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THE EFFECTS OF INCOMPLETE INSURANCE MARKETS AND TRADING COSTS IN A CONSUMPTION-BASED ASSET PRICING MODEL
by
John Heaton and Deborah Lucas
January 1992
Working Paper No. 3379-92-EFA

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The Effects of Incomplete Insurance Markets and Trading Costs in a Consumption-Based Asset Pricing Model

John Heaton*
Deborah Lucas**

Abstract

Incomplete financial markets, coupled with undiversifiable idiosyncratic shocks, have the potential to explain a number of asset pricing puzzles. We study a model in which agents have access to a limited set of securities markets, while facing aggregate and individual uncertainty. Trade is limited by the presence of transactions costs, borrowing constraints, and short sales constraints. We find a systematic relation between the extent and type of market frictions, and their implications for asset prices and consumption policy. With trading costs or binding borrowing constraints, the riskfree rate falls and the risk premium rises relative to the complete markets case, and the term structure exhibits a positive forward premium. However, with costless access to either the stock or bond market, agents effectively smooth out transitory income shocks, and equilibrium asset prices resemble those in the complete markets case.

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1. Introduction

The problem of predicting consumption patterns in an environment of random income fluctuations and incomplete insurance markets has traditionally been the domain of macroeconomists. Friedman's (1957) permanent income hypothesis provided the insight that transitory income disturbances should be smoothed through capital market transactions. In the extensive literature that develops this idea, it is generally assumed that consumers face an exogenous and constant interest rate. Starting from the perspective of the consumption-based asset pricing model of Lucas (1976), a number of authors have recently asked the related question of how income fluctuations and incomplete insurance markets influence predicted rates of return (Aiyagari and Gertler (1991), Constantinides and Duffie (1991), Heaton and Lucas (1991), Huggett (1991), Ketterer and Marcet (1990), Lucas (1991), Telmer (1991), Marcet and Singleton (1991)). Essentially, these recent papers imbed a permanent income type model in an infinite horizon general equilibrium setting.

There are a number of motivations for studying the implications of incomplete markets for asset pricing models. The failure of simple versions of the representative consumer consumption-based asset pricing model to predict the observed statistical properties of asset returns can be attributed in part to the low variability of aggregate consumption growth rates.¹

¹ Hansen and Singleton (1983) and Mehra and Prescott (1985), among others, identified the asset pricing puzzles in empirical investigations of the Lucas (1976)
Market incompleteness breaks the link between individual and aggregate consumption, and in particular allows for the possibility that individual consumption growth is more volatile than in aggregate. Furthermore, with incomplete markets agents engage in trade. Thus one can address questions about trading volume, the impact of transactions costs, and the nature of liquidity trading.

These models have several drawbacks as well. Critics argue that the ability to arbitrarily close markets results in a loss of discipline; small changes in assumptions can produce qualitatively different conclusions. Complexity is also an issue. The models can only be solved numerically, and very few general results have been derived. Finally, the data requirements for testing or calibrating the models are often escalated by the fundamental heterogeneity.

The purpose of this paper is to provide clearer intuition about the predictions of these models, with an emphasis on showing which implications are robust and which are not. To do this we study a relatively simple, three period, two person model in which a large number of cases can be easily analyzed and interpreted. We are particularly interested in comparing the impact on asset prices of different types of trading frictions, including borrowing constraints, short sales constraints, quadratic costs and proportional costs. Our analysis suggests that with trading frictions, this class of models has the potential to help resolve

model. Hansen and Jagannathan (1991) cleanly characterize the connection between consumption volatility and asset prices.
the equity premium and low risk-free rate puzzles and to explain the forward premium in the term structure. Calibration results, however, are likely to be quite sensitive to the assumed market structure.

The paper is organized as follows. In Section 2 we present the frictionless model, and explain why the assumed timing of markets is critical. Section 3 introduces various trading frictions. We show that: (1) If trading in some markets is costless, agents substitute almost entirely towards trading in these markets. Hence trading costs influence prices only if they are present in all asset markets; (2) The assumed market structure has a large and systematic impact on predicted asset prices. In particular, the equity premium is larger and the risk-free rate lower when only borrowers pay trading costs in the bond market, or when there is a binding borrowing constraint; (3) The implications of proportional costs are similar to those of quadratic costs. In Section 4 we demonstrate that the model with trading frictions predicts a positive forward premium in the term structure. Section 5 concludes.

2. Asset Pricing with Incomplete Markets and Frictionless Trade

This section presents two simple models that illustrate how small changes in assumed market structure can significantly affect predicted asset prices and consumption allocations when income shocks are uninsurable. The first model is in the spirit of Mankiw (1986), while the second captures the idea behind the infinite

These models have the following basic structure. Each economy contains two (groups of) agents who are distinguished by their income realizations. Stochastic income derives from two sources each period: an aggregate stock dividend, $\delta$, and individual labor income, $Y^i$. Initially each agent owns half of the stock and holds no debt. Riskfree bonds are in zero net supply at time 0. The stock price is $p^s$, and the bond price is $p^b$.

We assume that labor income is uninsurable in the sense that agents cannot write contracts contingent on the outcome of future labor income. This assumption can be motivated by considerations such as moral hazard and legal limitations on contracting, or the observation that complete insurance does not appear to be a feature of modern economies. To the extent that many capital assets have limited collateral value, income from these assets conceptually falls into the same category as labor income. Conversely, any future labor income that can be used to secure debt is equivalent to dividend income. The intuition behind the model only requires that some component of income is uninsurable, regardless of its

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2 For example, Hayashi, Altonji and Kotlikoff (1991) and Cochrane (1991) offer evidence to this effect for the U.S. economy. One could also think of a bad event as inducing a taste shock that increases the marginal utility of income. This appears to have similar implications for asset prices.
The key idea is that the inability to fully insure against idiosyncratic income shocks implies that individual consumption volatility may be much higher than aggregate consumption volatility.

