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AN EMPIRICAL IMPLEMENTATION

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SSM WP#1665-85 Revised: May 1985

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ABSTRACT

This paper studies household asset demands by allowing certain assets to contribute directly to utility. It estimates the parameters of an aggregate utility function which includes both consumption and liquidity services. These liquidity services depend on the level of various asset stocks. We apply these estimates to investigate the long- and short-run interest elasticities of demand for money, time deposits, and Treasury bills. We also examine the impact of open market operations on interest rates, and present new estimates of the welfare cost of inflation.

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This paper studies households' demand for different assets by allowing certain assets to contribute directly to household utility. We permit the utility function to capture the "liquidity" services of money, certain time deposits and even some government securities. Our approach yields estimates of the utility function parameters which can be used to study the effects of a variety of changes in asset returns. We investigate how asset holdings and consumption react to both temporary and permanent changes in real and nominal returns. We also study how the government's financial policy affects the returns on various securities.

Our approach provides an integrated system of asset demands of the form which Tobin and Brainard (1968) advocate for studying the effects of government interventions in financial markets. It provides an attractive and tractable alternative to the atheoretical equations which are commonly used to study the demand for money and other assets. Those equations cannot be interpreted as the rational response of any economic agent to changes in the economic environment. In particular, it is unlikely that they will remain stable in the face of changes in the supply of various non-monetary assets.

Our approach to studying asset demands is somewhat controversial. Its opponents argue that assets do not yield utility directly. They explain that rate of return dominated assets such as money are held because they reduce transactions costs, and argue that these should be modelled explicitly. Unfortunately, explicit models with transactions costs are too restrictive to be useful in analyzing aggregate data. Baumol (1952) and Tobin (1956) assumed that

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1. Theoretical work in monetary economics often makes this assumption. The Sidrauski (1967) model is part of most economists' standard tool kit; it has been extended by Fischer (1979), Calvo (1979), and Obstfeld (1984a, 1984b).
the individual receives a constant income and faces a constant interest rate. By assuming that the individual consumed at a constant rate they derived the optimal timing of financial transactions. If individuals are uniformly distributed over the time of their last visit to their financial intermediary, aggregate money holdings can be written as the average holdings of an individual, which are given by the famous square-root formula. This approach suffers from a number of drawbacks. Even assuming that consumption is constant, the optimal timing of individual transactions is extremely hard to compute when interest rates and income vary stochastically. Such a computation is well beyond the modern transactions-based models of Jovanovic (1982), Grossman and Weiss (1983), Romer (1984), and Rotemberg (1984). Moreover, the assumption of constant consumption cannot be justified if the individual is maximizing utility from consumption unless the real rate of return on money is equal to the discount rate. Thus, while Goldfeld (1973) appeals to transactions-based models to justify his money demand regressions, these models provide an unacceptable basis for empirical work.

On the other hand, the objections to estimating the utility flow of liquidity services seem to apply equally well to the estimation of the demand for many durable goods. Like many durables, money is not utilized constantly, but in bursts. However, just like some durables, even money which is not used provides some utility in the form of security. Whether or not money's services provide utility in the same fashion as other goods is a moot point. Various consumer goods provide different "types" of utility, and to single out money services as a particular variety which is unworthy of inclusion in a consumer's utility function seems arbitrary at best.
A number of researchers including Barnett (1980, 1983), Chetty (1969), Dieuwer (1974), Donovan (1978), Ewis and Fisher (1984), Husted and Rush (1984), Moroney and Wilbratte (1974), and Philips (1978) have attempted to estimate a utility function for assets. However, their attempts have encountered a number of difficulties. First, Chetty (1969) and his followers fail to recognize that when a consumer chooses to hold an asset with a relatively low rate of return, he will have to reduce his consumption at some point. To evaluate this loss in consumption, it is necessary to specify and measure the consumer's marginal utility of consumption.

A second problem, which affects all previous work, arises from the inherent uncertainty of the opportunity cost of money. The alternative to holding money or other assets which yield liquidity services is to hold assets with uncertain returns. Therefore, the opportunity cost of these assets is a random variable at the time when the consumer allocates his portfolio. This makes it inappropriate to model the consumer's portfolio allocation problem as one of choosing expenditures (opportunity cost times quantity held) on different assets. Yet, Donovan (1978), Barnett (1980), Ewis and Fisher (1984), Philips (1978), and Barnett (1983) model expenditure on individual assets as a function of expenditure on a bundle of goods and services which includes liquid assets.

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1. Feige and Pierce (1977) provide a partial survey of this literature.

2. This problem also plagues the construction of Divisia monetary aggregates, advocated by Barnett (1980, 1983). With standard nondurable goods, the rate of growth of a Divisia quantity aggregate equals the inner product of current expenditure shares and quantity growth rates. The expenditure on liquidity services, however, is unknown at the time the services are purchased. Divisia aggregation therefore appears infeasible. Any measure of the "expected" cost of liquidity services is likely to be problematic since risk adjustments of this measure are inherently arbitrary. On the other hand, the ex post cost of these services is extremely influenced by random events that play no role in the decision to consume liquidity. Thus, the only defensible liquidity aggregator seems to be one that relies on estimated parameters of a utility function as in Chetty (1969).
The first three studies focus only on asset choices, while the last two include consumption as well.

Our approach follows Grossman and Shiller (1981), Hansen and Singleton (1982, 1983), Mankiw, Rotemberg and Summers (1985), and others who attempt to estimate the parameters of a representative individual's utility function. We estimate the first-order conditions of this individual's maximization problem by instrumental variables. By analyzing a utility function which includes both consumption and asset holdings as arguments and by explicitly recognizing the uncertainty faced by this individual we circumvent a number of the problems which have plagued previous research in this area. Moreover, by studying consumption and portfolio choices simultaneously we can consider a broad class of consumer responses to changes in the economic environment.

This paper is organized into five sections. The first outlines the representative consumer model and explains the factors which motivated our choice of a parametric utility function. Section II describes our data and estimation procedure. Our basic results are presented in the third section, and the estimated parameters are used for comparative statics calculations in Section IV. A brief conclusion evaluates our findings on the usefulness of the assets-in-the-utility-function model, and suggests several directions for future work.

1. However, our approach, as previous ones, still requires some restrictions on the residuals which allows us to identify demand functions. In particular we require that any random shocks to preferences be uncorrelated with our instrumental variables.
I. The Theoretical Background

We maintain the convenient fiction that movements in per capita consumption, as well as real asset holdings, can be attributed to the optimizing behavior of a rational representative consumer. He is infinite-lived, has constant preferences, and derives utility by consuming and by holding assets. In principle, it would be possible to allow a wide variety of different assets to yield utility. We focus only on those which (i) constitute a substantial fraction of household wealth, and (ii) have easily measured market values and rates of return. This limits us to four asset classes: money, time deposits, short-term marketable government debt, and corporate equity.

We begin with a specification of preferences which is additively separable across time, and then examine a case in which costs of adjusting asset stocks violate this restriction. In the additively separable case, the consumer's expected discounted utility at time $t$ may be written

$$V_t = E_t \sum_{t=1}^{\infty} \rho^{t-t} U(C_t, \frac{M_t}{P_t}, \frac{S_t}{P_t}, \frac{G_t}{P_t}).$$  (1)

The expectations operator $E_t$ is conditional on information available at $t$; $\rho$ is a discount factor, assumed constant through time. The four arguments of the period-by-period utility function are real consumption, $C_t$, real money holdings, $M_t/P_t$, real savings and time deposits, $S_t/P_t$, and real holdings of short-term government debt, $G_t/P_t$.\(^1\) Equity holdings, represented as $Q_t$, provide the numeraire asset in defining preferences.\(^2\) They are not a direct

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1. Holdings of non-government short term debt are excluded from our analysis. Household holdings of open market commercial paper are much smaller than T-bills throughout most of our sample.

