EVALUATING THE QUALITY
OF MANAGEMENT INFORMATION

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The comments and criticisms of Dr. Jason L. Speyer at the Draper Laboratories, M.I.T. and Professor Paul R. Kleindorfer and Michael Scott-Morton of the Sloan School of Management, M.I.T. have been most helpful in writing this paper.
The remarkable advances in computation and communications technology over the past 20 years have led to systems which can bring information from many widespread sources immediately and accurately to decision makers. But will the global, immediate, and accurate information made possible by this technology improve the performance of decision making significantly? Successes in airline reservation systems and in continental defense say yes. However, research by Conway, Maxwell, and Miller (9, pp 238 and 241) and Carroll (7), and that presented below, suggest that the benefits from immediate, highly accurate, and global information probably will not justify the expensive technology needed to get it.

This ambiguity of the value of high quality information creates a dilemma for the designer of on-going, resource management systems. He doesn't know how to specify information quality in his designs. While he can calculate the costs of obtaining different levels of quality of information with relative ease, there is no useful body of theory or experience to help him evaluate the effects upon the performance of a resource management system of different levels of information quality.

On-going resource management decisions are a major part of any enterprise whether it be a business, a hospital, a school, a welfare department, or a military service. On-going resource management decisions determine manpower, sales effort, plant, and equipment. They schedule orders, allocate product, assign work, and man engineering projects. These decisions adapt resources to ever-changing circumstances. These decisions are repetitive and they are guided by policy. However, the frequency of the
decision making can vary widely from several times a day in the scheduling of work in a shop to once every few months for determining the amount of floor space needed. Similarly the precision of the policy which guides them can also vary widely from highly specified inventory reorder rules for an individual inventory item to a less precisely specified policy controlling aggregate production, workforce, and sales effort. What characterizes all of these decisions, however, is that they take place in a feedback framework where the information from the same sources is used repetitively in the decision making. This feedback framework will be discussed in much greater detail below. The on-going resource management decisions encompass most of the decisions that are made in the firm. They include both the operational control and the managerial control decisions discussed by Anthony (1). It is these decisions which most computerized management information systems are being designed to support.

The literature on information quality offers little practical help to the designer of resource management systems. He finds work that is either too abstract or too specific to be of use. The work of Marschak (16) looks at a static situation so it doesn't apply to the dynamic situation of on-going resource management. While Kriebel (15) has done excellent theoretical work which can become useful to system designers when developed further, currently it merely points to mathematical techniques which can evaluate information quality. However, the designer probably doesn't understand the mathematics and the mathematics require models far simpler than the real situations faced by the designer. On the other hand, the work by Boyd and Krasnow (3) points out that one can evaluate the effects of changes of information quality from a simulation model of a specific
situation. Unfortunately they do not articulate a theory of how to represent or analyze information quality. The information system designer is merely told to build simulation models. He is given no guidelines of what to include in the models nor is he told the relative impact of different types of information degradation. More importantly, system designers need rule of thumb guidelines to help them specify information quality so that they can avoid the expense of building simulation models to answer every design question.

A framework is needed upon which to erect a body of theory and experience which will help the designer specify information quality in resource management systems. Such a framework has several elements. It identifies the factors of resource management considered when evaluating information quality. It states how to represent information quality. A framework also presents a methodology for evaluating information quality in specific situations. More importantly, a framework is a structure which ties together pieces of research and applications to construct a coherent body of theory and practical experience upon which information system designers can draw.

Such a framework seems to exist. This paper discusses the framework, defines information quality in terms of the framework, presents an example of its application, and suggests paths of future work.
THE DYNAMIC CONTROL SYSTEM VIEWPOINT

The dynamic control system viewpoint is the appropriate framework for evaluating information quality in on-going resource management. The information flows, decision rules, and resources of the resource management process form a control system made up of feedforward flows and feedback linkages. By representing information quality in this control system, its effects upon dynamic behavior can be measured. Below, it is argued that the dynamic control system viewpoint is the only correct framework for evaluating information quality in on-going resource management.

Blumenthal (2), Carroll (3), Forrester (10), and Kriebel (14) all advocate a dynamic control system viewpoint for analyzing information in on-going resource management. But much research and application is needed to turn the advocacy into a body of knowledge useful to designers. Before defining this framework more fully, it should also be pointed out that this framework is appropriate for designing all segments of the resource management process: (1) the type of information to bring to decision points, (2) the decision rules that control resources, and (3) the characteristics of the processes by which resources generate results. However, a discussion of the general applicability of the dynamic control system view is beyond the scope of this paper. It also should be pointed out also that Anthony's (1) segmentation of decisions into his three categories of operational, managerial and strategic control while useful for some purposes is orthogonal to the question of evaluating information quality. Let us now return to the main topic.
On-going resource management is a dynamic control process made up of the three parts shown in Figure 1: i) information flow, ii) a set of decision rules controlling resource changes, and iii) the transformation of resources to results. In Figure 1 information is represented by the dashed lines, decisions rules are represented by the valve which controls resource changes (solid line), and the transformation of resources to results is shown by the double solid line. Any resource management process operates in an environment, represented by ovals, which affects each of the three parts as shown in Figure 1.

Information Flow

Both feedforward and feedback information enter the decision point. Feedforward information describes environmental variables which affect the decisions but which the decisions and resources being controlled cannot influence. Typical environmental variables are demand, customer attitudes, population age composition, general economic conditions, actions of competitors, laws, technical breakthroughs, and political atmosphere. Feedback information describes the resource posture of the organization and the results of the resources, both of which are affected by the decision rules. The resource posture is the amount of resources, men, money, capital equipment, etc. devoted to various purposes and in various places. The results of resources are the variables by which the performance of the resources is measured by customers, competitors, and the firm itself. For example, results of production resources show up in inventories, backlogs, and schedule condition. Results of product development are quality, profit
FIGURE 1. The Dynamic Control System of Resource Management
margin, sales rate and type of products. Results of advertising are awareness of the company's products, shelf-space in retail outlets, and sales rate.

**Decision Rules**

Using information about demand and comparing the resource posture with its desired values and the results against standards, the decision rules determine resource changes which set the amount and disposition of resources. For example, inventory and order rate information determine hiring, firing, or overtime. Information on the competitiveness of the product line, on technological developments, or breakdowns in the production process determine the shifting of engineers between new product development and solving production problems. In a computer simulation model, the decision rules are expressed as mathematical functions, tables, or algorithms.

