WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

A LINEAR PROGRAMMING MODEL FOR SHORT TERM FINANCIAL PLANNING UNDER UNCERTAINTY

G. A. Pogue[1] and R. N. Bussard[2]

576-71

MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

A LINEAR PROGRAMMING MODEL FOR SHORT TERM FINANCIAL PLANNING UNDER UNCERTAINTY

G. A. Pogue[1] and R. N. Bussard[2]

576-71

[1]Associate Professor of Finance, Sloan School of Management, M.I.T.

[2]Director of Systems Development, Dynabank Corporation, Atlanta, Georgia

A LINEAR PROGRAMMING MODEL FOR SHORT TERM FINANCIAL PLANNING UNDER UNCERTAINTY

G. A. Pogue and R. N. Bussard

I. INTRODUCTION

Short run financial planning deals with the problem of interfacing the short run cash requirements of the firm with the time stream of cash available from the firm's long run financing strategy. This task can be divided into two parts; the raising of funds required to supplement long term funds and the provision of short run financing and investment sources to buffer timing differences between subperiods of net cash outflows and inflows.

Given the long range plans of the firm, the essence of short term planning can be described as follows. First, determine the amounts of cash to be raised from short term sources during the planning horizon. These amounts are the differences between the stream of cash requirements resulting from the operating and capital investment plans of the firm and the cash sources provided from long run financing sources. Second, find the short term financing package which will provide the required funds at the lowest possible cost or, for subperiods where cash surpluses exist, determine the short term investment package (of acceptable risk level) that will maximize the expected return on available for investment surpluses. The financing investment strategy, however, must comply with any internally or externally imposed constraints which exist.

The short term financial planning problem can be structured within a mathematical programming framework, where the ojbective is to minimize the short run financing costs subject to the above noted constraints. This fact was first recognized by Robichek, Teichroew and Jones (RTJ) in their 1965 article, "Optimal Short Term Financing Decision" [1].

RTJ developed a model for optimal short term planning under uncertainty, i.e. they assume that all of the input data requirements of the model are known with certainty at the beginning of the planning horizon. These include the forecasted cash requirements during each subperiod of the planning period.

Our model is an extension of the RTJ formulation. The extensions to their model are the following.

(i) Treatment of the major source of uncertainty in the problem. We have reformulated the approach to allow explicit consideration of the uncertainty associated with the forecasted cash requirements. This extension results in the tendency to maintain liquidity buffers to protect against the possibility that future cash requirements may be higher than currently predicted.

(ii) Generalization of the financing options to include commercial paper and multiple period investment options.

(iii) Reformulation of the model in terms of stock variables rather than flow variables. This change simplifies the structure of the model and substantially adds to the clarity of the exposition.[3]

---

[3]James C. T. Mao has presented a partial reformulation of the RTJ model in terms of stock variables. See [2].

The remainder of the paper proceeds in the following fashion. In section II of the paper we present an overview of the short-term financial planning problem, focusing on the interaction between long-run and short-run decisions. In part III we develop a sample problem which will illustrate the data requirements for the model. In section IV of the paper the model formulation is presented. For clarity of exposition we present the reformulated RTJ model as well as our own extensions. Finally, in section V we present the solution to the sample problem.

II. AN OVERVIEW OF SHORT TERM FINANCIAL PLANNING[4]

As discussed above, short term financial planning has two major components. First, the projection of corporate cash requirements during the short-term planning horizon and second, the meeting of these requirements through a combination of short term financing sources. The funds to be raised from short term sources obviously are a function of level and timing of cash inflows from long run sources, such as long term debt and equity.

---

[4]The discussion of the relationship between short and long run financial planning is based on Chapter 7 of [3].

Figure 1 shows the connection between the short-run and long-run financing strategies for a typical firm. At the current time the firm has $300,000 committed to operations (current plus fixed assets). If the operating and capital investment program proposed by management is implemented, cumulative fund requirements over time as shown by the cyclical line X in Figure 1 would result. Two features of this projected funds requirement are prominent; a long-run growth and a regular seasonal pattern.

Assume that management is considering three possible long-run financing strategies, as indicated by lines A, B and C in Figure 1.

Under long-run strategy A, long-term funds would be used to meet all cash requirements (permanent plus seasonal). Under plan C, long-term funds would provide only for permanent requirements, leaving seasonal requirements to be met entirely by short term sources. Plan B is a compromise between plans A and C.[5]

As can be readily seen from figure 1, the nature of the short term financing problem changes dramatically depending on the long term strategy selected.

Under plan A, there is no need for any short term debt. The only requirement is to optimally invest cash surpluses which exist between periods of peak seasonal needs.

Under plan C, all seasonal requirements must be funded via short term instruments.

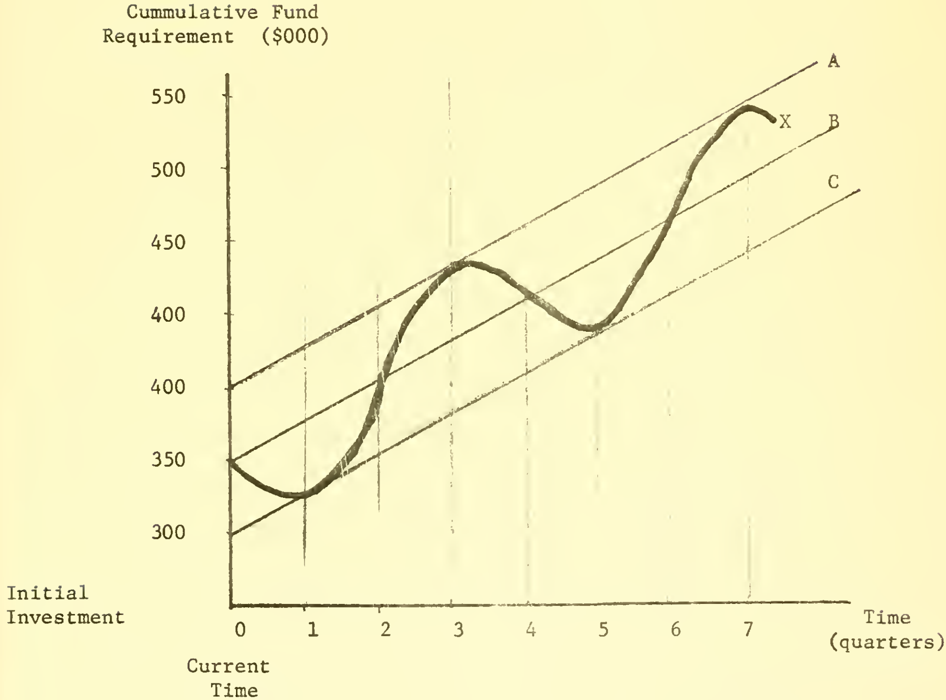
Under plan B, the firm alternates between periods of short term borrowing and lending.

---

[5] It is not inconceivable that short term sources would be used to provide a portion of permanent funds requirements. This would be equivalent to a line on figure 1 below line c.

Figure 1

RELATIONSHIP BETWEEN SHORT-RUN AND LONG-RUN
FINANCING DECISIONS



The proper mix of long and short term sources depends on the relative costs and risks of each source. Thus, as relative costs change, the best financing mix will also change. For the purposes of this paper, it will be assumed that the long term strategy has been previously selected. Given this decision, it remains to short-term planning to raise optimally any residual requirements and to invest any cash surpluses.

The Cash Budget

The next step is the projection of the cash requirements during the short term planning horizon. This involves the preparation of a pro forma cash budget. The cash budget is simply a period-by-period statement of all expected inflows and outflows of cash during the short term planning horizon.

A sample cash budget is shown in Table 1.[6] Table 1 shows by month, for twelve months, the total inflows and outflows of cash for a typical corporation with a seasonal financing problem.

Total receipts (line 6) are made up from payments for purchases,[7] and other disbursements. The latter category would include all

[6] This table is based on the sample cash budget in the RTJ article [1]. This table forms the basis of the sample problem developed in section III of our paper.

[7] The cash budget is prepared on the assumption that all payments for purchases are made on time -- i.e. all purchase discounts are taken. The decision as to whether payment should be delayed and, if so, for how long is treated explicitly as a decision variable in the model.

other cash outflows, including payment of dividends and interest and sinking fund payments on existing long-term debt.

Line 13 gives the net of total receipts less total disbursements. Line 13 is then adjusted by the projected change in the minimum operating cash balance to produce line 15 which is the month-by-month cash requirement.[8]

It should be noted that line 15 is obtained before consideration of any additional interest costs that will result from the financing of these requirements. The short-term financial planning model will adjust these cash requirements later for additional financing charges or for additional cash required to meet compensating balance requirements on bank loans.

Financing Alternatives

Now that the period-by-period short-term funds requirements have been determined, the next step is to identify the short-term sources available to meet these requirements. It is necessary to identify the costs (explicit interest costs and any implicit costs) associated with each source as well as the limitations on the amounts available from each source or a combination of sources.

The following financing options are short-term financing sources typically used by corporations to finance short-run requirements

---

[8]The minimum operating cash balances/(line 18) are assumed to be exogenously determined by the operating requirements of the firm and are not considered as a decision variable in the model.