2.1 Model 1: Asset Markets Close Prior to the Resolution of Uncertainty

We begin with a two period model similar to Mankiw (1986). At time 0 each agent $i$ chooses consumption, $c_0^i$, stock purchases, $s^i$, and bond purchases, $b^i$, to maximize:

$$U(c_0^i) + \beta EU(c_1^i)$$

subject to the budget constraints:

$$c_0^i = y_0^i + \frac{\delta_0}{2} - b^ip^b - s^ip^s$$

$$c_1^i = y_1^i + \frac{\delta_1}{2} + b^i + s^i \delta_1$$

Market clearing requires:

$$c_1^t + c_2^t = y_t^a + \delta_t \quad t=0,1$$

$$s^t + s^{t+1} = 0$$

$$b^t + b^{t+1} = 0$$

Asset prices satisfy:

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3 Of course, identifying the source of uninsurable income is important for empirical applications of these models. Due to the limited data on the composition of income, equating labor income with uninsurable income is a practical first approximation.
To characterize implied consumption patterns and asset returns, we must specify the exogenous stochastic process governing dividends and labor income, and further restrict the form of the utility function. Assume \( U'''(c) > 0 \). We will consider the implications of the following income process. At time 0 all agents have equal labor and dividend income: \( Y_0 + \delta_0/2 \). At time 1, aggregate output is high with probability \( q \), and low with probability \( 1-q \). The aggregate dividend is \( \delta_1 \epsilon [0, \delta_H] \), and aggregate labor income is \( Y_1^a \epsilon [Y_L, Y_H] \). If realized output is high, each agent receives \( Y_H/2 \). If realized output is low, one agent receives \( \lambda Y_L \), while the other receives \( (1-\lambda)Y_L; \lambda \geq 1/2 \). This can be interpreted as the increased probability of becoming unemployed in a recession. Ex ante each agent faces the same distribution of future labor income, and cannot write a contract contingent on the realization of this income. Notice that the case \( \lambda=1/2 \) corresponds to the standard representative agent model.

Under these assumptions it is straightforward to demonstrate that the equity premium may be much higher than in the
representative consumer economy.\textsuperscript{4} Because agents have the same wealth and information at time 0, it follows that no trades occur in the asset markets, so that consumption equals income in each period. The price of stocks and bonds is then found by substituting income into equations (4) and (5). The stock price is constant for all $\lambda$ since we have assumed that the stock only pays a dividend in the high income state. The convexity of the marginal utility function implies that the bond price increases in $\lambda$. Thus the premium, $E(\delta_t)/p^s - 1/p^b$, increases in $\lambda$. If we assume that $U'(0) = \infty$, the premium becomes arbitrarily large as the share of income, $\lambda$, received by the employed agent goes to one.

When the premium is written as a ratio of the stock and bond returns rather than their difference, it has been shown for more general income and dividend processes that the proportional premium increases in the level of idiosyncratic risk (Mankiw (1986), Weil (1992)).\textsuperscript{5} However, idiosyncratic risk has two offsetting affects on the level of the premium, which is the object of empirical interest. While idiosyncratic risk increases the required return on stocks relative to bonds, it also decreases the required return on both stocks and bonds because it creates a precautionary demand

\textsuperscript{4} Rietz's (1988) demonstration that the equity premium puzzle can be resolved with the assumption of a small probability of extremely low (aggregate) consumption relies on a similar mechanism. Mehra and Prescott (1988) argue that disasters of the requisite magnitude are not supported by the data. It seems more plausible, however, that individuals face a non-negligible probability of a very bad shock.

\textsuperscript{5} These results also depend on the sign of the third derivative of the utility function.
for assets. It should be noted that one can easily find examples in which the proportional premium increases (relative to the complete markets case), but the level of the premium falls.\textsuperscript{6}

This analysis suggests that a model with incomplete insurance markets has the potential to resolve the equity premium and riskfree rate puzzles. However, the next section demonstrates that even when idiosyncratic risk implies a large premium in this model, the result is not robust to realistic changes in the assumed structure of asset markets when income shocks are transitory.

2.2 Model 2: Markets Close After the Resolution of Uncertainty

Here we take into account the possibility that agents can use asset markets to partially offset idiosyncratic income shocks. To provide agents with a chance to trade, we extend the above model to three periods. At time 0 agents can trade in one and two period riskfree bonds, and in the stock. At time 1, old two period bonds can be resold, and new 1 period bonds can be issued, and stock can be bought or sold. The stock pays a stochastic dividend at times 1 and 2.

As before, agents are assumed to be identical at time 0. They potentially receive different shares of aggregate income at time 1 and time 2, however, due to the employment shock at that time.

\textsuperscript{6} For instance, let \( U(c)=c^{0.4}/-4, \beta=0.9, \delta \in [.1, .2], Y \in [.8, 1.2], \) and normalize consumption to one at time 0. Each realization \((Y, \delta)\) has probability 0.25. The implied equity premium is 12.7\%. Now at time 1 increase the idiosyncratic shock, so that \( Y \in [.7, 1.3] \). The implied premium falls to 9.0\%, while the percentage premium increases from 1.09\% to 1.10\%. 
Agents choose asset holdings and consumption in each period to maximize:

\[ U(c_i^0) + \beta EU(c_i^1) + \beta^2 EU(c_i^2) \]  

subject to the budget constraints:

\[ c_i^0 = y_0^i + \frac{\delta_0}{2} - b_0^{i1}p_0^1 - b_0^{i2}p_0^2 - s_0^{i} \]
\[ c_i^1 = y_1^i + \left(\frac{1}{2} + s_0^i\right)\delta_1 + b_0^{i1} - p_1^{i1} - s_1^{i}p_1^1 \]
\[ c_i^2 = y_2^i + \left(\frac{1}{2} + s_0^i + s_1^i\right)\delta_2 + b_1^{i1} + b_0^{i2} \]

where \( b_t^{in} \) is net purchases of the \( n \) period bond at time \( t \) by agent \( i \), \( p_t^n \) is the price of the \( n \) period bond at time \( t \), \( s_t^i \) is net purchases of stock at time \( t \), and \( p_t^s \) is the stock price.

