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WP 976-78 February 1978

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ABSTRACT

The word-of-mouth effect — the interaction between current and prospective product users — has an important impact on the sales of many products. A mathematical model is developed which explicitly incorporates such an effect. Procedures for estimating the parameters of the model are developed and data for two ethical drugs are used to validate the model.

The model is used to develop marketing policies for new product introduction, which have a short period of high marketing activity followed by a lower "maintenance level" of effort over the remainder of a planning horizon.
INTRODUCTION

In many product-marketing situations, the impact of brand promotional efforts is enhanced by a "word-of-mouth" effect -- that is, by the recommendation of the brand by current satisfied users to potential users. Examples of such situations are:

- satisfied viewers of a movie, or users of a restaurant or resort recommending it to their friends,
- doctors recommending a successful new drug to their colleagues,
- women recommending a new food store to other housewives.

In each of these examples, initial users are attracted by some sort of marketing effort -- advertising or sales promotion -- and then enhances the impact of that effort on a part of the potential user population.

In some situations it might be desirable to actually direct some of the initial marketing effort toward "opinion leaders," people who are more likely to try the new product and whose subsequent recommendations will carry more weight than the rest of the target population. Arndt, for example, [1] points to the importance of the word-of-mouth effect in developing advertising policies. Silk and Davis [5] review the literature dealing with influence processes in marketing situations, and stress the need for explicit understanding and measurement of these effects.

Thus, mathematical models of such marketing situations should explicitly consider the interaction between marketing expenditures and word-of-mouth effects, in order to develop good policies. However, the authors are not aware of any such models.
This paper hypothesizes and tests a model structure that explicitly includes the word-of-mouth effect. For the sake of definiteness we consider the marketing of an ethical drug, aimed at a certain specialty class of doctors. One of the most important components of the marketing mix employed by pharmaceutical companies is "detailing" -- i.e., personal selling by a force of "detailmen," who visit doctors and describe the portfolio of products produced by their company, provide free samples and literature, and of course, attempt to combat the efforts of detailmen from competing companies. Surveys performed over a number of years have indicated that physicians generally perceive detailmen as influential sources of information (Bauer and Wortzel [2]). Other components of the marketing mix include medical journal and magazine advertising and direct mail, but a smaller portion of the total marketing budget is devoted to these components than to detailing.

For a new product, the impact of company marketing effort is augmented by the word-of-mouth effect that occurs when doctors first prescribing the product find it satisfactory and recommend it to their colleagues.

One of the problems in testing such models is that data on word-of-mouth is hard to collect, and is usually not collected. Therefore, any model validation has to be indirect in nature -- i.e., we postulate the nature of the word-of-mouth effect and then, using the observed data available check to see whether the model is consistent with the data. Although this approach is not completely satisfactory, it is the only one available and is often used in econometric and behavioral research. Data
for two ethical drugs were used to validate the model in this indirect way, and yielded satisfactory results.

The model developed below explicitly considers only the detailing activity on behalf of, and against a new product, and the interaction of this effort with the word-of-mouth effect. Advertising and direct mail have been left out to simplify the exposition. The approach here differs from that developed by Montgomery, Silk and Zaragoza [4] in that we address the impact of word-of-mouth effects in the context of developing a long-term total detailing strategy. Montgomery et al. develop a more detailed, tactical procedure that is heavily dependent upon managerial judgment for calibration, i.e., a decision-calculus approach (Little [3]).

After validation, the model is used to develop "good" detailing policies. We call them "good" rather than "optimal," because they have been specified to be profit improving as well as easily implementable in the total detailing context rather than just profit maximizing. Management has to allocate detailmen's time across a variety of products; therefore a policy for a single product must be simple enough to be incorporated within the total portfolio. This, we believe, precludes policies that are highly state and time dependent, requiring frequent changes in effort allocation. A policy that seems to fit these marketing realities is of pulsed type -- i.e., a short period of high effort detailing during product introduction followed by a much lower "maintenance level" detailing over the remainder of the planning horizon.

