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A MODEL FOR ALLOCATING RETAIL OUTLET BUILDING RESOURCES ACROSS MARKET AREAS

Gary L. Lilien
Massachusetts Institute of Technology

Ambar G. Rao
New York University

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MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

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ABSTRACT

Many factors affect retail outlet profitability, including market potential, distribution and product costs, market pricing levels, cost (and availability) of land or space and the relationship between share of outlets and share of markets. A model is presented which was used to plan building decisions for outlets for a consumer product across time and across market areas. The model has been in use for a number of years and has provided important input for budgetting and planning decisions. The implementation process for this model is also discussed. The model and its use provide an example of what the authors believe to be "successful" management science application -- the characteristics of and reasons for this success are discussed.

I. INTRODUCTION

In a number of industries, products or services are offered to consumers through company controlled retail outlets; each outlet offers only the products or services of the company controlling it. Examples of such industries are retail banking, gasoline, and fast foods, where retail outlets are branch banks, service stations and franchised restaurants respectively. Companies in such industries grow by constructing or acquiring new outlets and one of the most important decisions faced by marketing management is the development of a plan for such construction.

The authors participated in a project to develop a systematic, model based approach to this planning decision. The approach was to provide guidelines on how many outlets should be built in each geographical market in each of the next 5 - 10 years. Traditionally, each year, district managers had submitted requests for construction of outlets on a number of sites that met company requirements in terms of anticipated profitability. These requests would be screened and then met subject to the availability of funds. The long term impact of construction on company profitability was never explicitly considered. The development of a model based approach was motivated by a top management desire to invest larger sums of money in outlet construction than it had in the past and by the recognition that the payback for such investments would occur over an extended time period. Thus the traditional approach was considered inadequate.

The profitability of a given site depends, among other factors, on its sales volume. Sales volume is affected by a number of site characteristics such as traffic flow and neighborhood population. When developing a long range plan, a list of specific sites is generally not available, so that one has to assume an "average" volume figure for each potential site. In implementing

the plan, only sites that satisfy this assumption are selected. More important from a planning viewpoint is the impact of the number of sites constructed on average volume per outlet, and thus, on market share. Total market demand in the product classes considered is rather inelastic -- new outlets divide essentially the same "pie". Thus, if a very large number of outlets were to be constructed in a single market, the average sales per outlet would be substantially depressed. Marketing management believed that a relationship did exist between the share of outlets s, and the (volume) share of market m enjoyed by a company, and that other things being equal outlets tended to have larger volumes in markets where s was larger than where s was small. The only quantitative work on the relationship between s and m that had been reported in the literature supported this belief. In their paper, "Brand Switching and Mathematical Programming in Market Expansion," Hartung and Fisher [1] showed that for $0< s<.2$, dm/ds and d^2m/ds^2 were both positive; thus all other things being equal, it is preferable to build in markets where s is high than where it is low. Hartung and Fisher do not consider the impact of saturation alluded to above, and their model has other more serious shortcomings, but their work formed a starting point for this analysis.

In this paper, we present a model for the relationship between s and m, and then show how the relationship was used to develop a model for the outlet construction decision. The output of this model was a specification of the number of outlets to be constructed each year in each market, given constraints on the total budget for construction and the availability of sites in each market in each year. These constraints are really estimates; thus, the initial output of the model is really a demand for refinement of these estimates. That is, once the model determines that n_{it} outlets should be

constructed in market i and year t, a search is conducted for such sites; if an adequate number cannot be found, then the constraint is revised and input to the model, a new solution obtained, etc. Given the crudeness of the various cost estimates and the mixture of hard and soft "data", an approximate procedure for solving the model was developed: this procedure, in most cases, provides optimal solutions, and always gives solutions close to optimal.

2. A MODEL OF THE OUTLET SHARE-MARKET SHARE RELATIONSHIP

Hartung and Fisher [1] model the sequence of purchases by a customer as a 2-state Markov Chain. The states are "purchase company brand" and "purchase some other brand". They assume that the probability that a customer will buy the company's brand on the t^{th} occasion, given he bought it at t-1 is k_1s and the probability that the customer buys the brand at t given he bought some other brand at t-1 is k_2s, where k_1 and k_2 are constants. After some algebra it can be shown that this model implies that

$$m = \frac{k_2 s}{(1-s) + (1 + k_2 - k_1)s}$$

The values of k_1 and k_2 are estimated from aggregate data and found to be 4.44 and .64 respectively. Although this model provides a good fit in the range of data available to Hartung and Fisher, it breaks down for s more than about 0.20; for $s = 1/k_1$, m=1.

In more recent work Naert and Bultez [3] question the robustness of the Hartung-Fisher model and suggest several alternative model structures. These structures are based on empirical evidence, not on fundamental behavioral hypotheses. For example, they do not question the Markovian basis for the

Hartung-Fisher model although they promise to explore it in later work.