Table 1. Sample Cash Budget

(Figures in Millions of Dollars)

Receipts	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12
Line 1: Accounts Receivable—Beginning	3.0	3.0	4.5	4.25	4.25	2.0	3.5	4.5	4.5	4.25	3.5	2.5
Line 2: + Sales on Accounts Receivable	2.0	3.0	2.5	2.5	1.75	3.0	3.0	3.0	2.25	2.25	2.0	2.5
Line 3: − Accounts Receivable—End	3.0	4.5	4.25	4.25	2.0	3.5	4.5	4.5	4.25	3.5	2.5	3.5
Line 4: Collections on Accounts Receivable	2.0	1.5	2.75	2.5	4.0	1.5	2.0	3.0	2.5	3.0	3.0	1.5
Line 5: Other Receipts (Cash sales, etc.)	1.5	1.0	1.25	2.0	1.0	4.8	1.0	1.0	1.0	1.6	1.9	1.7
Line 6: Total Receipts	3.5	2.5	4.0	4.5	5.0	6.3	3.0	4.0	3.5	4.6	4.9	3.2
Disbursements												
Line 7: Accounts Payable—Beginning	2.0	2.0	2.5	2.5	2.5	1.0	1.0	2.0	2.5	2.5	1.5	1.0
Line 8: + Purchases (Net)	2.0	2.5	2.5	2.5	1.0	1.0	2.0	2.5	2.5	1.5	1.0	2.0
Line 9: − Accounts Payable—End	2.0	2.5	2.5	2.5	1.0	1.0	2.0	2.5	2.5	1.5	1.0	2.0
Line 10: Payments for Purchases	2.0	2.0	2.5	2.5	2.5	1.0	1.0	2.0	2.5	2.5	1.5	1.0
Line 11: Other Disbursements	2.0	2.0	3.4	1.5	1.9	1.4	2.5	3.5	3.6	1.5	1.5	1.5
Line 12: Total Disbursements	4.0	4.0	5.9	4.0	4.4	2.4	3.5	5.5	6.1	4.0	3.0	2.5
Line 13: Receipts—Disbursements	(0.5)	(1.5)	(1.9)	0.5	0.6	3.9	(0.5)	(1.5)	(2.6)	0.6	1.9	0.7
Line 14: Change in Minimum Operating Cash Balance			(0.1)			0.1			(0.1)		0.1	
Line 15: Cash Requirement for Period Before Interest and Compensating Balance	(0.5)	(1.5)	(2.0)	0.5	0.6	4.0	(0.5)	(1.5)	(2.7)	0.6	2.0	0.7
Line 16: Cumulative Requirement Before Interest and Compensating Balance	(0.5)	(2.0)	(4.0)	(3.5)	(2.9)	1.1	0.6	(0.9)	(3.6)	(3.0)	(1.0)	(0.3)
Line 17: Std. Dev. of Cumulative Requirements	0.1	0.2	0.3	0.4	0.5	0.6	0.65	0.7	0.75	0.8	0.85	0.90
Line 18 Memo-Minimum Operating Cash Balance	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.2	0.1	0.1

Figures in parenthesis denote need to obtain funds and unbracketed figures denote excess cash.

(i) unsecured line of bank credit (with compensating balance requirements)

(ii) short-term loans secured to one (or more) of the current asset balances

(iii) trade credit resulting from delayment of payment for purchases for one or more periods.

(iv) intermediate term financing (term loans)

(v) short-term unsecured notes of various maturities (commercial paper)

For periods where cash surpluses exist, the firm has the option to invest in marketable securities of various maturities and expected yields.

The Objective

The goal is to find the short-run financing/investment package which meets the cash requirements at the lowest possible cost and is consistent with the constraints on various financing alternatives. An additional requirement is that the strategy contain sufficient liquidity reserves ("slack") that the firm in most cases would be able to absorb cash requirements which exceed the predicted requirements without undue strain. In other words, the solution must not be too finely "tuned." [9]

---

[9] The RTJ model produces "finely tuned", short-run strategies. This is due to the assumption that the cash requirements are known with certainty, in which case there is no requirement to maintain liquidity buffers. Our model considers the uncertainty associated with the cash requirements and produces a "looser" solution with sufficient liquidity buffers to meet up to a designated percentage of higher than predicted cash requirements. This "looser" solution would, of course, have higher expected costs than the finely tuned solution. The difference is the price of insurance against uncertainty.

III. A SAMPLE PROBLEM[10]

A sample problem is included at this point for several reasons- to give the reader a better grasp of the type of problems being considered, to make explicit the types of data required by the model and to serve as the basis for examples of model equations in section IV.

The sample problem is to develop the optimal short term financial plan, month-by-month, for a twelve-month planning horizon. The data for the example is based partly on table 1 and partly on the description of the financing options given below.

A. The Cash Budget

The month-by-month expected cash requirements are given in line 15 of the Cash Budget (table 1). These are the amounts which are projected to be required in each sub-period of the planning horizon.

The realized values however will typically differ from the fore-casted values. The uncertainty associated with the predicted requirements results from the uncertainty associated with factors affecting the component cash flows. For example, if sales were 10 percent higher than expected, the pattern of net cash requirements could differ substantially from the predicted values.

Given subjective estimates of the uncertainties associated with the cash flow components, measures of uncertainty associated with the net cash requirements can be obtained either analytically or through the use

---

[10] The sample problem is an extended version of the RTJ case study in [1].

of simulation techniques. For most practical cases, measures of the uncertainty of future cash requirements would be obtained by simulating the cash budget and observing the distribution of possible values of the requirements (lines 15 or 16).

We assume that the necessary simulations have been carried out and the estimated standard deviations for the cumulative cash requirements are given in line 17 of table 1.[11]

B. Financing Alternatives

It is assumed that there are five financing alternatives open to the financial officer. The amounts of financing available and the costs of each source are described below. In the description of the alternatives, it is assumed that all cash transactions take place at the beginning of the decision periods.

1. Pledging of accounts receivable

The firm can pledge its accounts receivable as security for a bank loan. The amount outstanding under this alternative is limited to a maximum of $3,750,000. The bank will lend up to 80% of the face value of the <u>average</u> amount of pledged receivables during any month. The cost of borrowing under this alternative is 9.0% per year (0.75% per month) on the amount of the loan outstanding during the month.

[11] The liquidity buffer constraints are formulated on a cumulative basis. Thus, the standard deviations of the cumulative as opposed to non-cumulative requirements are needed. The reasoning for this will be discussed in Section IV.

2. Unsecured Line of Credit

The company has available an unsecured line of credit from a commercial bank which permits it to borrow up to $1,500,000 at an interest rate of 8.5% per year (0.71% per month). The bank requires the company to maintain a compensating balance of not less than 20% of the amount actually borrowed. However, 100% of the firms operating cash balance (line 18, table 1) may be used to meet this requirement.

3. Stretching of Payables

The financial officer has the option to delay payment for purchases. This will result in additional financing being supplied to the firm from its vendors. If payments for purchases are delayed, the trade discounts allowed by the vendors will be lost. The discounts on the average amount to 3 1/2 percent of the face value of the bills. The firm can delay up to 80 percent of the payables first due in any month for one month, and 60 percent for two months. Stretching for the second month involves no further loss of discounts.

To the extent that stretching of payables generates vendor ill-will, the cost of this option must be adjusted upward to allow for this factor. Thus, the explicit cost (discounts lost) must be increased to measure the harm of possibly less favorable future service or credit terms from the firm's vendors.

In the sample problem the implicit cost of vendor ill-will is assumed to be 1 percent per year (0.08 percent per month) for payables stretched one month and 10 percent per year (0.83 percent per month) for payables stretched an additional month.[12]

[12] The implicit cost factors do not result in cash requirements during the planning horizon, (i.e. they do not appear in the sources and used constraints) but are included in the objective function to bias the selection of financing sources away from this option.

Thus, if a bill for $1,000 (discounted value $965) is delayed two months, the explicit financing costs would be 3.6 percent (based on the discounted value) in the first month and 0 percent in the second month. The implicit costs would be 0.083percent for the first month ($0.8) and 0.83 percent for the second month ($8.3). Thus, to obtain $965 for two months, the total cost would be $44, which is equivalent to an annualized borrowing rate of approximately 27 percent.

4. Term Loan

The firm has an offer of a term loan from its bank. The loan is available only at the beginning of the initial period, and is limited to a maximum $2,500,000. The principal of the term loan must be repaid in 10 equal installments beginning the seventh month with subsequent installments due at six month intervals. No speedup in repayment is permitted. The interest rate on the term loan is 10 percent per year (0.83 percent per month).

In addition the bank requires certain restrictions on the company's operations, such as limits on capital expenditures, financing actions, etc. These restrictions tend to reduce the flexibility of management during the period of the loan, and thus generate implicit costs in addition to the interest payment, as in the case of the stretching of payables option. Thus, it is again appropriate to assign an additional cost factor to this option in recognition of these intangible costs.

In the sample problem the term loan is assumed to carry an implicit interest cost of 2 percent per year (0.17 percent per month).