As above, since agents are identical at time 0, in equilibrium no assets are traded; \( b_0^{i1} = b_0^{i2} = s_0^i = 0 \). Without loss of generality, we assume that when aggregate output is low at time 1, the first agent receives the good idiosyncratic shock and the second agents receives the bad shock: \( Y_1^1 = Y_H \) and \( Y_1^2 = Y_L \), where \( Y_H > Y_L \). The equilibrium portfolio rule and asset prices are found by solving the first order conditions, (8)-(12), for \( i=1,2 \).

\[ p_0^{i1}U'(y_0 + \delta_0) = \beta E_0[U'(y_1^i + \frac{\delta_1}{2} - b_1^{i1}p_1^1 - s_1^{i}p_1^1)] \]  

\[ p_0^{i2}U'(y_0 + \delta_0) = \beta E_0[U'(y_1^i + \frac{\delta_1}{2} - b_1^{i1}p_1^1 - s_1^{i}p_1^1) (\delta_1 + p_1^1)] \]  

\[ p_0^{s}U'(y_0 + \delta_0) = \beta E_0[U'(y_1^i + \frac{\delta_1}{2} - b_1^{i1}p_1^1 - s_1^{i}p_1^1) (\delta_1 + p_1^1)] \]
\[ p_0^2 U'(Y_0 + \frac{\delta_0}{2}) = \beta^2 E_0[U'(Y_2^i + \delta_2 (\frac{1}{2} + S_1^i) + b_1^{i1})] \] (10)

\[ p_1^i U'(Y_1^i + \frac{\delta_1}{2} - s_1^i p_1^s - b_1^{ii} p_1^p) = \beta E_1[U'(Y_2^i + \delta_2 (\frac{1}{2} + S_1^i) + b_1^{i1})] \] (11)

\[ p_1^s U'(Y_1^i + \delta_1 - s_1^i p_1^s - b_1^{ii} p_1^p) = \beta E_1[U'(Y_2^i + \delta_2 (\frac{1}{2} + S_1^i) + b_1^{i1}) \delta_2] \] (12)

Market clearing implies that \( b_1^{i1} = -b_1^{21} \), and \( s_1^1 = -s_1^2 \) in each state.

In general, a closed form solution to the nonlinear system (8)-(12) cannot be found.\(^7\) We solve these equations numerically to illustrate some qualitative features of the model.\(^8\) We make the standard assumption of a constant relative risk aversion (CRRA) period utility function, \( U(c) = (c^{(1-\gamma)}-1)/(1-\gamma) \), \( \gamma > 0 \). It has the advantage of being relatively tractable, and of predicting stationary prices when the growth rate of income is stationary. Furthermore, these preferences have the desired property that increasing income volatility increases the required return on risky assets.

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\(^7\) The exception is quadratic utility. In this case it is easily shown that prices only depend on the aggregate endowment, independent of the wealth distribution. It should be noted, however, that even when prices are unaffected by the absence of insurance markets, welfare is reduced.

\(^8\) The model is solved using Newton's Method.
Whether asset markets will be used to smooth consumption depends critically on the persistence of the idiosyncratic shocks. Evidence from U.S. panel data suggests that income shocks have both a permanent and transitory component (e.g., Carroll (1991)). The ability to self-insure diminishes as shocks become more persistent, because more persistent shocks have a larger impact on permanent income and hence on desired consumption. We consider both cases, but focus primarily on transitory shocks.

a. Transitory Shocks

With transitory idiosyncratic shocks, the equilibrium has the characteristic that agents trade away from the initial allocation of equal stockholdings and no bonds in order to smooth consumption at time 1. At the initial allocation, agent 2 has a higher marginal rate of substitution between current and future consumption. Hence market clearing requires him to sell both assets to agent 1. The magnitude of the equity premium will depend on the residual variability of consumption after trading has taken place.

To calibrate the model, we assume that $\beta=0.95$, and $\gamma \in \{1, 3, 5, 7\}$. At time 0 each agent receives an equal share of dividend and labor income, with dividends comprising 12.5% of total income. Dividend and labor income grow by 2% on average each period. Aggregate dividends are high or low with equal probability, with $\delta_h = 1.3\delta_l$. When aggregate dividends are high, both agents receive equal labor income. When aggregate dividends are low at time 1,
the first agent receives income equal to 1.62 the income of the second agent.\textsuperscript{9} Time 2 labor income is also stochastic in the low aggregate state, and independent of the time 1 idiosyncratic shock: $Y_h = 1.10Y_L$. The lower variance at time 2 reflects the idea that if the economy were to continue for another period, the idiosyncratic income shocks would again be partially offset by trade.

Table 1 reports the average returns, the value of stocks and bonds traded in the low income state, and time 1 consumption variability. For comparison with section 2.1, we also report asset returns and consumption variability when time 1 is the terminal date, and time 1 consumption is constrained to equal income. These results demonstrate that despite the high individual income variability at time 1, the implied equity premium is substantially lower than in the no trade case. The fact that consumption variability is lower than income variability accounts for the difference; the coefficient of variation of consumption is about 9\%, while the coefficient of variation of income is 15.5\%.\textsuperscript{10} These results are qualitatively similar to those of the infinite horizon models of Lucas (1991), Heaton and Lucas (1991), and Telmer (1991).

\textsuperscript{9} The choice for the variability of individual income is loosely based on data from the Panel Study of Income Dynamics, as described in Heaton and Lucas (1991).

\textsuperscript{10} For comparison, the coefficient of variation of aggregate consumption is 2.64\%.
Table 1. Asset Returns, Trading Volume, and Consumption Volatility

<table>
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<td>$E(r^s)$</td>
<td>.069</td>
<td>.082</td>
<td>.062</td>
<td>.006</td>
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<tr>
<td>$E(r^b)$</td>
<td>.067</td>
<td>.073</td>
<td>.040</td>
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<td>$E(r^s - r^b)$</td>
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<td>.009</td>
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<td>$E(p^b)$</td>
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<td>.089</td>
<td>.086</td>
<td>.091</td>
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<td>.091</td>
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</tr>
<tr>
<td>$E(r^b)$</td>
<td>.049</td>
</tr>
<tr>
<td>$E(r^s - r^b)$</td>
<td>.006</td>
</tr>
<tr>
<td>$\sigma(c_1)/E(c_1)$</td>
<td>.155</td>
</tr>
</tbody>
</table>

*Volume is reported as a fraction of average per capita income at time 1.