Initial attempts to fit the Hartung-Fisher model to the data available to the authors were unsuccessful, even in the range $0 \leq s \leq .20$, and a new approach seemed advisable. In the product field being considered, there was practically no brand differentiation in terms of quality. The major reason customers gave for patronizing a particular retail outlet was "convenience", equated in this case to closeness to home or work. Modelling such a market using the concept of brand loyalty-disloyalty is questionable. A more plausible hypothesis was that buyer purchase behavior was really an independent trials process, the motivating factor in outlet selection on each purchase occasion being convenience -- that is, the outlet easiest to get to at a given time would be selected. Since most people have fairly fixed patterns of work and travel during a year, this hypothesis would encompass the case of a customer who patronized one outlet exclusively without attributing "brand loyalty" to him. In a market without product differentiation, brand loyalty implies consumer irrationality; our hypothesis avoids this implication while permitting apparently brand loyal behavior.

Formally, we hypothesize that:

H1. Purchase behavior is an independent trials process.

H2. The probability that a randomly chosen customer will select an outlet of a particular brand depends on whether an outlet of that brand is conveniently located for him.

In a given market suppose there are N brands with outlet shares s_i, $i = 1,\ldots N$. On a given purchase occasion suppose the outlets of more than one brand are convenient. Let J denote the subset of convenient brands.

H3. The probability that an outlet of brand i is selected is $s_i / \sum_{j \varepsilon J} s_j$, $i \varepsilon J$ and zero otherwise. Thus, an outlet is selected at random from the set of convenient outlets.

H4. The probability that an outlet of a brand with outlet share s_i will be convenient to a randomly chosen customer on a given purchase occasion is denoted by $f(s_i)$.

Obviously, $f(0) = 0$ and $f(1) = 1$; in addition, we assume only that $f'(s_i) \geq 0$, $0 \leq s_i \leq 1$, that is $f(s)$ is monotonic, non-decreasing. Since it is difficult to imagine a case when adding an outlet will make a brand less accessable, we feel this assumption quite realistic.

H5. The average quantity purchased per occasion is relatively constant over customers.

For the time being we shall also assume that the outlets of each brand are identical in size, and that they are all modern, well located facilities. This assumption will be relaxed later.

Now we explore the consequences of H1 – H5. First, consider a 2-brand market. For simplicity in notation let us drop subscripts and assume that the share of outlets of brand 1 is s and its share of market is m.

(1) Probability (A customer selects an outlet of brand 1) = Probability (Brand 1 convenient and Brand 2 inconvenient) + s Probability (Brand 1 and Brand 2 convenient) $= f(s)(1-f(1-s)) + s\, f(s) \cdot f(1-s) = m$ by H5.

Now suppose each brand has exactly equal numbers of outlets, i.e. $s = 1/2$. Then, so long as these outlets are essentially similar, they should share the market equally: that is $m = 1/2$. Therefore,

(2) $f(0.5)\,(1-f(0.5)) + 0.5\, f(0.5) = 0.5$, or $f(0.5) = 1$

By H4 this implies that

(3) $f(s) = 1,\ s \geq 0.5.$

Now substituting (3) in (1), the share

(4) $m = \begin{cases} s\, f(s), & s \leq 1/2 \\ 1 - (1-s)\, f(1-s), & s > 1/2 \end{cases}$

We have shown that $f(s) = 1$, $s \geq 1/2$; $f(s)$ may reach 1 earlier however, say at $s = s^* < 1/2$. Then $f(s) < 1$ for $s < s^* < 1/2$ and from equation (4), for small values of s, market share is less than outlet share, while for larger values of s, it is greater than outlet share, (which agrees with the operational part of the Hartung-Fisher model). For example, if we assume that $f(s) = 2s$, then:

$$(5) \qquad m = \begin{cases} 2s^2 & s \leq 1/2 \\ 4s - 2s^2 - 1 & s > 1/2 \end{cases}$$

This relationship is graphed in Figure 1.

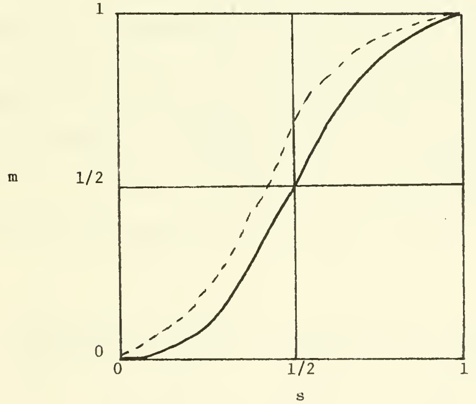

Figure 1

Relationship between m and s for a two-brand market assuming $f(s) = \begin{cases} 2s, & 0 \leq s \leq 1/2 \\ 1 & s > 1/2 \end{cases}$