2. If all assets give utility directly, one could redefine preferences to exclude the asset which gives the least utility and attribute its utility to future consumption.
source of utility. The utility function $U(\cdot)$ is concave and increasing in consumption and all three asset stocks.

Provided the optimal path for consumption and asset holding lies in the interior of the budget set, perturbation arguments may be used to establish some of its characteristics. In particular, along the optimal path the following first-order conditions must hold:

\[(EC): \quad E_t \left[ \frac{\partial U}{\partial C_t} - \rho \frac{P_t(1+r_{Et})}{P_{t+1}} \frac{\partial U}{\partial C_{t+1}} \right] = 0 \]  
\[(M): \quad E_t \left[ \frac{\partial U}{\partial K_t} - \rho \frac{P_t r_{Et}}{P_{t+1}} \frac{\partial U}{\partial C_{t+1}} \right] = 0 \]  
\[(S): \quad E_t \left[ \frac{\partial U}{\partial S_t} - \rho \frac{P_t(r_{Et}-r_{St})}{P_{t+1}} \frac{\partial U}{\partial C_{t+1}} \right] = 0 \]  
\[(G): \quad E_t \left[ \frac{\partial U}{\partial G_t} - \rho \frac{P_t(r_{Et}-r_{Gt})}{P_{t+1}} \frac{\partial U}{\partial C_{t+1}} \right] = 0 \]

where $P_t$ is the price of consumption at $t$, $r_{Et}$ is the nominal return on equity between $t$ and $t+1$, and $r_{Gt}$ and $r_{St}$ are the nominal returns on government debt and time deposits, respectively.

The Euler equation for consumption (EC) states that along an optimal path the representative individual cannot raise his expected utility by foregoing one unit of consumption in period $t$, investing his savings in equities, and consuming the proceeds in period $t+1$. The utility cost of giving up a unit of consumption in period $t$ is $\partial U/\partial C_t$. The expected utility gain from reducing $C_t$ is given by $E_t [\rho \frac{\partial U}{\partial C_t} \frac{P_t(1+r_{Et})}{P_{t+1}}]$. Equating the cost and gain from this perturbation
yields the first-order condition (EC). If there are several assets which are traded by the representative consumer but yield no utility, then (EC) should also hold with \( r_{Et} \) replaced by the return on any of these assets.

Euler equation (M) specifies that the discounted utility sum cannot be increased by holding one dollar less of money at time \( t \), investing the dollar in equities, and consuming the proceeds at time \( t+1 \). The foregone utility associated with a one dollar reduction in money holding is \( \left[ \frac{\partial U}{\partial P_t} \cdot \frac{1}{P_t} \right] \). Switching one dollar from money to equities at \( t \) increases real wealth at \( t+1 \) by \( r_{Et} \), since money yields no nominal return while equity does. The expected gain in utility if these higher proceeds are consumed in period \( t+1 \) is \( E_t\left[ C \cdot \frac{\partial U}{\partial C_{t+1}} \cdot \frac{r_{Et}}{P_{t+1}} \right] \).

Equating this to the foregone utility yields (M).

The last two Euler equations, (S) and (G), equate the costs and benefits of transferring one dollar from Treasury bills or savings deposits into equities for one period at time \( t \). The utility cost of holding one dollar less of government bonds is \( \frac{\partial U}{\partial G_t} \cdot \frac{1}{P_t} \). The expected utility contribution of the extra next-period consumption which results from this asset transfer is \( E_t\left[ C \cdot \frac{\partial U}{\partial C_{t+1}} \cdot \frac{(r_{Et}-r_{G_t})}{P_{t+1}} \right] \).

Equating these gains and losses yields equation (G); a similar argument can be applied to savings deposits to derive (S).

Given a specification of preferences, the budget constraint, and the conditional distributions of all future prices and rates of return, we could use (1) to find the representative consumer's consumption and asset holdings at time \( t \). However, solving the consumer's problem analytically is almost impossible in all but a few restrictive cases, such as the linear-quadratic-Gaussian. We shall
therefore follow previous authors in estimating the parameters of \( U \) from the first order conditions which result from maximizing \( (1) \) subject to a budget constraint.

One feature of equations \((2)-(5)\) is that if \( r_{Gt} \) and \( r_{St} \) are known at \( t \), then \((EC)\), \((M)\) and \((S)\), and \((EC)\), \((M)\), and \((G)\), may be combined to obtain two nonstochastic equations which should be satisfied at each moment:

\[
\frac{\partial U}{\partial S_t} - (1 + r_{St}) \frac{\partial U}{\partial M_t} = r_{St} \frac{\partial U}{\partial C_t} \tag{6}
\]

and

\[
\frac{\partial U}{\partial G_t} - (1 + r_{Gt}) \frac{\partial U}{\partial M_t} = r_{Gt} \frac{\partial U}{\partial C_t} \tag{7}
\]

The first equation requires that a consumer cannot raise his utility by reducing his holdings of money by \((1 + r_{Gt})\) dollars in period \( t \), raising his holdings of Treasury bills by one dollar to ensure that the original consumption plan is still feasible, and consuming the difference \( (r_{Gt}) \) today. Equation \((7)\) requires that a similar set of asset swaps, performed this time with time deposits and money, cannot raise utility.\(^2\) Equations \((6)\) and \((7)\) are similar to those considered by Chetty (1969) and his followers. However, since consumption is not part of their analysis, they either neglect or mismeasure the term on the right hand side of these equations.

\(^1\) In practice, over short intervals only \( r_{St} \) may be treated as known at \( t \).

\(^2\) There are even non-stochastic relationships between the marginal utilities of different asset stocks which must be satisfied by the optimal plan. For example, by manipulating \((EC)\), \((M)\), \((S)\), and \((G)\), one can show that:

\[
(\frac{\partial U}{\partial M_t} - \frac{\partial U}{\partial S_t})r_{Gt} = (\frac{\partial U}{\partial M_t} - \frac{\partial U}{\partial C_t})r_{St}.
\]
This analysis shows that if expectational errors are the only source of error in our equations, then our system of first order conditions can, by suitable linear combination, be transformed into two stochastic and two nonstochastic equations. This implies that the error covariance matrix for the system of equations which we estimate could be singular. If errors also result from random shocks to preferences, however, then this problem does not arise. For example, if the consumer's utility function includes terms such as \( v_{M_t}^N \) and \( v_{S_t}^S \), where the \( v_{M_t} \) and \( v_{S_t} \) are stochastic, then the marginal utility which the consumer derives from each asset will also be random. This would make the covariance matrix nonsingular.

We assume that the representative consumer's preferences are given by:

\[
U(C_t, \frac{N_t}{P_t}, \frac{S_t}{P_t}, \frac{G_t}{P_t}) = \frac{1}{\sigma} \left\{ C_t^{\beta} \cdot \left[ \delta_M^N \left( \frac{N_t}{P_t} \right)^{\gamma} + \delta_S^S \left( \frac{S_t}{P_t} \right)^{\gamma} + (1-\delta_S-\delta_M) \left( \frac{G_t}{P_t} \right)^{\gamma} \right]^{\frac{1-\beta}{\sigma}} \right\}.
\]  

This utility function is an amalgam of standard ingredients. It exhibits constant relative risk aversion in an aggregate of consumption and liquidity services. This aggregate is Cobb-Douglas in consumption and liquidity, ensuring that more consumption raises the marginal utility of liquidity and vice versa. Finally, our liquidity measure is a CES function of our three assets.\(^1\) Such functions have been pioneered by Chetty (1969) and used by Barnett (1980) and Husted and Rush (1984), among others.\(^2\)

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\(^1\) Assuming that the theoretical concept of money corresponds to our measure of liquidity, our utility function is identical to the one used in Fischer (1979), Calvo (1979), and Obstfeld (1984a, 1984b). Indeed, in Obstfeld's papers, the numerical values of \( \beta \) and \( \sigma \) are of critical importance.

