INTERTEMPORAL PORTFOLIO SELECTION MODEL
FOR BULK SHIPPING COMPANIES

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Acknowledgment. Thanks are due to Professor Peter Lorange for initiating some of the ideas in this paper and for his helpful comments on an earlier draft.
"...In any other industry, decisions on investments so huge would be wrapped in the full panoply of managerial science - economic forecasts, cashflow projections, feasibility studies, and the like. In shipping, however, the individual capitalist still reigns supreme ...and it is generally accepted that the head man is entitled to play his hunches."

Fortune August 1974

This paper explores a more systematic approach to making decisions on the big investments required. The paper presents a multiperiod portfolio selection model for companies operating in charter shipping markets.

Much has been written about the volatility of the world tanker (and, to a lesser extent, other charter) markets. Industry practice give owners several strategic options. Accepting a time charter guarantees the owner a definite revenue for a certain period. A single-voyage charter in the spot market, on the other hand, offers the possibility of extraordinary profits when there is a shortage of ships - at the risk of heavy losses when the delicate balance of supply and demand tips the other way. For example, during a three week period in May 1973 the spot rate for a 250,000 ton tanker rose from world scale$^1$ (WS) 92.5 to WS210; over a similar period in the fall of 1973 the rates fell from WS410 to WS57. Even shipowners who signed long term charters hoping to have a steady flow of income at low risk have been surprised by an era of inflation and currency chaos.

Faced with such uncertainty, ship owners have tended to develop distinctive strategies for operating in these markets. The strategies

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$^1$See Appendix (1) for a definition of this index. It is basically an index designed to equalize the revenue per ton per day on any voyage.
are mainly a function of their willingness to accept risk. Whatever their individual strategy, most shipowners tend to view the game as a gamble for high stakes where the key to success is to develop "a nose" for the right decisions. They seem to see little use for economic study or modelling.

Academicians on the other hand believe that much of the volatility in shipping markets can be predicted. There is a 2 to 5 year time lag between a new ship being ordered and its coming on the market. So that by studying the structure of the market and making supply demand balances it is possible to forecast rates many years into the future.\(^2\) This is probably true to a certain extent. However, unpredictable world events (such as, the closing of the Suez Canal, the Arab oil embargo, etc.) still leaves some residual uncertainty in the future rates.

To minimize the effects of this residual uncertainty it has been proposed that shipowners should view their fleet as a portfolio of assets whose risk can be minimized by diversification. Lorange and Norman\(^3\) have shown how a static single period model can be used for planning the investments in the portfolio. However, since time charters run many years and new ships take a long time to come on stream it would appear that the planning horizon of a shipping company should extend over a number of future periods. This paper presents such a multi-period portfolio selection model.

The following two sections give an overview of the shipping industry and a framework for looking at the decisions made by a shipping company.

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\(^3\)P. Lorange and V. Norman, "Portfolio Planning in Bulk Shipping Companies" in Shipping Management op. cit.
These section draw heavily on a case study⁴ of a U.S. shipping company's planning system. The next two sections present the Lorange-Norman model and some empirical evidence on the usefulness of the portfolio approach. This is followed by sections discussing the proposed model and some of the problems of implementing it. The final section summarizes the conclusions and suggests directions for future research.

The Shipping Industry

Ocean transportation is unique among transportation modes in the degree to which it has been unaffected by governmental regulation. Faced with this lack of regulation, the suppliers of ocean transportation have split into two quite disparate groups: chartered shipping and liner shipping. This paper is concerned with the management of a fleet of ships offering chartered shipping services.

A ship operating in the charter market is held available for lease or charter at any mutually agreeable terms between seller (shipowner) and buyer (shipper). The length of the charters arranged in these markets can range from a single voyage between two ports to as long as 15 years. Shippers are not generally offered a regularly scheduled service in the charter market. Also, there are no mechanisms for leasing portions of ships so that most contracts involve the entire ship. This attracts only large scale shippers (with full ship load consignments) to the charter market. Thus, most chartered shipping involves basic commodities which travel long distances (such as petroleum, ore and grain).

⁴"Gotaas-Larsen Shipping Corporation (Cases A and B)" by S. Alter and S. Anand, Copyright 1975, Sloan School of Management, MIT.
To serve various charter trades, specialized ships have been developed. Conventionally, the charter market is broken down into the following types of ships:

1. Oil tankers
2. Dry bulk carriers and combined carriers
3. Tramps or tween deckers
4. Specialized carriers: LNG, container vessels, etc.

Exhibit 1 indicates the relative size of the various shipping markets.

There are several types of contracts used in the charter market. A voyage charter is a contract to transport a commodity in a particular ship between two specified ports. All the expenses are borne by the owner. Voyage charters for commencement immediately are called sport charters while contracts commencing in the future are called forward charters. A term charter, on the other hand, refers to the rental of a ship and crew for a specified length of time varying from several months to 20 years. There are several variation on the above schemes; these are defined in Appendix 1.

The particular rate quoted in the charter market depends upon a number of variables. The important ones are:

- type of ship (i.e, tanker, bulk, etc.)
- size of ship
- length of time for which charter will run
- proposed route for the ship. This is generally important for a voyage charter or a contract of affreightment but not for a time charter.