Almost every decision is constrained by the environment or by other resources in the firm. Examples of environmental constraints are antitrust laws, social customs that discourage layoffs, tight money, scarce labor supply, or capacity shortages at suppliers. Examples of resources constraining a decision are cash and credit shortages limiting acquisition of plant and equipment, plant and equipment which limits hiring by placing a ceiling on manpower, or scarce technicians which limits expansion of sales and production of a high technology product. It is necessary to include decision rules and constraints in an evaluation of information quality because the performance of resource management is determined by the
total characteristics of the feedforward and feedback control processes of which the quality of information is only one part. The actual behavior depends upon how the characteristics of information interact with other parts of the feedback loops such as the decision rules and constraints. For example, the parameters on the feedback information in the decision rule largely control the steady-state influence of bias (Appendix B). These same parameters also help determine the propagation of random error (Appendix A). In another example, Breakwell (4) has shown that decision rules and information quality cannot be analyzed separately. He points out that the optimal decision rule in an aerospace example depends upon the random error (noise) in information. Finally, there are times when action is limited by constraints and in such a circumstance information quality can vary widely without effect and an incorrect evaluation of the effects of information quality would occur if constraints on action were ignored.

Transformation of Resource to Results

The last segment of the feedback process in resource management is the transformation of resources to results. For example, on a production line, resources of manpower, plant, equipment and raw materials are transformed into an inventory of finished products and achieve results of meeting customer-delivery requirements. An advertising agency, T.V. studio, actors, and a television network transforms an advertising budget into consumer awareness of a product. Technology is probably the major determinant of
the transformation of resources to results, but the legal, political, and social environment as well as the structure and psychological atmosphere of an organization also affect the transformation of resources to results.

This transformation is represented mathematically by a function which determines the results given a set of resources. Economists call it a production function. However, unlike the small and static production functions of microeconomic theory, the transformation function takes account of time, and it can show results in many dimensions such as schedule condition, inventory size at various locations in the production-distribution system, customer perceptions and attitudes about the firm, technological lead of products, costs, profits, customer service, delivery delay, etc. A model of a specific process will include only those result variables that we needed for feedback information to the decision point, for calculating costs, and for measuring performance.

One needs to include the transformation of resources to results in the framework for evaluating information quality for the same reason that the decision rules were included. The behavior of dynamic systems is determined not by components but by the total feedback loop and feedforward flow characteristics. For example, as Forrester (10, p. 33) has illustrated, behavior is altered little by delays in information flow which are small compared to delays in the transformation process.

**Performance and Costs**

To evaluate the quality of information, the analyst needs to include measures of performance and measures of cost in a model. Some of the cost
calculations or performance measures may be part of the resource management process. Others may have to be added to a designer's model or subjectively applied when choosing among alternative designs.

Information Quality

In this paper information quality is represented as degradation from perfect information by error, distortion, delay, and sampling.

1. **Error.** Error is the random deviation of information from reality. Information with error has the same mean as reality. Error can be introduced into information by measurement using a statistical sampling technique, by clerical mistakes, by lapses in memory, or by a "word of mouth" information channel. Error also exists when surrogate variables are measured. For example, the number of orders can be used as a surrogate of man-hours of work. Error arises if there is a less-than-perfect correlation between man-hours of work and number of orders.

In a mathematical model, error is represented by a random variable being added to or multiplied by the variable which represents perfect information. The probability density function of the random variable and the frequency of changing the random variable define the characteristics of error in the information flow.

Once the tolerance for error is known the information system designer can decide the sample size needed for statistical sampling. He can choose between using an easily measured surrogate variable or measuring, at high cost, the actual variable. He can decide if a quick visual check is adequate measurement. From an evaluation of forecast error, he can choose among various forecasting techniques.
2. **Distortion.** Distortion is the persistent displacement of information from reality where the displacement is often a function of the value of the information. Distortion has several forms. One form is threshold distortion in which reality is not transmitted until certain limits are exceeded as in exception reporting. A second form, called saturation, exists when information fails to reflect extremes of reality. Examples of saturation in information systems is the recent ignorance of brokerage houses about their liabilities and assets as heavy stock trading clogged their back office operations. A third form of distortion is bias in which information persistently undervalues or overvalues reality. One example of bias is an overestimate of working inventory due to neglecting inventory shrinkage and failing to consider slow moving items. Bias can arise from sources as diverse as statistical sampling techniques, the use of surrogate variables, or the organization's hopes and fears.

   In a mathematical model, distortion can be represented by multiplying the real state by a constant or variable. Where distortion depends upon the value of the information as in threshold or saturation, it can be represented by a non-linear function or table. Knowing the effects of distortion, the information system designer can set limits for exception reporting, select statistical sampling techniques, choose surrogate variables, or decide if distortion-prone qualitative impressions suffice for decision making.

3. **Delay.** Delay is a lag between reality and information at the decision point. Delay arises in two ways. First of all, it represents the fact that it takes time to collect data, process it into useful information, and transmit the information to the decision point. With
sophisticated technology the delay may be short. With manual data collection, clerical processing, and transmission by mail or phone, the delay is longer. A second source of delay is smoothing. Smoothing, whether it be formal, mathematical smoothing or the intuitive smoothing by a man inspecting a time series, inevitably introduces delay into the information.

Both forms of delay are represented as weighted averages of past data. Frequently, for ease of mathematical analysis or computer simulation, exponential weightings are appropriate.

The effect of delay upon performance tells the systems designer the incremental value of the most current information provided by on-line, real time measurement and transmission feeding a high-speed computer compared with the delayed information given by a few clerks. Evaluation of delay will also tell how to smooth time series data. For example, Thorsten (21), when estimating yields of butane from a catalytic cracking unit, found a short smoothing time on noisy yield data best even though it transmitted random error because it quickly detected any sudden long-term change in yields.

4. Sampling. Sampling is the periodic changing of information to a more recent value. It is caused by periodic measurement or the periodic up-date of information. Those who wish to investigate the differences between sampling and delay might read Truxal (22) Chapter 9.

Examples of sampling by periodic measurement are the counting of inventories once every six months and the surveying of a market once a year. Periodic reports, periodic up-date of a data base, or batch processing of data are all examples of the periodic up-dating of information.

Sampling is represented mathematically by a function which periodically
sets the information equal to a measurement and holds the value until the next sample time.

Since the frequency of measurement and of up-dating information can affect costs significantly, the information system designer needs to know the effect of sampling upon performance in order to find an economic design.

The measures of information quality discussed above differ from those of Forrester (11). He has described information quality in terms of error, bias delay, distortion, persuasiveness and cross-talk. Error here is defined as does Forrester. All the other terms have been redefined or dropped. Bias has been dropped since it is one form of distortion. We have defined smoothing as delay since they are mathematically identical, rather than under distortion as Forrester does. Sampling has been added since it is common and not described by any of Forrester's terms. Persuasiveness and cross-talk have not been included since they are not properties of information flow but are characteristics of decision makers and are represented in decision rules.