5. Commercial paper

The firm has the option to issue 30, 60 and 90 day commercial paper. The maximum amount which can be outstanding in any month (including accrued interest) in $1,000,000.

The cost schedule (including issue costs) is 9.2, 9.0 and 8.8 percent per year for 1, 2 and 3 month commercial paper respectively.

Joint Borrowing Constraint

Further constraints are placed upon the firm's ability to borrow from several sources at the same time. Specifically, the total borrowing from all sources (excluding stretching of payables) is not to exceed $4,000,000 in months 1 through 8 and $4,500,000 thereafter.

C. Short Term Investment Alternatives

The firm has the option of investing excess cash in either 30, 60 or 90 day securities. However, to insure a high degree of liquidity, at least 50 percent of the marketable securities portfolio must be invested in 30 day securities.

The yield schedule (net of transactions costs) is 7.0, 7.4 and 7.8 percent per year for 1, 2 and 3 month securities respectively.

D. Initial Conditions

Table 2 shows the initial values for the various asset and liability accounts as of the beginning of the planning horizon (before the first month's decisions).[13]

13Repayments of any initial loans from commercial paper and term loan options are assumed to be included as part of the input cash requirements (line 15 of table 1).

The "other current asset" and "other current liability" balances are assumed to remain at their initial values throughout the planning horizon.

E. Liquidity Reserve Requirements

Management wishes to insure that a sufficiently large liquidity buffer is built into the financial plan to provide a specific degree of protection against the uncertainties associated with the cash requirement projections. The liquidity reserve is to be composed of marketable securities and unused short term borrowing potential (line of credit, secured loans, stretching of accounts payable and commercial paper). The purpose of the reserve is to insure that funds will be potentially available to the firm in excess of the expected cumulative requirements (line 16 of table 1) in amounts related to the uncertainties of the projected figures.

The following requirements describe the liquidity reserve to be held in each month of the planning horizon.

(i) The minimum value must not be less than $500,000.

(ii) At least 50% of the minimum liquidity reserve must be held in marketable securities.

(iii) Management is willing to take no more than a 10% risk of not being able to meet the cash requirements in any period from planned sources (funds raised to meet expected requirements plus funds available from liquidity reserves).

Table 2

Summary of Initial Conditions

BALANCE SHEET AT BEGINNING OF PLANNING HORIZON

(thousands of dollars)

Current Assets:		
Cash		
Min. Operating Balance	100	
Additional Compensating Balance	0	
Total		100
Marketable Securities		
1-period maturity	250	
2-period maturity	0	
3-period maturity	500	
Total		750
Notes & Accounts Receivable		3000
Other Current Assets		4000
Total Current Assets		7850
Current Liabilities:		
Accounts Payable		
Not due in period 0	2000	
Payables delayed	700	
Total		2700
Commercial Paper		
1-period maturity	200	
2-period maturity	0	
3-period maturity	0	
Total		200
Lines of Credit		200
Loans secured by Accounts Receivable		350
Term Loan		
Current	0	
Non-Current	0	
Total		0
Other Current Liabilities		1750
Total Current Liabilities		5200

F. Current Ratio Constraints

To obtain the financing terms described above, management had to agree to maintain a current ratio (current assets/current liabilities) of at least 1.2 during each period of the planning horizon.

G. End Conditions

The short term planning model only looks ahead for a finite number of periods (typically 12 months) whereas the firm will continue well beyond this time. Thus, it is necessary to insure that the recommended capital structure at the end of the planning horizon be consistent with operations beyond that point.

A problem can arise if we make no attempt to interface the planning horizon of the model with the periods beyond. The model's recommendations might leave the firm in such an illiquid state that it would be unable to cope with the requirements beyond the planning horizon.[14]

There are a number of competing ways of treating this problem. One is to insure that substantial liquidity reserves exist in the terminal period. Another would impose terminal period current ratio constraints,

---

[14] These considerations are also important even in the cases where only the model's first period decisions are to be implemented since these decisions can be adversely affected by a myopic view of the life of the firm.

thus insuring expected terminal assets will exceed liabilities by a specified margin.

A third method is to constrain the terminal financial structure to a set of "reasonable" values. These values then constitute a target short term financial structure for the model. A set of "end cost" parameters are used to penalize deviations from these target values. By increasing the size of the end cost penalties, the terminal financial structure can be made to approach any designated (but feasible) values.

For the sample problem, we assume the firm has specified the target balances and deviation costs shown in table 3.[15]

Table 3

TARGET BALANCES & DEVIATION COSTS

Item	Target Balance ($000)	Cost per Dollar of Deviation (%)
Pledged Receivables Loan	350	0.6
Line of Credit	200	0.5
Stretched Payables	700	1.0
Commercial Paper	200	0.7
Term Loan	0	0.4
Marketable Securities	750	0.3*

*Credit

[15] While these three approaches provide overlapping functions, all are included in the sample problem to illustrate the most general formulation of the model in section IV.

For example, if the terminal period line of credit exceeds the $200,000 target by $1,000, the cost of the solution would be increased by $5.0. Note that in the case of securities, a credit is obtained for balances above the target value.

H. The Objective

The firm's objective is to minimize total relevant costs, that is, the sum of explicit financing costs plus any implicit costs associated with deviations from the target financial structure. The optimization is to be carried out subject to the limitations on financing sources, liquidity reserves and current ratios.

We now proceed to the formulation of the mathematical programming model which will produce the optimal short-term financial plan.

IV. MODEL FORMULATION

Before proceeding to the description of the model, we first summarize the major assumptions that have been used in the development.

(i) The only uncertain quantity is the future cash requirements. That is all of the uncertainty in the problem is confined to the forecasts of the period-by-period cash requirements. All other data (such as interest rates, receivables, payables, etc.) are assumed known at the beginning of the planning horizon.[16]

(ii) The financial manager is willing to specify the probability (e.g. 98%) with which future (actual) cash requirements are to be met from Planned sources (including

[16] This assumption allows us to deal with the major source of uncertainty in the short term planning problem within a computationally feasible framework. The other sources of uncertainty are typically of secondary importance and would substantially complicate the solution procedures.

liquidity reserves). The magnitude of this probability (combined with the measures of uncertainty associated with the cash requirements) will determine the magnitude of the liquidity buffers to be maintained.

(iii) The firm is able to identify the complete set of financing contracts available during the planning horizon. This description includes explicit financing costs, as well as estimates of any intangible costs. The cost factors are assumed to be directly proportional to the amounts borrowed.

(iv) The appropriate objective is to minimize the undiscounted sum of expected financing costs (explicit plus implicit).

(v) The model is terminated after some finite number of decision periods, but can be properly interfaced with the periods beyond the horizon such that the finite nature will have no impact on the model's decisions. All transactions are assumed to take place at the beginning of the decision periods.

While the assumptions regarding current knowledge of data from future time periods may seem restrictive, this should not be the case in practice, since typically only the first period decisions would be implemented. The model would be re-run at the end of the first decision period to prepare an updated plan using revised estimates of future cash requirements and financing costs. Thus, to the extent that forecasts change during the first period of the planning horizon, the firm has the opportunity to revise the second period decisions before they are implemented. The liquidity buffer maintained during the first period leaves the flexibility required to allow meeting of higher than expected second period cash requirements or to permit smooth modifications of the remainder of the short run plan.

Even though only the first period decisions are implemented, it is necessary, however, to prepare the plan for the full short term planning

horizon. This results from the dependence of the first period decisions upon expectations about conditions and courses of action in later periods of the planning horizon. The first period decisions represent the optimal first step in a multiple period short run plan based on the firm's best estimates of future events and the uncertainty associated with future cash requirements.

Using the assumptions discussed above, the short term financial planning problem can be formulated as a linear programming model.[17] Before proceeding with a detailed presentation, it is appropriate to first summarize the structure of the model. The reader who is not inclined to follow the complete description of the model can skip from the summary to section V without loss of continuity.

A. Summary of the Model

The objective of the model is to minimize the total financing cost, Z, over the planning horizon, where

Z	=	Sum of Explicit costs (interest plus lost Discounts)	+	Sum of Implicit Costs Over Horizon	+	End Costs

Subject to the following types of constraints.

(i) Constraints on the amount of financing available from each option; joint borrowing constraints on financing from several sources.

---

[17] The problem is actually a stochastic programming problem, which can be transformed into an ordinary linear programming formulation in several ways. The reasons for our choice of methods are discussed in note 1 of the appendix.

(ii) Mix constraints on the marketable securities portfolio

(iii) Sources and uses constraints; expected sources of funds must equal expected uses in each period.

(iv) Liquidity reserve constraints; the liquidity buffer must exceed minimum levels based on the uncertainty associated with the forecasted cash requirements.

(v) Financial Ratio Constraints; the financial plan must maintain a minimum current ratio in each period.

We now proceed with the details of the model. The constraints of the model will be described first, followed by development of the objective function.

B. Constraints on the Financing Options

The basic decision variables have the form X_{it} and L_{it} · X_{it} is the financing decision variable, representing the dollar amount of financing outstanding from source i in period t. L_{ij} is the contribution of financing option i to the liqidity reserve in period t.[18]

[18]Where the notation for any option differs from this format, it will be described in the text.

