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MONETARY POLICY AND COSTS OF PRICE ADJUSTMENT

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## MONETARY POLICY AND COSTS OF PRICE ADJUSTMENT

Julio J. Rotemberg\*

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Abstract: This paper studies the scope for monetary policy in an economy in which firms are concerned by the adverse reaction of their customers to price changes. First, the rational expectations equilibrium of such an economy is computed. Then, it is shown that the preferences of the monetary authority are time inconsistent as long as it can only respond slowly to changing aggregate statistics. The Strotz-Pollack equilibrium of the game between successive monetary authorities is computed and contrasted both to "optimal" feedback rules and to Friedman's constant growth rule.

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## I. Introduction

This paper focuses on the consequences of monetary policy in a model in which fully rational firms find it costly to change their prices. This model of firm behavior as well as its equilibrium has been introduced in Rotemberg [1981a]. It has the property that random shocks to the households' desire to hold real money balances affect both aggregate output and the rate of inflation for some time.

The monetary authority is assumed to attempt to minimize these effects by varying the stock of money balances. The preferences of the monetary authority over paths for the money supply are shown to be time inconsistent as long as it takes time for the government to observe the aggregate variables it wants to stabilize. This time inconsistency results from the effect future policies have on the impact of current disturbances which are observed only by the private sector. This effect will not be taken into account by the monetary authorities in the future as they choose the money supply corresponding to future periods.

Time inconsistency of preferences is a problem often encountered in models of decision making over time and it has been studied extensively by, for instance, Strotz (1955), Hammond (1976) and Goldman (1980). Kydland and Prescott [1977, 1980] and Calvo [1978] have presented other examples in which the preferences of the monetary authority turn out to be time inconsistent.

The model, which is presented in more detail in Rotemberg [1981b], is introduced in Section II. A consistent plan, also called a Strotz-Pollack equilibrium, is computed in Section III. This plan does not leave any serial correlation in aggregate output nor in the rate of inflation if it is carried out without

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error. In Section IV "better" plans which involve a precommitment on the part of the monetary authority are computed. These plans required that at a certain date a social contract which curbs the powers of the monetary authority be signed. The more desirable social contracts have the form of fixed feedback rules. The feasibility of such plans except for the simplest one (a constant money supply) is, of course, questionable.

Among the plans which require the monetary authority to precommit itself to a feedback rule the ones that stabilize output most generally involve"leaning with the wind", that is increasing the moncy supply when output is high. Instead those that stabilize the rate of inflation involve "leaning against the wind". In turn a constant money stock is preferable from the point of view of output stabilization to the consistent plan while the opposite is true for price stabilization. Section V is devoted to the conclusions.

#### II. The model

The model consists of n goods which are produced by n price setting monopolists who face demand curves of the form:

$$Q_{it}^{d} = A_{i} \left(\frac{P_{it}}{P_{t}}\right)^{-b} \left(\frac{M_{t}}{P_{t}F_{t}}\right)^{d} i \qquad i = 1, 2 \dots n \qquad (1)$$

where  $Q_{it}^{d}$  is the quantity of good i demanded at time t; A and b are firm specific constants; d is a parameter;  $P_{it}$  is the price of good i at time t;  $M_{t}$  is the level of aggregate money balances at time t while  $F_{t}$  is a time varying taste parameter akin to the inverse of desired velocity at time t.  $P_{t}$  is the price level at time t which is given by a geometric

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average of individual prices:

$$P_{t} = \begin{bmatrix} n & h_{i} & \frac{1}{\Sigma h} \\ i = 1 & \end{bmatrix}^{h_{i}} \begin{bmatrix} 1/\Sigma h_{i} \\ i \end{bmatrix}.$$
(2)

Firms use each other's output as inputs. Their technology has decreasing returns to scale to ensure that output is bounded. In particular the cost from producing the quantity Q<sub>it</sub> will be assumed to be equal to:

$$C(Q_{it}) = U_i P_t Q_{it}^2 / 2$$
(3)

where U<sub>i</sub> is a parameter.  $\frac{1}{}$ 

Therefore, in the absence of costs of changing prices, firm i would charge at t a price whose logarithm is p<sub>it</sub>:

$$p_{it}^{*} = s_{i} + p_{t} + d_{i}(m_{t} - f_{t} - p_{t})$$
(4)

where  $s_i$  is a function of the firm specific parameters and where lower case letters denote the logarithms of the respective upper case letters. It is now possible to approximate the difference between revenues from sales and costs of production by a quadratic function of the difference between  $p_{it}$  and  $p_{it}^*$ .

The firms will not always choose to change  $p_{it}^{*}$  because they are assumed to perceive convex costs to changing their prices. That price changes may be costly to firms has been pointed out by numerous authors including Nordhaus [1972], Barro [1972] and Sheshinski and Weiss [1977, 1979]. These costs are of two types. First, there is the cost of physically changing posted prices, which is probably a fixed cost per price change. This is the type of cost considered by Barro [1972] and by Sheshinski and Weiss [1977, 1979]. Second, and in my view more importantly, there is a cost to the firm that changes its prices, which results from the negative reaction of its customers. In particular, firms which change their prices often and by large amounts will see their reputations suffer and their customers turn to other firms.<sup>2/</sup> It seems plausible that customers react more strongly to large price changes than to recurrent small ones. I have, therefore, assumed that firms perceive a convex (quadratic) cost of changing prices. In particular, the expected value of discounted profits which firms are postulated to maximize is equal to:

$$\Pi_{it} = E \sum_{\tau=t}^{\infty} \rho^{\tau} [\Pi_{i\tau}^{*} - k_{i}(p_{i\tau} - p_{i\tau}^{*})^{2} - c_{i}(p_{i\tau} - p_{i\tau-1})^{2}]$$
(5)

where E is the expectations operator;  $\rho$  is the firm's discount factor;  $\Pi_{1\tau}^{*}$  are the profits that would accrue if the firm changed  $p_{it}^{*}$  and is therefore exogenous to the firm;  $k_{i}$  is the second derivative of the difference between revenues from sales and costs of production with respect to prices evaluated at  $p_{it}^{*}$  while  $c_{i}$  is a parameter. The ratio  $c_{i}/k_{i}$  is assumed to be constant across firms and equal to  $c_{i}^{3/2}$ 

