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THE EQUALIZATION OF SCHOOL EXPENDITURES  
IN MASSACHUSETTS

Charles B. Stabell, <sup>o</sup> Jerrold M. Grochow <sup>Marwick</sup>  
and Anders Haan <sup>o</sup>

606-72

June 1972

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## NOTE and ACKNOWLEDGEMENTS.

This is the report of a term project for the course 15.961, Seminar on Model Building, given by Professor John D. C. Little in the fall of 1971. The project, which has been continued and elaborated during the spring of 1972, does not represent a comprehensive study of the question of school expenditures and their financing. It however introduce new analysis and understanding of the problems and issues. Our purpose in writing the report is two-fold:

- 1) To distribute for comments and discussion our analysis and findings concerning the Massachusetts system of financing primary and secondary education.
- 2) To indicate, by means of a concrete example, the potential of interactive computer based models to help identify problems, clarify alternatives, and evaluate consequences with respect to issues facing public policy makers.

One issue that we have not addressed, and that we think needs to be looked into in future research, are the particular problems and possibilities that are encountered when dealing with public systems. An example of such an aspect is the diffuse policy making structure encountered in the area of education policy formation. It would seem that such a structure has important consequences for the organization and introduction of computer based decision support systems in this area.

During the project, we have received assistance from a variety of people. We especially wish to thank Dr. Paul Cook for advice and for financial support of computer time. The original study area was suggested by Mr. Lawrence Kotin, at that time on the Governors staff.

We have discussed the issues at length with Mrs. Charlotte Ryan of the Massachusetts Educational Conference Board. (Appendix A contains some of her comments to the report in its final form.) The results have been reviewed with Dr. Neil Sullivan and Mr. Everett Thistle of the Department of Education.

We especially wish to thank Meredyth A. Davies for trying to introduce some common sense in the report and for typing the numerous drafts of the paper.





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SUMMARY

This is a report of the results to date of a project concerned with the financing of primary and secondary education in the Commonwealth of Massachusetts. The central concern is the illustration of those aspects of the current system and proposals which have a bearing on the accomplishment of stated objectives. In particular, the objective with which we are most concerned is that of equalization between school districts of average school expenditures per child within school districts. The project has identified a number of problems with the existing system of state aid and with some of its central assumptions. In order to overcome these difficulties and develop a better system, a number of new areas of study are proposed.

The present NESDEC state aid formula, initially implemented in 1966, sought to equalize potential expenditure per child.<sup>1</sup> Current state aid proposals are mainly modifications of the formula approach. This study examines possible future effects of old and new formulas by performing simulations based on actual Chapter 70 data for the 351 school districts in Massachusetts. The simulations follow the effects of the formulas over a 7 year period into the future. In addition, mathematical analyses are also performed to highlight certain critical variables.

The general conclusion is that the existing system and current new proposals give very similar results with respect to equalization of expenditures. Their main differences lie in the share of the educational expenditures covered by the State. In terms of actual expenditures per child in the school districts, neither the historical development, nor

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<sup>1</sup>Potential expenditure equalization is an equalization when all school districts are giving the same local effort in terms of a uniform local school tax rate on property.



our simulation results concerning future developments, are encouraging. The general incentive character of the NESDEC formula does not solve the problem of equalization. It is generally considered that the incentive effect is lost on the poor districts, and is effective primarily in the rich districts, thereby increasing inequality.

Detailed analysis of the various effects which contribute to the performance (or non-performance) of the equalization aid formula include the following:

- (1) The constraints in the formula work against equalization (discussed in section 3.1).
- (2) The formula uses both the total school population and the public school population instead of the latter exclusively. This leads to distortions in equalization due to variations in the ratio of the two figures (as discussed in section 3.2).
- (3) The potential equalization is valid only at a unique uniform local tax rate in the state, given a level of funding and a set of formula parameters (as discussed in sections 3.3 and 3.4).

The third point above is the most important. It implies that a funded formula-based-system is not a general potential equalizer, but equalizes only under a very specific condition. It also implies that a decision about level of funding is in reality a decision about what should be the expenditure per pupil in the Commonwealth. It follows that in most cases an underfunded formula is disqualifying.

The Serano and subsequent court decisions seem to give a clear indication of what the objective of an equalization program should be. The decision emphasizes the rights of the child, rather the prerogatives of any arbitrary governmental unit. Governmental units have the responsibility to guarantee the rights of the child. Therefore, equalization of potential expenditures (as has been emphasized thus far in the formula



approach) is far less important than a greater equalization of actual expenditures.

It follows that attention must be focused on financial decision making at the local level. Any new proposal must consider the issue of local versus central control. Furthermore, it will be important to determine the impact of local control not only on financing but, insofar as possible, on other issues such as the quality of education.

The conclusion of the study is that new alternatives for achieving equalization should be created and analyzed. Equalization is a complex problem involving a number of interrelated issues that must be considered. Important issues are:

1. The method of raising funds.
2. The method of distributing funds.
3. The ability of the school districts to absorb additional funds effectively.
4. The dynamics of the school population and tax base.
5. Changes in educational costs.
6. The burden of other municipal services.

These questions become involved with organizational issues such as:

1. The size of school districts.
2. The balance of control between state and local district and within the district.
3. School desegregation.

The M.I.T team feels that a computer-assisted modeling approach using an appropriate Massachusetts data base can make a significant contribution to the analysis of existing and new alternatives for achieving equalization.





## 1 . Public School Funding: Components and Problems

The educational system in Massachusetts is supported by three major sources of revenue: local taxes, federal grants, and state grants. The effect of each on the level of school spending varies throughout the 351 cities and towns. One reason is that the local school tax rate on property is set by the city or town, determined independently of any external norm. The federal government dispenses grants for particular educational purposes: aid to depressed areas, aid to minority children, purchase of special educational materials, etc. Although this "categorical" aid is generally distributed with the advice of the state Department of Education, a good portion of federal aid goes to towns which either have a large number of military personnel or which are "poverty" areas. Similarly, the state has a number of categorical aid programs of its own, the most notable being the School Building Program. Here, there are often proportional matching funds constraints so that the money often goes where it is already abundant.

As can be seen from the above discussion, the funding system as so far described, provides many opportunities for an unequal distribution of funds per school child. The Commonwealth realized this and in 1966 established an "equalization" aid program. Through this vehicle the state attempts to equalize the ability of school districts to provide quality education despite differences in the relative wealth of their inhabitants. The basis for determination of aid is the relative property valuation per school child in a particular city to the state average property valuation per school child. It can readily be seen why this basis is used when one realizes that most local taxes are real estate taxes. In order to support a school expenditure of some number of dollars per child, the town must set a tax rate such that, when applied to its local pro-



erty valuation, will provide sufficient funds (in addition to possible grant revenue) to pay for the schools. Because property valuation per school attending child varies by some 300% across the state, we would expect corresponding tax rate variations if all localities spent the same amount per child. Since this would result in the poorest towns paying the highest tax rate, we find school quality (as measured by expenditures) often being sacrificed to obtain a lower tax burden. This then is the combined problem that state equalization aid, as determined by the NESDEC formula, was designed to solve: provide sufficient aid in proportion to each city or town's equalized property valuation such that there would be an equal expenditure on the schools per child at an equal tax rate. In order to show more exactly how the NESDEC formula attempted to solve these problems, the following paragraphs will describe the details of the formula. (See Appendix D for the exact format of the formula.)

The formula is based on the assumption that those cities and towns which can least afford to pay for schools should be given a greater share of the state funds available. The Massachusetts formula equalization aid is distributed as a percentage of the expenditures of the school district that are not supported by other state or federal grants. These expenditures are known as "reimbursable expenditures" and generally account for some 80% of the total school system's costs (although there is some variation across the state). The percentage paid back is based on two factors: the first is the relative percentage of an area's equalized property valuation to the state average. This is then multiplied by the fractional amount that the state wished to reimburse on the average. This "average coverage" figure is currently 35% although some have proposed that it be raised to 50%. In order to ensure at least some reimbursement to



wealthy towns and some expenditure by poor towns, the somewhat arbitrary limits on the percentage actually reimbursed have been set at 15 and 75%. (Recent proposals have also suggested varying these figures.) Therefore, the net effect of the current Massachusetts Equalization Aid Formula is that a town whose equalized property valuation per school attending child is equal to the state average would receive aid of 35% of its reimbursable expenditures. The town whose property valuation is 38% or less of the state's average will receive 75% while a town whose valuation is 131% or more than the average will receive 15%. (Additional limitations are imposed on reimbursable expenditures so that the base upon which the percentage aid is applied is between 85 to 110% of the state average).

The formula described above was implemented by legislation in 1965. It was felt at that time that the formula would, in fact, bring about a substantial equalization of school expenditures per child throughout the state. However, this has not proved to be the case. According to the latest figures available (1971) not only is there a 200% range in school tax rate but an even more astounding 250% difference in school expenditures per child. If the goal remains to equalize expenditures, several courses of action are now open to the legislature. These options vary from a complete takeover of school funding at a state level to the enactment of school legislation aimed at establishing state standards to the simpler but perhaps less effective, design of modifications to the existing formula-based approach. While interest has certainly been expressed in designing an entirely new system of financing, (in some cases involving a state property tax replacing local property taxes) there has been a considerable effort placed on the more politically tractable idea of modification to the NESDEC formula. Two bills have already been submitted



to the legislature regarding such modifications and more are expected. Since a good deal of the simulation experiments was designed to show the effects of these and other proposed modifications, the two bills will be described below.

State equalization aid is provided for in paragraph D of Section 2 of Chapter 70 of the General Laws of the Commonwealth. This was inserted in 1966. The two bills before the legislature, therefore, deal with changes to this Section. The first, an act "providing for corrective changes in the school aid formula" was submitted by Charlotte Ryan, Chairman of the Massachusetts Educational Conference Board (Senate bill 985, denoted "Ryan Bill" in this paper). The essence of the bill is as follows:

1. Change the 35% average coverage to 50% in three equal steps over the next three years (i.e., the average coverage constant should be .6 in 1973, .55 in 1974, and .5 in 1975).
2. The minimum aid as a percentage of reimbursable expenditures is reduced from 15% to 10%.
3. The stipulation that reimbursable expenditures are adjusted to fall between 80% and 110% of the state average is removed, and, therefore, the actual reimbursable expenditures are used in computing aid.

The second bill (Senate bill 958, denoted "Thistle Bill" in this paper) was entitled "an act to equalize fiscal ability for support of s schools and other municipal services."<sup>2</sup>

In addition to a number of provisions about the calculations of equalized property value, it provides for the establishment of a 4% sales tax such that 25% of the receipts from this tax shall be credited to a "Local Aid Fund". In addition, a number of changes are made to the dispersement of

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<sup>2</sup>An error that was drawn to our attention subsequent to the writing of this paper, bill 958 should be correctly termed the "Cauley Bill" as the bill was introduced by Senator Cauley.





categorical grants and the following changes are made to paragraph D of Section 2 of Chapter 70:

1. The average coverage is changed to 40% immediately (the average coverage constant in the formula becomes .6).
2. The minimum amount reimbursed is changed from 15% to 10%.
3. "No city or town shall be eligible for school aid which has made reimbursable expenditures per child in net average membership less than the smaller of (a) 90% of the average reimbursable expenditure per child for the entire state or (b) an amount equal to its reimbursable expenditures per child in the second preceding fiscal year increased by 5% and further increased by the percent of increase of the average reimbursable expenditure per child for the entire state...". That is, the cities and towns would be required to spend a certain amount of money in order to be eligible for aid (as opposed to the existing situation where if a city spends less than 80% of the average, that number is inserted in the formula rather than the true amount -- there is no requirement on the city to spend any particular amount).

As will be seen in the following sections, the essence of these two bills (as well as several other suggestions) was simulated in the computer runs.<sup>3</sup>

It can be seen from the above that a number of experts in the field of education and financing feel that significant steps towards equalization of school expenditure can be accomplished by varying one or more parameters of the NESDEC formula. The remainder of this paper will deal with our analyses by making use of computer simulation as well as

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<sup>3</sup> One aspect of the Thistle Bill that we have not evaluated is the third change listed, setting requirements for a town to be eligible to aid in terms of a minimum level of expenditure. The major reason for not evaluating this aspect of the bill is that we feel unable to predict the response of school districts to such a requirement. This is an issue that should be addressed in eventual follow-up studies.



analytical mathematical manipulations. We have attempted to highlight a number of the basic characteristics of this approach and also the consequences of the proposed actions. We will begin by discussing the results of the computer simulation.



