ESTIMATING MODELS OF PROMOTION-INDUCED
NON-COMPENSATORY CHOICE BEHAVIOR USING
UPC SCANNER PANEL DATA*

by
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Leigh McAlister**
Sloan School Working Paper #1625-85
March 1985
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** Ph.D. Student

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ABSTRACT

Promotional offers may induce non-compensatory decision processes among consumers in supermarkets. A phased decision strategy in which consumers screen choice alternatives on the basis of promotional offers and then make their choice from the remaining alternatives is modeled. Such models are estimated for a collection of consumers using UPC scanner data on their coffee purchases. The resulting models predict choices in a holdout period better than two reference models. Individual level compensatory decision models are estimated and their parameters are compared to those of the non-compensatory models. The non-compensatory models are shown to fit better and to have parameters which are more readily interpreted and more robust than those of the compensatory models for some consumers.
INTRODUCTION

A consumer is pushing her shopping cart down the first aisle of her favorite store. She has not yet made a decision whether to buy coffee during this shopping trip, and if so, which brand to buy. However, as she nears the end of the aisle, she notices a large display with cans of a particular brand of coffee stacked seven feet high. Although this is not one of her favorite brands, the consumer picks one of the cans, puts it in her cart, and moves down the second aisle. When she reaches the usual coffee area, she walks right by; the display near aisle one made it unnecessary to consider other brands.

In today's supermarkets, where different promotional devices shout at consumers from all directions, this type of behavior is quite plausible. Consumer promotions, often offering substantial price cuts, allow shoppers to make quick, fairly good decisions without processing all available information. Brand choice models rarely allow promotions to play this type of role. This study centers around the notion of choice restriction. We hypothesize that under certain conditions, some individuals will restrict their choice to a subset of available brands. The principle mechanism behind choice restriction is assumed to be the presence or absence of promotional offers.

Choice restriction stems from a phased decision strategy involving a non-compensatory decision rule. Few promotion response models have incorporated non-compensatory rules; most rely on linear functional forms that are more closely associated with compensatory processes.
Since the extent and magnitude of choice restriction can vary over a set of individuals, our model is defined and estimated at the individual level. We include promoted and unpromoted purchases in our analysis. Furthermore, at each purchase occasion, we take into account the promotional status of all relevant brands, not just the chosen brand. If an individual chooses an unpromoted brand when one or more other relevant brands are on promotion, we gain valuable insights about that individual's propensity to restrict choice to promoted brands.

We develop a simple brand choice model that incorporates choice restriction. A maximum likelihood procedure is used to estimate brand preferences and promotional sensitivities for 100 consumers. The estimated parameters are then applied to a holdout period of 20 weeks to predict brand choice decisions, and we compare the predicted probabilities to those of two reference models. Finally, we compare the parameters estimated by compensatory and non-compensatory models at the individual level. In many cases, phased decision making behavior is easily discernible, and the two models (compensatory and non-compensatory) represent these situations in different ways.

Background

Over the past twenty years, brand choice models have advanced dramatically. Yet such advances have allowed choice models to remain barely abreast of an ever-changing marketing environment. Today's consumers face a number of very complex choice situations, due in part to the rapid growth of consumer promotions. Two decades ago, promotion expenditure made up only 5
percent of a typical marketing budget and received very little academic attention. Since then, promotions have grown to 60 percent of the marketing budget (Bowman 1981).

Consumer promotions have become an area of great interest for marketing researchers. Recent studies have shown that promotions often have a greater impact on short-term sales than do simple price reductions (e.g. Cotton and Babb 1978; Wilkinson, Mason and Paksoy 1979; Guadagni and Little 1983). Yet as pointed out by Jones and Zufryden (1981), many brand choice models do not account for consumer response to promotions. When promotional effects are modeled, they are typically incorporated as a part of a compensatory choice model (e.g., Guadagni and Little 1983).

Compensatory models rest on the assumption that an individual compares and makes tradeoffs among relevant attributes when making choice decisions. Low scores for some attributes can be compensated by high scores for others. A number of recent studies have suggested that decision makers often apply non-compensatory heuristics when faced with too much information or a complex task environment (Wright 1975; Payne 1976). By resorting to non-compensatory rules, a decision maker need not examine all attributes for all relevant alternatives; therefore, the decision process can be greatly simplified.

In many circumstances, a phased decision rule might be used. Consumers might screen alternatives on one criterion and then evaluate the remaining alternatives with some other criteria. Tversky (1972) proposed such a strategy in his elimination by aspects model. We propose a similarly structured decision strategy. A consumer might first screen brands for
promotions. Having eliminated unpromoted brands, she might choose among the remaining brands.

While non-compensatory behavior is acknowledged to be used in a variety of choice situations, compensatory models have been shown to approximate non-compensatory behavior quite well. Einhorn (1970) used log-linear models that closely resemble non-compensatory response functions. More generally, Dawes and Corrigan (1974) theorized that in judgmental situations linear models are robust and can accurately approximate non-compensatory processes. In a more recent study, Johnson and Meyer (1984) found that a multinomial logit model showed strong predictive validity even when non-compensatory strategies were being used by subjects.

In this study, we develop a non-compensatory functional form to model the promotion-induced choice restriction phenomenon. There are no significant advantages to be gained by using compensatory approximations if behavior is truly non-compensatory. Powerful computer software makes parameter estimation a straightforward task. Furthermore, we have interest in the parameters estimated by the model. Compensatory models might fit the data as well, but valuable insights that the non-compensatory parameters reveal could be lost.