Each firm takes the other firms' prices as given. Rationing is assumed infinitely costly so that firms consider only pairs (P<sub>it</sub>, Q<sub>it</sub>) which satisfy their demand functions (1). It is therefore natural to think that the monopolists simply decide on a path for their prices to maximize (5). The solution to the maximization of (5) with respect to the prices set by firm i is a path for the prices firm i expects to change such that:

$$p_{it/t+k} = \alpha p_{it/t+k-1} + (1/\beta \rho c) \sum_{j=0}^{\infty} \left(\frac{1}{\beta}\right)^{j} p_{it/t+k+j}^{*}$$
(6)

where:

 $k = 0, 1, ... \infty$ 

$$\alpha \beta = 1/\rho \qquad \alpha + \beta = 1/\rho + 1/\rho c + 1$$

$$(7)$$

$$(\beta - 1)(1 - \alpha) = 1/\rho c \qquad ; \qquad \beta > 1 \ ; \quad \alpha < 1.$$

The i superscript denotes the expectation held by firm i. Furthermore,  $p^{*i}$  is the expectation held by i at t of the value at t+k of  $p_{i}^{*}$ . In any it/t+k given time period, firm i has an expectation of the path of the future  $p^*$ 's. This leads it to expect to charge a certain sequence of prices. It actually charges today the leading element of that sequence, (pit/t). Tomorrow it recomputes the expected p's using all the relevant information and comes to a decision as to the price to charge then. Therefore (6) describes the actual path of prices where the expectation of the  $p^*$ 's is taken at t. Firms choose to charge prices that are similar to the previous period's prices to avoid large costs of changing prices. They also want to keep their costs from future price changes low. Therefore they adjust their prices gradually towards prices which they expect will make marginal revenues from sales equal to marginal production costs.

I will solve for the sequence of price levels  $p_{t}$  that constitute a rational expectations equilibrium for this economy. In the first place firms are assumed to have rational expectations about the path of the exogenous variables m and f. Then, the path of expected price levels which appears on the right hand side of (6) is made consistent with the prices that the individual firms expect to charge given their current information. A fortiori, the current price level on the right hand side of (6) is made consistent with the prices that firms charge today, as is true in static equilibria.

This rational expectations equilibrium satisfies the following difference equation:

$$p_{t/t+k} = \gamma p_{t/t+k-1} + (D/\delta\rho c) \sum_{j=0}^{\infty} (\frac{1}{\delta})^{j} (m_{t/t+k+j} - f_{t/t+k+j} + S/D)$$
(8)  
ce: k = 0, 1, ...  $\infty$ 

wher

$$S = \frac{\Sigma h_i s_i}{\Sigma h_i} ; \qquad D = d \frac{\Sigma [h_i / (1 + b_i)]}{\Sigma h_i}$$
(9)

and:

$$\gamma \delta = 1/\rho$$
;  $\gamma + \delta = 1/\rho + D/\rho_c + 1$  (10)  
 $(\delta - 1)(1 - \gamma) = D/\rho_c$ ;  $\delta > 1$ ;  $\gamma < 1$ .

Here,  $m_{t/t+j}$  is the mathematical expectation of  $m_{t+j}$  conditioned on all information available at time t. Firms spread the costs of the price changes that become necessary as m and f change. Therefore, the price level at t,  $P_{t/t}$ is not only a function of the price level which would have prevailed today in the absence of costs to changing prices, but also is a function of the previous periods' price level and of the price levels firms expect would have prevailed in the future in the absence of costs to price changes. In the absence of these costs the price level at time t would be given by:

$$p_{t}^{*} = m_{t} - f_{t} + S/D$$
 (11)

which can be seen by aggregating (5).

I now define an index of aggregate output q\_:

$$q_{t} = \sum_{i=1}^{n} q_{it}$$
 (12)

Since in this equilibrium model output is always equal to output demanded, one obtains from (1):

$$q_t = \Sigma a_i + nd(m_t - f_t - p_t)$$
 (13)

As can be seen from (13) together with (11),  $q^*$ , the aggregate output when the price level follows the path  $p_t^*$  is unaffected by time and by variations in m and f. The deviation of output from  $q^*$  are proportional to  $(m_t - p_t - v_t)$ , where:

$$v_t = f_t - S/D \tag{14}$$

In the rest of the paper, it will be assumed that  $v_t$  follows a random walk:

$$\mathbf{v} = \mathbf{v} + \mathbf{\varepsilon} \tag{12}$$

where the  $\varepsilon$  are independently identically distributed normal variates with mean zero and variance  $\sigma^2$ . The qualitative results of this paper depend only on the process characterizing the first difference of v<sub>t</sub> having a stationary moving average representation.

The objective of monetary policy will be to reduce the impact of the sequence  $\varepsilon$  on output and on the rate of inflation. The monetary authority will be assumed to know (8) which can be characterized as the private sector's reaction function. In turn the private sector will know the rule the monetary authority will follow in setting both the current and future values of m. Monetary policy, even foreseen monetary policy, has a role to play here because changing prices is costly while changing money is not.

## III. Time consistent monetary policy

A path of monetary policies will be called time consistent if it will actually be carried out by a sequence of monetary authorities which all have loss functions W, Y or Z and which control only the money supply of their own period. These loss functions are given by:

$$W = E \sum_{t=0}^{\infty} r^{t} (m_{t} - v_{t} - p_{t})^{2}$$
(16)

$$Y = E \sum_{t=0}^{\infty} r^{t} (p_{t} - p_{t-1})^{2}$$
(17)

$$Z = E \sum_{t=0}^{\infty} r^{t} [(m_{t} - p_{t} - v_{t})^{2} + \theta (p_{t} - p_{t-1})^{2}] .$$
(18)

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Here r is the authority's discount factor and  $\theta$  is a parameter. W corresponds to the desire to stabilize output; Y to the desire to stabilize the price level and Z to the desire to stabilize both to a certain extent. Here price stabilization increases welfare since it reduces the costs borne by firms. Not only does it reduce the direct costs of price adjustment but also it decreases the gap between the prices firms charge and the prices that maximize profits from sales.

If the monetary authority is able to observe  $v_t$  at time t, it can make W, Y and Z equal to zero by setting:

$$m_t = v_t + p_{t-1} \tag{19}$$

By inspecting (8) and noticing that (10) implies that:

$$(D/\delta\rho c) \sum_{j=0}^{\infty} \left(\frac{1}{\delta}\right)^{j} = (1 - \gamma)$$
(20)

it is clear that  $p_t$  will be equal to  $p_{t-1}$  under this rule. Therefore Y will be equal to zero. Furthermore, if  $p_t$  is equal to  $p_{t-1}$  the rule (19) directly implies that W and hence Z is equal to zero.

