Organization Research Program

FURTHER EXPLORATIONS IN THE THEORY OF MULTIPLE BUDGETED GOALS

by

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Introduction

Implicit recognition of the existence of multiple goal structures possessed by individuals in organizations and organizations themselves is far from new. Even in the scientific management literature where employers' objectives are described as "....to secure a maximum output of standard quality at a minimum cost per unit...." (which is in itself a dual goal) mention is made of "such obvious benefits to the employer as having a better-satisfied labor force; resulting in minimum turnover." More recently, explicit recognition of the presence of multiple goal structures in an organizational setting has taken place. Cyert and March [12], and [13] have specifically postulated a mechanism within the organization through which a "ruling coalition"—a group at the top of the organizational hierarchy which is "in control"—sets short and long range goals for several areas of organizational activity. Cyert, Dill and March [11] have provided evidence in support of these hypothetical goal structures.

Such multiple goal structures may be considered in contradistinction to the "goal" of unconstrained profit maximization assumed in classical economic theory. As suggested by Cooper [10], the classical assumptions, however useful they may be for prediction of macroeconomic behavior provide

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1/ Lowry, Maynard and Stegemerten [34], pp. 6-7.
2/ Ibid.
neither guidelines for nor descriptions of the behavior of an individual firm. Viewing the firm as a whole, it (or its ruling coalition) may simultaneously form goals for a percentage of share of the market, level of employee welfare (e.g., as manifested by a "no layoff" policy), level of total assets and so on. It may be tautological, but nevertheless interesting, that it is impossible to simultaneously optimize two functions; one can, at best, optimize one, placing a constraint on the other; or, one can construct a super-functional which is some function (perhaps a weighted sum) of the initial functions. While it is frequently suggested that short-run constraints can be derived from an (unconstrained) optimization of long-run profit the usefulness of such an approach either for descriptive or normative analyses of micro-economic phenomena is questionable; from the evidence available\(^1\) firms both can and do construct constraints in the short run (or over a foreseeable horizon) which do not appear to derive from any explicit long run profit function.

Thus, in constructing short-run models of organizational behavior, one may postulate a process of profit maximization subject to certain constraints. These constraints may be stated deterministically or probabilistically. For example, in a short-run profit maximization,\(^2\) the optimal solution will drive ending inventory to zero (assuming positive selling prices) unless some lower bound is placed on it. This inventory constraint becomes the link between the correct short-run problem and that of the next period, such an extension being required to preserve the economic reality of the on going (dynamic) organization model without the necessity

\[^1\]See, for example, Charnes and Cooper [5], and Cyert, Dill and March [11].

\[^2\]As, for example, in a warehousing model. See Charnes, Cooper and Miller [8].
of constructing a (generally unavailable) long-run profit function.\(^1\)

In certain periods, this constraint may be extremely costly---e.g., if the price at which a portion of the ending could be sold is high relative to prices obtainable in subsequent periods. It may not be possible to assess the cost, however, because of uncertainty in future prices.\(^2\)

Thus it becomes advisable to state the constraint on ending inventory probabilistically---e.g., the probability that ending inventory will fall below 100 units will not exceed .05. The ending inventory figure of 100 units thus becomes a goal. It is something which one will try to achieve.

If several such chance constraints\(^3\) exist, the model may be considered a multiple goal model wherein a profit maximization goal is joined by these other goals which relate to the attainment of specific performance levels. The properties of several such models have been investigated.\(^4\)

In this paper, however, our focus will be on the description of a new class of models whose properties relate specifically to the allocation of resources so as to satisfy goal attainment motivations. We assume that an individual (or an organization, or a ruling coalition within an organization) constructs a set of goals. These goals may be aspiration levels internally arrived at or may be goals imposed by superiors, the market or the social environment.

\(^1\) In fact, a process similar to this is utilized in partitioning virtually insoluble long(er) run models into soluble short-run problems. See Charnes and Cooper [6],

\(^2\) The use of the term uncertainty here should be noted. If future prices are uncertain, it is impossible to construct even the expected cost function in terms of future prices. This should be distinguished from the frequently assumed, but rarely observed, case of risk where the distribution of future prices is known.

\(^3\) See Charnes and Cooper [5], and Charnes, Cooper and Symonds [9].