In this section we develop functional forms to represent compensatory and non-compensatory decision strategies that consumers might use in making choices from a product class in which there are frequent promotions. The compensatory model (multinomial logit) assumes that consumers trade off preference for the brands against preference for promotions in making their choices. The non-compensatory model assumes that consumers have a phased
 decision strategy. They decide first whether to react to promotions and then decide which brand to choose. We describe a technique for estimating the parameters of these models and present the results of that estimation.

Let:

\[ B_j = \text{Brand } j; j=1,2,...,J \]

\[ a_i = \text{Set of "relevant brands" for consumer } i, \text{ those brands that consumer } i \text{ ever chooses} \]

\[ \pi_{ij} = \text{Consumer } i's \text{ preference for } B_j, 0 \leq \pi_{ij}, \sum_{j \in a_i} \pi_{ij} = 1 \]

\[ \Theta_i = \text{Consumer } i's \text{ promotion sensitivity as estimated by the compensatory model, } \Theta_i \]

\[ \gamma_i = \text{Consumer } i's \text{ promotion sensitivity as estimated by the non-compensatory model, } \gamma_i \leq 1 \]

\[ P_{jt} = \begin{cases} 1 & \text{if } B_j \text{ is on promotion at time } t \\ 0 & \text{if } B_j \text{ is not on promotion at time } t \\ 1 & \text{if some brand acceptable to consumer } i \text{ is on promotion at time } t \end{cases} \]

\[ \delta_{it} = \begin{cases} 0 & \text{if no brand acceptable to consumer } i \text{ is on promotion at time } t \end{cases} \]

\[ C_{ijt} = \text{Probability that the compensatory model assigns to } B_j \text{ being chosen by consumer } i \text{ at time } t \]

\[ N_{ijt} = \text{Probability that the non-compensatory model assigns to } B_j \text{ being chosen by consumer } i \text{ at time } t \]

To develop the compensatory model, consider multinomial logit. This model suggests that the probability that consumer } i \text{ will choose } B_j \text{ at time ...
This "us over us plus them" formulation is compensatory in the sense that "consumer i's utility for B_j at time t" is generally expressed as a linear compensatory function of B_j's attributes at time t. In a simple compensatory model "i's utility for B_j at time t" is equal to \( \beta_i P_{jt} + \sum_{k \in A_i} \beta_i D_{ik} \)

where \( P_{jt} \) indicates whether B_j is on promotion at time t and D_k's are dummy variables indicating the brand under consideration (e.g., when estimating utility for B_j, \( D_j = 1 \) and \( D_k = 0 \) for all \( k \neq j \)). Hence i's utility for B_j at time t = \( \beta_i P_{jt} + \beta_i j \) and

\[
C_{ijt} = \frac{\beta_i P_{jt} + \beta_i j}{\sum_{k \in A_i} \beta_i P_{kt} + \beta_i k}.
\]

Letting \( \Theta_i = e^{\beta_i 0} \) and \( \pi_{ij} = e^{\beta_i j} \), we have:

\[
C_{ijt} = \frac{\Theta_i P_{jt} \pi_{ij}}{\sum_{k \in A_i} \Theta_i P_{kt} \pi_{ik}}. \quad (1)
\]

A simple example helps illustrate the model. Suppose consumer i has three relevant brands, \( (A_i = \{B_1, B_2, B_3\}) \) with preferences \( \pi_{i1}, \pi_{i2} \) and \( \pi_{i3} \) respectively. If, at time t, only B_1 and B_2 are on promotion, then:

\[
C_{ilt} = \frac{\Theta_i \pi_{i1}}{\Theta_i \pi_{i1} + \Theta_i \pi_{i2} + \pi_{i3}}
\]
To develop the non-compensatory model we assign probability $\gamma_i$ to the event that consumer $i$ will invoke an elimination strategy and restrict her choice to promoted brands, given that some brand acceptable to consumer $i$ is on promotion. With probability $1 - \gamma_i$ consumer $i$ will not invoke the disjunctive rule and will, instead, choose from all brands in $a_i$. We assume that a consumer chooses among brands with probabilities proportional to her preferences for those brands (Luce 1959). Therefore:

$$N_{ijt} = \gamma_i \frac{\sum_{k \in a_i} \pi_{ik} \delta_{kt}}{\sum_{k \in a_i} \pi_{ik}} + (1 - \gamma_i) \frac{\sum_{k \in a_i} \pi_{ik}}{\sum_{k \in a_i} \pi_{ik}} \quad (2)$$

If no relevant brand for consumer $i$ is on promotion at time $t$, $N_{ijt}$ collapses into the simpler expression

$$N_{ijt} = \gamma_i \frac{\pi_{ij}}{\sum_{k \in a_i} \pi_{ik}}$$

The first term of (2) disappears because $P_{jt} = 0$, and the presence of $\delta_{it}$ insures that the $1 - \gamma_i$ multiplier vanishes.

