Estimation of Axial Compressor Body Forces Using Three-Dimensional Flow Computations

by

Georg A. Reichstein

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Master of Science at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

This thesis presents an examination of body force distributions in a single stage low speed compressor. The body force distributions are developed using two different computational procedures, an axisymmetric streamline curvature calculation and an unsteady, three-dimensional flow simulation. A two-dimensional body force representation is defined as a benchmark to evaluate the departures of the computed forces from two-dimensional behavior. The most important contribution to this departure (for both the streamline curvature calculation and the three-dimensional simulation) is identified as the change in streamtube height across the blade rows. The magnitude of the departures increase with blade loading and, for the compressor examined, are smaller than five per cent of the two-dimensional estimate at design but show values up to 50 per cent near stall.

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Chapter 1

Introduction

Predicting axial compressor instability, which manifests itself as rotating stall and surge, has been a long standing problem in compressor design. This instability sets a limit on the useful operating range of gas turbine engines [1]. Current procedures for estimating the onset of instability are either based on correlations, or simplified analysis of the fluid dynamics. The goal of the present project is to create a methodology to predicts instability by using only geometry and flow conditions as input parameters.

A procedure for describing compressor instability was introduced by Gong [2], who used a body force representation of the blades to calculate the flow within the compressor. Geometric information is included into this procedure via the distribution of body forces that represent the forces exerted by the blades on the fluid. Gong showed that the body force description is sufficient to capture the scenario associated with the two inception routes to rotating stall: spike and modal.

A key element in determining the accuracy of the above method, is the body force description. To address this, Kiwada [3] developed a procedure that allows one to determine body forces using the flow fields provided from computational fluid dynamics (CFD). This methodology allows for the inclusion of three-dimensional flow effects, such as flow separation, reverse flow and tip leakage, into the body force distributions, and thus into the stability prediction. Kiwada used the extraction
procedure to provide qualitative information about the axial body force distributions at three spanwise locations: the hub of a single stage compressor, its mean radius, and its tip.

The body force database, which is the input to Gong’s procedure, contains the body forces obtained from various CFD solvers. The lack of converged solutions from unsteady three-dimensional flow solvers near and beyond stall creates the need for a body force database derived from multiple CFD solvers. Combining body force distributions from streamline curvature and three-dimensional computations requires assumptions; the influence of these assumptions on the accuracy of the stability prediction is not yet determined [4]. This issue will not be discussed in this thesis.

The purpose of this thesis is to address the flow dynamics that set the magnitude and spanwise distribution of body forces in a single stage axial compressor. Two-dimensional body force estimates derived based on constant streamtube height and constant radius are then compared to the actual body forces to estimate the influence of the three-dimensional flow effects such as reverse flow and tip leakage. These three-dimensional flow effects are then quantified and linked to the features of the streamsurface. Ultimately, understanding the influence of three-dimensional flow effects on body forces will help to improve the body force database and the stability prediction.

1.1 Contributions

1. Spanwise distributions of body forces, based on two- and averaged three-dimensional flow fields, are defined for a single stage compressor, and linked to pressure rise, turning and loss coefficients.

2. A two-dimensional body force estimate is defined to quantify the influence of changes in momentum flux and pressure rise on the actual body forces. The differences between the two- and three-dimensional forces are linked to the stream-
tube contraction across the blade passage.

3. A procedure to estimate three-dimensional body force distributions, for the range of flow coefficients, from design to low flow, based on two-dimensional estimates and streamtube contraction is derived.

1.2 Thesis Outline

The structure of the thesis is as follows:

Chapter 2 gives an overview of the literature on the use of body forces in stall prediction. The two computational fluid dynamics (CFD) applications that provide the flow field data for the body force analysis in this thesis, an axisymmetric streamline curvature code (SLC) [5] and a three-dimensional unsteady computation (TBLOCK) [6], are then briefly described. The stability prediction procedure and the body force extraction methodology [3] are discussed next; the latter closes the gap between the CFD results and the body force database.

In Chapter 3, a control volume based body force analysis is described. The two- and three-dimensional body force definitions from the body force analysis are the basis for the analyses in the following chapters.

Chapter 4 compares spanwise distributions of body forces from SLC and TBLOCK computations. Two-dimensional body force estimates are compared to three-dimensional axisymmetric body forces at the same radial position. This analysis determines the local significance of reverse flow, tip leakage and flow separation to the body force distributions.

Chapter 5 develops the links between the streamtube geometry and three-dimensional flow effects. A basic model is introduced to provide spanwise three-dimensional body force estimates based on streamline contraction. The general applicability of
the results beyond the examined configuration, is discussed based on Bernoulli's equation of energy conservation.

Chapter 6 presents the summary and conclusions.
Chapter 2

Background

This chapter provides background information on body force definitions, their processing and their application in stability assessment models. Section 2.1 introduces previous work on the topic of stability assessment. Section 2.2 explains the two sources of flow field data, a streamline curvature (SLC) solver and a three-dimensional unsteady flow solver. Section 2.3 describes the body force extraction methodology and the stability prediction model that requires the forces as an input.

2.1 Literature Overview

Marble [7] introduced body force as description of the flow in axial compressors. Horlock and Marsh [8] were among the first to examine the accuracy of different modeling and averaging methods with respect to body forces. The focus of the work by Horlock and Marsh was the behavior of the flow on the circumferential plane rather than on the radial variation. Later applications, shown below, of the body force representations use circumferentially averaged flow fields, making a detailed description of the flow field and body forces around the circumference unnecessary.

Longley [9] applied body force representations to the compressor stability assessment. He demonstrated the capabilities of a time accurate simulation of the compressible non-linear equations for the flow through an entire compressor based on
a body force representation. To simplify the computational requirements, Longley
only addressed instability with axial and circumferential resolution of moderate and
long lengthscale flow features, rather than spike type inception. The flow in the
non-blade regions was calculated with a two-dimensional Euler solver. The blade
rows were modeled with one-dimensional equations and additional body force terms.

One-dimensional compressible flow requires three equations, the conservation
of mass, axial momentum and energy. The change in tangential momentum due to a
tangential body force is determined by an additional equation. These four equations
constitute the body force equations as applied by Longley:

\[
\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho c \\ \rho e \end{bmatrix} + \frac{\partial}{\partial x} \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho u(e + p/q) \end{bmatrix} + \frac{\partial}{\partial y} \begin{bmatrix} \rho U \\ \rho u U \\ \rho v U \\ \rho e U \end{bmatrix} = \begin{bmatrix} 0 \\ F_x - F_y \tan \alpha_{rel} \\ F_y \\ U F_y \end{bmatrix} \tag{2.1}
\]

These equations describe a one-dimensional body-force blade row moving with speed
U. Stagnation pressure loss and flow turning are accounted for by F_x and F_y. The
body forces can only respond to the flow at the blade row inlet, and not to flow
variations in the blade passage.

Longley [9] points out that there is a benefit in having a continuous definition
of variables throughout the compressor and gradual turning of the flow rather than
discontinuous changes, as in the actuator and the semi-actuator-disk models. Doing
so avoids spurious reflections of pressure waves passing through the blade row are
avoided.

For the steady-state blade row performance, body forces were specified by curve-fits
for loss coefficient and flow turning [9]. The accuracy of the stall prediction is related
to the fidelity of the body force estimation, which Longley found difficult to gauge.
Although precise body force distributions are important for the use of Longley’s
model, he does not conduct a parametric study to determine the specific body force distributions in the blades, nor does he include a discussion of the dependence of body forces on the flow field, or vice versa. This lack led to the conceptual outline of this thesis.

Gong, et al. [10] presented a simple computational model with the capability of simulating both spike and modal stall inception in both single and multi-stage axial compressors. The following phenomena were captured: development of rotating stall, formation and evolution of localized short wavelength stall cells, transition from long to short wavelength stall inception as a function of inlet guide vane stagger angle, and occurrence of rotating stall inception on the negatively sloped portion of the compressor characteristic.