Theoretically, any combination of different values for each of these four variables would be a different asset and a different rate would be quoted for it in the market.
<table>
<thead>
<tr>
<th></th>
<th>Liner</th>
<th>Charter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Annual revenue ($B.)</td>
<td>Tanker</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>4-20</td>
</tr>
<tr>
<td>DWT $\times 10^6$</td>
<td>70</td>
<td>186</td>
</tr>
<tr>
<td>Cargo carried (DWT $\times 10^6$/yr.)</td>
<td>500</td>
<td>1200</td>
</tr>
</tbody>
</table>

Relative Size of different charter markets.

**Exhibit 1**

Source: MIT Department of Ocean Engineering.
The shipping industry is characterized by assets which are useable over long periods of time (the average ship provides undiminished service for more than 20 years). Although ships ordered from shipyards take 2 to 5 years to build, there is a substantial market for used ships. Thus it is possible for owners to change the composition of their fleet fairly rapidly. Unlike most other fixed assets, the cost of maintaining a ship even when it is not in use are very large so that there is a high premium to planning for many years into the future.

As mentioned earlier the rates quoted in the spot market fluctuate violently. The rates quoted for longer term charters seem to be more stable but are significantly influenced by the rate in the spot market. The market prices for new and used ships are also dependant on the spot charter rates. Most studies seem to indicate that the markets for buying and selling ships and the markets for chartering their services are competitive in that most of the participants are price-takers.

Overview of Management in Shipping

Generally, shipping companies operating in the charter market are large in terms of assets and income but small in terms of number of people employed. These companies tend to have highly centralized, functionally organized management structures. The management of these companies are called upon to make 3 types of decisions. The first are those that chart the business direction of the company and may be termed as portfolio planning decisions. The second are decisions to invest in

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5 Most of this section is based upon the case studies: "Gotaas-Larsen Shipping Corporation (Case A and B)" by Steven Alter and Sudeep Anand, op. cit.
particular ships and the third are operating decisions. The following are some key aspects of these 3 types of decisions:

---Portfolio planning: The portfolio planning problem has 3 main dimensions. The first is the choice of the financial structure of the firm. This involves not just deciding on the debt/equity ratio of the firm but also the manner in which the debt will be raised. The debt could vary from being tied to a particular vessel at one extreme or it could be a corporate bond issue at the other. The second important dimension is the allocation of capital to the different types of trade (i.e. tankers, bulk, etc.). Each trade has different characteristics and different returns to the capital invested. The third important variable is the chartering policy to be followed within each trade. As mentioned earlier, the chartering policy not only impacts on the rate of return earned but is an important determinant of the business risk to which the company is exposed.

---Investment decisions: The investment decision involves the buying and selling of ships within the framework of the portfolio decisions. Since many ships are owned jointly by a number of companies, the size of ship bought need not be determined by the amount of tonnage required in a particular line of trade.
Also, since there is a well developed market for second hand ships the average age of a company's fleet is a controllable variable. Another important aspect of the investment decision is determining when new technology is ready to be exploited commercially.

-- Operating decisions: These decisions are concerned with the day-to-day activities of running the fleet and include activities such as scheduling ships, finding crews, finding new charters, etc. Some of these decisions are made on board the ship. For example, the ship's captain controls voyage costs by controlling the ship's speed while the chief engineer is responsible for maintenance of the equipment aboard the ship. However, many of the operating decisions such as setting up dry docking schedules, finding new cargoes and hiring crews are done centrally.

In this paper, the main areas of concern are the portfolio planning (and to some extent the investment) decisions. Given the competitive nature of the markets in which charter shipping companies operate and the high capital intensity, the portfolio decisions are crucial to the success (even survival) of the company. The investment decisions may be viewed as part of the implementation of the portfolio decisions. Once the amount of tonnage required in a particular trade is decided, the investment decision consists of deciding in what size and age of ships
this tonnage should be acquired. These two variables determine the cost of acquiring this tonnage. The operating decisions are by and large routine and often even uncontrollable. Another interesting characteristic of the industry is that a large portion of its cost structure can be forecast with a fair amount of accuracy. Also, that part of the revenue that is expected from term charters is predictable. So that long range planning can be a meaningful exercise.

The problem of long range portfolio planning can roughly be broken down into two segments. The first deals with environmental scanning and prediction of prices and rates of return that will prevail in the shipping markets. There have been a number of studies of the structure of shipping markets\(^6\) to aid in the making of these predictions. The other component of long range planning is the best use of these forecasts once they have been generated. This paper is mainly concerned with the latter of these two tasks and assumes that forecasts are available.

**Previous Work:**

Previous work has suggested that classical portfolio selection theory be applied to the shipping company's strategic problem. Lorange and Norman\(^7\) have looked at the problem as a "one-shot" static decision. Their presentation is in terms of net present values. To make their formulation comparable with the model presented later in this paper, we show it in terms of rates of return.

The company is viewed as having an initial endowment of wealth \(W_0\) which it must invest in assets to maximize the utility of end of period

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\(^6\)See references (2).