Managerial use of the model in the context of a new product presents some novel aspects. Since the key period in the planning horizon occurs at
the beginning, when there is no marketing data on the product, even adaptive estimation of parameter values cannot be advocated as a model calibration strategy. We believe that the appropriate approach is to model a variety of products, obtaining the model parameters for each, use managerial judgement to select one of these pre-modelled products as being similar to the new product, and use the previously obtained parameters in setting detailing levels. Once the product has been in the market for some time, adaptive estimation becomes a feasible procedure.
2. **A Simple Model**

Assume there are \( N^* \) doctors in the prescribing class (psychiatrists for anti-depressants, e.g.) of which \( N(< N^*) \) may eventually prescribe the drug. We observe the number of prescriptions which we assume is a function of the number of prescribing doctors.

Assume a doctor prescribes a drug; some of his prescriptions will be new, others renewals. If he is a prescribing doctor, a greater fraction of his prescriptions will be new when he starts prescribing. After he stops prescribing, however, there will be some delay until all his prescriptions are switched to a new drug.

Assume these two effects (build up and decay) due to a doctor switching his prescriptions to a new drug) either are rapid or approximately balance. In either case; let:

\[
C_2(t) = \text{number of doctors prescribing the drug at (discrete time) } t.
\]

\[
K_1(t) = \text{number of new prescriptions observed at } t.
\]

\[
K_2(t) = \text{number of prescription renewals at } t.
\]

\[
W = \text{random variable, the number of patients actually using the drug class that a randomly chosen doctor has.}
\]

\[
C_1(t) = \text{number of doctors at } t \text{ not now prescribing who will, at some time, prescribe.}
\]

\[
C_3(t) = \text{number of doctors at } t \text{ who were prescribers at one time but who no longer prescribe.}
\]

Note that although the model is structured in terms of the number of prescribing doctors, the data we observe are the number of prescriptions. Hence, we assume that

\[
K_1(t) + K_2(t) = C_2(t) \cdot E(W)
\]
or

\[ C_2(t) = (K_1(t) + K_2(t))/E(W) \]

We describe the flows between these three classes of doctors (1 = never having prescribed, 2 = now prescribing, 3 = past prescriber), as follows:

The flow from \( C_1 \) to \( C_2 \) is affected

a) by level of detailing

b) by word-of-mouth effect related to the number of currently prescribing doctors

The flow from \( C_2 \) to \( C_3 \) is affected by

a) normal attrition, and

b) competitive detailing.

Flow back from state 3 to state 2 is accomplished (particularly, late in the life cycle) through

a) detailing.

The flow from 3 to 2 should be considerably less sensitive to detailing than from 1 to 2 unless the product has some new story to tell.

Figure 1 describes this process.
\( C_1(t) \)

\( C_2(t) \)

\( C_3(t) \)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{flow_model.png}
\caption{Flow Model Describing the Process}
\end{figure}
Let

\[ \overline{d}(t) = \text{competitive detailing level at } t \]
\[ d(t) = \text{level of detailing at } t \]
\[ \lambda_1(d(t)) = \text{detailing effect} \]
\[ \lambda_2(C_2(t)) = \text{word of mouth effect} \]
\[ \lambda_3(\overline{d}(t)) = \text{competitive detailing effect} \]
\[ \lambda_4(d(t)) = \text{late life cycle detailing effect} \]
\[ \lambda_5 = \text{natural decay rate or attrition} \]

Then we can characterize Figure 1 mathematically as

\[ C_2(t) = [\lambda_1(d(t-1)) + \lambda_2(C_2(t-1))] C_1(t-1) \]
\[ + [1 - \lambda_3(\overline{d}(t-1)) - \lambda_5] C_2(t-1) \]
\[ + \lambda_4(d(t-1)) C_3(t-1) \]

(1a)

\[ C_1(t) = [1 - \lambda_1(d(t-1)) - \lambda_2(C_2(t-1))] C_1(t-1) \]

(1b)

\[ C_1(t) + C_2(t) + C_3(t) = N \]

(1c)

The model described above has several important simplifying assumptions. The first is that \( N \), the number of doctors who eventually prescribe the drug, is not related to detailing effort. A second simplification is that all doctors are in the same "class" (psychiatrists versus general practitioners, for example). It is not difficult to amend the model to accommodate these modifications by:
(a) Considering $N/N^*$ as a function of (say) cumulative detailing effort (or effort share), and by

(b) constructing a series of parallel processes such as that in Figure 1, one for each class of doctors.