The three-dimensional aspects of the flow were simulated using a body force representation for the blade and the blade-free regions. The blade regions were assumed to consist of an infinite number of blades with the relative flow described by locally axisymmetric Euler equations. The body forces can respond to local, unsteady three-dimensional flow variations in the blade passage. In the work by Gong et al. each blade row is represented by body force vectors derived from the turning of the flow, and the blade’s total pressure rise ($\Psi_{tt}$):

$$\mathbf{F} = \mathbf{F}_{\text{turning}} + \mathbf{F}_{\Psi_{tt}} \quad (2.2)$$

Only the axial component of $\mathbf{F}_{\Psi_{tt}}$, $F_{\Psi_{tt},x}$, contributes to the pressure rise:

$$F_{\Psi_{tt},\theta} = 0 \quad (2.3)$$

$$F_{\Psi_{tt},x}(x,r,\theta) = f(x)\Psi_{tt}(\phi(x,r,\theta),r) \quad (2.4)$$

In Equation 2.4, $f(x)$ defines the body force distribution along the streamlines in the blade region. The total pressure rise, $\Psi_{tt}$, is expressed as a function of local flow
coefficient, \(\phi\), and blade radius. \(\mathbf{F}_{\text{turning}}\) is chosen normal to the local velocity \(\mathbf{V}\) so that it does not contribute to the change in total pressure:

\[
\begin{align*}
F_{\text{turning},\theta} &= cV_\theta \left( r\Omega + V_z \tan(\alpha) - V_\theta \right) \\
F_{\text{turning},x} &= \frac{V_z}{V_\theta} F_{\text{turning},\theta}
\end{align*}
\] (2.5)

The parameter \(c\) in Equation 2.5 is a scaling factor to ensure the correct deviation angle at the design flow conditions. The body force model can be summarized by the following equation:

\[
[F_x, F_\theta, F_r] = \mathbf{F}(\mathbf{V}(x, \theta, r), x, r)
\] (2.7)

This body force approach reproduces the input overall pressure rise and turning angle for steady axisymmetric flow.

Xu, Hynes and Denton [11] suggested a different approach to the body force modeling. They discuss the extraction of body forces from ‘stand-alone’ three-dimensional single passage steady calculations. The extraction of body forces from ‘stand-alone’ CFD can be difficult near and beyond stall and Xu et al. discussed two alternative possible approaches to modeling body forces. If the blade row characteristics are known, the construction of distributed body forces is possible, based on local pressure rise. The downside of this approach is the involved arbitrariness and empiricism in curve fitting and extrapolation used to obtain body force representations. A second suggestion to obtaining body force distributions, was to model the dynamic response of the body forces with a bulk body force, which split the force terms into a time mean term and a time fluctuating term, to capture unsteady dynamics. The latter can be derived as a function of the flow coefficient.

To limit the amount of arbitrariness in either approach, Xu et al. propose a ‘Viscous Body Force Model’ where the inviscid blade force is calculated for every blade passage, and only the viscous component of the force has to be modeled. In
other words, the unsteady flow is split into two parts, inviscid and viscous. The inviscid flow can be calculated without modeling, and the viscous contribution can be obtained as a linearized perturbation from the mean. This body force model is capable of dealing with unsteady disturbances from length scales on the order of the circumference to length scales of the blade passage. High accuracy can be maintained as the modeled, viscous part of the body force is small compared to the inviscid force.

Xu has further expanded on the topic of viscous body forces [12], showing that for small excursions from the mean, a simple drag coefficient model for the body forces is sufficient. The drag coefficient based viscous body force is described as follows:

\[ F = -C_d \frac{1}{2} \rho |u| |u| \text{Vol} \]  

(2.8)

In Equation 2.8, \( \rho \) and \( u \) are local density and velocity, ‘Vol’ is the volume of the mesh cell in CFD. The drag coefficient, \( C_d \), was obtained from fine mesh Reynolds-averaged, Navier-Stokes (RANS) computations. For small perturbations, the drag coefficient was assumed constant.

Xu states that the main benefit of a body force model, compared to an actuator disk model, is the ability to distribute the body force terms inside the blade row region. A continuous body force distribution inside the blade row regions allows the model to account for unsteady behavior and dynamic responses in the bladed regions of the compressor stage. Again, the main problem is obtaining an accurate body force distribution in the blade passage, because the distribution has to be inferred from steady axisymmetric characteristics and distributions at off-design will significantly deviate from the steady axisymmetric estimate. This, in Xu’s opinion, is a problem largely based on individual design and thus almost impossible to generalize.

To increase the accuracy of the body force distribution data base, Kiwada [3] devel-
oped a methodology for extracting body force distributions from three-dimensional computations (see 2.3.1). He presented the radial body force distributions at the hub, mean and tip radii inside the blade rows of a single stage compressor, allowing a qualitative link to flow features and loading. The four blade rows used in his computations are IGV, rotor, stator and OGV. In the IGV, he identifies the distribution of loading along the chord, with peak loading at 25% chord. The magnitude of the forces decreases with $\Phi^2$. In the rotor, the loading moves toward the leading edge as the flow coefficients approach stall. Negative forces and force vectors opposite to the direction of the main flow in the tip region are linked to the onset of rotor tip stall, which is accompanied by reverse loading associated with a local decrease in static pressure. Kiwada identified these regions as the onset of separation.

A body force database is the foundation of stability assessment models that represent axial compressors through body force distributions. The analyses and discussions so far have focused on the applicability of body forces to stall assessment, and the degrees of success and accuracy that can be achieved with different body force distribution approaches. An attempt to analyze the body force distributions inside the blade passages of an axial compressor beyond the first qualitative approach by Kiwada has not been made. Therefore this thesis will address radial body force distributions and their interdependence with the flow field.

2.2 Computational Fluid Dynamics (CFD)

This section describes the axisymmetric streamline curvature solver (SLC) and the three-dimensional time-accurate solver, used in the present work. Either solver can be used from the design flow coefficient to the peak in the characteristic. For flows lower than the peak coefficient, only the axisymmetric streamline curvature solver can be applied because the solutions from three-dimensional time-accurate solver do not converge. These two solvers provide the flow field data that is used in the body force extraction to create a database of body force distributions for the stability assessment.
2.2.1 Streamline Curvature

The axisymmetric streamline curvature (SLC) code described in this section was provided by Denton [5]. The inputs to SLC used here are loss coefficients and deviation angles that have been obtained from two-dimensional FLUENT cascade calculations at the hub, mean and tip radii of the IGV, rotor, stator and OGV blades [4]. In the SLC code the momentum and continuity equations are solved with entropy changes defined as functions of flow coefficients. Derivatives are defined, and equations are solved, on a meridional hub-to-tip surface. Quasi orthogonal calculating stations are placed at the leading and trailing edges of the blade rows. The meridional streamline curvature terms do not exert a large influence on the solution and are held constant after the first iteration that produces a radial equilibrium solution. The radial equilibrium terms are then iterated until a fully converged solution is achieved.

The code is designed to avoid entropy production near the walls, which can cause errors, especially when the flow begins to reverse. Instead of conserving the enthalpy, $H$, the entropy, $S$, and $rV_\theta$ along streamlines, a mixing model ensures that $H$, $S$ and $rV_\theta$ are conserved over the annulus. The axial velocity, $V_x$, is set to 5 % of the mean axial velocity when it attempts to drop below that value, in order to avoid reverse flow. SLC also does not include tip leakage effects. The flow effects mentioned above, tip leakage and reverse flow have an influence on the body force distribution and on compressor stability. To increase the accuracy of a stability assessment model, the flow effects mentioned above have to be a part of the body force distribution, therefore SLC cannot be the sole source of higher accuracy body force distributions.
2.2.2 Unsteady TBLOCK

TBLOCK is a finite volume, multiblock, Reynolds-averaged, Navier-Stokes solver developed by Denton [6]. It calculates the flow through a number of structured blocks, allowing the division of complex geometries into a series of blocks, each of which is solved as a separately structured grid. The blocks may be connected by a range of different boundary conditions. The code can handle shrouds, leakage paths, bleed slots, and various other geometric information.

TBLOCK uses the explicit “scree” algorithm [13] with spatially variable time-steps and three multigrid levels for faster convergence. This algorithm requires less artificial viscosity than other approaches. Turbulence is introduced using a mixing length model, while wall functions are used to obtain the surface skin friction.

The basic algorithm that TBLOCK solves is

\[ \Delta F = \left( 2 \left. \frac{dF}{dt} \right|^{n} - \left. \frac{dF}{dt} \right|^{n-1} \right) \Delta t, \]  

(2.9)

where \( F \) is the flow property being solved for, and \( n \) is the time step index. The maximum stable CFL limit for this algorithm is about 0.5. This value is lower than other, more complex, algorithms such as Runge-Kutta, but the speed and the low artificial viscosity make TBLOCK an attractive option. For the unsteady TBLOCK computations used in this thesis, a dual time-stepping method is used.

Body force distributions based on TBLOCK flow fields are of higher fidelity than those based on SLC. However, the range of convergence of TBLOCK solutions is limited to the negatively sloped part of the pressure rise characteristic. As low flows are of special interest to the stability assessment model, the body force database cannot be solely based on TBLOCK results. Due to the application of both solvers in the current approach to assess compressor stability, body force distributions from either solver, SLC and TBlock, are of interest and are part of the analysis in this
2.3 Body Force Extraction and Stability Assessment

This section gives an introduction to the body force extraction methodology and the stability prediction based on these forces.

