WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

A GENERAL ASSET THEORY

188 - 66

JAMES J. LINN

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139
A GENERAL ASSET THEORY

188 - 66

JAMES J. LINN

1 Assistant Professor of Management, Alfred P. Sloan School of Management, Massachusetts Institute of Technology.
# Table of Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
</tr>
<tr>
<td></td>
<td>The Problem</td>
</tr>
<tr>
<td></td>
<td>Structure of Suggested Solution</td>
</tr>
<tr>
<td></td>
<td>Scope of the Inquiry</td>
</tr>
<tr>
<td>II</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>The Asset Concept and Its Deficiencies</td>
</tr>
<tr>
<td></td>
<td>The Need for a General Concept of Asset</td>
</tr>
<tr>
<td></td>
<td>Theoretical Needs</td>
</tr>
<tr>
<td></td>
<td>Pedagogical Needs</td>
</tr>
<tr>
<td></td>
<td>Operational Needs</td>
</tr>
<tr>
<td></td>
<td>Conclusion</td>
</tr>
<tr>
<td>III</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>A Theory of Assets</td>
</tr>
<tr>
<td></td>
<td>The Important Aspects of Assets</td>
</tr>
<tr>
<td></td>
<td>A Theoretical Formulation of the Asset Concept</td>
</tr>
<tr>
<td></td>
<td>Integration with Existing Theory</td>
</tr>
<tr>
<td></td>
<td>Conclusion</td>
</tr>
<tr>
<td>IV</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>The Nature and Measurement of Services</td>
</tr>
<tr>
<td></td>
<td>Historical Review</td>
</tr>
<tr>
<td></td>
<td>The Nature of Services</td>
</tr>
<tr>
<td></td>
<td>Measurements of Services</td>
</tr>
<tr>
<td></td>
<td>Some Observations on Operating Assets</td>
</tr>
<tr>
<td></td>
<td>Conclusions</td>
</tr>
<tr>
<td>V</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>The Valuation of Services</td>
</tr>
<tr>
<td></td>
<td>The Types of Valuation Needed</td>
</tr>
<tr>
<td></td>
<td>The Jointness of Service Valuation</td>
</tr>
<tr>
<td></td>
<td>Valuations of Services</td>
</tr>
<tr>
<td></td>
<td>Approximation Analysis</td>
</tr>
<tr>
<td>VI</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>A General Asset Model</td>
</tr>
<tr>
<td></td>
<td>A Mathematical Formulation</td>
</tr>
<tr>
<td></td>
<td>The Problem of Computing and Valuing the Multiple Uses of an Asset</td>
</tr>
<tr>
<td></td>
<td>Integration with Models in Economics and Production</td>
</tr>
<tr>
<td>VII</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>An Illustration</td>
</tr>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td>Activity Level</td>
</tr>
<tr>
<td></td>
<td>Valuation</td>
</tr>
<tr>
<td></td>
<td>Units of Value</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
</tr>
<tr>
<td>-------------------------</td>
<td>------</td>
</tr>
<tr>
<td>VIII Summary and Conclusions</td>
<td>151</td>
</tr>
<tr>
<td>Summary</td>
<td></td>
</tr>
<tr>
<td>Further Research</td>
<td></td>
</tr>
<tr>
<td>Conclusions</td>
<td></td>
</tr>
<tr>
<td>Bibliography</td>
<td>156</td>
</tr>
</tbody>
</table>
CHAPTER I

Introduction

What follows is primarily an attempt to express the concept of asset in a fashion which is clear, complete, and quantitative. This will be the general asset theory. Also there will be a discussion and illustration of the practical applicability of this theory. This will be the general asset model.

I hope that a conceptual theory has been developed here which will facilitate comprehension and use of the term asset in the many areas of business administration and economies that utilize it. In addition, since the theory will quantify the asset concept to a greater degree than before, it will serve as a guide to the research and development necessary to implement the model in practice.

Financial accountants and economists concerned with the theory of value have studied the problem of asset valuation. Some of the reasons for their interest in this problem are income determination, pricing, and investment policy. As a result, a body of knowledge has grown up which treats both the theoretical and practical aspect of these problems.

There are other problems associated with assets. One of these, the use of assets, has received some attention in economics as the theory of production, and in business administration as production, operations research, and managerial accounting. It is the theoretical part of this aspect of assets—their use—that is the subject of this paper.
Asset usage is concerned primarily with the services an asset can
furnish. While asset usage is not concerned with asset valuation per se,
it is concerned with the valuation of the unit of service, for this is
what determines, in part, through the profitability of operations, asset
usage.

Asset valuation, on the other hand, is concerned with factors of
supply and demand which are the final determinants of asset value. In this
approach to assets the services the asset can furnish are considered only
indirectly through the demand mechanism which reflects the demand for
these services and the supply mechanism which reflects attempts to furnish
these services.

Structure of Suggested Solution

The theory suggested in this paper is based upon two aspects of
assets which have been unexploited. One is the separation of the physical
services of an asset and the valuation of these services and the other is
that the services combination obtained from an asset is but one of many
possible combinations of services the asset could be called upon to furnish.

These two aspects of assets can be utilized to develop a rich
asset theory. The separation of the services and their valuation permit
the use of different valuations for a given combination of services, while
the alternative service combinations permit the introduction of alternative
asset uses and the various costs and revenues associated with these uses.

Linear algebra is the language chosen to express these aspects,
their relationships and extensions in quantitative form. It was chosen
because it is rapidly becoming a primary quantitative tool in business and, more importantly, because it is well suited to express the relationships of the model developed in this paper.

The theoretical components of the theory came from many sources. Basically, the areas to which the theory would hopefully apply were scrutinized for the asset characteristics which were relevant to that field.

Scope of the Inquiry

The theory developed here is called a general theory. Generality is obviously a relative matter, for a theory can be general in one context and narrow in another. This theory is called general in the sense that it relates to several fields and disciplines rather than one. It is definitely not general in the sense of covering all present and future needs for an asset model.

The primary aim has been to develop a general asset theory which would be useful for managerial decision making. Other desirable considerations were that the theory should be useful to as many business fields as possible, that the theory should be consonant with theory in other fields and disciplines, if possible, and that the theory be potentially useful in an information theory of the firm.

This study consists of five major parts: Chapter II, the problem; Chapter III, the theory; Chapters IV and V, the nature and measurement of the major elements of the theory; Chapter VI, a model based upon the theory; Chapter VII, an illustration of the model.

The first major part of this study--Chapter II--is devoted to the
argument that at present there is no general asset concept or theory in business administration, economics, or industrial engineering. This line of argument is developed by examining the asset definitions in these areas and showing that they are quite narrow because they were developed for special purposes. In the process of developing this argument the asset characteristics which should be incorporated into a general asset theory become apparent.

Next a general asset theory is developed in Chapter III. This theory describes the origin, life, and retirement of assets generally. Its coverage includes management, economics, and industrial engineering.

Since the general asset theory\(^1\) relies upon measurements of asset services and measurements of the value of these services, the third major part of the study--Chapter IV, The Nature and Measurement of Services and Chapter V, The Valuation of Services--is devoted to these measurements and the problems encountered both in making and using them.

A general asset model is presented in the fourth major part of this study--Chapter VI, A General Asset Model. There are three methods, at least, by which the model could have been presented. One is the process by which it was conceived, another is by inference from present and future needs in business, economics and industrial engineering, and still another is by presenting the model as clearly as possible without dwelling upon its origin and development. The latter method of presentation was chosen.

\(^1\) The phrase "the general asset theory" refers to the model developed in this paper. With reference to all possibilities the model developed in this paper is properly "a general asset model"--one of many. The former phrase is used throughout this study to refer to the model in this paper.
The last major part of the study--Chapter VII, An Illustration--illustrates some of the possible uses of the model and the data which can be generated from it.
CHAPTER II

The Asset Concept and Its Deficiencies

The Need for a General Concept of Assets

There is no general theory of assets in existence today. There are many special-purpose asset formulations, but there is nothing which either explains the inter-relationships of these formulations or integrates them. Yet there is a need for a general theory of assets. This need has three sources: theory, pedagogy, and operations.

A general theory of assets would be useful to any field within business administration that deals with assets. Each field, and each user should find within this general theory asset characteristics relevant to his problems. At the same time, however, the asset characteristics relevant to problems in other fields would be apparent. This would facilitate integration among fields such as accounting and production and it would encourage integration between disciplines such as business administration and economics.

Business operations would also benefit from a generalized asset concept. At present, plans for organized and integrated data gathering exist for only a few isolated segments of any business. Quite often specific data are lost because they are unrecorded or because they are buried in an aggregation. Expensive redundancies occur when several segments of a business collect and process the same data independently. While a general asset concept would not eliminate these problems, it could be part of an integrated information system of the firm that could eliminate them.
My concern here is to develop a general theory of assets and a general asset model to represent this theory. Such a general asset model will also serve as an empirical analytical definition of assets since it will include those characteristics which are both necessary and sufficient for the existence of an asset.

Theoretical Needs

In business administration (industrial management) and economics there are many different concepts of assets, and these are rarely defined explicitly. They are revealed only in discussions and models that pertain to assets. Therefore, to determine the asset concept of each of these areas, it will be necessary to deduce it from writings in that area.

Accordingly, representative writings in accounting, production and industrial engineering, finance, economics, and operations research have been explored for the asset formulation or concept used. As in the fable of the blind men and the elephant, each definition defines an asset as an object with characteristics relevant to the area of interest for which the definition was made. This method of defining assets is useful, and the definitions developed under this approach are currently utilized in the theory and practice of these areas. But, taken as a group these definitions do not represent an asset as an integrated whole; it seems as if different concepts, not the same concept, are being defined.  

If a general theory of assets were developed, all fields of business

1 This is accurate if we say the asset concept in accounting, in production, etc., is being defined.
administration and economics could utilize the same definition and model. This could serve as an integrating force among the fields since it would facilitate interfield discussions about assets and facilitate interfield discussion generally. With respect to only one field, the aspects of the theory which relate to the others would be an indication of the needs and problems, with respect to assets, of these other fields. This characteristic of the model would increase the awareness of problems in other fields, related to assets.

At present, there is much discussion and work on the information system of the firm. The basis for the data introduced into these systems are the models of the various fields. It was just noted that there is a variety of these for assets. The system would be more efficient if a single general asset model were used instead of a series of special purpose asset models. At least some of the special purpose models require separate sets of data, since they partially duplicate each other.

Accounting

Only in the field of accounting has there been an effort to define the term asset. Unfortunately, this effort has been restricted to the financial point of view. There have been two types of asset definitions developed in accounting. One of these types is illustrative and the other is analytical.

An example of the illustrative type of definition is found in the Accountants' Handbook. There an asset is defined as "...a variety of money resources (accounts receivable, notes and bills, and marketable securities), physical resources (land equipment [sic] and inventory), and intangible
resources (patents, trademarks, and goodwill)."

This type of definition explains, in terms which are presumed to be understood, what the term asset means. It describes, by the use of representative items, the class of objects that the word asset represents.

As an elementary pedagogical device, this definition is useful. It has, however, one major defect. It does not describe the characteristics which distinguish an asset from other objects. Unless an object is listed as an asset or can be shown to be similar to some object listed as an asset it must be assumed to be a non-asset. But this assumption rests upon the validity and completeness of the list. How do we know the list includes all possible assets and how do we know only assets are included in the list? These questions, of course, bring up a major question. What is the criterion for inclusion in the list?

The analytical type definition is an attempt to formulate the criteria which would distinguish assets from other objects. Two species of this type are prominent in accounting. To distinguish them one will be called pragmatic and the other theoretical.

The pragmatic definition of an asset is best illustrated by the following statement from the Accounting Terminology Bulletins of the American Institute of Certified Public Accountants:

26. The word asset is not synonymous with or limited to property but includes also the part of any cost or expense incurred which is properly carried forward upon a closing of books at a given date. Consistently with the definition of balance sheet previously suggested, the term asset, as used in balance sheets, may be defined as follows:

Something represented by a debit balance—that is or would be properly carried forward upon a closing of books of account according to the rules or principles of accounting
(provided such debit balance is not in effect a negative balance applicable to a liability), on the basis that it represents either a property right or value acquired, or an expenditure made which has created a property right or is properly applicable to the future. Thus, plant, accounts receivable, inventory, and a deferred charge are all assets in balance-sheet classification. The last named is not an asset in the popular sense, but if it may be carried forward as a proper charge against future income, then in an accounting sense, and particularly in a balance-sheet classification, it is an asset.

According to this definition, an asset is a number calculated according to certain conventions. This is a case where the recordation of the representation (model) of assets is described instead of the thing itself. This definition identifies only the characteristics of the model which are of interest to those using the definition.

This characteristic is of course the adjusted historical cost or other objective basis (e.g., cost or market) of asset valuation. This valuation is of interest because it is the only acceptable valuation for financial reporting.

Cost is acceptable because it is (1) considered to be objective and (2) verifiable. It is considered objective because the transaction occurred between two profit seeking individuals, one of whom, the seller, intends to charge as much as possible while the other, the buyer, intends to pay as little as possible. Thus the resulting "cost" is objective in that it represents the result of an "arms-length" bargaining process and is not some figure useful only for dressing up the financial reports. Historical cost is verifiable because the original documents of purchase or sale can be examined.
Clearly a number will suffice to represent an asset for some purposes but not all, and even when a number will suffice the same number may not be equally useful for all purposes.

An example of the "theoretical" accounting definition of an asset is the following of the Committee on Accounting Concepts and Standards of the American Accounting Association: "Assets are economic resources devoted to business purposes within a specific accounting entity; they are aggregates of service-potentials available for or beneficial to expected operations." This is an attempt to recognize the characteristic of the object called assets that is relevant to business generally. This approach ignores the physical form of an asset and concentrates on the services expected from it. This definition is imprecise and incomplete. It does not acknowledge the difference between services and the value of these services, it does not reveal that the same physical object can be different as an asset when viewed by different people, and finally, such things as negative services (services required by an asset to continue operating, such as upkeep) and setup services are unmentioned.

In managerial accounting much attention is given to the different costs of an asset. For example, there are twenty-two specialized cost definitions listed in the Accountants' Cost Handbook. As yet there is no explanation of the relationship, if any, among these costs. The reason for these different valuations can be explained, but their derivation, even theoretically, cannot.
Production and Industrial Engineering

In production and industrial engineering an asset is perceived differently. The term asset is almost completely absent from the literature of this field.

Buffa in his *Modern Production Management* does not use the term asset. This text does use, undefined, the term "obsolescence" which is closely related to asset. Another author, Moore, in a discussion entitled "Ownership of Assets," mentions as examples of assets "Buildings," "Equipment," and "Accounts Receivable." However, neither the term asset nor these examples are defined.

In general, production terminology contains nominal definitions of the word asset. These definitions usually are for equipment, machines, or man-machine unit.

The asset concept of production and industrial engineering emphasizes the operating aspects of assets much as maintenance, output rates, and alternative types of output.

The monetary valuations of the outputs are not properly a part of the production and industrial engineering approach to the asset concept although other types of valuations, e.g., time, may be a part of the concept.

Finance

"Broadly speaking", state Guthman and Dougall, "business finance can be defined as the activity concerned with the raising and administering of the funds used in the business."¹

In pursuing this or similar objectives students of finance are concerned with the valuation of assets and the flow of funds involved in the acquisition, use, and disposal of assets. In discussing working capital Husband and Dockeray say: "The transmutation of a Company's working capital into income and profits, and back into working capital is one of the most dynamic and vital aspects of business operation."

Thus the characteristic of assets which is of importance in finance is a dollar value representation of assets. The following statements reflect this approach:

"capital, in terms of assets, as well as in use, may be classified as relatively fixed (or permanently installed) capital and working (or circulating) capital,"

current assets "...consist of cash and assets which will be converted shortly into cash in the operating cycle of business...," and

"thus the fixed assets may be thought of as being slowly converted into cash, and they can be regarded as an important source of current funds."

While it can be argued that the finance concept of assets includes the product of the physical services of assets and their value, it is more realistic to accept as the finance definition of assets some number representing historical cost, current appraised value or current realizable value.

**Economics**

Economists use two asset definitions; they are the factor of production concept used in the theory of the firm and the present value of future receipts concept of asset valuation used in capital theory.

The factor production definition according to Boulding is:

The prices which form the subject matter of the theory of functional distribution are the prices of the services of property, not of the property itself from
which the services are derived. These services are usually called the "factors of production," although this term is sometimes carelessly used to mean the property from which the service is derived. Thus, labor is a factor of production derived from the human body. Land service is a factor of production derived from land. The services of houses, of machines, and of other material equipment are factors of production derived from the forms of property which yield them. There is, indeed, a relation between the prices of the services of any particular piece of property and of that property itself.

Samuelson, on the other hand, in his *Foundations* does not use the word assets. Instead he uses the terms "inputs," "productive factors" and "productive services." He does not define these terms; he treats them as primitives. In effect the physical device embodying the services is ignored. Consequently, the problems of using the term asset are avoided; the rate at which the services are furnished, the length of time for which they are furnished, and maintenance costs are ignored.

**Operations Research**

Operations Research has not developed an explicit asset definition or concept. Instead, since it is generally an application of quantitative techniques to problems in the other fields of business administration, it utilizes the concept of the area involved, quantifying and supplementing it as necessary.

The typical operations research problems of inventory level, equipment replacement, sequencing, waiting lines, and programming, each require different asset characteristics.

In the inventory problem the output rate of the plant, the cost
of a unit of output, the cost of setup, and the inventory holding cost per unit are the desired asset characteristics. The equipment replacement characterization of an asset is a known life in years, yearly operating costs and yearly price. An asset for purposes of the sequencing and waiting line problem is a vector of the times required to perform different jobs. And for production linear programming, an asset is a vector of the different services available to the firm, which is similar to the economic theory of production.

Because of this borrowing from other fields, operations research has not emphasized asset characteristics which are unknown to these other fields. However, operations researchers have initiated the quantification of business data in a degree unknown in the past. This includes some asset characteristics which were known but not well documented, such as alternative uses and costs. Paul Kircher points out that:

Accountants will also have to improve their recording methods. The experience of the typical operations analyst is that regular accounting systems do not record or process the type of data he requires to build his models. Yet in many cases he can obtain the data by other means, so the data must be available.¹

Summary

From the foregoing it is clear that there are many explicit and implicit definitions of assets available but at present there is no general definition of asset. The available definitions are all descriptions of the same class of objects. While they are different, they are not inconsistent; each describes some aspect of the same concept. Assuming these are some of the important elements of assets, is it possible to develop an asset model which incorporates, at least, all of them? If this can be accomplished, what do we have and of what value is it?

If this can be accomplished, we will have a reasonably general asset theory and asset concept. And this alone will be valuable. The elements of the various fields dealing with assets will be embodied in a common model. The multitude of simple but apparently unrelated sets of concepts, definitions and models will be replaced by one, yet this one general set will contain and illustrate the special aspects of assets relevant to each field.

There are also practical applications. By quantifying the concept of assets in a model, the asset elements are defined. Definition—identification—of these elements is necessary as a guide to asset measurement.

Pedagogical Needs

One of the reasons for theory is the need to develop new concepts within the minds of individuals who are unfamiliar or only slightly familiar with a subject area. For this purpose a single general asset model and its
associated definition would be better than a multitude of special purpose definitions.

Assets are extremely important in business. Most business decision-making deals with assets in some manner, such as utilization, sale or acquisition.

Students are expected and required to understand assets and the analytical tools relating to them even though they have no clear concept of assets in the context--business--in which the term is used. The student therefore works with a concept which has not been adequately defined and, consequently, may not be clear. It would be much better if a thorough understanding of the concept of assets could be imparted to students. This would provide a common language and model between and among students and their teachers, and a greater appreciation of the interrelationships within a business organization. R. E. Pfenning suggested this in the April, 1962 issue of the Accounting Review when he said:

I am convinced, however, that we must develop in our future managers and specialists a deeper appreciation of the over-all business process or system, and the role played by the various professions that function in it. What is needed is the ability to understand the information requirements of the various parts of the business operation--to speak their language--and to understand their problems. Only then will good communication of information be feasible.

Gardner M. Jones reflected a similar view in the October, 1960 issue of the Accounting Review when he argued:

These myriad functions and duties and activities are integrated into a complex which we call "a business."

A lot of us forget that accounting, or marketing, or production control, or product engineering, is only
part of this complex. Marketing people become so obsessed
with getting volume, or percent of the market, or with
diversity of offerings, that profitability or the econ-
omy of production runs, or some other aspect of operations
is ignored. Accountants take so seriously the business
of cost minutiae that production foremen wish they'd
stay away. Production men want to avoid the personal
annoyances of uneven production, forgetting that the
fickle public is like a school of fish--you have to
catch them when they're biting, and that it is expensive
to tie up capital in stuff that you can't sell right
away. Others are so concerned with the "corporate image"
that internal management needs are subordinated to a
public relations emphasis. I am afraid that we in the
colleges are so steeped in our own particular specialties
that we are parties to this failure to recognize the
integrated-activity nature of the business firm.

Some of the benefits of a common language theory and model would be insight
into the general nature of assets with respect to business in general, a
model which would be incorporated into theories of business, a quantitative
device which could be manipulated by known quantitative methods, and a
starting point for other asset definitions, models, and concepts.