## 2. The Model

A model is a description of a system whose structure or behavior is sufficiently analogous so that it can be used for learning about the system in an economical manner. The purpose of a simulation study is to allow the researcher to gather data on the behavior of a real world system by reference to the behavior of a model of that system. In our case, the system being studied was that encompassing the allocation and expenditure of Chapter 70 School funds and its effect upon tax rate on the cities and towns. Because of the large factor that these funds represent in the total educational finance system, it was felt that most questions relevant to the problem of equalization of expenditure and tax rate could be addressed without bringing in the complications of state and federal categorical aid programs. The model that was designed used the NESDEC formula to represent the allocation process (this part of the model, therefore, is exactly the same as the real world system) and several different assumptions about the behavior of various localities in spending money and determining tax rates to represent the expenditure process. While a number of issues were addressed using the simulation process others were studied by mathematical analysis. For example, we could quite easily show the effect over a period of time of changing the various limits of the formula by use of simulation. In order to get an understanding, however, of the general effect of differences in the proportion of children in public and private schools, mathematical analysis was used. In general, where differences between school districts were to be studied, (in expenditures tax rate, etc.), simulation was used. Where general properties were of interest (relationships between the various factors in the formula), mathematical analysis was used. The following sections of the paper will address



each of these and their results in considerable detail.

One of the important parts of the model building process is deciding where to draw the boundary between the system under study and the rest of the world. Clearly, at a certain level everything is related to everything else. Boundaries should be drawn in such a way so that all factors influencing the system that have a bearing on the objectives of the study are included. For example, it is certainly true that changes in population or equalized property valuation effect the amount of money that a school district must raise for schools and effect the base upon which it can draw for these funds. It was decided, however, that since these items change fairly slowly in time (especially when percentage differences between towns are considered) they would only serve to complicate our study. On the other hand, the fact that the legislature often finds less total funds to distribute than application of the formula would require, seems to be a relevant factor. Thus, "underfunding" is discussed with an analytical treatment later in this paper. Our consultations with researchers in the field led us to believe that omitted factors were either of an unimportant nature or had predictable and regular effect.

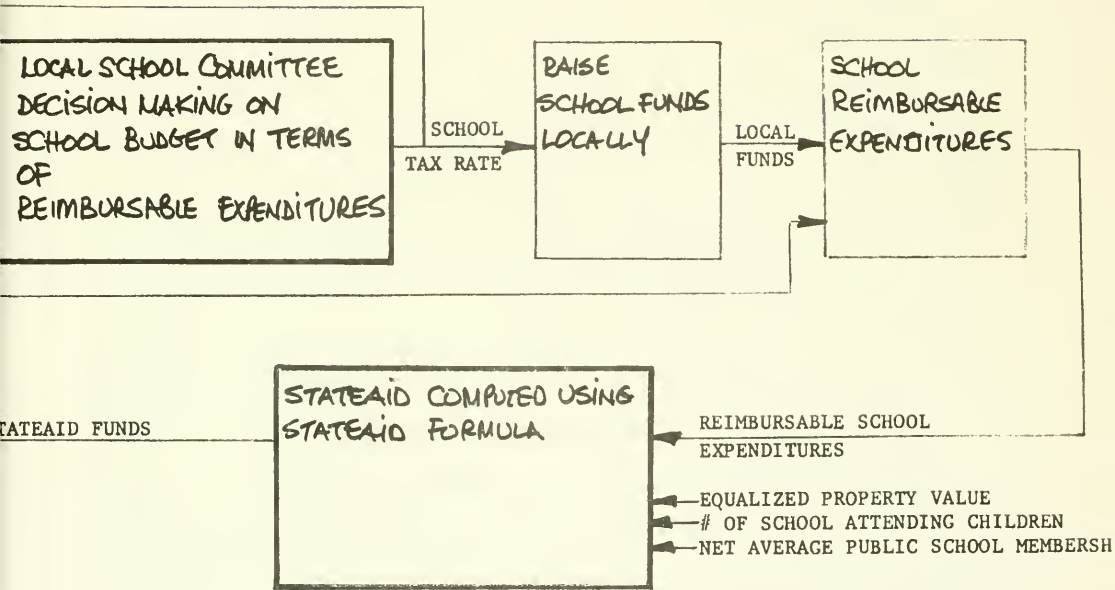
The simulation study began with a computer-based implementation of the expenditure-funding cycle. The program was written in the EXPRESS language and is included in Appendix B. It is not necessary, however, to study the computer program in order to follow the analysis. If reference is made to Fig. 2.1 on the following page, it will be seen that there are two major aspects of the model: first, is local decision making in terms of the school budget and the use of reimbursed expenditures; second is the computation of state aid using the formula. It can





Figure 2.1

The School District Financing Model





be seen that state aid funds can go either directly into school expenditures or can enter into the decision making process and lower the tax rate. The decision making process determines the tax rate which can be seen to determine locally available school funds. These funds (combined with the state aid funds) become the "reimbursable expenditures" which feed back in to the next year's computation of the state aid formula. The various data items input to and output from the model will now be enumerated.

The output of our simulation runs is a listing of state aid percentage, state aid, and school expenditures for each city and town over a period of seven years. Various statistics, such as mean and standard deviations were also computed. The input data consisted of the latest figures on equalized property valuation, number of school attending children, net average membership in the public schools, and current year reimbursable expenditures. These were taken from the report of 1971, Chapter 70, Equalization Aid prepared by the Division of Research of the Department of Education. Various parameters were input to each run of the model to simulate some proposed or possible modification of the formula. These decision variables were the average coverage percentage, the minimum and maximum aid percentage, and the floor and ceiling on reimbursable expenditures, if any. These parameters, when taken together with the local decision making behavioral assumptions, controlled the operation of the system.

Modeling the behavior of local decision makers could have been approached in a number of different ways. One approach would have been to gather interview data on how each locality used the different proposals



and how they would act in any particular case. It seemed, however, that a simpler approach might be considered because of the general propensity of a locality to act between two possible extremes. After it is determined how much money a school district is to receive, it is up to the individual district as to whether the money is immediately used to increase school expenditures, whether it is used to decrease the amount that local residents must provide via the property tax, or somewhere in between. We, therefore, decided to make several different assumptions of local decision making based on these two basic behavior patterns and to compare the effect caused by each. They can be expressed as follows:

1. All school districts preserve their existing local tax effort and use all additional state funds to increase total expenditures on schools. That is, total expenditures equal the local contribution plus the state aid. (This assumption is labeled "constant local effort".)
2. All school districts preserve their existing total expenditures per child and use all additional state funds to reduce their local school tax rate. (This is labeled "constant total effort".)

By setting an additional parameter in the model, we could vary behavior between these two extremes.

For the purposes of investigating certain statements about equalization of expenditures when all school districts have decided upon an equal tax rate, we allowed a third alternative known as "uniform local effort". In this behavior mode,<sup>4</sup> the tax rate was set uniformly across the state (possibly because of legislation forcing localities to do so) and state aid funds could not be used to change it. (This is different from the first behavioral assumption because all districts begin with an equal rate.) In this way we could test the potential quality of the different proposal for equalizing school expenditure under the best conditions.

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<sup>4</sup>This "behavior mode" was suggested by Mrs. Charlotte Ryan. As can be seen, some of the major findings in the report relate to this concept. We are grateful for having our attention drawn to this real issue.



Our modeling efforts to date tested the various behavioral assumptions but with all districts operating under the same assumption at any one time. It could be argued, and we would willingly agree that, it can not be assumed that behavior will be identical for all districts. One of our proposals for further study is concerned with this problem and, in general, with the more accurate determination of local decision making variables. This issue will not be dealt with further at this time.

The next section will discuss the various results of simulation runs performed with different values of the parameters mentioned above. Such changes in parameters can easily be performed on-line during the actual run and, in fact, this was done during several demonstrations. We found it extremely important to be able to achieve this flexibility in model design so that our analyses of proposed state aid formulas could be immediately available. One of our future aims would be to instruct the authors of legislation in the direct use of the system so that they too might be able to test out formula changes before actually proposing them. This, however, remains for the future.





### 3. Results

Having reviewed the development of the model of school funding, we will present here the results of the model manipulations. We start by reviewing the results of our initial computer based simulations. Thereafter, we will present the results of an analytical treatment of the model with some simulation runs that we performed to test the latter results. The last two sections deal with an analytical treatment of the question of underfunding of a formula based system, and with the analytical treatment of an issue that we have termed "the mirage of equal potential". As the title indicates, this last section suggests that potential equalization is a property of the formula based approach that disappears when one gets close up!

#### 3.1 Initial Simulation Runs

The complete summary of the results of the initial simulation runs are tabulated in the first six tables of Appendix C. Before reviewing the results, let us briefly present the different variables that we obtained as output from the simulation runs.

Each output variable is tabulated for the seven years (1971 to 1977) simulated, with one value for each year. Within each year, we computed the value of the following variables:

1. The mean and standard deviation of the reimbursable expenditures per net average number of public school attending children. These values are computed on a public school child basis, and not on the basis of school districts. I.e., it is the mean and standard deviation on a school district basis, weighted by the net average membership of each district. (If we use values that are not weighted in the sense above, the dispersion in expenditures per pupil increases enormously).
2. The mean and standard deviation of the local school tax rate on a school district basis.



3. The total amount of dollars required from the state to fund the programs.
4. A histogram representing the number of school districts within each of 20 reimbursable expenditure per net average membership ranges from \$400 or less, to \$2200 or more, in steps of \$100. This gives a frequency distribution of the expenditures in the Commonwealth for any particular year.

Each simulation run always starts with the same data in the first year, which is the most recent school district data available (from the Report of Chapter 70 State Aid Expenditures in Massachusetts in 1971 (published January 1972))

Table 3.1 summarizes the different parameter settings for the runs made. The parameter setting determines which program is being simulated and which local behavior model is being used. In all we made six runs initially, consisting of two behavioral models for each of the three programs studied. We do not report results of a third behavioral model described above, "constant total effort", as this model will not by definition result in any equalization of expenditure patterns.

Let us now turn to the results. They can best be reviewed by looking at a graphical representation. Figure 3.1 represents the development of the mean reimbursable expenditure per net average membership and the standard deviation of the same for the three programs, under the condition of "constant local effort". We find that there are three aspects that should be noticed:

1. All the programs have an upward sloping curve with time that tapers off after 3-4 years.
2. In terms of the standard deviation of expenditures, which can be considered a measure of equality of expenditures, the programs do not differ markedly.
3. The main difference between the programs is in the difference in the mean reimbursable expenditure per net average membership. The results in Appendix B concerning the total amount of funds required from the state, indicate how this



RUN #	FORMULA/ PROGRAM	LOCAL BEHAVIOR ASSUMPTION	CONSTRAINTS <sup>2</sup> ON FRACTION OF REEXP	CONSTRAINTS <sup>1</sup> ON REEXP (IN FRACTION OF STATE AVERAGE)	AVERAGE COVERAGE OF EXPENDITURES BY LOCAL EFFORT YEAR								
			<	>	'71 '72 '73 '74 '75 '76 '77								
1	existing	constant local effort	.75	.15	1.10	.80	.65	.65	.65	.65	.65	.65	.65
2	existing	uniform local effort	.75	.15	1.10	.80	.65	.65	.65	.65	.65	.65	.65
3	"Ryan"	uniform local effort	.75	.10	1000.	.0	.65	.60	.55	.50	.50	.50	.50
4	"Ryan"	constant local effort	.75	.10	1000.	.0	.65	.60	.55	.50	.50	.50	.50
5	"Thistle"	constant local effort	.75	.10	1.10	.80	.65	.60	.60	.60	.60	.60	.60
6	"Thistle"	uniform local effort	.75	.10	1.10	.80	.65	.60	.60	.60	.60	.60	.60

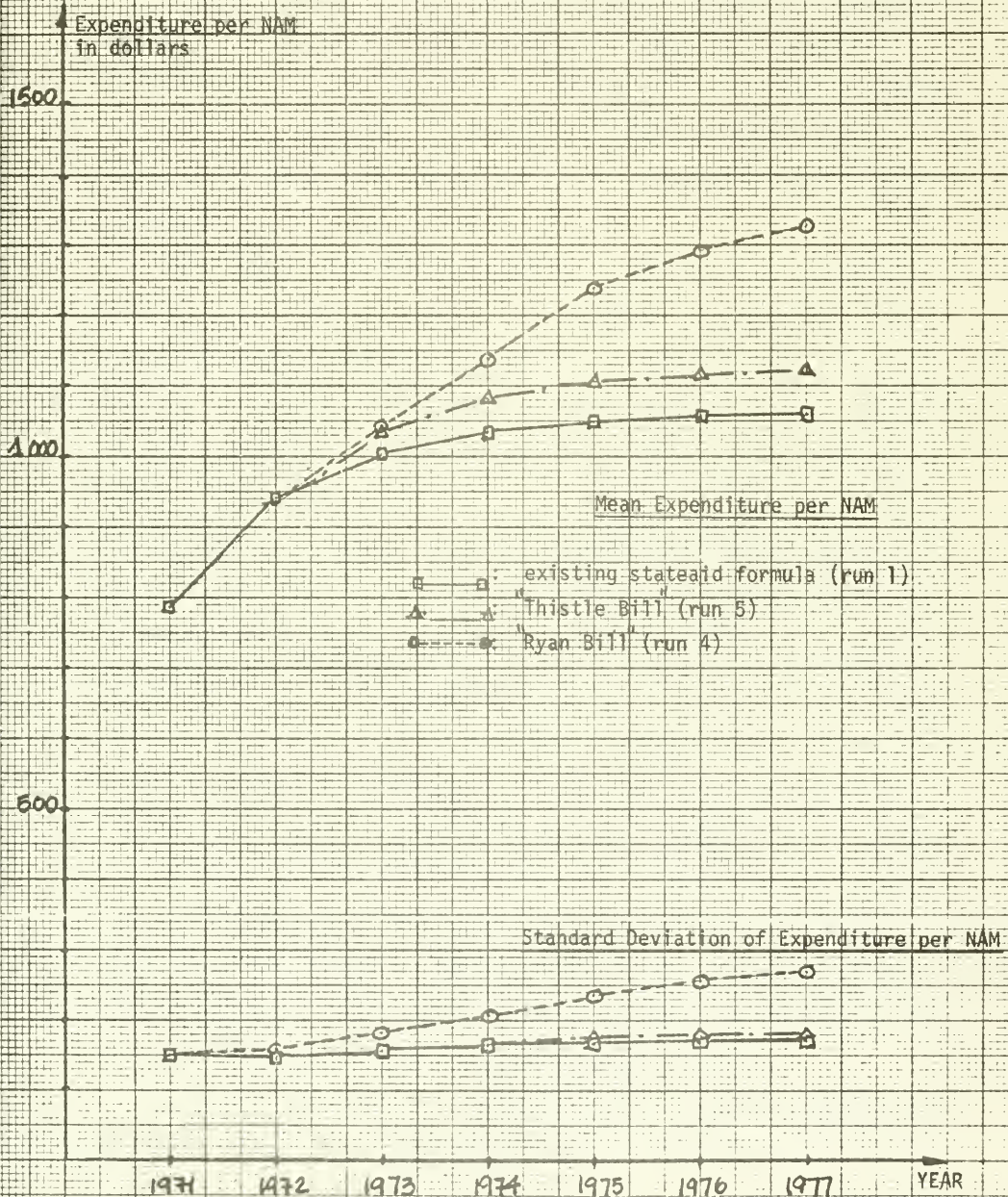
Table 3.1 A summary of the decision variable values used in initial simulation runs.