Now we generalize the relationship between m and s to the N brand case, N > 2. Using the same argument as in equation (1) the market share of Brand 1,

$$(6)\quad m_1 = f(s_1)\prod_{i\neq 1}(1-f(s_i)) + \sum_{i\neq 1}\frac{s_1}{s_1+s_i} f(s_1)\, f(s_i)\prod_{j\neq 1,i}(1-f(s_j))$$

$$+ \sum_{i\neq j\neq 1}\frac{s_1}{s_1+s_i+s_j} f(s_1)f(s_i)f(s_j)\prod_{k\neq 1,i,j}(1-f(s_k)) + \ldots +$$

$$\ldots + s_1\prod_i f(s_i)$$

Now, if each brand has exactly the same share of outlets, then as before they should have the same share of market; substituting $s_i = \frac{1}{N}$, $i = 1,\ldots N$ in (6) and setting $m_1 = \frac{1}{N}$ also, the relationship is satisfied if $f(\frac{1}{N}) = 1$. Thus, when a brand has 1/N or more of the share of outlets, it is conveniently located to all customers in the market. Again, it is easy to show that for some $0 \leq s_i \leq 1/N$, $m_i \leq s_i$. For example, suppose all brands except brand 1 have equal shares of market, that is, $s_i = \frac{1-s_1}{N-1}$ $i \neq 1$.

Then,

$$(7)\quad m_1 = \sum_{i=0}^{N-1}\binom{N-1}{i} f(s_1)\, f\left(\frac{1-s_1}{N-1}\right)^i \left[1-f\left(\frac{1-s_1}{N-1}\right)\right]^{N-i-1} \frac{(N-1)s_1}{(N-1-i)s_1 + i}$$

When $s_1 < \frac{1}{N}$, $s_i > \frac{1}{N}$, so that $f(s_i) = 1$, $i \neq 1$: similarly when $s_1 > \frac{1}{N}$, $f(s_1) = 1$, and $f(s_i) < 1$ $i \neq 1$. Thus,

$$(8)\qquad s_1\, f(s_1),\quad s_1 \leq \frac{1}{N}$$

$$= \sum_{i=0}^{N-1}\binom{N-1}{i} f\left(\frac{1-s_1}{N-1}\right)^i \left[1-f\left(\frac{1-s_1}{N-1}\right)\right]^{N-i-1} \frac{(N-1)s_1}{(N-1-i)s_1 + i},\quad s_1 > \frac{1}{N}$$

Again, assuming a linear form for $f(s_i)$ for $0 \leq s_1 \leq \frac{1}{N}$, we get $m_1 = Ns_1^2$, $s_1 \leq \frac{1}{N}$ in agreement with the operational part of the Hartung-Fisher Model.

So far we have assumed that the outlets of each brand are basically similar. Consider again, the two brand case. Supose now that the outlets of Brand 1 have been constructed more recently. Thus, it is likely that they are better located compared to older outlets. In the industry under study, the average life of an outlet could be 20 years; substantial changes in traffic patterns and neighborhoods occur during this time. Thus, we can hypothesize that if Brands 1 and 2 had the same <u>number</u> of outlets, Brand 1 would have a larger market share than Brand 2, because its outlets will be convenient to more people.

The model presented above can be easily modified to include this phenomenon. Let r_i be the effective share of outlets for brand i. In general r_i will differ from s_i, the share of outlets for brand i, depending upon the relative building rates of brand i and the rest of industry in the past. Operationally r_i may be computed as follows. Let us classify all outlets into recently built and old. The recently built ones, on the average, will be able to service more customers than the old outlets. Let n_{ir} and N_r be the number of recent outlets of brand i and the rest of industry respectively; similarly let n_{i0} and N_0 be the number of old outlets. Also suppose a recent outlet is capable of attracting k times as many customers as an old outlet. Then

$$r_i = \frac{kn_{ir} + n_{i0}}{k(n_{ir} + N_r) + n_{i0} + N_0}$$

$$= \frac{cn_i}{cn_i + N}$$

where c is a constant, and n_{ir} and $n_{i0} = n_i$, $N_r + N_0 = N$

Now replace s_i by r_i in equation (4) and equation (6). Assuming again a linear form for $f(r_i)$ and setting c = 2, the resultant relationship between market share m and outlet share s is shown by the dotted line in Figure 1 for the two brand case.

Thus we have identified another crucial factor impacting profitability: the age distribution of a company's outlets compared to competition. This reinforces the importance of developing a long range building plan rather than relying on the traditional "bottom up" approach to outlet construction described in the Introduction.

3. EMPIRICAL VALIDATION

In the previous section we demonstrated that the relationship between outlet share s and market share m is a non linear one; for small values of s, $m < s$ while for large values of s, $m > s$. This analysis tells us the type of function that should be fitted to data to empirically obtain the m-s relationship. In many empirical studies, the typical fitting procedure resembles a "fishing expedition" with no real idea of what shape of function theory dictates should be fit; the theoretical analysis avoids this pitfall and indicates that an s-shaped function should be employed. It also requires us to select a function that can be parameterized to be less or more steep, depending upon the outlet building rate in the market. The precise functional form e.g.cubic, Gompertz etc. -- is unimportant; any function that is likely to provide useful, useable results may be applied.