\(^7\)P. Lorange and V. Norman, op. cit.
wealth \( (w_1) \). An asset is a combination of trade \( i \) and charter policy \( j \). The objective function then is

\[
\text{Max } E\{u(\sum a_{ij} \hat{z}_{ij})\}
\]

where \( u = \) utility function for end of period wealth

\( a_{ij} = \) amount invested in trade \( i \), charter policy \( j \).

\( \hat{z}_{ij} = \) random variable predicted returns.

The maximization is subject to the budget constraint that the \( a_{ij} \)'s must sum to \( w_0 \). The initial wealth \( w_0 \) is the sum of the firm's equity and debt.

They then assume that the utility function \( u \) is quadratic so that only the first two moments of the \( \hat{z}_{ij} \) are important. This formulation is illustrated by solving a hypothetical example to produce the set of efficient (i.e., minimum variance for a given mean) portfolios.

Intuitively, the use of these kinds of models are premised on the fact that by investing in a portfolio of assets, the variance of the return to a portfolio reflects only that part of the variance in the return of each asset (in the portfolio) which is correlated to that of the other assets in the portfolio. The variance of the individual asset which is independent of that of the other assets can be reduced by diversification. To determine whether the class of models proposed by portfolio theory can be used profitably, the correlations between the (expected) returns to various types of assets in the charter market must be examined. If the returns to all the assets are highly correlated, there is little to be gained from using a portfolio approach.

Even if the portfolio approach is worthwhile, it is unlikely that the transaction interval, the decision interval and the company's planning
horizon are identical. For example, in the case of Gotaas-Larsen, "deals" are made in the charter market every day, the company normally updates its portfolio annually and its planning horizon is 10 years. If the portfolio is to be revised more than once during the planning horizon, it seems that a multi-period or intertemporal model is more suitable.

Further, Lorange and Norman have assumed that the capital structure decision (i.e., the debt/equity ratio) is taken independently and prior to the portfolio decisions. Since most shipping companies follow a policy of financing each ship separately, it would appear more reasonable to make the debt decision as part of the portfolio decisions. This can be done with relatively minor modifications to their formulation.

Value of the portfolio approach:

To examine the value of using a portfolio approach it is necessary to examine the correlation between the forecast returns on different kinds of assets in the charter market. Since it is difficult to determine the expectations of the people in the market we assumed that the correlations between the returns realized in the past are some indication of future expectations.

There was no readily available data of actual rates of return realized in the charter market. To get an idea of what historical rates of return have been, we had to put together data from different sources. As mentioned earlier, assets were defined in terms of trade, charter, size and route. Exhibit 2 shows that subset of the assets available that we chose to examine. This subset was chosen primarily because the data for them were easily available but they do represent a fairly important segment of the charter
<table>
<thead>
<tr>
<th>Asset No.</th>
<th>Trade</th>
<th>Size (10^3 tons)</th>
<th>Charter</th>
<th>Route</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tanker (D)*</td>
<td>22-40</td>
<td>Spot</td>
<td>P.G.-West</td>
</tr>
<tr>
<td>2</td>
<td>Tanker (D)</td>
<td>40-100</td>
<td>Spot</td>
<td>P.G.-West</td>
</tr>
<tr>
<td>3</td>
<td>Tanker (D)</td>
<td>100-175</td>
<td>Spot</td>
<td>P.G.-West</td>
</tr>
<tr>
<td>4</td>
<td>Tanker (D)</td>
<td>175+</td>
<td>Spot</td>
<td>P.G.-West</td>
</tr>
<tr>
<td>5</td>
<td>Tanker (D)</td>
<td>15-22</td>
<td>Spot</td>
<td>Carib.-U.S.A.C.</td>
</tr>
<tr>
<td>6</td>
<td>Tanker (D)</td>
<td>22-40</td>
<td>Spot</td>
<td>Carib.-U.S.A.C.</td>
</tr>
<tr>
<td>7</td>
<td>Tanker (C)*</td>
<td>15-22</td>
<td>Spot</td>
<td>P.G.-East</td>
</tr>
<tr>
<td>8</td>
<td>Tanker (C)</td>
<td>15-22</td>
<td>Spot</td>
<td>Med.-U.K.</td>
</tr>
<tr>
<td>9</td>
<td>Tanker (C)</td>
<td>15-22</td>
<td>Spot</td>
<td>Carib.-U.S.A.C.</td>
</tr>
<tr>
<td>10</td>
<td>Tanker (D)</td>
<td>30-40</td>
<td>5 year</td>
<td>NA</td>
</tr>
<tr>
<td>11</td>
<td>Tanker (D)</td>
<td>80-100</td>
<td>5 year</td>
<td>NA</td>
</tr>
<tr>
<td>12</td>
<td>Tanker (D)</td>
<td>200-250</td>
<td>5 year</td>
<td>NA</td>
</tr>
<tr>
<td>13</td>
<td>Tanker (C)</td>
<td>16-25</td>
<td>5 year</td>
<td>NA</td>
</tr>
<tr>
<td>14</td>
<td>Bulk</td>
<td>10-20</td>
<td>1 year</td>
<td>NA</td>
</tr>
<tr>
<td>15</td>
<td>Bulk</td>
<td>20-35</td>
<td>1 year</td>
<td>NA</td>
</tr>
<tr>
<td>16</td>
<td>Bulk</td>
<td>35-50</td>
<td>1 year</td>
<td>NA</td>
</tr>
<tr>
<td>17</td>
<td>Bulk</td>
<td>50+</td>
<td>1 year</td>
<td>NA</td>
</tr>
</tbody>
</table>

* D refers to dirty (i.e., for transport of crude)
C refers to clean (i.e., for transport of refined products and chemicals).