The disadvantage of such a common language and model are the lack
of emphasis upon asset characteristics not included in the model and the
tendency to accept the model and common language as a fait accompli rather
than develop others.

Operational Needs

Business operations also suffer from the lack of a general asset
concept. Data on assets are usually in a form that is relevant to a special
segment of the business. If some other department wants the same data, in
the same form or not, the data are collected anew. There are several reasons
for this. Often the data is aggregated and the detail lost, sometimes it is
in a form which must be laboriously translated before it is usable, some-
times it has been stored in such a manner that it is more economical to
collect the needed data anew rather than search, and sometimes the existence
of the data within another department is not known. These efficiencies could
be eliminated if data were collected once and stored in a basic form which
each user could process to develop the information he needed. This point
was emphasized also by Pfenning:

There presently are many places in a business where
information about the same thing is stored in different
places in different languages, so to speak. In the past,
when the information has moved from one place to another, it
has had to be translated from one language to another.
There appears to be an immediate possibility to reduce
much of this transfer and translation by maintaining closely
related information in but one place.

This same point was made also by Franklin G. Moore in his text on
production management where he said:

Computers can take over a great deal of the work of
compiling reports. But if they are to do it economically,
you need to have "integrated" systems and procedures. The
order form or record or report which each department uses must
be capable of being made from the records or reports that
come out of other departments.

Another operational problem is the determination of the unit of data.
Should the unit of data be the firm as a whole, individual assets, or components
of individual assets. With respect to accounting information, the Committee
on Management Accounting has stated broadly: "It is important that accounting
techniques involving the recording of basic economic data be kept flexible
so that the resulting information will lend itself to further and varied analysis." The key word "flexible" is not defined.

Another author, Goetz, in discussing the diverse information needs in a business in his *Management Planning and Control* said:

A basic record, meeting all these diverse requirements, must utilize a set of classes that constitute a sort of "greatest common divisor." Each primary class should contain data that are homogeneous with respect to every dimension. If this desideratum of classification is achieved, the primary classes can be used as building blocks to construct any desired configuration of totals or indexes.

The unit of information to be recorded will depend, basically, upon what information is needed. A model of a firm or part of a firm is a guide to data and information requirements because the model by its very nature as a model emphasizes only some of the characteristics—the relevant ones—of the object being represented. Those characteristics represented in the model, and the subsequent decision rules based upon the model, become guides to what data are important and how these data should be combined to form information for specific decision rules. A general asset model, since it is concerned with an important aspect of a firm—assets—will serve as one of these guides.

A general asset model can perform another important function, data selection. There exist much more data than will ever be recorded or need to be recorded. There are many details of every situation that are never perceived, much less recorded. We have learned to ignore what does not seem to be important. This process of filtering or selective perception is merely a process which aids us to operate. What we filter or, conversely,
perceive is primarily the result of our development by education, training or chance. A general asset model will contribute to this development. It will emphasize what is important and thus act as a guide in data perception and selection.

Accounting has been criticized often for its deficiencies in the area of managerial decision making. There are two aspects to this criticism, published accounting reports and managerial accounting theory. The Committee on Management Accounting of the American Accounting Association has stated in this connection:

Management accounting recognizes that accounting should not operate in a vacuum nor would it be conditioned entirely by specially oriented conventions such as many of those that have been fostered by authoritative organizations within the profession. Accounting based data generally will have considerable application with respect to all functions of management.

The committee believes that future development of management accounting should take a two-fold form. First, accountants should investigate with greater care the major management problems related to finance, production, and distribution. Second, accountants should investigate further and develop more appropriate techniques for the collection, analysis, interpretation, and use of quantitative data so that such data may be used effectively in management planning and control, pp. 211-212.

Published financial statements, generally auditor's reports, are designed for persons outside the firm. These reports are designed to meet the needs of all external decision makers generally, but no one of them specifically. The nature of the audience, the generality of the report and environmental constraints have combined to dictate fairly rigidly the content of these reports and the underlying measurements. As the information in these reports is ill-suited for specific external decision makers, it is
even less suited for internal decision making. Unfortunately, the accounting systems of most firms are designed for the collection of the accounting data and the production of the accounting information needed for these reports, and the data and information needs of internal decision makers have been met from the same systems. Consequently, the internal decision makers have been forced to utilize information that is not specifically designed for their purposes. In Kircher's words: "In our efforts to produce all-purpose statements we sometimes satisfy almost no one. In particular, a manager who wants to use the data may have to struggle to interpret it...."

At the same time this subordination of managerial needs has retarded the development of accounting theory in the area of managerial decision making. Managerial accounting, as it is called, is a loose collection of concepts, models and analysis. It lacks a theory. To some, even the ties between managerial and financial accounting appear tenuous.

A general asset model which reflects most of the characteristics of assets regarded as important to the various fields in business administration and economics would be, because of this, a guide to the recognition of data which are relevant to the decisions of the firm in these areas. To the degree that future unanticipated decisions require other unrecognized data a general asset model would be deficient.

The concept of an information system of the firm has received attention in recent years. It is a logical outgrowth of the increased use of and interest in computers by business and the increase (fostered partly by computers) in mathematical models of segments of the firm. Despite the fact that computers have fostered the growth of an information system of
the firm, it should not be assumed that computers are an essential ingredient of such a system. A system should exist separate from the tools available to implement it. Some simple applications of an information system using less complex data processing devices should be feasible. This view is shared by Jones who argued: "An accounting systems approach is a way for accountants to get moving toward a generalized system concept, and a computer system approach is one aspect of accounting systems."

Many of the attributes of an information system of the firm such as orientation to the information needs of the decision makers, homogeneity of data, common language and awareness of the problems and needs of other components of the firm, have been mentioned above. One point which has been mentioned deserves reiteration here. For an information system of the firm to exist there must be concepts of the firm and its elements, and models and definitions of these concepts and elements. The general asset theory which is proposed here could fulfill this need for the asset concept.

Conclusion

Assets are an important part of business and of some related disciplines such as economics and industrial engineering. Within business, its various fields, and these disciplines there are as many definitions of assets as there are areas of study. The existing state of asset theory is pedagogically confusing, removed from the real world, and operationally inadequate.

A general theory of assets is needed badly. Such a complete theoretical formulation can replace the multitude of partial treatments of assets.
It is obvious that at present no general asset theory is available. It is just as obvious that one is needed. It is one of the logically necessary developments at this stage of the development of business, accounting, and the information system of the organization.
CHAPTER III

A Theory of Assets

The Important Aspects of Assets

Introduction

A theory of assets should describe the origin, life, and retirement of assets generally. In it the term asset should be defined generally enough to encompass the characteristics important to the various fields in business administration, economics, and industrial engineering, and it should also encompass characteristics which will be of future use to the firm.

The theory that will be developed here will not replace existing bits of theory that relate to assets. It will, however, tie these together by filling in gaps and by presenting new aspects of old topics.

A Universe of Objects

The world we live in is familiar to us as a universe which contains people, animals, ideas, clothing, houses, airplanes, cars, etc. This also can be viewed as another kind of universe, one that is populated with objects. An object being anything that is tangible or intangible, animate or inanimate, capable of performing services, and requiring maintenance or upkeep. Everything is an object.

Objects can be assets, but not all objects are assets. For an object to be an asset three conditions must be satisfied. The object must apparently be capable of satisfying some need, it must be economically
feasible, and it must be owned. Each and every one of these conditions must be satisfied before an object is an asset. And each and every one of these conditions is satisfied by all existing assets.

Assets are assets only with respect to a particular person, or entity. To any individual the world is full of objects. A very few of these objects are assets of the individual, many of the other objects are assets of other individuals and some objects are as yet assets of no one. Some examples of objects which are regarded as assets by no one are (1) minerals on the ocean floor; (2) free ideas, inventions, processes; (3) air (in its natural state); and (4) people.

In this paper all objects owned by a firm are considered assets, including those with negative values or net negative satisfactions. When it becomes apparent that the asset value according to the relevant valuation system of the decision maker is negative, the decision maker can keep the asset, change its use (if this will be profitable), or retire the asset through sale or scrapping.

Since people are part of the universe of objects, it is interesting to note that whether or not they are assets is not a theoretical question, but a social question. To be an asset an object must be owned. If a society such as ours does not permit ownership of humans, then people cannot be assets. While the services of an individual can be purchased by contract, neither the individual nor his services are owned, only the right to the services of the individual, under certain, legally specified, conditions, is owned. Thus a contract for an individual's services, which can be owned, is an asset.
The satisfaction of want or need is the reason for acquisition and use of assets. Some assets such as clothing, food, shelter, transportation, and books satisfy wants directly. Other assets satisfy wants indirectly; they are a means to want satisfaction. A truck, a factory, or a business are also assets. In addition to possibly satisfying the desire for possession, or power, or independence, these assets produce other services or in combination with other assets, physical output. This output can be exchanged for dollars and these dollars can be used to acquire assets for personal want satisfaction or business needs.

An object capable of satisfying wants may not be worth the expenditure of economic goods necessary to acquire either the object or the right to its services for a period of time. First of all, there may be other objects capable of the same want satisfaction which cost less. For commercial concerns interested in profit, an object will not be bought unless it will bring to the business revenue greater than its costs, either individually or in conjunction with other assets. An individual makes a calculation similar to that of a business, but not identical, in calculating the worth and cost of an asset. Cost to an individual is composed of exchange assets and his own efforts. As in the case of the firm, opportunity cost is also considered.

In economics the asset selection problem on the individual level is portrayed by the use of indifference curves. With these the problem of exchange, asset cost, and opportunity cost are considered in addition to physical as well as psychic return. The formulation of this same problem in the theory of the firm is less inclusive. Psychic revenue is omitted and only units of value are considered to be revenue.
There is in the economic approach to this problem of object or asset acquisition an implicit assumption which is quite important. In both formulations above, indifference curves and theory of the firm, the individual's perception of the services available from the object, his decision as to which of these he desires, and his evaluation of these are all ignored.

It can be argued that they are not ignored, but are not important in view of the assumption of an economic man and so on. Such an argument postulates that there exists a best or optimal set of services from any object and these are the ones which will be considered in the revenue cost calculation. Such an assumption is valid only in the theoretical pedagogical world of pure competition.

The dichotomy between economics and the real world has been emphasized frequently and thoroughly. It has been established that many aspects of business have not been recognized in business theory but need to be. It is for this reason that I argue that the aspects of objects (potential assets) mentioned previously, perception of possible services, valuation, and choice, should be recognized and included in asset theory.

The important characteristics of assets are the services they can provide and the worth of these services. These data are sufficient to provide any asset information needed by any field of business administration, economics, and industrial engineering.

Ours is a society of property ownership. Consequently, the right to the services of an asset reside in ownership of that asset or its services
for a period of time. In the sense of asset recognition, the essential elements of an asset are want satisfying capabilities, economic feasibility, and ownership. Of these characteristics the first is possessed by objects as well as assets.

Another description of this want satisfying characteristic is service. That is, all assets (and objects) contain services. And it is for these services that the asset is valuable. The services an asset can provide in the future are potential services; those that have been provided are realized services. Services are a common element of all assets. Every asset is collections of potential services. This is why the asset is desired. Usually the asset services are physical and their corpus is immaterial. Consumer goods, however, are often wholly or partly prestige items and with prestige goods the physical embodiment rather than or as well as the incorporated services, is valued. Even this can be characterized as a type of service.

All assets, indeed all objects, have the additional common characteristic of services value. The services of these assets and objects have a worth. This worth or value may be positive, negative or zero. And it will vary, other things being equal, with how the asset is used, which potential services become realized services. Different services have different values, both cost and revenue, and this will be reflected in a different value for each different use of an asset.
A Theoretical Formulation of the Asset Concept

Asset Services

There are two types of assets with respect to services, multipurpose and single-purpose. Only the multipurpose type will be discussed since the single-purpose type is the degenerate case of the multipurpose type.

A multipurpose asset is an asset capable of performing in a given time period and for certain maintenance and operating policies some number of each type of two or more specified services. That is, if the asset were used only for one specific service during the whole time period, there would be some specific number of units of that type of service which the asset would furnish. And this is true for each type of service the asset can furnish. These quantities of services are exhaustive in the sense that only one of them can be obtained in the stated time period.

The asset can be used all during the time period to furnish only one type of service or sequence of services; it cannot furnish two services at once.\(^1\) Thus, one characterization of an asset would be the maximum units of each type of service the asset could furnish, for a given set of operating policies.

There are circumstances when an asset can furnish more than one unit of any of the several types of services it is capable of performing. This occurs when the operating time period is sufficiently long and the

---

\(^1\) It is conceivable that an asset can be constructed to furnish at the same time several of the services another asset can perform only singly. The second asset would be different from the first and would furnish different services since a unit of service of the second asset could include several types of the services the first asset could perform.
asset is multipurpose. For the remainder of the discussion an operating time period sufficiently long to allow the conditions in the discussion to occur (a day, a month or a year), is postulated.

Now a multipurpose asset is capable of furnishing a variety of services in a time period. It can furnish some of one type of service, some of another, some of still another, and so on. The limiting factor is the aggregate number of services possible in any given time period. This in turn is a function of the substitution ratio between services. The substitution ratio between services is the number of units less of one type of service the asset furnished to furnish one unit more of another type of service, given the number of each of the two types of services already being furnished.¹

Another way of stating this is in terms of activity levels. Complete utilization of an asset is indicated by 1. The activity level of the asset in each possible type of service is some number between or equal to either 0 and 1. If the sum of the activity levels of all the types of services an asset can furnish is less than 1, the asset is underutilized; if it is equal to 1, the asset is completely utilized; and if it is greater than 1, an error has been made because the asset is scheduled beyond its capacity.

For an asset, a time period, and a set of operating policies there will be a number of combinations of services the asset can furnish. And the asset, for the conditions just stated, can be represented by any one of these

¹ This is an application of the familiar marginal substitution ratio which is used extensively in economics in the theory of the firm.
combinations. The set of all those possible uses will be called the service possibility set for the asset.

The Subjective Perception of the Services

So far this discussion has assumed an omniscient point of view. Assets have been discussed as if we had full knowledge about all the services they can furnish. However, this is not so in reality. In reality assets are closely related to humans. Assets have no existence except to humans. They are of interest to us because of the services we perceive in them. However, the perception of asset services by a human is not necessarily perfect. A lack of knowledge may cause the human subjective perception of an asset (in terms of services) to be different from what it is in an absolute sense. This will result in a series of perceived possible asset uses, each of which represents some combination of services the asset can perform. This will be called the perceived service possibility set. It is analogous to the service possibility set. However, the two are not identical.

For comparative purposes the service possibility set will be the standard since it includes all services combinations the asset can supply. The perceived service possibility set can differ from the standard in two respects. First, not all of the combinations of services may have been perceived by the individual. This could occur if some types of services were

---

1 Some of these combinations will consist of zero amounts of all services but one and the maximum amount of that one.
unrecognized or because the amounts available of a specific service were underestimated. Second, there may be erroneous perception by the individual. This could be an erroneous perception of the types of services available or of the amount of the services of any one type available. Some services which the asset cannot furnish may have been perceived erroneously and more of some type of service than the asset can furnish may have been perceived erroneously.

It was pointed out above that an asset can furnish only one of the many elements of the possibility set since the elements are mutually exclusive. Which combination of services (element of the possibility set) is chosen as the planned services the asset will furnish is again a matter of individual subjective perception. The individual will select from his possibility space the combination which best fits his needs. In fact this was probably the basis for originally acquiring that particular asset: it was the asset which best (in some sense) furnished the combination of services the individual had in mind.

This results in what can be called the perceived services vector. In the mind of the individual, the asset is not a milling machine, or a car or a factory, but specific amounts of certain services which will be available in this and future time increments. Sometimes the individual will not think in discrete terms, but in continuous terms. In this case the services will be pictured as a rate. But a rate can be transformed into an amount per time period.

The concept of an asset furnishing services during a period of time has certain problems. Typically, services are sequential, not instantaneous.
An asset capable of processing twenty units a day will process these over the span of the whole day. Each unit may be processed before the next is started or all twenty may be simultaneously subjected to each step of the process. There is some governing time sequence for the services a machine produces, and when a machine has varying speeds at which services are furnished, each speed can be considered as a different service.

Another problem associated with the services vector is that the individual perception may change over time. One manner in which this can happen would be for the individually subjectively perceived service possibility set to have changed because of additional information. While this would not necessarily change the perceived services vector, it could change it.

Another manner in which the individually perceived vector may be affected is changing consumer tastes for the product being produced. If this forces a change in production plans during the time period instead of at the end or beginning, the individual subjective perception of the asset and, therefore, of the vector used to represent it may change.

As time passes (within the relevant time period), the perceived services vector will change. It will be smaller by the amount of expected services which have been realized. This of course is normal and expected.

There is another change possible in the perceived services vector. The composition of the vector may change as a result of a new changed perception of the asset. This should cause little trouble. One solution would be to reduce the relevant time period length to equal the time the
asset perception is constant. Another solution would be to sum these inter-period vectors, \textit{ex post}, and use the resulting composite vector as the services vector for the period. If a change in composition of the services vector coincides with a period end, the problem of interperiod change vanishes.

The asset model which has been suggested here can be either \textit{ex ante} or \textit{ex post}, or part of each depending upon where in the life of the asset one is working. The model can be compared to a budget whose data of the future is constantly being revised and whose data for the past is historical fact.

At acquisition an asset can be conceived as a series of vectors of expected services, one for each time period, extending some number of time periods into the future. As time passes, the individual vectors change for two reasons. One reason is that due to the passage of time, expectations become to some extent realized. Then the \textit{ex ante} vector is replaced by the \textit{ex post} vector. Another reason is that expectations change and one \textit{ex ante} vector is replaced by another revised \textit{ex ante} vector.

By itself this feature is meaningless. Its importance is that since the same type of data can be used for planning, control and reporting, this model presents a guide for what data are needed and a model form for using them.

\textbf{The Representation of an Asset as a Vector of Services Incorporated}

Just as it is possible to represent an asset for a period as a vector of services to be received or which were received, it is also possible
to represent an asset as a vector of the services used or utilized in the production of that asset. Thus, an individual would regard assets being used in production as a vector of services received or to be received while assets being produced would be regarded as vectors of services used in their production.

This representation of assets by the services utilized in their production is an *ex post* concept and each asset can be represented only by the unique combination of services contributed for its production. In this sense there is no set of possible combinations which can be associated with the asset since it has already been produced. There may be some problems of measuring the exact number and determining which types contributed to a specific production unit, but these are problems of measurement and do not alter the number or type of services utilized in its production. There is, however, an analogue to the subjectively perceived service possibility set.

Before a unit of output is produced there exists from an omniscient point of view a set whose elements consist of all possible combinations of services which will produce the unit of output.¹ There would also exist the related subjectively perceived set which would be a function of some individual's knowledge. This is of course an *ex ante* representation. Choice of which perceived combination to utilize would be a function of some valuation system (objective function). Once some combination was selected it could be costed by using the valuation set developed for the assets of the firm inasmuch

---

¹ A unit of output could be a whole production run. This would allow the relevant economies to be incorporated.
as the *ex ante* product must be stated as vector of services available.

**The Valuation of Services**

The chief attribute of assets is that they furnish services. These services are important because they have value. In a service representation of assets, value is attributable to the services because an economic good--the unit of service--is being considered and because value plays a prominent role in asset use.

There must be some valuation for each type of service. This includes the possibility of a zero value for free services or for services which have no value in particular circumstances. Once a value has been assigned to each type of service, a value for a combination of services can be calculated by summing the product of the services and their unit values. This will give a valuation of an asset in a particular use for a time period.

The general asset theory makes no pretext of explaining the valuation of a unit of a specific service. This is a problem of valuation theory. The general asset model accepts these data and utilizes them to develop information. The general asset model is an analytical framework for explaining the variations in the returns to the same asset in two different circumstances. It can offer no explanation or analysis of why a specific return to a unit of service is what it is.

Since the services combination and the valuation of the services are separate, it is possible to change either one and calculate various values for an asset for a time period. The various possible asset representations as service combinations has been discussed above. At this point the various
possible valuations will be developed.

For a specific asset a set of possible service combinations for each time period was developed. A set of unit valuations, one for each service and for each time period, can be developed also. This set of valuations is applicable to either an asset or a group of assets. Given a set of services and the valuations of these services the two can be combined. The result is a value for the set of services. Such a set of services might be a representation of an asset for a time period and the value would be the value of the asset for that time period. If the services are linear and the values are linear, the aggregation can be efficiently performed by the use of linear algebra.

It is well established that, for a number of reasons, the dollar is not constant as a unit of value and that the value of an item does not necessarily remain constant. Because of this, different valuations are necessary for different sets of decisions. For example, current costs might be desired to evaluate and control operations while future costs might be desired for planning purposes. For a single asset there is a set of possible valuations of that asset.

The set of possible valuations of an asset contains combinations based upon different valuation assumptions. It would seem that there is but one combination of values for each valuation assumption. This is not so. For each valuation assumption there is a class or subset of valuation combinations.