2.3.1 Body Force Extraction Methodology

The body force extraction methodology by Kiwada [3] is based on dealing with a steady axisymmetric flow. For streamline curvature (SLC), the flow field is already steady and axisymmetric. For unsteady and three-dimensional flow fields, the time- and \( \theta \)-dimensions are momentum averaged to create steady axisymmetric flow fields, which are then processed to extract body forces. The three-dimensional body forces, calculated by applying the extraction methodology, are the basis for the analysis in this thesis.

The blade forces are determined using a control volume approach to balance the momentum flux and pressure forces. The force exerted by the blades on the fluid is equated to the imbalance in momentum flux and pressure force across the control volume. The mass flow, impulse, and transfer of momentum terms are grouped in flux variable vectors, \( Q \), \( F \), \( G \), and \( H \), respectively. These flux variable vectors are equated to the source terms in \( S \) (compare Equation 2.1) to determine the blade forces:

\[
\frac{\partial}{\partial \theta} Q + \frac{\partial}{\partial x} F + \frac{\partial}{\partial \theta} G + \frac{\partial}{\partial r} H = S
\]  

(2.10)

In a steady, or time-averaged, flow the time derivative in Equation 2.10 is 0. For a \( \theta \)-averaged or axisymmetric flow field the derivative with respect to \( \theta \) is 0 as well. Equation 2.10 thus can be expressed as:
Figure 2-1: Axisymmetric Control Volume, aligning with a generic computational cell, [3]

\[
\begin{bmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}
\end{bmatrix}
= \begin{bmatrix}
\frac{\tau \rho V_x}{r} & \frac{\tau \rho V_y}{r} & \frac{\tau \rho V_z}{r} \\
\frac{\rho V_x^2 + rP}{r} & \frac{\rho V_y V_x}{r} & \frac{\rho V_z V_x}{r} \\
\frac{\rho V_x V_r}{r} & \frac{\rho V_y V_r}{r} & \frac{\rho V_z^2 + rP}{r}
\end{bmatrix}
\lambda = \begin{bmatrix}
0 \\
\lambda \rho F_x + rP \frac{\partial \lambda}{\partial x} \\
\lambda \rho F_y + \lambda^2 \rho F_\phi \\
\lambda \rho V_\phi^2 + \lambda P + \lambda \rho F_\phi + rP \frac{\partial \lambda}{\partial r}
\end{bmatrix}
\]

Equation 2.11

The quantity \( \lambda \) in Equation 2.11 accounts for the influence of metal blockage on the flow in the blade passage. With this framework, the forces are obtained by balancing flux variables on four surfaces of the axisymmetric control volume (Figure 2-1).

In Kiwada's approach [3], the control volume matches the computational cell from the CFD solver. The flow quantities are defined on the surface nodes of the cells, and are calculated as the mean values of the flow quantities at the respective cell corner nodes. The derivatives of the flux variables are calculated by using the flux through each of the projected surface areas, normal to the direction of the derivative, and dividing the flux by the cell volume. For example:

\[
\frac{\partial \mathbf{F}}{\partial x} = \frac{F_3 A_{3,axial} + F_4 A_{4,axial} - (F_1 A_{1,axial} + F_2 A_{2,axial})}{\text{Volume}_{\text{cell}}}
\]

Equation 2.12

The subscripts refer to the surface number, starting with the upstream surface of the control volume and continuing counterclockwise.
The force acting on the cell in each direction is determined from the expressions that define the source term, \( S \) (Eqn. (2.11)):

\[
\begin{bmatrix}
F_x \\
F_\theta \\
F_r
\end{bmatrix}
= \begin{bmatrix}
\frac{S_x - r P \frac{\partial \phi}{\partial \theta}}{\lambda \rho} \\
\frac{S_\theta}{\lambda r p} \\
\frac{S_r - (\lambda \rho \frac{\partial \theta^2}{\partial \theta} + \lambda F + r P \frac{\partial \phi}{\partial \phi})}{\lambda \rho p}
\end{bmatrix}
\]  

(2.13)

The extracted forces from the code are in units of force per unit mass.

### 2.3.2 Stability Prediction

The body force database is the input for Gong’s stability prediction methodology, UNSCOMP [2]. The body forces, extracted from the axisymmetric flow fields, and related to local flow coefficients, are input into an axisymmetric Euler solver to compute flow fields over the entire range of flow coefficients of interest. The blade geometry is captured through the form of body force distributions. To create a database of local flow coefficients and body forces, distributions from both flow solvers (SLC and TBLOCK) are combined [4]. The database has to include negative local flow coefficients, so that forces can be assigned to regions of reverse flow in the compressor stage. The necessary combinations of body forces and flow coefficients are difficult to obtain, and the estimations regarding the transition from one distribution of body forces to the other introduce arbitrariness, because little is known about the body force distributions at off-design flow conditions.

UNSCOMP is run in two stages. First axisymmetric solutions are obtained over the range of flows of interest. Then unsteady computations are carried out to define the instability point. In the unsteady runs, the flow is perturbed with local unsteady variations in the body force. The stability point is found by inserting these perturbations at different flow conditions until the disturbances grow into a rotating stall.
The accuracy of the stall prediction and the description of the stall onset as calculated by Gong’s UNSCOMP are determined mainly by the quality of the body force database. This database provides UNSCOMP with body force distributions in the entire computational domain. As the database also includes body forces from SLC, which does not provide a resolution of the body force distributions in the blade row, assumptions about the shapes of the distributions are necessary to generate detailed radial and axial distributions based on axially integrated body forces [4]. To provide these distributions, the axial distributions from two-dimensional cascade computations at three radial locations are interpolated to other radial positions, and are scaled to match the chordwise integrated body force at each corresponding position.
Chapter 3

Defining Body Forces

Body force distributions, extracted from axisymmetric flow fields using the extraction methodology described in section 2.3.1, represent flow fields with three-dimensional flow features. The influence of the three-dimensionality has not been quantified in earlier studies of body forces.

This chapter explains the control volume approach used to calculate the body forces. Two control volumes are used, a two-dimensional volume based on streamline information at the leading edge (see Figure 3-1), and a volume that corresponds to a streamtube (see Figure 3-2(a)). The upstream surfaces of the two control volumes are identical to allow for comparison between the calculated forces.

Using the two-dimensional control volume, a two-dimensional body force estimation is defined and compared to the actual three-dimensional body forces. The two-dimensional estimate neglects streamline shifts in the blade passage and assumes equal x-momentum flux at inlet and exit of the blade passage. The two-dimensional estimate is thus only an expression of the pressure rise across the blade.

For a two-dimensional incompressible flow in a compressor cascade, the axial force is equal to the pressure force from the passage inlet to the exit, and the circumferential force describes the change in tangential momentum. The axial and
Figure 3-1: Two-Dimensional Control Volume; The control volume is bound by the leading and trailing edge of the blade and two streamlines without radial shift.

Figure 3-2: Control Volume

(a) Generic Control Volume

(b) Areas in the Control Volume
circumferential forces per spanwise unit length are [14]:

\[ f_x = (p_2 - p_1) s \]  

(3.1)

\[ f_\theta = \rho V_x^2 \left( \frac{V_{\theta,2}}{V_x} - \frac{V_{\theta,1}}{V_x} \right) \]  

(3.2)

From here on the two-dimensional forces are labeled with the subscript '2D'. The body forces calculated applying the Kiwada extraction methodology [3], which uses the axisymmetric streamtubes, are labeled with the subscript ‘CFD’.

