OPTIMAL COMPETITIVE MARKETING BEHAVIOR IN OLIGOPOLY

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The problem of marketing mix optimization has received considerable attention in the literature. For the profit maximizing firm such an optimization is the extension of the "marginal revenue equals marginal cost" rule to determine optimal levels for all marketing instruments rather than just price. Dorfman and Steiner derived an optimization rule for a monopolistic firm [9], which was subsequently applied by Palda [27] and Lambin [20] in empirical studies.

A large number of normative models of competitive marketing behavior in oligopolistic markets have also been formulated. Various assumptions related to industry demand (stable versus expandable) and to the type of competitive reaction (follower versus leader) have lead to a large variety of models. Many of these are theoretical in nature and have not been directly applied, such as the competitive models, by Mills [23], Gupta and Krishnan [14, 15, 19], Shakun [29], Baligh and Richartz [2], Kotler [18], Naert [24], and others. Examples of empirical studies dealing with stable industry sales are Lambin (follower-type reaction) [22] and Telser (leader-type reaction) [30], and with expansible industry sales are Schultz (follower) [28] and Bass (follower) [3].

In those studies where competitive reaction is explicitly taken into account (leader), it is implicitly assumed that competitors react with the same marketing instrument as the one which
causes their reactions, that is, they react to a change in prices by a change in price, to a change in advertising by a change in advertising. We will identify this kind of reaction with the simple competitive reaction case. It is more realistic, however, and more in the spirit of the very concept of marketing mix, to consider what we will call multiple competitive reaction, that is for example, a competitor may react to a change in price not just by changing his price, but also by changing his advertising and, possibly other marketing instruments as well. To our knowledge no optimality conditions have been derived yet for the multiple competitive reaction case.

In this paper we will remain within the realm of static analysis. First, we will derive profit maximization conditions for the multiple competitive reaction case. We will then demonstrate through a series of corollaries how optimization rules, previously developed, are special cases of our more general rule. The second part of this article will be devoted to the estimation problems arising in a multiple competitive reaction framework. Through the analysis of the data collected on a stable industry demand market, evidence of the importance of considering multiple competitive reactions will be presented.
I. PROFIT MAXIMIZATION : A GENERALIZATION

First, we will derive optimality conditions in terms of the vector of total sales elasticities $E_{q,u}$. We will then show how to decompose $E_{q,u}$ in its various components related to industry sales effects and market share effects. The notation adopted is defined in Table I.

Consider the following company profit function:

(1) $\pi = q \cdot [p - c_i(q,x)] - s$.

For the profit maximizing producer, optimality conditions are obtained by setting the derivative of $\pi$ with respect to each of the decision variables equal to zero. Let $u' = (p,s,x)$,

$$\frac{\partial \pi}{\partial u} = \frac{\partial q}{\partial u} (p - c) + q \cdot (\frac{\partial p}{\partial u} - \frac{\partial c_i}{\partial u} - \frac{\partial c_i}{\partial q} \cdot \frac{\partial q}{\partial u}) - \frac{\partial s}{\partial u} = 0$$

Let $D_u$ be a $(n \times n)$ - here $(3 \times 3)$ - diagonal matrix of the elements of vector $u$. Premultiplying $\partial \pi/\partial u$ by $D_u/q$,

$$(p - c - q \cdot \frac{\partial c_i}{\partial q}) \cdot \frac{D_u \cdot \partial q}{q} + D_u \cdot \frac{\partial p}{\partial u} - D_u \cdot \frac{\partial c_i}{\partial q} \cdot \frac{D_u \cdot \partial s}{q} = 0$$