This is not caused by different valuations of the same service
unit. The basis for the different subsets of values is the different decisions which may arise. For some decisions certain services an asset may perform and the valuation of these will be irrelevant. In such a case the unit value or the number of units of that particular service are zero. This will effectively remove the irrelevant service from the calculation. For other decisions the services to be demanded from the asset may be different. For two different uses of an asset the valuation of the asset in each may be different although the unit service valuation remains constant.

Thus, the set of unit service valuation combinations consists of subsets. Each subset is based upon some valuation assumption. Each element of the subset consists of a valuation scheme relevant to a certain decision.

The valuation subsets are a function of a set of assumptions and then a set of decisions. The assumptions pertain not only to the valuation scheme utilized but also to the point of view adopted. For example, there could be a subset of current values based upon market values, which are considered objective and a subset of current values based solely upon subjective judgement. Each subset would be relevant to a different class of decisions, consequently, each is a valid subset of the set of valuations.

Integration with Existing Theory

Economics

In economics relevant asset characteristics are described from time to time. None of these characteristics nor the conclusions of the discussion
conflict with the general asset theory. Generally, this theory can be said to complement economics by providing explanation of certain phenomena. Three such explanations will be given below for certain aspects of rent, choice, and asset valuation. This is followed by a discussion of the relation between the general asset theory and national income accounting.

Rent

Economic discussions of rent are primarily aimed at explaining the existence of this phenomenon by describing causes. Some insight into transfer earnings and quasi-rent is also possible by considering their components.

Transfer Earnings—Quite often the same asset is useful in the production of any one of several products. And usually the return to the asset or attributable to the asset varies in each case. This return can be divided into two parts: transfer price and rent. Transfer price is the highest return the asset can attain in some alternate use. It is assumed that this is less than the present return. If it were more, the asset would be employed in producing the more profitable product, according to the assumption of rationality. Accountants call this opportunity cost. The remainder of the net return is called rent. These differences in returns to assets in different uses are explained in terms of demand and supply for the end product. What is not explained is the process by which this demand becomes a return to the factor of production and why these returns differ for different uses of the asset. If the supply and demand construction is pushed back one
more level, to the derived supply and demand for the services of the asset, the argument breaks down when the asset services are considered homogeneous in all uses unless some heroic assumptions such as market segmentation are introduced.

The explanation for this is that the asset is capable of furnishing several services and a larger number of service combinations. In one situation the asset is called upon to furnish one combination of services and in another situation the asset is called upon to furnish a different combination of services. The aggregate values of the combinations of services constitute the return to the asset. These are different because the asset was called upon to furnish different combinations of services in each situation. While the values of each service the asset could furnish were the same for both situations the aggregate value of each combination of services was different because the combinations were different.

Thus, different revenue to the same factor of production in different uses is explained by use of the general asset theory. The return to a unit of a service does not vary, but the combination of services an asset is called upon to furnish and, therefore, its return in different uses can vary.

Quasi-Rent--Rent is the general term for that part of the net return to an asset above what is necessary to employ the asset (cost). This occurs when there is inelastic supply of an asset and none of the of the supply is idle. When the excess of net return over cost is temporary (i.e., the long run supply is elastic), it is called quasi-rent. This is to distinguish this case from the case of rent for which the long run supply as well as the short run supply is inelastic.
Again, as in the case of transfer price, the reason for the existence and disappearance of the quasi-rent is given in terms of the supply and demand, in this case an initial change in demand and a slower corresponding change in supply. And again there is no explanation of the process by which this occurs.

In this instance the asset services involved do not change; the same services are used since the product remains unchanged. It is the valuation of these services which changes. As the supply of assets increases, so does the supply of the service which is in demand. And as the supply increases, the revenue per unit of service drops and so does the aggregate revenue and, consequently, the net revenue to each asset.

Asset Valuation

Economic theory, with one philosophy of accounting asset valuation concurring, has developed the conclusion that the present discounted value of future receipts is the value of an asset. This theoretical explanation takes as given the periodic revenue and expense attributable to the asset. The asset theory developed here can be used to explain these revenues and expenses and, thus, expand this aspect of economic theory.

Choice

The use of the subjectively perceived service possibility set and the separation of the services and their valuation allows the use of the general asset model in the economic theory of choice in two ways. First, given a desired combination of services for a number of future time periods and an objective function, the individual can search for the asset which
can fit or best fits this pattern. Second, given a specific asset, its possibility set and its valuation set, a best use (per an objective) for the asset can be chosen.

As the theory of choice is usually stated, the consumer's wants remain constant during the analysis. This would correspond with the first use of the general asset theory suggested above.

So long as the assumption that consumer wants are constant is maintained, nothing need be said about the alternative service combinations in the subjectively perceived service possibility set except that choice is made from this set and that the one selected will be the combination which best satisfies the wants of the consumer.

Once the assumption of constant wants is relaxed and consumer needs are allowed to change, the alternative service combinations (if any) an asset can furnish are again pertinent to the analysis as possibilities at least and perhaps choices.

**Decision Theory**

It is important to note that the structure of the general asset theory fits decision theory quite well. The perceived set of alternatives corresponds to the set of outcomes of acts of decision theory. The acts are the actual usage of the asset. The states of decision theory can have several meanings. They can be the environment states of the assets or they can be different assets to name two possibilities.
Accounting

Asset Definitions

The theoretical definition of an asset by the American Accounting Association is "assets are economic resources devoted to business purposes within a specific accounting entity; they are aggregates of service-potentials available for or beneficial to expected operations." While the general asset theory is more than this definition implies, it does satisfy the definition, and, therefore, it is consonant with this part of accounting theory.

The part of accounting theory which is concerned with valuation is relevant to the theory, but not a part of it just as is the economic theory of valuation. This would include the more pragmatic definitions of asset and the apparent denial of the theoretical concept by the Committee when they said,

The value of an asset is the money-equivalent of its service potentials. Conceptually, this is the sum of the future market prices of all streams of service to be derived, discounted by probability and interest factors to their present worths. However, this conception of value is an abstraction which yields but limited practical basis for quantification. Consequently, the measurement of assets is commonly made by other more feasible methods.

Different Classes of Costs for Different Decisions

Another and important area of accounting theory which is consonant with the general asset model is the area of cost. The different combinations of services an asset can furnish, in a time period, contained in the subjectively perceived service-potential set, the different valuations in the valuation set and the mechanism for combining these to calculate different
classes of values for different assumptions and different values for
different decision premises is an explanation of the phenomena of different
costs and revenues (values) relating to the same asset.

Theories of the Firm in Accounting

There are in the accounting literature today some theories of
the firm which have implications for accounting. These are usually
called theories of accounting, but this is a misnomer because they are
not theories of accounting; they are viewpoints of the firm. Their
importance to accounting lies in the implications these viewpoints have
for accounting theory and practice.

The most prominent of these theories are the proprietary theory,¹
the entity theory,² and the fund theory.³ Some minor ones are the commander
theory,⁴ and the enterprise theory.⁵

Each of these theories represents a point of view of the firm.
The model presented in this paper grew out of an attempt to devise some
method of showing that under these different viewpoints the same physical
assets were different.⁶

At present, accounting theory makes no statements about the same

¹ Stephen Gilman, Accounting Concepts of Profit (New York: The Ronald Press
² Ibid.
³ W. J. Vatter, The Fund Theory of Accounting and Its Implications for
⁴ L. Goldberg, "Some Basic Concepts for a Theory of Accounting" (Privately
circulated paper).
⁵ Waino W. Suojanen, "Enterprise Theory and Corporate Balance Sheets," The
⁶ I am indebted to Professor H. R. Anton for introducing me to this concept.
asset considered from two or more of these viewpoints. This furthers the misconception that the same physical asset appears the same under all these theories. This is due partly to the fact that no analytical apparatus for reflecting this has existed until now, and partly because few people visualized the problem. This lack of vision can be attributed in part to the absence of an analytical framework.

The general asset theory does provide such an analytical framework. By using this theory it is easy to understand how the same asset when considered from two different points of view can be two different things and have two different values.

Consider two individuals A and B, who are otherwise identical except for different viewpoints of the firm. For each there will be a subjectively perceived service possibility set and a valuation set for an asset.

Each individual will choose a series of service combinations for each of the future time periods, as his expectations for the asset. To each individual these expected services are the asset. Identical service combinations may be chosen, but this is not likely for several reasons. First, the subjectively perceived service possibility sets may not be the same and a combination of services which is in one set, but not the other could be selected. Second, the combination selected will be determined to a great extent by the services needed in the firm. However, the services needed in the firm are not absolute; they are perceived by the individual (A or B). Thus, if each, A and B, is in apparently identical firms at the instant we
are examining them, the firms will probably be non-identical if the perceptions of them by A and B could be compared.¹

The valuation set presents further opportunities for differences. Some but not all of the valuations will be different. Historical cost may be the same in each case. But it is at this point that it becomes clearly evident that valuation of the periodic asset representation will be different provided there is a different valuation vector for each point of view. It is conceivable that by accident the same combinations of services per period would be chosen by each individual. It is not conceivable that the same valuation vector will be chosen by each individual because the valuation vector represents, in part, the point of view.

It is conceivable that the values in the chosen vectors will be identical. If this occurs and if the chosen combinations are identical, then both the subjective perceptions of the asset and the individual valuations of these would be identical. That both can be identical is granted. What is important is that they need not and probably will not be identical.

As a result of this analysis, it is clear that when, according to the proprietary theory we write,

\[ \text{Assets} = \text{Proprietorship} + \text{Liabilities} \]

and, according to the entity theory we write,

\[ \text{Equities} = \text{Assets} \]

¹ If either A or B looked at both firms, he would say they were identical.
the term Assets does not necessarily stand for the same thing in each statement and as a consequence Equities is not necessarily equivalent to Proprietorship + Liabilities.

The Different Costs of the Same Asset

There is an anomaly in accounting which is unquestioned, I believe, because of its long standing existence. This anomaly is the existence of many costs for the same asset. Each cost can be and usually is different from every other. These different costs apparently have only one thing in common; they pertain to the same asset. But is this all? Is there not some other interrelation among these costs? How are they to be explained or are they only to be accepted? The general asset theory contains such an explanation.

The theory illustrates that for the various decisions different sets of services and services values for the same asset are considered. Since both the services and their values jointly determine the cost of an asset, it is easy to see how changes in either or both could produce the various costs.

Production and Operations Research

Man-Machine Systems

The general asset theory permits explicit recognition of factors affecting a man-machine output and it also includes differences between apparently identical machines; that is physically identical in terms of make, model, maintenance, etc.
Data of this sort are presently recorded, if recorded at all, by the production department. These data which are essentially data on actual and possible services remain in the production department and generally are used only by them.

If the general asset theory were to be utilized in a firm which had been using this sort of production data recordation system, the old system would not be supplanted, instead it would be supplemented. The results of the old system--data--would be available still to the production department. In addition the information furnished that department could be in the same form as before, and the data underlying the information would be collected and recorded by the production department.

The differences, if any, would be (1) the data would be available to anyone who wished to use them to develop information, (2) the production department may be required to collect some data previously uncollected, (3) if some sort of a large information processing system were in use, the data may be placed in the system rather than kept in the production department.

Attention Should be Focused on Service Attributes of Assets Not Physical Form

The essential attribute of an asset is usually the services it performs and the net value of these to the firm. Generally the physical form of the asset which embodies these attributes is unimportant.

The representation of assets in the general asset theory emphasizes these essential attributes--services--and ignores, unless it is important as a service, the physical aspects of an asset. The following statement by
Moore recognizes that this is an important approach to the asset concept.

Capital expenditures are (or should be) wholly forward looking. The past is nothing except as it helps you foresee the future. The point is that you are not just replacing machines. Not very often do you replace a wornout machine in the sense that you buy a new one which is like the old one. You are really buying machines to make products that customers will buy in the future.

But it is worthwhile to look at produced assets (output or product) as well as production assets in this fashion, since they (consumer goods, etc.) are acquired for the services they provide. By concentrating upon the service customers want or need, the thinking about product development and product innovation is freed from historical (traditional) physical embodiments of these services. This is a view recently put forth by Theodore Levitt when he said, "...industry is a customer-satisfying process, not a goods producing process,..." Business Week in the July 14, 1962 issue commented upon this when it reported "...the real job of the marketing man from now on will be to help his company decide its true relationship to the consumer and how it can make any product or perform any service the customer may want and it has the competence to supply." And, when talking about a management position to accomplish this, Business Week also said, "he must look at the characteristics of the population, what people seem to be thinking about wanting, what kind of devices or services technology promises for the future. In short, all business--not just marketing men--must become 'customer-oriented, not product-oriented.'"

Linear Programming

The general asset theory is formulated in a fashion which makes it compatible with linear programming. According to Churchman, Ackoff, and
Arnoff, "... linear programming can be used for optimization problems in which the following conditions are satisfied:

1. There must exist an objective such as profit, cost or quantities, which is to be optimized and which can be expressed as, or represented by, a linear function.

2. There must be restrictions on the amount or extent of attainment of the objective and these restrictions must be expressible as, or representable by, a system of linear equalities or inequalities."

The first requirement is not satisfied by the general asset theory, but the second is. This is acceptable since the first requirement is clearly outside the scope of the theory presented here. For example, in a production problem the constraint is the amount of services available, in physical terms, the objective is the desired amount of output (which is the additional data mentioned above) and the linearity characteristics of the general asset model fulfill the linearity requirements of linear programming.

Finance
Receipts

In Chapter VI by use of the general asset theory and the assumption of certainty, the classical economic formulation of asset value is derived. In a similar fashion, by the use of estimates in place of the assumption of certainty, present value calculations can be made. Also a system of rates of return can be developed.

Thus, by use of the general asset theory the type of information traditionally desired by finance can be calculated.
Investment Decision

More important, however, is the availability of additional information for the investment decisions. As indicated earlier, future requirements can be represented as future services needed. Both the aggregate amounts of services and the timing of the need for these services must be indicated. The form in which these services are embodied and other characteristics such as single or multiple-service assets will be a matter for production and finance to solve jointly.

Thus, data collected according to the general asset theory and information produced from these data are relevant to the investment decision process and the investment evaluation process. This latter includes of course current evaluation as well as the replacement problem and the make or buy problem.

Organization Theory

The development of a theory of business has received increasing attention in the last decade. As a result of this attention, there have been attempts to develop a theory of business and of segments of business.

The general asset model is consonant with organization theory for two reasons. First, it fills a void in business theory, to wit, it furnishes a theory of assets when none existed previously. Second, it has some of the desired characteristics of a business theory. These desired characteristics of business theory can be illustrated best by considering some of the criticisms leveled at other related theories--principally economic theory—which have long been used for business theory.
There have been two major criticisms of those theories as business theories. The first criticism is of the assumption of profit maximization. It has long been realized that businessmen do not maximize profits in the short run or the long run. One suggested alternative has been the satisficing objective suggested by Simon. Another major objection has been the lack of behavioral characteristics. Traditionally people were considered to be passive and have very few and limited characteristics.

In the general asset model no objective function, such as profit maximization, is ordained. The model can be used with whatever objective function management indicates so long as it can be quantified.

The model also incorporates behavioral assumptions. Maximum possible services of any one type in a time period—are a function of operator skill and operating policies among other factors. These are behavioral. This permits variations among people (operator skill) and allows variations in perception of the world, the selection of goals and means of attaining these goals.

**Information System of the Firm**

It is quite important that the general asset theory be consonant with the theory of the information system of the firm. As yet, none exists. Therefore, it will be necessary to speak in general terms and to anticipate some of the characteristics of such a system.

One characteristic which appears necessary to such a system is multi-purpose or multi-use data. Certain business data are of this nature. Other business data, such as aggregations of multi-use data, are not. It
would be important in an information system that multi-use data be recorded in their original form in the system, used as necessary in calculations, but not destroyed in the process of making these calculations. The general asset theory is consonant with these needs. It does not deal in aggregates, it suggests computations based upon multi-use data, and it relies upon the continued availability of these data in their original form for other computations. The general asset theory not only uses the data in their original form, but it requires that the original data be continuously available.

Since little is known at present of the structure and characteristics of an information system of the firm, any guides to these will be helpful. One source of such guides is theory. An information system of the firm is in essence a type of simulation of the firm. The structure of the simulation should be closely related to the theory. If it were otherwise, there would be a body of data relevant to one or no theoretical structure while the users of such data would be familiar with another theoretical structure.

The general asset theory fulfills this function. It indicates what is to be measured and what is to be valued. At the same time, however, it is sufficiently flexible to reflect the variations of individual firms and situations.

The general asset theory may also function as a guide to the indexing of data. According to the theory, each bit of asset data should indicate the time period, asset, and specific service. Value data are indexed by time period, type, and service.
This assumes of course that an indexed memory system will be used. There is another system, which is associative, and relies upon associations rather than indexing for retrieval. If an associative memory is used in the information system, it is likely that the data dictated by the general asset theory will be used even if the theory itself is not used as a retrieval model.

Conclusions

Thus, asset data, services and their values, are sufficient to provide most internal asset information desired by business firms. Information about the physical capabilities of assets can be generated from services data. The various type of cost and revenue data can be generated from a combination of services data and their valuation. These two aspects of assets are the key elements of the general theory of assets.
CHAPTER IV

The Nature and Measurement of Services

Historical Review

The idea of representing an asset in physical terms by vector notation is not new, and it is worthwhile to review its origin and development. Throughout the historical development of this idea, as will become apparent, emphasis has been upon the asset physical output and little is ever said about the valuation of these characteristics.

Leon Walras appears to be an early expositor of the idea that services come from factors of production and are used in producing goods and other services. The subsequent users of the services concept in either accounting or economics were exposed to Walras, his students, or his writings.

Perhaps the first accounting reference to assets as a group of potential services was made by Sprague when he said:

110. In another aspect all assets are the embodiment of services previously given; and in still another they are a storage of services to be received. Someone must have given labor in order to produce any wealth; but if it will not in the future command the services of labor, or save the expenditure of labor, or of its embodied results, it is worthless and not wealth at all.

113. To summarize this chapter, the assets comprising the debit side of a balance sheet may be considered in one or more of the following ways: ...

4. As the result of services previously given, or cost;

5. As the present worth of expected services to be received; ....
Sprague's use of the term service in his discussion of assets and the manner in which he uses it are similar to Walras's use of the word. It is clear that Sprague was familiar with the work of one of Walras's disciples, Fisher, for he also said: "On the other hand, a disservice (to use Professor Fisher's word) may have occurred...."

The next major writer in accounting to utilize this concept was Canning. Canning was also influenced by Irving Fisher. This influence of Fisher upon Canning was very great for in the preface to his classic work *The Economics of Accountancy* Canning thanks Fisher "for valuable critical notes and helpful comment on the manuscript of Chapter VIII," and then continues by writing: "I need not declare my obligation to Professor Fisher for the influence of his writings upon my thought--that obligation appears throughout the whole book." And this obligation does appear throughout the book for, according to the index, Fisher's name is used nine times and words, ideas, or definitions of his are used an additional seventeen times.

Canning's discussion of assets is probably the best in the accounting literature, which uses the service concept, for he defined an asset in terms of service, defined service and then described how valuations of these were made.

His definition of an asset is:

"An asset is any future service in money or any future service convertible into money (except those services arising from contracts the two sides of which are proportionately unperformed) the beneficial interest in which is legally or equitably secured to some person or set of persons. Such a service is an asset only to that person or set of persons to whom it runs."
This is, to use Canning's words, the professional accountants' implied definition. It must be remembered that Canning's sole objective was an explanation of what the professional accountant did and why. He was quick to see that there could be a negative aspect to services for he also said:

Associated with the capital sources, too, there are the undesirable events, disservices, the negative of income. To obtain fruit, human labor must be done in the orchard; tools and machinery must be purchased, operated and repaired; containers for the fruit and means of transporting the fruit to market are indispensable. The performance of the labor and the providing of means with which to obtain the fruit, to assure the coming in of fruit, are all disservices.

But it is not enough to define a term, it is also necessary to indicate how measurements of it can be made--how the definition can be implemented. There are two aspects to this. The first aspect is the physical unit of measure of service and the second is the valuation of these units.

The first aspect of this problem, defining a unit of service, Canning solved by giving a sample list.

Meaning of a Unit of Service,--There is little difficulty in assigning some appropriate measure of service, though it is sometimes inconvenient to obtain the record of service. But automobiles, trucks and busses are equipped to show a record of miles run; rolling stock statistics show like records for cars and locomotives; job records show machine hours under assigned operations, and so on. In all concerns in which any but the most sketchy cost accounting scheme is in use, the requisite data are found for other purposes in any event. General contractors' records show earth removed by various devices, yards of concrete mised, hoisted, and poured, and so on.

In the case of many types of asset, to be sure, useful life is more nearly controlled by mere lapses of
time than by service output. Thus, under light traffic, railway ties "wear out" because of rotting. A building site, so long as it is used for that purpose only, renders a continuous service.

He also explains:

The Income Must Be Convertible into Money.—One may have an enforceable right to the services of a thing or of a person and have no asset.... The service must either be itself a money income or it must have a money consequence. It must be commutable or convertible into money.

Two methods of valuation, direct and indirect, are described by Canning. Direct valuation includes items which are easily assigned a current money value. Examples of these are cash, accounts receivable, and merchandise. Indirect valuation includes those items none of whose "services consists in the direct and immediate bringing in of money," such as production machinery. However, he does not value services individually but only as aggregates. This is, essentially, a method of valuing assets—the embodiment of services—and indirectly the services rather than the services themselves and then the asset.

