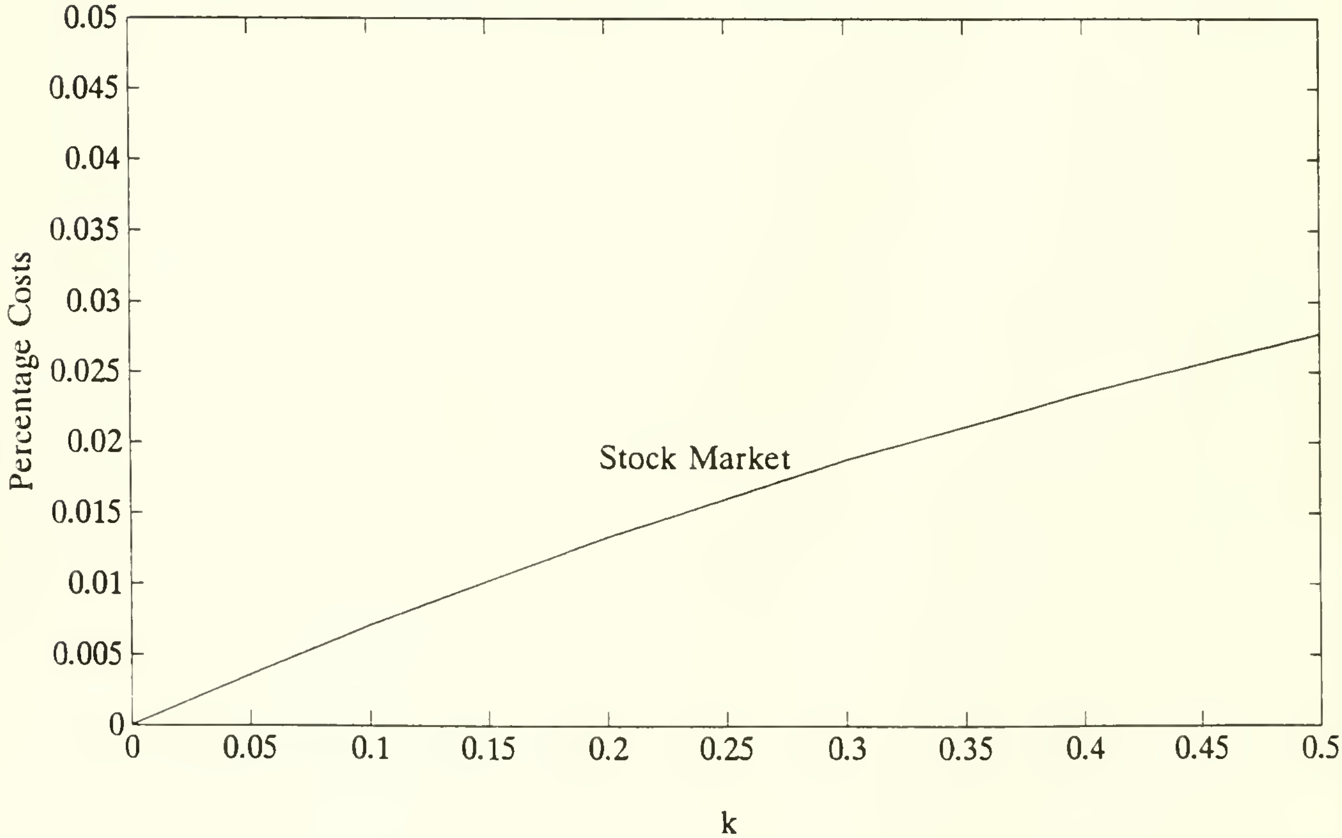


Figure 4.6c: B.C. No Bonds, Average Trading

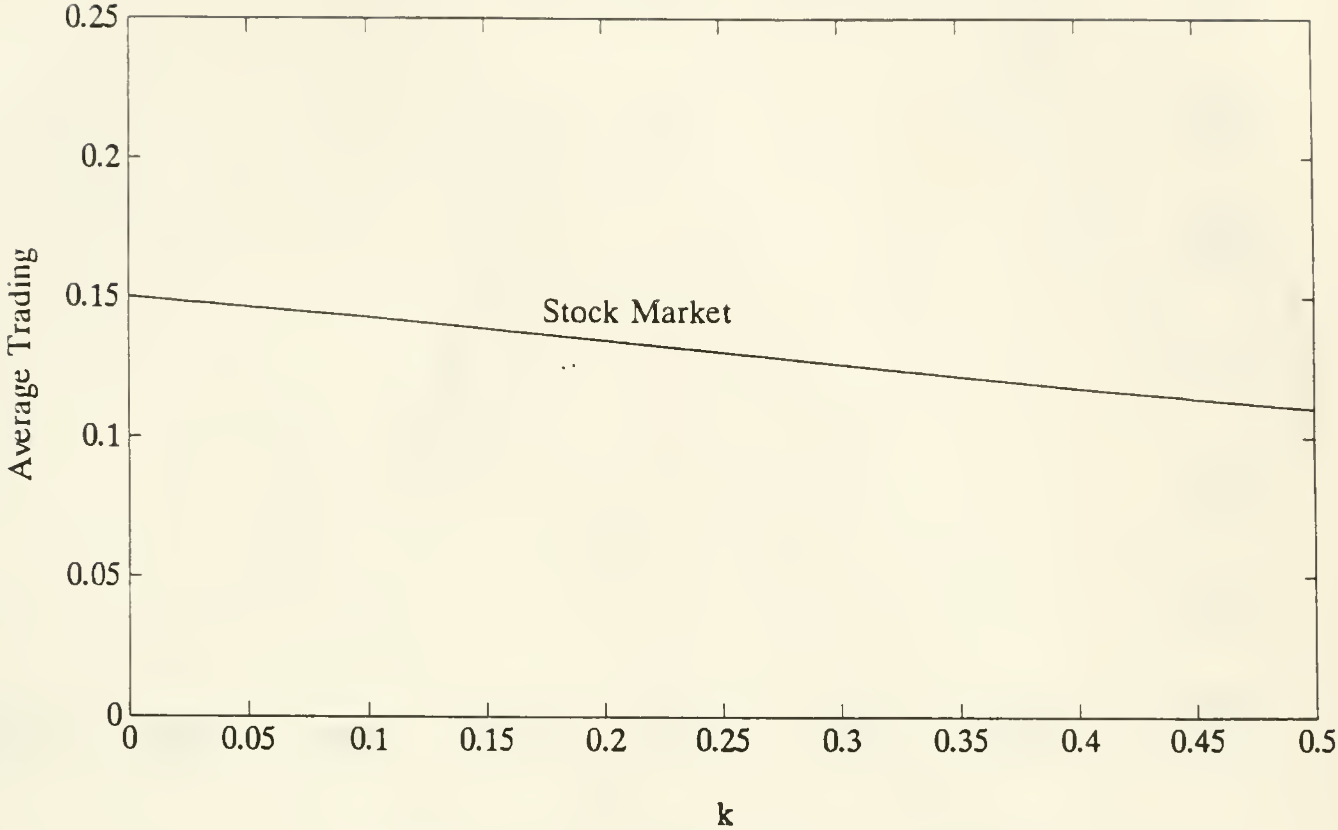