Note that the predicted returns in Table 1 are not monotonic in risk aversion. In general, increasing the risk aversion coefficient produces two offsetting effects: (1) it increases the precautionary demand for savings which tends to lower the interest rate, and (2) it lowers the elasticity of intertemporal substitution which tends to raise the interest rate in an economy that exhibits positive average income growth.\(^{11}\)

This precautionary demand for savings has been suggested as a resolution of the low risk-free rate puzzle. For instance, in an infinite horizon model with a continuum of agents and no aggregate uncertainty, Huggett (1991) shows that uninsurable idiosyncratic

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\(^{11}\) Kandel and Stambaugh (1991) emphasize these points.
risk lowers the risk-free rate relative to the complete markets case. The results here suggest that although a precautionary demand may substantially lower the riskfree rate, it lowers the required return on equity as well. One must therefore be careful not create a "high equity return puzzle" while explaining the low risk-free rate puzzle.

From these calculations, it appears that limitations on insurance markets alone are insufficient to explain the equity premium or the low risk-free rate puzzles for low values of the risk aversion coefficient when income shocks are transitory.

b. Permanent Shocks

In this model, a permanent idiosyncratic shock can be represented most easily by constraining labor income at time 2 to equal the realization of labor income at time 1, in the event that a low aggregate time 1 realization occurred. As expected, we find a marked decrease in trading volume, and an increase in the equity premium. These results are consistent with those of Constantinides and Duffie (1991), who derive a class of permanent income shock processes for which there is no trade and a potentially high equity premium in an infinite horizon model with many agents.

To summarize, if agents have no access to asset markets, or if idiosyncratic income shocks are permanent, standard utility specifications can produce a high equity premium and a low riskfree
rate. However, with access to frictionless asset markets agents effectively smooth out transitory income shocks, and implied asset prices are similar to the representative consumer case. We now turn to the question of whether more moderate frictions in the asset markets can help to explain asset prices when income shocks are transitory.

3. Asset Pricing with Incomplete Markets and Trading Frictions

In this section we modify the above model with transitory income shocks to allow for borrowing constraints, short sales constraints, and trading costs. "Borrowing constraints" and "short sales constraints" are fixed trading limits, while a "trading cost" is essentially a tax on trades. As one might expect, predicted asset prices are sensitive to the assumed form of the market friction. Here we hope to provide intuition about the systematic differences in the implications of the various cost structures.

For the base case, trading costs are assumed to take the quadratic form:

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12 This suggests that the two period models of Mankiw (1986) and Weil (1992) can be interpreted as representing permanent shocks in longer horizon models.

13 Aiyagari and Gertler (1991) examine the equity premium and riskfree rate puzzles in a related model with a continuum of heterogeneous agents, and proportional or fixed trading costs. Their analysis differs in that there is no aggregate risk, and the government controls the supply of liquid assets so as to peg the interest rate. Heaton and Lucas (1991) calibrate an infinite horizon version of the model of this section.
\[ \kappa_s(s, p^s) = \pi^s(p^s s)^2 \]
\[ \kappa_b(b, p^b) = \pi^b(p^b b)^2, \text{ or} \]
\[ \kappa_b(b, p^b) = \pi^b(p^b b)^2 I_{(b<0)} \]

where the function \( I_{(b<0)} \) indicates that the cost is paid by the borrower but not the lender. We use the quadratic form primarily for tractability. However, it also reflects the idea that as more assets must be sold, agents first exhaust their more liquid assets. In section 3.d below we study the alternative assumption of proportional costs. Two cost specifications are considered in the bond market. In the first, which we call the "symmetric case", we assume that the borrower and lender each pay an equal trading cost. Motivated by the substantial spread generally observed between the borrowing and lending rate for consumers, the second specification assumes that only the borrower bears the transactions cost. In the tables below, we report the cost parameters, and the percentage of the proceeds of the asset sale absorbed by the costs: \( \pi^s p^s s \) or \( \pi^b p^b b \).

Now agents choose consumption and asset holdings to maximize (6) subject to the constraints:

\[ c^i_0 = Y^i_0 + \delta_0^i - \delta^i_0 p^i_0 - b^i_0 p^2_0 - s^i_0 p^s_0 - \kappa_{b1}(b^i_0, p^1_0) - \kappa_{b2}(b^i_2, p^2_0) - \kappa_s(s^i_0, p^s_0) \]
\[ c^i_1 = Y^i_1 + (1 + s^i_0) \delta_1 + b^i_0 b^i_1 - p^i_1 b^i_1 - s^i_1 p^s_1 - \kappa_{b1}(b^i_1, p^1_1) - \kappa_s(s^i_1, p^s_1) \]
\[ c^i_2 = Y^i_2 + (1 + s^i_0 + s^i_1) \delta_2 + b^i_1 + b^i_2 \]
\[ s_t^i \geq S_t \quad \text{(15)} \]
\[ b_t^{in} \geq B_t^n, \quad n=1,2. \]

The determination of asset prices depends on whether the short sales or borrowing constraints in equation (15) bind. In states in which both agents are unconstrained, prices are determined by the marginal rates of substitution of both agents. If a constraint binds, the price is determined by the marginal rate of substitution of the unconstrained agent, and the quantity equals the constrained value. The first order conditions for the unconstrained agents are similar to (8) - (12), but now include the trading costs. For reference, the Appendix lists these modified equations. Note that the returns of interest are now those realized between times 1 and 2.

Trading at time 1 in either market can be effectively eliminated by setting the appropriate constraint in (15) to be binding (i.e., \( B_t = 0 \), or \( S_t = 0 \)), or by making the transactions cost \( \pi^b \) or \( \pi^s \) sufficiently large. One might suppose that either mechanism for restricting trade would have similar implications for asset prices in this extreme case. As illustrated by Propositions 1 and 2, however, predicted asset prices are sensitive to the form of trading frictions. In particular, either a binding borrowing constraint or transactions costs borne entirely by the borrower tends to lower the implied lending rate relative to the case in which both the borrower and lender directly pay a cost.
Proposition 1. Assume that there is no trading in the stock at time 1; \( s^i_1 = 0 \). (a) \( p^t_1 \) will be lower in the case in which \( \pi^b = \infty \) and the cost is symmetric than in the case in which \( B_1 = 0 \). (b) \( p^t_1 \) is the same if \( \pi^b = \infty \) and only borrowers pay a transactions cost, or if \( B_1 = 0 \).