This work is probably the only theoretical explanation of accounting practice which exists. While it is very good, it falls short of being a general theory in many respects. It does not include assets which are normally unrecognized and in this respect is bound by the then conventional accounting. It does not include services which an asset can provide regardless of whether or not the asset is called upon to furnish these services. It does not provide a method for valuation on any other basis than current value, if in dollars or other units than dollars. For these reasons the
work of John B. Canning can be called an outstanding piece of work but not a general definition of assets.

Chronologically the next mention of asset services in a major accounting work was by Patton and Littleton in An Introduction to Corporate Accounting Standards who state:

'Service' is the significant element behind the accounts, that is, service-potentialities, which when exchanged, bring still other service-potentialities into the enterprise.

Behind Accounting's array of figures, which laymen may think represent values of money, or, at best, price, lie the tangible and intangible embodiments of service.

In the same work these authors use the term "price-aggregate," in the sense of the summation of the services (from an asset or person) times the value of these services, as being a broader valuation term than cost. This is similar to an aspect of the general theory developed in these pages. That is the calculation of asset value for a period according to some valuation assumptions in which the product of the different ex post or ex ante services of the asset and their unit values are summed. This could be labeled an "aggregate."

And to some degree the general asset thing was anticipated when these authors said:

The concept of accounting subject matter as price-aggregates resulting from exchanges thus becomes a much-needed device for coordinating a number of related concepts. Instead of appearing as a confused mixture of elements--costs, assets, revenues, liabilities, investments, surpluses--accounting can be viewed as dealing with various aspects of a single subject matter.
William J. Vatter in his Fund Theory of Accounting and Its Implications for Financial Reports also discussed the concept of assets as a store of services as follows:

An approach to the problem of defining accounting concepts from the point of view of homogeneity of substance may be made by observing that the aim of all economic (or financial) activity is want satisfaction—"a state of change in an economic subject, produced and maintained over some period of time by an economic agency." Wants are satisfied by the conversion or transformation of services released from (rendered by) economic agencies, such as persons or things. Thus the underlying material of economics is the aggregation of services devoted to want satisfaction through the processes of storage, transformation, and exchange. By means of these processes services are built into various forms and are released to various individuals or activities. Wealth is thus an aggregation of service-potentials, and the business system is a mechanism to promote the separation, combination, transfer or other metamorphosis proceeding toward the satisfaction of wants, as they are finally expressed in various market situations. Commodities, productive agencies, money and other financial claims; special advantages, opportunities, or capabilities have thus a common significance—they are service potentials, capable of satisfying wants through the processes of business.

Vatter was not developing a theory of assets when he wrote these lines. He was developing a theory of the firm--The Fund Theory--which hopefully could be used as a guide in the development of accounting reports. As part of this theory, he needed some element (having "homogeneity of substance") common to all assets. The service concept has this characteristic and he therefore utilized it.

The next and most explicit recent use of the service-potential concept was by the American Accounting Association Committee on Concepts and Standards Underlying Corporate Financial Statements who defined assets as "...economic resources devoted to business purposes within a specific accounting entity; they are aggregates of service-potentials available for
or beneficial to expected operations."

However, it is apparent that the committee as a whole felt that there was little practical use in the concept and its associated asset definition for they state later:

Measurement

The value of an asset is the money-equivalent of its service-potentials. Conceptually, this is the sum of the future market prices of all streams of service to be derived, discounted by probability and interest factors to their present worths. However, this conception of value is an abstraction which yields but limited practical basis for quantification. Consequently, the measurement of assets is commonly made by other more feasible methods.

The term "value" is used here only in the sense of value for financial reporting and in this context the contention in this paragraph cannot be denied.

There is another professional group whose work on the measurement of assets should not be overlooked. This group is composed of economists interested in social accounting and the measurement of national income and wealth. Goldsmith has suggested, as a means of standardizing national wealth estimates for the purpose of comparability:

...a simple physical dimension as a rough indicator of quantity; e.g., the cubage of buildings, the horsepower of certain types of machines and transportation equipment, or the surface of soil of a given quality. It will usually be more difficult also to determine the weights to be applied to the different types of tangible assets. Notwithstanding these difficulties we should often be able to derive a measure that could be used as a check on deflated current national wealth estimates. Geer Stuvel, in a paper, "Development of Stock of Capital Goods in Six Countries Since 1870," presented at the 1949 meeting of the International Association for Research in Income and Wealth, made an attempt in this direction.

In addition he anticipated to some extent the model developed in this study when he said:
There are basically two possibilities for measuring the stock of reproducible tangible assets, retrospectively as the man-made resources that remain embodied in the stock; and prospectively as the economic services still expected from the stock. The first of these alternatives evaluates R.T.W. [real tangible wealth] by expenditures on durable tangible assets reduced to a constant price level, cumulated, and depreciated on the basis of the expected life of the different types of assets. The second alternative measures it as the market value of each asset, or the nearest substitute to it. These two values, of course, are not unrelated; but neither are they equal, nor necessarily always near each other.

This group has also devoted attention to the problem of the various possible asset valuations both at a point in time and over time.

It should be noted however that these suggestions for asset measurement are only two of several alternatives suggested by this group in their discussions on possible improvements of national income and wealth statistics.

It is clear from the work of Canning, The Walrasian School, and others that the economic concept of service has been fairly well developed. However, an economic concept of assets utilizing the service concept has not been developed.

On the other hand, accountants, except for Canning and his restricted theoretical model, have not been able to develop any theoretical formulation of assets. Part of this is the result of past emphasis upon financial accounting and the subordination of managerial accounting to financial accounting. Now managerial accounting is being recognized in its own right, but more important the concept of an information system of the firm is being developed. As a consequence, tools, concepts, and definitions are being fashioned for the recordation and manipulation of data which is generally basic to a firm.
There is a fairly serious discrepancy between the economic and accounting use of the services concept. Services are used in accounting to indicate the nature and potential of an asset. Economists, on the other hand, use services to reflect the components of the production function. This is a perfectly satisfactory use. However, by not discussing the corpus, the source of the services, the presumption is that the services are available separately in any rates and stocks desired. This is not so and is an important factor in long run production.

The Nature of Service

This study relies heavily upon service measurements of asset performance. It is not clear, however, if measurements of asset services exist or if they can be made. There are essentially two questions to be answered in order to measure services:

What services of assets are to be measured?

How can these measurements be made?

To answer these questions, operational definitions of services must be devised and operational illustrations of the answers to the questions above must be furnished.

What Characteristics of Assets Are To Be Measured?

To ascertain what asset services should be measured, prospective use of the information should be the guide. Vance argues: "In generic and nontechnical terms, capacity is simply the ability of an individual, an organization or a piece of equipment to be of service in the performance of
acts requisite to the attainment of predetermined objectives." The decisions we wish to make will require data with specific characteristics. By considering the decisions to be made in a firm and the asset service data needed for these decisions, it should be possible to deduce something about the general type of service measurements which will be most useful for these decisions.

The basic decisions which are made about assets are:

Which asset to buy
Whether or not to use an existing asset
Whether or not to replace an asset

Businessmen as purchasers, users, and manufacturers of assets discuss them in terms of the end service obtained, such as ton-miles of truck hauling. Which of the services of an asset will be used is a matter of managerial decision making. Which of the services management intends to use is a matter of managerial planning. Since intent and results are not always identical, and since planned decisions and actual decisions are not always the same, the services which should be measured are all those associated with an asset not just those which management intends to use.\(^1\) That is, all those services associated with the asset and relevant to the operations of the firm are to be measured. For example, ton-miles of capacity in the desert or the arctic are irrelevant to a firm operating in Florida, but bulk-miles in Florida are relevant to a Florida firm which is hauling only high density cargo in Florida.

\(^1\) This is consistent with the statements above which argue that those services which are the most useful—but not necessarily those used—should be measured.
What is relevant and what is irrelevant in a real situation is hard to determine. Ideally, all asset services should be recognized. In the theoretical formulation this can be done by assuming that the number of different services is finite.

In reality there are certain restrictions which must be recognized. One is the capacity of the physical apparatus storing and processing the data. Another is the cost of each unit of data even if sufficient capacity is available. Another important problem is the amount of output (data) the decision making system can absorb.

If all the services of an asset can be recorded without violating any of the restrictions listed above, there is no need to determine because of data capacity restrictions what is relevant and what is not, since all the data on all services of each asset can be contained in the data storage of the information system. However, when all the services of an asset cannot be included in the data section of the total system, some criterion for selecting what is to be included and what is not to be included must be formulated.

Services relevant to current and planned future operations should be given first consideration (assuming the model is being utilized to produce information about current and future operations), and after these, other services could be considered. A general rule might be to rank the services by the probability they will be needed for the decisions the firm expects to make and then select the top portion of this list as the relevant services. This question of what services to measure is only one aspect of the problem.
of what to measure. Two other important aspects remain. These are the type of service to be measured, intermediate or final, and the boundaries of the unit of data.

**Type of Service to be Measured**

The problem of specifying what to measure can be approached in a manner analogous to the Taylor time and motion studies. The Taylor approach is to define and identify the basic elemental movements in all operations. Then any operation can be described as some combination of these basic movements.

The same could be done with services. Asset services could be broken into their basic elements and the asset services described in terms of these basic units. A variation of this approach would be to define as a service not the basic elements, but some combination of these basic elements. In basic units a truck would be defined in terms of brake horsepower, traction, etc., while as a combination of these basic units the services of the same truck could be described in terms of ton-miles or bulk-miles in the operational environment.

Another approach to the problem of what to measure is measurement of the physical contribution of the asset. This is similar to the economic notion of value added without the valuation of what has been added (i.e., the "added" is stated in physical rather than in monetary terms).

**Boundaries of the Unit of Data**

What are the physical boundaries of an asset? What is considered an asset to one firm may be an asset component to another. For example, one
firm may consider a truck an asset while another firm may consider the motor, tires, and chassis of an identical truck separate assets. On the other hand, a complete plant can be considered to be an asset even though it consists of land, buildings, and machines.

If a company chooses to consider components of a piece of equipment as separate assets, instead of regarding the whole equipment as an asset, the services measured would be those of the separate components. If the decision-makers of the firm wish to have data relating to the piece of equipment as a whole, these data could be calculated also. A plant considered as a whole asset is never carried on the books as such by the operating company. The components that make up the whole plant are carried on the books separately.

The Interpretation of the Service an Asset Furnishes

From a service point of view it can be argued that an asset always furnishes the same service. What causes apparently different services to be produced is either the object acted upon or the environment. For example, a stake truck can haul people, low density materials, high density materials or a combination of all three. Whatever type of load the truck carries, the basic service of hauling a mass is being performed. The number of people carried is restricted by the need of humans to have breathing space and move around. The amount of high density matter the truck can carry is restricted by the load capacity of the truck and the low density capacity is determined by its volume limitations.

In this sense, an asset always furnishes the same type of service
not different types. Yet we speak of an asset as furnishing different services (e.g., for a truck, ton miles or passenger miles). Why is this so? From the point of view of what the asset does there is only one type of service. From the point of view of what is received from the asset, there are different types of services. The service received varies as the type of object which is subject to the service varies.

Both these methods of labeling the services of an asset are logical. Only one of the two should be chosen for use. There is no problem: the choice has already been made. Services are labeled with respect to the object which is subject to the service. This is convenient and operational. We think in terms of a number of people or tons of material to be moved. If we wish to think in terms of the service an asset furnishes, we must first translate people and material into units of the appropriate service. Some day, when we have a greater choice in the devices available to us, we may think in this manner.

Services Classification

It is the services of assets to objects that are to be measured. These services fall into one of the four classifications: operating asset services, operand asset services, monetary asset services, and rights assets services.

Operating Asset Services

An operating asset transforms operand assets. For example a truck transports a product from a factory to a store or a machine shapes a piece of metal. This term includes production assets but is not limited to these.
For example, a typewriter is an operating asset even when it is an administrative office.

The services of operating assets have been discussed above.

Operand Asset Services

Operand assets are assets that are transformed. In the previous illustration the product and the piece of metal are examples of this type of asset.

To calculate operand services, the general question of the nature of inventory services must be considered. Raw material and in-process inventories furnish the medium upon which the transformation assets operate. This is a service. However, it is different from the service available from a transformation asset. Thus, operand services are the amount and quality of the medium, raw materials and in-process inventories, which are transformed.

Finished goods inventories are exchangeable into dollars and that is the service they furnish. The services of finished goods should be measured in the standard units normally used for them, such as gallons, pounds, tons, etc.

This appears to present a paradox, since the purchaser may calculate the services of an asset differently from the seller. To the seller, a gallon of paint has a service in bringing dollars to the firm, but to the buyer, the same gallon of paint may have a different service, such as a wall covering. This is not really a paradox since the service calculation is made with reference to the firm, its activities and especially the intended use of the asset in question.
Monetary Service

Monetary assets are cash, claims to cash, and cash debts. The services of this type asset are each one. The value of this one unit of service, like the others, is determined in and by the market.

To the seller an asset is a monetary asset no matter what its past or future use, and it is valued as such. As a monetary asset, its service is the number of units-of-account it will bring into the firm and its value is the product of this number and the value of a unit; this is usually one. The value of a unit of account, say a dollar-unit-of-account, is a given and generally is one. It is the number of these units of account that must be determined.

This is done by considering the market if there is one for this type product. This is a variation of the usual procedure for valuing assets. Normally, an element of the perceived possibility set is selected as the set of expected services to be gained from the asset. It is the characteristics of the asset which determines not only what these are but their number. For monetary assets the number of services (dollar-units-of-account) are a function of the market not the asset.

Rights Service

Rights assets are patents, copyrights, research ideas, developed formulas, processes, advertising, etc. The services of rights assets are still different from those of the other types of assets. In general rights assets perform the service of eliminating or restricting competitive entry
into a specific product market. Thus, when a copyright is up or a patent has lapsed or a trade secret is no longer a secret, the worth of a previously valuable rights asset is zero. It is not zero because of a drop in value of the service that was performed. It is zero because there are no more services in the rights asset (i.e., the sum of the remaining services is zero).

Discussion

The character and number of services are a function of the asset—how it can be used. The valuation of these services on the other hand is a function of variables external to the asset, the market forces.

Measurements of Services

Measures of asset services can be divided into two general categories, measurements of past asset performance and estimates (measurements) of future asset performance. Measurements of past asset performance are measurements based upon the actual operation of the asset (or its duplicate) being measured. Estimates of asset performance are based upon performance of similar equipment, laboratory tests of equipment components, and experience with related but not identical equipment. Examples of actual asset performance measure data are engineering studies, historical records, and statistical sampling. Another type of asset performance data is pilot plant operation data. This type of data seems to be a combination of the two categories.

These diverse measures and their varying accuracy may seem objectionable, but there is really no choice in the matter. If a measure of something
is desired, the best measure available must be accepted. Even if this estimate is subject to large but unknown errors, as soon as operations involving the asset commence, historical data is accumulated which will serve the double purpose of permitting an evaluation of the original measure, and the development of other measures of the asset services.

**Measuring Services**

In the proceeding discussion it was clear that operating assets and rights assets are the two asset classes which present problems in service measurement. The services of operating assets are a function of past occurrences and therefore are theoretically measurable, if all of the past occurrences and their effect upon future services are known. These things are rarely known completely, although they are known with various degrees of accuracy and completeness for different pieces of equipment.

The services of rights assets are not solely a function of the effect of past events on the asset. Some outside factors such as demand, competition, and sources of capital also affect them, but these factors are difficult to measure. It seems feasible at this time to use the same technique for rights assets as was suggested for monetary assets, to calculate and value the associated services (e.g., dollars of account).

The ensuing discussion of the measurement of services will be devoted to operating assets alone. The measurement of the services of the other three classes of assets has been discussed and general solutions to their measurement problem have been formulated.

There exist many measurements of equipment in terms of services.
There is one slight drawback to these data. They are not in the exact form we would prefer. Invariably operating asset measurements are rate measurements. This, of course, is because managers are not interested only in the service the asset will perform, but they are also interested in the rate at which this service will be performed. The specified rate is of course one of many possible rates. It is assumed when this rate is calculated that there is a certain operator skill, a certain maintenance policy, etc. And finally, it is assumed that the variations between machines are such that any one machine will have a rate which is close to the mean of the distribution of the predicted rates. The operating rate data are developed for the type of asset discussed later—the constant-quality-service asset which has for practical purposes an unending life.

Each rateDatum is a derivative which is also constant. Thus, to find the services an asset is capable of delivering over some period of time, it is necessary only to integrate the stated rate with respect to time. However, since the derivative involved is a constant, integration is the same as multiplying the value of the derivative by the time period length to calculate the desired, unknown service.

The term time period length is used to represent the expected operating time of the asset in the production of a service. This excludes setup time which will be handled separately. The expected operating time is a multiple of the daily shift length less the time allowed for personnel breaks, the expected breakdown time and the expected down-time for preventive maintenance. Any downtime that occurs outside of operating shift time is not to be deducted from shift time used to calculate the services.
Maintenance data are usually in the form of a rate also, such as ten hours maintenance for fifty hours of operation. All of these rates are statistical averages and have variances whose size depends upon the equipment and operating conditions.

There remains the type of asset which has a declining quality service. For the most part this type of asset is a component rather than a unit asset. The operating life and services of this type of asset can be established by statistical testing.

Intuitively, we feel that a definite number of services should characterize every asset. It is also reasonable to expect every asset to have some definite life span. This life span should be measureable in some manner, such as time or services. It is somewhat shocking to realize that most assets wear out a part at a time so that in a sense they are never worn out at any one time even though they may not be functioning for short time intervals. However, it appears that this is the situation with many assets. (The implications of this aspect of assets on the valuation problem will be considered later.)

Part of this study will be devoted to the problem of calculating current costs by using a general asset model based on the general asset theory with the service concept, measures of services, and the valuation of services. The indefinite life of the asset units will have no bearing on current cost calculation since current costs must be calculated for a time period which is usually smaller than the life of the asset. It is possible to calculate the services associated with an asset for the type
of time periods, such as a week, a month, or a year, that will be considered. However, if we attempt a calculation involving an indefinite period of time, such as the useful life of the asset, there is a problem. The useful life of this type of asset is not determined by the services it can perform, but by the value of these services and the cost of maintaining the asset in operation. The valuations of the services both positive and negative are not a function of the physical characteristics of the asset, but of the state of the economy in general and the specific industry in particular. Consequently, since we never know the life of an asset, we can never foretell its value except in a statistical sense by giving an expected value which would undoubtedly contain a large subjective element.

**Factors Causing Variations Between Otherwise Identical Equipment**

The amount or rate of services forthcoming from an asset and the maintenance it requires are not a function of the asset type alone. There are a great many factors which affect the performance of equipment. Those which appear to be generally the most important, but not necessarily important in every case, are:

1. Climate
2. Maintenance policy
3. Type of work
4. Operator skill
5. Variations between otherwise identical assets operating under similar conditions.
Climate

The climatic factor is characterized by such things as temperature, humidity, and atmospheric conditions, such as snow, dust, and rain. Usually equipment is produced to operate in specific geographical areas. Automotive equipment, for example, is usually designed for operation in the temperate zones. Often these special conditions are not recognized as such because they are a part of the normal environment of the producer and the designer. For example, the Volvo, a Swedish designed and produced automobile, has a manually operated radiator shield. This mechanism enables the operator, on extremely cold days, to shorten the time needed for engine warm-up and while operating to maintain engine temperature. Explicit recognition of this environmental conditioning usually occurs when services are needed in an environment which is clearly different. When a piece of equipment is modified to operate in a new or different environment, the modified equipment is different from the original equipment and the services are also different, although similar.

A specific piece of equipment in a given environment, for which the equipment was designed, may still vary in performance as the environment varies within its own climate limits. For example, truck operation in a New England winter is different from truck operation in a New England summer. When the difference in service affects the calculations for which the measurements are made, the measurement units must reflect this.¹

¹ When differences in services, due to variations in any factor, are significant with respect to the decisions, the differences must be measured.
Maintenance Policy

While the following remarks on maintenance policy refer primarily to only one of the four classifications of assets—operating assets—the concept applies to the other asset classifications. This class of assets can be divided into two types. The division is based upon the quality of the service or the expected quality of the future service, of the asset. Type I asset services have a uniform quality while Type II asset services have a quality which decreases as the number of services obtained increases. These are illustrated by Figures 1 and 2.¹ A Type I asset is exemplified by a DC-3 aircraft. The service it can perform is relatively unchanged since the late 30's. This is so despite the fact that many of these aircraft have been in service for many years.

A saw is an example of a Type II asset. Through use it becomes worn and its cutting efficiency drops. The quality of cut decreases and identically the quality of each service decreases. Quality of service is restored to the original high level when the saw is sharpened.

¹ I am indebted to Professor Malcolm Gotterer of the Pennsylvania State University for suggesting this graphical representation.
Maintenance on a Type I asset would affect the amount of time the asset is inoperable and when it is inoperable. The effect of maintenance on a Type II asset would be the same as on a Type I asset and in addition it may affect the rate at which the quality changes. This would cause a variation in the slope of the curve.