- 1) Constraints imposed on the REEXP figure to be used when computing the actual stateaid received. e.g., a school district's reimbursable expenditure figure, if larger than 1.1 times the state average figure, will be adjusted to being equal to 1.1 times the state average.
- 2) Constraints imposed on the fraction of REEXP (to reimbursed) computed from the stateaid formula.
- 3) REEXP: reimbursable expenditure.



Figure 3.1

Mean and Standard Deviation of Expenditure per NAM  
Constant Local Effort







difference in mean expenditure is arrived at: by an increase in the level of funding by the state.

We feel it appropriate to underline that the results discussed above were obtained by assuming constant local effort. Figure 3.2 gives the presentation of the same results as Figure 3.1 for the case of uniform local effort (at \$2.25 per \$100 of equalized property value). These results should indicate how well the different programs achieve potential equalization.

The results indicate that there is no difference in the equalization of expenditures (as measured by the standard deviation) comparing the case of uniform local effort to the case of constant local effort. In actual fact, the standard deviation has increased slightly, with slightly lower mean expenditure. Looking at some of the histograms obtained, the issue is illuminated.

Figure 3.3 A and B give the histograms for run 6 (ref. table 3.1) respectively for the year 1971 and 1975 (the dotted line indicates the mean expenditure per net average membership). Figure 3.3A represents the distribution of the actual expenditures in 1971. Although the distribution is somewhat skewed, the mean and the median seem to be almost identical. However, in the case of Figure 3.3 B, which is the equal potential picture simulated for 1975, the distribution is vastly more skewed. At first sight, this result is not surprising. One would suspect that the constraints on the fraction of expenditures reimbursed in the program would give approximately such an effect. However, this does not explain the skewness of the distribution locally around the mean. Potential equalization should clearly result in a very tight distribution around the mean.

The fact that our simulations did not give the hypothesized effects with respect to potential equalization, led us to initially suspect that



Figure 3.2  
Mean and Standard Deviation of Expenditure per NAM  
Uniform Local Effort