NECESSITY OF EMPLOYING THE DYNAMIC CONTROL SYSTEM VIEWPOINT

In order to evaluate correctly the impact of information quality upon on-going resource management, one must utilize the dynamic control system viewpoint. The reason for this is that on-going resource management is a dynamic, feedforward, feedback process and has has been amply shown, the effect of error, distortion, delay, and sampling are all affected by the feedforward-feedback nature of the process. The effects of random error are a function of both the feedforward and feedback elements of a system.
(Bryson and Ho, 5). The effects of bias, which is but one form of distortion, are also strongly influenced by feedback processes (Chestnut and Meyer, 8, p. 224). Similarly the effects of delay in information which is but one part of the total delay in a system depends upon the remainder of the system as has been illustrated by Forrester (10, p. 33) and which can be seen by looking at the solutions of the equations of a linear feedback process which includes delay in information. Finally, the effect of sampling must also be looked at in a feedback context because of the tendency of sampling to destabilize feedback loops (Truxal, 22, Chapter 9).

**Common Structures**

Two observations about complex dynamic systems make this viewpoint even more useful to the design of on-going resource management systems. First of all, it seems that even for very complex systems the analysis can focus upon one to three feedback loops at a time (Swanson, 18). Secondly, it appears that certain control structures are common and appear again and again, e.g. Forrester (10), Swanson (20), Chestnut and Mayer (8, p. 230). Since analysis focuses upon a few feedback loops and since certain structures appear often, there exists a relatively small set of relatively simple common structures which can form the base for understanding many different specific situations. These common structures provide the link between theory and practice and the place where knowledge can accumulate. Thorough analysis of any one structure whether done in application or as theory becomes widely applicable. For example, the simplest and most common of the structures are single feedback loops for which extensive theory exists in the servo-mechanism and modern control literature.
An Example

One of the most common structures that exists in resource management is shown in Figure 2. The particular example shown here is the control of aggregate manpower, production rate, and inventory. The common elements in the structure of Figure 2 are (1) feedforward from demand (sales rate), (2) integral feedback information (inventory), and (3) controlling two resources -- one fast acting and expensive (overtime) and the other cheaper but less responsive (workforce). Integral feedback control is the most common type because of its ability to cope with bias and long term changes (Chestnut and Mayer, 8, p. 233). It is called integral control because the result that is measured to feedback to the decision point is the integration of the resource actions. In this case inventory integrates production rate.

Below we investigate the effect of error, distortion, delay, and sampling in the information flow from sales rate (feedforward) and from inventory (feedback) in the structure of Figure 2. The investigation is performed by degrading the information in a simulation model and looking at the impact upon total costs. The simulation model and its cost functions are based upon the classic work by Holt, Modigliani, Muth, and Simon (13). The model differs somewhat from theirs. Most importantly, the cost functions are piecewise linear curves which fit the raw cost data better than do the quadratic functions used by Holt et. al.

In order to insure that the effect upon total cost of degradation in information quality are not due to poor decision rules, the decision rules used are the same for each test and are near optimal for perfect information. I can't claim them to be optimal because they were found using simulation. However, with the piecewise linear cost functions, the rule used is three percent cheaper than the Holt et. al. decision rules which are
Figure 2: The Management of Workforce Overtime and Inventory: An Example of a Common Resource Management Structure
based upon quadratic costs. In addition, Swanson (19) found that when parameters in the rule are doubled or cut in half, costs increase less than one percent. Thus, there seems ample reason to believe that the decision rules used for the tests are very near optimal. For a more detailed explanation of the model see Swanson (19) and see Appendix C for the computer program listing.

The investigation of the effect of information quality follows. First we see the impact upon total costs of each type of information degradation, then we look at a more realistic case where all forms of degradation exist simultaneously and see the effects of reducing the degradation along the different dimensions.

Random Error

Random error is introduced by multiplying the true value by a random number which is Gaussian distributed and which is changed once a month. Figures 3A and 3B show the percentage increase in total costs in five-year simulation runs as the standard deviation of the random error is increased. The solid lines show the costs of degradation in inventory information and the dashed lines the costs of degradation in sales rate information. For each run the sales rate is generated by a Gaussian distribution of mean 500 and standard deviation of 150. To maintain comparibility, the same time series of pseudo random numbers in sales rate, inventory error, and sales rate error are used for each run within a graph. Only the standard deviation was changed.
The insensitivity of total costs to error is striking. With a reasonable error such as a standard deviation of 10 percent the total costs are raised less than one percent and usually less than one-half of one percent. Only in the case of sales rate error in Figure 3A do costs rise more than 10 percent and then only for error with a standard deviation far beyond reasonable expectation. The reason for the small increase in costs due to error in information is that error adds little variability to the variables that generate costs. The mathematical analysis of the effects of random error in information (Appendix A) makes this clear. When the data was examined (not shown) the standard deviation of workforce, overtime, and inventory increased little as the standard deviation of error increased in the simulation runs, which corroborates the analysis in Appendix A.

In Figure 3A costs increase about three times as fast as they do in Figure 3B as the standard deviation of the error in sales rate information increases. The difference between Figure 3A and 3B is that the time series of sales and information error are different although the mean and standard deviations are nearly identical. The cause of the rapid increase in total costs can be traced to a ten-month period when a series of random numbers produced a continuing underestimate of sales which is a form of information distortion that resembles bias rather than random error. As we see below persistant underestimate of sales rate is very expensive.

Distortion. There are many forms of distortion. A common form is bias which is examined here. Figure 4 shows the percentage increase in total costs for five-year simulation runs with different levels of bias in the inventory information and sales rate information. Bias is represented by a number which multiplies the actual value of inventory and sales rate. The
Figure 3A: Increase in Total Costs due to random error in information; Sales rate varies randomly about a constant mean (PC2)

Figure 3B: Increase in Total Costs due to random error in information; Sales rate varies randomly about a constant mean (PC3)
sales rate input is identical in each run and it is the pseudo random time series that was the input for the runs in Figure 3A.

Figure 4 shows that bias in sales rate information has a significant effect upon costs while bias in inventory information has far less impact. As shown in Appendix B, this is a general result. Let us see why. If the sales rate is underestimated, the production rate will be low and inventory will drop until the gap between desired and actual inventory becomes sufficiently large to compensate for the underestimate of the sales rate. To overcome bias in sales rate information, inventory moves three and three-tenths times the bias in the estimate of the sales rate (see Appendix B). If on the other hand, inventory information is biased, the inventory will change only by the amount of the bias (Appendix B). When a statistically identical but a different sales rate input is tested, the costs are essentially unchanged from those in Figure 4.