List of Assets Studied

Exhibit 2
market. The realized rates of return on these assets were computed for the 18 quarters from January 1970 through June 1974. Appendix 2 shows the sources of the data used and the methods for calculating the rates of return.

Over the 4½ year period considered, correlations between the rates of returns realized on different assets were computed. This correlation matrix is reproduced in Exhibit 3. A cursory examination of the exhibit shows that many of the correlation coefficients are substantially different from 1. This would seem to indicate that the portfolio approach does hold promise.

However, there are also a number of assets whose rates of return are almost perfectly positively correlated indicating that there is no need to differentiate amongst them in doing the portfolio calculations. Our limited sample indicates that returns to assets differentiated on the basis of route and charter policy are highly correlated. This suggests that assets should only be differentiated on the type of vessel (i.e., trade) and size in doing portfolio planning.

In looking at these results it should be borne in mind that the data analysed only covers a 4½ year period. In particular, charter markets seem to exhibit a cyclical behavior. There are long periods (3-4 years) when rates are low interspersed with short spells (3-6 months) when the rates go up dramatically. The period we have looked at covers the peak of 1970 and most of the 1973-74 peak. It is not clear what effect the choice of the time period looked at here has on the conclusions reached. To verify these conclusions, a longer time span needs to be studied and the data more systematically factor analysed.
Correlation matrix between rates of return on assets in charter shipping markets.

**Exhibit 3**
Intertemporal Portfolio Selection:

The model developed in this section is based upon the problem faced by one of the larger U.S. shipping companies. The company is a subsidiary of a holding company which manages its subsidiaries mainly through financial control and expects to receive dividends from each of the companies every year. The shipping company therefore has a dividend and a growth objective. Its problem is to decide on how to invest its resources in assets in the coming period. Since the company has decided that it will remain in the ocean charter market, the available set of assets consists of combinations of tonnage in different lines of trade and under different charter policies. The company has a 10 year planning horizon and re-evaluates its portfolio every year.

In going through the model that follows, it should become apparent that the same model with relatively minor modifications can be applied to most shipping companies.

Budget and accumulation equations: At any time t, the manager knows the price of each of the menu of assets available. He also knows the market value of the firm's (tangible) asset wealth $A(t)$. If there are $n$ assets available, define:

$$z_i(t) = \text{forecast return per unit of the } i^{th} \text{ asset.}$$

$$c_i(t) = \text{forecast net cash flow per unit of the } i^{th} \text{ asset (i.e., revenue - operating expenses).}$$

$$p_i(t) = \text{price per unit of the } i^{th} \text{ asset.}$$

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Thus \[
    z_i(t) = \frac{c_i(t) + p_i(t+1)}{p_i(t)}
\]

At time \( t \), the manager chooses the amount of dividend \( d(t) \) he wants to pay and pays it. At this point he has \( I(t) \) left to invest (i.e., \( I(t) = A(t) - d(t) \)). He selects a portfolio from amongst the \( n \) assets available and purchases \( N_i(t) \) units of the \( i^{th} \) asset at price \( p_i(t) \) per unit. The budget constraint implies:

\[
    I(t) = \sum_{i=1}^{n} N_i(t) p_i(t)
\]

This done, time rolls on; cash \( c_i(t) \) accrues to the firm from the assets it owns and a new set of prices \( p_i(t+1) \) are established for the assets in the market place. The firm now has assets worth:

\[
    A(t+1) = \sum_{i=1}^{n} N_i(t) [c_i(t) + p_i(t+1)]
\]

and the process moves on.

Let \( w_i(t) \) be the fraction of the portfolio invested in the \( i^{th} \) asset. Then,

\[
    A(t+1) = \sum_{i=1}^{n} w_i(t) \frac{I(t)}{p_i(t)} [c_i(t) + p_i(t+1)]
\]

\[
    = I(t) \sum_{i=1}^{n} w_i(t) z_i(t)
\]

\[
    = \sum_{i=1}^{n} w_i(t) z_i(t) [A(t) - d(t)]
\]

The budget constraint implies that the \( w_i(t) \)'s must sum to 1. The individual \( w_i(t) \) may take on any value. If the \( w_i(t) \) for a particular ship is less than zero, it means that the company charters in the ship and if it is greater than zero it owns the ship and charters it out.
Let one of the assets (say the $n^{th}$ one) be the riskless asset with return $R(t)$. Then, if $z(t)$ is the return on the total portfolio,

$$z(t) = \sum_{i=1}^{n} w_i(t) z_i(t)$$

$$= \sum_{i=1}^{n-1} w_i(t)[z_i(t)-R(t)] + R(t)$$

(2)

Substituting (2) into (1) and letting $m = n-1$

$$A(t+1) = z(t)[A(t)-d(t)]$$

$$= \left\{ \sum_{i=1}^{m} w_i(t)[z_i(t)-R(t)] + R(t) \right\}[A(t)-d(t)]$$

(3)

In (3) since the constraint on the sum of the $w_i(t)$ have been substituted out, the $w_i(t)$'s are unconstrained (i.e, need not sum to 1).