1. Pledging of Accounts Receivable

The loan available is limited to the minimum of \$3,750,000 and 80 percent of the average receivables outstanding during the decision period. Thus,

$$X_{1t} + L_{1t} \leq \min \begin{cases} \$3,750,000 \\ (0.8)\left[\dfrac{AR_t + AR_{t+1}}{2}\right] \end{cases}$$

$$t = 1, \ldots, 12$$

where AR_t is the expected receivables balance at the beginning of period t. (See line 1 of table 1.)[19]

2. Unsecured Line of Credit

The \$1,500,000 unsecured line of credit with the 20 percent compensating balance requirement generates the following constraints.

(a) Upper bound constraints

$$X_{2t} + 1.25L_{2t} \leq 1,500,000$$

$$t = 1, 2, \ldots, 12$$

where L_{2t} is the contribution of the unused line of credit to the liquidity reserve <u>net</u> of the full 20 percent compensating balance requirement.[20]

---

[19] Since the values AR_t are assumed known for each period of the planning horizon, the minimum value of \$3,750 and the average receivables can be computed (prior to execution) and inserted as the right hand side of the constraint for each month.

[20] This is the most conservative assumption, since it assumes that no cash balances will be available to support an additional line of credit. Thus, if the line of credit is increased at some future date to meet an unanticipated cash requirement, only 80 percent of the amount borrowed is assumed to be available to the firm.

The assumption is made here to simplify the exposition, and later in section V for computational efficiency. The exact formulation, which considers the cash balances available when computing L_{2t} is described in note 2 of the appenc

(b) Compensating balance requirement

$$C_t + b_t \geq 0.20(X_{2t}) ,$$

$$t = 1, 2, \ldots, 12 .$$

where

C_t = the proportion of the minimum operating cash balances available to meet the compensating balance requirement. (See line 18 of table 1.)

b_t = the additional amount borrowed at the beginning of period t to meet the compensating balance requirement.

3. Stretching of Accounts Payable

The firm can stretch up to 80% of the payables first due in any period for one month, and 60% for two months.

The constraints which limit the amount of payables which can be stretched one and two months are[21]

$$X_{3t1} + X_{3t2} + L_{3t1} = 0.8AP_t ,$$

$$X_{3t2} + L_{3t2} \leq 0.6AP_t ,$$

$t = 1, \ldots, 12.$

where

X_{3t1} = the amount of payables first due in month t which are stretched one month before payment (i.e., paid in period t + 1)

[21]Note that the first constraint is an equality, rather than an inequality as in the case of the previous financing options. This results from the fact that the stretching option does not enter into the joint borrowing constraint and thus the liquidity reserve is equal to the slack on this constraint. This point will be discussed further in connection with the joint borrowing constraint.

X_{3t2} = the amount of payables first due in period t which are delayed 2 periods before payment

AP_t = the amount of payables first due in period t (See line 10 of table 1)

L_{3t1} = the contribution to period t liquidity reserve of unused stretching potential

L_{3t2} = the contribution to period t + 1 liquidity reserve of payables delayed one period (i.e. due in t + 1) which can be further stretched into period t + 2.

For L_{3t2} to be a contribution to the liquidity reserve in period t + 1, it is necessary that some amount of payables be stretched in period t for planned payment in period t + 1 (i.e., $X_{3t1} > 0$). L_{3t2} must be no larger than the minimum of X_{3t1} and the difference between X_{3t2} and $0.6AP_t$. Thus, a further series of constraints limiting the size of L_{3t2} are required.

$$L_{3t2} \leq X_{3t1} , \qquad t = 1, \ldots, 12 .$$

The contribution of the stretching option to the period t liquidity reserve, L_{3t}, is given by[22]

$$L_{3t} = L_{3t1} + L_{3,t-1,2} , \qquad t = 1, \ldots, 12.$$

[22]For computational efficiency, only the L_{3t1} component of the stretching liquidity reserve is included in the solutions shown in section V.

The amount of accounts payable outstanding during month t (i.e. after period t decisions have been made), X_{3t}, is given by

$$X_{3t} = PN_t + X_{3t1} + X_{3t2} + X_{3,t-1,2} \qquad t = 1, \ldots, 11$$

where

PN_t = the payable outstanding but not due in period t (includes purchases made but not paid for in period t)(See line 9 of table 1).

4. Term Loan

The firm has the option of a \$2,500,000 term loan in period 1, with a 10 percent capital repayment requirement each 6 months thereafter.

The limit on the term loan is given by [23]

$$X_4 \leq 2,500,000$$

The term loan decision variable is not subscripted since it is assumed to be available only in period 1 of the planning horizon.

The amount of the term loan outstanding in period t is given by $a_{4t}X_4$, where a_{4t} is the proportion of the term loan outstanding in that period.

In the example, a_{41} through a_{46} are equal to 1.0, a_{47} through $a_{4,12}$ = 0.90, $a_{4,13}$ through $a_{4,18}$ = 0.80, etc.

5. Commercial paper

The firm has the option of issuing 30, 60 and 90 day commercial paper. The value of commercial paper outstanding in period t (including accrued interest), X_{5t}, is given by

$$X_{5t} = X_{5t1} + X_{5t2} + X_{5t3}$$

$$+ [1 + \delta_2]\, X_{5,t-1,2} + [1 + \delta_3]\, X_{5,t-1,3} + [1 + \delta_3]^2 X_{5,t-2,3}\ ,$$

t = 1, 2, . . ., 12.

---

[23]Note that the unused portion of the term loan has not been included as part of the liquidity reserve. This results from the assumption that increasing the term loan will involve renegotiation with the bank and can not be accomplished as quickly as obtaining funds from the other financing sources.

where

X_{5tj} = the amount of j period paper issued in period t, (j=1, 2, 3).

δ_j = the monthly interest rate for j period commercial paper (including amortize issue costs)

The constraints on the total amount of paper which can be issued are given by

$$X_{5t} + L_{5t} \leq 1{,}000{,}000 \ , \quad t = 1, \ldots, 12.$$

6. **Total Borrowing Constraint**

The maximum borrowing from all sources (excluding stretching of payables) is not to exceed $4,000,000 in months 1 through 7 and $4,500,000 thereafter. These restrictions are given by

$$X_{1t} + X_{2t} + a_{4t}X_4 + X_{5t} + L_{1t} + (1.25)L_{2t} + L_{5t} \leq 4{,}000{,}000,$$

$$t = 1, \ldots, 7.$$

$$X_{1t} + X_{2t} + a_{4t}X_4 + X_{5t} + L_{1t} + (1.25)L_{2t} + L_{5t} \leq 4{,}500{,}000,$$

$$t = 8, \ldots, 12.$$

Note that the sum of the liquidity reserve contributions of the pledging, line of credit and commercial paper options is constrained to be less than or equal to the unused total borrowing capacity. Thus, if the sum of the borrowings from these options is equal to the limit, they make no joint or individual contribution to the liquidity reserve, i.e.

$L_{1t} = L_{2t} = L_{5t} = 0$[24]

[24] It is for this reason that the upper bound constraints on the individual financing options were stated as inequalities rather than equalities. The limit on line of credit in a given period, for example, may not be reached, yet the option will have a zero L_{2t} if the joint upper bound constraint is binding in that period.

C. Marketable Securities Constraints

The firm has the option of investing in 30, 60 and 90 day securities

The value of the marketable securities portfolio held during month t (including accrued interest, net of transactions costs), X_{6t}, is given by

$$X_{6t} = X_{6t1} + X_{6t2} + X_{6t3}$$

$$+ [1 + \rho_2]\, X_{6,t-1,2} + [1 + \rho_3]\, X_{6,t-1,3} + [1 + \rho_3]^2 X_{6,t-2,3} ,$$

$$t = 1, \ldots, 12 .$$

where

X_{6tj} = the amount of j period securities purchased in period t, (j=1,2,3)

ρ_j = the monthly interest rate for j period marketable securities (net of amortized transactions costs).

The amounts of 1, 2 and 3 month securities outstanding (X^j_{6t}, j = 1, 2, 3) are given by

$$X^1_{6t} = X_{6t1} + (1 + \rho_2)X_{6,t-1,2} + (1 + \rho_3)^2 X_{6,t-2,3},$$

$$X^2_{6t} = X_{6t2} + (1 + \rho_3)X_{6,t-1,3},$$

$$X^3_{6t} = X_{6t3},$$

$$t = 1, 2, \ldots, 12.$$

To insure that the portfolio remained highly liquid, it was required that at least 50 percent of the portfolio be invested in the shortest maturity securities. These portfolio mix constraints are given by

$$X^1_{6t} \geq X_{6t} \, ,$$

$$t = 1, 2, \ldots, 12.$$

D. Sources and Uses Constraints

In each period of the model, the expected sources of funds must equal the expected uses.

The expected cash requirements for each period, before any adjustment for the costs of additional financing, are given in line 15 of table 1. These requirements must be adjusted for additional interest expense, lost discounts, interest earned on the marketable securities portfolio plus any additional cash required for compensating balances.