These modifications are beyond the scope of our current objectives, however.
3. **Parameter Estimation and Validation**

Two issues will be addressed in this section. The first relates to managerial use -- ideally, one would like to have an idea of the structure of the process very early, so that appropriate detailing action can take place. To meet this end, the model should be calibrated using the first few data points.

To determine if the structure is valid, one needs to look over the full data stream, however. For this purpose, a different approach to parameter estimation or evaluation is required.

The issues here are subtle, but important. A review of the analytical structure of the model indicates that, early on, the settling down or steady state nature of the model will be obscured by the growth of the product; calibration from early data alone will be hopeless. In this section we develop the following arguments

(a) The model should be calibrated on a set of different, full product-data streams. This will demonstrate the validity of the model as well as suggesting what types of growth patterns are possible.

(b) To use the model, one needs to be concerned with two parts of the model -- the transient and the stationary portions.

(b1) For the transient portion, one should first estimate parameters judgmentally (from experience with past products) and then update those judgments as data become available.

(b2) For the stationary portion, one could look at past products and, again, calibrate judgmentally. Hard data may not be
available for some time, however -- one might try to calibrate the "repeat" nature of prescriptions using early, perceptual information gathered from field survey. (Good correlates to the trial/repeat nature of consumer product purchases have been obtained using similar concepts -- see Silk and Urban [6], e.g.). This method of calibration is more speculative, however.

To validate the model, several problems need resolution. Equation (1) assumes that \( \lambda_1, \lambda_2, \lambda_3, \) and \( \lambda_4 \) are general functions of \( d(t), \bar{d}(t) \) and \( C_2(t) \). Assume that response to detailing is a quadratic function of detailing effort, i.e.

\[
\lambda_1(d(t)) = \lambda_{11}d(t) + \lambda_{12}d^2(t).
\]

This particular functional form has the following properties:

(a) \( \lambda_1(0) = 0 \)

(b) Neither increasing nor decreasing returns to scale are specified a priori; however, if \( \lambda_{12} \) is \(< 0\) (as would be expected) then \( \lambda_1(d(t)) \) has an upper bound of \( -(\lambda_{11}^2 / 4\lambda_{12}) \) and detailing above the level of \( -\lambda_{11} / 2\lambda_{12} \) results in decreasing response. So, a practical upper bound on detailing is established.

Certainly, more complex forms of \( \lambda_1 \) could be generated; this simple quadratic form is rather flexible and leads to a simple estimation procedure. We assume that \( \lambda_2, \lambda_3 \) and \( \lambda_4 \) are linear functions of their arguments, i.e.

\[
\lambda_2(C_2(t-1)) = \lambda_2C_2(t-1), \text{ etc.}
\]
Note that we only observe $K_1(t) + K_2(t)$ directly and not $N(=C_1(0))$, $C_1(t)$ or $C_3(t)$. We can resolve this issue as follows:

Let us assume that our time intervals have been chosen such that it is not unreasonable to assert that

$$C_2(t) = \frac{K_1(t) + K_2(t)}{E(\omega)}$$

Now, using that value in place of $C_2(t)$, choose a time $t=T$ such that $C_3(t)=0$ for $t \leq T$. The structure of the model suggests that for modest values of $T$, this assumption should be reasonable. Then

$$C_1(t) + C_2(t) = N \quad t \leq T$$

and

$$C_2(t) = \left[ \lambda_{11}d(t-1) + \lambda_{12}d^2(t-1) + \lambda_{2}C_2(t-1) \right] [N-C_2(t-1)] + C_2(t-1)$$

or

$$\Delta C_2(t) = N\lambda_{11}d(t-1) + N\lambda_{12}d^2(t-1) + N\lambda_{2}C_2(t-1) - \lambda_{11}C_2(t-1)d(t-1) - \lambda_{12}d^2(t-1) - \lambda_{2}C_2^2(t-1).$$
\[ \frac{\Delta L_x}{L_x} = \frac{(0.14)^2}{10} \]
Equation (4) is of the form

$$ Y = a_1 x_1 + a_2 x_2 \ldots + a_6 x_6 $$

where

$$
\begin{aligned}
Y &= \Delta C_2(t) \\
x_1 &= d(t-1) \\
x_2 &= d^2(t-1) \\
x_3 &= C_2(t-1) \\
x_4 &= d(t-1)C_2(t-1) \\
x_5 &= d^2(t-1)C_2(t-1) \\
x_6 &= C_2^2(t-1) \\
\end{aligned}
$$

(6)

and

$$
\begin{aligned}
a_1 &= N\lambda_{11} \\
a_2 &= N\lambda_{12} \\
a_3 &= N\lambda_2 \\
a_4 &= -\lambda_{11} \\
a_5 &= -\lambda_{12} \\
a_6 &= -\lambda_2 \\
\end{aligned}
$$

(7)

\[ a_1 \ldots a_6 \] can be estimated by least squares and \[ \lambda_{11}, \lambda_{12}, N \] can be developed from equation (7), also by least squares.

If we assume now that we can use \[ \lambda_{11}, \lambda_{12}, \lambda_2 \] and \[ N \] as fixed, known quantities in the remainder of the estimation, then as long as \( \{C_2(t)\} \) is known, \( \{C_1(t)\} \) can be constructed recursively from (1b) and \( \{C_3(t)\} \) from (1c).

Assuming linearity, either \( \lambda_3, \lambda_4 \) and \( \lambda_5 \) or all of \( \lambda_1 \) to \( \lambda_5 \) can be estimated by least squares using (1a), accepting \( \{C_1(t)\}, \{C_2(t)\}, \ldots \{C_3(t)\} \) and \( \{d(t)\} \) and \( \{\tilde{d}(t)\} \) as input data streams.
If \( \lambda_{11}, \lambda_{12} \) are re-estimated using the last half of the data, we may test the hypothesis that detailing and/or word-of-mouth effectiveness are lower later in the product's life cycle (i.e. Ho: \( \lambda_1(t > T) < \lambda_1(t \leq T) \) \( \lambda_2(t > T) < \lambda_2(t \leq T) \)).

The disadvantage of re-estimating \( \lambda_1, \lambda_{12} \) and \( \lambda_2 \) is that 3 degrees of freedom for estimation are lost. The estimation scheme chosen, then, should balance the value of added insight on changes in the estimates of \( \lambda_1 \) and \( \lambda_2 \) with the loss in degrees of freedom for estimation.

We now must choose \( T \). \( T \) is best chosen empirically. If \( T \) is too small, \( C_2(t) \) will be small and the estimate of \( \lambda_2 \) is likely to be unstable. If \( T \) is too large, the effects of \( C_3 \) will become substantial and all parameters will be unstable. In practice, there is a range of \( t \) during increases in sales, where the parameters are stable, and \( T \) should be chosen to lie in that range. For simplicity, in the examples discussed in the next section, \( T=4 \) is chosen, giving an exact solution for the parameters.