\(^2\) Chetty (1969) actually uses a slightly more general functional form in which each asset is allowed its own \( \gamma \). However, since he focuses only on the instantaneous utility function, he cannot identify the exponent of this CES aggregate.
The system of Euler equations which results from this parametrization is:

\[
(\text{EC}): \quad E_t[\rho \frac{P_t(1+r_{Et})}{P_{t+1}} \left( \frac{C_{t+1}^{\sigma_{\beta}-1}}{C_t} \right)^Y - \frac{\sigma(1-\beta)}{C_t} L_{t+1}^Y] = 0 \quad (9)
\]

\[
(M): \quad E_t[\sigma_{\beta} L_t^Y \delta_M(\frac{M}{P_t})^Y - \frac{\sigma(1-\beta)}{C_t} \delta_M \frac{P_t r_{Et}}{P_{t+1}} C_{t+1}^{\sigma_{\beta}-1} L_{t+1}^Y] = 0 \quad (10)
\]

\[
(S): \quad E_t[\sigma_{\beta} L_t^Y \delta_S(\frac{S}{P_t})^Y - \frac{\sigma(1-\beta)}{C_t} \delta_M \frac{P_t (r_{Et} - r_{St})}{P_{t+1}} C_{t+1}^{\sigma_{\beta}-1} L_{t+1}^Y] = 0 \quad (11)
\]

\[
(G): \quad E_t[\sigma_{\beta} L_t^Y (1-\delta_M - \delta_S)(\frac{G_t}{P_t})^Y - \frac{\sigma(1-\beta)}{C_t} \delta_M \frac{P_t (r_{Et} - r_{St})}{P_{t+1}} C_{t+1}^{\sigma_{\beta}-1} L_{t+1}^Y] = 0 \quad (12)
\]

where we use the shorthand

\[
L_t = [\delta_M(\frac{M}{P_t})^Y + \delta_S(\frac{S}{P_t})^Y + (1-\delta_M - \delta_S)(\frac{G_t}{P_t})^Y]^Y
\]

(13)

to denote total liquidity services n period t. We report estimates of \(\{c, \rho, \gamma, \beta, \delta_M, \delta_S\}\) based on (9)-(12) in Section III.

In usual representative consumer models, \(\rho = 1/(1+Y)\) where \(Y\) is the consumer's rate of time preference. In our specification, however, \(\rho\) could also capture shifts over time in consumer preferences for consumption versus liquidity. For example, if \(\lambda_t\) is a parameter defining taste shifts and

\[1. \text{ The (EC) equation corresponds to the (EC) equation above divided through by } \frac{\partial U}{\partial C_t}.\]
The estimated time preference parameter will equal \( \rho / \lambda^\beta \sigma \).

The second set of preferences which we consider allows for costs of portfolio adjustment.\(^1\) We assume that individuals face utility costs proportional to the square of the percentage change in their nominal asset holdings.\(^2\) Their expected discounted utility is therefore

\[
V_t' = E_t \sum_{\tau=t}^{\infty} \rho^{\tau-t} [U(C_{\tau}, \frac{M_{\tau}}{P_{\tau}}, \frac{S_{\tau}}{P_{\tau}}, \frac{G_{\tau}}{P_{\tau}}) - \frac{\Theta_M}{2} \left( \frac{M_{\tau}-M_{\tau-1}}{M_{\tau-1}} \right)^2 \\
- \frac{\Theta_S}{2} \left( \frac{S_{\tau-1}}{S_{\tau-1}} \right)^2 - \frac{\Theta_G}{2} \left( \frac{G_{\tau-1}}{G_{\tau-1}} \right)^2].
\]

The first order conditions which must be satisfied by the optimal consumption-portfolio plan corresponding to these preferences are:

\[
(EC'): E_t \left[ \frac{\partial U}{\partial C_t} - \rho \frac{P_t(1+\tau E_t)}{P_{t+1}} \frac{\partial U}{\partial C_{t+1}} \right] = 0.
\]

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1. This may well be the implicit justification for the inclusion of quasi-differences of assets in Barnett's (1980) utility function.

2. If it is a matter of physically adjusting one's asset stock the nominal and not the real magnitude is relevant. However, a better specification would recognize adjustments to money that are caused by consumption expenditures.
\( (K') : E_t \frac{\partial U}{\partial K_t} = -\frac{P_t E_t}{t+1} \frac{\partial U}{\partial C_{t+1}} - \frac{K_{t+1}}{K_t} \cdot (\frac{K_{t+1}-K_t}{K_t}) = 0 \) \( (18) \)

\( (S') : E_t \frac{\partial U}{\partial S_t} = \rho_p \left( \frac{r_{E_t} - r_{S_t}}{P_{t+1}} \right) \frac{\partial U}{\partial C_{t+1}} + \rho_{G_S} \left( \frac{S_{t+1} - S_t}{S_t} \right)^2 = 0 \) \( (19) \)

\( (G') : E_t \frac{\partial U}{\partial G_t} = \rho_p \left( \frac{r_{E_t} - r_{G_t}}{P_{t+1}} \right) \frac{\partial U}{\partial C_{t+1}} + \rho_{G_{G_t}} \left( \frac{G_{t+1} - G_t}{G_t} \right)^2 = 0 \) \( (20) \)

Note that in this case there is no transformation of the first order conditions which holds nonstochastically.\(^1\) Section III reports estimates of this system of equations assuming \( U(*) \) is given by (16).

The first-order conditions \((EC), (K), (S), \) and \((G)\) characterize a single individual's optimization problem. Their application to aggregate data is more problematic. Rubinstein (1975) showed that if all individuals have identical, separable utility functions belonging to a certain class, and if all risky assets including human capital are freely traded, then households will have perfectly correlated consumption streams and the representative consumer model may be applied to aggregate data. These conditions are of course not met in reality. All individuals do not choose perfectly correlated consumption streams. In particular, one person's spending of his money tends to raise someone else's money holdings. Taxes also induce systematic differences in the opportunity sets available to different agents. However, numerous previous studies have tried to measure the violence induced by the representative consumer model; our investigation may be viewed as a further study in that tradition.

\(^1\) Note that with costs of adjustment it is even harder to follow the two-step budgeting procedure used by Bernett (1980). This occurs because the marginal rate of substitution between two assets at \( t \) depends on the expected levels on assets at \( t+1 \) which, in turn, depend on assets at \( t \).
II. Data and Estimation

II. A. Data

We employ aggregate time series data on asset holdings by the household sector. These data, computed each quarter by the Federal Reserve Board and published in the Flow of Funds sector balance sheets, are available since the first quarter of 1952. Our money variable, $M_t$, is the sum of demand deposits and currency. We measure $S_t$ as households’ total holdings of time and savings deposits, and $G_t$ is defined as holdings of short-term marketable government debt.\(^1\)

The time series for $G$ is based on unpublished Federal Reserve data which were not collected after the second quarter of 1982.