These adjusted financing requirements must then be met via net new borrowing (new borrowing less repayments) plus net security sales in each period.

Increases in the amount of any short term liability outstanding or decreases in the value of securities held represent sources of funds Conversely, decreases in short term liabilities or increases in security balances represent uses of funds. For the problem to have a feasible solution, the amounts of short term securities and liabilities outstanding must change in each period so that the net change is identically equal to the adjusted requirements. Therefore, for each period (t = 1, . . ., 12) we require

Expected Cash Requirements + Financing Costs + Additional Cash required for Compensating Balances = Net New Borrowing + Net Sales of Securities

(a) The expected financing costs in period t are the (cash) payments made for all financing obtained in previous periods which matures in period t.

(i) Interest on receivables loan paid in t = $r_1 X_{1,t-1}$

where r_1 = the monthly interest rate

(ii) Interest on line of credit loan paid in t = $r_2 X_{2,t-1}$

where r_2 = the monthly interest rate

(iii) Discounts lost on payables stretched one and two periods ago which are paid in t = $r_3 [X_{3,t-1,1} + X_{3,t-2,2}]$

where r_3 are the discounts lost on stretched payables, expressed as a proportion of discounted values

(iv) Interest on term loan paid in period t = $r_4 a_{4,t-1} X_4$

where r_4 = the monthly interest rate

(v) Interest paid on commercial paper issued 1, 2 and 3 periods ago which matures in period t $= \delta_1 X_{5,t-1,1} + [(1 + \delta_2)^2 - 1.0] X_{5,t-2,2}$ $+ [(1 + \delta_3)^3 - 1.0]\ X_{5,t-3,3}$

(vi) Interest earned on securities purchases 1, 2, and 3 periods ago which matures in period t = $\rho_1 X_{6,t-1,1} + [(1 + \rho_2)^2 - 1.0]\ X_{6,t-2,2}$ $+ [(1 + \rho_3)^3 - 1.0]\ X_{6,t-3,3}$

The net financing charges in period t are the sum of (i) through (v) less (vi).

(b) The change in compensating balance requirement, which can not be met with existing cash balances, is given by

$$b_t - b_{t-1}$$

(c) Net new borrowing in period t is given by

$(X_{1t} - X_{1,t-1})$ increase in receivables loan

$+ (X_{2t} - X_{2,t-1})$ increase in line of credit

$+ (X_{3t1} + X_{3t2} - X_{3,t-1,1} - X_{3,t-2,2})$ increase in stretched payables

$+ (a_{4t} - a_{4,t-1})X_4$ increase in term loan

$+ (X_{5t1} + X_{5t2} + X_{5t3} - X_{5,t-1,1} - X_{5,t-2,2} - X_{5,t-3,3})$

increase in commercial paper

(d) Net security sales are given by

$$(X_{6,t-1,1} + X_{6,t-2,2} + X_{6,t-3,3} - X_{6,t,1} - X_{6,t,2} - X_{6,t,3})$$

E. Liquidity Reserve Constraints

The firm's liquidity reserve is composed of the firm's marketable securities balances plus the unused short-term borrowing capacity. The unused capacity is included only if it can be assumed to be immediately available without further negotiations. In the sample problem, this is assumed to include all of the options with the exception of the term loan.

The liquidity reserve in period t, L_t, is given by

$$L_t = L_{1t} + L_{2t} + L_{3t} + L_{5t} + X_{6t}$$

Minimum reserve constraints

The firm has specified that the minimum liquidity reserve must meet two requirements (a) exceed \$500,000 and, (b) be large enough so that the chance of not being able to meet the actual cash requirement in any period from normal sources (i.e. planned sources plus funds from the liquidity reserve) does not exceed 10%.

The first part of the requirement is given by

$$L_t \geq \$500{,}000, \; t = 1, \ldots, 12 .$$

The second part results in a constraint of the form

$$L_t \geq (1.5)\, \sigma(\tilde{C}R_t), \; t = 1, \ldots, 12 .$$

where $\sigma(CR_t)$ is the standard deviation of the cummulative funds required up to and including period t (see line 17 of table 1). The 1.5 factor insures that the probability the planned sources will exceed adjusted requirements exceeds 90 percent.[25]

Composition of the liquidity reserve.

The requirement that at least 50% of the liquidity reserve be held in marketable securities is given by

$$X_{6t} \geq \text{maximum} \begin{cases} 250{,}000 \\ 0.75\sigma(\tilde{C}R_t) \end{cases} ,$$

$$t = 1, \ldots, 12$$

[25] Justification for this statement is contained in note 3 of the appendix.

F. Financial Ratio Constraints

It is required that the adopted financial plan maintain an expected current ratio during the planning horizon of 1.2. This requirement is specified as follows

$$[C_t + X_{6t} + AR_t + \begin{matrix}\text{other}\\ \text{current}\\ \text{assets}\end{matrix}] \geq 1.2[X_{1t} + X_{2t} + X_{3t} + \begin{matrix}\text{current}\\ \text{term}\\ \text{loan}\end{matrix} + X_{5t} + \begin{matrix}\text{other}\\ \text{current}\\ \text{liabilities}\end{matrix}]$$

$$t = 1, \ldots, 12 ,$$

where the current term loan is the sum of the installment payments due on the term loan during the next 12 months (and thus by accounting conventions a current liability).

G. The Objective Function

The objective function is made up of three cost components, the explicit costs (interest and lost discounts), the implicit costs assigned to the intangible effects of the stretching and term loan options, and the end costs associated with deviations of the ending working capital structure from the designated target. The goal of the firm is to minimize the sum of these costs during the planning horizon.[26]

---

[26] As previously noted the liquidity reserve constraints, the current ratio requirements and the end costs are all methods which can be employed to induce a "reasonable" terminal financial mix. Since they tend to perform overlapping functions, then obviously not all three are jointly required in any problem. All have been included in the sample problem however simply to illustrate the most general formulation of the model.

The explicit costs in the objective function are computed on an accrual basis (as opposed to the cash basis used in the sources and uses constraints). For example, interest charges on loans outstanding are charged to the periods for which the borrowing was in force and not to the periods in which the interest was actually paid.[27]

The explicit costs associated with period t financing and investment decisions, E_t, are given by

$$D_t = r_1X_{1t} + r_2X_{2t} + r_3(X_{3t1} + X_{3t2}) + r_4a_{4t}X_4 + \delta_1X_{5t1} + \delta_2X_{5t2} + \delta_3X_{5t3} - \rho_1X_{6t1} - \rho_2X_{6t3} - \rho_3X_{6t3} ,$$

$t = 1, . . ., 12$.

The implicit costs associated with period t financing decisions are given by

$$D_t^* = g_{31}X_{3t1} + g_{32}X_{3,t-1,2} + g_4a_{4t}X_4 .$$

where g_{31} = the intangible cost assigned to payables first due in period t but stretched one period before payment.

= 0.083% per month

g_{32} = the intangible cost assigned to payables first due in period t - 1 but stretched 2 periods before payment (and thus providing additional financing in period t)

= 0.833% per month

g_4 = the intangible cost assigned to term loan borrowing

= 0.166% per month

The end cost associated with deviations from the target financial structure, E, is given by

[27] Interest charges for financing options which exceed beyond the time horizon are truncated of the horizon (i.e., at the end of the twelfth period). Thus, three period paper issued in period 12, would be charged only one period of interest in the objective function.

$$E = h_1(X_{1,12} - X_1^*) + h_2(X_{2,12} - X_2^*) + h_3(X_{3,12} - X_3^*)$$
$$+ h_4(a_{4,12}X_4 - X_4^*) + h_5(X_{5,12} - X_5^*) - h_6(X_{6,12} - X_{12}^*)$$

where X_k^* = the target balance for financing or investment option k

$k = 1, \ldots, 6.$

h_k = the cost per dollar of deviation from the target balance

(see table 4 for the sample problem numerical values).

The objective function is thus to minimize, Z, where

$$Z = \sum_{t=1}^{12} (D_t + D_t^*) + E$$

V. SOLUTION TO THE SAMPLE PROBLEM

The solution to the sample problem is presented in Figure 2 and tables 4 through 7. After obtaining this solution, the sample problem was modified by deleting the current ratio requirements and re-run. A summary of this solution is shown in figure 3 and table 8. The reports were produced by an interactive time-shared computer program called SHORTER.[27]

A. Description of Reports

Report #1: <u>Borrowing Summary</u> (See Figure 2)

---

[27] The SHORTER program is written in Fortran IV for use on the PDP-10 and IBM 360/67 time-shared computers. It is a self-contained package with interactive input routines, a matrix generator, a linear programming subroutine and a report generator. The report generator also produces sensitivity analyses for the objective function costs and the right hand sides of the model constraints which have not been reproduced here for reasons of brevity.

This report provides a period-by-period summary of the amounts and sources of the short term financing used in each decision period. The "period 0" figures shown relate to the amounts initially outstanding.

For example, in the second month of the planning horizon, the optimal plan contains $2,786,000 of short term financing, of which approximately $2,000,000 comes from the term loan, $400,000 from stretching of payables and $400,000 from the line of credit (precise figures for the amounts of financing used are shown in tables 5 and 6).