Delay. Figure 5 shows the percentage increase in costs for five-year simulation runs as inventory information alone (solid line) and inventory and sales rate information together (long and short dashed line) are delayed. To maintain comparability, the exogenous sales rate input is identical for all runs represented in Figure 5 and for the runs in Figures 3A and 4. Delay is represented by a third-order exponential delay (Forrester, 10, p. 89) of the actual values. As we see here, if only inventory information is delayed, the effect upon costs are marginal. But when both inventory and sales rate are delayed, there is no channel for recent data to reach the decision point and costs increase more rapidly. With a not uncommon information delay of one month, costs increase 5 percent. One percentage point increase in costs represents $6,000 per year. A reduction of the delay of
FIGURE 4: Effect of Bias upon Total Costs
both inventory and sales rate information from one month to one week saves nearly $22,500 a year.

One would expect information delay to increase costs significantly if sales rate were to change its average level suddenly. Figure 6 shows the percentage increase in total costs as information delay increases. The costs are taken after one year of a simulation run when sales rate increased from a constant 500 units/month to a constant 750 units/month at six months. The increases in costs are significant. Since one percentage point represents $7,200, a reduction of delay in both sales rate and inventory information from one month to one week represents an annual savings of $43,600. Further reductions in information delay can reduce costs less than $14,400 per year indicating that even where reduction of delay has the greatest benefits, reducing the delay to hours or minutes with very sophisticated information technology would have meager benefit relative to costs.

**Sampling.** Figures 7A and 7B show the percentage increase in total costs of five year simulations when sales rate and inventory are sampled. The standard run against which costs are compared assumed a sampling time of one-quarter of a month -- about one week. When checked, reduction of the time between samples to about one-quarter of a week (0.0625 month or 16 times a month) reduced cost a negligible 0.24%. The costs in Figure 7A are taken from simulation runs with the same pseudo random time series of sales rate as in Figures 3A, 4, and 5. Costs in Figure 7B come from runs with a sales rate which is statistically identical to Figure 7A but which is a different time series. The most striking characteristic of these two graphs is the small increase in costs until a two-month sample time at which point costs vary widely with no discernable pattern. These simulation results
FIGURE 5. Effect of Information Delay upon Total Costs when Sales Rate Varies Randomly about a Constant Mean
FIGURE 6. Effect of Information Delay upon Total Costs with a Step Change in the Sales Rate
are corroborated by Speyer's (17) analysis of the effect of data sampling upon the performance of a missile. In both the simulations and in Speyer's analysis, frequent samples degrade performance little. However, as the time between samples increases, the chance for serious degradation of performance increases. This degradation can come from two sources: (1) the destabilizing influence of low frequency sampling in a negative feedback loop (Truxal, 22, p. 524) and (2) the loss of information that occurs when the sampling frequency is less than twice the highest frequency of interest in the sampled signal (Truxal, 22, p. 505). The variability of the results for the same sampling period is explained by luck.

If events occur such that the sample cannot perceive them in time for effective response then performance is significantly degraded. On the other hand, with luck, events occur when the infrequent samples can perceive them and respond effectively. Luck explains the wide variations in cost for two-, three-, and four-month sampling in Figures 7A and 7B. However, the conclusion is clear. Infrequent sampling runs the risk of a significant reduction of performance.

**Typical Degradation**

Information normally is degraded all four ways and the information system designer wants to know what to improve in order to obtain cost-effective gains in performance.

Table 1 shows the percentage increase in total costs for different mixes of information degradation. Column 1 and column 2 represent the results of
FIGURE 7A. Effect of Sampling upon Total Costs (PC2)

FIGURE 7B. Effect of Sampling upon Total Costs
Different Time Series of Sales Rate
different pseudo random inputs to sales rate but which have identical statistics. The sales rate is Gaussian with mean 500 and standard deviation of 150. The time series for the sales rate for runs reported in Column 1 is identical to the sales rate of the simulations for Figures 3A, 4, 5, and 7A.

The degradation of the "typical information" case is: 1) Gaussian random error in both sales rate and inventory information with a standard deviation of ten percent and changed once a month, 2) no bias in sales rate information but a twenty percent overestimate of inventory, 3) delay of one-half of a month in both inventory and sales rate information, and 4) sampling of both inventory and sales rate every one-half month. In one case the result is a 7.5% increase in total costs in the other 4.7%. From Table 1 it is clear that reducing information error has little or no effect upon performance. The greatest improvement comes from eliminating the delay in information. The second largest improvement comes from eliminating delay in inventory information. In this example each percentage point represents $6,000 per year. Eliminating delay in both sales rate and inventory information saves an average of $21,000 per year. Eliminating the delay in inventory information alone saves an average of $16,800 per year. The third greatest savings averages $12,000 per year by eliminating inventory bias. Eliminating information delay and inventory bias together saves 5.5% or $33,000 for the sales rate in Column 1. This reduction of delay would require expensive technology or quick preliminary estimates. Since random error has little effect upon performance, quick estimates by educated guess or small samples may be an effective, low cost way to reduce delay. Thorsten (21) in a specific application found that quick perception of events, even if random error is increased, gives improved performance. In both cases the slow
<table>
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<th>SALES RATE TIME SERIES 1</th>
<th>SALES RATE TIME SERIES 2</th>
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<td>&quot;Typical&quot; Degraded Information</td>
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<td>10% random error</td>
<td>7.5</td>
<td>4.7</td>
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<td>20% overestimate of inventory</td>
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<td>1/2 month delay</td>
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<td>1/2 month sample time</td>
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<td>Improvement from &quot;Typical&quot; Degraded Information</td>
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<tr>
<td>No inventory error</td>
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<td>No delay, no bias in inventory</td>
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TABLE 1. Percentage Increase in Total Costs over Perfect Information for Combinations of Degradation in Five-Year Simulations
perception of and response to major changes were much more costly than responses to random error in information. But if these quick but noisy estimates are not possible and if immediate information is too expensive we see that a one-week delay in inventory and sales rate information and no bias reduces the costs of degraded information more than half and is about as effective as no delay in information. Assuming that information (1) sampled twice a month, (2) containing random error with a ten percent standard deviation (3) having no bias, and (4) delayed one week can be produced by a clerical or a batch computer information system, then the additional costs of perfect information would have to be less than $21,000 a year to be economical.

The information quality needed to support the management of aggregate production, inventory, and workforce in this example does not justify even moderate expenditures for sophisticated information technology and certainly not on-line, real time systems. Clerical or batch information processing is adequate. Bias can be kept small by proper definitions of measurements and care in recording data. A delay of one week in information is acceptable and delays can be shortened further by estimates and telephones.

More research is necessary in order to establish the generality of the conclusions even for this example. The effects upon information quality needs of different cost structures, of disaggregation, of imbedding the structure in a more complex system, of different delays of changing resources and producing the product, and of resource constraints need to be examined.
FURTHER RESEARCH

The above example shows clearly the need to use analytic or simulation techniques to evaluate information quality. Few, if any, possess both the incisive intuition and compelling persuasiveness to articulate convincingly, without mathematical analysis or simulation, the results shown here, such as the insensitivity to random error but the high cost of bias, or the high cost of delay, or the role of luck when sampling is infrequent.