### 3.1 The Axial Force

The non-dimensional axial force for the two-dimensional control volume is:

\[ F_{x,2D} = \frac{(p_{2,2D} - p_1) A_{CV}}{\rho \text{Volume}_{CV} \left( \frac{0.5U_{\text{wheel}}}{L_{\text{compressor}}} \right)} = \frac{(p_{2,2D} - p_1) A_{CV}}{N} \]  

(3.3)

The pressure at the leading edge surface, \( p_1 \), for each control volume is the mean of the values at the radii \( r_i \) and \( r_o \) at station \( \odot \) (see Figure 3-1). For the trailing edge, the pressure, \( p_{2,2D} \), is the mean of the values interpolated from the available CFD data to the radial locations \( r_i \) and \( r_o \) at station \( \odot \). Equations 3.4 and 3.5 show the pressures used in the calculation of two-dimensional forces, with the subscripts indicating the numbered station and the radial position:

\[ p_1 = \frac{p_{1,r_o} + p_{1,r_i}}{2} \]  

(3.4)

\[ p_{2,2D} = \frac{p_{2,r_o} + p_{2,r_i}}{2} \]  

(3.5)

For the two-dimensional force estimation, the upstream and downstream surface areas of the control volume are equal:

\[ A_{CV} = \pi (r_o^2 - r_i^2) \]  

(3.6)
The actual body forces are calculated using a control volume aligned with the streamlines. The difference between the two-dimensional body force estimate and the extracted forces is defined as $\Delta F$. For the axial force, $\Delta F_x$, is:

$$\Delta F_x = F_{x,\text{CFD}} - F_{x,2D}$$ \hspace{1cm} (3.7)

The quantity $\Delta F_x$ given in Equation 3.7 is due to two effects. The first effect is the change in momentum flux across the control volume. The second effect influencing $\Delta F_x$, is a difference in pressure rise across the control volume. In this thesis, the two contributors, change in momentum flux and change in pressure rise, are examined separately, so $\Delta F_x$ is divided into two components, as below in Equation 3.8:

$$\Delta F_x = \underbrace{F_{x,\text{mom}}}_{\text{due to momentum flux}} + \underbrace{F_{x,\text{contr&shift}}}_{\text{due to pressure gradient}}$$ \hspace{1cm} (3.8)

The momentum flux body force term in Equation 3.8, based on the momentum flux through the upstream and downstream surface of the control volume, is defined as:

$$F_{x,\text{mom}} = \left( \frac{\rho v_1^2}{2} A_1 + \frac{\rho v_2^2}{2} A_2 \right) \frac{1}{N}$$

The first term of Equation 3.9 is the mean of the momentum flux at the corners 1 and 2 of the control volume, multiplied by the area of surface 1; this is the momentum flux due to axial velocity entering the control volume through the upstream surface. The subscripts n1 through n4 in Equation 3.9 are associated with the labels at the four corner nodes of the generic control volume as shown in Figure 3-2(a). They label the flow quantities at the respective node. The surface areas of the control volume are numbered 1 to 4. The numbered surfaces correspond to the areas in Figure 3-2(b).

The pressure force is calculated from the difference in pressure acting on the areas projected normal to the axial direction at the trailing edge and the leading
edge of the blade:

$$F_{x,\text{contr\&shift}} = \left( \frac{[p]_{n3} + [p]_{n4}}{2} A_2 + \frac{[p]_{n1} + [p]_{n3}}{2} A_{1,3} + \frac{[p]_{n2} + [p]_{n4}}{2} A_{2,4} \right) - \left( \frac{[p]_{n1} + [p]_{n2}}{2} A_1 \right) \frac{1}{N} - F_{x,2D}$$

(3.10)

$F_{x,\text{contr\&shift}}$ accounts for the difference in pressure rise across the blade compared to $F_{x,2D}$. The subscripts share the same connotation as in Equation (3.9).

### 3.2 The Circumferential Force

As introduced in Equation 3.2, the circumferential body force describes the flow turning in the blade passage. For a two-dimensional flow the circumferential force is:

$$F_{\theta,2D} = \frac{\rho V_x (V_{\theta,2} - V_{\theta,1}) A_{CV}}{N}$$

(3.11)

The surface area is given in Equation 3.6, the factor $N$ in Equation 3.3. The density, $\rho$, and the axial velocity, $V_x$, are taken to be constant in the control volume. They are calculated by averaging the values at the four corners of the control volume:

$$\rho = \frac{[\rho]_{n1} + [\rho]_{n2} + [\rho]_{n3} + [\rho]_{n4}}{4}$$

(3.12)

$$V_x = \frac{[V_x]_{n1} + [V_x]_{n2} + [V_x]_{n3} + [V_x]_{n4}}{4}$$

(3.13)

Analogous to $\Delta F_x$, a difference between the two-dimensional force and the actual force is:

$$\Delta F_{\theta} = F_{\theta,\text{CFD}} - F_{\theta,2D}$$

(3.14)
\( \Delta F_\theta \) arises due to a difference in momentum flux in the circumferential direction. That is:

\[
\Delta F_\theta = F_{\theta,\text{mom}} = \left( \frac{[\rho v_2 v_\theta]_{n=3} + [\rho v_3 v_\theta]_{n=4}}{2} A_2 - \frac{[\rho v_2 v_\theta]_{n=1} + [\rho v_3 v_\theta]_{n=2}}{2} A_1 + \right)
\]

\[
\left( [\rho v_2 v_\theta]_{n=3} + [\rho v_3 v_\theta]_{n=4} A_{3,4} - \frac{[\rho v_2 v_\theta]_{n=1} + [\rho v_3 v_\theta]_{n=2}}{2} A_{1,2} \right) \frac{1}{N}
\]

(3.15)

The subscripts refer to Figure 3-2 and are the same as in the previous equations.

The terms \( \Delta F_z \) and \( \Delta F_\theta \) can be analyzed with respect to the relative change of the streamtube surface area, which is given below:

\[
\frac{\Delta A}{A_1} = \frac{A_2 - A_1}{A_1}
\]

(3.16)

The relative area change, \( \Delta A/A_1 \), in Equation 3.16 is positive for expanding streamtubes. The subscripts 1 and 2 refer to the leading and trailing edge respectively, and correspond to Figure 3-2(b).

Two other relevant parameters are the fractional change in mean streamtube radius and the relative change in streamtube height:

\[
\frac{\Delta r}{r_1} = \frac{r_{2,m} - r_{1,m}}{r_{1,m}} = \frac{\left( \frac{r_{n3} + r_{n4}}{2} \right) - \left( \frac{r_{n1} + r_{n2}}{2} \right)}{\left( \frac{r_{n1} + r_{n2}}{2} \right)}
\]

(3.17)

\[
\frac{\Delta h}{h_1} = \frac{h_2 - h_1}{h_1} = \frac{r_{n3} - r_{n4} - (r_{n1} - r_{n2})}{(r_{n1} - r_{n2})}
\]

(3.18)

The subscripts in Equations 3.17 and 3.18 correspond to the nomenclature in Figure 3-2. The fractional area change, \( \Delta A/A_1 \), is chosen as main reference parameter for the force analysis in the following chapters, because it includes both the change in streamtube height and mean radius [15].

The body force descriptions, as defined in this chapter, are applied to the re-
sults of the two flow solvers, SLC and TBLOCK. The next chapters include the analysis of the axial and circumferential body forces as well as the body force components that are defined to represent three-dimensional flow effects.
Chapter 4

Spanwise Body Force Distributions

This chapter presents a discussion of the spanwise body force distributions based on the CFD results. The analysis focuses on two operating points: design and off-design. From the flow fields obtained from streamline curvature, the flow at design and at 75 % of the design flow coefficient, close to the peak of the characteristic (see Figure 4-1), are chosen for examination. The analysis of forces at the design flow coefficient establishes a reference body force distribution. At off-design, there is a clear deviation from the two-dimensional flow as three-dimensional flow features, such as reverse flow, gain more influence on the pressure and the velocity field.

The unsteady and three-dimensional TBLOCK solution has been time- and circumferentially averaged. The analysis of the time- and circumferentially averaged TBLOCK flow fields will focus on the comparison between TBLOCK and SLC results. Two operating points are chosen for the analysis: 95 % and 80 % of the design flow coefficient, because for these flow coefficients solutions from SLC and TBLOCK exist (see Figure 4-1).

4.1 Streamlines

In the axisymmetric flow fields under examination streamlines reflect the pressure and the velocity field, and are used to discuss the SLC and TBLOCK flow fields.
Figure 4-1: Stage Characteristic, SLC and TBLOCK
4.1.1 SLC

The streamlines in the compressor stage calculated by SLC are shown in Figures 4-2 and 4-3. The shift in the streamlines increases with decreasing flow coefficient. Larger flow angles at the leading edges of the blades increase the possibility of flow separation in the blade passage, and a local deceleration of the flow is visible in the OGV near the hub. The upward shift of the streamlines can be observed in the rotor. Comparing the flow field at design and off-design indicates that the streamline shift originates in the OGV and propagates upstream with decreasing flow coefficient.