The column labelled "ratio" gives the expected current ratio in each month of the planning horizon. Note that the ratios in months 3 and 9 are as the minimum allowable values of 120.

Report #2: Cost Summary (See table 4)

This report summarizes the period-by-period accrued financing charges associate with the optimal financing decisions. The total cost for the recommended financing and investment plan is $316,232, which is made up of $267,278 of direct financing costs (interest expense plus lost discounts) plus $48,954 of implicit costs associated with stretching of payables, the term loan and deviations from the target financial structure.

Report #3: Sources and Uses of Funds (See table 5)[28]

This report describes the optimal short-term financial plan. It shows month-by-month how cash deficits were financed and how cash surplus were disposed of.

[28] The reader should carefully distinguish between the accrued costs shown in table 4 and the interest payments shown in table 5.

The cost summary corresponds to the terms in the objective function, i.e. costs are assigned to periods in which the financing is outstanding (i.e. accrued costs). For example, payables stretched for two periods in month 6 would result in lost discounts charged to month 6 (accrued interest column) plus implicit costs assigned to months 6 and 7.

The sources and uses statement is prepared on a cash flow basis. (It corresponds to the expected sources and uses constraints in the model). Thus, for example, in the above stretching example, the lost discounts associated with the period 6 stretching would not show up on the sources and

For example, the projected cash deficit for period 3 was $2,000,000 (see line 15 of table 1). This requirement is adjusted to include $19,000 of financing charges on previous borrowing due in period 3. The $2,019,000 adjusted requirement is met through $2,018,000 of new borrowing plus $1,000 of security sales. The Requirements Financed (bottom line of table) is the sum of the Net New Borrowing plus the difference between Securities Sold and Securities Purchased.

Report 4: <u>Pro-forma Balance Sheets</u> (See table 6)

This report gives the pro-forma balance sheets for each month of the planning horizon (at the beginning of each month <u>after</u> the decisions have been executed). The upper portion of the balance sheet shows the expected distribution of current assets during the month (including the maturity composition of the marketable securities portfolio). The lower portion shows the amounts borrowed from each short term source plus the term loan borrowing (divided into current and non-current balances).

Report 5: <u>Liquidity Reserve Summary</u> (See table 7)

This report shows the composition of the liquidity reserve in each month. This report provides a summary on an item-by-item basis of where additional funds could be obtained to meet unexpected future cash requirements.

The top line of the report shows the minimum required reserve specified by the firm. The bottom line shows the available reserve. Note that marketable securities always compose at least 50% of the required liquidity buffer.

In period 1, for example, the required reserve is $500,000, which is substantially surpassed by the actual reserve of $4,573,000. The actual reserve is composed of $2,006,000 of unused pledging capacity, plus $1,600,000 of potential for delaying payments for purchases (stretching) plus $967,000 of marketable securities balances.

B. Analysis of the Solution

The major portion of the required financing is provided by the term loan option, which at first consideration is surprising since this option is the second most expensive (after one month stretching of payables). The model recommends a $1,994,000 term loan in month 1, which is reduced to $1,795,000 in period 7.

For the remaining requirements, the model makes use of all of the sources at one time or other. The amount of the line of credit used never exceeds 1,000,000 in periods where operating cash balances are $200,000 and $500,000 in periods where operating cash is $100,000. Thus, the model only uses the line of credit where the compensating balance requirements are covered by operating cash resulting in an effective interest cost of only 8.5 percent per annum.

The stretching option is only used in periods where the joint borrowing constraint is binding (the reader will recall that stretching was not included in this constraint). Thus, in period 3 for example, the total financing used is 4,804,000 of which 804,000 comes from stretching of payables. A similar situation exists in periods 4 and 9 where the joint borrowing constraint is also binding.[29]

---

[29] An exception is period 2, where stretching accounts for $400,000 of total financing. Total borrowing from sources other than stretching however amounts to only $2,386,000 well below the $4,000,000 limit. The resolution of this interesting paradox is left to the interested reader (However, see note (4) of the Appendix for a hint).

Figure 2 -- BORROWING SUMMARY

```
PER  RATIO       TOTAL AMOUNT OUTSTANDING

  0  0.00   1250 :SSSSSSSLLPPP
  1  1,94   1994 :TTTTTTTTTTTTTTTTTTTT
  2  1,35   2786 :TTTTTTTTTTTTTTTTTTTTSSSSLLLL
  3  1,20   4804 :TTTTTTTTTTTTTTTTTTTTCCCCCCCCCCSSSSSSSSLLLLLLLLLL
  4  1,24   4390 :TTTTTTTTTTTTTTTTTTTTCCCCCCCCCCSSSSLLLLLLLLLL
  5  1,74   3902 :TTTTTTTTTTTTTTTTTTTTCCCCCCCCCCLLLLLLLLL
  6  2,71   1994 :TTTTTTTTTTTTTTTTTTTT
  7  2,25   1795 :TTTTTTTTTTTTTTTTTT
  8  1,75   2722 :TTTTTTTTTTTTTTTTTTCCCCLLLLL
  9  1,20   4849 :TTTTTTTTTTTTTTTTTTCCCCCCCCCCSSSLLLLLLLLLLPPPPPPP
 10  1,46   4323 :TTTTTTTTTTTTTTTTTTCCCCCCCCCCLLLLLLLLLLPPPPP
 11  2,19   2392 :TTTTTTTTTTTTTTTTTTCCCCCC
 12  1,46   4500 :TTTTTTTTTTTTTTTTTTLLLLLPPPPPPPPPPPPPPPPPPPPPP

LEGEND:
    T = TERM LOAN
    C = COMMERCIAL PAPER
    S = STRETCHED ACCOUNTS PAYABLE
    L = LINES OF CREDIT
    P = PLEDGED ACCOUNTS RECEIVABLE
```

Table 4 -- COST SUMMARY

PERIOD	ACCRUED INTEREST	--------- IMPLICIT COSTS --------- STRETCHING	TERM LOAN	END COST	TOTAL COST
1	10,775	0,000	3,323	0,000	14,098
2	32,366	0,333	3,323	0,000	36,023
3	44,099	3,670	3,323	0,000	51,093
4	29,151	3,363	3,323	0,000	35,837
5	27,991	0,000	3,323	0,000	31,315
6	1,784	0,000	3,323	0,000	5,108
7	4,252	0,000	2,991	0,000	7,243
8	14,250	0,000	2,991	0,000	17,241
9	43,954	0,293	2,991	0,000	47,239
10	29,473	0,000	2,991	0,000	32,464
11	15,422	0,000	2,991	0,000	18,413
12	13,760	0,000	2,991	3,408	20,159
TOTAL	267,278	7,659	37,887	3,408	316,232

Table 5 -- SOURCES AND USES OF FUNDS

PERIOD	1	2	3	4	5	6
REQUIREMENT	500.	1500.	2000.	-500.	-600.	-4000.
INTEREST:						
PLEDGING OF RECEIVABLES	3.	0.	0.	0.	0.	0.
LINE OF CREDIT	1.	0.	3.	7.	7.	7.
STRETCHING OF PAYABLES	25.	0.	0.	15.	15.	0.
COMMERCIAL PAPER	0.	0.	0.	0.	0.	22.
TERM LOAN	0.	17.	17.	17.	17.	17.
LESS: INCOME	2.	7.	1.	2.	1.	3.
TOTAL INTEREST	27.	9.	19.	36.	37.	42.
PLUS: ADD'L COMP BALANCE	0.	0.	0.	0.	0.	0.
ADJUSTED REQT FOR PERIOD	527.	1509.	2019.	-464.	-563.	-3958.
NEW BORROWING:						
PLEDGING OF RECEIVABLES	0.	0.	13.	0.	0.	0.
LINE OF CREDIT	0.	384.	616.	0.	0.	0.
STRETCHING OF PAYABLES	0.	400.	404.	0.	0.	0.
COMMERCIAL PAPER	0.	7.	986.	0.	0.	0.
TERM LOAN	1994.	0.	0.	0.	0.	0.
TOTAL NEW BORROWING	1994.	792.	2018.	0.	0.	0.
REPAYMENTS:						
PLEDGING OF RECEIVABLES	350.	0.	0.	13.	0.	0.
LINE OF CREDIT	200.	0.	0.	0.	78.	922.
STRETCHING OF PAYABLES	700.	0.	0.	400.	404.	0.
COMMERCIAL PAPER	0.	0.	0.	0.	7.	986.
TERM LOAN	0.	0.	0.	0.	0.	0.
TOTAL REPAYMENT	1250.	0.	0.	413.	488.	1908.
NET NEW BORROWING	744.	792.	2018.	-413.	-488.	-1908.
PLUS: SECURITIES SOLD	390.	843.	125.	123.	175.	186.
LESS: SECURITIES BOUGHT	607.	126.	124.	174.	249.	2236.
REQUIREMENT FINANCED	527.	1509.	2019.	-464.	-563.	-3958.