\(^4\) Ibid.
Imposed Goals and Aspiration Levels

It is indeed difficult to understand why a dichotomy has so often been assumed to exist between an "imposed" goal and an internally generated aspiration level. The latter may be defined as the level of performance whose attainment leads an individual to experience "success" and whose non-attainment leads him to experience "failure" (Becker and Siegel [3]). Frank [16] has defined the aspiration level as "that level which an individual, knowing his level of past performance, explicitly undertakes to reach." Lewin, Dembo, Festinger and Sears [21] have described construction of an aspiration level from the maximization of the expected utility (or valence) of success (diminished by the expected disutility (negative valence) of failure).

Simon [26] has used a utility function with a discontinuity at the aspiration level—performance at or above the aspiration level possesses positive utility, below it, negative utility. There is nothing in any of these definitions in widespread use which would preclude the possibility of the incorporation of goals imposed by a superior as a factor in influencing aspiration levels. Particularly if one appeals to the notions which are related to utility theory, the contribution to utility of attaining a level which is associated with reward (promotion, prestige, salary, or bonus) as may well be the case with "imposed" goals, must certainly establish that level as a lower bond on aspiration provided the reward is sufficiently large. The effects of various factors on aspiration level—need achievement (Rotter [24]), social norms (Chapman and Wolkman [4]), reality-irreality (Diggory [14])—and previous success (Festinger [15], Becker and Siegel [2]) have been investigated, to cite only a sample. Although Siegel and Fouraker [25] showed that the experimenter's goal was accepted
by subjects and translated into performance and Stedry [27] has investigated the frequency of acceptance of experimenter-presented goals as stated aspiration levels, such examples are rare. In general, superior-initiated goals have received attention through having their existence deplored by many writers including McGregor [23], Argyris [1], and Becker and Green [3].

It is not clear that a goal set by a superior is any more "imposed" than a social norm or need be more abhorrent. An individual's need for achievement derives at least in part from his external environment; that the affectors may be circumstances of an earlier environment would not seem to make them any less imposed than a contemporary superior, but rather the opposite since they are less easy to change. Finally, it is difficult to imagine an environment more imposed than the random elements present in a game against an impersonal nature in an uncertain world which result in a pattern of success and failure; while the long-run effects of the effort devoted to a task may be observable, for any single period, increased effort may be almost as likely to lead to failure as success if the amount of random noise is great. To accept these environmental factors as determinants of "internally generated" aspiration levels but reject superior-initiated goals on the basis that they are "externally imposed" would seem to be highly artificial.

Therefore models with which we shall deal here do not distinguish between goals that are set by an individual (or organization) which are the presented goals of a superior and those which are set with reference to other external or (presently) internal forces. The goals which are accepted by an individual or organization, however arrived at, and the rewards perceived as being associated with them, whether tangible
or intangible, are of interest. As will be revealed from analysis of the solutions to the goal problems posed, some areas of endeavor will not receive any effort toward goal attainment because a combination of too great difficulty of attainment in these areas and too small associated reward with them will render alternative areas more attractive. This withholding of effort is tantamount to goal rejection. Thus, if goals set by a superior fall into the category of effort allocation withdrawal because the reward associated with their attainment is inadequate these goals will appear to be rejected. Our models thus embrace both cases where superior-initiated goals are accepted and where they are rejected. We are thus constrained to accept neither the assumptions of the standard budgeting literature that (budgeted) goals are accepted without question nor the assumptions of more recent literature in budgeting and social psychology that only participatively set goals will be accepted. We readily accept the notion that the presence of participation in goal setting may, under certain conditions, increase the amount of personal satisfaction associated with goal attainment and thus raise the overall level of perceived reward. It is not clear from recent evidence, however, that participation in goal setting is advantageous as to preclude the inclusion of non-participatively set goals in behavioral models.

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1/ See Vroom [33], French, Israel and As [17], French, Kay and Meyer [18], and Stedry [27].
A Stochastic Representation of Goal Attainment as a Function of Effort

Let us assume that an individual (or organization) has a set of goals \( \{ g_j \} \) with whose attainment are associated a set of rewards \( \{ r_j \} \) for attainment and penalties \( \{ p_j \} \) for non-attainment. The index \( j \) designates the activity area for which a performance goal has been set. Finally, let \( \{ x_j \} \) represent the set of performances in the \( n \) activity areas and let performance behavior be stochastically described by the function:

\[
P \{ x_j \geq g_j \} = k_j (1 - e^{-\alpha_j p_j}) , \quad j = 1, \ldots, n
\]

where the left hand side represents probability of goal attainment in the \( j^{th} \) area, \( \ell_j \) the search effort allocated to the \( j^{th} \) area and \( k_j \) and \( \alpha_j \) are parameters dependent upon the difficulty of the selected goal \( g_j \) with \( 0 < k_j \leq 1 \) and \( \alpha_j > 0 \).