Using the simple example just described ($a_i = \{B_1, B_2, B_3\}$, $B_1$ and $B_2$ on promotion), we have:

$$N_{ilt} = \gamma_i \frac{\pi_{il}}{\pi_{il} + \pi_{i2}} + (1 - \gamma_i) \frac{\pi_{il}}{\pi_{il} + \pi_{i2} + \pi_{i3}}$$
Comparing the two model structures we see that they imply very different consumer choice strategies. In the compensatory model, preference for promoted brands is boosted by a factor of $\Theta_i$ and consumers choose from acceptable brands with probabilities proportional to altered preferences. In the non-compensatory model promotion is used as a signal for screening alternatives. The two models differ with regard to how often a consumer will respond to promotions. The non-compensatory model suggests that $1 - \gamma_i$ of the time consumer $i$ will deviate from her choice restriction strategy. The compensatory model allows for no deviations from its prescribed decision strategy; it assumes that preference is always boosted by the presence of promotions. In many cases (as noted in Footnote 1), this assumption might not hold true.

The possible ranges of the two promotion sensitivity parameters are also quite different. $\gamma_i$ takes on values between 0 and 1 while $\Theta_i$ can take on any value from 0 to infinity. Values of $\Theta_i$ between 0 and 1 indicate "promotion aversion". Such a value would indicate that consumer $i$ would be less likely to choose brands that were promoted. The compensatory and non-compensatory models are indistinguishable from one another for consumers who are not promotion sensitive ($\Theta_i = 1$ and $\gamma_i = 0$) and for consumers who are absolutely promotion sensitive ($\Theta_i$ "very large" and $\gamma_i = 1$). It is difficult to precisely define what is meant by a "very
large" value of $\Theta_i$. Once $\Theta_i$ exceeds a certain threshold value (approximately one million), choice probabilities are usually unaffected by further increases in $\Theta_i$. A consumer with $\Theta_i = 10^{30}$ is usually no more promotion sensitive than a consumer with $\Theta_i = 10^{20}$. Comparing the compensatory model's promotion sensitivity parameters across consumers is therefore a difficult task. $\gamma_i$ (the probability that consumer $i$ restricts choice given that a promotion is present), on the other hand, is comparable across consumers and, further, has an interpretation directly relevant to managers.

It is interesting to note that the compensatory model incorporates an assumption of independence of irrelevant alternatives while the non-compensatory model does not. (See Appendix A for a mathematical explanation). Complex multinominal logit-like models have been designed to account for brands' similarities (see Currim 1982) and the models presented here could be adapted to do the same. We avoid that issue here in order to focus on the differences between the compensatory and non-compensatory structures.

**Estimation**

We estimate the models using Universal Product Code (UPC) scanner panel data for 200 consumers prepared by Guadagni and Little (1983). This data, collected at supermarket checkout counters, creates a detailed record of each purchase occasion. Scanner data also contain comprehensive information about price, availability and promotional activity for each brand for every week.
We estimate the individual level compensatory model using multinomial logit and estimate the individual level non-compensatory model using a more general maximum likelihood routine. A likelihood function is obtained for consumer i by creating an expression using equation (2) for the probability of consumer i selecting each actually chosen brand and multiplying these probabilities together. We use a FORTRAN program with a quasi-Newton algorithm to choose optimal values for $\gamma_i$ and $\pi_{ij}$.

The models are fit for the 100 Guadagni and Little calibration sample panelists with the first 57 weeks of data. The last 20 weeks of data are held out for predictive testing. As a benchmark we also estimate two reference models. The first of these models is estimated at the individual level. Each consumer is assumed to ignore promotions and to choose among brands with probabilities proportional to her choice share as revealed in the first 57 weeks of her purchase history. The second of these models is Guadagni and Little's (1983) aggregate model of consumers' response to promotion. Their model assumes a single level of promotion responsiveness for all consumers in the sample but allows idiosyncratic preferences among brands with brand- and size-loyalty predictor variables. This model was shown (Guadagni and Little 1983) to fit this data well and to have very good predictive ability.

The individual level compensatory and non-compensatory models fit the 32 weeks of calibration data better than the individual level choice shares or aggregate Guadagni and Little models. Log likelihoods for the four models are: individual level compensatory = -601, individual level non-compensatory = -595, individual level choice share = -1116, aggregate level Guadagni and Little = -972. (Log likelihoods closer to zero indicate a better fit.) These
results are not surprising since the individual level compensatory and non-compensatory models estimate at least 100 more parameters than the two benchmark models. More importantly, the individual level compensatory and non-compensatory models also make better predictions in the hold out period than do the two benchmark models. The average probability assigned to the brands actually chosen in the holdout period by the four models are:

individual level compensatory = .548, individual level non-compensatory = .545, individual level choice shares = .385, aggregate Guadagni and Little = .494.

The differences between the benchmark models and the two individual level models are significant at the .01 level. The difference between the individual level compensatory and individual level non-compensatory models is not significant at the .1 level.

Having thus examined the predictive ability of these models, we reestimated each consumer's compensatory and non-compensatory models using the entire purchase history. Because of possible parameter bias induced by maximum likelihood estimation with small samples (average number of purchases per consumer = 21) we ran a test for bias on the non-compensatory model's parameters.

A Monte Carlo procedure, similar to the one used by Chapman and Staelin (1982) takes as input a $\gamma$ and set of $\pi$'s. It then chooses a number of random promotional environments and calls the maximum likelihood procedure. The result is a different set of parameters, $\hat{\gamma}$ and a set of $\hat{\pi}$'s. Complete details are in Appendix B.