Initially, two types of data sources were located within the company. The first was a Retail Competitive Survey which was conducted annually by company salesmen. This survey provided information on outlet numbers and estimated sales volumes by brand. This data source was considered by company management

to be much more reliable than commercially available data of the same type. The second data source was the New Outlets Openings Report, which was a record of all new outlet openings for the last ten years. Data from both sources were initially available for thirty markets and these were used for estimation purposes. Outlets that were less than five years old were classified as recently built. This admittedly arbitrary classification provided the best fit and also agreed with the intuition of marketing management. A variable called "aggressiveness" was defined as

$$a = \frac{\dfrac{\text{\# of recently built company outlets}}{\text{Total company outlets}}}{\dfrac{\text{\# of recently built industry outlets}}{\text{total industry outlets}}}$$

and a function $m = g(a,s)$ was fitted to the data. Figure 2 shows contours of this fitted function for $a = 1.25$ and $a = 0.85$. Also shown is the fitted Hartung-Fisher model for the same data set. While in the range $0 < s \leq .14$ there is not too much difference between this model and the Hartung-Fisher model, beyond $s = .14$ substantial differences occur. A high R^2 ($> .8$) was obtained and the impact of building rate found to be highly significant. The proprietary nature of the data precludes a fuller discussion of the estimation procedure or presentation of those data.

It should be noted that the results presented were initial ones. In practice the curves are reestimated each year to reflect the most recent data available. The most recent curves differ somewhat from those shown in Figure 2, but their general character is as illustrated.

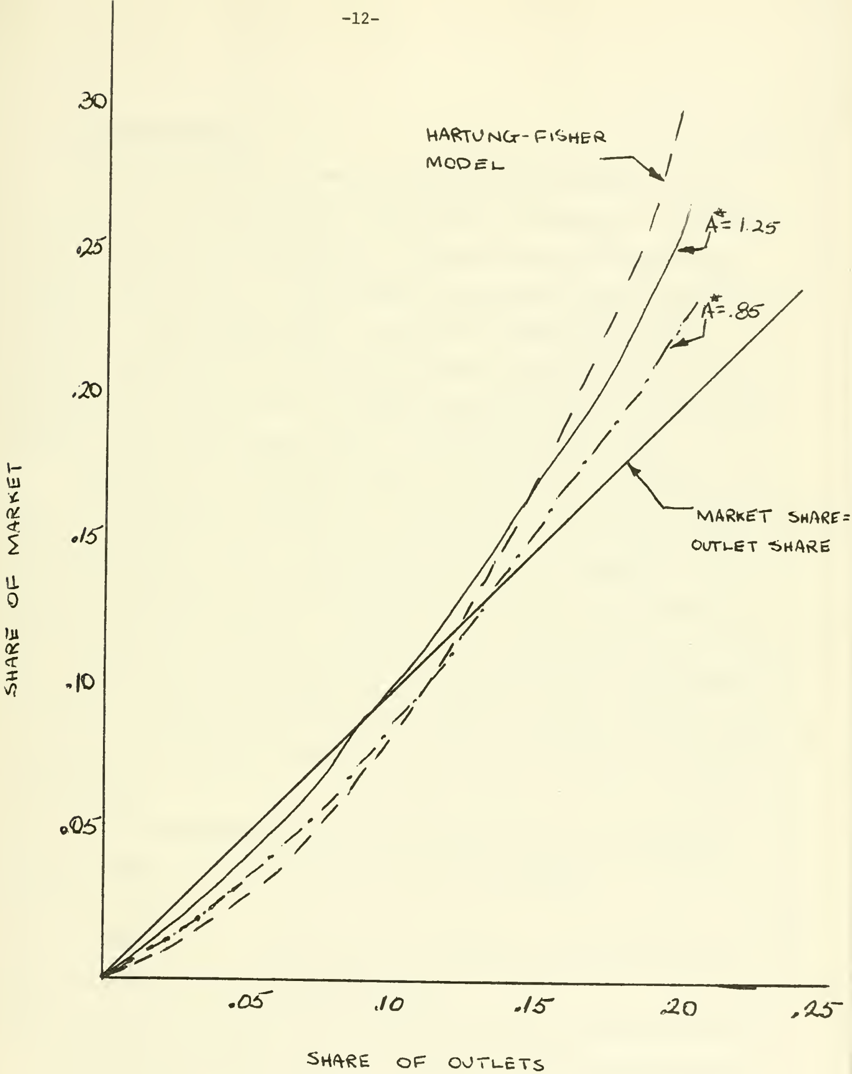

FIGURE 2

(A* = AGGRESSIVENESS)

4. ALLOCATION PROCEDURE

As mentioned before the model discussed here was designed to aid in the analysis of a planning problem. The output of the model was to aid management in constructing a building plan -- how many outlets the company should expect to build in each of a large number of market areas during a several-(usually 5) year planning period. The first year results become budget items -- building funds are allocated in accordance with plan "year 1". The following year results are used to prepare profit plan projections and to help allocate outlet-site procurement funds (in anticipation of building).