Type of Work

The service performed by a truck in carrying a cargo over a secondary road is different from that performed on a freeway; the service performed in carrying cargo in the mountains is different from that performed in a valley. This difference in the task performed can be considered also as a difference in the services furnished. By defining the services appropriately, what were considered performance variables become different services. This will change the problem of type of work, but not eliminate it. Within the limits of each newly defined service, there will be a performance variation. If the variation is significant with respect to the uses of the data resulting from the measurements, then type-of-work is still a variable in asset performance. If the variation is not significant, then type-of-work as a factor affecting asset performance can be ignored.

Operator Skill

Except for completely automatic machines, the operator affects the rate and quality of machine output. This is clearly recognized in time and motion study—a method of measuring potential output. It is so important that a name—man-machine—was developed for the operator-equipment
combination. There is also clear recognition of the differences between individuals of approximately the same skill and also between individuals of varying grades of skill. Differences between individuals are recognized in time and motion study when several individuals, instead of one, are studied to develop a standard. The skill differential is recognized when different standards are devised for individuals who have been on the job different lengths of time (e.g., trainees).

Variations Between Otherwise Identical Assets Operating Under Similar Conditions

Variations between otherwise identical assets which are caused by minor variations in their manufacture have been thoroughly documented statistically. This will cause different performance records for assets otherwise identical and operated under identical conditions. Again the error involved may be insignificant with respect to the purposes for which the data are being developed, but if this is not so, it will be desirable to forecast statistically what the error may be and what the spread of this error around the average may be.

The above factors are not always the ones affecting performance in every case. Generally, they are the most important. Since they are variable over time and independent of each other, it is obvious that the services embodied in an asset at any time are not only a function of the asset and the amount of use it has received, but they are also a function of any other factors affecting services, and the manner and amount in which they have occurred.
Any operating asset can have a wide range of factor service combinations during its life. This does not complicate the estimation of services unduly. In any specific situation the range of the variables affecting asset performance will be greatly restricted. Generally, operations will be performed in one climate; only two or three maintenance policies will be pursued; type-of-work can have many varieties which could be redefined and thereby reduced; operator skill is normally a continuum which can be arbitrarily divided into classes; and variations between otherwise identical assets can be reduced to discrete amounts which are few in number. In addition, records of past performance and the relevant variables will facilitate estimation of asset services.

Some Observations on Operating Assets

Asset Retirement

Let us consider as an asset some complex piece of equipment, such as a box-making machine or a truck. Now if we maintain this equipment, and it does not become accidentally wrecked, it will have what appears to be an indefinite life. However, the asset in later years may not have many of the physical components it had initially, because these components have been replaced. The early 1930 trucks the post office replaced some years ago are an example of this. Another example is old typewriters that are still operational. But assets are replaced, so if this type of asset does not wear out, why is it replaced? It is replaced for two reasons. One reason is related to operating costs and the other to operating revenue.
A firm will replace an existing asset with a newer asset, when both furnish the same services, if the savings in operating expenditures will pay for the newer asset. In other words the type of asset we are discussing here does not wear out; it becomes too expensive to operate.

A different reason for asset replacement or down-grading is that a newer asset embodying a service, which was previously unavailable, is now available. Piston engine aircraft furnish today the same service they performed when they were new. They have been replaced on the major routes by jet-engine aircraft because the jet-engine aircraft furnish a different and better service which was previously unavailable.

**Asset Interaction and Cooperation**

It is recognized that there is a difference between a collection of assets each considered separately and the same collection considered as a group. In accounting this has been expressed in the maxim, "The whole is greater than the sum of its parts." This characteristic of assets and the general asset theory are consonant.

When an individual physical unit is described, the services and their values are used. When a group of assets are described jointly, it is their joint services and their values which are used. At this point there is no difference between practical realization and the general theory. The practical point is that the collection of assets listed in a balance sheet represent the physical aspect of the production of the firm. This production would not be forthcoming from these same assets if they were dispersed and operating independently.
There is another way of stating this point. It has not been shown that the independent outputs of a group of assets can be summed analytically to equal the output of the group as a whole. This is a research problem: can empirical results be achieved analytically?

Two points mentioned earlier are pertinent to this problem. Services are perceived in terms of what is desired by the firm rather than what is given by the asset. Thus a truck performs the same hauling service for people as for flour. Yet one type of service is described in terms of passenger miles and the other in terms of ton miles.

Assets are generally multipurpose to some degree. When a group of assets are operating jointly, a specific element of the services possibility set of each has been chosen as the expected service output of that asset. The service an individual asset can perform is unaffected, except for upkeep and repair, by the service any other asset performs. However, the selection of an element of the perceived service possibility set of an asset as the expected services output of that asset is affected partly by the output of other assets and primarily by the goals of the firm.

Another pertinent point is that a balance sheet does not show all of the differences between a group of assets considered separately and jointly. A firm is more than a group of assets and includes some assets which are not reflected in the balance sheet. The training and knowledge of the work force is an important and valuable contribution to production. The difference in the spatial location of the assets of a firm and the same assets as independent units is not disclosed. It is well known that good will is generally ignored quantitatively. The knowledge and ability of
management are not disclosed or treated as an asset.

If these factors; services perception in terms of need, the multi-service character of most assets, and differences between separated and grouped assets which are not revealed are taken into consideration, the disparity between the same assets considered independently and as a group can be explained analytically.

Conclusions

All classes of assets can be measured in terms of services. These measurements exist today in one form or another. The most important class of assets, operating assets, has been thoroughly measured. These measurements are not in the form desired for the general asset model. However, the form in which the measurements of the two types of this class are expressed can be restated in the desired form.
CHAPTER V

The Valuation of Services

The term valuation as used in this paper has a special meaning. It is used in the sense of valuing some specific thing, usually a unit of service. Occasionally, it will mean something else such as the aggregation of services an asset furnishes in a time period. This is not the traditional use of the term valuation. Traditionally, the term is used implicitly to designate the value of an asset in current terms. As used here, valuation is not necessarily of assets and not necessarily in current terms. Billy E. Goetz pointed out the necessity for this when he said:

Conventional cost accounting assumes a unique truth, independent of purpose and of many aspects of situation. Each cost item at every stage of the process of division, qualitative change, and recombination, is assumed to have a single, absolute value easily ascertainable by ordinary arithmetic processes of division and addition. Cost items should be given different values according to the managerial purpose to be served, e.g., one set of values in determining prices, another in deciding whether to make or buy parts. Values also vary with situations; e.g., in pricing, it makes considerable difference whether plant capacity is fully or only partially utilized.

Paul Kircher also realized this for he pointed out the same thing in the following statement:

As a first step in developing theory in these areas, I propose an expanded concept of the framework of accounting. This framework should consider the basic content of accounting theory to include all the value measurements made for economic purposes. Therefore, it should range from the goals desired by the manager through all the techniques the accountant can use to help the manager achieve these goals.
The Types of Valuation Needed

While valuation is usually conceived to be monetary, it can be couched in other terms which reflect important considerations besides money. For discussion purposes these types of valuations will be classified either as monetary or non-monetary.

It must be emphasized, however, that non-monetary valuations can be converted, if necessary, into monetary terms so that all valuations could be stated in monetary terms. However, to do so would introduce an unnecessary complication into decision making.

Monetary valuations are necessary because money is an important good which we wish to conserve and increase. It is the same with non-monetary valuations. They are used because the valuation media (e.g., time) is an important good and must be considered by the decision maker. Putting non-monetary valuations in monetary terms would needlessly complicate matters because the decision data would be stated in terms which were not pertinent to the critical production factor and would require interpretation.

Monetary

Monetary valuations of services are naturally subdivided between costs of services and revenues attributable to services. In addition, there are two other groups of valuations which are sufficiently important to merit separate discussion; they are mixed values and partial values.

Cost

There are many specialized cost definitions. Many of these are partial combinations of common sets of data based upon the same valuation scheme. Specialized cost definitions will not be discussed at this time.
Instead three general valuation categories will be discussed. They cover the past, the present, and the future. They are commonly called historical cost, current cost, and future cost. It will be shown subsequently that these include all monetary valuations; all specialized costs are derived from these.

**Historical Cost**—Accounting is in part fiduciary. One aspect of financial accounting is concerned with reporting on the flow of dollar units of account through a business and reporting on the number of these that the business has earned. It might be noted that all historical costs were current at some time. Thus, if current cost data are always recorded, historical cost data are always available without any additional search. The primary reason for historical cost data is financial reporting. The irrelevance of historical costs for decision making was emphasized by Horngren when he said:

Under our definition of relevant costs, book value of old fixed assets is always irrelevant in making decisions. This proposition is by far the most difficult for managers and accountants alike to accept. The real concept of importance here is deeper than this subordinate issue: it is the concept that all historical costs are irrelevant. At one time or another we all like to think that we can soothe our wounded pride arising from making a bad purchase decision by using the item instead of replacing it. The fallacy here is in erroneously thinking that a current or future action can influence the long-run impact of a past outlay. All costs are down the drain. Nothing can change what has already happened.

**Current Cost**—Control is broadly concerned with all current operations in a business. Control includes checking operations to find how much actual results differ from planned results as well as acting when there are differences of importance. One method of evaluating operations is through
the use of current cost data. This cost basis shows whether current operations are profitable in current terms. In recent years there has been increased agitation for the presentation of accounting information in current terms. Current cost data available on a continuous basis and for all assets would facilitate the preparation of such information.

Future Cost—Planning, another important function in modern organizations, needs estimates of the future. Part of these future estimates are values such as those related to operating certain assets and producing goods and some are physical data such as the type and amount of services needed for future production. This sort of data will be available also from the general asset model.

Revenue

Since revenue is the analog of cost, the reasons for its classifications are similar to those stated for the cost classifications.

One point deserves discussion. This is the matter of the association of costs and revenues. Accountants have traditionally taken the position that costs attach and set those off against revenue when it is realized. However, in the area of current costs and future costs such an offset is undesirable. For control and planning, historical costs are irrelevant and the important values are those costs and revenues which are relevant to the decisions being made. These values are invariably those which exist currently or will exist in the future.
Mixed

There is only one mixed valuation. It is known as opportunity cost but paradoxically, it is a net revenue. It is defined, in the Accountant's Cost Handbook, as "...the measurable advantage foregone as a result of the rejection of alternative uses of resources, whether of materials, labor, or facilities."

Horngren comments upon the concept at greater length:

Opportunity cost is the measurable sacrifice of rejecting an alternative; it is the amount foregone by forsaking an alternative; it is the maximum alternative earning that might have been obtained if the productive good or service had been applied to some alternative product or use.... Thus, opportunity cost is the measure of the rejection of alternative uses of resources.

Opportunity costs are rarely found as a part of conventional accounting records, nor do they ordinarily appear on income statements. Such costs represent incomes foregone by rejecting alternatives; therefore, opportunity costs do not involve cash receipts or outlays. Accountants usually confine their recording to those events that ultimately involve asset exchanges. Accountants confine their history to those alternatives selected rather than those alternatives rejected, primarily because it is either impractical or impossible to accumulate meaningful data on "what might have been.

To illustrate this, consider another use of an asset. Then calculate total revenue and total cost of using the asset in this manner for some time period. The difference between total revenue and total cost is net revenue foregone in not using the asset in this other manner. (The classical example is planting a field in one crop rather than another.) Therefore, opportunity cost is a net revenue and a mixed concept, since it is a combination of cost and revenue.
Partial Values

In cost accounting it is recognized that there are "different costs for different purposes" and it is recognized that there are different valuations for different purposes.

Within a valuation class there are many possible partial valuations each relevant to a different purpose. For example, if a decision rests upon the total amount of out-of-pocket current costs to be incurred, any current costs which are not out-of-pocket are irrelevant; yet the costs which are used in the calculation are of the class, current costs.

Non-Monetary

At present only two non-monetary costs come to mind. They are time and opportunity cost in physical terms.¹ Non-monetary costs are needed, as previously stated, because there are decisions in which the unit of importance is something other than money. For those decisions data stated in dollar terms would obscure important aspects of the solution.

The Jointness of Service Valuation

In preceding sections of this study it has been demonstrated that for complex assets, the availability of services is a function of the asset, company operating policies, and its operator. In addition it has been shown that complex assets furnish services at a rate instead of in fixed amounts. These characteristics of complex assets, in addition to the fact that complex

assets are purchased and maintained as whole units, cause the valuation of services to be a joint cost problem.

The joint cost problem is the situation in which a series of products are produced together from some single factor of production. The problem is the assignment, to the separate output products, of the cost of the single factor of production utilized to produce the series of products.

In our case the output is services, and the factor of production is the asset; and, the problem is how to identify the value of each service produced by a complex asset. The cost of the asset may be a starting figure. How is this amount to be allocated? There are several problems. The total amount of the services to be rendered by a specific asset are not known when the asset is acquired. The effect of maintenance and repair upon the cost of the asset services is not known. If the exact number of services to be rendered and the maintenance and repair costs were known, the problem would not be solved because of the timing of the services.

The services are furnished at different times, since they are neither available nor desired all at the same time. This creates many problems. Should all services be valued the same (an average) or are some of greater value than others? What discount rate should be used to find the present value of services available over time? A solution to the joint valuation of services might be attempted by valuing the complex asset components and then, through these, the services.

By definition a complex asset is composed of basic elements. The expected service from each of these elements can be determined statistically
and the number of services from each of these basic elements needed for a unit of service of the complex asset can also be determined. It follows then that if a value can be set upon the services of the basic elements of a complex asset, the services of the complex asset can be valued. However, the valuation of the basic elements cannot be determined except in one improbable case. This one case would be when each basic element is purchased separately and there was no assembly cost to putting all the basic elements together to form the complex asset.\(^1\) So long as a lump sum payment was made for the whole complex asset, the allocation of the joint costs problem exists. A lump sum payment has been made for a collection of objects—the basic elements of the complex asset in this case—with no indication of which part of the lump sum payment is to be attributed to which component.

While there is no cause and effect solution to the problem of allocating joint costs there are possible arbitrary solutions which can be evaluated. This evaluation procedure is referred to in a later section of this chapter entitled Sensitivity Analysis.

Existing Valuations of Services

It must be understood at the outset that nearly all existing valuations of services are approximations. Those which are not, are the values of services for which a market exists. The existence of markets for assets is quite common, but the existence of a market for specific services is uncommon. For example, most cities have car rental agencies which rent cars

\(^1\) Even in this example, a critical assumption must be made. It is that the unit as a whole is priced by summing the price of its components.
for a fixed fee plus mileage fee, but such markets account for only a small part of the automobile services used in an advanced industrial economy. (Aside from the problem of the fixed component.)

The remaining valuations are approximations for two reasons. First, many are part of a joint situation (either cost or revenue or both) and secondly the approximations are averages. The characteristics and problems of the joint value situation have been discussed. The problem of the valuations as average approximations will be discussed here.

It is possible that the cost of the first service is not the same as the second or third or last service, and what is more important, that this would affect the decision reached by the decision makers of an organization. At present, we use averages and ignore these possible variations.

Most of us wish precise answers and presume they can be founded upon precise data. Churchman says, "Few (I'd be inclined to say 'No') costs are known with certainty," and Moore says:

But you make money and stay in business only if you sell things for more than they cost. So you still have to do the best you can—-even if your cost measures of what it costs to make each kind of item are crude. You need cost figures so that you can get selling prices. And inside the factory you need cost figures for budgets and controls.

These comments apply not only to the valuation system suggested here, but to traditional accounting as well. Each criticism mentioned can be applied to accounting generally.

But what is the justification for using such figures? The justifi-
cation is not that it has been done successfully in accounting for years.
Some pertinent comments by Moore on this in a discussion of the use of approximations in Operations Research follows:

O. R. needs figures before it can get answers. You have to answer the following questions with answers that are numbers....
To answer every one of these questions is partly guesswork. To the extent that it is guesswork, the O. R. answer is also a guess....

...you ought not to throw out O. R. because it has to work with quantified intangibles some of the time. These "guesstimates" are the very same figures that you have to try to weigh in your mind if you discard O. R. On the other hand, having used crude figures in O. R., don't forget it. Answers should be used as approximations, not as exact answers.

Supporting this is the argument by Anshen, Holt, Modigliani, Muth and Simon:

One common misunderstanding about the language of mathematics is the belief that precise numerical expression requires equal precision in reporting "facts." Mathematics can be an effective decision-making tool even in circumstances in which the values assigned to costs represent no more than approximations.

This is the real reason for using approximations. They are the best we have. Admittedly, they are quite crude, especially in first attempts at quantifying hitherto unquantified things. Despite their crudity, they must be utilized if analytical methods are to be applied to organizational problems.

Approximation Analysis

Although using approximations has been justified, it is worthwhile to investigate the possibility of learning how to control the effects of error. This means that we should learn how error affects the analyses we use and also it may mean that we must change some of our techniques to
minimize the effects of errors. Churchman has commented upon this by saying:

The errors in cost figures (or return figures) are clearly important in decisions of management. They are important in deciding how much effort to put into the task of gathering cost information and transmitting it. The errors are also important in understanding the risks entailed in decisions. Yet company data-processing systems do not generate information about the errors of estimated costs, and in this respect, again, we lack a sound basis for verifying managerial decisions based on costs.

**Sensitivity Analysis**

Sensitivity analysis applied to service value estimates is a means of analyzing the error introduced by these estimates. The underlying idea of sensitivity analysis is not new although the term may be unfamiliar.

In economics for example it is known that there are ranges of parameter values for which systems of equations are stable and unstable. In business administration the effect of a change in expense upon a profit can be quickly calculated. These are types of sensitivity analysis.

A type of sensitivity analysis (known as parametric programming)

---


2 "SENSITIVITY ANALYSIS: PARAMETRIC LINEAR PROGRAMMING
In the formulation and solution of linear-programming problems, one essentially assumes, at least initially, that all values of the co-efficients are given and exact. Actually, such coefficients are derived from analyses of data and usually represent average values or best-estimate values. Accordingly, it is most important to analyze the sensitivity of the solution to variations in these coefficients or in the estimates of these coefficients. Stated still
similar to the type which will be used in conjunction with the general asset model is being used in operations research and specifically the area of inventory control.

Such work as this is needed to develop the relationship between measurement error and the effect this error has upon the results obtained from analytical tools, and the decision criteria based upon these results.

**Decision Criterion Redesign**

Once the possible magnitude and the effect of errors has been determined it will be necessary to look at the decision rules we have been using to determine if they should be reformulated to reduce the effect of error. Thus, the error magnitude and distribution of measurements of services as well as services valuations must be made. By combining these findings with sensitivity analysis, it will be possible to determine the degree of measurements accuracy necessary to minimize the effects of errors.

CHAPTER VI

A General Asset Model

In this chapter a general model of assets is developed. The model is mathematical and relies upon basic concepts and operations of linear algebra.

This chapter consists of the mathematical notations and formulations needed to express the model for single assets and collections of assets, for one time period and for many time periods. The resulting model and the two applications, asset valuation for one time period and the calculation of asset present value, are theoretical.