The objective function: Assume that the manager has a $T$ period planning horizon. His criterion function is then based upon the dividends he pays over this horizon and the growth rate of the assets of the firm. The total value of assets at the end of the planning horizon may be viewed as a proxy for the growth objective. The manager has a utility function that can be characterized as:

$$V[d(0),d(1),\ldots,d(T),A(T),T]$$

where $d(i) = \text{dividends in the } i^{th} \text{ period}$

$$A(T) = \text{value of assets at the end of } T \text{ periods.}$$

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9 In the case of the shipping company this asset is the amount of debt in the company's capital structure. Investment in the riskless asset corresponds to paying off some of the debt while a reduction in the portfolio weight invested in this asset corresponds to increasing the debt/equity ratio.
To make the problem tractable, we shall assume that the manager's utility function belongs to the "additively-separable" class so that:

\[ V[d(o), d(1), \ldots, T] = \sum_{t=0}^{T-1} U(d(t), t) + B(A(T), T) \]

We shall also make the "normal" assumptions about \( u \) and \( B \) (continuity, concavity, etc.).

The manager's objective function can then be expressed as:

\[
\text{Max } T^{-1} \sum_{t=0}^{T-1} u(d(t), t) + B(A(T), T)
\]

The \( E_0 \) is a conditional expectation at time 0. The expectation is conditional on knowledge of past prices and returns, current assets and current estimates of future returns.

**Derived utility function:** Define a function \( J \) such that

\[
J(A(t), t) = \text{Max } E\left\{ \sum_{s=t}^{T-1} u(d(s), s) + B(A(T), T) \right\}
\]

This "derived" utility function has been assumed to depend only on current asset wealth for simplicity. It could in general be a function of the investment opportunity set available to the manager. The "max" refers to choosing the decision variables (i.e., dividends \( d(s) \) and portfolio investments \( w_i(s) \)) for periods \( t, t+1, \ldots, T \) so as to maximize the expected value of the terms in curly brackets; the expectation being conditional on knowledge of events before and including \( t \).

By the definition of \( J \),

\[
J(A(T), T) = \text{Max } E_T \{ B(A(T), T) \} = B(A(T), T)
\]

and

\[
J(A(0), 0) = \text{Max } E_0 \left\{ \sum_{t=0}^{T-1} u(d(t), t) + B(A(T), T) \right\}
\]
Consider the manager's decision problem when he has one period left in his planning horizon (i.e., at T-1).

\[ J(A(T-1), T-1) = \max \ {u(d(T-1), T-1) + B(A(T), T)} \]
\[ d(T-1), w_1(T-1) \]

As of T-1, d(T-1) is not stochastic so that

\[ J(A(T-1), T-1) = \max \ {u(d(T-1), T-1) + E_{T-1}B(A(T), T)} \]
\[ d(T-1), w_1(T-1) \]

In the derivation that follows, we shall omit the time subscript when there is no scope for confusion. Also from (3),

\[ A(T) = \left[ \sum_{i=1}^{m} w_i(T-1)(z_i(T-1) - R(T-1)) + R(T-1) \right] (A(T-1) - d(T-1)) \]

To solve this problem, we differentiate (4) with respect to the decision variables (i.e., d(T-1), w_1(T-1)) and set them equal to zero.

\[ \frac{\partial}{\partial d(T-1)} : 0 = \frac{\partial u(d, T-1)}{\partial d} + E_{T-1} \left[ \frac{\partial B(A, T)}{\partial A} \right] \cdot \frac{\partial A}{\partial d} \] --------------(5)

From (3),

\[ \frac{\partial A}{\partial d} = \left[ \sum_{i=1}^{m} w_i(T-1)(z_i(T-1) - R(T-1)) + R(T-1) \right] \]

Substituting this into (5) and letting \( d^* \) and \( w_1^* \) be the optimal values for d(T-1) and \( w_1(T-1) \), we have

\[ 0 = \frac{\partial u(d^*, T-1)}{\partial d} - E_{T-1} \left[ \frac{\partial B(A, T)}{\partial A} \right] \sum_{i=1}^{m} w_i^*(z_i - R) + R \] --------------(6)

\[ \frac{\partial}{\partial w_1(T-1)} : 0 = E_{T-1} \left[ \frac{\partial B(A, T)}{\partial A} \right] \cdot \frac{\partial A}{\partial w_1} \] for each \( i = 1, 2 \cdots m \)

\[ = [A(T-1) - d(T-1)] \cdot E_{T-1} \left[ \frac{\partial B(A, T)}{\partial A} \right] \cdot (z_i - R) \] for \( i = 1, 2 \cdots m \) --------------(7)

From (7), since \( d(T-1) = A(T-1) \) would be a trivial solution, it must be that

\[ 0 = E_{T-1} \left[ \frac{\partial B(A, T)}{\partial A} \right] \cdot (z_i - R) \] for \( i = 1, 2 \cdots m \) --------------(8)
Substituting (8) into (6) we get

\[ 0 = \frac{\partial u(d^*, T-1)}{\partial d} - E_{T-1} \left[ R \frac{\partial B}{\partial A} \right] \] \hspace{1cm} (9)