All parameters of the system can, theoretically be estimated simultaneously. Writing down those equations shows that this results in a complex set of interaction terms of all orders and numerical, non-linear estimation procedures need to be used. The authors feel this added computational complexity does not justify its implementation difficulty and cost.
4. **Model Properties and Validation**

The long run sales of the product can be calculated easily if we assume \( \bar{d}(t) = \bar{d}, d(t) = d \) for all \( t \). In that case,

\[
C_2 = [1 - \lambda_3 \bar{d} - \lambda_5]C_2 + \lambda_4 d (N - C_2)
\]

or,

\[
\bar{C}_2 = \frac{N\lambda_4 d}{\lambda_3 \bar{d} + \lambda_5 + d\lambda_4}
\]

The dynamic properties of the model are quite complex and are best evaluated via simulation model.

**Case 1:** Here, quarterly data were available for an ethical drug from 1968 – 74. The data are disguised for proprietary reasons.

The parameter estimation procedure used was as follows: Four parameters: \( N, \lambda_{11}, \lambda_{12} \) and \( \lambda_2 \) were calculated from the first 4 data points. Three parameters -- \( \lambda_3, \lambda_4 \) and \( \lambda_5 \) were calculated from the last 3. Table 1 lists the parameter values; Table 2 lists the smoothed data.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>1624</td>
<td>3662</td>
</tr>
<tr>
<td>( \lambda_{11} )</td>
<td>( 83 \times 10^{-4} )</td>
<td>( 1.9 \times 10^{-4} )</td>
</tr>
<tr>
<td>( \lambda_{12} )</td>
<td>( -1.1 \times 10^{-6} )</td>
<td>( -9.2 \times 10^{-7} )</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
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<td>( 2.7 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>( 1.1 \times 10^{-4} )</td>
<td>--</td>
</tr>
<tr>
<td>( \lambda_4 )</td>
<td>( 6.1 \times 10^{-4} )</td>
<td>--</td>
</tr>
<tr>
<td>( \lambda_5 )</td>
<td>( 4.4 \times 10^{-2} )</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 1 Parameter Estimates
These parameters were then used to "forecast" points 6 through 22, which were not used for estimation.

We can test the goodness of fit of this model as follows: Using the calculated values of the parameters, estimate

\[ \hat{C}_2(t) = \text{estimated value of } C_2, \text{ for } t = 6, \ldots, 22. \]

Then, we can test the following hypotheses: the model gives no predictive information. For this purpose we form the ratio:

\[ R = \frac{\sum (C_2(t) - \bar{C}_2(t))^2}{\sum (\hat{C}_2(t) - \hat{C}_2(t))^2} \]

where \( \bar{C}_2(t) = \frac{1}{17} \sum_{t=6}^{22} C_2(t) \).

Under the null hypothesis, \( R \) will be distributed approximately as \( F \) with 16 and 1 degrees of freedom. \( R \) is calculated to be 4.7, so we can reject the hypothesis of no relationship at the level, \( \alpha = .001. \)

Figure 2 displays the results.
<table>
<thead>
<tr>
<th>Quarter</th>
<th>$d$</th>
<th>$\bar{d}$</th>
<th>$c_2$</th>
<th>$c_2^*$</th>
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<tr>
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<td>53</td>
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<td>2</td>
<td>47</td>
<td>417</td>
<td>65</td>
<td>-</td>
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<td>417</td>
<td>303</td>
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<td>466</td>
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</tr>
<tr>
<td>24</td>
<td>22</td>
<td>472</td>
<td>522</td>
<td>-</td>
</tr>
</tbody>
</table>
Case 2: This example concerns a drug which has not "peaked-out."

We cannot use the same procedure as above and the number of data points is too small to use a simultaneous estimation scheme. 21 quarters of data were available and are once again disguised.

The first 4 data points were again used to calculate the first 4 parameters. Table 1 lists these parameter estimates. As a "guess," the same values of parameters $\lambda_3 - \lambda_5$ were used to generate $\hat{C}_2(t)$ as above. Those values are displayed in Table 3.

These data were used to forecast points 6 through 21; using the same procedure as in case 1, $R = 2.44$ and we reject the hypothesis of no relationships at $\alpha = .05$.