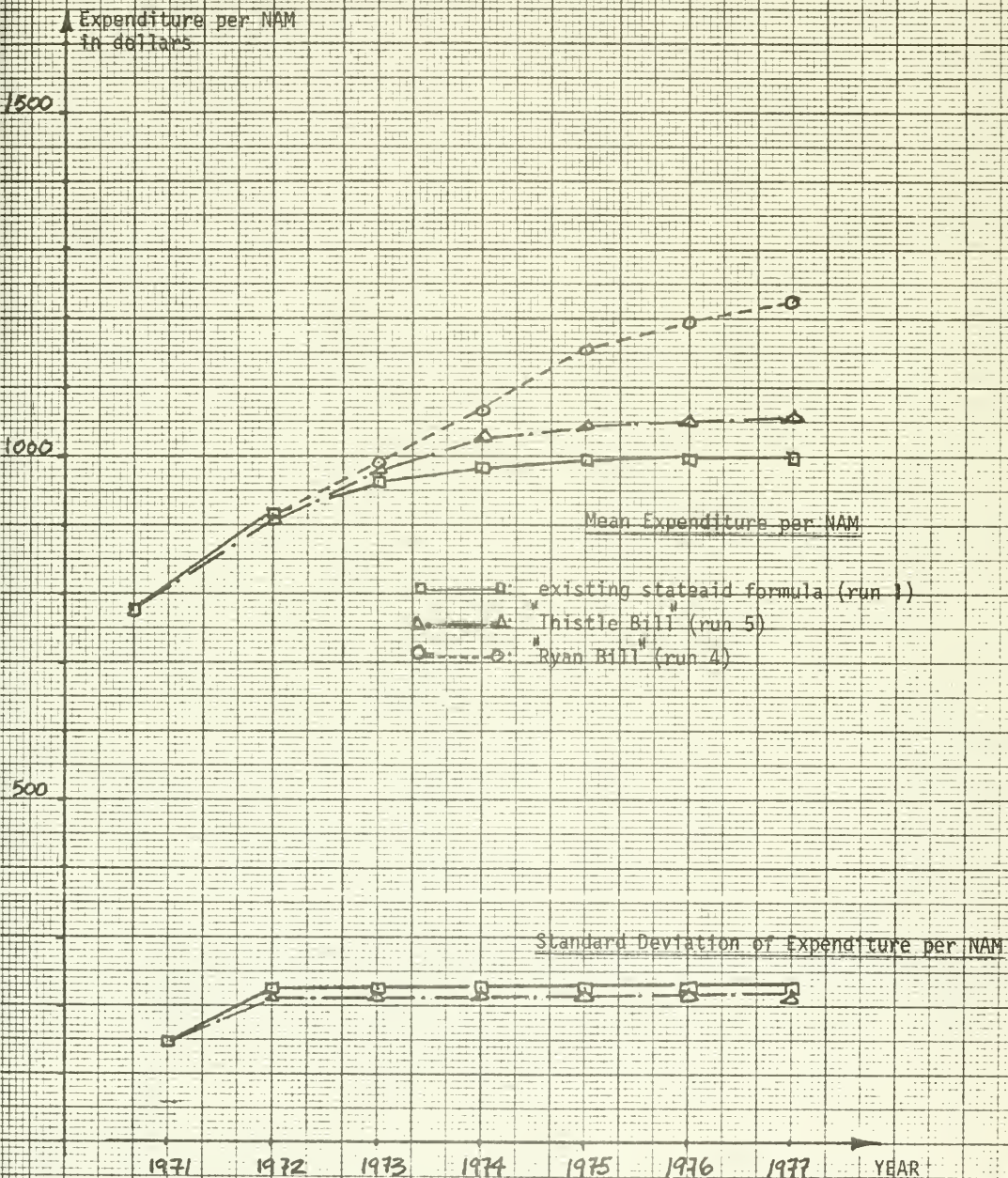




Figure 3.3A

Frequency Distribution of Expenditure per NAM per School District

1971 actual figures

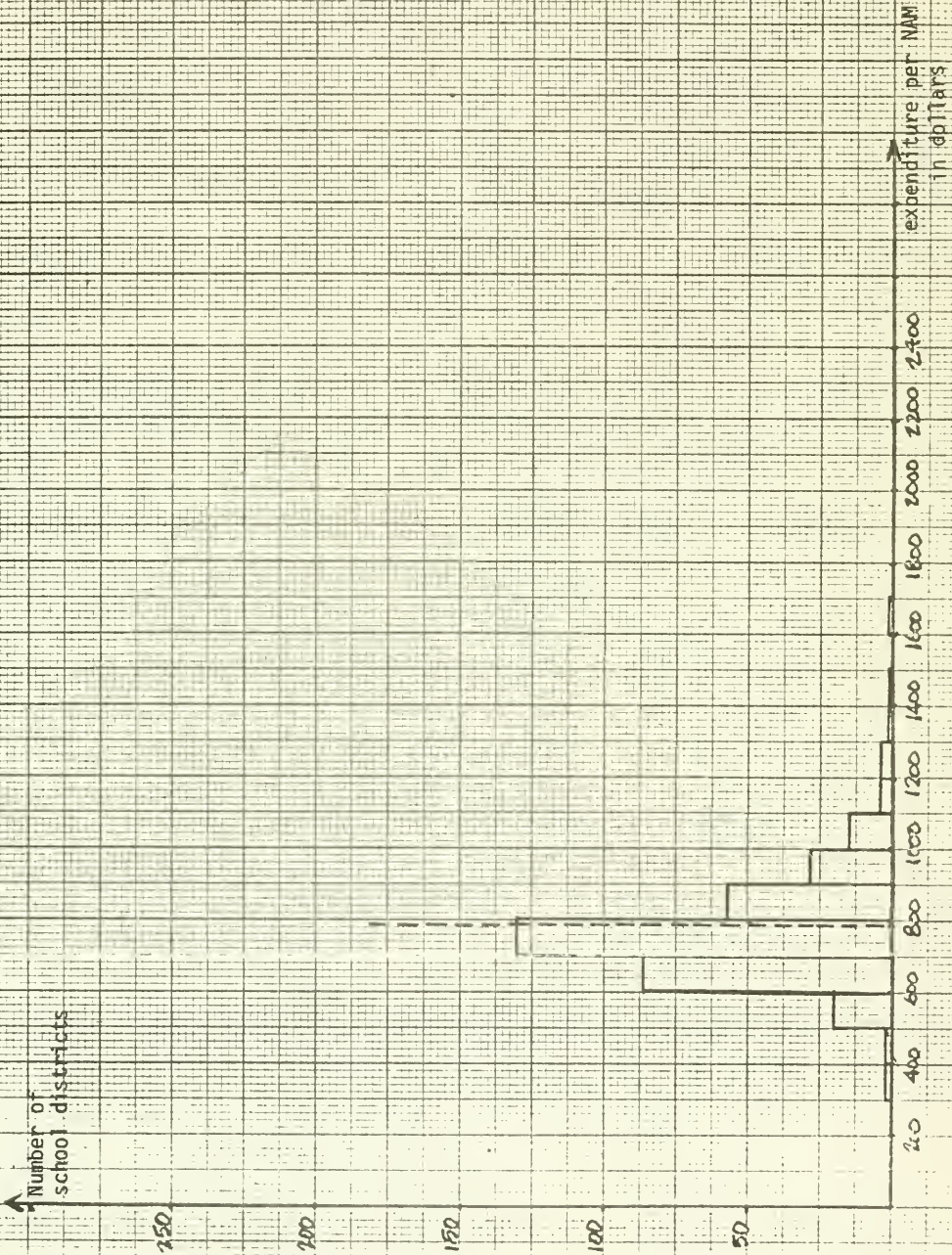


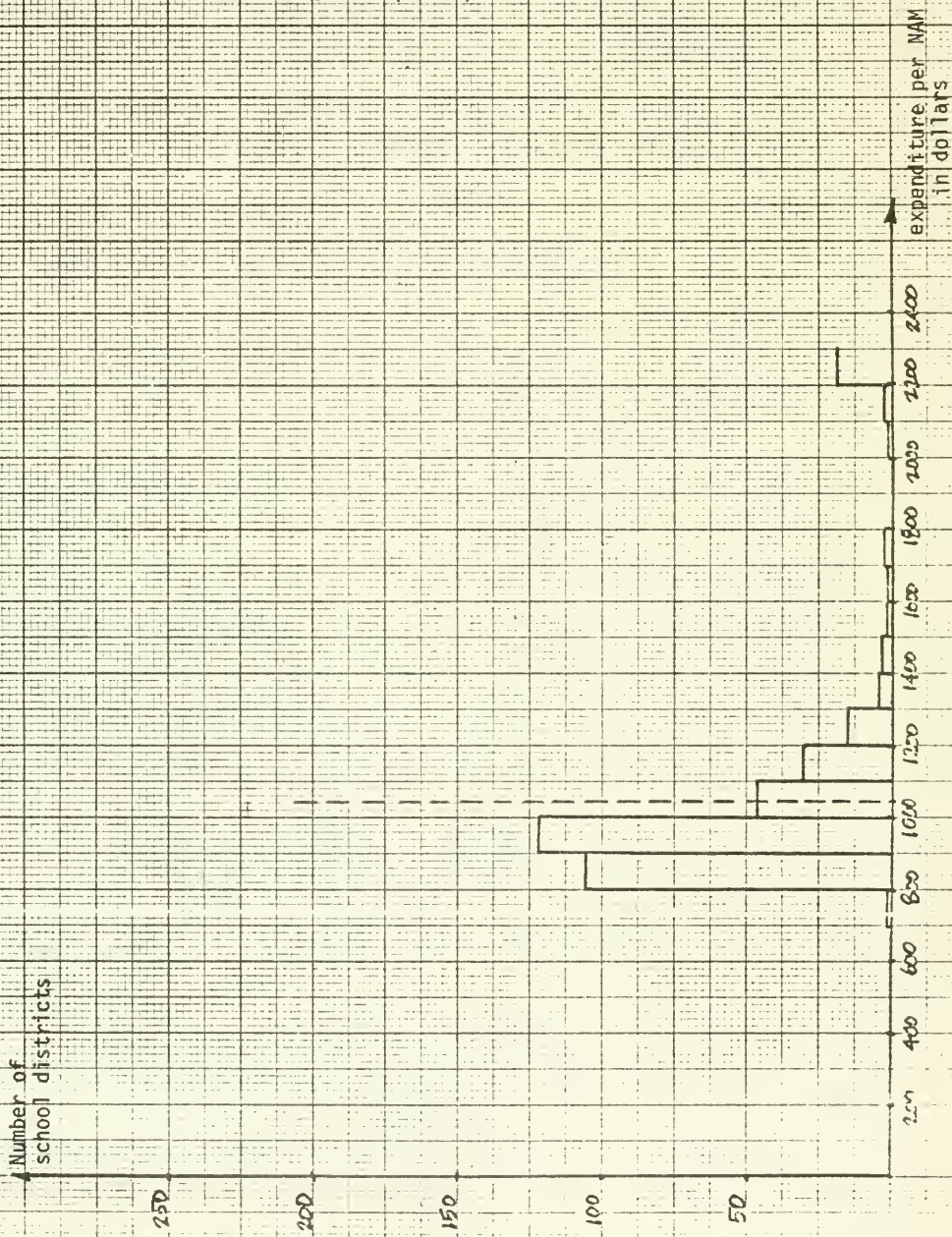


Figure 3.3B

Frequency Distribution of Expenditure per NAM per School District

Thistle Bill - Uniform Local Effort

Year 1975 (run 6)







there was a "bug" in our computer model. However a meticulous analysis of the model did not uncover any bugs, and we therefore were led to hypothesize that there was an unknown structural property of the formula system that led to the results observed. This led subsequently to an analytical treatment of the model that did uncover a property whose effect in terms of potential equalization had not been recognized previously. We have termed it the NAM/SAC effect.

### 3.2 Identification and Analysis of the NAM/SAC Effect

In an attempt to identify the cause of the lack of potential equalization in the formula based programs, we developed an expression of the reimbursable expenditures (REEXP) per net average number of public school attending children (NAM) for a single school district. The basic assumption that enabled us to derive an expression for the REEXP per NAM was that the district had reached an equilibrium REEXP/NAM where it was receiving the exact amount of state aid corresponding to its school tax rate (TAXRATE) and equalized property valuation (EQPVAL). The analytical treatment is presented in detail in Appendix D. Here we will solely discuss the main result, which was the following expression:

$$\text{REEXP} / \text{NAM} = \frac{\text{TAXRATE}}{\text{AVGCOV}} \cdot \frac{\sum \text{EQPVAL}}{\sum \text{SAC}} \cdot \frac{\text{SAC}}{\text{NAM}}$$

The additional variables are the number of school attending children in the district (SAC), and the average fraction of expenditures to be covered locally (AVGCOV).

From the formula we observe that REEXP/NAM is not a function of local wealth. However, it is the function of the ratio of the total school population of the district to the net average membership. This ratio varies over the state for different school districts, and would therefore



seem to account for some of the dispersion in REEXP/NAM observed when a uniform local tax rate is assumed. The ratio is introduced by the use of the EQPVAL per SAC figure in the formula when computing the fraction of expenditures that the state is to reimburse.

The hypothesis above was tested by changing the formula in the simulation model to a formula that solely used the NAM figure. Three runs were made to test the effects of this last change on the potential equalization capability of the three programs considered. Table 3.2 summarizes the parameters for the three runs. Figure 3.4, a histogram representing 1975 conditions in run 8, clearly shows the hypothesized effect of the formula change. The expenditure per pupil distribution is now much more clustered around the mean. The effect becomes even more noticeable after two more years (ref. the full listing of the results in Appendix C).

Figure 3.5 gives an indication of the effect on the mean and standard deviation of REEXP/NAM by the change. The main point here is probably to notice that the change only results in a 10% reduction of the standard deviation. The remaining dispersion is due to the constraints on the fraction of REEXP/NAM to be reimbursed. The effect is identical for all three programs.

The equation above shows a property of the formula approach that our first simulation runs indicated: a change in the average fraction of the expenditures covered does not change the potential equalization properties of the program.

### 3.3 The Effect of Underfunding the Formula

We here report an analytical treatment of the question: is it



RUN #	FORMULA/ PROGRAM	LOCAL BEHAVIOR ASSUMPTION	CONSTRAINTS <sup>2</sup> ON REEXP (IN FRACTION OF STATE AVERAGE)		CONSTRAINTS <sup>1</sup> ON REEXP (IN FRACTION OF STATE AVERAGE)		AVERAGE COVERAGE OF EXPENDITURES BY LOCAL EFFORT						
			REEXP <sup>3</sup> <	>	<	>	YEAR						
							'71	'72	'73	'74	'75	'76	'77
7	existing	uniform local effort	.75	.15	1.10	.80	.65	.65	.65	.65	.65	.65	.65
8	"Thistle"	uniform local effort	.75	.10	1.10	.80	.65	.60	.60	.60	.60	.60	.60
9	"Ryan"	uniform local effort	.75	.10	1000.	.0	.65	.60	.55	.50	.50	.50	.50

Table 3.2 A summary of the decision variable values used in the simulation runs performed to test the NAW/SAC effect.

(See Table 3.1 for comments)



Figure 3.4  
Frequency Distribution of Expenditure per NAM per School District

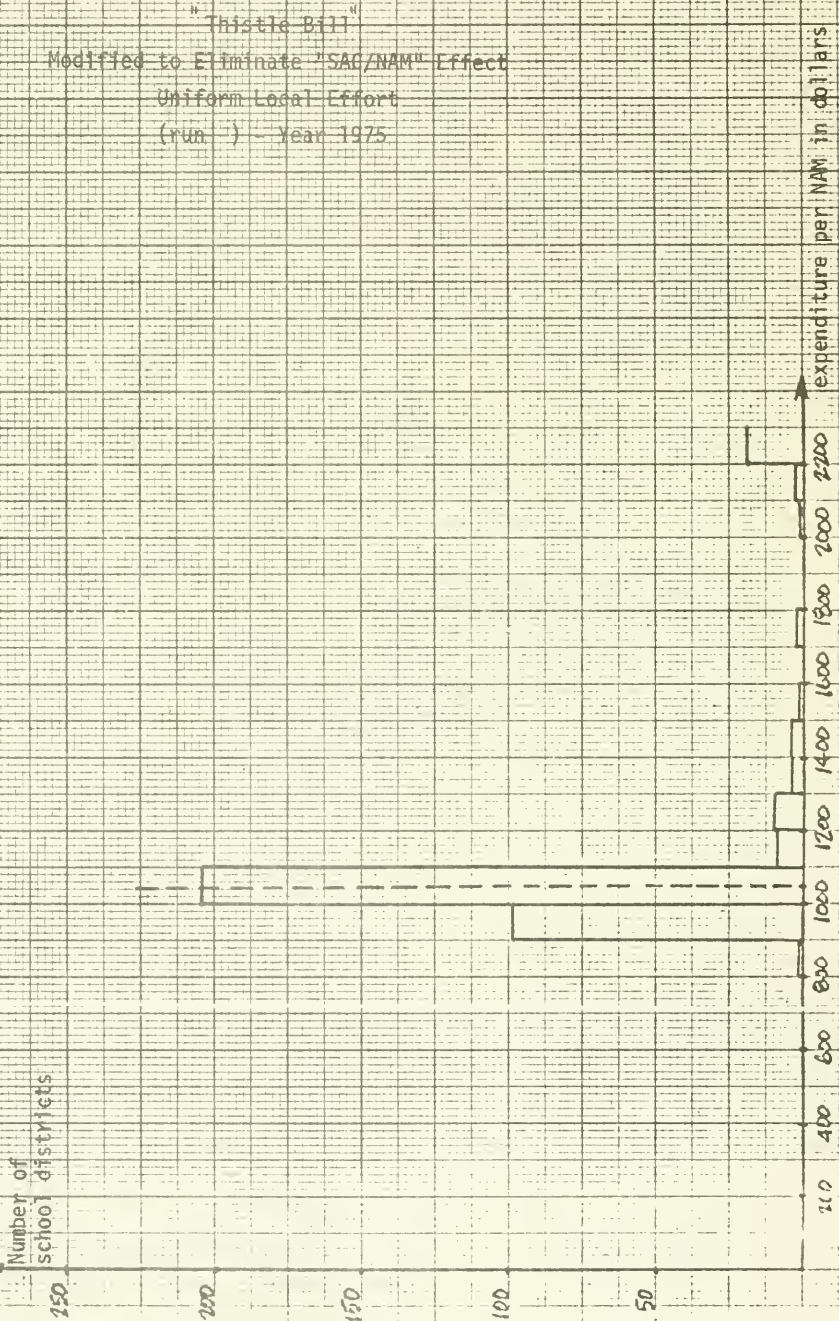
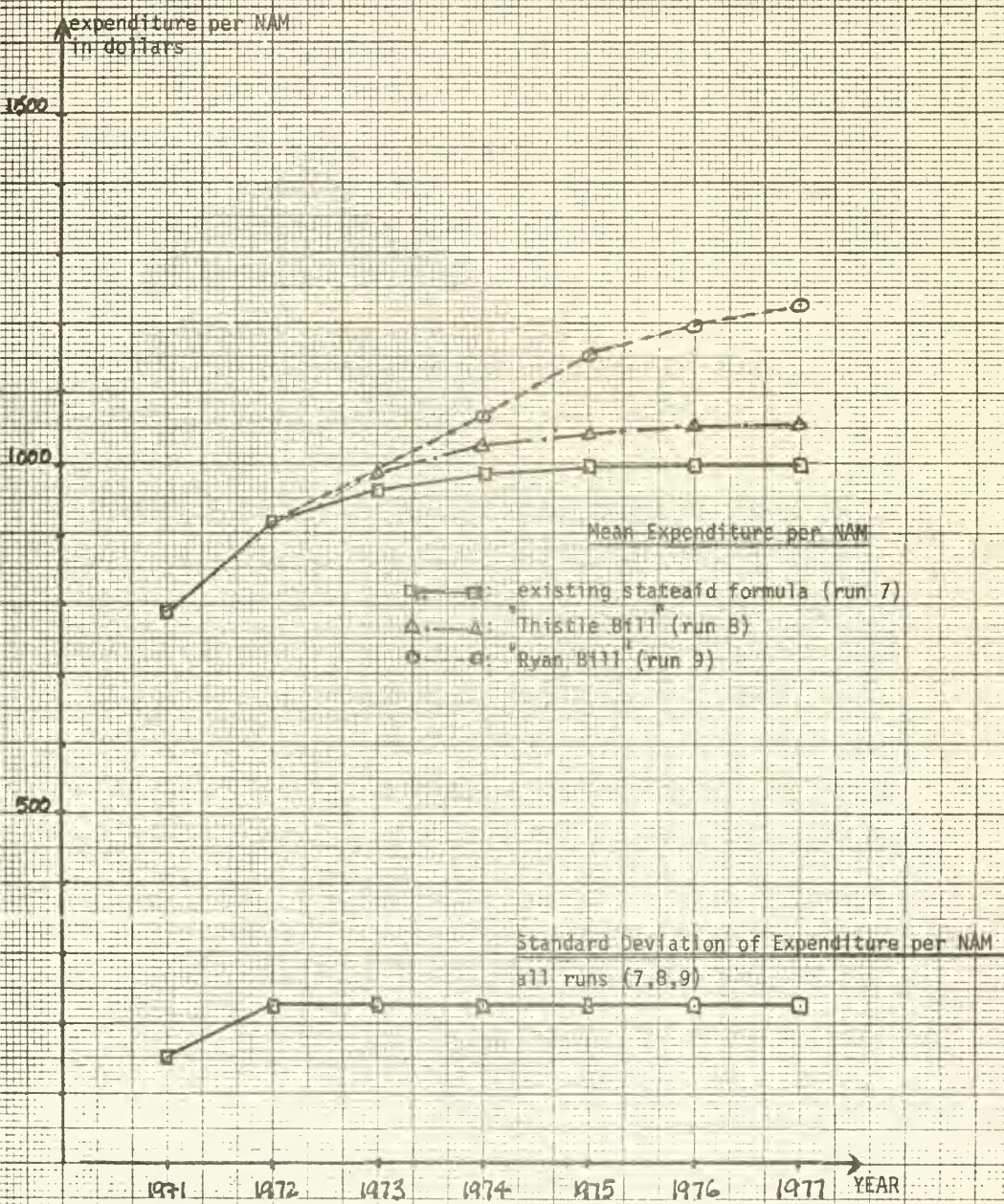






Figure 3.5  
 Mean and Standard Deviation of Expenditure per NAM  
 Uniform Local Effort with SAC/NAM Effect Eliminated





equalizing or disequalizing to underfund<sup>5</sup>a formula, as opposed to changing the formula so that it is fully funded? (This when the scarce funds are allocated on an equal percentage basis to all districts.)