There are several problems with our data series on asset holdings. First, household currency holdings are computed as a residual after subtracting corporate currency holdings from the outstanding currency stock. Errors can arise if currency has flowed abroad, since it will be allocated mistakenly to the U.S. household sector. This problem is particularly acute if there are movements through time in the fraction of the U.S. currency stock which is held abroad, since this will induce spurious variation in measured household money holdings. Despite this difficulty, these data have been used in almost all previous investigations of money demand.\(^2\)

A second problem which is less significant for money than for other assets is that the "household sector" encompasses households as well as personal trusts and nonprofit institutions. Relative to individuals, the last two groups are

\(^1\) We also experimented with another measure of short-term debt, computed as the sum of Treasury bill holdings, open market paper, and money market mutual fund accounts. The results were similar to those reported below.

\(^2\) Previous studies using these data include Goldfeld (1973) and Mankiw and Summers (1984).
likely to hold little cash and a small quantity of demand deposits. However, their holdings of short-term Treasury bills could be substantial. While personal trusts may be aggregated with households on the grounds that persons are the beneficial owners of their assets, this argument seems inappropriate for nonprofit groups. The biases which result from this measurement problem are unclear.

Our measure of consumption, $C_t$, is seasonally-adjusted real personal expenditures on nondurables from the National Income and Product Accounts. Our choice of nondurable consumption raises further aggregation issues. Nondurables are only a part of total consumption, excluding both the service flow from durables and services which are purchased directly. We implicitly restrict the utility function to be additively separable between nondurable and other consumption. We deflate each of our asset stocks, as well as consumption expenditure, by the personal consumption deflator and convert to a per capita basis by dividing by the total population over age 16.

We calculate quarterly equity returns ($r_{Et}$) using data on both the dividend yield and the level of the Standard and Poors' 500-Stock Composite Index, $SP_t$. We define the one-quarter capital gain as $g_t = (SP_{t+1} - SP_t)/SP_t$. Standard and Poors' data on dividend yields at annual rates, $DY_t$, are used to calculate the quarterly yield $d_t = (1+DY_t)^{1/4} - 1$. The total pretax return is $r_{Et} = g_t + d_t$. We also construct estimates of the after-tax rates of return.

1. There is substantial seasonal variation in the growth rate of consumption which cannot be explained by seasonal movements in real returns. To prevent this paradox from coloring our results, we work with the seasonally-adjusted series.

2. If the utility function is homothetic and the relative price of different types of consumption are fixed, or if different goods are consumed in fixed proportions, then any subset of aggregate consumption can be used as a proxy for total consumption.
The after-return on equity may be written as 
\[ r_{Et} = (1 - \tau_d)d_t + (1 - \tau_ge_t \text{ where} \]
\[ \tau_d \text{ is the dividend tax rate and } \tau_g \text{ is the effective capital gains tax rate.} \]

Our tax rates are drawn from Feldstein, Dicks-Mireaux, and Poterba (1983).

Returns on T-bills and savings deposits are computed in a similar fashion. The annual interest rates on these securities are reported each quarter in the Federal Reserve Bulletin. We convert each to a quarterly return and then multiply by \( (1 - \tau_d) \) to obtain the after-tax return.\(^2\) Yields on savings deposits are available beginning in the first quarter of 1955; this determines the beginning of our estimation period.

Table 1 shows the average values of per capita asset holdings and quarterly rates of return for our data period. The average value of real per capita consumption is $1,900 1972 dollars. Real holdings of currency and demand deposits are just over $800 per capita, while T-bills and savings deposits average $1,623 and and $2,790, respectively. The most striking feature of the table, however, is the difference between the pre- and post-tax rates of return. The tax-adjusted real rate of return on equity during our period averaged .5\% per quarter, or just over two percent per year. The tax unadjusted real equity return is 6.1\% per year. Average after-tax real returns on both government

\(^1\) We assume perfect loss-offset in the taxation of capital gains. Assuming that the losses on equity could not have been offset against other taxable income would change our rate of return series very little. Since \( \tau_g \) varies between .04 and .07, there is little difference between our series and the series:

\[ r_{Et} = \begin{cases} 
(1 - \tau_d)d_t + (1 - \tau_g)e_t & \text{if } e_t > 0 \\
(1 - \tau_d)d_t + e_t & \text{otherwise} 
\end{cases} \]

\(^2\) The holdings of bonds, savings deposits, and equity are not identically distributed in the household population. Previous calculations of weighted-average marginal tax rates for these assets have therefore shown different tax rates. In the spirit of the representative consumer model we apply the dividend tax rate to all interest and dividend income.
bonds and savings deposits are negative. The holder of T-bills lost an average of 1.4 percent per year, while the savings account investor lost 1.9 percent annually during our sample period. The differences between tax adjusted and tax unadjusted annual returns on these assets are 0.8% and 0.1%, respectively.

The rate of return measures which we employ are subject to several limitations. First, within each of the asset aggregates which we use there are actually a variety of different assets which yield different rates of return. For example, demand deposits and currency includes some interest-bearing NOW accounts. Demand deposits also vary with respect to their implicit service flow, as well as their liquidity. Within the time deposits category, we include both large time deposits at commercial banks, which may pay interest at market rates, with savings and loan institutions.¹

II. B. Estimation

We estimate the parameters \( \{\sigma, \gamma, \rho, \beta, \delta_M, \delta_S\} \) by fitting the implied first order conditions (EC), (M), (G), and (S) to the time series data using three stage least squares. The residuals in our equations are, at least partially, forecast errors uncorrelated with information available at \( t \). The other component of our residuals, the \( \nu \)'s, are assumed to be i.i.d. and thus uncorrelated with our instruments. We employ two different sets of instruments. The first includes a constant term, two lagged values of the real returns on equity, time deposits, and T-bills, as well as the growth rates in money, consumption, time deposits and T-bills. The second includes a constant term, two

¹ More than sixty percent of all assets in this category are invested at S&Ls as opposed to commercial banks. For a large part of the sample period, savings deposits are likely to be the dominant asset in this class. This is why we use the yield on savings deposits to measure the return in this asset category.
lagged values of equity, savings deposit, and time deposit returns, as well as two lagged values of consumption, money, time deposit, and T-bill holdings. The difficulty with the second instrument set, and the reason we will focus primarily on the first, is that some of the instruments may be nonstationary. The appeal of the second set is that the variables in it are probably more correlated with the variables which appear in our equations.

For each system of equations, we report Hansen's (1982) J-statistic for the validity of our overidentifying restrictions. This statistic is distributed as \( \chi^2(kq-m) \) under the null hypothesis of valid restrictions, with \( k \) equal to the number of instruments, \( q \) the number of equations, and \( m \) the number of parameters estimated. Since we have four equations, fifteen instruments, and six free parameters, it is distributed \( \chi^2(54) \).

We constrain our estimates of the utility function parameters two ways. First, we require that \( \delta_M, \delta_S, \) and \( (1-\delta_M-\delta_S) \) be positive by estimating \( \alpha_0 \) and \( \alpha_1 \), where

\[
\delta_M = \cos^2(\alpha_0) \tag{21}
\]

\[
\delta_S = [1 - \cos^2(\alpha_0)] \cdot [1 - \cos^2(\alpha_1)] \tag{22}
\]

Second, we require \( \beta \) to be positive and between zero and one by defining \( \beta = \cos^2(\alpha_2) \) and estimating \( \alpha_2 \). Standard errors for the parameter transformations which define \( \delta_M, \delta_S, \) and \( \beta \) are obtained by standard asymptotic methods.
III. Estimation Results

Table 2 shows the results of estimating our systems of Euler equations for the case of time-additive preferences. We report four sets of estimates, corresponding to each of the two instrument sets using both pre-tax and post-tax returns. The estimates are remarkably stable across specifications. The J-statistics for each system are well within the ninety-five percent confidence bounds, so we can never reject the validity of our over-identifying restrictions.