Research needs to be performed in order to develop a body of knowledge useful to the designer of resource management systems. Fortunately much of the basic work has already been done. The understanding of simple feedback systems is well advanced in the engineering field but needs to be adapted to management. Techniques such as those of Kriebel can yield results which can highlight approaches to good designs. Modern control theory shows the way to deal optimally with information quality in many situations. For an excellent survey and bibliography, see a paper by Ho (12). Simulation methodologies (10) are well developed. Simulation models to aid design can be constructed relatively quickly and cheaply.

However, while much of the theoretical base exists, practical knowledge useful to the designer faced with time and budget constraints, still needs to be developed. The practical knowledge would include an investigation of common structures, such as that started above, and a set of applications. The most important objective of this work is to create design guidelines so that the designer can specify information quality without having to construct a model. A second objective, and one likely to be realized sooner, is to give the designer guidelines when constructing models to evaluate information
quality. Needed is further investigation of aggregate resource management problems such as that above, common resource management structures found in growth situations (Swanson, 20), and more complex but common logistical systems (Forrester, 10, Chapters 15 and 17). The investigation of information quality in the management of individual resource units and items as typified by Westerman (23) and Carroll (7) needs to be conducted. But the designer faced today with significant questions about information quality need not wait for the development of the practical knowledge. He can construct a simulation model of the resource management process and can analyze the effects of information quality upon performance as illustrated in the above example.
Appendix A

Analysis of Random Error in Information

Mathematical analysis can show the effect of random error in information upon the variance of states in the model. The mathematics require linear systems and additive Gaussian noise. In the model the production rate is independent of workforce; consequently the inventory and workforce equations can be considered separately. First, two, first order, exponential smoothing equations which help define the production rate and set desired inventory are:

\[ A_{k+1} = (1-a)A_k + aS_k + aX_k \]  
\[ B_{k+1} = (1-b)B_k + bS_k + bX_k \]  

The balance equation of inventory is:

\[ I_{k+1} = I_k + P_k - S_k \]  (inventory)  

and production rate is:

\[ P_k = A_k + u[dB_k - (I_k + Y_k)] \]  (production rate)  

Inserting (4) into (3) yields:

\[ I_{k+1} = A_k + duB_k + (1-u)I_k - S_k - uY_k \]  (5)
where:

A Exponential smoothing of sales information for production 
(units/month)

B Exponential smoothing of sales information for desired inventory 
(units/month)

C Exponential smoothing of sales information for workforce 
(units/month)

H Hiring (Firing) rate (men/month)

I Inventory (units)

W Workforce (men)

X Random error in sales information (units/month)

Y Random error in inventory information (units)

P Production rate (units/month)

a Smoothing constant for A (1/month)

b Smoothing constant for B (1/month)

c Smoothing constant for C (1/month)

d number of months of sales desired in inventory

r Productivity of workers [(units/month)/man]

f Fraction of desired workforce filled with full-time workers 
(dimensionless)

h Fraction adjustment of workforce to desired level per month (1/month)

u Fraction of the gap between desired and actual inventory adjusted 
per month by production rate (1/month)

v Fraction of the gap between desired and actual inventory affecting 
desired workforce adjusted per month (1/month)
Thus equations 1, 2, and 5 form the system of difference equations to be solved to find the impact of random error in information upon inventory.

The workforce dynamics follow.

First, a first order exponential smoothing of the sales rate is taken as the set point for workforce.

\[ C_{k+1} = (1-c)C_k + cS_k + cX_k \]  

(6)

Then the workforce balance equation is:

\[ W_{k+1} = W_k + H_k \]  

(workforce)  

(7)

Finally the hire-fire rate adjusts workforce to the desired workforce at a fraction (h) per month. The desired workforce is a fraction (f) of the workforce required to fill the sales rate and inventory adjustment needs with full-time workers and no overtime.

\[ H_k = h[(f/r)(C_k + v(dB_k - (I_k + Y_k))) - W_k] \]  

(hire-fire rate)  

(8)

Substituting 8 into 7 one obtains:

\[ W_{k+1} = (hfvd/r)B_k + (hf/r)C_k + (1-h)W_k - (hvf/r)I_k - (hvf/r)Y_k \]  

(9)

The equations 2, 6, and 9 form the system to be solved after finding inventory variance in order to find the variance of workforce. By solving for the variance of inventory \((I')\) and workforce \((W')\) as a function of variance in sales rate \((S')\), variance of sales information error \((X')\) and variance of error in inventory information \((Y')\), we can see how random error in information affects the states of inventory and workforce.
Solving first for inventory variance (I'), we set up the equations for propagation of the covariance matrix (M) from Equations 1, 2, and 5. (See Bryson and Ho, 5, p. 320-326).

\[
M_{k+1} = \begin{bmatrix}
1-a & 0 & 0 \\
1-b & 0 & 0 \\
1 & ud & 1-u \\
\end{bmatrix} M_k 
\begin{bmatrix}
1-a & 0 & 1 \\
0 & 1-b & ud \\
0 & 0 & 1-u \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
a & a & 0 \\
b & b & 0 \\
-l & 0 & -u \\
\end{bmatrix} \begin{bmatrix}
S' & 0 & 0 \\
0 & X' & 0 \\
0 & 0 & Y' \\
\end{bmatrix} \begin{bmatrix}
a & b & -1 \\
a & b & 0 \\
0 & 0 & -u \\
\end{bmatrix}
\]

Solving for the steady-state and assuming that:

\[
a = .25 \\
b = .083 \\
c = .167 \\
\]

\[d = .8 \\
u = .4 \\
h = .25 \]

\[I' = 1.547S' + .845X' + .25Y' \]

Solving in the same way for the variance of workforce, W' from equations 2, 6, and 9 we find:

\[W' = .0018S' + .0017X' + .00018Y' \]

As we can see from this analysis, random error in information and noise in the sales rate have very little effect upon the workforce, but greater effect upon inventory. Confirming the simulation, we see that random error in sales rate information has a greater effect than does random error in inventory information, but the randomness in sales has the greatest effect upon the variance of inventory.
APPENDIX B

ANALYSIS OF BIAS

Figure 4 shows that costs are relatively insensitive to bias in inventory information while they are quite sensitive to bias in sales rate information. We can examine the steady-state deviation of inventory, workforce, and overtime from values with perfect information to see the effect of bias.