We show that in the case of actual expenditures, it is difficult to give a general answer to this question. However, we argue that with the existing expenditure pattern in the state, it is highly disequalizing to underfund the formula. In terms of potential equalization, we clearly show that underfunding reduces the equalization effect of the formula approach.

The effect of underfunding is studied by looking at the change in state aid a school district receives as the result of going from a fully funded formula to a partially funded formula. We assume, for simplicity, that the total reimbursable expenditure of the school district, REEXP, remains the same. The ratio of its equalized property value per school attending child to the state average, RATIO, stays constant. The effect of underfunding results from going from a fully funded formula with an average local effort coefficient,  $AVGCOV_1$ , that results in a state aid,  $STAID_1$ , to a formula with a lower average local effort coefficient,  $AVGCOV_2$  ( $AVGCOV_2 < AVGCOV_1$ ), that results in a state aid,  $STAID_2$ , that is only a fraction P of the state aid implied by RATIO,  $AVGCOV_2$  and REEXP. The change in state aid,  $\Delta STAID$ , is determined as follows:

$$(1) \quad STAID_1/NAM = (1 - AVGCOV_1 \cdot RATIO) \cdot REEXP/NAM$$

$$(2) \quad STAID_2/NAM = P \cdot (1 - AVGCOV_2 \cdot RATIO) \cdot REEXP/NAM$$

from (1) and (2) we have

$$(3) \quad \begin{aligned} \Delta STAID/NAM &= STAID_2/NAM - STAID_1/NAM \\ &= ([AVGCOV_1 - P \cdot AVGCOV_2] \cdot RATIO - [1-P]) \cdot REEXP/NAM \end{aligned}$$

<sup>5</sup>i.e., that the legislature appropriates only a fraction of the funds due to the school districts as derived by the formula and their expenditures.



Assuming that all districts have an equal reimbursable expenditure per child, REEXP, we can determine  $\Delta$ STAIID as a function of RATIO, a measure of local wealth per school attending child. The functional relationship is represented graphically in Figure 3.6.

In order that underfunding has no effect,  $\Delta$ STAIID should be zero for all districts, i.e., for all values of RATIO. The actual relationship is a monotone increasing curve with the value of RATIO, i.e., the effect of underfunding the formula benefits the wealthier districts at the expense of the poorer districts.

Looking at the actual expenditure pattern in Massachusetts, we note that with respect to the discussion above, the assumption of equal reimbursable expenditure per school child is not valid. We note, however, that the change in state aid in equation (3) is directly a function of the reimbursable expenditure per NAM: with larger REEXP per NAM, the larger  $\Delta$ STAIID, all other things equal. Given that the wealthier districts tend to have higher REEXP/NAM in the case of actual expenditures in Massachusetts, the underfunding will have a compounded effect of both the RATIO value and of the high REEXP/NAM. The result is even greater disequalization than in the case of potential equalization.

### 3.4 The Mirage of Equal Potential

The analysis of the effect of underfunding the formula, directed our attention to the question of what is a fully funded formula. Our discussion assumes that we have rid the formula of the disequalizing effects of constraints and of the use of SAC figures.





$$\text{RATIO}_{\Delta \text{STAID}/\text{NAM}=0} = (1-P) / (\text{AVGCOV}_1 - \text{AVGCOV}_2 * P)$$

$$\Delta \text{STAID}/\text{NAM}_{\text{RATIO}=0} = (1-P) * \text{REEXP}/\text{NAM}$$

Figure 3.6 The effect of formula underfunding on the stateaid received by a school district in terms of change in stateaid ( $\Delta \text{STAID}$ ). The school districts are represented by the ratio of their equalized property value per SAC to the average for the Commonwealth ( $\text{RATIO}$ ).





We show that given a decision on the average coverage coefficient in the formula (e.g., 35% in the existing NESDEC formula), and on the level of funding of the program, the legislature has made an implicit decision on the uniform local tax rate and uniform reimbursable expenditure per public school attending child at which there is full equality in the Commonwealth. At any other uniform local tax rate, the formula is either underfunded or overfunded. The implication of this observation is that a funded formula does result in equalization solely if all school districts decide to operate with a school tax rate at a specific uniform value. A funded formula is not a general potential equalizer, but equalizes only under a very specific condition. It also implies that a decision about funding is in reality a decision about what should be the expenditure per pupil in the Commonwealth.

To show the above assertion, it is only necessary to observe that the total state aid required to fund the formula is a linear function of the school tax rate of the individual school districts. With the total funds available determined, there is therefore a unique uniform local tax rate that satisfies the relationship. In more analytical terms, we have for district  $i$

$$(1) \text{ STAID}_i = A_i \cdot \text{TAXRATE}_i \text{ and } (2) \text{ REEXP}_i = B \cdot \text{TAXRATE}_i$$

The sum of the state aid for each individual district should equal the funds available, FUNDS, i.e.,

$$(3) \text{ FUNDS} = \sum \text{STAID}_i$$

If the districts have a uniform and equal local school tax rate, TAXRATE, i.e.,

$$(4) \text{ TAXRATE}_i = \text{TAXRATE} \quad \text{for all } i = 1, 2, 3, \dots, 351$$



we have

$$(5) \text{ FUNDS} = \sum_i \text{STAID}_i = \sum_i A_i \cdot \text{TAXRATE}_i = \text{TAXRATE} \cdot \sum_i A_i$$

i.e.,

$$(6) \text{ TAXRATE} = \text{FUNDS} / \sum_i A_i$$

and

$$(7) \text{ REEXP}_i = B \cdot \text{TAXRATE} = \text{REEXP} \quad \text{for all } i = 1, 2, 3, \dots, 351$$

We see from equation (6) that the uniform local school tax rate is determined at a unique rate, given a specific amount of total funds appropriated.

#### 4. Discussion of the Results of the Simulation and Analytical Modeling

We will first discuss the results of the model testing and manipulation. Then we will introduce some issues that need to be addressed, and that are plausible to attack within the general framework of the model. However, the further we go, the more we are getting away from the simplicity of our model and the closer to the real world. The discussion will lead us to a statement of what we think the issues are that have to be approached before coming up with a diagnosis of and a cure for the financing and equality problems facing the educational system.

When discussing the concrete results of our simulation and analytical treatment of the school financing model, we must necessarily restrict ourselves primarily to the discussion of the results in terms of a potential equalization objective. For an analysis of actual expenditure equalization, we are totally dependent on an accurate model of local decision making with respect to school financing. This is a problem we will expand on later. Let us first see how the existing NESDEC formula system and



current proposals fare with respect to equal potential.

The most important result of the study in terms of potential equalization is that we show that general potential equalization is a completely theoretical concept in a formula based approach to state equalization aid. When the formula approach is bound to the real world by a specific funding decision, only then will there exist potential equalization at a unique, specific and uniform (by definition of equal potential) local school tax rate. For example, the decision of the legislature to fund Chapter 70 at a level of A dollars is, in terms of potential equalization, identical to the legislature deciding that full equalization will only arise if all school districts decide to have a school tax rate of B dollars. Ironically, funding the formula is equivalent to deciding what the school districts should spend per pupil.

We have also shown that even at the unique school tax rate, all current formula based systems do not fully equalize. This is due to two major causes:

1. The constraints in the formula on the fraction to be reimbursed.
2. The uses of both the total school population and the public school population, instead of the latter exclusively. This leads to distortions due to variations in the ration of the two figures.

Of the two causes for nonequalization, the first is dominant. This is shown by the fact that a change in the formula to remedy the second cause has only a small effect on the measure of dispersion in reimbursable expenditures per public school attending child.

The two current proposals for new formula parameters both incorporate a change in the constraints mentioned above. In this sense the proposals can be said to be better than the existing NESDEC formula system.

However, our results also clearly show that the change in the size



of the average percentage of school reimbursable expenditures that the state will support has no effect on the equalization properties of the formulas. Our analysis of the effects of underfunding indicate that one might obtain quite the opposite effect of equalization. An increase in the average coverage in the formula might increase the chances of the formula being underfunded. Our results show that underfunding has a disequalizing effect on the distribution of state aid funds.

Let us now focus on equalization in terms of actual expenditures. With this shift of emphasis, we start to expose the insufficiencies of our present model, and thereby set the stage for recommendations for further work.

In order to predict future actual expenditure patterns, it is necessary to predict the decisions of the local decision making units - the school committees. In our model we assumed a uniform behavior for school districts, such as e.g., "all school districts continue with the same local school tax rate throughout the prediction period". In terms of the existing and current proposals for new systems, our results indicate no substantial shift in equalization of expenditures. The model does show that the implicit equalization effect of the formulas will, under certain behavioral assumptions take some periods to manifest itself fully.

##### 5. Objectives and Directions for Future Work

The Serano and subsequent court decisions seem to give a clear indication of what the objective of an equalization program should be. The decisions emphasize the rights of the child, rather than the prerogatives of any arbitrary governmental unit. Governmental units have the responsibility to guarantee the rights of the child. Therefore, equalization of





potential expenditures (as has been emphasized thus far in the formula approach) is far less important than a greater equalization of actual expenditures.

It follows that attention must be focused on financial decision making at the local level. Any new proposal must consider the issue of local versus central control. Furthermore, it will be important to determine the impact of local control not only on financing but, insofar as possible, on other issues such as the quality of education.

The conclusion of the study is that new alternatives for achieving equalization should be created and analyzed. Equalization is a complex problem involving a number of interrelated issues that must be considered. Important financial issues are:

1. The method of raising funds.
2. The method of distributing funds.
3. The ability of the school districts to absorb additional funds effectively.
4. The dynamics of the school population and tax base.
5. Changes in educational costs.
6. The burden of other municipal services.

These questions become involved with organizational issues such as:

1. The size of school districts.
2. The balance of control between state and local district and within the district.
3. School desegregation.

The M.I.T. team feels that a computer-assisted modeling approach using an appropriate Massachusetts data base can make a significant contribution to the analysis of existing and new alternatives for achieving equalization.



Once a state aid program has been decided upon, models developed in the analysis phase could be used by the appropriate decision makers to track the actual development of expenditures and other variables of interest. Based upon such other information, one could determine if the program is meeting its objectives. If not, a change of program, program implementation, and/or model would be required. The result of such a continuous effort would hopefully be a fundamentally better understanding of the situation and problem one is seeking to govern.



APPENDIX A - Comments to report by Mrs. Charlotte Ryan

110 Bridge St.  
Manchester, Mass. 01944  
June 19, 1972

Mr. Charles B. Stabell  
c/o Professor John Little  
53-350  
Massachusetts Institute of Technology  
Cambridge, Mass. 02138

Dear Charles:

I appreciate having a copy of your report, as I have also appreciated the help of your model in working out some of my own questions in this business. Let me say that you and your colleagues have developed an admirable tool.

You ask for comments. What you have done is wholly intelligible, despite my fear; also it is significant, and likely to have influence. Thus I have read with care. It will not be surprising to you that we have differences in views, but as I read it occurred to me that there are questions of interpretation here that may never have been spelled out clearly.

- 1) The paper seems to be based throughout on the premise that equalization means equalized expenditure. Even though on page 35 you speak of a "shift of emphasis" to equalized expenditure, you have already on page 1 stated this as your own objective. Even though also on page 1 you recognize that the NESDEC formula seeks to equalize potential expenditure, it seems to me that you have still judged the various proposals in terms of equalized expenditure throughout the report. I read both Serrano and Dusartz as requiring state provision for equal fiscal ability in school districts, also defined as fiscal neutrality on the part of the state; both specifically deny intent to require equal expenditure.
- 2) The percentage feature of this formula is not intended as "incentive" to spending, but rather as giving flexibility to expenditure. It serves the same purpose as the weighting given to differing costs of educating students of different ages and course requirements by the National Education Finance Project for use in foundation programs.
- 3) So far as I can determine from careful reading, the intent of the 50% proposal remains obscure. You note that the chief effect seems to be to increase state funding. This is true, but carries the corollary that the property tax burden would decrease accordingly, which you have not addressed. The other purpose is to get as many towns as possible within an equalizing range where state aid can make up the difference between districts. At present 83 towns - almost a quarter of the whole number - are protected by the 15% minimum and thus outside the equalizing range; at the 40% average, 71 towns would be protected by 15% minimum and 60 at 10%; at the 50 % average, all but 46 would be covered at 15%, 45 at 10%, and all but 32 at zero minimum. Above 50% the issue of local control enters the discussion.
- 4) I strongly agree with you that the underfunded formula is disequalizing and several years running used actual figures to show this in testimony at the State House. In the past we have proposed that the legislature change the average coverage figure to fit available funds, but were told it was too complicated. I could agree that a higher percentage in the formula might be more subject to underfunding in the present



situation, but suggest that contemplated changes in the tax structure can be expected to change the situation. I do not agree that the change in percentage "does not change the potential equalization properties of the program" (page 25) as indicated above.