We are interested in the differences $\hat{\gamma}_i - \gamma_i$ and $\hat{\pi} - \pi$. Since the $\pi_{ij}$'s always
sum to one, it is meaningless to consider $\hat{\pi}_{ij} - \pi_{ij}$ for all $j$. Accordingly, we choose a random brand for each (randomly chosen) individual. Gamma bias (GBIAS) is measured by performing a two-sided $t$-test on the $\hat{\gamma}_i - \gamma_i$, while bias for the $\pi_{ij}$'s (PBIAS) is measured analogously. The sample size consists of 100 randomly-chosen individuals. The table below shows the mean values of the biases.

<table>
<thead>
<tr>
<th>Number of Simulated Purchases</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBIAS</td>
<td>-0.108*</td>
<td>-0.047</td>
<td>-0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>PBIAS</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-0.005</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

*Significant at $p<0.01$. All others insignificant at $p=0.1$.

The $\pi_{ij}$'s appear to be completely unaffected by bias, but when sample sizes become very small, the gammas are biased downward. This bias results from overfitting. When five or fewer observations are used to estimate about three parameters, at least one parameter becomes useless. The $\pi_{ij}$'s alone explain most of the variance present in such small samples. When a consumer makes so few purchases, it is difficult to make strong conclusions about her promotional sensitivity. Fortunately, most members of the panel make enough purchases to avoid these small sample problems.

**COMPARISON OF NON-COMPENSATORY AND COMPENSATORY MODEL PARAMETERS**

From the last section we see that these individual level models improve predictive ability over reference models and that they produce unbiased parameters. However, neither of the two individual level models seems to dominate the other in the aggregate. This may be because some consumers use compensatory decision rules while others use non-compensatory strategies. Consistent with this, we see that the non-compensatory model fits better for
9 panelists, the compensatory model fits better for 80 panelists, and there are 71 ties. Although the compensatory model fits better for more panelists, the non-compensatory model tends to win by larger margins. For the 49 panelists who fit better with the non-compensatory model, the mean difference (LL_N-LL_C) is 1.21 and the median is 1.06. The 80 panelists for whom the compensatory model fits better have a mean and median LL_C-LL_N of 0.69 and 0.34 respectively. Thus, these two effects cancel each other out. Over the full set of 200 panelists, the average LL_N-LL_C is 0.02 indicating a very slight advantage for the non-compensatory model.

As mentioned earlier, the length of a panelist's purchase history often influences the promotion-sensitivity parameters for that panelist. When a purchase history is short, the promotion-sensitivity variable is likely to move toward an extreme. The issue of overfitting becomes relevant again. Certain combinations of π's and γ's or θ's lead to perfect or near-perfect fits, even if these parameter estimates are unrealistic. Over half the panelists with 12 or fewer purchases have extreme θ's (above 1 million or below 1), but only a quarter of the remaining panelists have extreme θ's.

The correlation between number of purchases and relative model fit (LL_N-LL_C) is only 0.098. The only noteworthy trend between these two variables is that the non-compensatory model tends to outperform the compensatory model for panelists with long purchase histories. Of the 20 panelists who make at least 40 purchases, the non-compensatory model fits better ten times, fits worse six times, and ties four times. One possible explanation is that heavy users of a product may rely on simplifying (i.e., non-compensatory) decision strategies more frequently than light users do.
In this section we examine the advantages to be gained by modeling the phased decision strategy rather than forcing the non-compensatory consumers' purchase histories into a compensatory model structure. In order to contrast the non-compensatory and compensatory model structures, we consider actual purchase histories at the individual level. The examples chosen highlight the advantages to be gained from allowing non-compensatory choice behavior to manifest itself.

**Hierarchical Preference**

A consumer might have a brand (or set of brands) with which she is happy and that she is willing to buy at full price. She might have another brand (or set of brands) that she finds marginally acceptable and will buy only at a reduced (promoted) price. The consumer's preference for the brands with which she is "happy" should be substantially larger than preference for "marginal" brands. We describe such a consumer as having hierarchical preferences. The compensatory model can only reflect such hierarchical preferences for consumers who are extremely promotion sensitive ($\Theta_i$ very large). The non-compensatory model, on the other hand, can identify such a preference structure for consumers with any non-zero level of promotion sensitivity.

Consider the purchase history depicted in Figure 1. Each column lists the promotional status of each relevant brand for panelist 10037 at each purchase occasion. A "P" indicates that the brand is promoted. The absence of a "P" indicates that the brand is not promoted at that purchase occasion. The chosen brand at each purchase occasion is represented by a box. For instance,
at the first purchase occasion, B_3 was on promotion alone, and was chosen. On the second purchase occasion B_1 was chosen and none of the three brands were promoted.

[Figure 1 About Here]

Figure 1 also reports the estimates of promotion sensitivity and brand preferences under the non-compensatory model and under the compensatory model. The log-likelihoods (LL's) show that the non-compensatory model fits better than the compensatory model. According to the non-compensatory model, this panelist will restrict her choice to promoted brands 52 percent of the time when faced with promotions. The values of γ and Θ are moderate compared to the other 199 people. (The median values of γ and Θ, respectively, are .691 and 23.1.) Hence both models suggest that this panelist is moderately promotion sensitive.