Proposition 2. Assume that there is no trading in the bond at time 1; \( b^{i*}_1 = 0 \). Then \( p^s_1 \) will be lower in the case in which \( \pi^s = \infty \) and the transactions cost is symmetric than in the case in which \( S_1 = 0 \).

(The proofs of propositions 1 and 2 are in the Appendix.)

These results suggest that the predictions of this class of models may be very sensitive to the assumed cost structure. To examine the implications of less extreme costs, we calculate equilibria for a variety of parameterizations and market structures.

a. Trading Costs in the Stock Market

Participation in the stock market is often thought to be more costly than participation in the bond market for a typical investor or borrower. One way to represent this idea in the model is to impose a trading cost in the stock market only \((\pi^b = 0, \pi^s > 0)\).
Table 2. Asset Returns, Trading Volume, and Consumption Volatility with Costly Trading in Stocks and Frictionless Trading in Bonds

<table>
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<td>.076</td>
<td>.163</td>
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<td>.161</td>
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<tr>
<td>E(rᵣ₋rᵇ)</td>
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<td>.003</td>
<td>.017</td>
<td>.016</td>
<td>.015</td>
</tr>
<tr>
<td>% Stock Trade Cost*</td>
<td>.00%</td>
<td>.02%</td>
<td>.03%</td>
<td>.00%</td>
<td>.08%</td>
<td>.16%</td>
</tr>
<tr>
<td>E(pᵣs)</td>
<td>.006</td>
<td>.000</td>
<td>.000</td>
<td>.006</td>
<td>.001</td>
<td>.001</td>
</tr>
<tr>
<td>E(pᵇb)</td>
<td>.093</td>
<td>.099</td>
<td>.099</td>
<td>.086</td>
<td>.091</td>
<td>.091</td>
</tr>
<tr>
<td>σ(cᵢ)/E(cᵢ)</td>
<td>.088</td>
<td>.088</td>
<td>.088</td>
<td>.093</td>
<td>.093</td>
<td>.093</td>
</tr>
</tbody>
</table>

* The ratio of total trading costs to the value of shares traded.

Table 2 shows that imposing a trading cost in the stock market alone has little affect on asset prices or consumption allocations. For comparison, columns 1 and 4 report the results for the case of zero costs, in which agents trade stocks as well as bonds. Note that the cost parameter πᵣ is exogenous, but the realized trading costs, "% Stock Trade Cost," is endogenous since it depends upon the quantity traded. By substituting towards trading in the bond market, agents continue to smooth consumption without incurring trading costs.

Interestingly, agents prefer to substantially alter their portfolio composition (relative to the frictionless market) rather than pay transactions costs or tolerate more volatile consumption. This is consistent with Constantinides' (1986) result that transactions costs have little affect on asset prices when the only reason to trade is portfolio rebalancing.
Similar results obtain (but are not reported here) when there is costly trading in bonds and free trading in stocks. In this case agents cease to trade in the bond market, and smooth consumption by selling stock. Asset prices again remain largely unaffected in the absence of binding short sales constraints.

b. Trading Costs in Both Markets

From the above results, it appears that to increase consumption volatility, costs must be imposed in both markets simultaneously. Table 3 reports results for a range of small costs, under the assumption that only the borrower bears a cost in the bond market.

Table 3. Asset Returns, Trading Volume, and Consumption Volatility with Costly Trading in Stocks and Bonds, $\pi^s = .2$, Asymmetric Transactions Costs in the Bond Market

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^b$</td>
<td>.05</td>
<td>.20</td>
<td>.40</td>
<td>.05</td>
<td>.20</td>
<td>.40</td>
</tr>
<tr>
<td>$E(r^s)$</td>
<td>.076</td>
<td>.077</td>
<td>.077</td>
<td>.164</td>
<td>.171</td>
<td>.175</td>
</tr>
<tr>
<td>$E(r^s-r^b)$</td>
<td>.005</td>
<td>.010</td>
<td>.013</td>
<td>.020</td>
<td>.032</td>
<td>.041</td>
</tr>
<tr>
<td>% Bond Trade Cost</td>
<td>.41%</td>
<td>1.28%</td>
<td>1.79%</td>
<td>.38%</td>
<td>1.48%</td>
<td>2.48%</td>
</tr>
<tr>
<td>% Stock Trade Cost</td>
<td>.11%</td>
<td>.38%</td>
<td>.73%</td>
<td>.10%</td>
<td>.15%</td>
<td>.37%</td>
</tr>
<tr>
<td>$E(p^s)$</td>
<td>.006</td>
<td>.022</td>
<td>.041</td>
<td>.006</td>
<td>.008</td>
<td>.021</td>
</tr>
<tr>
<td>$E(p^b)$</td>
<td>.091</td>
<td>.072</td>
<td>.050</td>
<td>.086</td>
<td>.083</td>
<td>.069</td>
</tr>
<tr>
<td>$\sigma(c_1)/E(c_1)$</td>
<td>.088</td>
<td>.092</td>
<td>.094</td>
<td>.093</td>
<td>.094</td>
<td>.095</td>
</tr>
</tbody>
</table>
Table 4. Asset Returns, Trading Volume, and Consumption Volatility with Costly Trading in Stocks and Bonds, $\pi^s = .2$, Symmetric Transactions Costs in the Bond Market