The nature of the managerial decision is such that a near-optimal solution to the mathematical formulation of the problem is quite adequate. All the planned outlets cannot or are not always built due to changing local building codes, construction difficulties, lack of sites, etc. And if an extra "choice" site comes available in a desirable area, an outlet will be constructed on it immediately, even if no money was originally allocated. What management is concerned with here is whether it should acquire five sites or twenty sites in an area; the difference between five sites and six often washes out during implementation.

It has been demonstrated that the firm's market share m is related to the aggressiveness a and share of outlets s by a relationship

(9) $m = f(a,s)$

In general m, a, s as well as f will be known for a particular market. Thus, for consistency the following is assumed:

A-1: In equation (9) market share (m), aggressiveness (a) and the function (f) are known with certainty, while outlet share (s) is to be determined from the equation.

A-1 gives an operational definition of outlet share which may seemingly be different from the one observed. This could be due to (a) differences in the size and effectiveness of outlets in the market (as discussed in Section 2), (b) marketing factors, (c) random, or other factors. The reason for the differences need be of no concern in general; specific, significant differences should be brought to the attention of management for purposes of control.

Given this starting point ($m = m_0$, $s = s_0$, $a = a_0$), and an assumption about non-firm building rate, one can now calculate the annual expected market share for a given building plan for each year of a planning horizon. This is not the whole story, of course: a host of other data (growth rates, discount rates, cost factors, margins, etc.) are needed to choose an economically optimal building plan for a particular market. The details of the economic evaluation will vary from application to application. The highlights of one such application are sketched here.

The problem of determining an optimal building plan was originally formulated as a dynamic programming problem. The procedure was cumbersome, computationally inefficient and was not able to handle several of the constraints. An empirical market-by-market analysis of the relationship between cumulative NPV and building investment indicated that most such curves were nearly concave. Thus the dynamic programming approach was scrapped and the following algorithm developed.

The objective of the algorithm is to maximize the total net present value (NPV) of a Y-year building program subject to restrictions on the total number of outlets that can be built (a) within a market, (b) across all markets in a given year and (c) during the Y-years, where NPV is defined as:

$$(10) \qquad NPV = \sum_{j=1}^{J} \sum_{i=1}^{\tau} \frac{CF_{ij}}{(1+R)^{i-1}}$$

where CF_{ij} = cash flow associated with market area j in year i

R = discount rate

J = market areas considered in the plan

τ = planning horizon ($\tau > Y$)

To do this, the procedure selects the group of outlets in the market which has the highest average NPV per outlet. It then selects the next highest NPV group and so on until all allowable outlets have been allocated.

It will be assumed that if one knows, for a particular market:

- the firm's building/investment plan,
- the firm's current market share,
- market growth rate,
- discount rate
- margin,
- competitive building/investment plans,
- current age distribution of firm/industry outlets,
- other financial information: land costs, improvement and equipment costs, depreciation methods, working capital needed, etc,

then it will be straightforward together with equation (9) to calculate cash flows, and, hence the NPV associated with any particular building plan. The following assumptions have been used in practice in making such NPV calculations; though they are somewhat arbitrary, we trust they seem reasonable.

A-2: "New" outlets, used in the definition of aggressiveness are defined as those five years old or newer. In year 3 of the building plan, outlets built in years -1 (last year), 0 this year), 1, and 2 are included in the definition of aggressiveness.

The building plan is designed for Y years (where Y usually equals 5); the planning horizon is set for τ (generally 20) years. Because of the non-linear relationship between outlet share and market share, if one assumed no building after year Y it could seriously understate the profitability of the building plan. On the other hand, it would be a mistake to assume the continuation (and reap the model-profits) of an aggressive building plan in years after Y (with no capital outlay). As a compromise:

A-3: The model assumes, after Y years, that the firm will build enough outlets to maintain its market share: $m_k = m_Y$, $k = Y + 1, \ldots.$ Thus aggressiveness is assumed = 1:($a_k = 1$) $k = Y + 1 \ldots.$

Note, that were an infinite planning horizon, and an infinite building horizon being considered, A-3 would not be necessary. There are also some minor end-off problems (which can be taken account of by properly defining salvange values) which this finite horizon approach entails. However, since the model was developed as an operational tool for managers, it had to conform to the planning practices currently in use. The inconveniences encountered in such modelling are rather minor and the implementation benefits are considerable.