The results provide strong support for the view that liquidity is a direct source of utility. We estimate $\beta$, the share of expenditure which is devoted to consumption, to be between .961 and .979. In three of the four equations we reject the hypothesis that $\beta=1$ at the .05 confidence level. This null hypothesis corresponds to the idea that our included assets yield no utility. This test is thus a test of whether a utility based model can rationalize any observed rate of return dominances.

Our estimate of $\gamma$, the exponent in our CES liquidity aggregator function, is .27 when we use our preferred instrument set and pre-tax returns, and .19 with post-tax returns. These point estimates argue against linear aggregation of currency and demand deposits, time deposits, and short term debt to form a monetary aggregate. However, we cannot reject the null hypothesis that $\gamma=1$ which would imply that additive aggregation is appropriate. When we use Instrument Set II, the estimates of $\gamma$ increase. With pre-tax returns, this instrument set yields point estimates of $\gamma$ very close to unity. The elasticity of substitution between different assets, $1/(\gamma-1)$, is estimated to be 1.23 using post-tax returns and 1.37 with pre-tax returns. These substitution elasticities, which are obtained using Instrument Set I, are larger than those reported by Husted and Rush (1984) but less than those of Boughton (1981) and Chetty (1969). However,
when we use Instrument Set II, the substitution parameter becomes much larger. With pre-tax returns, the most extreme case, $1/(\gamma-1)$ equals one hundred.

Within our monetary aggregator, the coefficients on the various assets are estimated with relatively large asymptotic standard errors. These standard errors overestimate the imprecision of our estimates because they do not take into account that the $\delta$'s must lie between zero and one. The general pattern which emerges from the point estimates is $\delta_S > \delta_M > 1-\delta_S-\delta_M$. If all real asset stocks were of equal size, this would imply that the marginal utility associated with another dollar of time deposits would exceed that from another dollar of demand deposits or currency. However, it is essential to recognize that at current asset levels, with time deposits five times larger than demand deposits and currency, rather different conclusions emerge. In 1981:4 our estimates from the first column of Table 2 imply that the marginal utility of money is twice that of savings accounts and four times that of government securities. The estimates in Column 3 of Table 2 imply even larger differences.

Although we have allowed government securities to provide liquidity services, our estimates do not suggest a major liquidity role for these assets. Indeed, when we reestimate our system imposing the constraint that $\delta_S = 1-\delta_M$, the value of our objective function deteriorates very little. This means we cannot reject the hypothesis that Treasury bills are not a direct source of utility. Yet, Mehra and Prescott (1985) show that the riskiness of equities is not sufficient to explain their high expected rate of return relative to T-bills. They use a utility function like (8), imposing $\beta = 1$ so liquidity services play no role.

There are two ways of reconciling Mehra and Prescott's findings with ours. First, it may be impossible to capture the rate of return dominance of equities
over T-bills in our utility-based framework. For example, the correct model for the utility services from T-bills may be different from (8). Second, their rate-of-return dominance puzzle may only have arisen because they misspecified the aggregate utility function by excluding liquidity services. Our failure to reject the overidentifying restrictions provides some evidence for the second possibility.

Our results also provide estimates of the intertemporal elasticity of substitution, $\sigma$, which has been the focus of many previous studies in the representative consumer framework. Earlier estimates range between -.8 (Hansen and Singleton, 1983) and -6.0 (Grossman and Shiller, 1981). Our estimates are at the edge of this range; they vary between -6.2 and -5.6. Moreover, they are estimated quite precisely with standard errors of about .60. Our estimates of the discount factor, $\rho$, all exceed unity. However, since these estimates capture both the consumers' rate of time preference as well as trends over time in the structure of preferences, they do not imply that the pure rate of time preference is negative.

Table 3 reports estimates of four additional sets of estimates corresponding to preferences which incorporate costs of adjusting asset stocks. To allow us to perform hypothesis tests these estimates are obtained using the same estimates of the residual covariance matrix as in Table 2. The differences between the J-statistics reported here and in Table 2 are distributed $\chi^2(3)$ under the null hypothesis that adjustment costs are unimportant. The pattern of coefficients \{$\sigma, \rho, \gamma, \beta, \delta_M, \delta_S$\} does not change significantly when adjustment costs are

\[1\] In Obstfeld (1984b), $\sigma < 0$ implies that anticipated disinflation leads to the kind of capital inflows that have been experienced in the Southern cone, rather than to capital outflows. In Obstfeld (1984a), uniqueness of the economy's rational expectations equilibrium requires that \((1-\sigma) < \beta/(1-\beta)\). This condition is always satisfied by our estimates.
introduced. More importantly, however, we can never reject at the 95% level the joint null hypothesis that all of the adjustment cost parameters, $\hat{\theta}_W$, $\hat{\theta}_G$, and $\hat{\theta}_S$, are zero.

Adjustment costs are often invoked as casual justification for the quasi-differencing that is common in the asset demand literature. In a utility based framework, adjustment costs have somewhat more complex implications since they make leads, as well as lags, important. This may explain our inability to obtain significant adjustment costs. More generally our results show a very small role for dynamics since lagged variables appear uncorrelated with our residuals. This lack of dynamics is puzzling in light of the pervasive differencing and quasidifferencing which is typical in other studies of asset and especially money, demand. It is possible that these lags in others' studies capture expectations of returns and future consumptions which are important in our formulation. Since we find little evidence of adjustment costs, the parameters which we use for our calculations in the next section are drawn from Table 2.2

---

1. The Durbin-Watson statistics calculated from our estimated residuals ranged between 1.15 and 1.9. Although their statistical properties in our estimation procedure are unknown, this may provide some evidence of dynamic misspecification.

2. Our calculations are based on estimates using Instrument Set I.
IV. Comparative Statics

The parameters which we estimate in Section III can be used to study the effects of changes in interest rates and inflation on consumption and asset holdings. To fully characterize the consumer's responses to random shocks, we would need to find a closed form solution to the stochastic control problem posed in Section I. Since such solutions remain intractable, we concentrate on the effects of various changes in deterministic environments, asking how the representative consumer would respond to various changes if he maximized (1) and believed the world to be deterministic.

We consider the responses to two types of changes which we label "short-run" and "long-run". The short run responses are responses at $t$ to changes in interest rates from $t$ to $t+1$. We analyze these responses by neglecting the effect of these changes on choices after $t+1$. We study not only how the demands for assets depend on the vector of returns but also how the vector of returns is affected by changes in the supply of assets. These are obviously equivalent exercises. However, the latter focuses attention on the difficulties which mar the interpretation of the interest elasticities which are derived from standard money demand equations. If many assets yield utility then these interest elasticities cannot be used directly to infer the responses of interest rates to open market operations. The long run responses are derived by considering steady states with different interest rates. Across different steady states consumption and asset holdings for a given level of wealth are different. We study these differences holding lifetime wealth constant.

Our short run effects are similar in spirit to those analyzed by Mankiw, Rotemberg, and Summers (1985). Their neglect of the effect of changes in interest rates at $t$ on decisions at $t+1$ is necessarily incorrect. However, it is likely to be a good approximation since the changes in subsequent periods are
mediated through changes in future wealth. For consumers with long horizons, future wealth is essentially unaffected by changes in current decision variables.¹

IV. A. Short-Run Responses

We compute two types of short run responses. The first fixes consumption at \( t \), as well as all future choices. This is very much in the spirit of money demand studies which hold the transactions variable fixed when computing interest elasticities. The second short-run calculation allows consumption at \( t \) to vary optimally, while fixing all choices in future periods. The implied consumption responses are akin to those studied by Hansen and Singleton (1982). However, intertemporal consumption decisions now depend on nominal rates as well as real rates since nominal rates affect asset choice which affects the marginal utility of consumption.