The local shift of the streamlines can be seen in Figure 4-4 for the rotor and stator. The small shift in streamlines in the rotor (about 1% of the radial position at the leading edge, Figures 4-4(a) and 4-4(b)) implies that the two-dimensional estimates of the body forces should be close to the actual force, due to little three-dimensional influences. The streamline shift in the stator is more visible at off-design conditions (Figures 4-4(c) and 4-4(d)), where the two-dimensional estimate is expected to deviate from the actual forces. The change in stator streamtube area at the hub is roughly 5% near design and 45% in the stator at off-design. The streamline shift in the SLC flow field is due to hub separation, so $\Delta F_x$ and $\Delta F_\theta$ can be interpreted...
4.1.2 T_BLOCK

The streamlines based on the T_BLOCK solution are shown in Figures 4-5 and 4-6. The shape of the streamlines within the blade passages is the most obvious difference between the SLC and the T_BLOCK results. In the tip region of the rotor and the stator, the streamline shift is on the order of 20% of the annulus height. The shift is due to two reasons: reverse flow in the rotor tip region, and a local acceleration of the flow at the stator leading edge. The local acceleration is accompanied by a local...
Even close to design (95 % $\Phi_{des}$, Figure 4-7(a)), the streamlines in the tip region of the rotor show signs of reverse flow, which is not observed for the SLC streamlines (Figure 4-7(b)), which give no detail within the blade passage. The radial positions of the streamlines at the leading and trailing edge of the rotor differ between SLC and TBLOCK by 8 % of the annulus height. In contrast to the forces based on SLC, the forces from TBLOCK are expected to include three-dimensional effects in the tip region and to deviate from the two-dimensional estimate.

At the off-design (80 % $\Phi_{des}$, Figure 4-7(c)), the shift of the TBLOCK streamlines in the blade passage is increased by 5 % of the annulus height over the shift at design. Both the reverse flow in the rotor tip region and the local unloading of the stator near the hub are increased. These changes in the velocity and pressure field increase the three-dimensional force components.

The TBLOCK streamlines in the rotor and stator are within 5 % compared to each other. The radial positions of the streamlines at the trailing edges are within
Figure 4-7: Streamlines, Design and Off-Design, Rotor, TBLOCK and SLC
Figure 4-8: Streamlines, Design and Off-Design, Stator, TBLOCK and SLC

10% of the annulus height comparing SLC to TBLOCK streamlines. Though no reverse flow occurs in the tip region of the stator for SLC and TBLOCK, there is a local streamline shift on the order of 20% near the design and off-design for the TBLOCK streamlines (see Figures 4-8(a) and 4-8(c)). The SLC streamlines at the trailing edge are within 5 to 10% of the TBLOCK streamlines with respect to the annulus height. Though SLC is limited in it’s capability to represent flow features such as reverse flow and tip leakage, the difference in streamline positions at the leading and trailing edges of the blade rows are within 10% compared to TBLOCK. The influence that these differences have on the body force distributions is discussed in section 4.2.2.

The flow fields calculated by the axisymmetric streamline curvature and the averaged
unsteady three-dimensional TBLOCK seem differ up to 20 % in the blade rows comparing streamlines from each solver. The leading and trailing edge radial positions of the streamlines agree within 10 % for both approaches. Excluding the tip region of the blades from this comparison, the agreement is on the order of 2 % between SLC and TBLOCK streamlines.

4.2 Body Forces

This section describes the spanwise distributions of the body forces and their relation to the pressure rise, turning and loss coefficient. We initially discuss results based on the axisymmetric streamline curvature and then address the body force distributions based on TBLOCK.

4.2.1 SLC and Two-Dimensional Estimate

Figure 4-9 shows the axial and circumferential force computed from SLC as a function of passage height, along with the two-dimensional approximation defined in Chapter 3. The right hand side of the plot (x-axis top to bottom) shows turning in degrees, loss coefficient, normalized pressure rise and non-dimensional axial force. The two dimensional approximation (white circles) follows the trend of the actual force (black) at the design, but overestimates the actual force over the entire span. The difference will be discussed in detail in section 4.2.3. As expected, the axial force follows the pressure rise. The circumferential force is almost constant over the range of the span, indicating a radially constant turning of the flow. The total pressure loss coefficient is below 0.01 over the annulus, and is considered to be of negligible influence on the force distribution.

Figure 4-10 shows the body forces and blade row characteristics at off-design conditions. The circumferential force varies more over the span than at the design, and the two-dimensional approximation of the body force is not as close to the actual force as at the design (the difference is increased from 2 % to 5 %). Both
Figure 4-9: Forces and Flow Characteristics, $\Phi = \Phi_{\text{des}}$, spanwise, Rotor

The circumferential and the axial force are within 2.5% of the two-dimensional approximation in the center span region. The axial force at the hub is increased by 5% compared to design. At the casing the axial force is increased by 50% compared to the design, and the turning in the upper half of the span is almost doubled. A larger axial and circumferential force means more work, in terms of energy input to the flow, i.e. turning and pressure rise, in the rotor. The increase in mechanical work compared to the design is mostly realized in the upper half of the span, because the local increase of forces is larger than at the hub. The two-dimensional interpretation of the body forces is able to reproduce this increase. The flow field in the stator at both flow conditions is shown in Figures 4-11 and 4-12. Again, the two-dimensional estimate is within 5 to 10% of the actual forces. As in the rotor, a flow separation in the blade passage can be identified for the lower 50% of the span.

Since the largest difference between the two-dimensional estimate and the actual body force is on the order of 10%, it can be concluded that the axial force is mainly driven
Figure 4-10: Forces and Flow Characteristics, $\Phi = 0.75 \cdot \Phi_{des}$, spanwise, Rotor

Figure 4-11: Forces and Flow Characteristics, $\Phi = \Phi_{des}$, spanwise, Stator
The observable differences between the two-dimensional estimate and the actual body forces are the result of hub separation in the OGV, to the extent that they are captured by SLC. The influence of three-dimensional flow effects, such as hub separation, will be quantified in Chapter 5.

### 4.2.2 Comparisons of TBLOCK and SLC

We first compare the body forces from the averaged TBLOCK to the results from the axisymmetric streamline curvature. Smaller axial forces in the TBLOCK computations are expected because the pressure rise is lower than given by SLC (see Figure 4-1). The ratio of axial forces obtained from TBLOCK and SLC is approximately 80% near the design and near stall. Equations 4.1 compare the sum of axial force, in the rotor and stator obtained from TBLOCK to the sum of forces from SLC:

\[
\frac{F_{x,\text{TBLOCK}}}{F_{x,\text{SLC}}} \Phi=0.95 \Phi_{\text{des}} = 0.81 \quad \frac{F_{x,\text{TBLOCK}}}{F_{x,\text{SLC}}} \Phi=0.8 \Phi_{\text{des}} = 0.8 \quad (4.1)
\]
The axial forces in the rotor close to the design (Figure 4-13(a)) agree with each other within 10% at the tip when comparing the TBLOCK and the SLC calculations. At off-design flow conditions (Figure 4-13(b)), the TBLOCK axial force is smaller than the SLC force, the difference being on the order of 50%. This is because the TBLOCK stage pressure rise is only 70% of that given by streamline curvature. The lower pressure rise is due to flow effects, such as reverse flow, not captured in the SLC solution. In the stator (see Figures 4-13(c) and 4-13(d)) the difference to the SLC solution and to a two-dimensional flow is observable as well. The axial body force calculated from TBLOCK near design, is on the order of 66% of SLC, over the entire span. At off-design, the TBLOCK axial body force is lower than the SLC force by roughly 25% for 60% of the span, while in the tip region, it is larger by 15%. These differences correspond to the local unloading of the stator identified in the streamline analysis.

The circumferential forces (Figure 4-14) are in better agreement than the axial forces between TBLOCK and SLC. The turning of the flow inside the blade passage is less affected by the three-dimensional flow effects than the contributors to the axial force. The hub and tip regions show signs of separation and reverse flow, that locally reduce the turning, with the turning as calculated by TBLOCK being about 20% less than in SLC. The reduced circumferential force can be observed over 80% of the span.

4.2.3 Three-Dimensional Flow Effects

This section discusses the distributions of $\Delta F_z$ and $\Delta F_\theta$ along with the fractional changes of the streamtube parameters, radial position, $r$, height, $h$, and cross sectional area, $A$. The analysis aims at identifying possible relations between the body force distributions and the changes in streamtube parameters.
Figure 4-13: $F_x$, Design and Off-Design, Rotor and Stator, TBlock and SLC
Figure 4-14: $F_\theta$, Design and Off-Design, Rotor and Stator, TBLOCK and SLC
4.2.3.1 SLC representation of $\Delta F_x$ and $\Delta F_\theta$

Near design, in the rotor (Figure 4-15), $\Delta F_\theta$ (white circles) is the largest in the casing region, with 2% of the two-dimensional estimate. The difference in axial force, $\Delta F_x$, ranges between 2.5% and 5% of the two-dimensional estimate. Locally accelerated flow is accompanied by a decrease in the pressure rise, and thus decrease the axial force, seen from comparing the two-dimensional estimate to the actual force. To explain this, the right-hand side of Figure 4-15 shows the relative radial shift, $\Delta r/r_1$, the relative streamtube contraction, $\Delta h/h_1$, and the streamtube area change, $\Delta A/A_1$. The spanwise distribution of these three parameters reinforces the observation, made in the analysis of $\Delta F_x$ and $\Delta F_\theta$: that the influence of distortions of the flow field on body forces are mostly limited to the tip region.