(8) and (9) are the optimality conditions with 1 period to go in the planning horizon. Since by definition R is not stochastic, these conditions are

\[ R(T-1) E_{T-1} \left\{ \frac{\partial B(A(T), T)}{\partial A(T)} \right\} = E_{T-1} \left( z_{i}(T-1) \cdot \frac{\partial B(A(T), T)}{\partial A(T)} \right) \quad i = 1, \ldots, m \] \hspace{1cm} (10)

\[ \frac{\partial u(d^*, T-1)}{\partial d(T-1)} = R(T-1) E_{T-1} \left\{ \frac{\partial B(A(T), T)}{\partial A(T)} \right\} \] \hspace{1cm} (11)

Note that A(T) is a function of the optimal portfolio weights \( w^*_i(T-1) \) and (10) and (11) are m+1 equations in the m+1 unknowns \( w^*_i \) and \( d^* \). These first order conditions are implicitly used by Lorange and Norman in deriving their set of efficient portfolios.

To look at some of the characteristics of the derived utility function, substitute (3) into (4) and differentiate with respect to A(T-1)

\[ \frac{\partial J}{\partial A} = \frac{\partial u}{\partial d} \frac{\partial d^*}{\partial A} + E_{T-1} \left\{ \frac{\partial B}{\partial A} \cdot \left[ \sum_{i=1}^{m} (z_{i} \cdot R) \frac{\partial w^*_i}{\partial A} (A(T-1) - d(T-1)) \right] \right. \\
\left. + (\sum_{i=1}^{m} w^*_i(z_{i} \cdot R) + R) \left( 1 - \frac{\partial d^*}{\partial A} \right) \right\} \]

Substituting (10) and (11) into this expression

\[ \frac{\partial J(A(T-1), T-1)}{\partial A(T-1)} = \frac{\partial u(d^*(T-1), T-1)}{\partial d} \] \hspace{1cm} (12)

(12) is the envelope condition and says that optimally the manager would equate the marginal utility of dividends to the marginal utility of future benefits from current assets.
The multiperiod case: Now consider the case with 2 periods to go (i.e.,
at time (T-2)).

\[ J(A(T-2), T-2) = \max \mathbb{E}_{T-2} \{ u[d(T-2),T-2] + u(d(T-1),T-1) + B(A(T),T) \} \]

Because, relative to T-2, d(T-2) and A(T-2) are known

\[ J[A(T-2),T-2] = \max \{ u[d(T-2),T-2] + \mathbb{E}_{T-2} \{ u[d(T-1),T-1] + B(A(T),T) \} \} \tag{13} \]

Note: (1) If \( x \) is any stochastic variable

\[ \mathbb{E}_{T-2} \{ \mathbb{E}_{T-1} (x) \} = \mathbb{E}_{T-2} (x) \]

by the definition of conditional expectation.

(2) By the principle of optimality, an optimal set of decisions
(over a multiperiod horizon) has the property that, whatever the first
decision, the remaining decisions are optimal with respect to the outcome
of the first decision. Hence, if "x" is to be maximized over 2 periods:

\[ \max (x) = \max_1 \{ \max_2 (x) \} \}

\[ \text{given what happened in 1} \]

Substituting these two conditions into (13):

\[ J[A(T-2), T-2] = \max_{d(T-2), w_1(T-2)} \left\{ u[d(T-2),T-2] + \mathbb{E}_{T-2} \left\{ \max_{d(T-1), w_1(T-1)} \{ u[d(T-1),T-1] + B(A(T),T) \} \right\} \right\} \]

From the definition of the "J" function

\[ J[A(T-2), T-2] = \max_{d(T-2), w_1(T-2)} \{ u[d(T-2),T-2] + \mathbb{E}_{T-2} J[A(T-1),T-1] \} \tag{14} \]

Equation (14) is analogous to eqn. (4) with "B" replaced by "J". Hence the
optimality conditions are:
\[
\frac{\partial u}{\partial d} \bigg|_{T-2} = R E_{T-2} \left[ \frac{\partial J}{\partial A}(A(T-1), T-1) \right] = \frac{\partial J}{\partial A}[A(T-2), T-2] \quad (15)
\]

\[
E_{T-2} \left[ \frac{\partial J}{\partial A}(A(T-1), T-1) \cdot z^1 \right] = R E_{T-2} \left[ \frac{\partial J}{\partial A}(A(T-1), T-1) \right], \quad i=1, 2, \ldots, m \quad (16)
\]

Equations (14), (15) and (16) can be generalized to any period \( t \) where \( t = 0, 1, \ldots, T-1 \)

\[
J(A(t), t) = \text{Max} \left\{\left[ u(d(t), t) + E_j \{J[A(t+1), t+1]\}\right] \right\} \quad (17)
\]

where \( E_j \) is the conditional expectation, conditional on knowing \( A(t) \) and \( p_j(t) \) and past history. The optimality conditions are

\[
\frac{\partial u}{\partial d}(d^*(t), t) = R(t) E_j \left[ \frac{\partial J}{\partial A}(A(t+1), t+1) \right] = \frac{\partial J}{\partial A}[A(t), t] \quad (18)
\]

\[
E_j \left[ z^1(t) \cdot \frac{\partial J}{\partial A}(A(t+1), t+1) \right] = R(t) E_j \left[ \frac{\partial J}{\partial A}(A(t+1), t+1) \right] \quad (19)
\]

As is normally done in dynamic programming, we start at the end of the planning horizon where \( J(A(T), T) = B[A(T), T] \). This is substituted into (17)-(19) to obtain \( J[A(T-1), T-1] \) which is again substituted into (17)-(19) to obtain \( J[A(T-2), T-2], \) etc. This is continued until \( J[A(0), 0] \) is obtained.