In this model m and v have symmetric effects. An increase in the desire to hold money balances is exactly equivalent to a decrease in the available level of money balances from the point of view of their effect on prices and output. Therefore the monetary authority can totally offset the effect of changes in v by changing the money stock in the same proportion.

It seems more reasonable to suppose, as will be done in the rest of this paper that the monetary authority (or Federal Reserve) only reacts to the disturbances with a one period lag. This lag may be due to the time involved in obtaining information on v indirectly by collecting data on prices and quantities traded  $\frac{4}{}$  as well as to the time required to evaluate these data and implement a policy.

This asymetry between the information available to the private sector  $(v_t)$  and the information available to the public sector  $(v_{t-1})$ has the property of making monetary policy less potent. It also leads the Federal Reserve today to desire a different monetary policy for tomorrow from the one the Federal Reserve will implement tomorrow if free to set m at that time. In other words, as will be shown in the next section, the preferences of the Fed over sequences  $[m_t]$  are time inconsistent.

In this section, the Federal Reserve will, at a given point in time, know that the money supply corresponding to future time periods will be chosen by a different entity. It will pick the money supply corresponding to its own period to minimize W, Y, or Z given this knowledge as well as the knowledge of the way in which the future authorities will react to its choice of m. $\frac{5}{-}$  Proposition 1 provides an equilibrium for this game between generations of Federal Reserve Boards in which the strategic variables are the values of m<sub>t</sub>. This equilibrium is a Strotz-Pollack equilibrium as defined in Pollack (1968) and Goldman (1980)  $\frac{6}{-}$ .

Suppose that the monetary authority will use a given feedback rule to set the money supply from time one onwards. It is then possible to compute the optimal monetary rule at time zero given the rule that will be used in the future. If the optimal rule at time zero coincides with the rule that is expected to be followed in the future then the rule constitutes an equilibrium. It will also be optimal for the monetary authority to use this rule at time one if it expects it will be used in the future. Therefore the rule will actually be used forever.

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PROP. 1: The rule:

$$\mathbf{m}_t = \mathbf{p}_{t-1} + \mathbf{v}_t \tag{21}$$

is a Strotz-Pollack equilibrium of the game among the sequences

of monetary authorities with loss functions W, Y or Z. $\frac{7}{}$ 

<u>PROOF</u>: I assume (21) will be followed for  $t \ge 1$  and show that it will be used at time t = 0. The proof proceeds in two steps. First it is shown that  $m_0$  has no effect for  $t \ge 1$  on  $E_0(m_t - p_t - v_t)^2$  nor on  $E_0(p_t - p_{t-1})^2$ . Then it is shown that setting  $m_0$  equal to  $(v_{-1} + p_{-1})$ minimizes both  $E_0(m_0 - p_0 - v_0)^2$  and  $E_0(p_0 - p_{-1})^2$ .

(1) 
$$t \ge 1$$

Using (14), (15) and the rule (21) in (8), one obtains the price level at t as:  $\infty$  i

$$p_{t} = \gamma p_{t-1} + \frac{(\delta - 1)(1 - \gamma)}{\delta} \varepsilon_{t} + (D/\rho c \delta) \sum_{j=0}^{\infty} \left(\frac{1}{\delta}\right)^{j} p_{t+j}^{e}$$
(22)

where at time t the expectation, by firms, of the future price level is given by:

$$p_{t+j}^{e} = \gamma p_{t+j-1}^{e} + (D/\rho c \delta) \sum_{k=0}^{\infty} p_{t+j+k}^{e} \quad j = 1, 2..., \infty$$
(23)

The only sequence of expected price levels that solves (23) is  $p_{t+j}^e = p_t$ . Using this solution in (22), one obtains:

$$p_{t} = p_{t-1} - (1 - \frac{\gamma \delta}{\delta + \gamma - 1}) \varepsilon_{t}$$

Therefore, by (21):

$$m_t - p_t - v_t = -\left(\frac{\gamma\delta}{\delta + \gamma - 1}\right) \varepsilon_t$$

These last two formulas establish that  $E_0(m_t - p_t - v_t)^2$  and  $E_0(p_t - p_{t-1})^2$  are independent of  $m_0$  even though the sequence  $p_t$  certainly is not. In other words, an increase in  $m_0$  will lead to higher prices forever. But if (21) will be followed,  $m_0$  will not affect future rates of inflation nor future levels of output.

(2) 
$$t = 0$$

Given that the rule (21) will be used from time one onwards, and using (14) and (15), (8) yields for the price level at time zero:

$$p_{o} = \gamma p_{-1} - (1 - \gamma)v_{o} + \frac{(\delta - 1)(1 - \gamma)}{\delta}m_{o} + \frac{(1 - \gamma)}{\delta}(p_{o} + v_{o})$$

where the last term on the RHS results from the firms belief that the future price level is expected to be equal to p<sub>0</sub>. Therefore:

$$p_{o} - p_{-1} = (1 - \frac{\gamma \delta}{\delta + \gamma - 1})(m_{o} - v_{-1} - p_{-1} - \varepsilon_{o}) \text{ and:}$$

$$m_{o} - p_{o} - v_{o} = \frac{\delta \gamma}{\delta + \gamma - 1}(m_{o} - v_{-1} - p_{-1} - \varepsilon_{o})$$
(24)

Hence both the minimization of  $E_0(m_0 - v_0 - p_0)^2$  and that of  $E_0(p_0 - p_{-1})^2$ leads to setting  $m_0$  equal to the sum of  $v_{-1}$  and  $p_{-1}$ .

And this completes the proof.

The equilibrium therefore involves a rule that sets the expectation by the monetary authority of the price level equal to both its previous value and to the value that makes output equal to  $q^*$ . That the rule is the same whether output or price stability is sought is one of the special features of this equilibrium.

This proposition describes the paths of the money supply, output and the rate of inflation in an economy in which the monetary authority pursues stabilization policies. It is, therefore, fair to inquire whether the equilibrium paths of m,q and p resemble the paths of these variables in economies like the U.S. In fact, the paths of p and q are different in the U.S. from those implied by prop 1. In particular, when the authorities follow rule (21) both output and inflation become serially uncorrelated random variables while, in the U.S., these variables are serially correlated. Therefore, it appears that the equilibrium computed in this section is not a satisfactory description of the interaction between the monetary authority and the rest of the U.S. economy.