Locally higher pressure and lower axial velocity in the hub region result in a radial redistribution of streamlines over the entire annulus height. The radial shift will lead...
to a force component due to the change in momentum flux over the entire span. At off-design flow conditions (Figure 4-16), both the relative shift and the contraction are more visible than near the design. Three-dimensional effects are no longer local to the tip region, but contribute on the order of 5% of the two-dimensional force along more than 60% of the annulus, affecting especially the hub region. This can be attributed to the growing upstream influence of the OGV hub separation.

$\Delta F_x$ and $\Delta F_\theta$, along with the shift, the contraction and the area change, are shown in Figures 4-17 and 4-18 for the stator at design and off-design respectively. At design, the circumferential force is estimated to be within a maximum of 2% of the actual force. The axial forces are in agreement with each other in the mid span region, but deviate in the region toward the hub and the casing. The axial momentum is increased in the tip region along with the narrowing in the streamtubes. Both the relative contraction and the area change are on the order of
7.5% at the hub. Towards stall, the area changes range from -25% to 45%. The relative streamline shift influences the entire span. At around 20% of the passage height, the relative shift and the contraction balance each other, which results in no net change in area. $\Delta F_x$ is on the order of 2.5% of the two-dimensional force. The circumferential force shows differences of up to 10% between the two-dimensional and the actual force. In summary, the upstream influence of the OGV hub separation is more developed than in the rotor, causing a decrease in the axial momentum in the hub region. In addition to the rotor analysis, it is observed that the area change in the stator near stall is about 30 percentage points larger than in the rotor. All of the effects above make up the deviation from the two-dimensional flow and thus the two-dimensional body force estimate. The body force distribution has to be modified to account for the changes in the flow field discussed in this section.

The implications of larger area changes are more developed three-dimensional flow
effects, to the extent that SLC can capture these. \(\Delta F_x\) and \(\Delta F_\theta\), which are a representation of the three-dimensional flow effects, increase with the increase in streamtube area. In both the rotor and the stator, the spanwise distribution of \(\Delta F_\theta\) resembles the spanwise distribution of the relative area change. For the axial body force component, a similar resemblance cannot be detected. In addition to a change in momentum flux, \(\Delta F_x\) also includes a change in pressure rise. An analysis of the single components of \(\Delta F_x\), \(F_{x,\text{contr&shift}}\), and \(F_{x,\text{mom}}\), is presented in section 4.2.3.3.

**4.2.3.2 TBlock representation of \(\Delta F_x\) and \(\Delta F_\theta\)**

This section illustrates \(\Delta F_x\) and \(\Delta F_\theta\) as indicators of three-dimensional flow effects, and their relation to the spanwise change in streamtube geometry for the TBlock flow fields.

Figures 4-19 to 4-22 show both \(\Delta F_x\) and \(\Delta F_\theta\), and the relative changes in
streamtube geometry. The TBlock results show larger differences with the two-dimensional estimate than the SLC results. Near design, the two-dimensional estimate is 5% larger than the actual axial force over the lower 80% of the span. These 80% of the span correspond to the region that is affected by a contraction of streamlines due to the reverse flow in the tip region. The local acceleration of the flow causes a drop in the pressure. \( \Delta F_x \) is on the same order of magnitude at design and off-design conditions. However, at off-design the turning, as described by the circumferential force, changes on the order of 5% compared to the two-dimensional estimate. Both the axial and the circumferential force change by more than 10% at the tip. The tip region is discussed in section 5.3. For the stator, it is mainly the axial force that is affected by the three-dimensional flow effects. The change in the circumferential force is under 2.5% for the larger part of the span.

These observations lead to the hypothesis that a two-dimensional estimate of the circumferential force might be sufficient to represent the blade turning. The axial force, on the other hand, is more influenced by the three-dimensional effects. A two-dimensional estimate for the axial body force might not be sufficient. A sensitivity study is being carried out by Walker [4].

Being based on the averaged unsteady three-dimensional flow fields, \( \Delta F_x \) and \( \Delta F_\theta \) from TBlock capture more three-dimensional influences than those from SLC. The changes in momentum flux and pressure rise (e.g. in the region of reverse flow at the rotor tip) are reflected by the streamlines, and \( \Delta F_x \) and \( \Delta F_\theta \), respectively. The trend in the spanwise distribution of \( \Delta F_\theta \) to resemble the spanwise distribution of the relative area change, that is observed for the SLC results, is continued for TBlock.

### 4.2.3.3 Pressure and Momentum Flux in \( \Delta F_x \) and \( \Delta F_\theta \)

\( \Delta F_\theta \) describes only the change in the circumferential momentum (see Equation 3.15). The \( \Delta F_x \) combines the change in axial momentum and the change in the pressure rise. Therefore, \( \Delta F_x \) is split into two terms: a pressure rise term (see Equation 3.10),
Figure 4-19: $\Delta F$ and Streamtube Change, $\Phi = 0.95 \cdot \Phi_{des}$, spanwise, Rotor, TBLOCK

Figure 4-20: $\Delta F$ and Streamtube Change, $\Phi = 0.80 \cdot \Phi_{des}$, spanwise, Rotor, TBLOCK
Figure 4-21: \( \Delta F \) and Streamtube Change, \( \Phi = 0.95 \cdot \Phi_{\text{des}} \), spanwise, Stator, TBLOCK

Figure 4-22: \( \Delta F \) and Streamtube Change, \( \Phi = 0.80 \cdot \Phi_{\text{des}} \), spanwise, Stator, TBLOCK
and a term that equals the change in momentum flux (see Equation 3.9). These two terms allow for a more detailed analysis, which is discussed in this section. All figures in this section show $F_{x,\text{mom}}$ and $F_{x,\text{contr\&shift}}$ on the left-hand side and the right-hand side of these plots shows the fractional area change, $\Delta A/A_1$, to point out the similar shapes of the spanwise distribution of body forces and fractional streamtube area change.

At the design, the three-dimensional flow features affect only the pressure component of $\Delta F_x$, $F_{x,\text{contr\&shift}}$, on the order of 0.5% compared to the two-dimensional force estimate. In the casing region, the offset is on the order of 2.5% to the two-dimensional estimate. This is due to a local shift of streamlines toward the hub. The change in streamtube area is on the order of 2.5% and only affects the top 10% of the span. The change in momentum flux is a result of the blockage in the OGV, which has an upstream effect on the flow in the rotor. At off-design (Figure 4-24), the deviation
Figure 4-24: $\Delta F_x$, $F_{x,\text{mom}}$, $F_{x,\text{contri&shift}}$, $\Delta A/A_1$, $\Phi = 0.75 \cdot \Phi_{\text{des}}$, spanwise, Rotor

Figure 4-25: $\Delta F_x$, $F_{x,\text{mom}}$, $F_{x,\text{contri&shift}}$, $\Delta A/A_1$, $\Phi = \Phi_{\text{des}}$, spanwise, Stator
from the two-dimensional flow is increased. The three-dimensional flow effects now influence both the pressure field and the momentum flux. An increase in the axial velocity (and hence momentum) is accompanied by a local decrease in pressure. The impact of the three-dimensional flow effects on $\Delta F_x$ is on the order of 2.5%. The components of $\Delta F_x$, $F_{x,mom}$ and $F_{x,contr\&shift}$, can locally increase up to 5 or 10%, but mostly cancel each other out. The increase of the upstream influence of the OGV is now stronger than any potential downward shift of the tip region in the rotor.

The stator (Figure 4-25 and 4-26) experiences an upward shift of the streamlines at design. As has been observed in the rotor, the pressure force component is not affected by more than 2%. The observable $\Delta F_x$ is 90% due to a change in momentum flux. This is decreased in the hub region, where expanding streamlines indicate slowing flow, and is increased in the casing region, where the flow is locally accelerated. At off-design, the OGV hub separation has reached the stator. The
separation, expressed by an upward shift in the streamlines, reduces the axial momentum flux, while the pressure force is increased. Unlike the rotor tip, the $\Delta F_z$ in the stator is only affected in the lower 50% of the span.