The optimal rules will be of the form:

\[
d^*(t) = g[A(t), t]
\]

\[
w^*_1(t) = h[A(t), t]
\]

These are contingent rules and the actual values of \( d^* \) and \( w^*_1 \) will depend on the value of \( A(t) \) actually obtained.

**Implementing the model:**

The model has turned out to be a fairly complex stochastic dynamic program. In trying to implement such a model, two major problems come to
mind. The first is that even with the help of a computer, the amount of calculations involved would be infeasible. The other problem concerns determining the forms of the utility functions involved.

In most dynamic programs of this sort, the amount of computation required to find a solution depends upon the number of state variables. In the case of the model proposed, there is only one state variable (i.e. the market value of the assets) so that at least on this score the calculations would appear to be feasible. At each stage of the dynamic program, a system of $m+1$ equations has to be solved. The value of $m$ depends on the number of assets. Taking all possible combinations of charter policy, type, route and size of ship, there would appear to be a hopelessly large number of assets. However, an examination of the limited empirical data presented in a previous section shows that many of these assets have returns which are highly correlated with each other. This would suggest that these assets are essentially close substitutes and the number of different assets that need to be considered is fairly small. Carter\(^{10}\) has tried developing computer programs to select efficient portfolios in an (essentially) static environment and his results seem to indicate that the burden of computation is not excessive. Further, it may be possible to reduce the computations by constraining the policies allowable to reflect the realities in the company considered. For example, in the case of one company we looked at, since it is part of a larger group, its choices of debt/equity ratios are bounded and there is also a minimum level of dividends it is obliged to pay out.

The second problem is considerably more difficult. As is normally the case, we can use unrealistic but mathematically tractable forms or try to be more realistic. Lorange and Norman in their paper have

used a quadratic form of the utility functions and we could make a similar assumption. A similar result would obtain if the returns had distributions whose third and higher moments were negligible as compared to the first two moments. It is hypothesized that rates of returns on stocks traded in the U.S. belong to such families of compact distributions if trading intervals are small. We have not examined the data on rates of return on ships to ascertain if this is true of shipping charter markets.

Another approach might be to try and estimate the shapes of the utility functions by interview and questionnaire techniques. Preliminary work on this approach has been done by Lorange and Norman. They estimated the utility functions of 17 Scandinavian shipowners through interviews using lottery tickets under conditions of good and poor liquidity. Their experiment seemed to be reasonably successful but again it was in an essentially one period context. If it is feasible in a multiperiod context, it could be used to determine the utility functions and solve the dynamic program analytically. Alternatively, the program could be discretized and solved numerically.

A third approach might be to set up the model and have it produce portfolios with different risk return characteristics. These would then be presented to the manager who would choose the ones he wanted without explicitly stating his risk preferences.

Conclusions and Implications for Future Research:

There are two major conclusions of this paper. The first is that, within the limitations mentioned earlier, an examination of empirical

---

data indicates that it makes sense to look at a fleet of ships within a portfolio framework. Paying attention to diversification in the long range planning process is likely to be a worthwhile exercise. In applying the portfolio approach, if care is taken in defining assets, considerable economy can be obtained in the number of assets that need be considered. Specifically, it seems that assets only need to be differentiated in terms of line of trade and size.

The second major conclusion is that, given the nature of the industry, the portfolio approach should be applied in a multiperiod context. To demonstrate this approach, a general model is presented. Given that the model tries to abstract from being too specific to any one company, it is difficult to use it to produce useful "pat rules of thumb" for direct use by managers. To do this would require tailoring the model more specifically to the conditions prevailing in a particular company.

In terms of directions for future research, the first thing to be done would be to test the conclusions of this paper using a larger body of data which would allow more thorough statistical analysis. Beyond this, there are two general areas of study:

--The first is to test different methodologies for implementing the model in a real company. The various alternatives that need to be tried are outlined in the implementation section of this paper.

--The other general area of interest is determination of the effects on the structure of the charter markets if the use of the model presented here becomes widespread. Theoretical constructs to answer a similar question for US capital markets are now being developed in the Economics literature.
Appendix 1

Data and Calculations for rates of return

In general, the rate of return on an asset for a period is defined as the sum of the cash flow and the capital gain over the period divided by the original investment. Since the calculation of returns are over one month periods which are small, we have assumed that there is a negligible change in the value of the asset. In our case we have taken an asset to be 1 ton of a ship of a particular type, of a specific size, operating on a particular route and under a specific charter policy.