This stochastic function, as shown in Figure 1, incorporates a limiting probability of goal attainment \( (k_j) \) as \( p_j \) grows large. This represents a formal recognition of the infeasibility of eliminating all random noise from the system determining success and failure by mere application of additional effort. The parameter \( \alpha_j \) determines the rate of ascent to the limiting probability as effort increases—the effort required to obtain a probability of attainment equal to a specified proportion of \( k_j \) is inversely proportional to \( \alpha_j \).

This probability function would appear to possess several desirable properties. It reflects diminishing returns, has sufficient flexibility to describe a wide variety of hypothetical situations and, from the standpoint of solving problems, is both monotone increasing and strictly convex.
Although other functions might both represent behavior in specific situations it would appear that extensions to other probability functions are less interesting, at least in exploratory work, than observations of model behavior change under different motivational assumptions. At the level of allocation of individual or organizational search effort at a motivational level, the limitations on human ability to perceive subtle refinements in probability functions would seem to preclude the desirability of constructing models on an overly complex base. It would seem that a more precise specification or greater complexity in the function would be required for consideration of practical budgetary planning problems—
e.g., problems of allocating funds to several competing projects or investment opportunities—but that even here, the difficulties inherent in estimating probability functions from real data in an organizational setting may render superfluous much greater complexity.
Models of Effort Allocation for Optimal Reward

The concept of effort used here is adopted from Simon [26], and March and Simon [22]. We refer not to physical effort, but rather to search effort—effort which results in problem solving, innovation and change. While actual time allocation to various areas of endeavor may be indicative of the search effort allocation (and, because of the difficulties of measurement inherent in the latter, may be required as an operational definition for observation) it is recognized that certain kinds of routine or stereotyped activity do not lead to goal attainment.

Moreover, it is worth noting that, whether or not search effort is an easily measurable quantity, measurement of the allocation of time to tasks of a non-repetitive nature is an unsolved problem. Development of a theory of rational (or quasi-rational) effort allocation, it is possible that the problems of measurement may be at least partially solved through imputations of effort from performance measures—in which case search effort (defined circularly as that effort which results in innovation) becomes the more likely operational measure.

Toward this theory, we postulate a set of models predicated on an assumption of expected reward maximization. It has been observed above that profit maximizing does not adequately describe organizational behavior. It is possible, however, that the maximization of reward associated with discrete goals may provide a better description of behavior.

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1/ See, e.g., Stogdill and Shartle [32].
Short Term Maximization of Expected Reward Subject to an Effort Constraint

We can define a reward function over the probability function given above by observing that reward, $R$, over the $n$ goal areas may be expressed as:

$$R = \sum_{j=1}^{n} r_j z_j^+ - \sum_{j=1}^{n} p_j z_j^-$$

where:

$$z_j^+ = \begin{cases} 1 & \text{for } x_j \geq g_j \\ 0 & \text{for } x_j < g_j \end{cases}$$

and

$$z_j^- = \begin{cases} 0 & \text{for } x_j \geq g_j \\ 1 & \text{for } x_j < g_j \end{cases}$$

indicated that the contribution to reward of the $j^{th}$ performance area will be $r_j$ if the goal, $g_j$, is attained and $-p_j$ otherwise. To formulate expected reward, we observe that:

$$E(z_j^+) = (1) P \{x_j \geq g_j\} + (0) [1 - P \{x_j \geq g_j\}]$$

(4) and

$$E(z_j^-) = (0) P \{x_j \geq g_j\} + (1) [1 - P \{x_j \geq g_j\}] = 1 - E(z_j^+)$$

Recalling that

$$P \{x_j \geq g_j\} = k_j (1 - e^{-\alpha_j x_j})$$

we obtain for the expected reward,

$$R = E(R) = \sum_{j=1}^{n} (r_j + p_j) k_j (1 - e^{-\alpha_j x_j}) - \sum_{j=1}^{n} p_j$$

$$= \sum_{j=1}^{n} (r_j + p_j) k_j e^{-\alpha_j x_j} + \sum_{j=1}^{n} [r_j (1 - k_j) p_j]$$
The second term in the finally obtained expression for R may be viewed as a long-term factor relevant to considerations of the inducements relevant to an individual's remaining in the industry. It may also affect the amount of effort, ρ, which is available for allocation among the competing areas but, as we will consider ρ to be constant in the short run, this facet will not be relevant to the problem posed. The second term, constant in the short run, may thus be ignored and our problem stated as:

\[(6a)\] \[
\text{Minimize } \int \rho = \sum_{j=1}^{n} \eta_j e^{-\alpha_j \rho_j}
\]

\[\text{Subject to:}\]

\[(6b)\] \[
\sum_{j=1}^{n} \rho_j = \rho
\]

\[(6c)\] \[
\rho_j \geq 0 \quad , \quad j=1,...,n
\]

where \(\eta_j = (r_j + p_j)k\) and the non-negativity conditions (6c) reflect the inability to "borrow" effort from one area beyond no effort at all to provide more capacity for another. We have given a formal derivation of the solution to this problem and a computational algorithm elsewhere [30]; the derivation is an extension of a solution of Charnes and Cooper [6] based on the Kuhn-Tucker conditions. The latter become:

\[(7a)\] \[
\eta_j \alpha_j e^{-\alpha_j \rho_j} = \mu \quad \text{for } j \in J
\]

\[(7b)\] \[
\eta_j \alpha_j \leq \mu \quad \text{for } j \notin J
\]

where \(J = \{ j / \rho_j^* > 0 \}\) -- i.e., the set of areas at optimum to which some effort is allocated. The process followed for computational solution begins
by ordering the $\eta_j \alpha_j$ from the largest ($j=1$) to the smallest ($j=n$). It will be observed that thus quantity is a product of the reward for attainment $^1/ (r_j + p_j)$, the limiting value for possible probability gain ($k_j$) and the proportionality constant for return to effort ($\alpha_j$)—basically an overall measure of potential gain from effort allocated to the $j^{th}$ area. It is intuitively clear that effort should be allocated first to those with the highest potential gain. The formal algorithm does this, first allocating all of the available effort to activity 1 and, utilizing the conditions (7a) and the overall constraint on activity expressed in terms of those conditions, 

\begin{align}
(8a) & \quad -\alpha_j \rho_j + \ln \eta_j \alpha_j = \ln \mu \\
\text{subject to} & \quad \rho_j^* = -\frac{1}{\alpha_j} \ln \mu + \frac{1}{\alpha_j} \ln \eta_j \alpha_j \\
\text{so that:} & \quad \rho = \sum_{j \in J} \rho_j = -(\ln \mu) \left( \frac{1}{\alpha_j} \sum_{j \in J} \frac{1}{\alpha_j} \right) + \sum_{j \in J} \frac{1}{\alpha_j} \ln \eta_j \alpha_j \\
\ln \mu = & \quad \sum_{j \in J} \frac{1}{\alpha_j} \ln \eta_j \alpha_j - \rho \\
\end{align}

\(^1/\)Including the evidence of a penalty.

to obtain a possible value for $\mu$. This value for $\mu$ is then tested against the constraint (7b) for the largest of the remaining areas (initially, activity 2). Observing (8b) it will be noted that $\mu$ will increase in size with each additional activity added to the set $J$ of positive effort allocation activities. Thus $\mu$ will be small initially and unless $\eta_2 \alpha_2$ is much smaller than $\eta_1 \alpha_1$, the condition (7b) will be violated for at least $j=2$. Then a new trial value for $\mu$ will be computed from (8b) with the
set $J$ now containing two elements. The process continues until a value of
$\mu$ is found that is sufficiently large that is is larger than the maximum of
the $\eta_j \alpha_j$ for $j \notin J$ (and hence all of the others). The conditions of (7b) are
now fulfilled and the conditions of (7a) may be fulfilled by solving for
the values of the individual $\rho_j$ by (8a).

Although the formal process is complex and the proof that this pro-
cedure will arrive at a solution is non-trivial, an heuristic interpretation
is possible. Certainly an ordering of activity areas according to potential
gain is not infeasible. Then "data breaks"—natural divisions formed as a
result of a difference between two consecutive values of the parameter being
substantially greater than the differences between values in the groups
formed between the decisions—are examined. Then the "top priority" group
can be examined along with the next group in priority to determine
whether all of the effort should be allocated to the former or whether
allocating some to the latter will improve the overall goal attainment
"picture." The process can be continued until the items of sufficiently
low priority (either because of low reward attached or because of
tremendous amount of effort required to attain a reasonable probability
of success) are decided upon as worthy of ignoring (or postponing).