In particular, it may well be the case that in spite of the government's claims, their loss function is fundamentally different from W, Y and Z. Instead, for political reasons, the government sometimes seems to pursue high levels of output which are obtainable by increasing m. At other times the monetary authority seems to be concerned with the rate of inflation and reduces the growth of the money stock.

In the next section, I will consider monetary rules which are potentially more desirable than the comsistent plan. It turns out to be simpler to consider alternative rules in which the money stock depends on the history of the  $\varepsilon$ 's rather than to consider those in which the money stock is set as a function of the past state variables v and p. I will, therefore, study rules of the former type. For the purpose of comparing the consistent plan to these other rules, I first establish Proposition 2:

PROP. 2: The consistent plan is equivalent to:

$$m_{o} = p_{-1} + v_{-1}$$
 (25)

$$m_{t} = m_{t-1} + \mu_{t} \varepsilon_{t-1} \qquad t \ge 1$$
(26)  
with: 
$$\mu_{t} = \frac{\gamma \delta}{\gamma + \delta - 1} = \overline{\mu} \qquad \text{for all } t.$$

<u>PROOF</u>: Equation (25) is obviously equivalent to (21) for t = 0. I now show that (26) is equivalent to (21) for t = 1. This is done in two steps. First I show that if rule (26) with  $\mu_t = \bar{\mu}$  is going to be followed from time equals one onwards, then the money supply has the property (21) at t = 1. Then I show that if rule (21) is used from t = 1 onwards, then the money supply has property (26) with  $\mu_t = \bar{\mu}$  at t = 1.

1) If (26) with  $\mu_t = \overline{\mu}$  is followed for  $t \ge 1$ , then the price level at time zero is equal to:

$$p_{o} = \gamma p_{-1} + (1 - \gamma)(m_{o} - v_{o}) + \frac{(1 - \gamma)}{\delta} \bar{\mu} \epsilon_{o}$$

 $= p_{-1} - \frac{(\delta - 1)(1 - \gamma)}{\delta + \gamma - 1} \varepsilon_{0}$  which implies that:  $v_{0} + p_{0} = v_{-1} + \varepsilon_{0} + p_{-1} - \frac{(\delta - 1)(1 - \gamma)}{\gamma + \delta - 1} \varepsilon_{0}$  $= m_{0} + \overline{\mu}\varepsilon_{0}$ 

which establishes that  $m_1 = v_0 + p_0$ .

2) If (21) is followed for all t, then, using (24):

$$p_{o} = p_{-1} - (1 - \overline{\mu})\varepsilon_{o}$$
 and:  
 $v_{o} + p_{o} = v_{-1} + p_{-1} + \overline{\mu}\varepsilon_{o} = m_{o} + \overline{\mu}\varepsilon_{o}$ 

which establishes that  $m_1 = m_0 + \bar{\mu}\epsilon_0$ . The equivalence for periods 2, 3,... is shown inductively.

If the monetary authority had full information on the current disturbance  $\varepsilon_t$ , it would increase  $m_t$  by  $\varepsilon_t$  thereby completely offsetting the shock. When the Fed has to wait one period, its reaction is, in equilibrium, smaller than the shock in the sense that  $\overline{\mu}$  is smaller than one by (10). At time t, the price level does move in the opposite direction from  $\varepsilon_t$  somewhat, thereby reducing the effect of the shock on aggregate output. Therefore, at t+1 it seems logical for the monetary authority to offset only that part of  $\varepsilon_t$  that is not already incorporated in the price level at t. This consistent plan then makes  $p_t$ , on average, the price level at t+1.

## IV. Social Contracts

### A. Generalities

The previous section studied the equilibrium that results from giving the monetary authority the power to change m from day to day. It is conceivable that society could expect to be better off if the government precommitted itself to a different rule. The consequences of rules of the form (25) and (26) with various sequences  $[\mu_t]$  will be studied in this section. Indeed, sequences  $[\mu_t]$  different from the consistent plan will turn out to minimize the expectation at time zero of W, Y and Z.  $\frac{8}{}$ 

Before proceeding to compute these optimal sequences, it is important to point out that no matter what the sequence of  $[\mu_t]$ , as long as equation (26) is used to set the money supply in the future, the monetary authority would employ rule (25) to set the current value of m. This is formalized in Proposition 3.

<u>PROP. 3</u>: Rule (25) minimizes W, Y and Z for t = 0 as long as rule (26) will be used for t  $\geq$  1 independently of the sequence  $[\mu_t]$ . <u>PROOF</u>: By using (8), (14), (15) and (26) and solving forwards for the price level:

$$P_{t} = \gamma^{t} P_{-1} + (1 - \gamma^{t}) (m_{o} - v_{-1}) + (1 - \gamma) [\epsilon_{t} (\frac{\mu_{t+1}}{\delta} - 1) + \sum_{j=0}^{t-1} \epsilon_{j} [(\mu_{j+1} - 1) (\frac{1 - \gamma^{t-j}}{1 - \gamma}) + \gamma^{t-j} (\frac{\mu_{j+1}}{\delta} - 1)]]$$
(27)

Therefore:

$$\begin{split} \mathbf{m}_{t} - \mathbf{v}_{t} - \mathbf{p}_{t} &= \gamma^{t} (\mathbf{m}_{o} - \mathbf{v}_{-1} - \mathbf{p}_{-1}) + \varepsilon_{t} [ -1 - (\frac{\mu_{t+1}}{\delta} - 1)(1 - \gamma)] \\ &+ \sum_{j=o}^{t-1} \varepsilon_{j} [(\mu_{j+1} - 1)(1 - (1 - \gamma^{t-j})) - (\frac{\mu_{j+1}}{\delta} - 1)(1 - \gamma)\gamma^{t-j}] \end{split}$$

$$= \gamma^{t} (m_{o} - v_{-1} - p_{-1}) + \varepsilon_{t} [\mu_{t+1} (\frac{\gamma - 1}{\delta}) - \gamma] + \sum_{j=0}^{t-1} \varepsilon_{j} \gamma^{t-j} [\mu_{j+1} (\frac{\delta + \gamma - 1}{\delta}) - \gamma].$$
(28)

While:

$$p_{t} - p_{t-1} = (\gamma^{t-1} - \gamma^{t})(m_{o} - v_{-1} - p_{-1}) + \varepsilon_{t}(1 - \gamma)(\frac{\mu_{t+1}}{\delta} - 1) + \sum_{j=0}^{t-1} \varepsilon_{j}\gamma^{t-j-1}(1 - \gamma)[\mu_{j+1}(\frac{\delta + \gamma - 1}{\delta}) - \gamma]$$
(29)

Therefore:

$$\frac{dE_{o}(m_{t} - p_{t} - v_{t})^{2}}{dm_{o}} = \gamma^{t}(m_{o} - v_{-1} - p_{-1}) \quad \text{and}$$

$$\frac{dE_{o}(p_{t} - p_{t-1})^{2}}{dm_{o}} = \gamma^{t-1}(1 - \gamma)(m_{o} - v_{-1} - p_{-1}).$$

Therefore setting  $m_0 = v_{-1} + p_{-1}$  minimizes W, Y, and Z with respect to  $m_0$ , as claimed.