The non-compensatory model further suggests that this panelist has hierarchical preferences: she is "happy" with B_1 and B_3, she finds B_2 only "marginal". We see that she only chooses B_2 when no brand with which she is "happy" is on promotion and B_2 is on promotion. To understand why the non-compensatory model reflects this phenomenon while the compensatory model does not, consider the non-compensatory choice probability expression for B_2 (individual subscript suppressed):

\[ N_{2,6} = \gamma \frac{\pi_2}{\pi_2} + (1-\gamma) \frac{\pi_2}{\pi_1 + \pi_2 + \pi_3} = \gamma + (1-\gamma) \frac{\pi_2}{\pi_1 + \pi_2 + \pi_3} \]

Therefore, even if \( \pi_2 \) is negligible, \( N_{2,6} > \gamma \). Virtually no preference is needed for a brand such as brand B_2 to get sizeable choice probabilities when promoted alone under the non-compensatory model.
FIGURE 1: PURCHASE HISTORY AND ESTIMATED PARAMETERS FOR PANELIST 10037

<table>
<thead>
<tr>
<th>Purchase Occasion</th>
<th>Brand B₁</th>
<th>Brand B₂</th>
<th>Brand B₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>P</td>
</tr>
</tbody>
</table>

Non-Compensatory

\[
\begin{align*}
\gamma &= 0.516 \\
\pi_1 &= 0.605 \\
\pi_2 &= 1.63 \times 10^{-12} \\
\pi_3 &= 0.395 \\
LL &= -4.95
\end{align*}
\]

Compensatory

\[
\begin{align*}
\Theta &= 7.46 \\
\pi_1 &= 0.622 \\
\pi_2 &= 0.070 \\
\pi_3 &= 0.308 \\
LL &= -5.36
\end{align*}
\]
On the other hand, the compensatory model cannot be so flexible. The compensatory tradeoffs made between Θ and π₂ imply that:

\[ C_{2,6} = \frac{Θπ₂}{π₁ + Θπ₂ + π₃} \]

Since Θ is not particularly high, π₂ must be pushed upward by the compensatory model to yield a reasonably high choice probability for purchase occasion 6. Even with its higher value for π₂, the compensatory model estimates a lower choice probability than does the non-compensatory model when brand B₂ is promoted alone (\( N_{2,6} = .52, C_{2,6} = .36 \)).

In general, any individual who chooses a brand only when that brand is promoted alone will have "hierarchical preferences" according to the non-compensatory model.

There is no reason why the preference hierarchy should be restricted to two levels. Multi-level hierarchical preferences are consistent with the descriptive analytical technique known as a decision net (Bettman 1979). For instance, a consumer might be characterized by a strictly lexicographic decision process at each occasion as in Figure 2.

[Figure 2 about here]

Panelist 30106 (Figure 3) exhibits choice behavior consistent with the decision net in Figure 2. This panelist has LL's of zero, indicating a perfect fit for each model. (Panelists with LL=0 for either or both models are somewhat common: about ten percent of all panelists. These panelists usually have very few purchases or always choose the same brand.) Brand B₁
FIGURE 2. A DECISION NET REPRESENTATION OF PANELIST 30106'S DECISION PROCESS

Is $B_2$ on promotion?

Yes → Choose $B_2$

No

Is $B_3$ on promotion?

Yes → Choose $B_3$

No

Is $B_1$ on promotion?

Yes → Choose $B_1$

No → Choose $B_2$
is clearly a "marginal" brand, since it is only chosen when promoted alone. Any time \( B_1 \) is promoted simultaneously with another brand, the \( B_1 \) promotion is ignored. The \( B_1 \) promotions at purchases 2 and 3 can therefore be ignored, so the two \( B_3 \) purchases are made when brand \( B_3 \) is promoted "alone". It follows that \( B_3 \) is also a "marginal" brand, except when compared to brand \( B_1 \). The non-compensatory model resolves this confusing situation with a three-level hierarchy. \( B_1 \) and \( B_3 \) are both "marginal" brands, but \( B_1 \) is much more "marginal" (i.e., less preferred than ) brand \( B_3 \). To be more precise, \( \pi_1 = 8.69 \times 10^{-20} \) and \( \pi_3 = 1.59 \times 10^{-10} \).

Regardless of the number of levels in the hierarchy, the non-compensatory model separates the \( \pi \)'s by several orders of magnitude to make the hierarchy clear.

[Figure 3 About Here]

Modeling strict lexicographic behavior implies that all choice probabilities will be zero or one. The decision net shown above implies a deterministic choice rule. Should this panelist deviate from that rule in future periods, the estimated model would assign a probability of zero to that event. However, lexicographic behavior need not always be so strict. It is possible, for instance, to have hierarchical \( \pi \)'s with \( \gamma < 1 \). Choice probabilities would then move away from 0 and 1.