Within the groups of relatively similar potential, effort may be
allocated as demanded to provide roughly equivalent probabilities of
success, a simple and reasonable approximation to the relationship of (8a).
Perhaps the most difficult problem to attach without recourse to the
formal methodology is the distribution of effort between groups—i.e.,
how high is "high" priority. Returning to earlier terminology, the risk
of failure to attain a goal, at optimum, may be expressed as:

$$k_j e^{-\alpha_j \rho_j} = \frac{\mu}{(r_j + p_j) \alpha_j}$$

(9)

to provide a guideline: the risk of failure should be inversely proportional both to the reward available and to the responsiveness of the activity to effort. Since $k_j$ would not be expected to vary excessively—only values between, say, .7 and 1.0 making very much sense—the activities in the groups formed between the data breaks in $\eta_j \alpha_j$ should have similar $(r_j + p_j) \alpha_j$. Thus effort can be allocated between groups in such a way that the risks of failure to achieve goals in any group is approximately inversely proportional to a multiplicative measure of reward and difficulty.

It has been observed that tolerable probabilities of failure to attain goals are more easily arrived at by organization executives than the cost of failure. Charnes, Cooper and \cite{9} found this to be the case in scheduling heating oil where the goal was satisfying all customers. Stedry and Griswold \cite{11} observe that supply control officers in a military organization could more easily establish risk categories than costs of stock out. In both cases the researchers observed a willingness to alter the acceptable risk level where it was pointed out that the cost (generally of inventory investment and storage) would be excessive in certain cases. Thus the groupings of risk were found \textit{a priori} and the mathematical models were used to determine the operating procedures required to attain them, subject to inventory investment (i.e., effort) constraints. We thus are able to provide evidence for the viability of the
approximate solution to our reward maximization model\(^1\) in one form—i.e., where the establishment of acceptable risks of goal attainment provide the missing data for approximate reward maximization even where the rewards attached to the individual goal attainments in the functional cannot be specifically stated.\(^2\)

**Optimal Reward Subject to Risk Constraints**

The evidence presented for risk limitation strongly suggests an extension to the expected reward maximization model. Risk constraints may be placed on specific activity areas of the form:

\[
P \left\{ x_j < g_j \right\} \leq \delta_j , \quad j \in J'
\]

where \(J'\) is a subset of the \(n\) activity areas represented in the functional and \(\delta_j\), the acceptable risk of non-attainment in the \(j\)\(^{th}\) area, is a constant such that \(0 < \delta_j < 1\).\(^3\) Defining these constraints over the

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\(^1\) Or a profit maximization model which, in [30], we have shown under certain conditions to be equivalent.

\(^2\) Some evidence from the study of children is also suggestive. Hoppe [ ] found that the goal to "get dressed without help" received less effort (i.e., more requests for help were observed) when a reward was offered for attainment of the goal to "get dressed." It should be noted that this interpretation is somewhat different from that offered by Lewing et al. where the clear dissimilarity between these two competing goals has been ignored, resulting in the misleading conclusion that reward lowers aspiration levels.

\(^3\) An alternative formulation would allow \(\delta_j = 1\) and the constraints (10) could apply for \(j=1,\ldots,n\) where \(\delta_j = 1\) for \(j \in J'\).
probability function, we obtain:

\[ 1 - k_j(1-e^{-\alpha_j \rho_j}) \leq \delta_j \]

\[ \frac{1-\delta_j}{k_j} \leq 1-e^{-\alpha_j \rho_j} \]

\[ \frac{1-\delta_j - k_j}{k_j} \leq e^{-\alpha_j \rho_j} \]

\[ \rho_j \geq \frac{1}{\alpha_j} \ln \left( \frac{k_j}{1-k_j-\delta_j} \right) \]

If we now let:

\[ \bar{\rho}_j = \frac{1}{\alpha_j} \ln \left( \frac{k_j}{1-k_j-\delta_j} \right) \quad j \in J' \]

and perform the transformation:

\[ \rho'_j = \rho_j - \bar{\rho}_j \quad j \in J' \]

\[ \rho'_j = \rho_j \quad j \notin J' \]

\[ \rho' = \rho - \sum_{j \in J'} \bar{\rho}_j \]

we have the problem in a form which is identical to that of (6) above with \( \rho'_j \) substituted for \( \rho_j \) and \( \rho' \) for \( \rho \). (Clearly if \( \rho' \) is negative the constrained problem is infeasible.)

The new set \( J \) will contain only those activity areas where effort is to be allocated in excess of the constraints for \( j \in J' \). If the parameters of the original problem are preserved, elements formerly in \( J \) but whose
solution provided $p_j^* < \bar{p}_j$ will not now be in J. It is intuitively clear (and can be proved) that the amount of effort available for allocation to the unconstrained areas is thus diminished so that no additional elements will be added to J. Indeed, if $p'$ is sufficiently small, the problem is effectively constraint-determined and the decisions as to which areas to allocate some (additional) effort may be made at the periphery. This distinction may be noted in the military terminology which distinguishes between "military-essential" and "nice-to-have" categories of endeavor and the observed avoidance of effort allocation to the latter.