Figure 4.6d: B.C. No Bonds, STD of Consumption

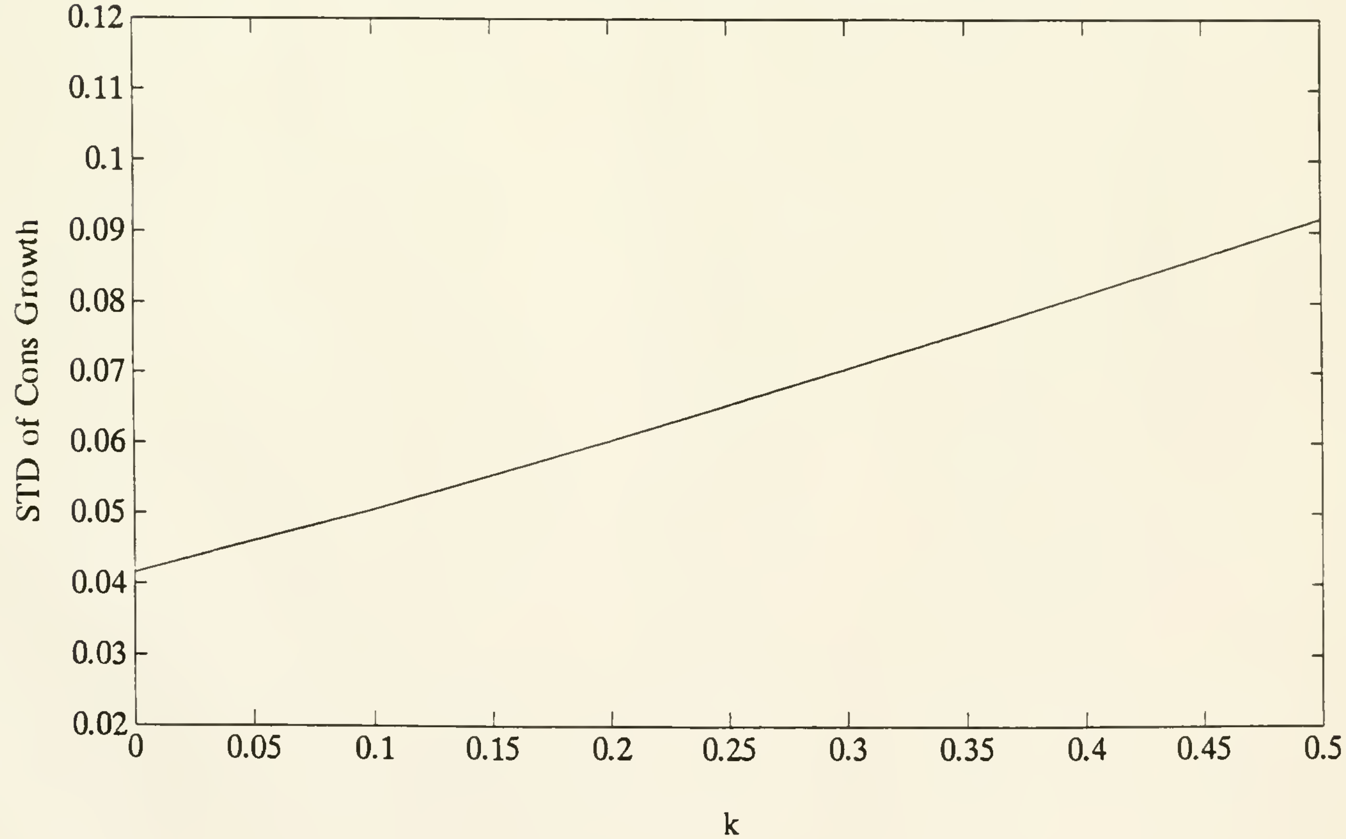
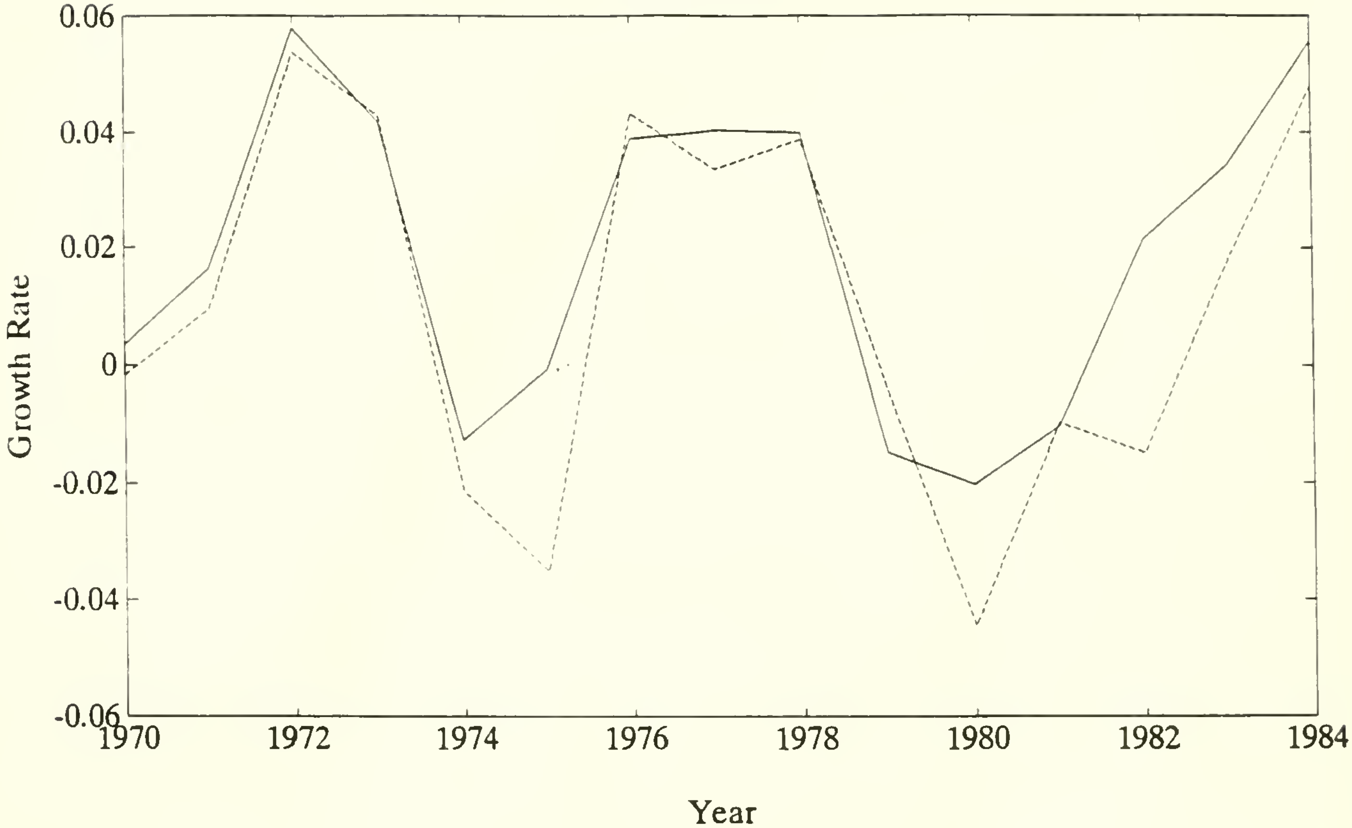


Figure A.1



0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Growth Rate
1970
1972
1974
1976
1978
1980
1982
1984
Year