By altering this purchase history slightly, a clear picture of the differing model sensitivities develops. Figure 4 presents the estimates for both models as the promotional environment is varied slightly.
FIGURE 3: PURCHASE HISTORY AND ESTIMATED PARAMETERS FOR PANKLIST 30106

<table>
<thead>
<tr>
<th>Purchase Occasion</th>
<th>Brand B₁</th>
<th>Brand B₂</th>
<th>Brand B₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>P</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>P</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>P</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>P</td>
<td>P</td>
<td></td>
</tr>
</tbody>
</table>

Non-Compensatory

\[
\begin{align*}
\gamma &= 1.00 \\
\pi_1 &= 8.69 \times 10^{-20} \\
\pi_2 &= 1.00 \\
\pi_3 &= 1.59 \times 10^{-10} \\
LL &= 0.00
\end{align*}
\]

Compensatory

\[
\begin{align*}
\Theta &= 8.57 \times 10^{29} \\
\pi_1 &= 2.48 \times 10^{-19} \\
\pi_2 &= 1.00 \\
\pi_3 &= 1.20 \times 10^{-9} \\
LL &= 0.00
\end{align*}
\]
Variant 1: Suppose there were a \( B_1 \) promotion present at purchase occasion 1, but the rest of the purchase history were unchanged. \( \gamma \) drops to \( 4/5 \) since a \( B_1 \) promotion is ignored at purchase occasion 1. Promotions are available at all five purchase occasions, but are used only four times. The hierarchy of \( \pi \)'s remains unchanged, but the decision net now has a probabilistic element as shown in Figure 5.

The top right panel of Figure 4 shows that this variant in the choice history results in a lower value of \( \gamma \) and a reduced the log-likelihood (from 0.0 to -2.50) for the non-compensatory model. The compensatory model changes similarly, still yielding hierarchical \( \pi \)'s, but \( \Theta \) and the \( \pi \)'s move closer together. The compensatory model actually has a better LL than the non-compensatory model for this variant.

In variants 2 and 3 (the bottom two panels of Figure 4), the non-compensatory model produces a better fit and more stable parameter estimates than the compensatory model. The non-compensatory model is not dependent on whether the promoted brand is \( B_1 \) or \( B_3 \) (or both). The non-compensatory parameter estimates remain constant from one variant to another. The compensatory model is sensitive to the changes in variants 2 and 3. The estimated parameters and their relative magnitudes change. The third level of the hierarchical structure of the \( \pi \)'s disappears, and LL continues to fall.
**FIGURE 4: ESTIMATED PARAMETERS FOR DIFFERENT PROMOTIONAL ENVIRONMENTS AT PURCHASE OCCASION 1 FOR PANELIST 30106**

<table>
<thead>
<tr>
<th>Actual History</th>
<th>Variant 1</th>
<th>Variant 2</th>
<th>Variant 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-Compensatory</strong></td>
<td><strong>Compensatory</strong></td>
<td><strong>Non-Compensatory</strong></td>
<td><strong>Compensatory</strong></td>
</tr>
<tr>
<td>$\gamma$ = 1.00</td>
<td>$\theta$ = 8.57x10$^{29}$</td>
<td>$\gamma$ = 0.80</td>
<td>$\theta$ = 2.84x10$^{21}$</td>
</tr>
<tr>
<td>$\pi_1$ = 8.69x10$^{-21}$</td>
<td>$\pi_1$ = 2.48x10$^{-19}$</td>
<td>$\pi_1$ = 3.46x10$^{-20}$</td>
<td>$\pi_1$ = 7.03x10$^{-22}$</td>
</tr>
<tr>
<td>$\pi_2$ = 1.00</td>
<td>$\pi_2$ = 1.00</td>
<td>$\pi_2$ = 1.00</td>
<td>$\pi_2$ = 1.00</td>
</tr>
<tr>
<td>$\pi_3$ = 1.59x10$^{-10}$</td>
<td>$\pi_3$ = 1.20x10$^{-9}$</td>
<td>$\pi_3$ = 2.28x10$^{-11}$</td>
<td>$\pi_3$ = 8.29x10$^{-13}$</td>
</tr>
<tr>
<td>LL = 0.00</td>
<td>LL = 0.00</td>
<td>LL = -2.50</td>
<td>LL = -1.91</td>
</tr>
</tbody>
</table>

**Variant 1:** Brand B$_1$ on promotion at purchase occasion 1.
**Variant 2:** Brand B$_3$ on promotion at purchase occasion 1.
**Variant 3:** Brands B$_1$ and B$_3$ on promotion at purchase occasion 1.
Figure 5: A probabilistic decision net representation of panelist 30106's decision process under variant 1 of the choice history.

- Is $B_2$ on promotion? Yes → Choose $B_2$
  - No
    - Is $B_3$ on promotion? Yes, $p=0.8$ → Choose $B_3$
      - $p=0.2$ → Choose $B_2$
        - No
          - Is $B_1$ on promotion? Yes, $p=0.8$ → Choose $B_1$
            - $p=0.2$ → Choose $B_2$
              - No → Choose $B_2$
From this example, we see that: (1) decision nets can be drawn even when choice probabilities are between zero and one; (2) the compensatory model may yield hierarchical \( \pi \)'s under some conditions, but minor changes can destroy the structure; and (3) the non-compensatory model produces parameters that are more stable than those of the compensatory model.

**Robustness**

The last observation made above — parameter stability despite a changing purchase history — reflects a desirable trait that the non-compensatory model often exhibits. A robust model should be able to withstand small changes in a purchase history; the non-compensatory model is less sensitive than the compensatory model to the addition or deletion of purchase occasions that involve brands with "marginal" preferences. For example, in Figure 1, if purchase occasion 6 were thrown out, the maximum likelihood estimate for \( \pi_2 \) would be zero. The non-compensatory model would miss only by \( 1.63 \times 10^{-12} \), while the compensatory model would require more substantial changes in its \( \pi \)'s.