A Mathematical Formulation

Notations Used

Physical Representation of Assets

\( t \) time period \( t = 1, \ldots , g \)

\( i \) asset \( i = 1, \ldots , m \)

\( j \) service \( j = 1, \ldots , n \)

\( \alpha_{tij} \equiv \) total amount of service \( j \) asset \( i \) could furnish in time period \( t \)

\( \beta_{ti k:h} \equiv \) substitution ratio between services \( k \) and \( h \) for asset \( i \) in time period \( t \)

\( b_{tij} \equiv \) the percent of the total possible service \( j \) asset \( i \) will furnish in time period \( t \)

\( \beta_{ti} \equiv \) a vector of substitution ratios for asset \( i \) in time period \( t \)

\( \beta_{ti} = (\beta_{ti k:1}, \ldots , \beta_{ti k:h}, \ldots , \beta_{ti k:n}) \)
\[ a_{tij} = \text{amount of service } j \text{ asset } i \text{ will furnish in time period } t, \quad a_{tij} = a_{tij} \cdot b_{tij} \]

\[ B_{ti} = \text{a diagonal matrix whose diagonal elements } b_{tij} \text{ relate to only the } i^{th} \text{ asset.} \]

\[
\begin{bmatrix}
    b_{tii} & 0 \\
    \vdots & \ddots \\
    0 & \cdots & b_{tjj} \\
    0 & \cdots & b_{tii} \\
\end{bmatrix}
\]

\[ B_{t} = \text{matrix of matrices, each matrix is of the type } B_{ti}. \]

\[
\begin{bmatrix}
    B_{t1} & 0 \\
    \vdots & \ddots \\
    0 & \cdots & B_{ti} \\
    0 & \cdots & 0 \\
\end{bmatrix}
\]

\[ \alpha_{ti} = \text{vector of total possible services of each type the } i^{th} \text{ asset could furnish in time period } t. \]

\[ \alpha_{ti} = (\alpha_{t1}, \alpha_{t2}, \ldots, \alpha_{tn}) \]

\[
\begin{bmatrix}
    \alpha_{t1} \\
    \vdots \\
    \alpha_{ti} \\
    \vdots \\
    \alpha_{tm} \\
\end{bmatrix}
\]
\[ \alpha_{ti} \equiv \text{vector of services the } i^{th} \text{ asset will furnish} \]
in time period \( t \).

\[ a_{ti} = \alpha_{ti} b_{ti} = (\alpha_{til} b_{til} \ldots \alpha_{tij} b_{tij} \ldots \alpha_{tin} b_{tinn}) = (a_{til} \ldots a_{tij} \ldots a_{tin}) \]

\[ A_t \equiv \text{vector of vectors each indicating the services} \]
the \( i^{th} \) asset will furnish in time period \( t \)

\[
A_t = \begin{bmatrix}
a_{ti} \\
\vdots \\
\vdots \\
a_{tm}
\end{bmatrix}
\]

\[ A^* \equiv \text{vector whose elements are } A_t \text{ for a sequence of time} \]
periods

\[ A^* = (A_1 \ A_2 \ldots \ A_t \ldots \ A_g) \]

\[ a_{i}^* \equiv \text{vector whose elements are } a_{ti} \text{ for a sequence of time} \]
periods. (The analog of \( A^* \) for a single asset instead of a collection of assets.)

\[ a_{i}^* = (a_{1i} \ a_{2i} \ldots \ a_{ti} \ldots \ a_{gi}) \]

Valuation of Services

\( c, c \) Cost
\( R, r \) Revenue
\( t \) time period of valuation \( t = 1, \ldots, g \)
valuation system \( e = 1, \ldots, w \)
valuation subsystem \( f = 1, \ldots, m \)
service valued \( j = 1, \ldots, n \)

e_{tefj}(r_{tefj}) \equiv \text{cost (revenue) of service } j, \text{ for time period } t \text{ according to valuation system } e

c_{tef}(r_{tef}) \equiv \text{column vector of service costs (revenue) for time period } t

c^* (r^*) \equiv \text{column vector of column vectors of service costs (revenues) for a sequence of time periods}

d_t \equiv \text{the discount factor for time period } t

D \equiv \text{a diagonal matrix whose diagonal elements are the discount factors for the time periods } 1, \ldots, g

The Physical Representation

Assume an asset (A) which has the following characteristics:

(A1) It can provide only two services, \( x_1 \) and \( x_2 \);

(A2) The amount (\( \alpha_1 \) and \( \alpha_2 \)) of either service available in a time period is a function of time and nothing else, \( \alpha_1(x_1) = f(t) \) and \( \alpha_2(x_2) = f(t) \);

(A3) In any time period the asset can furnish either type of service or a combination of the two;

(A4) The substitution ratio (\( \beta \)) between any two services is constant, \( \frac{\alpha_1}{\alpha_2} = \beta_{1:2} \).
Simple Model

The time subscript is omitted until later (page 108) where multiple time periods are considered.

For a given time period \( t \) the asset \( a \) can furnish so much of the service \( x_1 (a_1) \), or so much of the service \( x_2 (a_2) \), or some linear combination of the two.

Then \( a \) can be represented as a vector

\[
a = (a_1, a_2)
\]

subject to the restrictions

\[
a_1 + \beta_{1:2} a_2 \leq a_1
\]

and

\[
\frac{1}{\beta_{1:2}} a_1 + a_2 \leq a_2
\]

because

\[
\beta_{1:2} = \frac{a_1}{a_2}
\]

Graphically this is illustrated by Figure 1.
The representation of asset use during the time period can lie anywhere within or on the boundary of the shaded area. Thus, the points in the shaded area and on its boundary represent all possible asset uses. Any point not on the line $a_1 \ a_2$ can be considered inefficient since services otherwise available are not utilized.

Many Dimensions

Assumption A1 will now be generalized from 2 to k services. The asset use representation then becomes

$$a = (a_1 \ a_2 \ldots a_j \ldots a_k)$$ (6.05)

$$a_1 + \beta_1:2 \ a_2 + \beta_1:3 \ a_3 + \ldots + \beta_1:k \ a_k \leq \alpha_1$$ (6.06)

where

$$\beta_1:2 = \frac{\alpha_1}{\alpha_2}, \ \beta_1:3 = \frac{\alpha_1}{\alpha_3}, \ldots, \ \beta_1:k$$

$$= \frac{\alpha_1}{\alpha_k}$$ (6.07)

and the $a_1 \ldots \ a_k$ represent the maximum amount of each of the k services available in the time period. This imparts generality to the model by broadening it to include assets which can perform multiple, rather than two, services.

Many Assets

The original assumption of only one asset will now be relaxed. Several assets $a_1, a_2, \ldots, a_m$ will be considered. To do this the notation must be expanded. Thus (6.01)

$$a = (a_1 \ a_2 \ldots a_j \ldots a_k)$$ (6.08)
becomes in the new notation

\[ a_1 = (a_{i1} a_{i2} \ldots a_{ij} \ldots a_{ik}) \]  
(6.09)

and a collection \((A)\) of assets is represented in the following manner:

\[
\begin{align*}
  a_1 &= (a_{i1} a_{i2} \ldots a_{ij} \ldots a_{ik}) \\
  &\vdots \\
  a_m &= (a_{m1} a_{m2} \ldots a_{mj} \ldots a_{mh})
\end{align*}
\]  
(6.10)

\[
A = \begin{bmatrix}
  a_1 \\
  \vdots \\
  a_i \\
  \vdots \\
  a_m
\end{bmatrix}
\]

The restrictive \(\alpha\)'s assume an identical notation \((a_{ij})\) and the substitution ratios become

\[ \beta_{i1:2}, \beta_{i1:3}, \ldots, \beta_{i1:k} \] for the \(i^{th}\) asset.

The vectors (6.01) and (6.02) are a representation, in physical terms, of the assets of a firm. In this representation any specific \(a_{ij}\) may be zero or positive. It will be zero if the \(i^{th}\) asset cannot perform this service or if the \(i^{th}\) asset will not be called upon to perform this service.
Extension of the Service Concept to Include Services Needed by the Asset

It may be that in addition to furnishing services an asset may require services in the sense of upkeep. The difference between a service and an upkeep service is that a service is furnished by an asset while an upkeep service is required by an asset. In addition, for the purposes of this model an upkeep service will not have any revenue attributed to it.

These could also be setup services. They are represented by the $k^{th}$ to the $n^{th}$ services. The vector (6.09) then becomes

$$a_i = (a_{11}, a_{12}, \ldots, a_{1k}, a_{1k+1}, \ldots, a_{1n})$$

(6.11)

where

$$a_{11}, \ldots, a_{1k} = 0$$

and

$$a_{1k+1}, \ldots, a_{1n} \geq 0$$

This extension applies to the remaining vectors, $a_2, \ldots, a_m$.

Multiple Variables

An original assumption (A2) stated that $\alpha$ or $\alpha_{ij}$ as it is in the new notation is a function of time $t$ alone, ($\alpha = f(t)$). This can be relaxed as follows:

$$\alpha_t = f(u_{t-1}, u_{t-2}, \ldots, u_{t-t}, m_{t-1}, \ldots, m_{t-t}, s, \ldots.)$$

(6.12)

where $u_{t-1}$ is service potentials realized in period $t-1$ and $m_{t-1}$ is maintenance in period $t-1$.
s is operator skill
  
  etc.

Relaxation of this assumption does not change the formulation
developed above. It does affect the value of the constraint \( a_{ij} \) and it
complicates the task of determining the value each constraint takes for each
time period. Since the \( a_{ij} \) are parameters of this model and not model var-
ables (unknowns), they must be computed outside of the model.

The Constant Substitution Ratio

The term substitution ratio has been used when discussing the
different services available from the same asset under one given set of
operating policies of the management of the firm. It is to be emphasized
that the term substitution ratio does not refer to a relationship between
the services of two different assets or of the same asset under different
management operating policies.

Given the above distinction an argument can be made for assuming
a constant substitution ratio between any two types of services the same
asset can produce. To produce a service an asset must perform certain
smaller tasks which we have called elemental services. A second or third
or one hundredth service can be produced in the same manner or, to put it
another way, there is a linear relationship between the number of elemental
services needed to perform one service and multiple services of the same
type.
In any time period there will be a finite number of elemental services available from an asset, given the assumptions above. In addition, there is a finite amount of each type of service possible from the asset in the same time period. Since each of these amounts of services is linearly related to the individual unit service, it follows that each type is linearly related to every other type.

The constant substitution ratio will not hold between two assets unless the assets are identical. If two assets are not identical, but produce the same services, there is no reason to suppose that there would be a constant substitution ratio between the services of the assets. This is so despite the fact that for either of the assets there is a constant substitution ratio between its services.

A simple example will illustrate this

<table>
<thead>
<tr>
<th>Services</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset A</td>
<td>8</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Services ratio</td>
<td>1/2</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>Substitution ratio of A</td>
<td>2/1</td>
<td>4/3</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1/3</td>
<td>1/4</td>
<td></td>
</tr>
</tbody>
</table>

First of all this example illustrates that the idea of a substitution ratio as it has been used here has a different meaning when applied between two assets. The substitution ratio for an asset gives an answer to the question of how much more of one service can we produce if we produce one unit less of another service (which is being produced).
A substitution ratio between assets can exist only for services identical to both assets not the assets themselves. In addition, the ratio has no use unless some economic restraint exists. In the single asset case only so much asset use (in any combination of services is possible). For two assets, there is no problem as to which service to produce inasmuch as both assets can produce the service.

A type of comparative advantage situation could exist where it is necessary to produce as much as possible of services one and two in the illustration. Then of course asset A would be used to produce service one and asset B to produce service 2. Such a conclusion does not rest solely upon a consideration of substitution ratios, but could also be reached by considering the absolute amounts involved.

Review

It is helpful to review the asset model as developed to this point.

Assumptions of the model:

A. A finite collection of assets is being described.

B. Each asset is capable of furnishing a finite quantity of each of n services.

C. For any asset there is a constant substitution ratio between any two of the services it furnishes. This results in the following matrix.
in which the $a_{ij}$ indicate the amount of the $j^{th}$ service the $i^{th}$ asset furnishes in the time period.

**Time Period**

The model above was developed for a single time period. It will be desirable, however, to use the model for more than one time period. This can be accomplished easily by expanding the index of the asset collection representation (6.13).

Thus (6.13), in the new notation, would be

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1k} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2k} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mk} & \cdots & a_{mn}
\end{bmatrix}
\]  

(6.13)

\[
A = \begin{bmatrix}
  a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  a_{m1} & \cdots & a_{mj} & \cdots & a_{mn}
\end{bmatrix}
\]  

(6.14)

\[t = 1, \ldots, g\]
Let
\[ A^* = (A_1 A_2 \ldots A_t \ldots A_1) \quad (6.15) \]
and
\[ a^*_i = (a_{1i} a_{2i} \ldots a_{ti} \ldots a_{1i}) \quad (6.16) \]

This notation allows the representation of single assets (6.16) or collections of assets (6.15) for several time periods.

Calculation of the \( a_{tij} \)'s

The relation of the \( a_{tij} \) and the \( a_{tij} \) has been discussed above. However, no scheme was presented for calculating the \( a_{tij} \) from the \( a_{tij} \). Such a scheme is desirable, although not necessary.

Since there is no direct relation between the \( a_{tij} \) and the \( a_{tij} \), the calculating procedures start from the \( a_{tij} \). If the information about assets contained in the \( a_{tij} \) is desired in the information system, it can be included, but it will be of nominal value. Eventually, since emphasis will be concentrated upon the \( a_{tij} \), the important limits embodied in the \( a_{tij} \) may be lost or ignored through lack of use. This can be remedied, however, by using the \( a_{tij} \) as a starting point and deriving the \( a_{tij} \) from them.

To do this it is necessary to translate the plans of management for use of an asset from expected services of each type to the percentage of services of each type.\(^1\) The \( b_{tij} \) represent this percentage (\( b_{tij} \leq 1 \)). Let the \( b_{tij} \) be the elements of a diagonal matrix.

---

\(^1\) This may be awkward if management thinks in terms of numbers of services rather than a percentage of the total possible service available.
Then

\[ a_{ti} B_{ti} = \begin{bmatrix} a_{til} & \cdots & a_{tij} & \cdots & a_{tin} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & b_{tij} & \cdots & b_{tin} \end{bmatrix} \]

\[ = \begin{bmatrix} a_{til} \ b_{till} & \cdots & a_{tij} \ b_{tij} & \cdots & a_{tin} \ b_{tin} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & b_{tij} & \cdots & b_{tin} \end{bmatrix} \]

\[ = \begin{bmatrix} a_{til} \ a_{tij} \ a_{tin} \\ \vdots \\ 0 \ a_{tin} \end{bmatrix} = a_{ti} \]

Note that \( a_{ti} B_{ti} = a_{ti} \)

where \( a_{ti} \) is \( 1 \times n \)

and \( B_{ti} \) is \( n \times n \)

The check on over all individual asset use still can be made as follows:

\[ a_{ti} B_{ti} \beta'_{ti} = \begin{bmatrix} a_{ti} B_{ti} \end{bmatrix} \beta'_{ti} \]

\[ a_{ti} B_{ti} \beta'_{ti} = a_{ti} \beta'_{ti} \]

Since \( a_{ti} \) is \( 1 \times n \)

and \( \beta'_{ti} \) is \( n \times 1 \) (since \( \beta_{ti} \) is \( 1 \times n \)),

their product has dimension \( 1 \times 1 \) and is a number which must be, according to the constraint, smaller than or equal to 1.
\[ a_{t_i} \beta_{t_i} \leq 1. \]
(Note: \( \beta_{t_i} = [\beta_{t_i:1}, \ldots, \beta_{t_i:i'}, \ldots, \beta_{t_i:n}] \))

**Valuation Assumptions**

**B1.** Each separate service furnished by an asset can be valued for each valuation system (e.g., historical cost, current cost, and future cost) for each time period.

**B2.** For a given asset and a given time period all units of any service furnished by the asset have identical value. These are portrayed by the column vector.¹

\[
v_e = \begin{bmatrix}
  v_{el} \\
  \vdots \\
  v_{ej} \\
  \vdots \\
  v_{en}
\end{bmatrix} \quad -\infty < v_{ej} < \infty
\]

for the time period and the valuation system \( e \).

If more than one time period is being considered, a time index must be used.

---

¹ The letters \( V \) and \( v \) are used instead of \( C \), \( R \), \( c \) and \( r \) to represent unit service value for purposes of discussion only.
The Problem of Computing and Valuing the Multiple Uses of an Asset

The Calculation of Services Combinations

The different services of a multiple-purpose asset can be obtained in many different combinations. Furthermore, the total amount of services of any one service type, and therefore combinations of these, is affected by certain variables. This means that there are a large number of possible uses for a multiple-purpose asset for each set of operating conditions (variables). The problem of measuring these uses appears formidable. However, this is a case in which appearances are deceiving. It appears that each of the large number of possible uses must be measured separately if they are to be measured at all. Actually, it is necessary only to make a few measurements and to use these to compute the measures of all uses. Thus, the original problem has been changed to a problem of less measurement plus some computation.

Assume an asset with two services \( (x_1 \text{ and } x_2) \) and two variables (A and B). Variable A is assumed to have three discrete values, and variable B, four. There are twelve combinations of the variables A and B. Each combination of variables constitutes a set of operating conditions. Corresponding to each combination of the variables is a set of possible asset uses. Each possible use is a combination of the services. It is a measure of these uses or combinations.
of services which is desired.

Given the maximum value for each type of service in a possibility set, it is possible to calculate any one or more of the uses in this possibility set. This is illustrated in Figure 2. This figure is the positive quadrant of a cartesian coordinate system. The axes are units of service 1 and service 2. The area bounded by the axes and the diagonal line connecting them represent the set of possible uses of the asset under the conditions specified \((a_j, b_k)\). Each use can be identified by the services associated with it. These are also the coordinates of the point representing the use.

The diagonal boundary of the space would be a straight line if the services are continuous and a stepped line if the services were discrete. This stepped effect occurs in the discrete case when the calculations give a fractional number of service units and because of the characteristic of discreteness the fraction must be dropped.

For any desired amount of one type of service the possible amounts of the other service are easily obtained.

Figure 3 is another graphic presentation of part of the total data on the asset. This graph illustrates the effect of changing variable A and holding variable B constant. Since A is discrete, only the points marked by o's are relevant. However, if A were continuous from \(a_1\) to \(a_3\) the whole broken line including the o-points would be relevant.

By adding to Figure 3 a third axis--the B axis--the variation caused by the values of the variable B would be presented. The new values would be represented by points in the resulting three dimensional space.
Maximum amount of service 2

for some $a_j$, $b_k$

Maximum amount of service 1

Figure 2
If the variables and the services were all continuous, a surface would represent all the amounts of service possible for all values of A and B, and the whole set of all possible values would be a solid figure. This figure would be the vertical "stacking" of all possible individual surfaces.

Figure 4 is another graphical presentation which illustrates the effect that the combinations of variables have upon the total services available.

These graphs portray the measurement of every possible asset use under each possible set of conditions. It is important to note that it was not necessary to measure the services available in each use under all conditions. It is necessary to measure certain services and to measure the effect changes in the variables had upon these services. Thus, the measurement problem is much simpler than it appears.

Periodic Valuation

For a given valuation system (o), time period and physical asset representation a, we can compute net asset value for the time period (omitting the time index).¹

¹ The index e on the valuation vectors and their elements will be omitted since only one valuation system is being used.
Figure 3

- 60,000 units of service 1
- 70,000 units of service 1
- 80,000 units of service 1
- 90,000 units of service 1
Figure 4
\[ a_i v = \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix} \]  
\[ = a_{i1} v_1 + a_{i2} v_2 + \cdots + a_{in} v_n \] (6.17)

This last quantity is a number which is the sum of the products of each service and its valuation. This final number is asset valuation for the time period.

The matrix product 
\[ A_t \cdot v_t = \begin{bmatrix} A_{t1} v_t \\ \vdots \\ A_{tn} v_t \end{bmatrix} \]  
(6.18)

where

- \( A_t \) has dimension \( m \times n \) and 
- \( v_t \) has dimension \( n \times 1 \),

is the vector of valuation for a collection of assets such as a firm. That is, it is the vector of values (6.18) when a collection of assets (a) rather than a single asset is being valued.

To determine net revenue attributable to an asset during a time period, it is of course necessary to deduct asset costs for the time period from asset revenues for the time period.
Let
\[
\begin{bmatrix}
c_1 \\
. \\
. \\
c_n
\end{bmatrix}
\]
\(c_t\)
be the cost valuation vector for some time period \(t\) and
\[
\begin{bmatrix}
r_1 \\
. \\
. \\
r_n
\end{bmatrix}
\]
\(r_t\)
be the revenue valuation vector for the same time period \(t\).

Then the net return to asset \(i\) during this time period is
\(a_{ti} (r_t - c_t)\)
and the net return to the collection of assets which is the firm is
\(A_t (r_t - c_t)\)

Acquisition Cost and Accounting Depreciation

Acquisition cost and present value to the firm, due to the vagaries of the market and the advantages and disadvantages an individual firm encounters, are not identical except by chance. After an asset has been acquired, and acquisition cost becomes a sunk cost, only current and future values are of importance to the firm. These are measures of the current and future revenues and costs the firm will receive because of this asset. Thus, to restate a well-known point as it applies to this case, acquisition cost, since it is a sunk cost, is not relevant to operational decision making.

Acquisition cost would have no place in this general asset model if the model were devoted exclusively to the data needed for internal decision
making. But the model is for all the data needs of the firm with respect to this asset and historical cost is one of these. Also necessary is the accumulated depreciation of this historical cost. Both of these must be included. The problem is how to do this.

It cannot be done by utilizing the expected services vector and an appropriate valuation vector. Acquisition cost cannot be calculated from the expected services and valuation vectors alone since it is also a function of other factors external to the asset. Depreciation is usually an arbitrary calculation and not based upon any expiration of asset services.

The end desired is two vectors whose product will give acquisition cost and accumulated depreciation. Increase the services vector \( a_i \) by two elements and relabel all elements so that these elements are \( a_k \) and \( a_{k-11} \) and these values are 1 and some number between 0 and -1 respectively. Then construct a valuation vector, the historical cost vector \( c_{tal} \) whose elements are all zero except \( v_k \) which will be equal to historical cost of the asset. The needed other valuation vector \( c_{ta2} \), the accumulated depreciation vector, has all zero elements except \( a_{k+1} \) which is also historical cost. Then:

\[
\text{historical cost} = a_i c_{tal} \\
\text{accumulated depreciation} = a_i c_{ta2} \\
\text{and} \\
\text{historical cost less depreciation} = a_i (c_{tal} + c_{ta2})
\]

The \( k^{\text{th}} \) element of the services vector is always 1, but the \( k+1^{\text{st}} \) element decreases from 0 to -1. This value is determined exogenously for each time period.
Integration with Existing Models in Economics and Production

It was argued earlier that a general asset theory should incorporate the asset definitions of the various fields of business administration and economics and reconcile their apparent differences. This will be done below.

Economics

Theoretical Economic Valuation

In economics it is stated that

\[ PV = \sum_{t=1}^{g} d_t R_t \]  

(6.19)

where

- \( PV \) = present value of an asset
- \( d_t \) = the discount rate for period \( t \)
- \( R_t \) = the net receipts of the asset for time period \( t \)

However, no statement is made concerning the computation of \( R_t \).