For a given path of consumption, the demand for the three assets we consider depends on the three differences between the return on equities and the return on the assets which give utility. Thus, it depends on \( u_{Mt} \), \( u_{St} \), and \( u_{Gt} \) which are given by:

\[
\begin{align*}
  u_M &= r_{Et}^t P_t / P_{t+1} \\
  u_S &= (r_{Et}^t - r_{St}^t) P_t / P_{t+1} \\
  u_G &= (r_{Et}^t - r_{Gt}^t) P_t / P_{t+1}
\end{align*}
\]

¹. Note that if one estimates a static system of demand equations based on total expenditure on liquid assets (as in Barnett (1980)), then one can only obtain responses to interest rates holding these expenditures on liquidity constant. This is a less appealing approximation to the consumer's action at \( t \).
In the short run we allow $M_t$, $S_t$ and $G_t$ to change in response to the $u$'s. The effects are given by differentiation of $(M)$, $(S)$ and $(G)$:

$$
\begin{vmatrix}
\frac{\partial^2 U}{\partial K^2} & \frac{\partial^2 U}{\partial K \partial S} & \frac{\partial^2 U}{\partial K \partial G} \\
\frac{\partial^2 U}{\partial K \partial S} & \frac{\partial^2 U}{\partial S^2} & \frac{\partial^2 U}{\partial S \partial G} \\
\frac{\partial^2 U}{\partial M \partial G} & \frac{\partial^2 U}{\partial S \partial G} & \frac{\partial^2 U}{\partial G^2}
\end{vmatrix}
= \begin{bmatrix}
\frac{dK}{du} \\
\frac{dS}{du} \\
\frac{dG}{du}
\end{bmatrix}
\begin{bmatrix}
\frac{dM}{du} \\
\frac{dS}{du} \\
\frac{dG}{du}
\end{bmatrix}
\begin{bmatrix}
\frac{du}{du} \\
\frac{du}{du} \\
\frac{du}{du}
\end{bmatrix}
$$

In Table 4 we present the results of this differentiation for the estimates obtained using our first set of instruments. There we consider the percent change in the assets held in the fourth quarter of 1981 when the $u$'s increase by one hundred basis points holding constant asset stocks and consumption for the first quarter of 1982.\(^1\) The first column can be interpreted as the effect of inflation in a world in which the Fisher effect describes the behavior of all interest rates except that on money which is zero. In this scenario, 

$$(1+r_{Et})_tP_t/P_{t+1}, (1+r_{St})_tP_t/P_{t+1}, \text{and} (1+r_{Gt})_tP_t/P_{t+1}$$

are unaffected by inflation, while $r_{Et}P_t/P_{t+1}$ increases by approximately the increase in the inflation rate.\(^2\) Such an increase in inflation reduces money holdings and promotes the use of other liquid assets. Nonetheless, total liquidity falls substantially.

The response of money to $u_M$ is the closest analogue in our model to "the" interest elasticity of money demand since, if all nominal interest rates rise by

---

1. To actually differentiate these equations we must first modify them to make them hold without error. To do this we compute the value of the $u$'s which make $(M)$, $(G)$ and $(S)$ hold exactly. These can be interpreted as the expected returns which rationalize actual subsequent consumption and asset holdings. Then we use these $u$'s instead of the actual $u$'s.

2. The constancy of the real rate implies that $(1+r_{Et})/(1+\Pi)$ is constant, where $\Pi = P_{t+1}/P_t - 1$. This implies $\frac{d(r_{Et})}{1+\Pi} = -\frac{d(1)}{1+\Pi}$, establishing that $r_{Et}P_t/P_{t+1}$ rises by essentially the increase in inflation.
the same amount, only $u_N$ is affected. Indeed we find that our semielasticities which are between .6 and .8, are very similar to those obtained by Mankiw and Summers (1984) using consumption as the transactions variable in an aggregate money demand equation.

The second and third columns of Table 4 give the responses to changes in the return premia of time deposits and T-bills. As we move from money to time deposits to T-bills, i.e. towards assets that yield less liquidity service at the margin, the own semielasticity with respect to the return premium increases. In some sense, these assets are increasingly good substitutes for equity.

The results of Table 4 can be presented in an alternative way which is illustrative of their policy implications. We can use our parameter estimates to calculate the response of the interest rate spreads to changes in the asset stocks.\(^1\) These results are presented in Table 5. As can be seen in this table, the increase in the stock of any liquid asset tends to depress the yield on equities relative to that of the other assets. With more of these assets, their yield relative to that on equity must rise. Nonetheless, the biggest effect is on the "own" yield spread. Thus an increase in money has the biggest depressing effect on the nominal equity yield. As a result, increases in household money which the government finances by buying back some government bonds tend to depress nominal rates even if money is exchanged for bonds on a one-to-one basis. In practice the money multiplier exceeds one so that the effect is even larger.

Alternative measures of households' short-run responses to the rate of return movements can be obtained by letting consumption at $t$ vary as well. These

---

\(^1\) This neglects some effects due to the uncertainty in returns. In particular it ignores the effect on expected returns which would be brought about in the standard CAPM when asset supplies change with a constant covariance matrix of returns.
responses can be obtained by differentiating all four first order conditions with respect to decisions at \( t \) and returns from \( t \) to \( t+1 \):

\[
\begin{bmatrix}
\frac{\partial^2 U}{\partial M^2} & \frac{\partial^2 U}{\partial M \partial S} & \frac{\partial^2 U}{\partial M \partial G} & \frac{\partial^2 U}{\partial M \partial C} \\
\frac{\partial^2 U}{\partial M \partial S} & \frac{\partial^2 U}{\partial S^2} & \frac{\partial^2 U}{\partial S \partial G} & \frac{\partial^2 U}{\partial S \partial C} \\
\frac{\partial^2 U}{\partial M \partial G} & \frac{\partial^2 U}{\partial S \partial G} & \frac{\partial^2 U}{\partial G^2} & \frac{\partial^2 U}{\partial G \partial C} \\
\frac{\partial^2 U}{\partial M \partial C} & \frac{\partial^2 U}{\partial S \partial C} & \frac{\partial^2 U}{\partial G \partial C} & \frac{\partial^2 U}{\partial C^2}
\end{bmatrix}
\begin{bmatrix}
\frac{dE_t}{dt} \\
\frac{dS_t}{dt} \\
\frac{dG_t}{dt} \\
\frac{dC_t}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{du_{M_t}}{dt} \\
\frac{du_{S_t}}{dt} \\
\frac{du_{G_t}}{dt} \\
\frac{d(1+r_{Et})P_t}{P_{t+1}}
\end{bmatrix}
\]

The results of this differentiation for the fourth quarter of 1981 are given in Table 6. The liquid assets respond to the nominal yield spreads in much the same way they do when consumption is held constant in Table 4. A one hundred basis point increase in the real rate has only a mild depressing effect on consumption due to our high estimate for the coefficient of relative risk aversion. In turn, precisely because this coefficient is so large, the reduction in consumption depresses instantaneous utility and raises substantially the marginal utility provided by the Cobb-Douglas consumption-liquidity aggregator. This, in turn, raises the marginal utility of liquidity and thus promotes a slight increase in liquid assets. Similarly, reductions in liquid assets which are prompted by increases in the yield spreads lower instantaneous utility, increasing the marginal utility of consumption. Savings rise therefore when nominal yield spreads shrink or when inflation falls. This finding is potentially of great policy importance, since it suggests that anti-inflationary policies promote savings.
IV. E. Long-run Effects