I note that you have based the bulk of your paper on work done in response to certain requests I made of you early on, although you do not credit them. You will recall I asked you to make the seven-year projections in order to test the effect of the formula over time, and gave you some work I did last year on a dozen or so towns which indicated that expenditures tended to equalize in this manner. Secondly, I asked you to make the test on a uniform effort simply to remove one variable, not because we intended to rely upon uniform effort.

One outcome of this was support for dropping the minimum percentage to zero, which does not appear in your paper, and I think this would affect your findings. For instance, the runs you gave me last week bore out others I had from R & D. I think it significant that val/NAM runs give 1975 standard deviations of 158 on 50% aid run at 75%~~0%~~, and 184 on 40% aid. Again, the 40% histogram, even with expenditure controls, show 48 towns above the mean on val/NAM and the 50% histogram only 26. I suggest this results from the wider equalization range I referred to above.

Third, I pointed out the weighting effect of the val/SAC in some work I did comparing another group of towns using both val/SAC and val/NAM, and asked that it be tested for the whole group, which you did in February. What is not mentioned in the report, and apparently was not made clear, is that the SAC was used as a municipal overburden factor. It seems obvious that any weighting will distort a formula; the question is, does it serve a useful purpose? There are three proposals for municipal overburden factors at the moment - the SAC, a nonschool tax factor, and the LVW's valuation per capita. R & D ran the latter two, but on top of the SAC, which was hard to evaluate, and we have yet to look at them separately.

Returning to the notion of testing the various formulas for intent, it seems to me it should be made clear that the 40% bill does lean to equalizing expenditure by maintenance of the constraints on reimbursable expenditures, while the 50% bill tries to extend the equalizing range. It should be recognized that the 40% bill contemplated the same dollar total as the 50% bill, with the balance in straight municipal aid. Since the intent of the 50% bill is to shift a portion of school funding to state revenues, I would propose that, instead of comparing the two for mean expenditure on the same local tax base, you run the 50% bill at 75% to 0% with corresponding effect in lowering the property tax to arrive at average expenditures equivalent to those at the 40% level (can you do this? you refer to the procedure on page 53 but did not use it) and then note any differences in equalization of potential expenditure due to the differences between use of constraints on the one hand and extending the equalizing range on the other.

Some other points that concerned me:

1. The membership weighting you refer to on page 16 is not specified and would be interesting to know.
2. If you wish to test the past effectiveness of the formula, despite its underfunding, on real-world expenditures, it would be important to add categorical programs which are significant in some cases.
3. I have trouble with the notion that the level of state aid is a legislative decision about the level of local expenditure. This might be so in a situation of full state





funding, but it is hard to accept in a state where local costs are state-funded at less than 30 percent while they are increased by minimum-salary legislation, mandated kindergarten, and other legislated programs, on the expectation that the property tax will bear the increased burden. Indeed, the prospect of that sort of legislative decision-making is one reason why full state funding has little support here.

4. A third cause bears even more heavily on equalization with all versions of the formula than the two you have mentioned on page 34. This is the wide variations in current expenditures from which we start. What seems significant to me is the strong movement toward equalized expenditures all these proposals take. What would have happened at 7 plus x years?

I would agree with the suggestion on page 36 that the NESDEC formula supports local control and is intended to do so. The issues you cite are all pertinent, but I feel all can be studied in a NESDEC framework. At the same time it is essential to note that real equality of opportunity is not to be attained by equalization of expenditure alone nor yet by equalization of fiscal potential alone. Effective use of funds, diverse approaches for diverse needs, high quality of educational resources, and strong intent to help children learn, all enter into what you might call the dynamics of local decision-making, and have as much to do with good education as money. Under these realities, you may call equal potential a mirage; I propose it is a necessary base for equity in funding the differing expenditures that enter into providing equal opportunity.

Thank you for letting me discuss these matters with you.

Sincerely yours,

*Charles*

Mrs. Edward F. Ryan



APPENDIX B - EXPRESS SIMULATION MODEL AND COMPUTER PROGRAM LISTING

STATE AID TO EDUCATION MODEL  
 DEVELOPED BY (IN ALPHABETICAL ORDER):  
 BROCHOW, A HAAN, C STABELL  
 NOVEMBER 18, 1971  
 TITLE MODEL LEVEL 1000  
 STATE AID TO EDUCATION  
 SUB LEVEL 100  
 TOWN SUBSCRIPT SMIN 1 SMAX 352 SVN TNAME1  
 YEAR SUBSCRIPT SMIN 1 SMAX 15 SVN YRNAME  
 NCINTRV SUESCRIP SMIN 1 SMAX 352  
 DATA.BAS LEVEL 100  
 TOWN DATA INEUT //  
 SAC INTEGER <TOWN, YEAR>  
 NUMBER OF SCHOOL ATTENDING CHILDREN  
 TOWN INTEGER <TOWN, YEAR>  
 PERCENT AVERAGE MEMBERSHIP IN PUBLIC SCHOOLS  
 EQPVAL REAL <TOWN, YEAR>  
 EQUALIZED PROPERTY VALU  
 REEXP REAL <TOWN, YEAR>  
 REIMBURSABLE SCHCOL EXPENDITURES  
 STATEID REAL <TOWN, YEAR>  
 STATE AID PER TOWN BASED ON PREVIOUS YEARS EXPENDITURES  
 TNAME CHAR <TOWN>  
 TOWN NAME  
 TNAME1 CHAR <TOWN>  
 TOWN SUBSRPT NAME  
 YRNAME CHAR <YEAR>  
 REAL YEAR  
 DECISION LEVEL 100  
 DECISION VARIABLES//  
 NTOWN INTEGER  
 NUMBER OF TOWNS TO BE CONSIDERED:  
 NYEAR INTEGER  
 NUMBER OF YEARS TO BE CONSIDERED  
 LBFRAC REAL  
 LOWER BOUND ON FRACTION OF REEXP:  
 LBFRAC REAL  
 LOWER BOUND ON FRACTION OF REEXP:  
 LBEXP REAL  
 LOWER BOUND ON EXPENDITURE BASE:  
 LBEXP REAL  
 LOWER BOUND ON EXPENDITURE BASE:  
 LBEXP REAL  
 LOWER BOUND ON EXPENDITURE BASE:  
 LBEXP REAL  
 AVERAGE COVERAGE REAL <YEAR>  
 DESIRED AVERAGE COVERAGE:  
 UNIRATE BCCLEAN  
 INDICATE BY REPLYING YES OR NO /IF YOU DESIRE TO RUN THE MODEL WITH -  
 UNIFORM LOCAL SCHCOL TAX RATE FOR THE WHOLE STATE (ANSWER YES) / -  
 IF YOU DESIRE TO RUN UNDER CONDITIONS WITH DIFFERENT LOCAL TAX RATES -  
 (ANSWER NO).  
 TAXRATE REAL  
 IF YOU HAVE ANSWERED YES ON THE UNIRATE QUESTION INDICATE HERE THE -  
 FORM TAXRATE DESIRED.  
 DISPOSE REAL  
 PERCENTAGE OF STATEAID THAT IS USED TO ALLEVIATE GENERAL LOCAL TAX -  
 IF UNIRATE HAS BEEN ANSWERED YES, SHOULD USE A DISPOSE VALUE OF 0.0



STATEAID INPUT P1

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LISTO BOOLEAN  
HISTOGRAM OF EXP PR NAM DESIRED?ANSWER YES OR NO  
OWEXP REAL  
NEK BOUNDARY CN EXPENTIEURE HISTOGRAM  
VWEXP REAL  
PENDITURE HISTOGRAM INTERVAL  
CINTR INTLGER  
MBER OF INTERVALS IN EXPENDITR HISTOGRAM  
RTYEAR INTEGER  
AR TO BE SCRTED  
TPUT LEVEL 100  
SUITS CF MCODEL CALCULATION  
ASAC REAL <TOWN, YEAR>  
ATE AID PER SCHOCI ATTENDING CHILD  
AFRAC REAL <TOWN, YEAR>  
CTICN TC BE PAID BY STATE AID BEFORE APPLYING BOUNDS  
ALSAC REAL <TOWN, YEAP>  
VALIZLD PFCPPERTY VALUE PER SAC  
PSAC REAL <TOWN, YEAR>  
MEUSABLE EXPENDITURE PER NAM  
PHISTO INTEGER <NOINTRV, YEAR>  
PENDITURE PR NAM HISTOGRAM  
PMAN REAL <YEAR>  
N EXPENDITURE PER NAM  
PVAE REAL <YEAR>  
NDARD DEVIATION OF EXPENDITURE PER NAM  
DSUM REAL <YEAR>  
AL SCHOCI AID FFCM STATE  
HTAX REAL <TOWN, YEAR>  
AL SCHCCL PROPERTY TAX RATE  
XMEAN REPL <YEAR>  
N LCCAL TAX RATE  
XSTD REAL <YEAR>  
NDARD DEVIATICN CF LCCAL TAX RATE  
EXP1 <NOINTRV, YEAR>  
ALSAC1 <NCINTRV, YEAR>  
ASAC1 <NOINTRV, YEAR>  
AME2 <NCINTRV>  
T.END LEVEL 100  
EL GC ERCC NAME STAID  
RT PROC NAME SORTING



STAIID FORTRAN P1

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

```

SUBROUTINE STAIID                                     STAC0010
REAL*8 NCINTR/'NCINTRV'/                             STAC0020
REAL*8 YES/'YES'/                                     STAC0030
REAL*8 SCTAX1/'SCHTAX'/,RELIEF/'DISPOSE'/,UNIFRM/'UNIRATE'/ STAC0040
REAL*8 TAXMEN/'TAXMEAN'/,TAXVAR/'TAXSTD'/,TAXRTE/'TAXRATE'/ STAC0050
REAL*8 AIDSUM1/'AIDSUM'/,EXPM1/'EXPMAN'/,EXPV1/'EXFVAR'/ STAC0060
REAL*8 LOW/'LOWEXP'/,INTER/'INVEXP'/,ITOP/'NCINTE'/ STAC0070
REAL*8 EXPHST/'EXPHSTC'/                             STAC0080
REAL*8 STFR1/'STAFRAC'/                              STAC0090
REAL*8 STASC1/'STASAC'/                              STAC0100
REAL*8 TCWN1/'TCWN'/,EQPVL1/'EQPVAL'/,OYEAR1/'NYEAR'/ STAC0110
DIMENSION LCLEXP(355)                                STAC0120
REAL*8 CTCWN1/'NTOWN'/,REEXP1/'REEXP'/,NAM1/'NAM'/ STAC0130
REAL*8 EXPSC1/'EXPSAC'/                              STAC0140
REAL*8 SAC1/'SAC'/,PVLSC1/'PVALSC'/,AVGCV1/'AVGCCV'/ STAC0150
REAL*8 STAIID1/'STAIID'/                             STAC0160
REAL*8 UBFR1/'UBFRAC'/,LBFR1/'LBFRAC'/              STAC0170
REAL*8 UBEXP/'UBEXP'/,LBEXP/'LBEXP'/               STAC0180
REAL*8 YEAR/'YEAR'/,TOWN/'TCWN'/                   STAC0190
CALL EQU(YEAR,J)                                     STAC0200
CALL EQU(NCINTR,KH)                                 STAC0210
CALL EQU(TOWN,I)                                    STAC0220
NTOWN=IV(CTOWN1)                                    STAC0230
NYEAR=IV(OYEAR1)                                    STAC0240
NTOP=IV(ITOP)                                       STAC0250
NAMS=0                                              STAC0260
SACS=0                                              STAC0270
EQPVLS=0                                            STAC0280
IHISTO=0                                            STAC0290
DO 55 J=1,NYEAR                                     STAC0300
DO 55 KH=1,NTOP                                     STAC0310
CALL IUTIV(EXPHST,IHISTO)                          STAC0320
CONTINUE                                           STAC0330
DO 7 I=1,NTOWN                                      STAC0340
DO 7 L=1,NYEAR                                      STAC0350
J=1                                                 STAC0360
SEQPVL=V(EQPVL1)                                   STAC0370
SNAM=V(NAM1)                                        STAC0380
SSAC=V(SAC1)                                        STAC0390
J=L                                                 STAC0400
CALL PUTV(EQPVL1,SEQPVL)                           STAC0410
CALL PUTV(NAM1,SNAM)                               STAC0420
CALL PUTV(SAC1,SSAC)                               STAC0430
CONTINUE                                           STAC0440
DO 1 I=1,NTOWN                                      STAC0450
J=1                                                 STAC0460
NAMS=NAMS+IV(NAM1)                                 STAC0470
SACS=SACS+V(SAC1)                                  STAC0480
EQPVLS=EQPVLS+V(EQPVL1)                          STAC0490
DO 1 J=1,NYEAR                                     STAC0500
PVALSC=V(EQPVL1)/IV(NAM1)                        STAC0510
CALL PUTV(PVLSC1,PVALSC)                          STAC0520
CONTINUE                                           STAC0530
COMPUTATION OF CONSTANT LOCAL EFFORT              STAC0540