Actually, \( R_t \) has been computed already. It is (6.17) and (6.18). However, the problem of combining these for separate time periods and introducing a discount rate remains.

Let \( d_t \) be the discount rate for time period (\( t \)). Then let

\[
D = \begin{bmatrix}
  d_1 & \cdots & 0 & \cdots & 0 \\
  1 & \cdots & & & \\
  \vdots & \ddots & \ddots & & \\
  \vdots & & \ddots & \ddots & \\
  0 & \cdots & d_t & \cdots & 0 \\
  \vdots & \ddots & & \ddots & \\
  \vdots & & \ddots & & \ddots \\
  0 & \cdots & 0 & \cdots & d_g \\
\end{bmatrix}
\]
be the matrix of discount rates for the sequence of time periods \( l, \ldots, g \).

Let

\[
C = \begin{bmatrix}
c_1 \\
\vdots \\
c_t \\
\vdots \\
c_g
\end{bmatrix}
\]

be a column vector whose elements are the vectors of services costs for the sequence of time periods \( (1, \ldots, t, \ldots, g) \), and let

\[
R = \begin{bmatrix}
r_1 \\
\vdots \\
r_t \\
\vdots \\
r_g
\end{bmatrix}
\]

be a column vector whose elements are the vectors of services revenues for the sequence of time periods \( (1, \ldots, t, \ldots, g) \).

Then,
since $d$ is a number and not a vector, its order in each element of (6.21) can be changed without affecting the multiplication operation to be performed or the value of the element. This is illustrated in (6.22) and the following proof.

\[ a_{ti} d_t (r_t - c_t) = d_t a_{ti} (r_{ti} - c_t) \]  

Proof:

\[ a_{ti} d_t (R_t - C_t) = \sum_j A_{tij} (r_{tj} - c_{tj}) = d_t \sum_j A_{tij} (r_{tj} - c_{tj}) = d_t [a_{t1i} \ldots a_{tim}] \]

\[ = d_t a_{ti} (r_t - c_t) \]

Now (6.21) can be rewritten as
\[ a_i \cdot D \cdot (R - C) = d_1 \left[ a_{li} \cdot (r_l - c_l) \right] + d_2 \left[ a_{2i} \cdot (r_l - c_l) \right] + \ldots + d_g \left[ a_{gi} \cdot (r_g - c_g) \right] \] (6.23)

But \( a_{ti} \cdot (r_t - c_t) \) is identical to \( R_t \) of (6.19). Therefore (6.23) can be rewritten as

\[ a_i \cdot D \cdot (R - C) = d_1 \cdot R_1 + d_2 \cdot R_2 + \ldots + d_g \cdot R_g = \sum_{t=1}^{g} d_t \cdot R_t \] (6.24)

by substituting

\[ R_t = a_{ti} \cdot (r_t - c_t) \]

This demonstrates that the economic formulation of asset present value can be formulated from the model developed here.

The method of making the computation has been demonstrated and, in addition, the components of \( R_t \) can be examined if desired, since these components are generated as a step in the computational process.

The present value of a collection of assets (\( A^* \)) can be formulated as follows:

\[ A^* \cdot D \cdot (R - C) = \begin{bmatrix} a_{1} & \ldots & a_{ti} & \ldots & a_{gm} \\ \vdots & \ddots & \vdots & & \vdots \\ \vdots & & \ddots & & \vdots \\ a_{lm} & \ldots & a_{tm} & \ldots & a_{gm} \end{bmatrix} \cdot \begin{bmatrix} d_{1} & \ldots & 0 & \ldots & 0 \\ \vdots & & \ddots & & \vdots \\ \vdots & & \ddots & & \vdots \\ 0 & \ldots & d_t & \ldots & 0 \\ \vdots & & \ddots & & \vdots \end{bmatrix} \cdot \begin{bmatrix} r_{1} - c_{1} \\ \vdots \\ \vdots \\ r_{t} - c_{t} \\ \vdots \end{bmatrix} \]
The dimension of $A^*$ is $m \times g$

The dimension of $D$ is $g \times g$

The dimension of $(R - C)$ is $g \times 1$

The result of $(6.25)$ is a column vector of present values of the collection of $m$ assets. Each value is computed as in $(6.21)$, $(6.22)$, and $(6.23)$.

**Production Theory**

It has been shown already that the part of economics which deals with production -- production theory -- largely ignores assets and discusses the services assets furnish. In this context assets, if they exist at all, are considered as inexhaustible sources of one combination of services, are considered as small discrete units each furnishing one unit of one service, or are considered as furnishing only one service per asset. The general asset model contains the service aspect of assets and in this sense it is consonant with economic-production theory.

The introduction of the model as a combination of services for a time period will not negate any part of production theory although it is possible that it will complicate the theory. This complication would arise as the result of the need to handle the discreteness of combinations of services.
CHAPTER VII

An Illustration

This chapter is devoted to an illustration of the model. Since it is an illustration, a small part of a firm rather than a total firm will be used. Such a simple example has the advantage of completely illustrating the model while at the same time avoiding the complex interrelations of a total firm which would obscure the important aspects of the illustration.

Data

Production

The illustration will cover six time periods of which the fourth is the present time period. This segment of the firm consists of two assets, \( A_1 \) and \( A_2 \), and it produces two products, \( X \) and \( Y \). Asset \( A_1 \) is replaced at the end of period three.\(^1\)

The representation of the assets in terms of their possible service\(^2\) for the periods one, two, and three is given as follows:\(^3\)

\[
A_1 = \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \end{bmatrix} = \begin{bmatrix} 3 & 6 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 12 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}
\]

\( A_1 = A_2 = A_3 \)

The first four columns (1-4) represent services the assets furnish. The next two columns (5 and 6) represent labor services incorporated into the products \( X \) and \( Y \). Columns 7 and 8 represent services the assets require

---

1 The same as the beginning of period four.
2 Columns 1 through 4, only, give the possible services.
3 This is history, since we are in period four.
to remain operational. Column 9 represents a service the products may require such as storage.\footnote{This would presume that we speak of products X and Y just before they leave the firm physically rather than just after they were produced.}

The representation of the new asset $A_1$ and the old asset $A_2$ in terms of their possible services in period 4 is shown below:

$$
\begin{bmatrix}
  a_{41} \\
  a_{42}
\end{bmatrix}
= 
\begin{bmatrix}
  5 & 8 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
  0 & 0 & 4 & 12 & 0 & 0 & 0 & 3 & 0
\end{bmatrix}
$$

$$
A_4 = A_5 = A_6
$$

The columns represent the same services as before. Note, however, that the new asset contains more of each type of service than its predecessor and these services have a different substitution ratio.

\textbf{Product}

The production functions for one unit of X and Y are given below:

Product X \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 4 & 0 & 0 & 1 \end{bmatrix}

Product Y \begin{bmatrix} 0 & 2 & 0 & 3 & 2 & 1 & 0 & 0 & 1 \end{bmatrix}

Product X, for example, requires one unit of service 1 from asset $a_1$ and one unit of service from asset $a_2$. In addition it requires one unit of labor service 5, four units of labor service 6, and one unit of upkeep service 9. Product Y is similar.

Service 7 and 8 are not shown as contributors to the production of X and Y. However, they do contribute by maintaining assets $a_1$ and $a_2$ which are used to produce these products. These upkeep costs can be thrown either into an overhead calculation or the upkeep can be allocated to each product on some basis by replacing the zeros with appropriate numbers.
For example, if allocation on the basis of activity levels\(^1\) and units produced were chosen, the new vectors would be:

\[
\begin{align*}
\text{Product X} & \quad [1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 4 \quad 4/3 \quad 3/2 \quad 1] \\
\text{Product Y} & \quad [0 \quad 2 \quad 0 \quad 3 \quad 2 \quad 1 \quad 1/3 \quad 3/4^2 \quad 1]
\end{align*}
\]

According to the activity levels developed later in this chapter, one unit of product X and two of Y will be produced in each of the first three periods. Using the figures above, the calculations

\[
\begin{align*}
1 \cdot 4/3 + 2 \cdot 1/3 &= 2 \\
1 \cdot 3/2 + 2 \cdot 3/4 &= 3
\end{align*}
\]

show that all the upkeep on the two assets is allocated to the total output for the period.\(^3\)

### Activity Level

#### Discussion

The activity level of an asset is the percentage (or fraction) utilization of the asset in each type of service the asset can perform.

There are several possible methods of calculating activity levels. Each method is useful under certain circumstances, which in turn render the others unfit. These will each be discussed, briefly.

---

1. The activity level data on which this calculation is based are given later in this chapter.
2. This should be 1/8, but the aggregate activity level was less than one for this asset, so it was increased enough to allocate all upkeep expense to the products.
3. In subsequent representations of the products X and Y the allocations of upkeep are omitted as being unnecessary detail.
Linear Programming of Production

When we can sell all we can produce and if costs and revenues are linear, we can use a linear programming model to determine activity levels.

Inventory Scheduling

When the maintenance of inventory levels is crucial to the firm then the activity levels are determined by the answers from the inventory model—how much and when to produce.

Equipment Utilization

Another method of computing activity levels is by considering some optimum method of utilizing equipment.

Company Policy

Company policy may dictate a unique activity level which is the result of some non-quantifiable factors affecting the firm.

Demand

The demand function facing the firm (or at least the interpretation of this by a firm) can be the cause of an activity level solution.

Thus an activity level for a group of assets can be computed from a variety of conditions.

Use of the Activity Level Transformation

Let us assume that the activity level for each asset has been
determined as indicated by $B_{11}$ and $B_{12}$. These activity levels are represented by the first four elements of the diagonal of the $B_{ij}$ transformations.

If the total services an asset performs in any combination of services is represented as $1^2$ then the sum $3$

$$
\sum_{j=1}^{4} b_{tij} = 1 \text{ as an indication of asset utilization. If the sum is 1, then the chosen activity levels utilize the asset completely. On the other hand, if the sum is less than one, the chosen activity levels under-utilize the asset.}
$$

The Activity Transformations

The company has used the same activity level ($\overline{B}_1$) transformation for the first three periods.

$$
\overline{B}_1 = \overline{B}_2 = \overline{B}_3
$$

$$
\overline{B}_1 = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{12} \end{bmatrix}, \overline{B}_2 = \begin{bmatrix} B_{21} & 0 \\ 0 & B_{22} \end{bmatrix}, \overline{B}_3 = \begin{bmatrix} B_{31} & 0 \\ 0 & B_{32} \end{bmatrix}
$$

---

1 Our concern is not with how the activity levels are determined but with how they are used in the model.

2 That is total asset utilization can be represented as 1.

3 This sum is also known as the trace of $B_{ij}$ and can be shown as

$$
\text{tr}(B_{ij}) = \sum_{j=1}^{4} b_{tij}
$$
The diagonal elements of this transformation are

\[
B_{11} = \begin{bmatrix}
\frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
B_{12} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

For example, \( B_{11} \) indicates that for the first three periods asset \( A_1 \) will be completely used since \( \sum_{j=1}^{4} b_{11jj} = \frac{2}{3} + \frac{1}{3} + 0 + 0 = 1 \). On the other hand, asset \( a_2 \) is under-utilized since
\[ \sum_{j=1}^{4} b_{12jj} = 0 + 0 + 1/2 + 1/4 = 3/4 < 1 \]

The last three elements of the diagonals (the 1's) serve to transform unchanged the upkeep services required by the asset.

The Transformation Operation

\[ A_1 = \{a_1^{-1} \bar{B}_1\} = \left[ \begin{array}{cc} a_{11} & a_{12} \\ B_{11} & 0 \end{array} \right] \cdot \left[ \begin{array}{c} 0 \\ B_{12} \end{array} \right] = \left\{ \begin{array}{cc} a_{11} & B_{11} \\ a_{12} & B_{12} \end{array} \right\} \]

\[ a_{11} B_{11} = \]

\[
\begin{bmatrix}
2/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[ = [2, 2, 0, 0, 0, 0, 2, 0, 0] \]
The transformation for $A_2$ is similar.

The resulting transformations for each asset and the composition in services of the products $A$ and $B$ are shown below for each of the first three time periods.

$$A_1 = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

Prod A
1 0 1 0 1 4 0 0 1
Prod B
0 2 0 3 2 1 0 0 1

$$A_2 = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

Prod A
1 0 1 0 1 4 0 0 1
Prod B
0 2 0 3 2 1 0 0 1

$$A_3 = \begin{bmatrix} a_{31} \\ a_{32} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

Prod A
1 0 1 0 1 4 0 0 1
Prod B
0 2 0 3 2 1 0 0 1

New Activity Levels

At the beginning of the present period (4) a new asset was acquired to replace the old $a_{tl}$, $t<4$. This new asset is also designated $a_{tl}$, $t \geq 4$.\(^1\)

---

\(^1\) These two assets can be differentiated by the time periods they were in the firm.
Because of the additional services and the new substitution ratios of asset $a_{t1}$ the activity level was recalculated.\footnote{Again we will ignore the specific calculation process.} The new activity levels are:

\[
B_{41} = \begin{bmatrix}
2/5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
B_{42} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
With these new levels asset $a_{t1}$ is 1/10 underutilized and asset $a_{t2}$ is completely utilized. Assuming that the planning horizon for these assets and products is two periods and that the firm plans to hold the activity levels constant for the next two periods (5 and 6), the expected asset uses and the services required by Products X and Y are shown as follows:

\[
A_4 = \begin{bmatrix} a_{41} \\ a_{42} \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}
\]

Prod X
\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 4 \\
0 & 2 & 0 & 3 & 2 & 1 & 0 & 0 & 1
\end{array}
\]

Prod Y

\[
A_5 = \begin{bmatrix} a_{51} \\ a_{52} \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}
\]

Prod X
\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 4 \\
0 & 2 & 0 & 3 & 2 & 1 & 0 & 0 & 1
\end{array}
\]

Prod Y

\[
A_6 = \begin{bmatrix} a_{61} \\ a_{62} \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}
\]

Prod X
\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 4 \\
0 & 2 & 0 & 3 & 2 & 1 & 0 & 0 & 1
\end{array}
\]

Prod Y

This asset data at the beginning of period 4 is planning data for periods 4, 5, and 6. At the end of period 4 the actual data will be incorporated into the records and the expectations for periods 5 and 6 will be reevaluated.

\footnote{In periods 1, 2, and 3 expectations were realized.}
Valuation

Interestingly enough only one set of valuation data\(^1\) for each valuation system for each time period is needed. All subsystem valuation vectors are combinations of this basic data; this will be illustrated.

Data

The following matrices are the basic cost (C), revenue (R), and net revenue (R-C) data for our firm. Each column represents the data for one time period. Each element of the columns represents the value of a service (cost or revenue or net revenue). The first three columns are historical data. Column 4 is present data and columns 5 and 6 are future data.\(^2\)

\[
\begin{align*}
\mathbf{t} & = & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{align*}
\]

\[
\begin{bmatrix}
8 & 8 & 8 & 10 & 10 & 10 \\
5 & 5 & 5 & 7 & 7 & 7 \\
8 & 8 & 8 & 11 & 11 & 11 \\
3 & 3 & 3 & 6 & 6 & 6 \\
3 & 3 & 3 & 3 & 3 & 3 \\
2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\(\mathbf{R}_{t21}\)

\(^1\) Cost and revenue.

\(^2\) \(e = 2\) is the valuation system in use, and \(f = 1\) is current.
\[ t = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\
4 & 4 & 5 & 5 & 5 & 5 \\
2 & 2 & 4 & 4 & 4 & 4 \\
3 & 3 & 6 & 6 & 6 & 6 \\
1 & 1 & 2 & 2 & 2 & 3 \\
\end{bmatrix} \]

\[ C_{t21} = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1.3 & 1.3 & 1.3 & 1.3 \\
1 & 1 & 1.3 & 1.3 & 1.3 & 1.3 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix} \]

\[ t = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\
4 & 4 & 3 & 5 & 5 & 4 \\
3 & 3 & 1 & 3 & 3 & 3 \\
5 & 5 & 2 & 5 & 5 & 5 \\
2 & 2 & 1 & 4 & 4 & 3 \\
\end{bmatrix} \]

\[ (R_{t21} - C_{t21}) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1.3 & -1.3 & -1.3 \\
-1 & -1 & -1 & -1.3 & -1.3 & -1.3 \\
-1 & -1 & -1 & -1 & -1 & -1 \\
\end{bmatrix} \]
Calculations

By combining this valuation data with the services data, it is possible to calculate valuation information about the assets and the products.

Net Revenue Attributable to Assets

Asset $a_{11}$ earned in period 1 twelve units of value.

$$a_{11}(R_{121} - C_{121}) = [2, 2, 0, 0, 0, 2, 0, 0] \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = 8 + 6 - 2 = 12$$

In a similar manner the table of net values for each time period can be calculated.

\[
\begin{array}{ccccccc}
  t & = & 1 & 2 & 3 & 4 & 5 & 6 \\
  a_{t1} & = & 12 & 12 & 6 & 19.4 & 19.4 & 17.4 \\
  a_{t2} & = & 13 & 13 & 4 & 30.1 & 30.1 & 25.1 \\
  \text{Prod } X^1 & = & 13 & 13 & 9 & 14 & 14 & 13 \\
  \text{Prod } Y^1 & = & 14 & 14 & 7 & 20 & 20 & 17 \\
\end{array}
\]

This is current net revenue for each period. For those periods in which $t = 4$ these are estimates. Also those data relate to one asset $a_{t1}$ for the periods

1 If the production functions of products X and Y included the upkeep allocations calculated earlier the net values for X and Y would be:

\[
\begin{array}{ccccccc}
  t & = & 1 & 2 & 3 & 4 & 5 & 6 \\
  \text{Prod } Y & = & 11.92 & 11.92 & 5.92 & 18.59 & 18.59 & 15.59 \\
\end{array}
\]
$t = 1, 2, 3$ and another asset for $t_1$ for the periods $t = 4, 5, 6$.

**Other Calculations**

This section will be devoted to explaining the earlier statement that the data given above are sufficient for all calculations. The estimates of future services and values are subject to revision, of course.

While all the data for all valuations, in the same value unit (say dollars), are given above some of the data must be relabeled or recombined for subsystem requirements such as different current costs.

**All Valuations are Current at Some Point in Time**

All valuations are current at some point in time. The cost of a service in period one was a current cost in period one. In period four it is a past cost. A future cost is an estimate which is subject to change until an estimate or measure of the actual phenomena is made. At that time it becomes a current cost for an instant and then becomes a past cost.

**Past Costs**

In period 4, for example, the past cost of asset utilization is the cost of asset utilization in period 1, the period in which the firm acquired the assets.

In period 4 the past cost of asset utilization is a combination of period 1 and period 4 costs because a new asset with new historical costs was acquired in period 4. This is a case of recombination of data.

**Price Level Adjustments**

General—To adjust for a general price level, the particular
values involved must be multiplied by a constant, let us call it \( L \).

\[
a_{ti} \left( LR_{tef} - LC_{tef} \right) = a_{ti}L \left( R_{tef} - C_{tef} \right)
\]

\[
= L \left[ a_{ti} \left( R_{tef} - C_{tef} \right) \right]
\]

But this is the same as multiplying the final answer by the same constant. Thus, an adjustment for price level can be made by applying the appropriate constant to the period values being adjusted.

**Specific:** To adjust for a specific price level we traditionally use another constant, calculated on a different basis. The general asset model allows us to discard this approach and use a better method. In any price level adjustment we try to introduce base year values to present year assets for comparative purposes. The specific price level adjustment is used to adjust for the price changes that have occurred in the assets being adjusted. By use of the general model the asset in terms of services can be valued at a given base year set of values. For example, adjusted for the specific price level, with period 1 as a base, product A net revenue per unit in period 4 is the product A vector of services incorporated multiplied by the period 1 net revenue vector.

**Valuation Families**

Partial data such as out-of-pocket costs or sunk costs are obtained by recombining the existing data.

For example, if the first four values of the valuation vector represent sunk costs, then a calculation of out-of-pocket costs would require
only that the first four elements of the valuation vector be zero. Using period four data, this out-of-pocket cost of operating asset $A_1$ is

$$a_{41}c_{423} = [2, 4, 0, 0, 0, 2, 0, 0] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ 1.3 \\ 1.3 \\ 1 \end{bmatrix} = 2.6$$

Opportunity Cost

Opportunity cost is defined as "the measureable advantage foregone as a result of the rejection of alternative uses of resources, whether of materials, labor or facilities."\(^2\)

Initially in the general asset model a vector of possible services plus needed upkeep was developed. By combining this with a net revenue matrix for a period, opportunity cost can be computed. To illustrate this opportunity costs for asset $A_1$ in period four will be calculated.

The net revenue matrix $R_{425}$ contains the same data as the net revenue vector $(R_{421} - C_{421})$ in a slightly different arrangement.

$$(R_{421} - C_{421}) = \begin{bmatrix} 5 \\ 3 \\ 5 \\ 4 \\ 1 \\ 1 \\ -1.3 \\ -1.3 \\ -1 \end{bmatrix}$$

\(^1\) $f = 3$ is out-of-pocket cost.