An allocation algorithm for a single building plan can now be developed. It will then be extended to Y years and theoretical justification for why the procedure is, at least, near optimal will be given. First, some notation:

Let

X_i = number of outlets built in market i

n_i = market building constraint

T = overall building constraint

V_{ik} = Incremental net present value (NPV) of the kth station in market i.

$\sum_{k=i}^{j} V_{ik}$ = cumulative NPV of the first j stations in market i, $j = 1, 2, \ldots n_i$.

$$W_{ij} = \begin{cases} \sum_{k=1}^{j} V_{ik}/j & = \text{average NPV of the first j stations in market } i,\ j = 1, \ldots n_i. \\ -M & j > n_i,\ M \text{ is large positive number.} \end{cases}$$

M = Number of markets

$N = \max_i [n_i].$

The single year problem is to

M X$_i$

Massachusetts Institute of Technology
Alfred P. Sloan School of Management
50 Memorial Drive
Cambridge, Massachusetts, 02139

Erratum: Page 17, Theorem: (should read) if, in every market, NPV is a concave function of the number of outlets...

Theorem: if, in every market, NPV is

lets built, then a simple allocation according to incremental NPV yields an optimal building plan.

Proof: let

$g_j(X_j)$ = cumulative NPV associated with building X_j outlets in market j, j=1, ... J and assume g_j is concave for all j.

A building plan can be considered a vector $\overline{X} = (X_1, X_2, \ldots, X_J)$ and the NPV of that building plan is simply

$$G(\overline{X}) = \sum_{j=1}^{J} g_j(X_j).$$

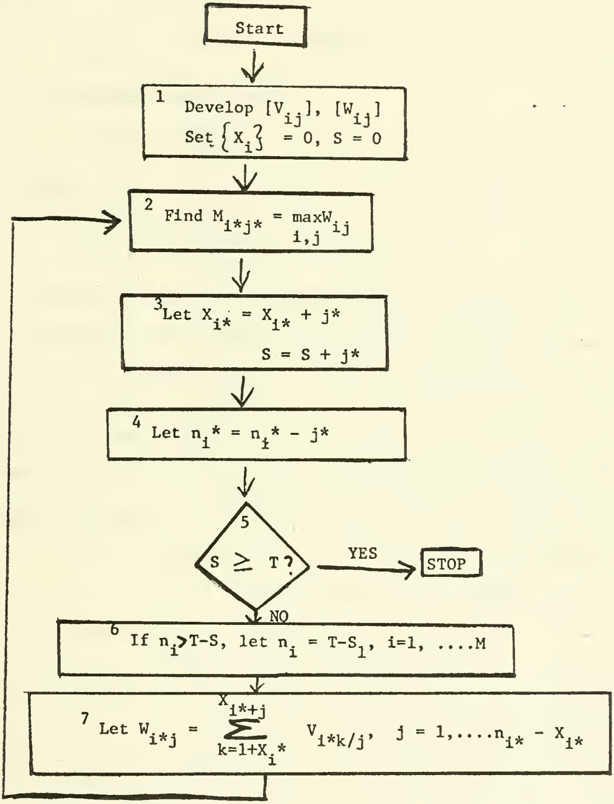

FIGURE 3: ALLOCATION ALGORITHM

A resource constraint exists:

$$\sum_{j=1}^{J} X_j = K \text{ (Assuming management uses all building resources).}$$

The Lagrangian can be formed:

$$L(\overline{X},\lambda) = G(\overline{X}) + \lambda(k - \sum_{j=1}^{J} X_j)$$

Setting partial derivatives of L = 0:

$$\frac{\partial L}{\partial X_j} = 0 = g_j'(x_j) - \lambda, \quad g_j'(x_j) = \lambda \ \forall_j$$

Thus the NPV maximizing solution has equal incremental NPV's for each market. The solution is a global optimal since $L(\overline{X},\lambda)$, a sum of concave functions, is concave. This completes the proof.

In general, the cumulative NPV curves may not be concave; thus the W_{ij} matrix is constructed and only entries of maximal size are allocated. This forms a <u>concave envelope</u> for the cumulative NPV curves (transforming them into concave functions). Then maximal W_{ij} entries, the only ones chosen for allocation, always correspond to a feasible point. As an example consider Figure 4 with the solid line indicating the concave envelope.

Point A in Figure 4 would be a maximal entry for market i'. Thus 5 stations would built in market i' (assuming it had the highest current $[W_{ij}]$ entry) and then Step 7 in the algorithm would move the origin, 0, to point A where the algorithm is repeated. The next set of stations picked in this market will correspond to point B, i.e., nine stations (or 4 additional). Note that if the slope from 0 to B $(\frac{0B}{9})$ were greater than that from 0 to A $(\frac{0A}{5})$, either the entire set of 9 outlets would be included in the building plan or none would be at all (i.e., $W_{i\,9}$ would be the largest entry in the i'th row). A numerical example is included in the next section which illustrates this procedure.