```





```

DO 5 I=1,NTOWN
J=1
LCLEXP(J)=V(RFEXP1)-V(STAID1)
SCHTAX=100.*LCLEXP(I)/V(EQPVL1)
CALL PUTV(SCTAX1,SCHTAX)
CONTINUE

```

SETTING ALL VARIABLES THAT ARE TO BE CONSTANT OVER YEARS EQUAL TO VALUE FOR YEAR 1

LOOPING FOR EACH YEAR TO CALCULATED FOR

```

DO 100 J=1,NYEAR
REEXPS=0
AIBSUM=0
EXPSQ=0
EXPSCS=0
TAXSUM=0
TAXSQ=0

```

LOOPING FOR EACH TOWN TO COMPUTE EXPENTITURE IN CURRENT YEAR WHICH IS A FUNCTION OF LOCAL FUNES (HERE CONSTANT) AND STATEAID RECEIVED ON LAST YEARS EXPENDITURE

```

DO 10 I=1,NTOWN
IF(J.EQ.1) GO TO 305
IF (IV(UNIFORM).EQ.1) GO TO 300
J=J-1
REFXP3=V(REEXP1)
J=J+1
LCLEXP(I)=LCLEXP(I)-V(RELIEF)*(LCLEXP(I)+V(STAID1)-REFXP3)
GO TO 305

```

LCLEXP(I)=V(TAXITE)\*V(EQPVL1)/100.

```

REFXP=LCLEXP(I)+V(STAID1)
SCHTAX=100.*LCLEXP(I)/V(EQPVL1)
CALL PUTV(SCTAX1,SCHTAX)
TAXSUM=SCHTAX+TAXSUM
TAXSQ=SCHTAX**2+TAXSQ
REEXPS=PEEXPS+REEXP
EXPSAC=REEXP/IV(NAM1)
KH=(EXPSAC-V(LOW))/V(INTER)
IF (KH.GT.NTOP) KH=NTOP
IF (KH.LE.0) KH=1
IHISTO=IV(EXPHST)+1
CALL PUTIV(EXPHS1,IHISTO)
EXPSQ=EXPSQ+IV(NAM1)*((EXPSAC)**2)
EXPSCS=EXPSCS+(EXPSAC*IV(NAM1))
CALL PUTV(PEEXP1,REEXP)
CALL PUTV(EXPSC1,EXPSAC)
CONTINUE
EXPMN=EXPSCS/NAMS
CALL PUTV(EXPM1,EXPMN)
EXFVR=SQRT((EXPSQ/NAMS)-EXFMN**2)

```

STAC0560  
STAC0570  
STAC0580  
STAC0590  
STAC0600  
STAC0610  
STAC0620  
STAC0630  
STAC0640  
STAC0650  
STAC0660  
STAC0670  
STAC0680  
STAC0690  
STAC0700  
STAC0710  
STAC0720  
STAC0730  
STAC0740  
STAC0750  
STAC0760  
STAC0770  
STAC0780  
STAC0790  
STAC0800  
STAC0810  
STAC0820  
STAC0830  
STAC0840  
STAC0850  
STAC0860  
STAC0870  
STAC0880  
STAC0890  
STAC0900  
STAC0910  
STAC0920  
STAC0930  
STAC0940  
STAC0950  
STAC0960  
STAC0970  
STAC0980  
STAC0990  
STAC1000  
STAC1010  
STAC1020  
STAC1030  
STAC1040  
STAC1050  
STAC1060  
STAC1070  
STAC1080  
STAC1090  
STAC1100



STAID        FORTRAN    P1

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CALL PUTV (EXPV1, EXPVE)	STAC1110
TAXMN=TAXSUM/NTOWN	STAC1120
CALL PUTV (TAXMEN, TAXMN)	STAC1130
TAXVR=SQRT ( (TAXSQ/NTOWN) -TAXMN**2)	STAC1140
CALL PUTV (TAXVAR, TAXVR)	STAC1150
DOING FOR EACH TOWN TO COMPUTE STATEAID	STAC1160
	STAC1170
	STAC1180
AVGEXP=RFEXPS/NAMS	STAC1190
LOWERE=V (LBEXP) *AVGEXP	STAC1200
UPPERE=V (UBEXP) *AVGEXP	STAC1210
DO 30 I=1,NTOWN	STAC1220
STAFRC=1.-V (AVGCV1) *(V (PVLSC1) / (EQPVLS/NAMS))	STAC1230
IF (STAFRC.LE.V (UBFRC1) .AND. STAFRC.GE.V (LBFRC1)) GO TO 21	STAC1240
IF (STAFRC.LT.V (LBFRC1)) ACTFRC=V (LBIRC1)	STAC1250
IF (STAFRC.GT.V (UBFRC1)) ACTFRC=V (UBFRC1)	STAC1260
GO TO 22	STAC1270
ACTFRC=STAFRC	STAC1280
CALL PUTV (STFRC1, STAFRC)	STAC1290
IF (V (REEXP1) .LE. (UPPERE*V (NAM1)) .AND. V (REXPE1) .GE. (LOWERE*V (NAM1)	STAC1300
C) ) GO TO 23	STAC1310
IF (V (REEXP1) .LT. (LOWERE*V (NAM1)) ) RXP=LOWERE*V (NAM1)	STAC1320
GO TO 29	STAC1330
CONTINUE	STAC1340
IF (V (REEXP1) .GT. (UPPERE*V (NAM1)) ) RXP=UPPERE*V (NAM1)	STAC1350
GO TO 28	STAC1360
RXP=V (REEXP1)	STAC1370
STAID=RXP*ACTFRC	STAC1380
AIDSUM=AIDSUM+STAID	STAC1390
CALL PUTV (AIDSM1, AIDSUM)	STAC1400
STASAC=STAID/V (SAC1)	STAC1410
J=J+1	STAC1420
CALL PUTV (STAIE1, STAID)	STAC1430
J=J-1	STAC1440
CALL PUTV (STASC1, STASAC)	STAC1450
CONTINUE	STAC1460
CONTINUE	STAC1470
END	STAC1480



-APPENDIX C - SIMULATION COMPUTER RUN OUTPUTRUN # 1

COV)	DESIRED AVERAGE COVERAGE:						
	1971	1972	1973	1974	1975	1976	1977
	0.65	0.65	0.65	0.65	0.65	0.65	0.65
MAN)	MEAN EXPENDITURE PR NAM						
	1971	1972	1973	1974	1975	1976	1977
	787.269	940.947	1006.38	1036.66	1050.56	1056.97	1060.12
VAR)	STANDARD DEVIATION OF EXPENDITURE PER NAM						
	1971	1972	1973	1974	1975	1976	1977
	149.954	146.121	156.346	164.204	168.3	170.144	171.032
MEAN)	MEAN LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	2.40374	2.40374	2.40374	2.40374	2.40374	2.40374	2.40374
STD)	STANDARD DEVIATION OF LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	0.732952	0.732952	0.732952	0.732952	0.732952	0.732952	0.732952
SUM)	TOTAL SCHOOL AID FROM STATE						
	1971	1972	1973	1974	1975	1976	1977
	332.465M	406.186M	440.299M	455.958M	463.177M	466.736M	468.603M
HISTO)	EXPENDITURE PR NAM		HISTOGRAM				
	1971	1972	1973	1974	1975	1976	1977
TRV	2	0	0	0	0	0	0
	2	0	0	0	0	0	0
	20	4	3	0	0	0	0
	86	11	8	10	9	9	8
	130	62	33	32	29	27	27
	57	91	73	51	52	51	52
	28	104	89	85	77	73	71
	15	51	82	77	74	76	73
	4	17	45	65	71	69	72
	4	6	10	23	30	37	39
	1	3	5	5	6	6	5
	1	1	1	1	1	1	2
	0	0	1	1	1	1	1
	1	1	0	0	0	0	0
	0	0	1	1	1	1	1
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0



RUN # 2

COV)	DESIRED AVEPAGE COVERAGE:						
	1971	1972	1973	1974	1975	1976	1977
	0.65	0.65	0.65	0.65	0.65	0.65	0.65
MAN)	MEAN EXPENDITURE PR NAM						
	1971	1972	1973	1974	1975	1976	1977
	787.269	915.533	961.675	982.337	992.197	996.667	998.793
VAR)	STANDARD DEVIATION OF EXPENDITURE PER NAM						
	1971	1972	1973	1974	1975	1976	1977
	149.954	224.027	224.45	225.362	226.173	226.431	226.472
MEAN)	MEAN LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	2.40374	2.24983	2.24983	2.24983	2.24983	2.24983	2.24983
STD)	STANDARD DEVIATION OF LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	0.732951	0.023478	0.023478	0.023478	0.023478	0.023478	0.023478
SUM)	TOTAL SCHOOL AID FROM STATE						
	1971	1972	1973	1974	1975	1976	1977
	332.465M	384.45M	407.728M	418.841M	423.874M	426.266M	427.514M
HISTO)	EXPENDITURE PR NAM HISTOGRAM						
	1971	1972	1973	1974	1975	1976	1977
FRV	2	0	0	0	0	0	0
	2	0	0	0	0	0	0
	20	0	0	0	0	0	0
	86	6	0	0	0	0	0
	130	95	57	39	33	29	28
	57	144	155	161	162	164	163
	28	42	57	59	61	58	60
	15	15	27	31	29	33	32
	4	8	13	17	22	23	24
	4	10	8	9	9	9	9
	1	2	5	6	6	6	5
	1	4	4	4	4	4	5
	0	1	0	0	0	0	0
	1	0	1	1	1	1	1
	0	1	1	1	1	1	1
	0	1	1	1	1	1	1
	0	0	0	0	0	0	0
	0	1	0	0	0	0	0
	0	2	2	1	1	1	1
	0	19	20	21	21	21	21





RUN # 3		DESTRFD AVERAGE COVERAGE:						
		1971	1972	1973	1974	1975	1976	1977
COV)		0.65	0.6	0.55	0.5	0.5	0.5	0.5
DESTRFD AVERAGE COVERAGE:								
PMAN)	MEAN EXPENDITURE PR NAM							
R		1971	1972	1973	1974	1975	1976	1977
		787.269	911.612	990.402	1067.06	1151.85	1197.	1222.66
PVAR)	STANDARD DEVIATION OF EXPENDITURE PER NAM							
R		1971	1972	1973	1974	1975	1976	1977
		149.954	223.397	230.05	225.46	221.757	220.83	221.441
(KMEAN)	MEAN LOCAL TAX RATE							
R		1971	1972	1973	1974	1975	1976	1977
		2.40374	2.24983	2.24983	2.24983	2.24983	2.24983	2.24983
(KSTD)	STANDARD DEVIATION OF LOCAL TAX RATE							
R		1971	1972	1973	1974	1975	1976	1977
		0.732954	0.023478	0.023478	0.023478	0.023478	0.023478	0.023478
(DSUM)	TOTAL SCHOOL AID FROM STATE							
R		1971	1972	1973	1974	1975	1976	1977
		328.048M	416.815M	503.185M	598.721M	649.594M	678.501M	695.611M
(PHISTO)	EXPENDITURE PR NAM HISTOGRAM							
R		1971	1972	1973	1974	1975	1976	1977
INTRV								
		2	1	0	0	0	0	0
		2	0	1	0	0	0	0
		20	0	0	1	0	0	0
		86	11	0	0	1	1	1
		130	92	21	4	0	0	0
		57	147	160	57	10	3	2
		28	39	76	144	87	45	25
		15	17	37	62	125	140	133
		4	6	13	32	53	70	87
		4	7	12	14	31	38	43
		1	5	2	8	9	17	18
		1	1	4	4	9	8	11
		0	1	0	0	1	3	5
		1	0	1	1	1	2	2
		0	2	0	0	0	0	0
		0	0	2	2	2	2	2
		0	0	0	0	0	0	0
		0	1	0	0	0	0	0
		0	2	1	1	1	1	1
		0	19	21	21	21	21	21