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^b$</td>
<td>.05</td>
<td>.20</td>
<td>.40</td>
<td>.05</td>
<td>.20</td>
<td>.40</td>
</tr>
<tr>
<td>$E(r^s)$</td>
<td>.076</td>
<td>.077</td>
<td>.074</td>
<td>.167</td>
<td>.165</td>
<td>.167</td>
</tr>
<tr>
<td>$E(r^s-r^b)$</td>
<td>.003</td>
<td>.003</td>
<td>.003</td>
<td>.020</td>
<td>.016</td>
<td>.017</td>
</tr>
<tr>
<td>% Bond Trade Cost</td>
<td>.35%</td>
<td>.78%</td>
<td>1.11%</td>
<td>.38%</td>
<td>.87%</td>
<td>1.28%</td>
</tr>
<tr>
<td>% Stock Trade Cost</td>
<td>.30%</td>
<td>.84%</td>
<td>1.02%</td>
<td>.10%</td>
<td>.73%</td>
<td>.95%</td>
</tr>
<tr>
<td>$E(p^s)$</td>
<td>.017</td>
<td>.044</td>
<td>.057</td>
<td>.006</td>
<td>.041</td>
<td>.053</td>
</tr>
<tr>
<td>$E(p^b)$</td>
<td>.079</td>
<td>.047</td>
<td>.031</td>
<td>.085</td>
<td>.049</td>
<td>.036</td>
</tr>
<tr>
<td>$\sigma(c_1)/E(c_1)$</td>
<td>.090</td>
<td>.094</td>
<td>.096</td>
<td>.094</td>
<td>.095</td>
<td>.096</td>
</tr>
</tbody>
</table>

Table 4 repeats the experiment of Table 3 under the assumption that borrowers and lenders both pay a transactions cost in the bond market. As one might anticipate from proposition 1, the interest rate is much higher than when only the borrower pays, and the equity premium is correspondingly lower.

To understand these results, notice that transactions costs have both a direct and indirect affect on the premium. The direct effect is that a lender requires a higher rate of return as costs increase, and conversely for the borrower. When only the borrower pays the trading cost, the decrease in the demand for funds causes the equilibrium borrowing rate to fall. The indirect effect is
that higher costs reduce the volume of trade.\textsuperscript{14} This indirect effect increases consumption volatility, and hence the predicted premium. For the parameters considered, the direct effect dominates. It must be stressed that while these qualitative results appear to be fairly general, we do not consider the reported level of returns informative.\textsuperscript{15}

One might expect that higher trading costs would cause the premium to increase more steeply for higher levels of risk aversion. Table 4 suggests that this need not be the case. An offsetting influence is that with higher risk aversion, agents are willing to incur higher transactions costs to smooth consumption. The net affect of risk aversion on the interaction between the premium and transactions costs is therefore ambiguous.

c. Borrowing or Short Sales Constraints

In this section we look at the impact of borrowing and short sales constraints on asset prices. The borrowing constraint might be thought of as representing the type of credit rationing suggested by Stiglitz and Weiss (1981), who show that consideration of moral hazard may cause lenders to deny credit even if borrowers promise a high interest rate. The short sales constraint reflects

\textsuperscript{14} The analysis of Aiyagari and Gertler (1991) examines the direct effect only, since there is no aggregate risk to differentiate stocks and bonds in their model.

\textsuperscript{15} For quantitative implications of a related model, see Heaton and Lucas (1991).
the idea that individuals may find short-selling stock prohibitively costly.

Table 5 reports the results of limiting the maximum number of bonds issued by varying the level of the borrowing constraint, while permitting costly trading in stocks. With a sufficiently restrictive borrowing constraint, the model predicts a large premium and a low riskfree rate. Recall that when the borrowing constraint binds, the bond price is determined by the marginal rate of substitution of the lender, who in equilibrium must be content to lend no more than the maximum allowed. The interest rate must therefore fall relative to the frictionless case to reduce desired lending.

Although these results are encouraging, we have several reservations about this case. First, the restriction on borrowing required to generate a high premium may be unrealistically severe. For cases in which the reported equity premium is high, borrowers would willingly pay a substantial spread over the market rate. For instance, for the parameters in the third column of Table 5, the interest rate implied by the marginal rate of substitution for the constrained borrowers is 11.3%, while the market rate is only 4.4%. Secondly, the results are quite sensitive to the constraint specification. Small changes in the borrowing limit in Table 5 cause large changes in implied prices. In experiments not reported here, we find that the opposite assumption of a binding short sales constraint in the stock market and costly trading in bonds tends to produce a negative premium, since limitations on the amount of
stock sold increase its price relative to the unconstrained case. It seems then, that an accurate empirical measure of borrowing constraints must be found before it can be persuasively argued that borrowing constraints resolve the equity premium and low risk free rate puzzles.

Table 5. Asset Returns, Trading Volume, and Consumption Volatility with Costly Trading in Stocks, and Binding Borrowing Constraints in the Bond Market

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1</th>
<th>1</th>
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<th>5</th>
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</thead>
<tbody>
<tr>
<td>γ</td>
<td>0.08</td>
<td>0.05</td>
<td>0.00</td>
<td>0.08</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>πₕ</td>
<td>0.076</td>
<td>0.077</td>
<td>0.079</td>
<td>0.163</td>
<td>0.167</td>
<td>0.174</td>
</tr>
<tr>
<td>E(rₕ)</td>
<td>0.008</td>
<td>0.019</td>
<td>0.035</td>
<td>0.026</td>
<td>0.041</td>
<td>0.065</td>
</tr>
<tr>
<td>% Stock Trade Cost</td>
<td>0.51%</td>
<td>1.46%</td>
<td>3.08%</td>
<td>0.81%</td>
<td>1.90%</td>
<td>3.74%</td>
</tr>
<tr>
<td>E(pₕs)</td>
<td>0.012</td>
<td>0.033</td>
<td>0.069</td>
<td>0.018</td>
<td>0.043</td>
<td>0.084</td>
</tr>
<tr>
<td>E(pₕb)</td>
<td>0.083</td>
<td>0.053</td>
<td>0.000</td>
<td>0.073</td>
<td>0.046</td>
<td>0.000</td>
</tr>
<tr>
<td>σ(c₁)/E(c₁)</td>
<td>0.091</td>
<td>0.098</td>
<td>0.112</td>
<td>0.094</td>
<td>0.096</td>
<td>0.110</td>
</tr>
</tbody>
</table>

d. Proportional Costs

What if trading costs in the stock market do not increase with the volume traded? To consider this possibility, we substitute proportional costs for quadratic costs by replacing equation (13) with:

\[
\kappa_s(s, p_s) = \pi_s p_s |s|
\]

\[
\kappa_b(b, p^b) = \pi^b p^b |b| I(b < 0), \quad \text{or}
\]

\[
\kappa_b(b, p^b) = \pi^b p^b |b|
\]
Solving the model with proportional costs is complicated by the discontinuity in the marginal cost functions. The Euler equations (A1)-(A5) can be solved as before to determine prices and quantities under the assumption that agents trade in both markets. It must then be checked whether given these prices, agents prefer to trade in both markets, one market, or not at all.