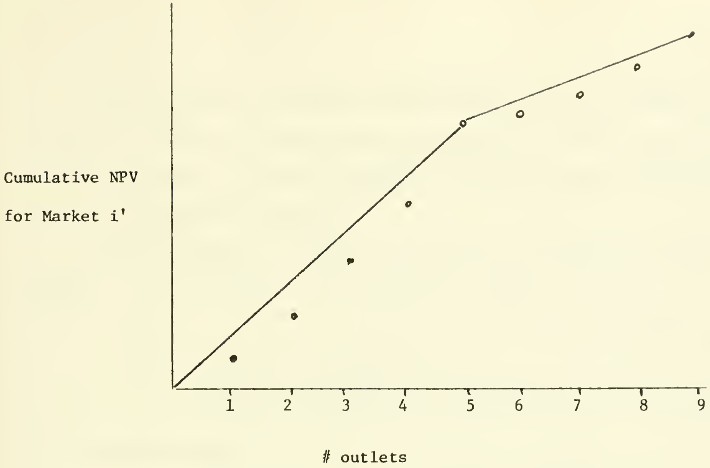

Figure 4

Assume the process continues until T outlets had been selected (and ignore Step 6 for the moment). Two events are possible:

(a) S, the running total of outlets, = T

(b) S > T.

If (a) occurs the resulting X_i is optimal by the theorem. If (b) occurs, an optimal solution has been found for problem (4.3) with S replacing T. This is not feasible for the original (4.3), but S is usually close enough to T to be acceptable for planning purposes.

An alternative which has been used is to insert a set of steps, (2a) in the algorithm:

(2a) If $S + j^* \leq T$ continue to 3.

If $S + j^* > T$, Find $M_{i'j} = \underset{i}{\text{Max}}\ W_{i'j'}$

such that $j' = T-S$. Then let i' replace i* and go to 3.

This may lead to a less than optimal solution and is, in essence, an algorithm "end effect." The end-effect problem has not proved nearly important enough in practice to justify the dynamic programming solution which would guarantee theoretical optimality.

Let us now consider a building program which can span several years ($Y > 1$).

Define: X_{it} = number of outlets built in market i in year t.

V_{ijt} = NPV of the cumulative jth outlet built in market i, given it is built in year t.

T_t = cumulative number of outlets which can be built up through year t. $t = 1....Y$.

All other quantities are altered by adding a subscript, t, to the prior symbol. The problem becomes:

(4.4) $$\max Z = \sum_{i=1}^{M} \sum_{t=1}^{Y} \sum_{j=1}^{X_{it}} V_{ijt}$$

subject to

$$0 \leq X_{it} \leq n_{it} \qquad i = 1...M,\ t=1...Y.$$

$$\sum_{k=1}^{t} \sum_{i=1}^{M} X_{ik} \leq T_t, \qquad t = 1, \ldots Y$$

$\{X_{it}\}$ integer

The multi-year problem is slightly more complicated that the single year problem. Two assumptions make the problem more tractable, however. Assume:

(A4) V_{it} is independent of the <u>time</u> at which outlets j-1 were built.

(A5) $V_{it} > V_{ij(t+1)}$ -- the earlier an outlet is built, the greater its NPV.

Then the algorithm for the multi-year case is very similar to that for the one-year case; the main difference is that the cumulative NPV matrix is formed from a three dimensional NPV matrix $[V_{ijt}]_{MXNXY}$ where $N = \max_{i,t} [n_{it}]$.

A problem which would seem to arise here (the reason for assumption A5) is that even though $V_{i,t}$ is independent of the time at which other outlets are built, the cumulative value of the first j stations does depend on the time at which the first (j-1) outlets are built (due to the aggressiveness definition among other things.)

Since it has been assumed that $V_{ijt} > V_{ij(t+1)}$, the cumulative value is the greatest when outlets are built as fast as constraints allow. Thus, the algorithm will always assume stations are built as soon as possible, and there is no ambiguity in calculating NPV's.

We still have the "end-effect" problem mentioned above in the multi-year problem and the comments made earlier apply here as well. In addition, another problem rests with the assumption that $V_{ijt} > V_{ij(t+1)}$. This cannot always be assumed in advance, although a large discount rate (internal rate of return) will almost always lead to this event. Large market growth rates or profit growth rates could lead to this assumption being violated.

Experience with this procedure has indicated that management generally concedes that the assumptions are reasonable, if debatable. Violation of the assumptions seem to be rare and when they occur, are slight and have little effect on allocation. And, as stressed earlier, the type of planning decision which the procedure is designed to support will not be grossly affected by small variations from optimal solutions.

5. NUMERICAL EXAMPLE AND COMPUTATIONAL EXPERIENCE

A small, two-market, example indicates basically how the algorithm works:

# outlets	Cumulative NPV Market A	Cumulative NPV Market B
1	5	4
2	8	9
3	12	16
4	14	20.5
5	15	22

Assume $n_1 = n_2 = 5$ and $T = 5$.