A similar effect can be noted at an experimental level in the aspiration level determination of subjects in a goal-oriented situation. In a series of time trials, subjects were given performance goals with monetary rewards attached. Two competing goals for effort could be discerned. The first was to "do a good job" or "solve many problems." The second was to obtain the monetary reward. Competition for effort expenditure (apparently) ranged over the several time trials as well. The perceived difficulty of attainment of the experimenter's goal (with monetary reward attached) in early trials would provide (as the experiment was designed) a good estimate for future goal difficulty. It was observed that as goal difficulty was increased (across subjects) that the aspiration levels for individual trials stated aspiration levels down

\[1\] Stedry [27] and [28].
relative to the experimenter's goals.\footnote{1} A possible interpretation arises from the suppression of the goal to "do well" in a given time trial (short run) which would require effort which could otherwise be allocated to reducing the risk of non-attainment of the monetary rewards in later periods. This interpretation is one of several possible explanations of the observed behavior but suggestive, nevertheless, of the possible application of multiple-goal models to the analysis of dynamic phenomena.

\footnote{1}{Although not, as frequently assumed, in an absolute sense. Apparently the increasing goal difficulty had some beneficial effect in raising the overall effort expenditure for most subjects, at least as observed by performance, although an interaction effect was observed between goal difficulty and manner of presentation which was independently varied.}
A Constraint on the Risk of Unacceptable Performance

Frequently of interest rather than (or in addition to) the risk of non-attainment of individual goals is the risk of unacceptable performance where acceptable (or satisfactory) performance is considered to be the simultaneous attainment of minimally acceptable performance levels in several activity areas. These levels, say $h_j$, would be lower than the corresponding $g_j$. They may be viewed as minimum standards rather than desirable levels, thus establishing a goal hierarchy defining different degrees of satisfactory performance.

Ordering the n areas so that the first $m$ of them are represented in the acceptable performance criterion, the constraint may be expressed as:

$$\prod_{j=1}^{m} k_j (1-e^{-\alpha_j \rho_j}) \geq \delta$$

which, noting that $(\prod_{j=1}^{m} k_j)$ may be factored out of the expression and making the substitution

$$W = -\ln\left(\frac{\delta}{\prod_{j=1}^{m} k_j}\right)$$

we obtain:

$$\sum_{j=1}^{m} \ln(1-e^{-\alpha_j \rho_j}) \leq W$$

A value of $W$ for which the constraint is useful as a policy will be quite small. An acceptable risk of unacceptable performance would not seem sensible were $\delta$ less than, say, .8. Even if the $k_j$ were close to 1, reasonable values of $W$ would not exceed .2.
Clearly, a feasible solution requires that the maximum value attainable for the product of the probabilities under the effort constraint be equal to or greater than δ. This value can be obtained in the manner presented by us in [30] and if this value is equal to δ the solution to the reward maximization is entirely constraint determined and may be obtained by the methods of the earlier paper. If the maximum value is greater than δ, the reward maximization may be stated as:1/

Minimize \[ \sum_{j=1}^{n} \eta_j e^{-\alpha_j \rho_j} \]

Subject to:

\[ - \sum_{j=1}^{m} \ln (1-e^{-\alpha_j \rho_j}) \leq W \]

\[ \sum_{j=1}^{n} \rho_j = \rho \]

\[ \rho_j \geq 0 , \quad j = m+1, \ldots, n \]

The constraint on ρ is stated as an equality inasmuch as it is monotone increasing, and the functional monotone decreasing, in ρ_j. The non-negativity conditions are required only on the ρ_j which are not included in the constraint on W whose satisfaction requires ρ_j > 0 for W finite. (Intuitively, if even one activity involved as a criterion of acceptable performance is ignored, overall acceptable performance cannot be attained.)

\[ 1/ \text{A more formal treatment of this model will be available in a forthcoming paper.} \]
It can be shown that the constraint or $W$ may be replaced by equality unless the optimal solution to the problem without this constraint (but with the non-negativity conditions on the first $m$ of the $\rho_j$ restored) overfulfills it, in which case the problem solution is obtained by the methods used above.