PERIOD	7	8	9	10	11	12
REQUIREMENT	500.	1500.	2700.	-600.	-2000.	-700.
INTEREST:						
PLEDGING OF RECEIVABLES	0.	0.	0.	5.	4.	0.
LINE OF CREDIT	0.	0.	4.	7.	7.	0.
STRETCHING OF PAYABLES	0.	0.	0.	13.	0.	0.
COMMERCIAL PAPER	0.	0.	0.	0.	9.	12.
TERM LOAN	17.	15.	15.	15.	15.	15.
LESS: INCOME	8.	8.	17.	2.	6.	2.
TOTAL INTEREST	9.	7.	2.	39.	30.	26.
PLUS: ADD'L COMP BALANCE	0.	0.	0.	0.	0.	0.
ADJUSTED REQT FOR PERIOD	509.	1507.	2702.	-561.	-1970.	-674.
NEW BORROWING:						
PLEDGING OF RECEIVABLES	0.	0.	713.	0.	0.	2205.
LINE OF CREDIT	0.	500.	500.	0.	0.	464.
STRETCHING OF PAYABLES	0.	0.	352.	0.	0.	0.
COMMERCIAL PAPER	0.	427.	562.	0.	0.	0.
TERM LOAN	0.	0.	0.	0.	0.	0.
TOTAL NEW BORROWING	0.	927.	2127.	0.	0.	2670.
REPAYMENTS:						
PLEDGING OF RECEIVABLES	0.	0.	0.	174.	539.	0.
LINE OF CREDIT	0.	0.	0.	0.	964.	0.
STRETCHING OF PAYABLES	0.	0.	0.	352.	0.	0.
COMMERCIAL PAPER	0.	0.	0.	0.	427.	562.
TERM LOAN	199.	0.	0.	0.	0.	0.
TOTAL REPAYMENT	199.	0.	0.	526.	1931.	562.
NET NEW BORROWING	-199.	927.	2127.	-526.	-1931.	2108.
PLUS: SECURITIES SOLD	1380.	859.	856.	281.	296.	336.
LESS: SECURITIES BOUGHT	672.	279.	281.	317.	336.	3118.
REQUIREMENT FINANCED	509.	1507.	2702.	-561.	-1970.	-674.

Table 6 -- PRO FORMA BALANCE SHEET

)	1	2	3	4	5	6
IT ASSETS:						
I OPERATING BALANCE	100.	100.	200.	200.	200.	100.
)'L COMP BALANCE	0.	0.	0.	0.	0.	0.
'AL	100.	100.	200.	200.	200.	100.
'ABLE SECURITIES						
'ERIOD MATURITY	483.	125.	125.	175.	187.	1380.
'ERIOD MATURITY	360.	124.	1.	125.	0.	189.
'ERIOD MATURITY	123.	1.	124.	0.	188.	856.
'AL	967.	250.	250.	300.	375.	2425.
& ACCTS RECEIVABLE	3000.	3000.	4500.	4250.	4250.	2000.
CURRENT ASSETS	4000.	4000.	4000.	4000.	4000.	4000.
CURRENT ASSETS	8067.	7350.	8950.	8750.	8825.	8525.
T LIABILITIES:						
TS PAYABLE						
DUE IN PERIOD	2000.	2500.	2500.	2500.	1000.	1000.
ABLES PAST DUE	0.	400.	804.	404.	0.	0.
AL	2000.	2900.	3304.	2904.	1000.	1000.
CIAL PAPER						
ERIOD MATURITY	0.	0.	0.	7.	1000.	0.
ERIOD MATURITY	0.	0.	7.	993.	0.	0.
ERIOD MATURITY	0.	7.	986.	0.	0.	0.
AL	0.	7.	993.	1000.	1000.	0.
OF CREDIT	0.	384.	1000.	1000.	922.	0.
SECURED BY A/R	0.	0.	13.	0.	0.	0.
OAN						
RENT	399.	399.	399.	399.	399.	399.
-CURRENT	1595.	1595.	1595.	1595.	1595.	1595.
AL	1994.	1994.	1994.	1994.	1994.	1994.
CURRENT LIABILITIES	1750.	1750.	1750.	1750.	1750.	1750.
CURRENT LIABILITIES	4149.	5440.	7458.	7052.	5071.	3149.

Table 6 (cont.) -- PRO FORMA BALANCE SHEET

)	7	8	9	10	11	12
IT ASSETS:						
I OPERATING BALANCE	100.	100.	200.	200.	100.	100.
)'L COMP BALANCE	0.	0.	0.	0.	0.	0
'AL	100.	100.	200.	200.	100.	100.
'ABLE SECURITIES						
'ERIOD MATURITY	862.	867.	281.	300.	336.	1711.
'ERIOD MATURITY	862.	0.	281.	0.	302.	0
'ERIOD MATURITY	0.	279.	0.	300.	0.	1711.
'AL	1724.	1147.	563.	600.	638.	3422.
& ACCTS RECEIVABLE	3500.	4500.	4500.	4250.	3500.	2500.
CURRENT ASSETS	4000.	4000.	4000.	4000.	4000.	4000
CURRENT ASSETS	9324.	9747.	9263.	9050.	8238.	10022
IT LIABILITIES:						
ITS PAYABLE						
' DUE IN PERIOD	2000.	2500.	2500.	1500.	1000.	2000.
'ABLES PAST DUE	0.	0.	352.	0.	0.	0.
'AL	2000.	2500.	2852.	1500.	1000.	2000.
'CIAL PAPER						
ERIOD MATURITY	0.	0.	0.	434.	570.	0.
ERIOD MATURITY	0.	0.	431.	566.	0.	0.
ERIOD MATURITY	0.	427.	562.	0.	0.	0.
AL	0.	427.	993.	1000.	570.	0.
OF CREDIT	0.	500.	1000.	1000.	36.	500.
SECURED BY A/R	0.	0.	713.	539.	0.	2205.
OAN						
RENT	399.	399.	399.	399.	399.	399.
-CURRENT	1396.	1396.	1396.	1396.	1396.	1396.
AL	1795.	1795.	1795.	1795.	1795.	1795.
CURRENT LIABILITIES	1750.	1750.	1750.	1750.	1750.	1750.
CURRENT LIABILITIES	4149.	5576.	7706.	6188.	3755.	6854.

Table 7 -- LIQUIDITY RESERVE SUMMARY

PERIOD	1	2	3	4	5	6
REQUIRED RESERVE	500.	500.	500.	600.	750.	900.
PLEDGING OF RECEIVABLES	2006.	1614.	0.	6.	84.	2006.
LINE OF CREDIT (NET)	0.	0.	0.	0.	0.	0.
STRETCHING OF PAYABLES	1600.	1200.	1597.	2000.	2000.	800.
COMMERCIAL PAPER	0.	0.	0.	0.	0.	0.
MARKETABLE SECURITIES	967.	252.	252.	300.	375.	2425.
AVAILABLE RESERVE	4573.	3064.	1847.	2306.	2459.	5231.

PERIOD	7	8	9	10	11	12
REQUIRED RESERVE	975.	1050.	1125.	1200.	1275.	1350.
PLEDGING OF RECEIVABLES	2205.	1778.	0.	166.	2099.	0.
LINE OF CREDIT (NET)	0.	0.	0.	0.	0.	0.
STRETCHING OF PAYABLES	800.	1600.	1648.	2000.	1200.	800.
COMMERCIAL PAPER	0.	0.	0.	0.	0.	0.
MARKETABLE SECURITIES	1724.	1147.	563.	600.	638.	3422.
AVAILABLE RESERVE	4729.	4525.	2210.	2766.	3937.	4222.

The major question, however, relates to the use of the expensive term loan option. Term loans carry a 10% per annum interest charge plus a 2% implicit cost. This results in a total cost per dollar borrowed which exceeds all of the other options (except one-period stretching).

The clue to this puzzle comes from observing the current ratios during the planning horizon. The solution is "binding" on the 1.20 minimum during periods 3 and 9. The term loan is the only option that would allow these constraints to be met. This results from the includion of only the current portion of the term loan in the denominator of the current ratio. Thus the term loan, in contrast to the other financing options, can be used to sufficiently increase net working capital to meet the minimum current ratio requirements.

The next question is how much does it cost the firm to meet the minimum current ratio constraints. The answer is provided in figure 3 and table 8, which show a summary of the same problem except for the deletion of the minimum current ratio requirements.

The borrowing summary (figure 3) shows a radical shift in the financing used. The term loan is no longer used, being replaced by primarily a combination of commercial paper and pledging of receivables. The average current ratio has declined. The former 1.2 minimum is now violated in four months (2, 3, 4 and 9) reaching a minimum in month 3 of 0.99.