The probabilistic nature of the choice restriction process also enhances the non-compensatory model's robustness. Consider the ways in which the two models allocate preference between \( B_1 \) and \( B_2 \) for panelist 10037 described in Figure 1. Both models agree that \( B_1 \) deserves more preference than \( B_2 \), although each brand is purchased three times. The key is purchase occasion 3. The non-compensatory model is again more robust than the compensatory model for this purchase. The non-compensatory model assigns a probability of .48 that the panelist is not restricting choice to promoted brands at purchase occasion 3 and hence is less moved by the choice of unpromoted brand \( B_1 \) when
brand $B_3$ is promoted. The non-compensatory model is therefore less sensitive to the presence of the ignored promotion at purchase occasion 3. If that ignored promotion were deleted and the models rerun, none of the non-compensatory $\pi$'s would change by more than .05. $^3$

**Non-Uniqueness of $\theta_i$'s**

One final difference between the two models parameters should be noted. When the compensatory model suggests hierarchical preferences, it is often the case that the value of compensatory model's promotion sensitivity parameter cannot be uniquely identified. Consider, for example, panelist 10293 in Figure 6. This panelist is almost perfectly loyal to $B_1$. She observes $B_2$ on promotion 4 times and opts to buy $B_2$ on promotion one of those 4 times.

[Figure 6 About Here]

The non-compensatory and compensatory models fit this purchase history equally well, but $\gamma$ and $\Theta$ differ. One can see why $\gamma = 0.25$: $B_2$ is promoted alone four times, but it is purchased only once. The non-compensatory model infers that this panelist restricts choice one-fourth of the time. $B_1$ promotions have no effect, since promotions of brands $B_1$ and $B_2$ do not overlap. $^4$ The interpretation of $\Theta$ is similar but less obvious. Consider the compensatory choice probability expression for brand $B_2$ when it is promoted alone:

$$\frac{\Theta \pi_2}{\pi_1 \Theta \pi_2} = \frac{\Theta \pi_2}{1 + \Theta \pi_2} = \text{Prob(choose } B_2 | B_2 \text{ on promotion alone)} = 0.25 .$$

$\pi_1$ must equal 1, but $\Theta$ and $\pi_2$ are chosen in such a way to insure that the above equality $(\Theta \pi_2/(1 + \Theta \pi_2) = 0.25)$ holds.
### Figure 6: Purchase History and Estimated Parameters for Consumer 10293

<table>
<thead>
<tr>
<th>Purchase Occasion</th>
<th>Brand B 1</th>
<th>Brand B 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td></td>
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<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>P</td>
</tr>
<tr>
<td>8</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>P</td>
<td></td>
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<tr>
<td>14</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>P</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Non-Compensatory**

- $\gamma = 0.25$
- $\pi_1 = 1.00$
- $\pi_2 = 4.04 \times 10^{-11}$
- $LL = -2.25$

**Compensatory**

- $\Theta = 5.81 \times 10^8$
- $\pi_1 = 1.00$
- $\pi_2 = 5.72 \times 10^{-10}$
- $LL = -2.25$
An infinite number of \((\Theta, \pi_2)\) combinations would work, and there is no reason that this pair is necessarily better than any other. However, certain constraints (e.g. \(\pi_1 = 1\) and \(\Theta \pi_2 / (1 + \Theta \pi_2) = 0.25\)) will still hold, and the optimal log likelihood will be unaffected.

**CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH**

We have developed and analyzed a non-compensatory model of consumer response to promotion. The primary tenet of this model is the notion that consumers do not always process information about all available brands; the presence of promotions can lead consumers to consider only promoted brands at certain times. The proposed non-compensatory model was estimated at the individual level, and the estimated parameters were used to predict brand choice decisions over a 20-week holdout period. The collection of individual level non-compensatory models was shown to fit and predict better than an individual level choice share model and also better than the aggregate multinomial logit model developed by Guadagni and Little (1983).

The collection of individual level non-compensatory models neither fit nor predicted statistically better than the collection of individual level compensatory models. We suggested that this might be the result of different consumers having different processing strategies. However, the non-compensatory model's promotion sensitivity parameter was shown to be more readily interpretable. Further, the non-compensatory model was shown to have more flexibility in representing hierarchical preferences and to be more robust to small changes in purchase histories.
Due to the non-linearity of the non-compensatory functional form, we used a general maximum likelihood procedure rather than more commonly used procedures such as logit or regression. While we could not take advantage of the computational efficiencies developed for those compensatory models, we did gain flexibility. We can change the non-compensatory model in virtually any way and still estimate parameters for it. The model can be tailored to fit many different databases, other marketing variables may be included, and additional behavioral hypotheses can be incorporated.

The usefulness of this model is not limited to analyzing consumer response to promotion. Various other (potentially) non-compensatory decisions can be modeled with the methodology presented here. For example, a common simplification made in brand choice modeling is the use of only one product subcategory (e.g. regular ground coffee). In reality, many consumers might rely on a non-compensatory decision rule to make the ground vs. instant decision. Furthermore, there might be interactions between the presence of promotions and the ground vs. instant choice. Estimating models with such interactions is a feasible extension of the proposed model.
1. A number of factors may affect a consumer's decision to restrict choice on any given shopping trip. He or she may be under time pressure and may not notice promotional cues. Alternatively, he or she may have specific instructions from a friend or household member to buy a particular brand. The type or quality of the promotion may also play a role in the choice restriction process.