Provided the problem is feasible and the constraint on $W$ is not redundant, the conditions of solution (Kuhn-Tucker) become:

$$\eta_j \alpha_j e^{-\alpha_j \rho_j} = -\nu \left( \frac{\alpha_j e^{-\alpha_j \rho_j}}{1-e^{-\alpha_j \rho_j}} \right) + \mu \quad j \in M$$

(18)

$$\eta_j \alpha_j e^{-\alpha_j \rho_j} = \mu \quad j \in K$$

$$\eta_j \alpha_j \leq \mu \quad j \in K'$$

where $M = \{j/j=1,\ldots,m\}$, $K = \{j/j=m+1,\ldots,n \text{ and } \rho_j^* > 0\}$ and $K' = \{j/j=m+1,\ldots,n \text{ and } \rho_j^* = 0\}$. For simplicity, let:

(19a) $\quad x_j = e^{-\alpha_j \rho_j}$

(19b) $\quad \mu_j = \frac{\mu}{\alpha_j \eta_j}$

(19c) $\quad \nu_j = \frac{\nu}{\eta_j}$

for all $j$. It will be observed that $k_j \cdot x_j$ represents the probability of non-attainment of $h_j$. Thus $x_j$ must be quite small for $j \in M$ in order that joint attainment of several goals will be meaningful, as suggested above.
The Kuhn-Tucker conditions and the constraints restated in terms of (19) appear as:

\[(20a)\]
\[x_j = -v_j \frac{x_j}{1-x_j} + \mu_j \quad j \in M\]

\[(20b)\]
\[x_j = \mu_j \quad j \in K\]

\[(20c)\]
\[1 \leq \mu_j \quad j \in K^\prime\]

\[(20d)\]
\[-\Sigma_{j \in M} \ln (1-x_j) = W\]

\[(20e)\]
\[-\Sigma_{j \in M} \frac{1}{\alpha_j} \ln x_j - \Sigma_{j \in K} \frac{1}{\alpha_j} \ln \mu_j = \rho\]

where the last is obtained by substitution in (19a), (19b) and (20b).

The computation of an exact solution to this problem is sufficiently complex and tedious that it is unlikely that it is of much value in terms of explanations of behavior. An approximate solution is, however, readily available. Multiplying both sides of (20a) by \((1-x_j)\) we obtain:

\[(21)\]
\[x_j - x_j^2 = -v_j x_j + \mu_j - \mu_j x_j\]

Clearly, for realistic cases \(W\) will be small---of the order of .2 or .1---for the constraint on it to be meaningful as a limitation of risk on unacceptable performance. If, for example, these minimum standards were

\[1/\text{Further explanation occurs in a forthcoming paper by Charnes and Stedry.}\]
a set of specifications set by a purchaser for acceptable product, traditional values of "producer's risk" would be even smaller--i.e., .05. Noting that:

\[ W = - \sum_{j \in M} \ln (1-x_j) \geq \sum_{j \in M} x_j \]

the individual \( x_j \)'s, particularly if numerous, must be extremely small.\(^1\)

It is clear that:

\[ 0 \leq x_j - x^2 \leq x_j \]

so that, utilizing (21):

\[ \frac{\mu_j}{1+\mu_j+v_j} \leq x_j \leq \frac{\mu_j}{\mu_j+v_j} \leq \frac{\mu_j}{v_j} \]

which, for small \( x_j \), requires that \( v_j \) be very large relative to \( \mu_j \). By substituting the lesser upper bound in (20d) we obtain:

\[ W = - \sum_{j \in M} \ln (1-x_j) \]

\[ \leq - \sum_{j \in M} \ln \left(1 - \frac{\mu_j}{1+\mu_j+v_j}\right) \]

\[ \leq - \sum_{j \in M} \left(\frac{v_j}{\mu_j+v_j}\right) \]

\[ \leq \sum_{j \in M} \ln \left(1+\frac{\mu_j}{v_j}\right) \]

\[ \leq \sum_{j \in M} \frac{\mu_j}{v_j} \]

\(^1\) By definition they may only take on values \( 0 \leq x_j < 1 \) for \( \rho_j > 0 \).
Recalling the definitions of \( \mu_j \) and \( v_j \), (25) yields:

\[
W \leq \sum_{j \in M} \frac{1}{\alpha_j} \left( \frac{\mu_j}{v_j} \right)
\]

or:

\[
\frac{\mu_j}{v_j} \geq \frac{W}{\left( \sum_{j \in M} \frac{1}{\alpha_j} \right)}
\]