In other words, independently of the  $\mu_t$  that will be chosen in the future, the monetary authority always desires the current money stock to be at a level such that the expectation of the current p is yesterday's p while the expectation of aggregate output is q<sup>\*</sup>. If a sequence of  $\mu$ 's different from the consistent plan turns out to be optimal from the point of view of minimizing W, Y or Z, it will never be enforced by a monetary authority whose loss function is W, Y or Z. In that sense, monetary rules like (25) and (26) with  $\mu_t$  different from  $\bar{\mu}$  are achievable only if the monetary authority precommits itself to abide by them in the future. In the next section it will be shown that some rules with  $\mu_t \neq \bar{\mu}$  are indeed superior to those with  $\mu_t = \bar{\mu}$ . Here I will provide an intuitive argument that explains this phenomenon.

As can be seen in equations (28) and (29) the impact of  $\varepsilon_0$  on current output and inflation depends on the future ( $\mu_1$ ) reponse by the monetary authority to  $\varepsilon_t$ . This results from the private sector's ability to forecast the activities of the government. The larger  $\mu_1$  is, the smaller the difference between the price the firm would like to charge at t=1 and the price it charged at t=-1. On the other hand, the monopolists do not want their prices at t=0 to differ too much from the prices they expect to charge at t=1 since they want to avoid price changes. Hence, the larger  $\mu_1$  is, the less current prices will respond to the current shock,  $\varepsilon_0$ . Naturally, a smaller response by prices means that at t=0 the rate of inflation is lower and the deviation of output from q\* is larger.

Therefore, the monetary authority at t=0 is concerned by the effect of  $\mu_1$  on current variables, it would like to control  $\mu_1$  . On the other hand, the monetary authority at t=1 will ignore the effects of  $\boldsymbol{\mu}_1$  on previous variables since "bygones are bygones." In this sense, the preferences of the monetary authority over the coefficients  $\mu_t$  vary over time, they are "time-inconsistent." Furthermore, the  $\mu$ 's that the monetary authority prefer at t=0 are superior from the point of view of minimizing W, Y or Z to those preferred at t=1. Therefore, society would be better off if the monetary authority precommitted itself somehow. That precommitment can be superior to the Strotz-Pollack equilibrium was pointed out already by Strotz (1955). In the literature on macroeconomics, various schemes that involve precommitment by the monetary authority have been called "optimal." (See for instance Calvo [1978], Kydland and Prescott [1977], [1980]). However, the feasibility of such a precommitment by the monetary authority is a far from trivial question. I imagine that these schemes would have to be written as social contracts into the laws governing the powers of the U.S. Congress and of the Federal Reserve Board. However,

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when these schemes are complicated feedback rules which are derived from barely tested models and where the measurements of many of the relevant variables are subject to errors and to interpretation, it seems absurd to pretend that these schemes could indeed by imposed in the foreseeable future.

Even so I will study the best social contracts of the form (25) and (26) which would be signed at time zero to minimize the functions W, Y and Z. $\frac{10}{}$  This is done both to prove more formally that the  $\mu$ 's desired by the monetary authority at time zero for the future are in general different from  $\bar{\mu}$  and to study the qualitative features of these "desirable" social contracts.

# IV. B. Output stabilization

In this subsection the sequence  $[\mu]_t$  is chosen at time zero to minimize W. Using (25), (28) and the properties of the  $\varepsilon$ 's:

$$E_{o}(m_{t} - p_{t} - v_{t})^{2} = \sigma^{2} \left\{ \left[ \mu_{t+1}(\frac{\gamma - 1}{\delta}) - \gamma \right]^{2} + \frac{t^{-1}}{\sum_{j=0}^{\delta}} \gamma^{2(t-j)} \left[ \mu_{j+1}(\frac{\delta + \gamma - 1}{\delta}) - \gamma \right]^{2} \right\}$$
(30)

And therefore using the definition of W:

$$W = \sigma^{2} \left\{ \sum_{t=1}^{\infty} \sum_{j=0}^{t-1} r^{t} \gamma^{2(t-j)} \left[ \mu_{j+1} \left( \frac{\delta + \gamma - 1}{\delta} \right) - \gamma \right]^{2} + \sum_{t=0}^{\infty} r^{t} \left[ \mu_{t+1} \left( \frac{\gamma - 1}{\delta} \right) - \gamma \right]^{2} \right\}$$

By changing the order of summation, one obtains:

$$W = \sigma^{2} \left\{ \sum_{j=0}^{\infty} \sum_{t=j+1}^{\infty} r^{t} \gamma^{2(t-j)} [\mu_{j+1}(\frac{\delta + \gamma - 1}{\delta}) - \gamma]^{2} + \sum_{j=0}^{\infty} r^{j} [\mu_{j+1}(\frac{\gamma - 1}{\delta}) - \gamma]^{2} \right\}$$
$$= \sigma^{2} \left\{ \sum_{j=0}^{\infty} r^{j} \left[ \frac{r\gamma^{2}}{(1 - r\gamma^{2})} [\mu_{j+1}(\frac{\delta + \gamma - 1}{\delta}) - \gamma]^{2} + [\mu_{j+1}(\frac{\gamma - 1}{\delta}) - \gamma]^{2} \right\}$$

The last term in brackets is, as can be seen by inspecting (28), the effect on output at t=j of the j<sup>th</sup> disturbance. W includes the discounted sum of these impact effects. The first term in brackets is the first after-effect of the j<sup>th</sup> disturbance. The j<sup>th</sup> disturbance also generates later aftereffects which are smaller by a factor  $\gamma$  Therefore, the first term in brackets gets weighed differently from the second term. W is proportional to  $\overline{W}$  where  $\overline{W}$  is:

$$\overline{W} = \sum_{j=0}^{\infty} \beta^{j} \left\{ \mu_{j+1}^{2} \left[ r \gamma^{2} \left( 1 + \frac{2(\gamma - 1)}{\delta} \right) + \left( \frac{\gamma - 1}{\delta} \right)^{2} \right] - 2\mu_{j+1} \left[ \gamma^{3} r + \frac{\gamma(\gamma - 1)}{\delta} \right] + \gamma^{2} \right\}$$
(30)