Inventory, workforce and overtime are defined by the desired workforce and desired production rate equations. Following the notation used in Appendix A, in steady-state the production rate is:

\[ P = A + u(dB - I) \]  \[B1\]

and the workforce is:

\[ W = (f/r)(C + v(dB - I)) \]  \[B2\]

Sales Rate Information Bias

Sales rate bias is examined first. If \( s \) is the sales rate bias multiplier, \( SS \) is sales rate information.

In steady-state production rate equals sales rate and all average
sales rate equal \( sS \), so

\[ S = sS + u(dsS - I) \]  \[\text{[B3]}\]

Solving for \( I \) and finding the deviation of inventory from its desired value \( Y_S \) is:

\[ \Delta I = I^* - I' = Y_S - \frac{S}{\beta}(s + \beta Y_S - 1) = S(1 - s)(\frac{1}{u} + d) \]  \[\text{[B4]}\]

\( I^* \) Inventory with perfect information

\( I' \) Inventory with biased information

The difference of workforce with degraded information \( (W') \) from its value with perfect information \( (W^*) \) in steady-state is:

\[ \Delta W = W^* - W' = (f/r)S - (f/r)(sS + v(dsS - \frac{1}{u})(sS + udsS - S) \]
\[ = S(f/r)(1 - s)(1 - \frac{v}{u}) \]  \[\text{[B5]}\]

The difference in overtime manpower when information is perfect, \( O^* \), and when sales rate information is biased, \( O' \), is the negative of the change in workforce

\[ \Delta O = O^* - O' = - \Delta W \]  \[\text{[B6]}\]

Using the values of the constants listed in Appendix 3 and assuming
s = .8, we find:

\[ \Delta I = (500)(1 - .8)(\frac{1}{.4} + .8) = 330 \]

\[ \Delta W = (500)(\frac{.92}{.67})(1 - .8)(1 - \frac{.2}{.4}) = 8.1 \]

\[ \Delta O = -8.1 \]

When I = 70, inventory costs are $12,750 per month greater than with normal inventory. The substitution of overtime for 8.1 men adds $1,380 per month to costs.

**Inventory Information Bias**

The same steady-state analysis for bias in inventory information can be carried out. In steady-state, production rate equals the sales rate. The bias multiplier for inventory information is i. Then

\[ S = S + u(dS - II) \]  \[ \text{[B7]} \]

Solving for I with I'' the inventory with inventory information degraded.

\[ \Delta I = I^* - I'' = dS(1 - \frac{1}{l}) \]  \[ \text{[B8]} \]
Then finding the steady-state workforce with degraded inventory information $W''$, the change in workforce is:

$$\Delta W = W^* - W'' = S(f/r) - (f/r)\left[S + v(dS - iI)\right]$$  \[B9\]

$$= S(f/r) - (f/r)\left[S + v\left(dS - (ids/i)\right)\right]$$

$$= 0$$

Thus bias in inventory information has no effect on the steady-state workforce and overtime.

Comparing Eqs. [B1] and [B8] we see that for reasonable values of $u$, i.e., less than one, the bias multipliers $s$ and $i$ must become unreasonably small before bias in inventory information has a greater effect upon inventory than bias in sales rate information. This explains the much lower costs of bias in the inventory information. It is generally true (Chestnut and Meyer, ) that bias in feedback information, in this case inventory, has less effect upon behavior than bias in feedforward information, in this case sales rate. Assuming $i = .8$, a constant sales rate of 500 and the parameter values of Appendix A, we find that

$$\Delta I = (.8)(500)(1 - \frac{1}{.8})$$

$$= -100$$

which is one-third of the change with the same bias in sales rate. With the costs assumed, the higher inventory raises costs $400$. 
APPENDIX C

DYNAMO Listing of Computer Simulation Model

******** PRODUCTION AND INVENTORY EQUATIONS ********

\[ I_{k+1} = I_k + J_k (P_{j} - S_{j}) \]
\[ I_0 = 0 \]

\[ P_k = (W_k + Q_k M_k) (A_{Pr} \cdot k) \]
\[ A_{Pr} \cdot k = (N_{Pr} (1 + R_{Pr} \cdot Noise) + \text{STEP}(C_{Pr}, S_{C_{Pr}})) \]
\[ N_{Pr} = 5.67 \]
\[ R_{Pr} = 0 \]
\[ C_{Pr} = 0 \]
\[ S_{C_{Pr}} = 12 \text{ MONTHS} \]

\[ N_{Pr} \cdot k = A_{Pr} \cdot k \cdot W_p \cdot k \]

****** DEGRADATION OF INVENTORY INFORMATION ******

\[ D_{G_{I} \cdot k} = \text{SAMPLE}(D_{G_{I} \cdot k}, S_{D_{G_{I}}}, N_{D_{G_{I}}}) \]
\[ S_{D_{G_{I}}} = 0.25 \text{ MONTHS} \]
\[ N_{D_{G_{I}}} = 1 \]
\[ D_{G_{I} \cdot k} = (1 - Z_{D_{I} \cdot k}) (1 - Z_{D_{I} \cdot k}) D_{G_{I} \cdot k} \]
\[ Z_{D_{I} \cdot k} = 1 \]
\[ B_{I} = 1 \]
\[ D_{L_{I} \cdot k} = D_{I} \text{ INF}_{3}(I_{k}, T_{D_{I}}) \]
\[ T_{D_{I}} = 1 \text{ MONTHS} \]
\[ N_{L_{I} \cdot k} = \text{SAMPLE}(N_{O_{m_{I}}}, S_{D_{O_{m_{I}}}}, S_{T_{O_{m_{I}}}}, N_{O_{m_{I}}}) \]
\[ S_{D_{O_{m_{I}}}} = 0 \]
\[ S_{T_{O_{m_{I}}}} = 1 \text{ MONTHS} \]

****** WORKFORCE AND HIRING LAYOFF EQUATIONS ******

\[ W_t = W_p \cdot k + W_t \cdot k \]
\[ W_p = W_p \cdot j + (D_t) (W_{B_P} \cdot j - W_{P_L} \cdot j K) \]
\[ W_{P_L} = (W_{F D S} (S / E_{P_R})) \]
\[ W_{B_P} \cdot k = W_t \cdot k / T_{D} \]
\[ T_{D} = 2 \text{ MONTHS} \]
\[ W_t = W_t \cdot j + (D_t) (W_{H_J} \cdot B_P \cdot j - W_{T_L} \cdot j K) \]
\[ W_{T_L} = 0 \]
\[ W_{H_L} = \text{MAX}(0, W_{H_L} \cdot k) \]
\[ W_{L} = \text{MAX}(0, W_{H_L} \cdot k) \]
\[ W_{T_L} = \text{MIN}(W_{T_L} / D_{T}, W_{L} \cdot k) \]
\[ W_{P_L} = \text{CLIP}(\text{MIN}(W_{P} \cdot k / D_{T}, W_{L} \cdot k), 0, 0, W_{T} \cdot k) \]
\[ W_{F} = W_p \cdot k + Q \cdot W_{k} \]