Approximations to \( x_j^* \), \( j \in M \) are readily obtained by substitution of the lower bound for \( \mu/v \) in the upper bound for \( x_j \), or:

\[
\hat{x}_j^* \approx \frac{\mu_j}{\mu_j + v_j} = \frac{1}{1 + \frac{v_j}{\mu_j}} \approx \frac{1}{\alpha_j} \sum_{j \in M} \frac{1}{\alpha_j} \frac{1}{W}
\]

or:

\[
\hat{x}_j^* \approx \frac{\mu_j}{v_j} \frac{1}{\alpha_j} \sum_{j \in M} \frac{1}{\alpha_j}
\]

Either of the approximate solutions, which differ from the true value by second and higher order terms only, is clearly independent of the \( \eta_j \).

The approximate solution to the effort allocation problem among areas involved in a minimum standard thus appears to depend only on the relative ease of attainment, \( \alpha_j \), and the ratio of an overall measure of difficulty, \( \sum_{j \in M} \frac{1}{\alpha_j} \), and the size of the risk, \( W \). It will be noted that the risk taken in individual areas varies approximately inversely with the ease of attainment--i.e., the easiest things to attain are allocated sufficient effort to be "very safe." In the second solution, it will be noted that the
sum of the estimates is equal to \( W \).

The approximate solution for \( x_j^* \), \( j \in M \), readily yields a solution for \( j \notin M \). If we denote the approximate amount of effort allocated to the \( m \) constrained areas by \( \tilde{\rho} \), we observe:

\[
\tilde{\rho} = - \sum_{j} \frac{1}{\alpha_j} \ln x_j^*
\]

so that we may now solve the reduced problem:

Minimize \[ \frac{\sum_{j=m+1}^{n} \eta_j e^{-\alpha_j \rho_j}}{\rho_j = \rho - \tilde{\rho}} \]

Subject to: \[ \rho_j \geq 0 \quad , \quad j = m+1, \ldots, n \]

by the method given in a previous section.

The pattern of behavior described by the second on the approximate solutions might result in the following instructions:

1) Choose a trial value for one of the \( \alpha_j x_j \).

2) Set individual risks for all of the areas subject to the joint risk constraint so that the \( \alpha_j x_j \) are constant across them.

3) Add the risks so obtained.

4) Multiply each of the trial \( x_j \)'s by \( n \) and divide by the sum found in (3) to find the estimated risks \( \tilde{x}_j^* \).

5) Allocate the remaining effort so as to approximately maximize reward over the areas not involved in the joint risk constraint. \(^1\)

\[^1\] As given in a section above.
The results are suggestive of a possible interpretation of the frequently observed tendency in organizations to "put out brushfires" rather than to work on areas where the effort would be of long-run benefit. The only solution which appears credible as an implicit behavior determinant where it is desired to hold the risk of unacceptable performance to some small value is one which allocates effort independent of the rewards attached to desirable goals. The relative risks of not attaining the minimum standard in individual areas are set approximately inversely proportional to \( \alpha_j \) -- i.e., the easiest thing to protect will be made the safest. Only where areas are not part of an overall acceptable performance criterion will effort be allocated in terms of desirability rather than safety. The computational difficulty inherent in taking into account both reward for a desired attainment and a joint risk constraint to obtain effort allocation for both purposes would seem to result in the elimination of one of the two criteria-- and the obvious one to eliminate is the "nice-to-have" criterion.
Summary and Conclusions

We have presented three models of optimizing behavior in response to a set of presented goals. It is interesting to note how closely the optimal solution to these reward (or profit) maximization models resemble behavior which may be termed "satisficing." In general, the addition of constraints on risk of individual activity areas or a subset thereof tends to quite easily subordinate the importance of the coefficients in profit function. Furthermore, this need for use of approximate solutions because of the complexity of the exact solution provides quasi-optimal behavior which depends even more heavily on the constraints at the expense of the coefficients in the reward function.

From the standpoint of capital budgeting models where the actual distributions are known and the computational complexity is no problem. The exact solutions on which we have concentrated elsewhere ([29] and [30]) are of interest. For predicting behavior patterns which would logically occur in response to desired ends when the effort allocation process is (more or less) implicit, the approximate solutions seem to be of greater interest. Systematic study of behavior in response to multiple budgets or goals is rare.\footnote{Data collected by one of the authors and E. Kay appear to approximate the behavior in response to the third model in an industrial budgeting system.} Observations at the non-systematic level, however, appear to lend credibility to the possibility that the approximate solutions do resemble behavior in response to the reward functions postulated.
BIBLIOGRAPHY


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