The second derivative of  $\overline{W}$  with respect to  $\mu_{j+1}$  is:

$$d^{2}\bar{W}/d\mu_{j+1}^{2} = 2\rho^{j} [\gamma_{r}^{2} (1 + \frac{2(\gamma - 1)}{\delta}) + (\frac{\gamma - 1}{\delta})^{2}]$$

This derivative is positive because  $(\gamma - 1)/\delta = (\gamma - 1)\gamma r > ... 5$  by (10). Therefore the first order conditions are sufficient for a minimum of  $\overline{W}$ . The first order conditions are the same for all  $\mu_i$ . They reduce to:

$$\mu_{w}[r\gamma^{2}(1 + \frac{2(\gamma - 1)}{\delta} + (\frac{\gamma - 1}{\delta})^{2}] = [\gamma^{3}r + \frac{\gamma(\gamma - 1)}{\delta}]$$
(31)

where  $\mu_{W}$  is the value of  $\mu$  which minimizes W for all t. Clearly  $\mu_{W}$  will in general be different from  $\overline{\mu}$ . The optimal  $\mu$ 's are constant because the shocks are uncorrelated and because, for every shock  $\varepsilon_{t}$  the valuation by W of the after-effects is always discounted by the same factor r relative to the valuation of the impact effects.

When r is equal to zero the monetary authority is concerned only with t = 0 and would like to set :

$$\mu_1 = \frac{\gamma o}{\gamma - 1}$$

which is negative and larger than one in absolute value by (10). This value of  $\mu_1$  ensures that the impact of  $\varepsilon_0$  on output at zero is nil by promising to "lean <u>with</u> the wind." The price level at zero is a linear combination of the previous price level, the current p\* and the expected future p\*'s. It will be equal to the present p\* only if either there is no disturbance at zero or the expected future p\*'s move by more than  $v_0$  and in the same direction. Note that this rule will generate a large impact of  $\varepsilon_0$  on output at t=1.

As r increases the monetary authority becomes increasingly concerned by the secondary effects of the shocks and therefore raise  $\mu_w$  towards  $\bar{\mu}$ .  $\bar{\mu}$  is, of course, the value of  $\mu$  that makes the secondary effects on output of any given disturbance zero. The effect of changing the discount factor can be ascertained by examining the sign of  $d\mu_w/dr$ :

$$\operatorname{sgn}\left\{ d\mu_{w}/dr\right\} = \operatorname{sgn}\left\{ \left[1 + \frac{2(\gamma - 1)}{\delta}\right]\gamma^{5}r - \left(\frac{\gamma - 1}{\delta}\right)^{3}\gamma \right\}$$

which is indeed positive.

Clearly  $\mu_w$  is superior from the point of view of W to both  $\bar{\mu}$  and to the rule proposed by Friedman (1948) which is here interpreted as requiring that  $\mu_t = 0$  for all t.<sup>11/</sup> However a rule with  $\mu = \mu_w$  is extremely hard to implement. Therefore the only realistic choices available to the government may well be the Friedman rule (which is simple) and the consistent plan (which requires simply that the Fed be given discretionary powers to set  $m_t$ ).<sup>12/</sup> I now compare the merits of these two rules. Setting  $\mu_t = 0$  for all t leads to:

$$\bar{W} = \gamma^2 / (1 - r)$$

Whether  $\overline{W}$  is larger (i.e. worse) under the consistent plan depends on the sign of:

$$\operatorname{sgn}\left\{ \frac{-r}{\mu^{2}} \left[ r\gamma^{2} \left( 1 + \frac{2(\gamma - 1)}{\delta} + \left( \frac{\gamma - 1}{\delta} \right)^{2} \right] - 2\overline{\mu} \left[ \gamma^{3} r + \frac{\gamma(\gamma - 1)}{\delta} \right] \right\} = \operatorname{sgn}\left\{ -r\gamma^{2}\delta(\delta - 1) + (1 - \gamma)(\delta + (\delta + \gamma - 1)) \right\}$$
(32)

Clearly, for r = 0, (32) is positive. The Friedman rule dominates the consistent plan since the impact effect at time zero will be lower when the policy involves no intervention than when it involves any "leaning against the wind". As the discount rate increases the consistent policy becomes relatively better but it may still be the case (depending on  $\gamma$  and  $\delta$ ) that even when the discount factor is unity W is smaller under Friedman's rule.

## IV. C. Price stabilization

In this subsection the sequence  $[\mu]_t$  which minimizes the expectation at time zero of Y is computed. Using (25) and the properties of the disturbances in (29) one obtains:

$$E_{o}(p_{t} - p_{t-1})^{2} = \sigma^{2} \left\{ (1 - \gamma)^{2} (\frac{\mu_{t+1}}{\delta} - 1)^{2} + \frac{t^{-1}}{\sum_{j=0}^{\gamma} \gamma^{2} (t^{-j-1}) (1 - \gamma)^{2} [\mu_{j+1} (\frac{\delta + \gamma - 1}{\delta}) - \gamma]^{2} \right\} (33)$$

By substituting (33) into the definition (17) the expectation of Y at t = 0 becomes:

$$Y = o^{2}(1 - \gamma)^{2} \left\{ \sum_{t=1}^{\infty} \sum_{j=0}^{t-1} r^{t} \gamma^{2(t-j-1)} \left[ \mu_{j+1} \left( \frac{\delta + \gamma - 1}{\delta} \right) - \gamma \right]^{2} + \sum_{t=0}^{\infty} r^{t} \left( \frac{\mu_{t+1}}{\delta} - 1 \right)^{2} \right\}$$

And by reversing the order of summation of the first term:

$$Y = \sigma^{2}(1 - \gamma)^{2} \left\{ \sum_{j=0}^{\infty} \sum_{t=j+1}^{\infty} r^{t} \gamma^{2}(t-j-1) \left[ \mu_{j+1}(\frac{\delta + \gamma - 1}{\delta}) - \gamma \right]^{2} + \sum_{t=0}^{\infty} r^{t} (\frac{\mu_{t+1}}{\delta} - 1)^{2} \right\}$$

$$= \sigma^{2}(1 - \gamma)^{2} \left\{ \sum_{j=0}^{\infty} r^{j} \left[ \frac{r}{1 - r\gamma^{2}} \left[ \mu_{j+1} \left( \frac{\gamma + \delta - 1}{\delta} \right) - \gamma \right]^{2} + \left( \frac{\mu_{j+1}}{\delta} - 1 \right)^{2} \right] \right\}$$