\(^2\) Bedford, 1-11.
$R_{425} = \begin{bmatrix}
5 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-1.3 & -1.3 & -1.3 & -1.3 & -1.3 & -1.3 \\
-1.3 & -1.3 & -1.3 & -1.3 & -1.3 & -1.3 \\
-1 & -1 & -1 & -1 & -1 & -1
\end{bmatrix}$

In general if the dimensions of any net revenue matrix \(R_{te5}\) are \(m \times n\), \(m\) is equal to the number of services (valued) available from assets and humans and \(n\) is equal to the total number of services (valued) in the net revenue vector. ¹

The calculation of opportunity costs for asset \(A_{41}\) is

$$a_{41} R_{425} = [5, 8, 0, 0, 0, 0, 2, 0, 0]$$

$$= [22.4, 21.4, -2.6, -2.6, -2.6, -2.6]$$

¹ If \(f = 5\) is opportunity cost matrix.
This vector can be interpreted as follows:

The opportunity cost of not using asset \( a_{11} \) to furnish only service 1 is 22.4 units of value.

The opportunity cost of not using asset \( a_{11} \) to furnish only service 2 is 21.4 units of value.

The opportunity cost of not using asset \( a_{ti} \) to furnish any one of the services 3-4 is -2.6 units of value. This last statement is correct because the asset is not designed to furnish these services and there is no revenue to be obtained from the asset if it is used to produce these services. Since the asset cannot produce these services, there would be a loss--cost of upkeep.

**Rate of Return Over Cost**

Let us assume a calculation of the rate of return over cost is desired for some specific asset, say asset 1, which was acquired in period 1 and retired at the end of period 3 and which has been used at the same activity level in each period.

Then

\[
a_{11} \left( R_{112} - C_{112} \right) + a_{21} \left( R_{221} - C_{221} \right) + a_{31} \left( R_{321} - C_{321} \right)
\]

represents the return per year less historical upkeep cost. By equating this with cost, introducing an unknown interest factor, and solving for this unknown discount factor rate of return over cost can be calculated.

**Planning**

Planning for future production will not be a clear cut allocation or optimization problem. It is more likely to be an iterative process in which some assumptions are made and a related group of answers based upon these figures obtained. The assumptions are then modified as the difference
between these computed answers and the desired answers are considered (the desired answers may also be adjusted as part of this interactive process).

In general this iterative process can start with either an assumed level of assets or services which are changed for successive iterations or an assumed share of the market to determine the best path and company structure to achieve the company goals.

These processes will be similar, differing only in the assumptions, which are flexible, and the starting point. As a consequence of this similarity, the possible uses of the general asset model in either process is the same.

There are three general uses of the general asset model data. These are (1) determination of activity levels; (2) valuations of asset utilization; (3) determination of asset needs.

Since the first two general uses have been illustrated above, only the third use will be illustrated here.

Matrices of Service

Future asset requirements can be represented in two forms. One is a matrix of services. When this method is used, a matrix of expected services per period is developed. These services are shown by asset origin. That is in formulating the matrix of planned services some predetermination of the nature of the assets has been made. This predetermination extends to the minimum number of services each asset must furnish per time period. Certain assumptions about upkeep, labor as an input to the production process, and skill of operations is also made.
A comparison between the matrix of future required services and the matrix of present assets, which will be on hand, will indicate what additions, retirements and alterations will be necessary to the existing asset structure to achieve the desired asset structure. This comparison can be of a simple visual nature or of a more complex mathematical nature, such as a subtraction of identical assets.

**Illustration.** -- To illustrate this it will be assumed that the firm wishes to produce in period 6 three units of product X, one unit of product Y, and either one unit of product Z or one unit of product W. The production functions for these products in asset services are

Service (present numbering) 1 2 3 4 5 6 7 8 9 10

(Original numbering of services 1 2 3 4 5 6 7 8 9 10)

| Prod. X | [1 0 1 0 0] |
| Prod. Y | [0 2 0 3 0] |
| Prod. Z | [0 1 1 0 3] |
| Prod. W | [0 2 2 3 0] |

The asset services needed to produce the required amounts of the products are

<table>
<thead>
<tr>
<th>Service</th>
<th>1 2 3 4 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 units Prod. X</td>
<td>3 0 3 0 0</td>
</tr>
<tr>
<td>1 unit Prod. Y</td>
<td>0 2 0 3 0</td>
</tr>
<tr>
<td>1 unit Prod. Z</td>
<td>0 1 1 0 3</td>
</tr>
<tr>
<td>1 unit Prod. W</td>
<td>0 2 2 3 0</td>
</tr>
</tbody>
</table>

1Because product Z requires a new asset service which the company does not possess at present, the original services have been renumbered to include the new service.
Using this information, two asset groupings, one for each alternative, can be made. In each asset grouping the requirements for products X and Y are the same. The asset grouping reflecting the requirements for products X, Y, and Z is

\[
A^P_6 = \begin{bmatrix}
P^P_{a_{61}} \\
\vdots \\
P^P_{a_{63}} \\
\end{bmatrix} = \begin{bmatrix}
3 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & 3 & 0 \\
0 & 0 & 0 & 0 & 3 \\
\end{bmatrix}
\]

and the grouping for products X, Y, and Z is

\[
A^P_6 = \begin{bmatrix}
P^P_{a_{61}} \\
\vdots \\
P^P_{a_{62}} \\
\end{bmatrix} = \begin{bmatrix}
3 & 4 & 0 & 0 & 0 \\
0 & 0 & 5 & 6 & 0 \\
\end{bmatrix}
\]

The next step is to compare these needed services with the amounts on hand in period 6. This can be done two ways. One method is to see if the planned utilization of an asset exceeds its possible utilization. Using this method to evaluate the requirements for products X, Y, and Z we see that \(a_{61}\) would have utilization of \(3/5 + 3/8 = 24 + 15 = 39/40\) which shows planned utilization of asset \(a_{61}\) is less than the possible utilization. A similar computation for \(a_{62}\) gives \(4/4 + 3/12 = 1 1/4\) which indicates that planned utilization exceeds possible utilization for \(a_{62}\). There is no asset \(a_{63}\) so its planned utilization is an indication
that such an asset is needed.

This analysis shows that in order to produce the desired amounts of products X, Y, and Z in period 6 it will be necessary to acquire an asset or assets which will furnish, at least 3 units each of services 4 and 5 or, one unit of service \(^1\) and three units of service 5.

Alternative Computation

Construct a matrix of possible asset services

\[
A_e^6 = \begin{bmatrix}
    a_{61}^e \\
    a_{62}^e \\
    a_{63}^e
\end{bmatrix}
= \begin{bmatrix}
    3 & 3.3 & 0 & 0 & 0 \\
    0 & 0 & 4 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

This is constructed by using for each asset the amount of its first service in \(A_p^6\). This is three units of service 1 for \(a_{61}^e\) and four units of service 3 for \(a_{62}^e\). Then whatever allocation is needed to achieve 100 percent utilization is made to the remaining services.

The difference

\[
A_e^6 - A_p^6 = \begin{bmatrix}
    a_{61}^e \\
    a_{62}^e \\
    a_{63}^e
\end{bmatrix}
= \begin{bmatrix}
    0 & .3 & 0 & 0 & 0 \\
    0 & 0 & -3 & 0 & 0 \\
    0 & 0 & 0 & 0 & -3
\end{bmatrix}
\]

shows that we will have .3 units of service 2 available and that we need

\(^1\)For \(A_{62}^e\) 1 unit of service 3 equals three units of service 4.
three units each of services 4 and 5 or one unit of service 3 and three units of service 5.

A similar calculation for the asset grouping needed to produce products X, Y, and W shows that any one of the following grouping is needed:

\[
\begin{array}{ccccc}
\text{Service} & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 2/3 & 1 & 6 & 0 \\
1/2 & 0 & 1 & 6 & 0 \\
\end{array}
\]

Alternative groupings of units of services needed:

\[
\begin{array}{ccccc}
\text{Service} & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 2/3 & 3 & 0 & 0 \\
1/2 & 0 & 3 & 0 & 0 \\
0 & 2/3 & 2 & 3 & 0 \\
1/2 & 2/3 & 2 & 3 & 0 \\
\end{array}
\]

With these data and similar data for future periods management can evaluate the possible actions available to it.

Vector of Aggregate Services

The other possible general asset model representation of future asset requirements is in the form of a vector for each time period. Each element of the vector would be the sum of all the services of that type which would be needed during the time period. This vector and a similar one representing the asset structure as of the future time period if no additions or replacements were made could be compared to determine what additional assets in terms of services would be needed.
Illustration.—By using the data from the illustration in the preceding section this method will be illustrated here. To produce products $X$, $Y$, and $Z$ the aggregate services vector

$$\text{Service} = 1 \ 2 \ 3 \ 4 \ 5$$

$$a_{g1} = [3 \ 3 \ 4 \ 3 \ 3]$$

is required, and to produce products $X$, $Y$, and $W$ the aggregate services vector

$$\text{Service} = 1 \ 2 \ 3 \ 4 \ 5$$

$$a_{g2} = [3 \ 4 \ 5 \ 6 \ 0]$$

is needed.

The aggregate vector representing management plans for utilizing the assets it now has are

$$\text{Service} = 1 \ 2 \ 3 \ 4 \ 5$$

$$A_g = [2 \ 4 \ 2 \ 6 \ 0]$$

with asset $a_{61}$ $1/10$ underutilized.

Comparing these three vectors gives

$$A_g - a_{g1} = [-1 \ 1 \ -2 \ -3 \ -3]$$

$$A_g = a_{g2} = [-1 \ 0 \ -3 \ 0 \ 0]$$

which is essentially the same data calculated by the other method.

Units of Value

In this chapter the term units of value rather than dollars was deliberately used to illustrate that any unit of value which can be quantified could be used.
It appears that the revenue concept is important only in the dollar type valuation scheme. For other valuation schemes such as time or physical units only the cost concept is needed. Undoubtedly this is the result of considering these valuation units as measures of something to be minimized only and not measures of revenue.
CHAPTER VIII

Summary and Conclusions

Summary

The relatively new scientific field of business has many gaps in its theoretical structure. One of these gaps occurs in the area of asset theory. Until now, there has been no asset theory to explain the connections between the various asset descriptions in the various fields, the consumers view of assets, or the relations among the various asset values.

A theoretical framework which provides explanations and these relations has been presented in this paper. There are several benefits to be derived from any attempt, not just this one, to fill this theoretical void on assets.

First, an area of confusion is organized and the confusion is reduced. Second, the relationships between the various fields and disciplines are revealed and emphasized. Third, the theory will serve as a guide to operational models.

The confusion about assets arises because concepts and models relating to assets have been developed in response to specific needs. This has led to almost as many and as diverse partial asset models and concepts as there are needs; in a few cases the models and concepts serve multiple needs.

Assets, their transformation and their exchange are one of the major elements of business operations.

The general asset theory emphasizes the elements of assets which are most common to the different areas of business and emphasizes the interrelations, due to assets, of these areas. Essentially the general asset theory is a theoretical structure which portrays aspects of the asset structure of the real world which are well known.

-151-
In decision making models, measures of the quantities manipulated are often assumed, and primary concern for validity is focused on the model and its underlying assumptions. However, the measures of the manipulated quantities are as important to the final solution of a problem as the model employed and the assumptions underlying the model. While this assumption of measurability is valid in the extreme, since it is possible to attach a number to almost anything, its validity can be questioned when the criterion of usefulness is considered.

While much research has been done which assumes these quantities are measurable, there has been little research into making these measurements or developing models which indicate what measurements should be made. This is especially true in the area of assets. The general asset theory performs this function.

**Theory**

In the general asset theory all objects are viewed as collections of services either physical or psychic. Assets are a subset of this class of objects and have the additional characteristics of being economically feasible and owned.

A potential asset is viewed as a set of combinations of future services. The expected combination to be utilized and the valuation of the services in this combination can be combined and aggregated to give a value for this asset which can be compared to its cost.

The key elements of the theory are services and the valuation of these services. Assets are represented as sets of combinations of certain services. The specific combination of services desired from an asset is a function of the perception of the management of the firm owning the asset.

Representing assets as sets of mutually exclusive combinations of services permits insight into the reason why the same asset has different values in different uses, and different individuals regard the same asset differently. In addition,
the task of determining asset use is aided by this type of representation, since the perceived possible opportunities are also clearly shown.

Model

A linear asset model, based upon the general asset theory, has also been formulated to demonstrate a possible operational use of the theory. This model is linear because the empirical data on services and services values are mainly averages and consequently also linear. In addition the linearity of the model permits use of the quantitative techniques which have been developed in the various fields and disciplines concerned with asset utilization.

The existence of measurements of services and values of the type necessary to implement the model were documented. Service measurements are unfortunately given as a rate, but these can easily be converted into the desired form. Valuations also exist, however, they are not as thoroughly validated as services data.

Dollar cost and revenue or other quantitative valuation schemes can be used to value asset use in one or more time periods. Produced assets can also be valued by looking at the individual good as a collection of services received from other productive assets, humans and raw materials and then applying the desired valuation scheme.

More information about the range and size of errors in these measurements is desired generally and will be needed specifically to implement the general asset model.

Further Research

Theory

Other theories of assets are possible and should certainly be explored. One limitation of the present theory is the need to divide the flow of services into units which correspond to time periods. The possibility of an asset theory using flows of services is intriguing.
General Applications

One test of a theory and its model is operational use. Such research about this model is necessary for valuation and for clues about the direction additional model building should take. Similarly, research into the feasibility of using the model as a segment of an information system of the firm would be useful and necessary.

Specific Applications

Better data on assets are needed. With respect to the general asset model, this means principally the measurement and estimation of the services of assets and the development of valuation systems and the measurement of values. These measurements should include the magnitude and distribution of errors. Concurrent with this, research on the quantitative models using asset data is vital to indicate the allowable errors of measurement.

The areas of services valuation measurement and valuation schemes development seem to be areas of natural interest for accountants. They have a long history of cost finding and computing which involves many valuation schemes.

The general asset theory contains two major hypotheses which can be tested. The first hypothesis is that people explicitly or implicitly view assets as collections of services and select the asset which they believe can furnish a desired set of services economically. The choice mechanism is based upon a comparison of the cost and the expected return from the asset. An explicit mechanism for calculating the return has been formulated as part of the model.

The second hypothesis is that assets can be quantitatively represented by the general asset model. There are two aspects to this question. The first is that the model is a general operational representation of assets. Second, if this is so, it will be important to determine if such a construct has any operational validity. It is felt that the model is operationally valuable as an element of
analytical quantitative approach to assets and their use, and as an element in the data storage of the information system of the firm.

Conclusions

A general asset theory, oriented to asset use, has been developed in this paper. This theoretical structure can be utilized to explain asset recognition, acquisition and retirement; the common antecedents of the various values associated with assets; and the common elements of the various asset concepts. This theory neither contains nor implies any normative statements about the use of assets, but it does contain some normative statements about the type of asset data that should be collected and how it should be aggregated.
I. GENERAL BIBLIOGRAPHY

Books


Articles


Thirlby, G. F. 'The Subjective Theory of Value and Accounting "Cost"," *Economica* (February, 1946), pp. 32-49


Encyclopaedia Britannica. 14th Ed.

II. PARTIALLY ANNOTATED BIBLIOGRAPHY ON
MEASUREMENTS AND VALUATIONS
OF SERVICES


A section of this article (p. 555) is devoted to the life of equipment under various operating conditions.

This article contains charts of service rating factors which is a type of alternative service-potential listing.

An analytical method relying upon measurement and valuation of service potentials.


By the use of charts showing power-output per gallon of fuel, fuel consumed in different uses and fuel prices, the author illustrates that diesel engines now are competitors of gasoline engines and distillate engines in the farm tractor field.


An example of detailed equipment analysis.


A study utilizing service-potential data in rate form and the valuation of some of the service-potentials.


This study relies upon service-potentials and their valuation.


"The problem discussed is that of comparing the longevities of two or more types of equipment under operational conditions where it is not convenient to identify or keep records of individual items. Such a comparison can be made by adopting certain replacement policies and observing their effect on the composition of the population. Methods of estimating relative and absolute longevities are given for the case where k types of equipment are being compared and various logistical requirements are placed upon the replacement policies. Methods of making decisions and testing hypotheses concerning the relative and absolute longevities are also given. Replacement policies are given which, under certain conditions are optimum for purposes of studying longevity."

This study deals implicitly with the one component type of equipment. This is indicated by the example and the references, in fact the author acknowledges this in a footnote to the title of the article.

"Operating characteristics of several propeller-engine combinations for private-owner type aircraft are reviewed to compare performance, weight and cost.

It is concluded that, for airplanes having low wing loading and high power loading, controllable-pitch propellers improve take-off and climb performance. However, weight and cost considerations favor the simpler automatic 2-position propellers. Planes with high wing loading and low power loading definitely need controllable-pitch propellers."

Harris, A. P. and Parrott, K. W. "What Designers Should Know about Humidity," Electronics, 32, No. 44 (October 30, 1959)

Hercules Motors Corporation, Hercules Power Units, Canton, Ohio: January, 1948.

This catalog describes various motors. Part of each description is a suggested maximum working load for the motor under conditions of continuous and intermittent service for gasoline fuel and natural gas fuel.


This report has the following statistics:

a. The ratio of steam to diesel locomotives needed for switching operations (p. 35).
b. Passenger miles per pullman and coach car (p. 3).
c. Ton-miles per freight car for the years 1940-1950 (p. 51).
d. Average capacity of railroad owned freight cars in tons (p. 87).


This report contains the following statistics:

a. Table 29 (p. 84).
   1. Average load per car.
   2. Average miles per vehicle per annum.
   3. Tons of revenue freight.
b. Table 44 (p. 152).
   1. Average number of bus passengers in intercity revenue service.
   2. Bus miles per owned and leased vehicle per annum.
   3. Passengers per bus miles.
   4. Expenses per bus mile.

This article has service-potentials in chart form and valuations (costs and revenues) of these service-potentials.


"The author shows that the number of factors and the lack of data, as well as of a definite theory, make it impossible to do more than make a reasonable guess of life expectancy."


This article mentions life tests of diesel-engines and contains charts showing submarine-engine fuel performance and power output as a function of engine speed.


This article contains a table showing "expenses per car mile" and "Receipts per car mile."


Table I (p. 299) is a tabulation showing material, Gas, Steam and Power Requirements for Production of one ton of Steel Sections for: coke ovens, blast furnaces, gas cleaning, steel plant, cogging mill, roughing and finishing mills, finishing banks, waste heat boilers, fuel-fired boilers, power station, and others.

Kirchen, Calvin J. "Determination of the Necessary Sample Size in Life-Testing when the Length of Life is Exponentially Distributed," Industrial Mathematics, 7 (1956), 17-23.


This case study has in it service-potentials measurements made apparently by the industrial engineer Alden Elstrom and valuations of these service potentials.

On page 362 the following statement is made: "One of the first essentials for good labor estimating is a file of machine performance data."

This article has graphs depicting payload and direct operating costs versus range.


"The authors have found that of the many factors influencing ground and direct costs, these five have the greatest effect:
1. Payload
2. Ratio of payload to number of passengers
3. Number of ground stations
4. Average length of passenger journey
5. Administrative policy and efficiency

The effects of these factors on a sample airline are shown in a series of graphs.


This article contains some computed average costs and usage rates.


This article mentions statistical sampling as a method of testing reliability.


This book gives performance capabilities such as range, maximum altitude, top speed, and cruising speed, for some military aircraft.


Graphical measurements of helicopter service-potentials (payload, range, cruising speed) are contained in this article.


This article gives truck performance as a function of weight, horsepower speed, grade and some other technical factors.

This book contains detailed descriptions of industry equipment including service-potential data in rate form.


"The purpose of statistical analysis of operating experience has a four-fold purpose:
1. Product improvement
2. Manufacturing and quality improvement
3. Space parts forecasting
4. Overhaul and service life forecasting."


"Power-control requirements of agricultural implements are analyzed here from the standpoint of capacity and lifting rate."

Table 1 lists "Capacity requirements of Implements under Typical Field Working Conditions."


"A simple method of predicting truck performance in terms of grade ability at a given road speed, taking into consideration rolling resistance, air resistance, and chassis friction..."


This study presents "a method of calculating the mortality curve, the probable life curve, and the rate of renewals of particular examples and types of physical equipment. The method has been applied to 65 sets of original life data for property found in the following industries: water supply, telephone, telegraph, electric service, electric railway, steam railroad, agricultural implement, and motor vehicle.

Since the 65 mortality curves presented show very similar characteristics, they were redrawn so that the age was expressed in percent of average life, and grouped into 13 classes, from which 13 type characteristic curves were drawn. Mathematical equations were developed for some of the 13 type frequency and all of the total renewal curves. Fundamental relationships of the various characteristics of both the original 65 curves and the 13 type curves are given in tabular and graphical form.

The 13 type curves can be used as valuable aids in forecasting the probable future service lives of individual items and of groups of items of different kinds of physical equipment."

This article discusses problems of calculating man-machine output.

"Equipment and Accessories," Cargo Handling (January, 1956) 236-238.

Marine Engineering and Shipbuilding Abstracts, 22, No. 1-12 (January to December, 1959).

This is a description of a diesel-electric locomotive built to specifications for the Army.

"What the Military Expects from Future Components," Electronics, 32, No. 29 (July 17, 1959).