****** CONTROL POLICIES ******

\[ D_{P} \cdot k = S_{P_R} \cdot k + I_{A_P} \cdot k \]

PRODUCTION RATE CONTROL

********* PRODUCTION RATE CONTROL **********

DESIRED PRODUCTION RATE
FORMATION DEGRADATION, PROD.-INV. MODEL PC2 10/17/70

SSP.K = SSP.J + (DT (1/TSSP) (DEGS.J - SSP.J)) SMOOTH SALES FOR PRODUCTION
TSSP=4 MONTHS TIME TO SMOOTH SALES FOR PRODUCTION
SSP=5

TAP.K = TABLE (TAP.DI - DEGI.K - 600, 600, 100) INVENTORY ADJ. PROD. RATE
TAP=-240/-200/-160/-120/-80/-40/0/40/80/120/160/200/240

WORKFORCE CONTROL POLICIES

DW.K = FWDS*( ISSW.K + IAW.K ) / EPR.K
FWDS = .92

SSW.K = SSW.J + (DT (1/TSSW) (DEGS.J - SSW.J) ) SMOOTHED SALES FOR WORKFORCE
TSSW=6 MONTHS TIME TO SMOOTH SALES FOR WORKFORCE
SSW=DEGS

TIW.K = TABLE (TIW.DI - DEGI.K - 600, 600, 100) INVENTORY ADJ. WORKFORCE
TIW=-120/-100/-80/-60/-40/-20/0/20/40/60/80/100/120

EPR.K = EPR.J + (DT (1/TAPR) (PRM.J - EPR.J) ) ESTIMATED PRODUCTIVITY
FPR=NPR
TAPR=4

TIME TO AVERAGE PRODUCTIVITY

TIME TO AVERAGE PRODUCTIVITY

PROD.K = DW.K * FPR.K

DESIRED INVENTORY

DI.K = (SSI.K) (4MSDI)
MSDI = 4 MONTHS

SSI.K = SSI.J + (DT (1/TSSI) (DEGS.J - SSI.J) ) SMOOTHED SALES FOR INVENTORY
TSSI=12 MONTHS TIME TO SMOOTH SALES FOR INVENTORY
SSI=DEGS

OVERTIME CONTROL

OM.K = WP.K * TABLE (TOM, NMDP.K / WP.K, -6, 6, 2) OVERTIME MANPOWER EQUIVALENT
TOM=-6/-5/-4/-3/-2/-1/0/1/2/3/4/5/6
NMDP.K = 1 ((DP.K - WP.K * EPR.K) / EW.K) OVERTIME MANPOWER DESIRED PROD. FASO=1

RS.K = RS.J + (DT) (DP.J - P.J) FRACTION ADJ. OF SCHEDULE BY OVERTIME
RS=0

DP.K = OM.K * APR.K

OVERTIME PRODUCTION RATE

STEP IN SALES RATE

β.K = (NORMS) (1+7S1*STEP.K+ZS2*RAMP.K+ZS3*RNS.K) SALES RATE
NORMS=500
7S1=0
7S2=0
7S3=0
STEP.K = STEP (HS, TS)
HS = .5 MONTHS
TS = 6

RAMP.K = RAMP (SR, TRU) + RAMP (-SR, TRD)
SR = .15
TRU = 6
TRD = 36
RNS.K = SAMPLE (NORMP (0, SDNS), STNS, 0) RANDOM NOISE IN SALES RATE

SALES RATE
NORMAL SALES RATE
STEP IN SALES RATE
HEIGHT IN SALES STEP
TIME FOR SALES STEP
RAMP IN SALES RATE
SLOPE OF THE RAMP IN SALES RATE
TIME OF THE RAMP UP
TIME OF THE RAMP DOWN
RANDOM NOISE IN SALES RATE
SDNS=.3 Standard Deviation of Noise in Sales
STNS=1 Sample Time of Noise in Sales

***** Degradation of Sales Information *****

DFGS,K=SAMPLE(DFGS1,K,STDFGS,IDEGS) Degraded Sales Information
IDEGS=5
STDFGS=.25 Months

DFGS1,K=(S,K*7DS1+DELS,K*(1-7DS1)) (RIASS+NS,K) Deg. Sales Info 1
7DS1=1
RIASS=1
DELS,K=LINEAR(S,K,TDS)
TDS=1 Month

NS,K=SAMPLE(NORMRV(MNIS,SDNIS),STNIS,MNIS) Noise in Sales Information
MNIS=0
SDNIS=0
STNIS=1

***** Piecewise Linear Cost Function from Holt *****

TMC1,K=TMC1,J+(DT)(MCl,J)
TMC1=0
MCl.K=W,K*MR1
MR1=240 Dollars/Month/Man

THC1,K=THC1,J+(DT)(HC1,J)
THC1=0
HC1.K=HR1*KWH.JK
HR1=180 Dollars per Man

TLC1,K=TLC1,J+(DT)(LC1,J)
TLC1=0
LC1.K=LR1*KWL,K
LR1=360 Dollars/Man

THLCl,K=THCl,K+TLC1,K
THLCl=0

TOCl,K=TOCl,J+(DT)(OC1,J)
TOCl=0

TOLC1,K=1.5*MR1*MAX(0,0,M,K)

TIC1,K=TOCl,J+(DT)(IC1,J)
TIC1=0

TIC1,K=1000*TARLF(TARIC1,K,-840,1440,120) Inventory Costs/Month
TARIC1=96/87/78/69/60/51/42/33/24/15/15/17.4/19.8/22.2/24.6/27.2
9.4/31.8/34.2