Again Y consists of the discounted impact effects captured by the last term in addition to the secondary effects which get weighed differently. Y is proportional to  $\overline{Y}$  where  $\overline{Y}$  is :

$$\bar{Y} = \sum_{j=0}^{\infty} r^{j} \left\{ \mu_{j+1}^{2} [r(\delta - 1)(\delta - 1 + 2\gamma) + 1] - 2\mu_{j+1} [r(\delta - 1)\delta\gamma + \delta] + \delta^{2} \right\}$$
(34)

The second derivatives of  $\overline{Y}$  with respect to  $\mu_{i+1}$  are:

$$d^{2}\bar{Y}/d\mu_{j+1}^{2} = 2r^{j}[r(\delta - 1)(\delta + 2\gamma - 1) + 1]$$

and they all are positive. Therefore the first order conditions determine the  $\mu_{\rm v}$  which minimizes Y:

$$\mu_{v}[r(\delta - 1)(\delta + 2\gamma + 1] = r(\delta - 1)\gamma\delta + \delta$$
(35)

The policy which minimizes Y will therefore in general be different from  $\mu_t = \overline{\mu}$ . Proposition 3 then allows one to say that the loss function Y leads to time inconsistent preferences over the sequence  $[\mu]_t$ . What makes the sequence of  $\mu$ 's which minimize Y constant is also that the secondary effects of any shock get discounted by r relative to the impact effect of that shock.

A discount rate equal to zero leads the Federal Reserve to concentrate its efforts exclusively on the impact effect of  $\varepsilon_0$ . To encourage the firms to keep prices constant at zero the monetary authority must promise more than that it will offset the shock at time one. It must guarantee that it will increase m by more than  $\varepsilon_0$ . In fact, for  $\rho = 0$ :

$$\mu_{\rm v} = \delta > 1.$$

The feedback rule that stabilizes prices most is quite different from the rule that stabilizes output most. When the discount rate is zero, the best feedback coefficients  $\mu$  even lie on opposite sides of the unit circle. When committing itself to a social contract the authority faces a tradeoff beetween output and price stability at time zero. No such tradeoff exists when only consistent rules are considered. The tradeoff takes the following form: The same  $\mu$ , i.e.  $\bar{\mu}$ , eliminates the secondary effects of any shock on output and inflation. However, the impact effect on output of any given shock is reduced by encouraging the prices to respond more to the shocks, that is by promising to amplify the shocks in the future. Instead the impact effect of any given disturbance on the rate of inflation is reduced by making prices less sensitive to the current  $\varepsilon$ 's which is achieved by promising to counteract the current shock in the future.

The  $\mu$  that minimizes Y ought to decrease towards  $\mu$  as the discount rate increases and the secondary effects become more important. Indeed:

$$sgn [d\mu_y/dr] = sgn [(\delta - 1) - (\delta - 1)(\gamma - 1 + 2\gamma)] < 0.$$

I now turn to comparing from the point of view of  $\overline{Y}$  Friedman's rule and  $\overline{\mu}$ . Setting  $\mu_{t} = 0$  for all t leads to:

$$\tilde{a} = \delta^2 / (1 - r)$$

 $\widetilde{Y}$  will be larger under  $\widetilde{\mu}$  if the following sign is positive:

$$\sup \left\{ \frac{-2}{\mu^2} [r(\delta - 1)(\delta + 2\gamma - 1) + 1] - 2\mu [r(\delta - 1) + 1]\delta \right\} = \\ \operatorname{sgn} \left\{ -r[(\delta - 1)^2 \gamma] - (\delta + \gamma - 1) - (\delta - 1) \right\}$$

which is negative. Furthermore it becomes even more negative as  $\rho$  increases from zero towards one. Therefore the consistent plan always stabilizes prices more than the Friedman rule. This is due to two reasons: 1) Since firms

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know that the central bank will lean against the wind in the future they change their prices by less under the consistent plan than under a plan in which the monetary authority is committed to inaction; 2) the secondary effects on price changes of the disturbances are eliminated when the consistent plan is followed.

## IV. D. Output and price stabilization

If the central bank wishes to minimize Z, the characteristics of optimal policies depend crucially on the parameter  $\theta$ . This parameter captures the relative importance of price and output stability in the objective function of the monetary authority. When social contracts are considered, output and price stability become competing objectives. The parameter  $\theta$  determines which objective is given more weight.

If  $\mu_{\tau}$  is chosen optimally to minimize Z, it satisfies:

$$\lambda_{1}(\mu_{z} - \mu_{w}) + \theta \lambda_{2}(\mu_{z} - \mu_{y}) = 0 \qquad \text{or:}$$

$$\mu_{z} = \frac{\lambda_{1}}{\lambda_{1} + \theta \lambda_{2}} \mu_{w} + \frac{\theta \lambda_{2}}{\lambda_{1} + \theta \lambda_{2}} \mu_{y} \qquad (36)$$

with:

$$\lambda_{2} = (1 - \gamma)^{2} [r(\delta - 1)(\delta - 1 + 2\gamma) + 1]$$
 and:  
$$\lambda_{1} = r\gamma^{2} (1 + \frac{2(\gamma - 1)}{\delta}) + (\frac{\gamma - 1}{\delta})^{2}$$

where (10) ensures that  $\lambda_1$  and  $\lambda_2$  are positive. Therefore  $\mu_z$  is a convex combination of  $\mu_w$  and  $\mu_y$  with the weight of the latter increasing with  $\theta$ . As  $\theta$  increases, the optimal  $\mu$  becomes closer to the one that brings about the most price stability. An analogous argument can be applied to the derivative of  $\mu_z$  with respect to the discount rate. It is a convex combination of  $d\mu_w/dr$  and of  $d\mu_y/dr$  with the weight of the latter equal to  $(\theta\lambda_2)/(\lambda_1 + \theta\lambda_2)$  where this weight is obviously increasing in c.

Finally the choice beetween Friedman's rule and the consistent plan by a monetary authority with loss function Z depends on the parameter  $\theta$  as well as on the discount factor r. As price stability becomes more important and as the secondary effects of the shocks get weighted more heavily the consistent plan progressively provides more welfare relative to a constant money supply. Unfortunately it is not possible to rank uniquely  $\overline{\mu}$  and  $\mu = 0$ even when the parameters of the authority's loss function correspond to those of the loss function that is consistent with the private sector's behaviour.