We can also use our estimated utility function parameters to examine changes in steady-state asset holdings and consumption. Long run elasticities are computed by holding constant steady state real financial wealth, \( \frac{W_t}{P_t} \). By ignoring all assets and liabilities other than money, savings deposits, government securities, and equities, \( W_t \) is defined as

\[ W_t = Q_t + S_t + G_t + M_t \tag{28} \]

where each asset stock is measured at the end of the period. We also know that

\[ W_{t+1} = (1+r_{Et})Q_t + (1+r_{St})S_t + (1+r_{Gt})G_t + M_t - P_{t+1}C_{t+1} + P_{t+1}Y_{t+1} \]

\[ = (1+r_{Et})W_t + (r_{St}-r_{Et})S_t + (r_{Gt}-r_{Et})G_t - r_{Et}M_t - P_{t+1}C_{t+1} + P_{t+1}Y_{t+1}. \tag{29} \]

where \( Y_{t+1} \) is real income. Dividing through by \( P_{t+1} \) yields a difference equation for real wealth:

\[ \frac{W_{t+1}}{P_{t+1}} = \frac{W_t}{P_t} \frac{(1-r_{Et})P_t}{P_{t+1}} + \frac{(r_{St}-r_{Et})P_t}{P_{t+1}} \frac{S_t}{P_t} + \frac{(r_{Gt}-r_{Et})P_t}{P_{t+1}} \frac{G_t}{P_t} \]

\[ - \frac{r_{Et}P_t}{P_{t+1}} \frac{M_t}{P_t} - C_{t+1} + Y_{t+1}. \tag{30} \]

To find the long-run elasticities we totally differentiate \((M), (G), (S), \) and the budget constraint in \((30)\). This yields:
Table 7 reports the results of this differentiation for our data. We assume that the consumption and asset holdings of the fourth quarter of 1931 are steady state values. To ensure this we compute the u's and real return on equity needed to ensure that agents find it optimal to maintain these levels. We assume that \((1+r)_{t}P_t/P_{t+1}\) remains at \(1/\rho\) forever. However, we let the u's jump to new steady state values and we consider the percent change in \(C, K, S,\) and \(G\) as a result of a change in \(u\) by one hundred basis points. The first column again can be viewed as the effect of a permanent one percent per quarter change in the inflation rate.

The calculations show that consumption itself is relatively unaffected by changes in yield spreads. As any of the three utility-yielding assets becomes more expensive to hold, consumption falls. However, even in the case of time deposits where actual holdings are relatively large, a one hundred basis point increase in \(u_S\) reduces consumption by at most one fifth of one percent.

The results also show that the responses of asset holdings are basically the same as those in Table 4 and 5. Because consumption is relatively unaffected by changes in yield spreads, there is little difference between the marginal utility of asset holdings in Tables 4 and 5 and that in Table 7. Moreover, the fact
that we now let consumption and asset holdings in the future vary is of relatively minor consequence. This occurs because these changes affect the current holdings only to the extent that they affect the product of the yield spread and the future marginal utility of consumption. Thus, given that the yield spreads equal only a few percentage points, even relatively large changes in the future marginal utility of consumption have only moderate effects on the present.

We can use the first column of Table 7 to compute a measure of the welfare costs of inflation. This column gives the response of C, S, T and G to permanent inflation. By multiplying these changes by the marginal utilities of these variables we obtain an estimate of the instantaneous loss in utility. We then translate this loss in utility into the fall in consumption which would have produced the same loss. A one hundred basis point increase in inflation would lower utility by the same amount as a 0.4 percent fall in consumption.¹

¹. This obtains whether we use pre- or post-tax returns data.
V. Conclusions

We have presented a method of estimating systems of asset demand equations which are consistent in the Brainard-Tobin (1968) sense, and which permit analysis of a variety of government interventions in asset markets. While reduced form evidence suggests that these interventions change aggregate output, it does not clarify the mechanism by which they work. The need for empirical measures of the effects of open market operations was the original motivation for the estimation of structural money demand functions, which were supposed to capture the aggregate LM curve. As Tobin (1969) pointed out, however, in the presence of many assets that are imperfect substitutes, more complete modelling of the financial sector is needed. Our paper takes a step in that direction.

Our analysis suffers from several shortcomings. These are primarily limitations of our particular implementation of the assets-in-the-utility function approach, and not difficulties with the approach in general. First, it is difficult to maintain that the marginal utility of one liquidity-producing asset is independent of the holdings of other such assets. Yet, if many assets yield these services in substitutable forms, the exclusion of some assets from the analysis may bias conclusions about the importance of other assets. Eventually, our approach should therefore be extended to incorporate a broader range of assets. This will present measurement problems with respect to both asset stocks and rates of return. If liquidity services are a function of the market value of the assets, it is necessary to measure the market value of long term nominal assets, such as corporate bonds, with various maturities and risk characteristics. Measuring the return on these heterogeneous assets is also difficult.
A second, and related issue, is that the menu of important assets changes over time. Financial innovations, like the recent improvements in money-market mutual funds, allow assets to be repackaged to yield different liquidity services. Although our approach can in principle address these issues, this has been left for future research. An important policy issue which our pre-1982 data probably cannot address is the extent to which the new popularity of money market mutual funds has changed the power of open market operations.

A third direction for future work concerns the utility flows which assets provide. We have modelled the utility flows yielded by assets as a simple function of the level of these assets. This is similar to the traditional approach to modelling the demand for consumer durables. Recent studies of consumer demand have focused attention on the actual service flows yielded by these durables. For example, air-conditioners provide two services: they cool one's house, and they also yield the pleasure of knowing one's house need never be hot. The former, at least, is subject to measurement.¹ Similarly, the service flow from a liquid asset depends on the transactions it simplifies, as well as the help it might have provided had more transactions taken place. Again, the former might be measureable. This line of inquiry could potentially reconcile the view that these assets are held because they give utility with transactions-based models.

¹. Hausman (1979) employs data on the amount of cooling services actually provided by air conditioners to study the demand for air conditioners.
References


Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption (C_t)</td>
<td>1.871</td>
<td>.164</td>
</tr>
</tbody>
</table>

Asset Stocks:

| Demand Deposits and Currency (M_t/P_t) | .826   | .041               |
| Short-Term Government Debt (G_t/P_t)  | 1.623  | .711               |
| Time and Savings Deposits (S_t/P_t)   | 2.790  | .936               |

Quarterly Rates of Return:

<table>
<thead>
<tr>
<th>( r_Et )</th>
<th>Tax Adjusted, Real</th>
<th>0.51</th>
<th>5.79</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tax Adjusted, Nominal</td>
<td>1.57</td>
<td>5.46</td>
</tr>
<tr>
<td></td>
<td>Tax Unadjusted, Real</td>
<td>1.49</td>
<td>6.12</td>
</tr>
<tr>
<td></td>
<td>Tax Unadjusted, Nominal</td>
<td>2.57</td>
<td>5.79</td>
</tr>
<tr>
<td>( r_Gt )</td>
<td>Tax Adjusted, Real</td>
<td>-0.35</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Tax Adjusted, Nominal</td>
<td>0.66</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>Tax Unadjusted, Real</td>
<td>0.21</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>Tax Unadjusted, Nominal</td>
<td>1.23</td>
<td>0.72</td>
</tr>
<tr>
<td>( r_S^t )</td>
<td>Tax Adjusted, Real</td>
<td>-0.46</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Tax Adjusted, Nominal</td>
<td>0.54</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Tax Unadjusted, Real</td>
<td>0.02</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>Tax Unadjusted, Nominal</td>
<td>1.01</td>
<td>0.49</td>
</tr>
<tr>
<td>( P_t/P_{t+1} )</td>
<td>.990</td>
<td>.0092</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Real rates of return were calculated as \((1+r_{it})P_t/P_{t+1} - 1\), while nominal rates are just the average values of \( r_{it} \). Data cover the period 1955:1 through 1982:2.
Table 2: Estimates of Utility Function Parameters