Table 6. Asset Returns, Trading Volume, and Consumption Volatility, with Proportional Costs in the Stock Market, and a Short Sales Constraint in the Bond Market

<table>
<thead>
<tr>
<th>γ</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>.05</td>
<td>.05</td>
<td>.05</td>
<td>.05</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>π^s</td>
<td>.005</td>
<td>.010</td>
<td>.025</td>
<td>.005</td>
<td>.010</td>
<td>.025</td>
</tr>
<tr>
<td>E(r^s)</td>
<td>.081</td>
<td>.086</td>
<td>.103</td>
<td>.168</td>
<td>.175</td>
<td>.194</td>
</tr>
<tr>
<td>E(r^s-r^b)</td>
<td>.008</td>
<td>.014</td>
<td>.030</td>
<td>.024</td>
<td>.030</td>
<td>.046</td>
</tr>
<tr>
<td>E(p^s)</td>
<td>.047</td>
<td>.047</td>
<td>.047</td>
<td>.046</td>
<td>.046</td>
<td>.046</td>
</tr>
<tr>
<td>E(p^b)</td>
<td>.051</td>
<td>.051</td>
<td>.051</td>
<td>.045</td>
<td>.045</td>
<td>.045</td>
</tr>
<tr>
<td>σ(c_t)/E(c_t)</td>
<td>.088</td>
<td>.088</td>
<td>.089</td>
<td>.094</td>
<td>.094</td>
<td>.094</td>
</tr>
</tbody>
</table>

Table 6 reports expected returns, trading volume, and consumption volatility as a share of per capita income for proportional costs in the stock market between .5% and 2.5%, and a borrowing constraint set to approximately 5% of income. The results are qualitatively similar to those reported for quadratic costs in Table 5. Agents trade up to the maximum permitted in the frictionless bond market, and also trade in stocks. Comparing the second column in Table 5 with the third column in Table 6, more stocks are traded and consumption volatility is lower for proportional than for comparable quadratic costs. The equity
premium is similar in both cases, but the level of the returns is higher for proportional costs. The differences can be explained by the fact that with quadratic costs the marginal cost is double the average cost, which discourages higher trading volume for similar levels of average costs. In these examples, the equity premium appears to be largely determined by the trading costs in the stock market. The higher level of overall returns in the case of proportional costs reflects a lower precautionary demand due to more effective smoothing.

In simulations not reported here, we considered proportional costs in both markets without imposing a binding borrowing constraint. For many parameterizations, the discrete jump in marginal trading costs at zero motivates agents to trade exclusively in the lower cost market. This produces a multiplicity of equilibrium prices that support no trade in the higher cost market.

4. Incomplete Insurance Markets and the Term Structure

Related asset pricing puzzles arise in the literature on the term structure of interest rates. The representative agent model calibrated with aggregate U.S. time series data has difficulty matching the statistical properties of the term structure.\textsuperscript{16} One

\textsuperscript{16} Backus, Gregory and Zin (1989) demonstrate this failure by calibration, and a number of recent papers further examine this issue. Donaldson, Johnsen and Mehra (1990) consider the term structure implications of the one-good stochastic growth model. Although the model matches some characteristics of the data, it does not explain the forward premium. In a model with idiosyncratic shocks of
aspect of this is the failure of the standard model to predict the positive average forward premium. In this section we show that the model with uninsurable income shocks and trading frictions generates a positive forward premium.\footnote{17}

In our model, the implied forward rate is the one period rate on time 1 investments that can be locked in at time 0 by buying a two period bond and selling a one period bond of equal present value: \( f_0 = \frac{p_0^1}{p_0^2} - 1 \). The expected future spot rate is the one period rate on time 1 investments that agents expect to have available at time 1, based on their time 0 information: \( E_0(1/p_1^1) - 1 \). The forward premium can then be written as \( \frac{p_0^1}{p_0^2} - E_0(1/p_1^1) \). The forward rate and expected future spot rate at time 0 are found by solving equations (A1) - (A5).

Table 7 reports the implied forward and spot rates for the parameterizations used in Tables 3 and 4. The model consistently generates a positive forward premium that increases with bond trading costs. Similar results can also be obtained with borrowing constraints. Notice that when trading costs are zero, we get the

\[ f_0 = \frac{p_0^1}{p_0^2} - 1 \]

very high frequency relative to aggregate shocks, Mehrling (1991) shows that the forward premium will be positive if the aggregate shocks dominate the idiosyncratic shocks.

\footnote{17} Because Treasury securities pay a fixed nominal return, most models of the term structure incorporate money. For instance, Backus, Gregory and Zin (1989) add a cash-in-advance constraint to the Mehra and Prescott (1986) model. Since our results are suggestive rather than quantitative, for simplicity this analysis is in real terms.
standard result that with low risk aversion, the forward rate is approximately equal to the future spot rate.

Table 7. Forward Rates and Expected Future Spot Rates, $\pi^s = .2$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^b$</td>
<td>.05</td>
<td>.20</td>
<td>.40</td>
<td>.05</td>
<td>.20</td>
<td>.40</td>
</tr>
</tbody>
</table>

| One-sided bond costs | Implied forward rate | .073 | .075 | .075 | .158 | .164 | .172 |
| Expected future spot rate | .071 | .067 | .064 | .144 | .139 | .132 |

| Two-sided bond costs | Implied forward rate | .074 | .075 | .076 | .160 | .172 | .169 |
| Expected future spot rate | .073 | .074 | .074 | .147 | .151 | .150 |

As in the analysis of Section 3.b, there is a direct and indirect affect of trading costs, with a stronger direct effect when the trading cost is only paid by the borrower. The indirect effect again is through an increase in the variance of the marginal rate of substitution, which tends to increase the forward premium when (individual) consumption exhibits negative serial correlation.