The cost summary (table 8) shows a $30,000 reduction in direct financing costs, and an overall reduction of approximately $60,000 in the total solution costs. The $60,000 figure provides management with a basis for negotiation, if these constraints are externally imposed (i.e. try payment of higher interest rates to creditors to accept lower working capital requirements) or a basis for deciding whether this "risk-avoidance" cost

Figure 3 -- BORROWING SUMMARY

```
PER  RATIO        TOTAL AMOUNT OUTSTANDING

  Ø  Ø,ØØ   1250 :SSSSSSSLLPPP
  1  1,47   1265 :CCCCCSSSLLLLL
  2  1,Ø5   2777 :CCCCCCSSSLLLLLPPPPPPPPPPPPPP
  3  Ø,99   4789 :CCCCCCCCCSSSSSSSSSLLLLLLLLLLPPPPPPPPPPPPPPPPPPPP
  4  1,Ø1   4384 :CCCCCSSSSLLLLLLLLLLPPPPPPPPPPPPPPPPPPPPPPPPP
  5  1,33   39ØØ :CCCCLLLLLLLLLLPPPPPPPPPPPPPPPPPPPPPPPPP
  6  2,38      7 :
  7  1,88    542 :CCCCC
  8  1,44   2Ø76 :CCCCCCLLLLLPPPPPPPPPP
  9  1,Ø2   4822 :CCCCCCSSSLLLLLLLLLLPPPPPPPPPPPPPPPPPPPPPPPPPPPPP
 1Ø  1,2Ø   43Ø9 :CCCCCCCCCLLLLLLLLLLLPPPPPPPPPPPPPPPPPPPPPPP
 11  1,61   2367 :CCCCCCCCCLLLLLPPPPPPPPPP
 12  1,22   4486 :CCCCCCCCCLLLLLLLLLLLPPPPPPPPPPPPPPPPPPPPPPPPP

LEGEND:
    T = TERM LOAN
    C = COMMERCIAL PAPER
    S = STRETCHED ACCOUNTS PAYABLE
    L = LINES OF CREDIT
    P = PLEDGED ACCOUNTS RECEIVABLE
```

Table 8 -- COST SUMMARY

PERIOD	ACCRUED INTEREST	IMPLICIT COSTS STRETCHING	TERM LOAN	END COST	TOTAL COST
1	5,451	2,5ØØ	Ø,ØØØ	Ø,ØØØ	7,951
2	3Ø,699	Ø,34Ø	Ø,ØØØ	Ø,ØØØ	31,Ø38
3	41,931	3,719	Ø,ØØØ	Ø,ØØØ	45,65Ø
4	27,535	3,242	Ø,ØØØ	Ø,ØØØ	3Ø,776
5	26,459	Ø,ØØØ	Ø,ØØØ	Ø,ØØØ	26,459
6	-2,811	Ø,ØØØ	Ø,ØØØ	Ø,ØØØ	-2,811
7	Ø,953	Ø,ØØØ	Ø,ØØØ	Ø,ØØØ	Ø,953
8	11,924	Ø,ØØØ	Ø,ØØØ	Ø,ØØØ	11,924
9	41,645	Ø,275	Ø,ØØØ	Ø,ØØØ	41,919
1Ø	27,911	Ø,ØØØ	Ø,ØØØ	Ø,ØØØ	27,911
11	13,475	Ø,ØØØ	Ø,ØØØ	Ø,ØØØ	13,475
12	12,351	Ø,ØØØ	Ø,ØØØ	7,568	19,919
TOTAL	237,523	1Ø,Ø75	Ø,ØØØ	7,568	255,166

is justified if the constraints are internally imposed.

In this latter case, the additional protection provided by the current ratio constraints over the liquidity reserve requirements might well be questioned. The liquidity reserves were established to insure a minimum probability that future cash requirements could be met from existing sources of funds. The current ratio constraints attempt to duplicate this function in a less direct manner and thus are probably not worth the substantial additional cost.

VI. SUMMARY

The model presented in this paper permits the financial manager to incorporate his subjective estimates regarding the uncertainty of future cash requirements into the development of the optimal short-run financial plan. The liquidity reserve features of the model permit the maintenance of any desired degree of protection against the possibility of not being able to meet future cash requirements from planned sources of funds. The liquidity reserve contains the firm's marketable securities balances plus any unused borrowing potential on immediately available financing sources.

In most applications only the first period decisions would be implemented. The model would be re-run at the end of each month using revised estimates of future cash requirements and financing costs. It is necessary, however, to prepare the complete short term plan since the first period decisions will depend upon expectations about conditions and courses of action in later periods of the planning horizon.

We expect the model to be a useful aid to management in the development of short term financing decisions for any firm with an interesting short run planning problem. By interesting we mean alternating periods of cash surpluses and deficits, a variety of financing alternatives and constraints and explicit requirements for protection against the uncertainties of future cash requirements.

APPENDIX

Note (1)

The short term planning problem has been formulated as a chance constrained programming problem. This approach was used instead of the more desirable multi-stage linear programming approach, since the deterministic equivalent linear programming problem has the same size and structure as a deterministic version of the short-term planning model. Consequently, the computational burden of the stochastic version is no greater than for the deterministic version of the model. Also, the amount of information required about distribution of the cash requirements is less for a chance constrained as opposed to a multi-stage linear programming approach.

The principal weakness of chance constrained model is that it only indirectly evaluates the consequences of violating a constraint. The chance-constrained approach indicates no differential panalty between a small and a large violation of a constraint. In other words, specifying the correct values of the probabilities with which constraints can be violated should be part of the optimization problem.

Therefore, being faced with the choice between two approaches for modeling a multi-stage decision process, we have chosen the chance constrained approach (with its restricted meaning of optimality) as opposed to the multi-stage model (with its severe limitation on problem size). By making this decision we can solve interesting problems that would be beyond the bounds of computation with the multi-stage linear programming.

For a further discussion of stochastic programming methods, see [4], chapter 16.

Note (2)

The following formulation will reduce the contribution to the liquidity reserve by unused lines of credit by only the amount of any incremental borrowing required to meet the compensating balance requirement and not the full 20 percent.

(a) Upper bound constraints

$$X_{2t} + (L_{2t} + \gamma_{2t}) \quad 1{,}500{,}000, \qquad t = 1, \ldots, 12$$

where γ_{2t} = the incremental borrowing required to meet compensating balance requirements on L_{2t}

(b) Compensating balance requirements

$$C_t + b_t - S_t = 0.2(X_{2t}), \qquad t = 1, \ldots, 12.$$

where S_t is the excess cash available from operating balances after existing compensating balance requirements are met. Note that S_t will equal zero whenever b_t is greater than zero.

(c) Incremental borrowing constraints

$$S_t + \gamma_t \geq 0.2[L_{2t} + \gamma_t], \qquad t = 1, \ldots, 12.$$

Note that γ_t will equal zero whenever S_t is greater than 0.20 (L_{2t})

With this new formulation, L_{2t} is now the contribution to liquidity reserves net of any additional borrowing required to meet compensating balance requirements.

Note (3)

The stochastic constraint which is implicit in the liquidity reserve requirement is given by

$$\text{Prob}\ \{\tilde{CR}_t \leq CS_t + L_t\} \geq 1 - \varepsilon$$

where $\tilde{CR}_t$ = the cumulative funds requirements to period t (a random variable)

CS_t = the cumulative funds raised via short term financing to period t (a deterministic variable)

L_t = Liquidity reserve in t (a deterministic variable)

ε = the maximum acceptable probability of default.

The constraint is formulated on a cummulative rather than a non-cummulative basis so that the statistical relationship among the period by period cash requirements, $\tilde{R}_t$, can be explicitly considered. Also the cummulative formulation avoids the problem of double counting the same liquidity reserve against the liquidity requirements in several periods.

This constraint can be restated in terms of the normalized cumulative probability function for $\tilde{CR}_t$

$$F\left[\frac{CS_t + L_t - E(\tilde{CR}_t)}{\sigma(\tilde{CR}_t)}\right] \geq 1 - \varepsilon$$

or

$$CS_t + L_t - E(\tilde{CR}_t) \geq k(\sigma(\tilde{CR}_t))$$

where $k = F^{-1}(1 - \varepsilon)$ is determined from the distribution form for $\tilde{CR}$ or from a non-parametric relationship, such as Tchebyscheff's inequality. In the example it is assumed that $\tilde{CR}_t$ is normally distributed, thus $k \simeq 1.5$.

Note (4)

To see why the model stretched in period 2, rather than 3, it is necessary to consider the incremental borrowing costs of the various options as well as the marginal costs associated with an additional dollar of cash requirements in month 3. The following are the shadow prices (marginal costs) for the sources and uses constraints for each period in the planning horizon.

Marginal Costs
(Dollars per Additional Dollar)

Mo. 1	Mo. 2	Mo. 3	Mo. 4	Mo. 5	Mo. 6	Mo. 7	Mo. 8	Mo. 9	Mo.10	Mo.11	Mo.12
0.206	0.198	0.190	0.106	0.098	0.090	0.084	0.077	0.069	0.031	0.023	0.016

REFERENCES

1. Robichek, Teichroew and Jones, "Optimal Short Term Financing Decision," Management Science, Vol. 12, No. 1, September 1965, pp. 1-36.

2. James C. T. Mao, "Application of Linear Programming to Short Term Financing Decision," The Engineering Economist, Vol. 13, No. 4, pp. 221-241.

3. A. A. Robichek and S. C. Myers, Optimal Financing Decision, Prentice Hall, Englewood Cliffs, New Jersey, 1965.

4. Harvey M. Wagner, Principles of Operations Research, Prentice Hall, Englewood Cliffs, New Jersey, 1969.