For both the TBLOCK and the SLC results, the upstream influence of the hub separation of the flow in the OGV was identified. Unlike TBLOCK, SLC cannot compute flow separation and other three-dimensional effects, such as rotor tip leakage or reverse flow. The difference between the two-dimensional force estimate and the three-dimensional body force was explained by the influence that three-dimensional flow effects have on the momentum flux and the pressure rise. The changes in the momentum flux and the pressure rise were expressed as $F_{z,\text{mom}}$, $F_{x,\text{contr&shift}}$ and $F_{\theta,\text{mom}}$. The spanwise distributions of these terms correspond qualitatively to the related distributions of the area change, $\Delta A/A_1$. The following chapter analyzes this trend.
Chapter 5

The Influence of Streamtube Change

This chapter discusses $F_{x,\text{mom}}$, $F_{x,\text{contr\&shift}}$ and $F_{\theta,\text{mom}}$ as a function of the fractional streamtube area change, $\Delta A/A_1$. A conceptual approach to estimate these terms based on the streamtube area change is discussed.

5.1 SLC representation of $\Delta F_x$, $\Delta F_\theta$

This section discusses $F_{x,\text{mom}}$, $F_{x,\text{contr\&shift}}$ and $F_{\theta,\text{mom}}$ based on streamline curvature (SLC).

Figure 5-1 shows both terms of $\Delta F_x$, $F_{x,\text{mom}}$ and $F_{x,\text{contr\&shift}}$, plotted versus the fractional area change, $\Delta A/A_1$ as defined in Chapter 3. The data points are the 20 spanwise locations for all available flow conditions on the characteristic (Figure 4-1). $F_{x,\text{contr\&shift}}$ (white circles) as a function of the area change, resembles a linear trend, with a local deviation of 1.5 percentage points. With streamtube contraction (negative $\Delta A$), the two-dimensional pressure force is overestimated, scaling with 2.5 % of the two-dimensional force per 5 % contraction of the streamtube. For the streamtube expansion, the pressure force is larger than the two-dimensional estimate, again by 2.5 % per 5 % expansion.

The momentum flux related force terms versus the fractional area change deviate
Figure 5-1: $F_{x,mom}$ and $F_{x,contr&shift}$ versus $\Delta A/A_1$, Rotor, SLC; The gray lines represent a slope of 0.5 and -0.5 respectively.

Figure 5-2: $F_{x,mom}$ and $F_{x,contr&shift}$ versus $\Delta A/A_1$, Stator, SLC; The gray lines represent a slope of 0.5 and -0.5 respectively.

from a linear relation. For the range of positive values for $\Delta A/A_1$, fractional values for $F_{x,mom}$ can differ by 6 percentage points for the same value of $\Delta A/A_1$. An estimate of $F_{x,mom}$ based on $\Delta A/A_1$ cannot be as accurate as an estimate of $F_{x,contr&shift}$ based on the same value of fractional area change.

For the stator (see Figure 5-2), the linear trend between the pressure related force term and the streamtube contraction, can be observed as well. A linear relation between $F_{x,mom}$ and the streamtube contraction cannot be inferred from the results shown for the stator.

For the circumferential force, $F_{\theta,mom}$ is plotted against $\Delta A/A_1$ for the rotor in Figure...
5-3 and the stator in Figure 5-4. Over the range of streamtube changes in the rotor, a linear trend in the relation of $F_{\theta,\text{mom}}$ to the area change of the streamtube, can be observed.

In the stator, there is not a linear relation between $F_{\theta,\text{mom}}$ and the change in streamtube area. The data points associated with the stalled part of the characteristic (Figure 4-1) deviate up to 20% from a possible linear relation. This offset is due to the limitations of the SLC solver as mentioned in section 2.2.1. Enthalpy and entropy are not necessarily conserved along a streamline.

Figures 5-5 and 5-6 combine the previous results for both rotor and stator. Though the rotor and the stator experience different pressure rises and flow turning
across the blade passage, the respective streamtube changes and corresponding $\Delta F_x$ and $\Delta F_\theta$ terms follow the same trends.

5.2 TBLOCK representation of $\Delta F_x$, $\Delta F_\theta$

$F_{x, \text{contr\&shift}}$ and $F_{x, \text{mom}}$, based on the TBLOCK rotor, are plotted in Figure 5-7 versus the area change. The influences of three-dimensional flow on the pressure rise show a linear trend when plotted versus the streamtube area change. Excluding the tip force (gray circles in Figure 5-7), a linear relation to the streamtube area change can be seen for the change in momentum flux as well. These linear trends correspond to the observations made for the SLC results. The slope for $F_{x, \text{contr\&shift}}$ versus $\Delta A/A_1$ is 0.5. The change in momentum flux, as expressed in $F_{x, \text{mom}}$, plotted versus the
Figure 5-7: $F_{x,mom}$ and $F_{x,contr&shift}$ versus $\Delta A/A_1$, Rotor, TBlock; The gray lines represent a slope of 0.5 and -0.5 respectively.

area change is close to a slope of -0.5.

$F_{\theta,mom}$ associated with the tip streamtube (gray circles in Figure 5-8), follows a different trend than the other streamtubes. Excluding the tip region, $F_{\theta,mom}$ scales linearly with the streamtube area change on the order of 10 % per 20 % of $\Delta A/A_1$. This relation is more visible in Figure 5-9. Here the range of of the axis displaying $F_{\theta,mom}$ is limited to values from -0.5 to 0.5, so that the linear trend between $F_{\theta,mom}$, and the area change is more obvious. The four gray points correspond to the forces in the tip streamtube.

Similar results for the stator are given in Figures 5-10 and 5-11. Again, the pressure related force component, $F_{x,contr&shift}$, can be linked to the change of the streamtube area. The force versus the area change is linear with a slope of 0.5. For $F_{x,mom}$ the
Figure 5-8: $F_{\theta, \text{mom}}$ versus $\Delta A/A_1$, Rotor, TBLOCK

Figure 5-9: $F_{\theta, \text{mom}}$ versus $\Delta A/A_1$, Rotor, TBLOCK, Details; The gray line represents a slope of 0.5.
Figure 5-10: $F_{x,mom}$ and $F_{x,con&shift}$ versus $\Delta A/A_1$, Stator, TBLOCK; The gray lines represent a slope of 0.5 and -0.5 respectively.

gray data points correspond to the stator tip streamtube. The data points associated with the four operating points closest to the peak do not follow the general trend. $F_{\theta,mom}$ shows greater deviation from a linear trend than in the rotor. Excluding the tip (gray circles in Figure 5-11), the local range of $F_{\theta,mom}$ values for the same area change can be on the order of 5 to 10 %, limiting the accuracy of the estimation.

The trends for the rotor and stator agree, as the combination in Figures 5-12 and 5-13 shows. For the latter plot, the range of $F_{\theta,mom}$ values is limited from -0.5 to 0.5 so that the large variation of tip force components in the rotor do not obscure the other results.

The linear trends identified for the results based on TBLOCK computations are the same as those identified for SLC. The influence of the three-dimensional flow features
Figure 5-11: $F_{x,\text{mom}}$ versus $\Delta A/A_1$, Stator, TBLOCK; The gray line represents a slope of 0.5.

Figure 5-12: $F_{x,\text{mom}}$ and $F_{x,\text{contr}}$ versus $\Delta A/A_1$, Rotor and Stator, TBLOCK
on the body forces can be related to the change in the streamtube area. The observable deviation of some of the data points from the linear trend, is discussed in section 5.4.

5.3 Estimation of $\Delta F_x$ and $\Delta F_\theta$

The linear trends that $F_{x,\text{contr\&shift}}$, $F_{x,\text{mom}}$, and $F_{\theta,\text{mom}}$ follow in relation to the fractional change in streamtube area, can be applied to modify the two-dimensional body forces to account for the three-dimensional flow effects.

As the flow deviates from a two-dimensional flow, the body forces undergo changes due to the changes in both axial and circumferential momentum flux, and in the pressure rise within the streamtubes. The results presented in the previous section show that for a large range of flow coefficients and for most of the span, a direct linear relation between the change in streamtube area and $\Delta F_x$ or $\Delta F_\theta$, respectively, can be assumed. Based on a given two-dimensional estimate and three-dimensional effects quantified as streamline shift, three-dimensional body force distributions can be derived. As is discussed in section 4.1, three-dimensional flow effects can be captured in the streamline shift. This streamline shift can be translated into a streamtube area change across the blade row. With each fractional area change, a fractional $F_{x,\text{contr\&shift}}$, $F_{x,\text{mom}}$, or $F_{\theta,\text{mom}}$ term can be associated. This fractional force term can be used to adjust the two-dimensional body force estimate,
Figure 5-14: Flowchart on the estimation of $\Delta F_x$ and $\Delta F_\theta$ to resemble the actual three-dimensional body force.