This analysis suggests that uninsurable income shocks may help explain one of the more persistent term structure puzzles. However, by using a simple three period model, we have introduced an asymmetry between the pricing of one and two period bonds.
Since in equilibrium two period bonds are in zero net supply, there is no direct affect of transactions costs on their price, while there is for the one period bond at time 1. The question remains of whether the prediction of a relatively large positive forward premium will obtain in a long horizon model that treats bonds of different maturities more symmetrically.\footnote{Another issue that is better examined in an infinite horizon model is the differential volatility of returns across maturities. We plan to examine these questions using the model in Heaton and Lucas (1991).}

5. Conclusions

Incomplete financial markets have the potential to explain a number of asset pricing puzzles because the presence of undiversifiable idiosyncratic risk affects the precautionary demand for assets and individuals' attitude towards aggregate uncertainty. In this paper we examine a three period model in which individuals have access to a limited set of securities markets, while facing aggregate and individual uncertainty. The agents also face restrictions on their trading due to quadratic or proportional transactions costs, borrowing constraints, and short sales constraints.

This analysis, coupled with the results of the previous studies discussed earlier, suggests that there is a systematic relation between the extent and type of market frictions, and their implications for asset prices and consumption policy. The first important regularity is that when agents have unrestricted access
to any financial market, they effectively smooth out transitory idiosyncratic income shocks by trading primarily in the frictionless securities. The implied asset prices are then similar to the representative consumer case. With effective trading frictions in all markets, however, asset prices vary predictably with the assumed market structure. As the cost parameter increases with quadratic or proportional costs, the riskfree rate tends to fall and the risk premium tends to rise. The incidence of the costs, as well as their level, heavily influence the pricing predictions. In particular, the premium is larger and the riskfree rate is lower when only the borrower directly bears a transactions cost in the bond market. Finally, imposing a binding borrowing constraint in the bond market has the most dramatic affect in the direction of explaining the asset pricing puzzles.

The results reported in this paper are suggestive of the potential importance of market incompleteness for explaining asset market behavior. Clearly, though, many empirical and theoretical issues remain unresolved; we mention only a few. What constitutes "reasonable" magnitudes for the frictions in this type of model is an open question. Since the predictions of the model are most promising under the assumption of a binding borrowing constraint, endogenizing this constraint, perhaps by modeling a more fundamental credit market imperfection, would make this a more persuasive explanation for rate differentials. Finally, we have focussed primarily on the predictions about first moments. Whether
trading frictions will help to match higher moments of asset prices remains to be investigated.
Appendix

First order conditions for the maximization problem (7):

\[
\begin{align*}
    & p_0 U'(Y_0 + \frac{\delta_0}{2}) = \beta E_0 \left[ U'(Y_1 + \frac{\delta_1}{2} - b_1 p_1^i - s_i p_1^s - \kappa_b(b_1, p_1^i) - \kappa_s(s_1, p_1^s)) \right] \quad (A1) \\
    & p_0^s U'(Y_0 + \frac{\delta_0}{2}) = \beta E_0 \left[ U'(Y_1 + \frac{\delta_1}{2} - b_1 p_1^i - s_i p_1^s - \kappa_b(b_1, p_1^i) - \kappa_s(s_1, p_1^s)) \right] (\delta_1 + p_1^s) \quad (A2) \\
    & p_0^2 U'(Y_0 + \frac{\delta_0}{2}) = \beta^2 E_0 \left[ U'(Y_2 + \frac{\delta_2}{2} + s_i + b_1) \right] \quad (A3) \\
    & (p_1^i + \frac{\partial \kappa_b(b_1, p_1^i)}{\partial b}) U'(Y_1 + \frac{\delta_1}{2} - s_i p_1^s - b_1 p_1^i - \kappa_b(b_1, p_1^i) - \kappa_s(s_1, p_1^s)) = \beta E_1 \left[ U'(Y_2 + \frac{\delta_2}{2} + s_i + b_1^i) \right] \quad (A4) \\
    & (p_1^s + \frac{\partial \kappa_s(s_1, p_1^s)}{\partial s}) U'(Y_1 + \frac{\delta_1}{2} - s_i p_1^s - b_1 p_1^i - \kappa_b(b_1, p_1^i) - \kappa_s(s_1, p_1^s)) = \beta E_1 \left[ U'(Y_2 + \frac{\delta_2}{2} + s_i + b_1^i) \delta_2 \right] \quad (A5)
\end{align*}
\]

Proof of Propositions 1 and 2. (1) Let $Y_1^1 = Y_{H^1}$, and $Y_1^2 = Y_L$; $Y_H > Y_L$. As $\pi^b \to \infty$, $b \to 0$ and $\pi^b(b^1 p_1 b^2) \to 0$ (since $p_1^i$ is bounded away from $\infty$ when income is strictly greater than 0). Let $x$ equal the limit as $\pi \to \infty$ of $(\partial \kappa/\partial b)/p_1^i$. Then with symmetric trading costs and without a borrowing constraint, the bond price must satisfy:
\[ p_1^1 = \beta \frac{E_1[U'(Y_2 + \frac{\delta_2}{2})]}{U'(Y_h + \frac{\delta_1}{2}) (1 + x)} \]  

(A6)

and

\[ p_1^1 = \beta \frac{E_1[U'(Y_2 + \frac{\delta_2}{2})]}{U'(Y_L + \frac{\delta_1}{2}) (1 - x)} \]  

(A7)

When \( B_1 = 0 \), the bond price is determined by the marginal rate of substitution of the agent with the high time 1 shock:

\[ p_1^1 = \beta \frac{E_1[U'(Y_2 + \frac{\delta_2}{2})]}{U'(Y_h + \frac{\delta_1}{2})} \]  

(A8)

For the price in (A6) and (A7) to agree, \( x > 0 \) since \( U'' < 0 \). Then the price given by (A6) must be lower than the price given by (A8). This proves proposition 1(a). Part (b) follows from noting that when the lender (who must be agent 1) pays no costs, the bond price again must satisfy equation (A8). (2) An analogous argument establishes the second proposition.
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