2. Thus far we have implicitly assumed that each decision maker in our database is an individual. We might be more correct to use the term "household", since each panelist identification number refers to a specific household rather than an individual. Studies have emphasized and tried to account for differences between individual and household brand choice decisions (Davis 1976). As noted in Footnote 1 above, these individual/household distinctions may have some influence on the estimated parameters, but should not affect the general theory to a large extent.

3. If the ignored B3 promotion at purchase occasion 3 for panelist 10037 (see Figure 1) were deleted, the new parameter estimates for the two models would be:

<table>
<thead>
<tr>
<th>Non-Compensatory</th>
<th>Compensatory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.705$</td>
<td>$\theta = 8.19 \times 10^{13}$</td>
</tr>
<tr>
<td>$\pi_1 = 0.636$</td>
<td>$\pi_1 = 0.667$</td>
</tr>
<tr>
<td>$\pi_2 = 1.18 \times 10^{-10}$</td>
<td>$\pi_2 = 1.22 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\pi_3 = 0.364$</td>
<td>$\pi_3 = 0.333$</td>
</tr>
</tbody>
</table>

While the non-compensatory parameters change only slightly, the compensatory parameters, especially $\theta$ and $\pi_2$, change dramatically.

4. If any B2 promotions should overlap with B1 promotions, they would be ignored by the non-compensatory model. Whenever a low-preference hierarchy brand competes simultaneously against a brand higher in the preference hierarchy, the low preference hierarchy brand's promotions are completely ineffectual (according to the non-compensatory model). For instance, if there were a B2 promotion at purchase occasion 5 (see Figure 3), the non-compensatory parameters would be unchanged. There is no consistent analogue for the compensatory model.
The non-compensatory model's functional form

\[ N_{ijt} = \gamma \sum_{\pi \in \mathbb{A}_i} P_{ijt} + (1 - \gamma \delta_{ijt}) \sum_{\pi \in \mathbb{A}_i} P_{ijt} \]

can be rearranged as follows:

\[ N_{ijt} = \gamma \sum_{\pi \in \mathbb{A}_i} P_{ijt} \sum_{\pi \in \mathbb{A}_i} P_{jt} + (1 - \gamma \delta_{ijt}) \sum_{\pi \in \mathbb{A}_i} P_{ijt} \sum_{\pi \in \mathbb{A}_i} P_{jt} \]

Now the ratio \( \frac{N_{ijt}}{N_{jlt}} \) is:

\[ \frac{N_{ijt}}{N_{jlt}} = \gamma \sum_{\pi \in \mathbb{A}_i} P_{ijt} \sum_{\pi \in \mathbb{A}_i} P_{jt} + (1 - \gamma \delta_{ijt}) \sum_{\pi \in \mathbb{A}_i} P_{ijt} \sum_{\pi \in \mathbb{A}_i} P_{jt} \]

Consider the addition of a new brand, with preference \( \pi_{iM} \):

\[ \left( \frac{N_{ijt}}{N_{jlt}} \right)' = \gamma \sum_{\pi \in \mathbb{A}_i} P_{ijt} \left( \sum_{\pi \in \mathbb{A}_i} P_{jt} + \pi_{iM} \right) + (1 - \gamma \delta_{ijt}) \sum_{\pi \in \mathbb{A}_i} P_{ijt} \left( \sum_{\pi \in \mathbb{A}_i} P_{jt} + \pi_{iM} \right) \]

IIA requires \( \frac{N_{ijt}}{N_{jlt}} = \frac{N_{ijt}'}{N_{jlt}'} \) regardless of the presence of brand M.

A necessary and sufficient condition for equality is \( P_{jt} = P_{jlt} = 0 \). If, for example, \( P_{jt} = 1, P_{jlt} = 0, P_{Mt} = 0 \), then the numerator would contain an interaction between \( \pi_{ij} \) and \( \pi_{iM} \), while the denominator does not involve \( \pi_{iM} \). Given the existence of promotions, this equality will not always be true, and therefore the IIA does not hold in general for the basic non-compensatory model. (Special thanks to Professor James M. Lattin of Stanford for pointing out this property.)
Appendix B: The Bias Test Procedure

(1) Choose a $\gamma_i$ and set of $\pi_{ij}$'s by drawing a random consumer from the panel of 200. Estimate the proposed model using all 65 weeks. Call the resulting $\gamma_i$ and $\pi_{ij}$'s the "true parameters" $\gamma_i^T$ and $\pi_{ij}^T$ (j=1 through 8).

(2) Randomly choose a specific number of store environments from the 260 different environments (4 stores x 65 weeks) present in the database. The number of environments used, NENV, will be set to different values, ranging from 5 to 20.

(3) For each of the NENV purchase occasions, calculate choice probabilities for all eight brands by applying equation (1) to the parameters from step 1 and the promotional environments from step 2.

(4) Simulate a brand choice decision by using the choice probabilities from step 3 and choosing a random number.

(5) After steps 3-4 have been performed for each of the NENV synthetic purchase occasions, we have a full purchase history. There is now sufficient data to run the MLE procedure. The end result is a $\gamma_i^E$ and a set of $\pi_{ij}^E$'s. When there is no bias, $\gamma_i^E = \gamma_i^T$ and $\pi_{ij}^E = \pi_{ij}^T$ for all j.
REFERENCES