For the blade regions that do not follow the linear relation (e.g. the tip), the three-dimensional contribution can be inferred from the overall blade force. The overall blade force is obtained from a control volume that matches the blade. The overall blade force can also be interpreted as the sum of all local forces in the blade passage. Spanwise distributions of the body forces can be calculated excluding the tip. As the overall blade force does not depend on the accuracy of streamline calculations and local mass conservation, the body force in the tip streamtube has to match the difference between the overall body force and the summation of all other local body forces along the span.

The procedure is summarized in the flowchart in Figure 5-14.

5.4 Analysis of the Variations in $\Delta F_x$ and $\Delta F_\theta$

In this section we discuss the general applicability and quality of the results presented in the previous chapters. It is hypothesized that the linear trend observed in the relation between the $\Delta F_x$ and $\Delta F_\theta$ and the streamtube area change is a trend that
can be generally applied to corresponding body force analyses. The fact that the linear trend was observed for four different blade passages (rotor and stator for SLC and TBLOCK), with different characteristics, cannot be conclusive, but support the reasoning given below for a generally linear relation between three-dimensional flow effects and the streamtube contraction.

As an example, $F_{x,\text{mom}}$ will be discussed, due to its relative simplicity compared to $F_{\theta,\text{mom}}$ and $F_{x,\text{contr\&shift}}$. The contribution to the force term from the radial velocity, $V_r$, is one order of magnitude smaller in relation to the influence of the axial velocity, and is therefore neglected. Under the assumption of incompressibility, we can reduce $F_{x,\text{mom}}$ to:

$$F_{x,\text{mom}} = \frac{\rho (V_{x,1}^2 A_1 - V_{x,2}^2 A_2)}{N}$$ \hspace{1cm} (5.1)

Equation 5.1 follows the definitions in Chapter 3. In the analysis, $F_{x,\text{mom}}$ is discussed relative to the two-dimensional axial force estimate, which is given by:

$$F_{x,2D} = \frac{(p_2 - p_1) A_1}{N}$$ \hspace{1cm} (5.2)

In Chapters 4 and 5, $F_{x,\text{mom}}$ is shown relative to the two-dimensional estimate. Similarly, we divide $F_{x,\text{mom}}$, as in Equation 5.1, by its two-dimensional estimate, $F_{x,2D}$, as in Equation 5.2:

$$\frac{F_{x,\text{mom}}}{F_{x,2D}} = \frac{\rho V_{x,1}^2 A_1 - \rho V_{x,2}^2 A_2}{(p_2 - p_1) A_1} = \frac{\rho (V_{x,1}^2 A_1 - V_{x,2}^2 A_2)}{(p_2 - p_1) A_1} \frac{\text{Vol}_{2D}}{\text{Vol}}$$ \hspace{1cm} (5.3)

The second step in Equation 5.3 applies the definition of $N$ given in Equation 3.3, which reduces the ratio of the denominator terms to the ratio of control volumes. The control volume on which the actual force, and thus $F_{x,\text{mom}}$, is based, is different form the control volume of the two-dimensional estimate. Over the observable range of area changes, the volumes of the two control volumes are on the same order of
To derive the slope of the ratio of force terms, Equation 5.3 is compared to the Bernoulli equation for incompressible fluids. The Bernoulli equation can be written as:

$$\frac{d}{dx} \left( \rho \frac{v^2}{2} + p \right) = 0$$  \hspace{1cm} (5.5)\

Across the blade passage, along a streamline of length $\Delta x$, this relation can be approximated as:

$$\frac{d}{dx} \left( \rho \frac{v^2}{2} + p \right) \sim \frac{1}{2} \frac{V_{x,1}^2 - V_{x,2}^2}{\Delta x} + \frac{p_1 - p_2}{\Delta x} = 0$$  \hspace{1cm} (5.6)\

$$\frac{1}{2} \frac{V_{x,1}^2 - V_{x,2}^2}{\Delta x} = \frac{p_2 - p_1}{\Delta x}$$  \hspace{1cm} (5.7)\

$$\frac{1}{2} \frac{(V_{x,1}^2 - V_{x,2}^2)}{(p_2 - p_1)} = 1$$  \hspace{1cm} (5.8)

Comparing the approximated Bernoulli equation (Equation 5.3) to the ratio of body force terms (Equation 5.8), leads to:

$$\rho \frac{(V_{x,1}^2 A_1 - V_{x,2}^2 A_2)}{(p_2 - p_1) A_1} \sim \frac{1}{2} \frac{A_1 - A_2}{A_1}$$  \hspace{1cm} (5.9)\

$$\frac{F_{x,mom}}{F_{2D}} \sim \frac{1}{2} \frac{A_2 - A_1}{A_1} = \frac{-1}{2} \frac{\Delta A}{A_1}$$  \hspace{1cm} (5.10)

Equation 5.10 states that the influence of the three-dimensional flow effects on the change in momentum flux, relative to the two-dimensional force estimate, scales with the fractional streamtube area change. Plotting $F_{x,mom}/F_{x,2D}$ versus $\Delta A/A_1$ will result in a linear relation, with a slope of -0.5. This slope can be observed in Chapter 5.

Applying Bernoulli’s principle to the ratio $F_{x,contra\&shift}$ and its two-dimensional estimate, leads to:
\[
\frac{F_{x,\text{contr\&shift}}}{F_{2D}} \sim \frac{1}{2} \frac{A_2 - A_1}{A_1} = \frac{1}{2} \frac{\Delta A}{A_1}
\] (5.11)

This equation is consistent with our observations, as a decrease in axial velocity is accompanied by an increase in static pressure. The linear relation between \(F_{x,\text{contr\&shift}}/F_{x,2D}\) versus \(\Delta A/A_1\) can be observed in Chapter 5. The slope of 0.5 can be identified in the corresponding plots.

The observable deviation from a linear relation, especially for \(F_{x,\text{mom}}\), is due to local errors in the mass flow as computed by SLC and TBLOCK. Integrated over the annulus, mass is conserved for both the SLC and TBLOCK computations. The local definition of streamtubes can be less accurate, so that the error in mass flow in the streamtubes from leading to trailing edge of the blade passage can be on the order of up to 2\% for SLC for off-design flow conditions. The mass flow in the tip streamtube for the averaged TBLOCK flow field cannot be calculated correctly due to reverse flow. If the streamlines are not calculated correctly, and therefore the mass conservation in the streamtubes is not accurate, the ratio of \(F_{x,\text{mom}}\) and \(F_{\theta,\text{mom}}\) to their respective two-dimensional estimate, versus the streamtube area change, do not scale with a factor of -0.5.

The pressure, on the other hand, is a scalar quantity, and thus \(F_{x,\text{contr\&shift}}\), is unaffected by possible inaccuracies in the local mass flow. \(F_{x,\text{contr\&shift}}/F_{x,2D}\) versus \(\Delta A/A_1\) follows a linear relation with a slope of 0.5 for all discussed flow fields.

Based on Bernoulli’s equation, we are able to conclude that the linear trends that can be observed for the relation between the components of \(\Delta F_x\) and \(\Delta F_y\), and streamtube area change, are generic. The results presented in this chapter are not limited to the configuration under examination.
Chapter 6

Summary and Conclusions

The effects, associated with non-two-dimensional flow aspects, on the body force representation of compressor blade-rows has been quantified for a single stage compressor. The quantification is based on assessing the results from an axisymmetric flow computed using a streamline curvature procedure and from time- and circumferentially-averaged unsteady three-dimensional flow fields, against those from two-dimensional flow estimates. This work constitutes a first known attempt to interpret and quantify changes in body force representations of compressor blade-rows from one based on two-dimensional flow assumptions. The outcome would have potential utility in assessing results from the use of blade-row by blade-row body force representations of compressors in multi-stage compressor instability analysis. The key results are as follows:

1. At design the two-dimensional body force estimate is within 5 % of the actual axisymmetric body force.

2. Near stall conditions the two-dimensional body force estimate is typically on the order of 15 % of the actual body force. The blade tip region shows local differences on the order of 40 %. The difference between the body force representations is roughly two thirds due to a change in momentum flux and one third due to a change in pressure rise associated with non-two-dimensional effects.
3. For flow conditions ranging from design to close to stall, body force terms accounting for the difference between the two-dimensional estimate and the actual axisymmetric body forces, can be linearly related to the change in streamtube area.

4. The difference between the two-dimensional force estimate and the actual body forces is due to reverse flow, tip leakage and hub separation. These flow effects can be included into a two-dimensional body force distribution, following the linear relation.
Bibliography