Note that although the fit is satisfactory, actual sales ($C_2$) seem to be consistently above the forecast sales ($\hat{C}_2$). This suggests that an appropriate, managerial procedure would be to update the "guess" at $\lambda_3$, $\lambda_4$, and $\lambda_5$ for forecasts beyond this period. Such a procedure would improve the fit in periods 6 through 21 and would provide a better base for policy development for this drug in the future.

These two cases illustrate the general structure of the model and demonstrate its descriptive validity: it seems to explain the data in these two cases adequately.
### Table 3 Case 2 Detailing Data

<table>
<thead>
<tr>
<th>Period</th>
<th>Detailing</th>
<th>Competitive</th>
<th>Smoothed Sales</th>
<th>'Forecast' Sales</th>
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<td></td>
<td>d</td>
<td>( \bar{d} )</td>
<td>( C_2 )</td>
<td>( \hat{C}_2 )</td>
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<tr>
<td>1</td>
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<td>-</td>
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<td>-</td>
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5. **Determination of Detailing Policies**

In principle, the profit maximizing policy over a planning horizon $T$ periods long can be obtained by solving a dynamic programming problem which has two state variables, namely $C_1$ and $C_2$ (see equations 1a - 1c). Computation of such a policy requires some assumptions about competitive detailing activity during the planning period, but these assumptions can probably be made, and the sensitivity of the policy to these assumptions examined.

We believe, however, that this approach will lead to policies that are complicated to implement and also unrealistic, as follows:

a. Because of competitive reasons it is usually desirable to drive the market share of the new product up as quickly as possible, and then to maintain it at that level. As will be shown below, this would imply a pulse of detailing activity during the introductory phase of the detailing campaign, followed by a much reduced "maintenance" level of detailing during the life of the product.

b. Product management is dealing with a portfolio of drugs, all of which are promoted by the same detailing force. Highly time dependent policies, calling for a different amount of effort on each drug in each period are difficult to implement or control. These are the types of policies that are likely to be yielded by a dynamic programming, profit maximization formulation. Assuming a sequence of new product introductions by the company, an approximate "steady state" policy for the detailing force would be to devote a certain fraction of its effort to new products and the balance to "maintenance detailing."
In view of the above we shall develop the parameters of a policy of the following type: "Drive the market share of the produce up to some level \( m \), and then maintain it at this level."

Developing the parameters of an exact policy of this form is analytically complex; however an approximation can be obtained rather simply, based on the following observation: During the product introduction period, \( C_3 \) (those who used to prescribe) is very small, certainly in comparison to \( C_1 \) or \( C_2 \). During the mature sales period of the product, \( C_1 \) (never having prescribed) is likely to be small in comparison with \( C_2 \) and \( C_3 \). Therefore in computing detailing levels during the introduction phase, \( C_3 \) will be set to zero, and for the "maintenance" detailing level computation, \( C_1 \) will be set to zero.

The introductory phase goal then is to reach a desired share \( m \) as quickly as possible. We can operationalize this by computing policies that maximize \( m \) at the end of \( t \) periods, where \( t \) can be a variable to be selected to provide the desired \( m \).

Setting \( t=1 \), it is easy to show that the optimal detailing level \( d^*_1 = \frac{1}{2} \lambda_1 \). Because the objective function as now set up is separable between periods, we can show (after some algebra) that \( d^*_j = \frac{1}{2} \lambda_1 \), \( j = 1, 2, \ldots, t \), maximizes \( m_t \), the market share at the end of \( t \) periods. Thus, during the introductory phase, the detailing level should be maintained at \( \frac{1}{2} \lambda_1 \) until the desired or target share is achieved. In order to compute the values of \( m_t \), an assumption must be made regarding competitive detailing levels.