## V. Conclusions

The results of this paper bear on two issues. First, they describe the alternatives open to the monetary authority in the context of a fairly realistic model of a monetary economy. This model implies a Phillips curve; periods of relatively high output are also those with relatively large inflation rates. Furthermore deviations of output from long run output tend to persist for some time.

Second, this paper points out yet another difficulty for the choice of monetary policies in models in which producers have rational expectations. These models require the private sector to know the rules that the monetary authority will follow both today and in the future. When the <u>future</u> rules affect the current decisions of producers, then the preferences of the

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monetary authority will typically be time-inconsistent. This point is also made in Fischer (1980). In the future, the monetary authority will think that its rules do not influence the past and will therefore desire a different rule from the one it would like to precommit itself to today. If private agents observe the nominal shocks as they occur and know that the Federal Reserve will react to them in the future, as is assumed in this paper, then indeed the rule with which the central bank will set the money supply in the future will often affect the current behavior of the private sector. In the model of the paper in which firms spread their price increases due to costs to changing prices, the future monetary rule does influence today's pricing decisions and the preferences of the monetary authority turn out to be time inconsistent.

The monetary authority would therefore like to precommit itself. Ideally it would like to precommit itself to a feedback rule. This feedback rule will dominate a totally neutral rule like the one prescribed by Friedman. Within a certain class of rules the one that most stabilizes output generally requires that more money be supplied when output is relatively high. This will create an incentive for firms to adjust their prices substantially as they observe a nominal shock and thereby decrease the impact effect of the shock on output. Instead, the rule that most stabilizes prices requires that more money be supplied when output is relatively low. Then the firms will only slightly alter their prices in response to a nominal shock.

It is often thought that when the preferences of the monetary authority are time inconsistent Friedman's rule which involves a neutral policy is better than the consistent plan. In this paper the Strotz-Pollack

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equilibrium of the game among generations of central banks with full discretionary powers over the money stock actually dominates keeping the money supply constant as long as the monetary authority is sufficiently concerned about price stability. Instead, when the authority is worried mainly about output stability, it will prefer to precommit itself to a neutral policy over following the consistent plan.

## FOOTNOTES

- 1. This cost function suggests that only goods are required to produce goods. In fact the model can be extended to include a classical labor market without affecting any of the results below. This extension is presented in Rotemberg [1981b].
- A search theoretic version of this argument is presented in Stiglitz [1979].
- 3. This assumption ensures that all firms adjust their prices at the same speed. In the absence of this assumption the equilibrium of this economy would be intractable.
- 4. In this model the government would only need to awn one firm to discover the current value of v since all shocks are aggregate shocks. If firms were faced with mixtures of aggregate and firmspecific disturbances the qualitative results of this paper would probably be obtained in terms of the difference beetween the actual aggregate shocks and those inferred by the government.
- 5. The Fed today is the dominant player of this game since it makes its move before the other players and knows their reactions. However, in a sense, the Fed today has very little leverage on the future Feds. After t, the Fed at t has no tools with which to threaten them. Besides its ability to pick m<sub>t</sub> the Federal Reserve at t has no other mechanism with which to influence future m's.
- Goldman (1980) shows that the Strotz-Pollack equilibrium exists under quite general conditions.

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7. I do not know whether this equilibrium is unique. When the planning horizon of the monetary authority is finite and ends at T, the optimal rule at T and hence for all previous periods is indeed (21). This fact does not depend on whether the firm's planning horizon ends at T or not.

If v follows a more complicated stochastic process then the one given by (15) then the consistent plan would be:

 $m_{t} = E_{t-1}(v_{t}) + P_{t-1}.$ 

8.

Buiter (1980) uses rules with the same structure as (25) and (26) to solve a different problem.

9.

Therefore, in the absence of precommitment the money supply follows the same path wether the Federal Reserve today takes into account the reactions of the future authorities (in Hammond (1976)'s terminology "sophisticated" choice) or wether it naively believes the future Federal Reserves will pick the  $\mu$ 's that are currently preferred. This property of the preferences of the monetary authority over sequences  $[\mu]_t$  is called "essential consistency" by Hammond. The preferences in the Kydland and Prescott (1977) "Inflation vs. Unemployment" and in the Calvo (1978) models also exhibit this property. "Essential consistency" ensures that the choices of the consistent plan can be derived from some well behaved loss function albeit not necessarily the actual loss function of the monetary authority.

10.

More complicated social contracts of the form:

$$m_{o} = E_{-1}(v_{o}) + p_{-1}$$
(i)  
$$m_{t} = m_{t-1} + \sum_{j=0}^{t} \mu_{tj} \varepsilon_{t-j}$$
(ii)

could have been studied. While such social contracts could decrease W, Y and Z relative to the levels attainable with contracts of the form (25) and (26) the best such contracts would have the same qualitative features as the best contracts of the type studied in this paper.

On the other hand, if  $v_t$  follows a process whose first difference has a stationary MA representation then (ii) can be made equivalent to (26) with v following a random walk.

Let:

$$\mathbf{v}_{t} = \mathbf{v}_{t-1} + \sum_{j=0}^{\infty} \omega_{j} \varepsilon_{t-j} \qquad \qquad \omega_{0} = 1.$$
(iii)

Then, letting  $\mu_{tj} = \omega_{j}$  for j different from zero immediatly makes the  $\mu_{to}$  of (ii) equivalent to the  $\mu_{t}$  of (26).

Chow [1980] proposed another type of social contract in which a stationary feedback rule on the state variables was chosen by the monetary authority at time zero. For the model of this paper Prop. 3 makes it clear that stationarity is not a desirable property for the monetary rule from the point of view of the Federal Reserve signing the social contract. In particular the **station**ary feedback rule most preferred by the central bank at time zero will depend on the initial conditions. Instead the optimal  $\mu$ 's of (26) will not depend on any initial conditions as will be seen below.

11.

Friedman [1974] and his followers have actually advocated a constant growth rule for the money stock where the growth rate of M is supposed to equal the "natural" growth rate of GNP. In the model of this paper output doesn't grow and therefore the rule corresponding to Friedman's ideas would indeed be a constant money stock. Furthermore, deterministic growth rates of money do have real effects in this model as is shown in Chapter II.

12. Kydland and Prescott [1980] also stress that when the best rule from the point of view of the monetary authority at time zero involves time inconsistency one should study simple rules with "good operating characteristics."

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