Data on rates of return realized in the shipping industry are not readily available. Therefore data from different sources had to be brought together. In doing this we found that the sizes of the ships for which revenue numbers were available did not correspond to the sizes for which cost data was available. We therefore used the cost data to generate regressions which were then used to obtain costs for the required ship sizes.

In the sections that follow, the data and calculations are shown. Since the revenue numbers used were the actual rates realized, we give these. Since the costs were gotten from regressions, we give the regression equations used. Since we found capital costs in many of the periods considered, we put time as an independent variable in the regression equation. For other costs we took the costs in mid-1973 and used U.S. Government (Maritime Administration) escalation factors.

Revenue: All the data was gotten from "Shipping Statistics and Economics". The rates are shown in Exhibit 1. The data for bulk carriers is in $/ton per month and was used as such. The rate for tankers is in World Scale rates.
The world scale rate is a percentage of world scale 100. World Scale 100 is the cost to a "standard" shipowner of accepting a voyage charter on a particular trade route.\(^\text{1}\) To get the actual revenue:

$$\text{Revenue/trip} = \frac{(\text{W.S.} \times \text{base rate})}{100}$$

$$\text{Trips/month} = \frac{(\text{Speed} \times \text{hrs./month})}{(\text{2 x distance})}$$

$$\text{Revenue/ton per month} = (\text{Revenue/trip}) \times (\text{trips/month})$$

**Capital cost:** The raw data was obtained from "U.S. Department of Commerce (Maritime Administration) News". The final regression used was

$$y = 0.137 + 0.0165Q + 0.568T - 0.127S^{0.5}$$

$$R^2 = 0.93$$

$$Y = \text{capital cost in $MM/MDWT}.$$  
$$Q = \text{time for cost estimate (t stat. = 6.1)}$$  
$$T = 0 \text{ for a tanker}$$  
$$\quad = 1 \text{ for a bulk carrier}$$  
$$S = \text{size in MDWT (t. stat. = -4.3)}$$

**Insurance:** The raw data was obtained from estimates made by the MIT Department of Ocean Engineering. The regression:

$$Y = -0.0021 + 0.00145S^{0.5}$$

$$R^2 = 0.90$$

$$y = \text{annual insurance cost in $MM/MDWT}$$

$$S = \text{size in MDWT (t stat. = 8.7)}$$

The escalation factor was 8%.

**Maintenance:** The data was obtained from estimates made by the MIT Department of Ocean Engineering. The regression used:

$$\log Y = -3.74 - 0.326 \log S$$

$$R^2 = 0.98$$

$$Y = \text{Annual maintenance cost in $MM/MDWT}$$

$$S = \text{Size in MDWT (t. stat. = -20.7)}$$

The escalation factor was 5%.

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\(^{\text{1}}\) For the exact calculations see, for example, "Worldwide Tanker Nominal Freight Scale", Association of Ship Brokers and Agents, Inc.
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Revenue for assets considered

Exhibit 1
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</table>

* Bulk rates in $/ton per month. Tanker rates in World Scale

Exhibit 1 (contd.)
**Crew:** The data was obtained from estimates made by the MIT Department of Ocean Engineering. The cost was a fixed number ($300,000 per year in 1973) independent of size of the vessel. A 6% escalation factor was used.

**Fuel:** The data was obtained from "Tankers and the US Energy Situation: An Economic and Environmental Analysis" in Marine Technology, Oct. 1974.

The regression equation was:

\[ Y = 22.53 - 1.085 \times S^{0.5} \]

\[ R^2 = 0.98 \]

\[ Y = \text{annual fuel cost in $/DWT} \]

\[ S = \text{size in MDWT (t-stat. = -18.2)} \]

The escalation factor used was 5%. In the Fall of 1973 an escalation of 325% was used.
Five types of charters are used commonly in shipping:

--Time Charter: This is the most common charter arrangement. The vessel is chartered for a specified period of time at a specified rate per deadweight ton of capacity per calendar month. Operating costs for crew, stores, maintenance, insurance, etc. are commonly paid by the owner, while voyage costs such as fuel, port and cargo charges, etc. are paid by the charterer.

--Bareboat Charter: This is similar to a time charter, except that the shipper charters the boat "bare", provides his own crew, and pays all operating expenses. This is generally regarded as being analogous to leasing as a financing arrangement.

--Consecutive Voyage Charter: Under this arrangement, the ship is chartered for a specified number of consecutive voyages among specified ports over a specified time period. The charterer pays an agreed freight rate per ton of cargo lifted on each voyage. All expenses, including voyage costs, are paid by the owner.

--Single Voyage Charter: This is commonly known as a "spot market" charter. The vessel is chartered to lift a cargo from one or more loading ports to one or more discharging ports at a specified freight rate. Single voyage charters are identical to consecutive voyage charters in all respects except their one-time nature.
Contracts of Affreightment: This type of contract provides for the shipment of a specified quantity and type of cargo from and to specified ports over a specified period of time at a certain freight payment per ton. It usually requires the owner to use vessels of certain minimum/maximum sizes. As in the case of single voyage charters, the owner pays all transportation costs.

A relet refers to an arrangement where a company charters in tonnage and in turn charters it out again.