<table>
<thead>
<tr>
<th>Instrument Set:</th>
<th>Returns without Tax Adjustment</th>
<th>Tax-Adjusted Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-6.091</td>
<td>-6.247</td>
</tr>
<tr>
<td></td>
<td>(0.653)</td>
<td>(0.666)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.007</td>
<td>1.006</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.269</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>(1.169)</td>
<td>(0.305)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.965</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\delta_M$</td>
<td>0.316</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>(0.184)</td>
<td>(0.262)</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>0.515</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>(0.273)</td>
<td>(0.256)</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>0.168</td>
<td>0.231</td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>$J$</td>
<td>41.005</td>
<td>47.432</td>
</tr>
</tbody>
</table>

Notes: Estimates correspond to the utility function $V_t = \sum_{i=0}^{\infty} \rho^i U_{t+i}$ where $U_{t+i}$ is defined by (8) in the text. The estimation period is 1955:1 - 1982:1 (100 observations) in each case. Standard errors are shown in parentheses.
Table 3: Estimates of Cost-of-Asset-Adjustment Models

<table>
<thead>
<tr>
<th>Instrument Set:</th>
<th>Returns without Tax Adjustment</th>
<th>Tax-Adjusted Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-6.109</td>
<td>-6.469</td>
</tr>
<tr>
<td></td>
<td>(0.583)</td>
<td>(0.659)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.007</td>
<td>1.007</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.604</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.802)</td>
<td>(1.429)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.962</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\delta_M$</td>
<td>0.366</td>
<td>0.309</td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
<td>(0.252)</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>0.408</td>
<td>0.509</td>
</tr>
<tr>
<td></td>
<td>(0.184)</td>
<td>(0.321)</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>0.225</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>(0.955)</td>
<td>(0.127)</td>
</tr>
<tr>
<td>$\theta_M$</td>
<td>-0.011</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>$\theta_S$</td>
<td>0.513</td>
<td>0.649</td>
</tr>
<tr>
<td></td>
<td>(0.472)</td>
<td>(0.296)</td>
</tr>
<tr>
<td>$\theta_G$</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$J$</td>
<td>39.768</td>
<td>42.560</td>
</tr>
<tr>
<td>$J$ (Table 3)</td>
<td>1.237</td>
<td>4.872</td>
</tr>
<tr>
<td>$J$ (Table 2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Estimates correspond to the lifetime utility function defined in (16), with $U$ given by (8). Standard errors are shown in parentheses. All equations are estimated for 1955:1 - 1982:1 (109 observations).
Table 4: Short-Run Return Semi-Elasticities of Asset Demand

<table>
<thead>
<tr>
<th>Asset Stock</th>
<th>Yield Spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity-Money</td>
</tr>
<tr>
<td>Demand Deposits and Currency</td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>-.587</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>-.732</td>
</tr>
<tr>
<td>T-Bills</td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>.048</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>.070</td>
</tr>
<tr>
<td>Time Deposits</td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>.071</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>.065</td>
</tr>
</tbody>
</table>

Notes: All estimates are based on parameter values reported in Table 2. We use estimates corresponding to Instrument Set 1. See text for further description of elasticity calculations.
Table 5: Short-Run Effect of Changes in Asset Supply

<table>
<thead>
<tr>
<th>Yield Spreads:</th>
<th>Demand Deposits and Currency</th>
<th>Time Deposits</th>
<th>T-Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity-Money</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Tax</td>
<td>-2.256</td>
<td>-.174</td>
<td>-.058</td>
</tr>
<tr>
<td>Post-Tax</td>
<td>-1.752</td>
<td>-.083</td>
<td>-.012</td>
</tr>
<tr>
<td>Equity-Time Deposits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Tax</td>
<td>-.174</td>
<td>-.289</td>
<td>-.029</td>
</tr>
<tr>
<td>Post-Tax</td>
<td>-.083</td>
<td>-.195</td>
<td>-.006</td>
</tr>
<tr>
<td>Equity-T-Bills</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Tax</td>
<td>-.058</td>
<td>-.029</td>
<td>-.145</td>
</tr>
<tr>
<td>Post-Tax</td>
<td>-.011</td>
<td>-.006</td>
<td>-.028</td>
</tr>
</tbody>
</table>

Notes: Each entry in the table shows the change in the yield spread in basis points, which results from a one thousand (1972) dollar increase in per capita asset stocks. Calculations are based on parameter estimates using Instrument Set 1, pre-tax and post-tax returns, as reported in Table 2. The calculations are described in the text.
Table 6: Short-Run Return Semi-Elasticities of Consumption and Asset Holdings

<table>
<thead>
<tr>
<th>Percent Change In:</th>
<th>Equity-Money Yield Spread</th>
<th>Equity-Time Deposits Yield Spread</th>
<th>Equity-T Bill Yield Spread</th>
<th>Equity Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Deposits and Currency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax</td>
<td>-.602</td>
<td>.256</td>
<td>.110</td>
<td>.008</td>
</tr>
<tr>
<td>- Post-Tax</td>
<td>-.751</td>
<td>.218</td>
<td>.165</td>
<td>.009</td>
</tr>
<tr>
<td>Time Deposits</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Pre-Tax</td>
<td>.055</td>
<td>-1.067</td>
<td>.106</td>
<td>.009</td>
</tr>
<tr>
<td>-Post-Tax</td>
<td>.047</td>
<td>-1.496</td>
<td>.170</td>
<td>.009</td>
</tr>
<tr>
<td>T-Bills</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Pre-Tax</td>
<td>.032</td>
<td>.146</td>
<td>-2.60</td>
<td>.009</td>
</tr>
<tr>
<td>-Post-Tax</td>
<td>.045</td>
<td>.241</td>
<td>-13.087</td>
<td>.010</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Pre-Tax</td>
<td>.325</td>
<td>1.680</td>
<td>1.215</td>
<td>-.179</td>
</tr>
<tr>
<td>-Post-Tax</td>
<td>.342</td>
<td>1.682</td>
<td>1.536</td>
<td>-.169</td>
</tr>
</tbody>
</table>

Notes: Calculations based on parameter estimates using Instrument Set 1, pre-tax and post-tax returns, reported in Table 2. The calculations are described in the text.
### Table 7: Steady-State Return Semi-Elasticities of Asset Demand

<table>
<thead>
<tr>
<th>Percentage Change in:</th>
<th>Yield Spreads</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity-Money</td>
<td>Equity-Time Deposits</td>
<td>Equity-Treasury Bills</td>
</tr>
<tr>
<td>Demand Deposits and Currency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>-.635</td>
<td>.275</td>
<td>.104</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>-.761</td>
<td>.232</td>
<td>.144</td>
</tr>
<tr>
<td>Time Deposits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>.596</td>
<td>-1.098</td>
<td>.101</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>.049</td>
<td>-1.523</td>
<td>.283</td>
</tr>
<tr>
<td>T-Bills</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>.030</td>
<td>.135</td>
<td>-2.708</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>.041</td>
<td>.380</td>
<td>-113.125</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Pre-Tax Returns</td>
<td>-.011</td>
<td>-.173</td>
<td>-.071</td>
</tr>
<tr>
<td>- Post-Tax Returns</td>
<td>-.012</td>
<td>-.106</td>
<td>-.0450</td>
</tr>
</tbody>
</table>

Note: All estimates are based on parameters estimated using Instrument Set 1, reported in Table 2. See text for further discussion of the elasticity calculations.