TC1,K=TMC1,K+THLC1,K+TOCl,K+TIC1,K Total Cost 1

ATC1,K=TC1,K+(AC1,K)(320-1,K)
AC1.K=TC1,K/(CUMP,K,E)
E=.0001

CUMP,K=CUMP,J+(DT)(P,K)
CUMP=0

***** Output Request *****

PRINT 1 S, I, DI
PRINT 2 P, DP, SSP, IAP, NP
PRINT 3 APR, EPR, RS, SDW, SDOM
PRINT 4 WP, WT, W, MWM, MOM
PRINT 5 DW, SSW, IA, VW, VDM
PRINT 6 WHL, WTL, WPL, WH
PRINT 7 S, DEGS, DEGS1, DELS, MNS, VNS
PRINT 8 I, DEGI, DEGI1, DEL1, MNI, VNT
### Summary Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS.K</td>
<td>Cumulative Sales</td>
</tr>
<tr>
<td>SDS.K</td>
<td>Standard Deviation in Sales Rate</td>
</tr>
<tr>
<td>VS.K</td>
<td>Variance of Sales Rate</td>
</tr>
<tr>
<td>MI.K</td>
<td>Cumulative Inventory</td>
</tr>
<tr>
<td>SDL.K</td>
<td>Standard Deviation in Inventory</td>
</tr>
<tr>
<td>VI.K</td>
<td>Variance of Inventory</td>
</tr>
<tr>
<td>MOP.K</td>
<td>Cumulative Overtime Production</td>
</tr>
<tr>
<td>SDOP.K</td>
<td>Standard Deviation of Overtime Production</td>
</tr>
<tr>
<td>MOM.K</td>
<td>Cumulative Overtime Manpower</td>
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<tr>
<td>SDOM.K</td>
<td>Standard Deviation of Overtime Manpower</td>
</tr>
<tr>
<td>MW.K</td>
<td>Cumulative Workforce</td>
</tr>
<tr>
<td>SDW.K</td>
<td>Standard Deviation of Workforce</td>
</tr>
<tr>
<td>WN.K</td>
<td>Cumulative Normal Production</td>
</tr>
<tr>
<td>SDNP.K</td>
<td>Standard Deviation of Normal Production</td>
</tr>
<tr>
<td>MP.K</td>
<td>Mean of Production</td>
</tr>
</tbody>
</table>

---

**Mean Sales Rate**

\[
MS.K = \frac{CUMS.K}{(TIMF.K+E)}
\]

**Cumulative Sales**

\[
CUMS.K = CUMS.J + (DT)(S.J)
\]

**Standard Deviation in Sales Rate**

\[
SDS.K = CUMS.K - (MS.K*MS.K)
\]

**Variance of Sales Rate**

\[
VS.K = CUMSS.K + (DT)(S.J*S.J)
\]

**Mean Inventory**

\[
MI.K = \frac{CUMI.K}{(TIME.K+E)}
\]

**Standard Deviation in Inventory**

\[
SDI.K = CUMI.K - (MI.K*MI.K)
\]

**Variance of Inventory**

\[
VI.K = CUMSI.K + (DT)(I.J*I.J)
\]

**Mean Overtime Production**

\[
MOP.K = \frac{CUMOP.K}{(TIME.K+E)}
\]

**Standard Deviation of Overtime Production**

\[
SDOP.K = CUMOP.K - (MOP.K*MOP.K)
\]

**Variance of Overtime Production**

\[
MOM.K = \frac{CUMOM.K}{(TIME.K+E)}
\]

**Standard Deviation of Overtime Manpower**

\[
SDOM.K = CUMOM.K - (MOM.K*MOM.K)
\]

**Variance of Overtime Manpower**

\[
MW.K = \frac{CUMW.K}{(TIME.K+E)}
\]

**Mean Workforce**

\[
CUMW.K = CUMW.J + (DT)(W.J)
\]

**Standard Deviation of Workforce**

\[
SDW.K = CUMW.K - (MW.K*MW.K)
\]

**Variance of Workforce**

\[
WN.K = \frac{CUMWN.K}{(TIME.K+E)}
\]

**Mean of Normal Production**

\[
MNP.K = \frac{CUMNP.K}{(TIME.K+E)}
\]

**Standard Deviation of Normal Production**

\[
SDNP.K = CUMNP.K - (MNP.K*MNP.K)
\]

**Variance of Normal Production**

\[
MP.K = \frac{CUMP.K}{(TIME.K+E)}
\]

**Mean of Production**
ORMATION DEGRADATION, PROD.-INV. MODEL PC2 10/17/70

\[ SDP.K = \text{SORT}(VP.K) \]
\[ VP.K = \left( \frac{\text{CUMSP.K}}{\text{TE.K}} \right) - (\text{MP.K} \times \text{MP.K}) \]
\[ \text{CUMSP.K} = \text{CUMSP.J} + (\text{DT}) \times (\text{P.J} \times \text{P.J}) \]
\[ \text{CUMSP} = 0 \]
\[ TF.K = \text{TIME.K} + E \]

**TIME PLUS EPSILON**

**OUTPUT EQUATIONS**

\[ \text{CUMOTM.K} = \text{CUMOTM.J} + (\text{DT})(\text{OTM.J}) \]
\[ \text{CUMOTM} = 0 \]
\[ \text{OTM.K} = \text{MAX}(0, \text{OM.K}) \]
\[ \text{CUMUTM.K} = \text{CUMUTM.J} + (\text{DT})(\text{UTM.J}) \]
\[ \text{CUMUTM} = 0 \]
\[ \text{UTM.K} = \text{MAX}(-0, \text{OM.K}) \]
\[ \text{AIL.K} = \frac{\text{CUMLI.K}}{(\text{TIME.K} + E)} \]
\[ \text{CUMLI.K} = \text{CUMLI.J} + (\text{DT}) \times \left( \text{MAX}(240 - I.J, 0) \right) \]
\[ \text{CUMLI} = 0 \]

**AVGHRN INVENTORY LOW**

\[ \text{AIL.K} = \frac{\text{CUMHI.K}}{(\text{TIME.K} + E)} \]
\[ \text{CUMHI.K} = \text{CUMHI.J} + (\text{DT}) \times \left( \text{MAX}(I.J - 480, 0) \right) \]
\[ \text{CUMHI} = 0 \]

**AVGHIHRN INVENTORY HIGH**

\[ \text{MNS.K} = \text{CUMNS.K} / (\text{TIME.K} + E) \]
\[ \text{CUMNS.K} = \text{CUMNS.J} + (\text{DT})(\text{NS.J}) \]
\[ \text{CUMNS} = 0 \]

**VARIANCE OF NOISE IN SALES**

\[ \text{VNS.K} = \left( \frac{\text{CUMNS.K}}{(\text{TIME.K} + E)} \right) + (\text{MNS.K} \times \text{MNS.K}) \]
\[ \text{CUMNS.K} = \text{CUMNS.J} + (\text{DT})(\text{NS.J} \times \text{NS.J}) \]
\[ \text{CUMNS} = 0 \]

**MEAN OF NOISE IN INVENTORY**

\[ \text{MNI.K} = \frac{\text{CUMNI.K}}{(\text{TIME.K} + E)} \]
\[ \text{CUMNI.K} = \text{CUMNI.J} + (\text{DT})(\text{NI.J}) \]
\[ \text{CUMNI} = 0 \]

**VARIANCE OF NOISE IN INV.**

\[ \text{VNI.K} = \left( \frac{\text{CUMNI.K}}{(\text{TIME.K} + E)} \right) - (\text{MNI.K} \times \text{MNI.K}) \]
\[ \text{CUMNI.K} = \text{CUMNI.J} + (\text{DT})(\text{NI.J} \times \text{NI.J}) \]
\[ \text{CUMNI} = 0 \]

\[ 00-PB2 \]
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