Initially,

Step 0. $$W_{ij} = \begin{bmatrix} 5 & 4 & 4 & 3.5 & 3 \\ 4 & 4.5 & 5.3 & 5.1 & 4.4 \end{bmatrix} \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; S = 0.$$

The maximum entry is 5.3 for Market B, 3 outlets. Three outlets are added to X_2 and S and W_{ij} are updated:

Step 1. $$W_{ij} = \begin{bmatrix} 5 & 4 & 4 & 3.5 & 3 \\ 4.5 & 3 & -M & -M & -M \end{bmatrix} \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \; S = 3.$$

The maximum entry here is 5 for Market A, 1 outlet. $\bar{X}$, W_{ij}, S are updated.

Step 2. $$W_{ij} = \begin{bmatrix} 3 & 3.5 & 3 & 2.5 & -M \\ 4.5 & 3 & -M & -M & -M \end{bmatrix} \quad \bar{X} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad S = 4.$$

The maximum entry is 4.5, Market B, 1 outlet. S, X are updated and S = T so the procedure stops with the allocation $\bar{X}$ = 1 outlet in Market A, 4 in Market B.

If this procedure were for a one-year plan the above would be complete. For a multi-year problem, an additional check has to be made after each allocation to be sure no single year constraint is violated. Otherwise the procedure is identical.

The algorithm is simple and efficient. Including set-up calculation of NPV's, a 170 market, 5-year problem, allocating 600 outlets has been run in under five minutes on an IBM 360-75. The bulk of that time is I/O and NPV Calculation; the allocation procedure itself took less than one minute. This makes update runs and sensitivity analysis quite inexpensive.

6. IMPLEMENTATION

This system has been used as an aid in outlet building plans at a major U.S. Corporation since 1969. For planning purposes the company breaks the U.S. down into seven operating regions, with each regional manager providing a five-year "building proposal" for markets in his region. (A region might contain as many as 35 markets). These proposals are then considered at a building-plan meeting, presided over by the Marketing Vice President. Invariably the individual proposals add up to considerably more building requests than the company annual constraints allow. Prior to the development of the model, political considerations and pseudo-quantitative arguments preceded an executive decision which left little room for reconsideration.

After the model was developed, the regional managers still produced manual proposals. But the model results, produced in parallel, became an additional input at the building plan meetings. The model inputs, as well as the outputs, were plain for all to see. Initial runs were rarely close to the proposals made by the regional managers -- input items were changed for further runs and building proposals were updated. After several iterations, model output and regional proposals were close enough so that the few differences could be resolved by hand. This process is schematically represented in Figure 5.

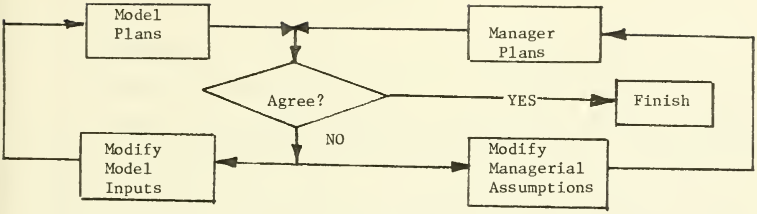

Figure 5: The "Implementation" process

There are a number of things to be learned from this implementation process. This model-interaction and input revision classifies the model as a "decision-calculus" type. (See Little [2]). The model does not replace or transcend the manager here; rather the interaction process provides more meaningful model-inputs and leads to more useful outputs. The managers are involved at every stage of shaping the final results; managers trusted the model because they could control it.

The authors consider the process outlined in Figure 5 to be indicative of successful model implementation. Model results are rarely (and should rarely) be used as they are. They are one input into the decision making process and the assumptions behind the model should be screened and adapted until they seem reasonable. During the screening and updating process, managers learn a great deal about their own decision-situation and, the authors have found, become more secure in their decisions.

7. CONCLUSION

A model was developed to help plan retail outlet building. From some very simple hypotheses about buyer behavior, an S-shaped outlet share-market share relationship was devised. This relationship was then one input in a resource allocation algorithm which efficiently produced optimal or near optimal plans.

The results of the study were "implemented" in the sense that they had an important influence on the decision-making process. Through use, management became more comfortable with the procedure and it became an integral part of the planning procedure.

REFERENCES

1. Hartung, P. H. and J. L. Fisher, "Brand Switching and Mathemaiical Programming in Market Expansion", Management Science, Vol. 2, August 1965, pp. 16-4 .

2. Little, J.D.C., "Models and Managers: The Concept of a Decision Calculus," Management Science, Vol. 16, No. 8 (April 1970), pp. B466-485.

3. Naert, P. A. and A. V. Bultez, "A Model of Aggregate Distribution Network Performance," Management Science, (forthcoming).

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