RUN # 4

COV)	DESIRED AVERAGE COVERAGE:						
	1971	1972	1973	1974	1975	1976	1977
	0.65	0.6	0.55	0.5	0.5	0.5	0.5
MAN)	MEAN EXPENDITURE PR NAM						
	1971	1972	1973	1974	1975	1976	1977
	787.269	937.027	1042.83	1139.28	1239.88	1295.14	1327.23
VAR)	STANDARD DEVIATION OF EXPENDITURE PER NAM						
	1971	1972	1973	1974	1975	1976	1977
	149.954	158.589	181.32	207.155	232.914	253.15	268.265
(MEAN)	MEAN LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	2.40374	2.40374	2.40374	2.40374	2.40374	2.40374	2.40374
(STD)	STANDARD DEVIATION OF LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	0.732954	0.732954	0.732954	0.732954	0.732954	0.732954	0.732954
(SUM)	TOTAL SCHOOL AID FROM STATE						
	1971	1972	1973	1974	1975	1976	1977
	328.048M	447.251M	555.925M	669.264M	731.532M	767.75M	789.587M
PHISTO)	EXPENDITURE PR NAM HISTOGRAM						
	1971	1972	1973	1974	1975	1976	1977
	2	1	1	0	0	0	0
	2	1	1	1	1	0	0
	20	5	3	4	4	4	4
	86	14	9	6	4	5	5
	130	64	31	20	15	14	14
	57	91	57	30	15	14	14
	28	98	72	56	36	23	20
	15	49	89	70	52	48	46
	4	18	51	72	66	58	49
	4	6	26	52	65	58	56
	1	2	7	29	49	53	53
	1	0	2	7	24	38	37
	0	1	1	2	14	21	32
	1	0	0	1	4	11	13
	0	1	1	1	1	3	6
	0	0	0	0	1	0	1
	0	0	0	0	0	1	0
	0	0	0	0	0	0	1
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0







RUN # 6

COV)	DESIRIED AVEPAGE COVERAGE:						
	1971	1972	1973	1974	1975	1976	1977
	0.65	0.6	0.6	0.6	0.6	0.6	0.6
MAN)	MEAN EXPENDITURE PR NAM						
	1971	1972	1973	1974	1975	1976	1977
	787.269	909.287	987.747	1024.43	1042.27	1050.7	1055.04
VAR)	STANDARD DEVIATION OF EXPENDITURE PER NAM						
	1971	1972	1973	1974	1975	1976	1977
	149.953	216.006	210.347	209.988	210.604	210.789	210.858
MEAN)	MEAN LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	2.40374	2.24983	2.24983	2.24983	2.24983	2.24983	2.24983
STD)	STANDARD DEVIATION OF LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
	0.732954	0.023478	0.023478	0.023478	0.023478	0.023478	0.023478
SUM)	TOTAL SCHOOL AID FROM STATE						
	1971	1972	1973	1974	1975	1976	1977
	325.427M	413.824M	455.161M	475.26M	484.752M	489.651M	492.31M
HISTO)	EXPENDITURE PR NAM			HISTOGRAM			
	1971	1972	1973	1974	1975	1976	1977
TRV	2	0	0	0	0	0	0
	2	0	0	0	0	0	0
	20	0	0	0	0	0	0
	96	6	0	0	0	0	0
	130	97	19	3	1	1	1
	57	149	163	125	106	94	92
	28	39	77	111	122	127	125
	15	17	39	43	46	52	54
	4	5	12	25	30	30	28
	4	7	10	13	15	15	19
	1	6	5	5	3	4	4
	1	0	1	1	3	3	3
	0	1	1	1	1	1	1
	1	0	0	0	0	0	0
	0	2	2	2	2	2	2
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0
	0	1	1	1	1	1	1
	0	2	2	2	2	2	2
	0	19	19	19	19	19	19





RUN # 7

	MEAN EXPENDITURE PR NAM						
	1971	1972	1973	1974	1975	1976	1977
MEAN)	787.269	917.054	960.938	980.125	989.314	993.967	996.423
	STANDARD DEVIATION OF EXPENDITURE PER NAM						
	1971	1972	1973	1974	1975	1976	1977
VAR)	149.954	217.597	210.563	206.381	204.381	203.396	202.891
	MEAN LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
MEAN)	2.40374	2.24983	2.24983	2.24983	2.24983	2.24983	2.24983
	STANDARD DEVIATION OF LOCAL TAX RATE						
	1971	1972	1973	1974	1975	1976	1977
STD)	0.732952	0.023478	0.023478	0.023478	0.023478	0.023478	0.023478
	TOTAL SCHOOL AID FROM STATE						
	1971	1972	1973	1974	1975	1976	1977
SUM)	334.177M	383.62M	405.235M	415.59M	420.832M	423.599M	425.106M
	EXPENDITURE PR NAM		HISTOGRAM				
	1971	1972	1973	1974	1975	1976	1977
TRV	2	0	0	0	0	0	0
	2	0	0	0	0	0	0
	20	0	0	0	0	0	0
	86	1	0	0	0	0	0
	130	45	3	1	1	1	1
	57	172	105	39	8	2	0
	28	73	180	247	278	284	286
	15	11	13	12	11	11	11
	4	8	7	9	10	10	10
	4	10	9	8	8	8	8
	1	2	5	6	6	6	5
	1	4	4	4	4	4	5
	0	1	0	0	0	0	0
	1	0	1	1	1	1	1
	0	1	1	1	1	1	1
	0	1	1	1	1	1	1
	0	0	0	0	0	0	0
	0	1	0	0	0	0	0
	0	2	2	1	1	1	1
	0	19	20	21	21	21	21



RUN # 8

DESIBED AVERAGE COVERAGE:							
	1971	1972	1973	1974	1975	1976	1977
(COV)	0.65	0.6	0.6	0.6	0.6	0.6	0.6
MEAN EXPENDITURE PR NAM							
	1971	1972	1973	1974	1975	1976	1977
(MAN)	787.269	911.553	989.17	1025.05	1042.8	1052.06	1057.07
STANDARD DEVIATION OF EXPENDITURE PER NAM							
	1971	1972	1973	1974	1975	1976	1977
(VAR)	149.954	209.229	195.518	188.242	184.513	182.603	181.629
MEAN LOCAL TAX RATE							
	1971	1972	1973	1974	1975	1976	1977
(MEAN)	2.40374	2.24983	2.24983	2.24983	2.24983	2.24983	2.24983
STANDARD DEVIATION OF LOCAL TAX RATE							
	1971	1972	1973	1974	1975	1976	1977
(STD)	0.732951	0.023478	0.023478	0.023478	0.023478	0.023478	0.023478
TOTAL SCHOOL AID FROM STATE							
	1971	1972	1973	1974	1975	1976	1977
(SUM)	327.979M	415.429M	455.857M	475.859M	486.286M	491.935M	495.089M
EXPENDITURE PR NAM HISTOGRAM							
	1971	1972	1973	1974	1975	1976	1977
(HISTO)	2	0	0	0	0	0	0
(TRV)	2	0	0	0	0	0	0
	20	0	0	0	0	0	0
	86	1	0	0	0	0	0
	130	45	1	1	0	0	0
	57	172	53	7	1	1	1
	28	78	206	162	98	38	13
	15	12	46	133	204	264	289
	4	5	6	9	8	7	7
	4	7	8	8	9	10	10
	1	6	5	5	3	3	3
	1	0	1	1	3	3	3
	0	1	1	1	1	1	1
	1	0	0	0	0	0	0
	0	2	2	2	2	2	2
	0	0	0	0	0	0	0
	0	0	0	0	0	0	0
	0	1	1	1	1	1	1
	0	2	2	2	2	2	2
	0	19	19	19	19	19	19



RUN # 9

DESIRE		AVERAGE COVERAGE:						
	1971	1972	1973	1974	1975	1976	1977	
SCOV)	0.65	0.6	0.55	0.5	0.5	0.5	0.5	
MEAN EXPENDITURE PP NAM								
	1971	1972	1973	1974	1975	1976	1977	
PMAN)	787.269	913.925	991.875	1068.1	1151.5	1195.92	1220.88	
STANDARD DEVIATION OF EXPENDITURE PER NAM								
	1971	1972	1973	1974	1975	1976	1977	
PVAR)	149.954	216.58	215.69	204.01	192.868	185.068	180.161	
MEAN LOCAL TAX RATE								
	1971	1972	1973	1974	1975	1976	1977	
(MEAN)	2.40374	2.24983	2.24983	2.24983	2.24983	2.24983	2.24983	
STANDARD DEVIATION OF LOCAL TAX RATE								
	1971	1972	1973	1974	1975	1976	1977	
(STD)	0.732954	0.023478	0.023478	0.023478	0.023478	0.023478	0.023478	
TOTAL SCHOOL AID FROM STATE								
	1971	1972	1973	1974	1975	1976	1977	
(SUM)	330.65M	418.478M	504.354M	598.33M	648.382M	676.505M	692.9M	
EXPENDITURE PR NAM HISTOGRAM								
	1971	1972	1973	1974	1975	1976	1977	
(HISTO)								
TRV								
	2	1	0	0	0	0	0	
	2	0	1	0	0	0	0	
	20	0	0	1	0	0	0	
	86	7	0	0	1	1	1	
	130	39	4	0	0	0	0	
	57	171	51	6	0	0	0	
	28	77	200	82	7	2	1	
	15	12	47	197	102	17	5	
	4	6	7	24	178	198	139	
	4	7	10	10	32	102	174	
	1	5	2	2	2	2	2	
	1	1	4	4	4	4	4	
	0	1	0	0	0	0	0	
	1	0	1	1	1	1	1	
	0	2	0	0	0	0	0	
	0	0	2	2	2	2	2	
	0	0	0	0	0	0	0	
	0	1	0	0	0	0	0	
	0	2	1	1	1	1	1	
	0	19	21	21	21	21	21	



APPENDIX D

An Analytical Analysis of NESDEC and Similar Formulas

The analysis assumes that there are no limits imposed on the general NESDEC formula of type

$$1. \quad \text{STAFRAC}_i = \left[ 1 - \text{AVGCOV} \frac{\text{EQPVAL}_i / \text{SAC}_i}{\sum_i \text{EQPVAL}_i / \sum_i \text{SAC}_i} \right]$$

where (using notation in EXPRESS model)

$\text{STAFRAC}_i$  = fraction of reimbursable expenditures that state will cover for school district  $i$

$\text{AVGCOV}$  = coefficient determining the average amount of total reimbursable expenditures that the state will cover with this average amount being equal to  $(1 - \text{AVGCOV})$

$\text{EQPVAL}_i$  = equalized property value in school district  $i$

$\text{SAC}_i$  = number of school attending children in school district  $i$ .

$\frac{\sum_i \text{EQPVAL}_i}{\sum_i \text{SAC}_i}$  = average for the state of the equalized property value per school attending child

In addition, if we define the following variables,

$\text{REEXP}_i$  = total reimbursable expenditures for district  $i$

$\text{NAM}_i$  = net average membership for district  $i$

$\text{TAXRATE}_i$  = school tax rate on local, equalized property value in district  $i$

we have at an equilibrium position (defined as when a school district is receiving exactly the amount of stateaid to its education that is is entitled to by the formula and by its local effort in terms of local funds raised through the school tax rate) that

$$2. \quad \text{REEXP}_i [1 - \text{STAFRAC}] = \text{EQPVAL}_i \cdot \text{TAXRATE}_i$$

substituting the value of  $\text{STAFRAC}_i$  from 1 into 2 we obtain:





$$\text{REEXP}_i \left[ 1 - (1 - \text{AVGCOV} \cdot \frac{\text{EQPVAL}_i / \text{SAC}_i}{\sum \text{EQPVAL} / \sum \text{SAC}}) \right] = \text{EQPVAL}_i \cdot \text{TAXRATE}_i$$

which gives

$$3. \quad \text{REEXP}_i \cdot \text{AVGCOV} \cdot \frac{\text{EQPVAL}_i / \text{SAC}_i}{\sum \text{EQPVAL} / \sum \text{SAC}} = \text{EQPVAL}_i \cdot \text{TAXRATE}_i$$

we see that as suggested by the proponents of the NESDEC formula, the total expenditure is not a function of local property value,  $\text{EQPVAL}_i$ , but a function of local effort in terms of the school tax rate,  $\text{TAXRATE}_i$ , as  $\text{EQPVAL}_i$  is eliminated on both sides of the equation 3 giving

$$\text{REEXP}_i = \text{TAXRATE}_i \cdot \frac{\sum \text{EQPVAL} / \sum \text{SAC}}{\text{AVGCOV}} \cdot \text{SAC}_i$$

However, we see that the total reimbursable expenditures is also a function of the number of school attending children,  $\text{SAC}_i$ . By dividing both sides by the net average membership for the district,  $\text{NAM}_i$ , we see that the total reimbursable expenditure for NAM is both a function of the local tax rate and the ratio of  $\text{SAC}_i$  to  $\text{NAM}_i$ , i.e. the inverse of the proportion of the school children population in public schools:

$$\text{REEXP}_i / \text{NAM}_i = \text{TAXRATE}_i \cdot \frac{\sum \text{EQPVAL} / \sum \text{SAC}}{\text{AVGCOV}} \cdot \frac{\text{SAC}_i}{\text{NAM}_i}$$

From this result we can draw the following conclusions:

A. In order that a uniform taxrate over the state will give a uniform, equal expenditure per NAM, it is necessary that the proportion of children in public schools be constant all over the state. (Studies show that the ratio has a standard deviation of approximately 8% around a mean of 78% for the state)

B. The effect of changing the AVGCOV constant in the formula, keeping all other factors constant except local school tax rate, will result in a general uniform lowering of the school tax rate (i.e., the factor AVGCOV in the formula has no leverage on the equalization effects of the formula, in terms of equal potential equalization). Its main effect is the general transfer of the costs of education to the state.



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