To compute the approximate "maintenance detailing" level, set \( C_1 = 0 \), and \( C_2(t) = C_2 \). We must make some assumption about competition as before. Assuming that \( d(t) = d \), we get.
\[ C_2 = (1 - \lambda_3 \bar{d} + \lambda_5)C_2 + \lambda_4 d(N-C_2) \]

which yields

\[ d = \frac{(\lambda_3 \bar{d} + \lambda_5)C_2}{\lambda_4(N-C_2)} \]

Policies of this type will frequently lead to an overshoot of the target market share, even if all detailing were to be suspended when \( m \) is reached. This is because, for moderate values of \( m \), there are still a substantial number of doctors in the "never prescribed" pool who are brought into the "prescribing" pool by the word of mouth effect. If the rate of attrition from "prescribing" to "used to prescribe" is small, the new prescribers dominate the "used to prescribe" group for some time, leading to an overshoot. As the target market share increases, and as the attrition rate grows in comparison to the word of mouth rate, the overshoot will be reduced, and for some combinations of parameters may not occur.

Let \( D_1(m) \) represent the detailing cost per period during the introductory phase, and \( D_2(m) \) the detailing cost per period in the maintenance phase. Figures 3 and 4 show the relationship of detailing levels to desired target share levels, as suggested by the proposed policies, using the data in example 1. In order to determine the profit maximizing value of \( m \), \( m^* \) say, we first note that the profit over the life of the product \( T \) is given by

\[ P(m) = \frac{\sum_{t=1}^{T} \left( C_2(t) \cdot \frac{\# \text{ of prescriptions}}{\text{doctor}} \cdot \text{margin} - D_1(m) \right) \left( \frac{1}{1+i} \right)^t}{\sum_{t=1+1}^{T} \left( C_2(t) \cdot \frac{\# \text{ of prescriptions}}{\text{doctor}} \cdot \text{margin} - D_2(m) \right) \left( \frac{1}{1+i} \right)^t} \]

where \( i \) is the discount rate.
Assuming that adequate detailing staff is available, $m^*$ is found easily by a one dimensional search over $0 \leq m \leq 1$.

To illustrate the above discussion, and to show the overshoot phenomenon, we used the data in the example to simulate the effect of using the suggested policies. We assume that margins and costs are such that $m^* = 0.3$. Then the optimal policy is to detail at 400 per period for 3 periods and at 45 per period for the remaining periods. The expected sales are shown in Figure 5.
Figure 5: Effect of Policy $d=400$, $t=1,\ldots,3$, $d=45$, $t > 3$, using parameters from example 1.
6. **Detailing Force Implications**

In the last section we showed how detailing policies can be computed for a single product line. If we can assume that the different product lines constituting the portfolio of product offerings are independent of one another, then portfolio profit maximization can be achieved by selecting the optimal market share for each product line individually, so long as the total number of detailers required does not exceed the available force.

In general however, the portfolio maximization problem, given a fixed detailing force \( D \) can be addressed as a lagrangian problem. If \( P_{i,t}(m) \) is the profit associated with the \( i^{th} \) product line in period \( t \) with a steady state share \( m \), and \( d_{i,t}(m) \) is the detailing force required in the same period (note that given \( m \) and our policy as in the previous section, \( d_{i,t} \) can take only one of two possible values), we wish to

Maximize \[ \sum_{i,t} P_{i,t}(m) \]

subject to \[ \sum d_{i,t} \leq d. \]

Detailing manpower and detailing cost will be assumed to be linearly related, a reasonable assumption given that some detailing will always be done. Market share is a concave function of detailing activity both for the introductory phase and the maintenance phase, as is illustrated in Figures 3 and 4. Therefore \( P_{i,t}(m) \) is convex in \( d_{i,t} \). This implies that solutions to the lagrangian problem

\[
\mathcal{L}(d) = \sum_{i,t} P_{i,t}(m) - \sum_{t} \lambda_{t} \left( \sum_{i} d_{i,t} - d \right)
\]

will be unique. In addition \( \lambda_{t} \) will provide us the marginal value of additional detailers.
Conclusion

This paper develops an approach toward modeling the word-of-mouth effect in marketing situations. The model was developed to be consistent with normally-collected sales data and used in the development of easy-to-implement marketing policies.

Use of the model can offer insights into the impact of informal communication on development of marketing strategy and product-introduction planning.
References


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