With marginal cost $MC = c + q \cdot (\partial c_i/\partial q)$, and observing that
\[
\frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial u} = E_{q,u} \quad \text{the following optimality condition obtains}
\]

\[
(2) \quad E_{q,u} \cdot (p - MC) \cdot I : \left( \begin{array}{c} p, -s/q, -x \cdot \frac{\partial c_i}{\partial x} \end{array} \right) = 0
\]

where \( I \) is the identity matrix. From which we can deduce:

\[
(3) \quad -\eta_{q,p} = \left( \frac{pq}{s} \right) \cdot \eta_{q,s} = \left( \frac{p}{x \cdot (\partial c_i/\partial x)} \right) \cdot \eta_{q,x} = \frac{1}{w^*}
\]

Equation (3) is generally known as the Dorfman-Steiner theorem (or rule). In fact, it merely states that at optimality, marginal revenue must equal marginal cost for each marketing instrument. The Dorfman-Steiner rule is generally written in the following form:

\[
(4) \quad -\eta_{q,p} = \mu = \eta_{q,x} \cdot \frac{p}{x} = \frac{1}{w^*}
\]

where \( \mu = p \cdot \frac{\partial q}{\partial s} \), and \( \eta_{q,x} = \frac{\partial q}{\partial c/\partial x} \cdot \frac{c}{q} \).

Equation (4) holds, since:

\[
\frac{pq}{s} \cdot \eta_{q,s} = \frac{pq}{s} \cdot \frac{s}{q} \cdot \frac{\partial q}{\partial s} = p \cdot \frac{\partial q}{\partial s} = \mu, \quad \text{and}
\]

\[
\frac{p}{x} \cdot \frac{\eta_{q,x}}{\partial c/\partial x} = \frac{p}{x} \cdot \frac{\partial q}{\partial c/\partial x} \cdot \frac{x}{q} = \frac{p}{x} \cdot \eta_{q,x}
\]

Equations (3) and (4) are thus equivalent.
From equation (2) one can easily derive the optimal values for the decision variables. In vector notation, we obtain:

\[
(5) \quad u = \left( \frac{MC}{1 + \eta_{q,p}} \right) \cdot D_{E_{q,u}} \cdot \left( 1, -q, -\frac{1}{\partial c_1/\partial x} \right),
\]

where \( D_{E_{q,u}} \) is a diagonal matrix of the elements of vector \( E_{q,u} \).

We now want to show how the vector \( E_{q,u} \) of total sales elasticities can be decomposed into various elements in the case of an oligopoly. First, we can distinguish between a market share effect and an industry sales effect. And secondly, we can separate direct effects (assuming no competitive reaction) from indirect effect (competitive reaction effects). The theorem proposed below traces the links between total sales elasticities, industry demand, market share, and competitive reaction elasticities.

**Theorem**

The vector of elasticities of company sales with respect to its decision variables is equal to a matrix partitioned into an identity matrix and the matrix of competitive reaction elasticities postmultiplied by the sum of the vectors of industry sales and market share elasticities, i.e. \( E_{q,u} = [I, R] \cdot [E_{Q,T} + E_{m_i}] \)

**Proof:** Company sales can be written as industry sales times market share:
The derivative of \( q \) with respect to the decision vector \( u \) is:

\[
\frac{\partial q}{\partial u} = m \cdot \frac{\partial Q}{\partial u} + m \cdot \frac{\partial Q}{\partial u} + Q \cdot \frac{\partial m}{\partial u} + Q \cdot \frac{\partial m}{\partial u},
\]

where \( \frac{\partial U}{\partial u} = \left( \frac{\partial U_1}{\partial u}, \frac{\partial U_2}{\partial u}, \ldots, \frac{\partial U_n}{\partial u} \right) \). Premultiplying by \( D_u/q \), we obtain:

\[
\frac{D_u \cdot \partial q}{q \cdot \partial u} = \frac{D_u}{Q} \cdot \frac{\partial Q}{\partial u} + \frac{D_u}{Q} \cdot \frac{\partial Q}{\partial u} + \frac{1}{m} \cdot D_u \cdot \frac{\partial m}{\partial u} + \frac{1}{m} \cdot D_u \cdot \frac{\partial m}{\partial u},
\]

which reduces to:

(6) \[ E_{q,u} = E_{Q,T,u} + R \cdot E_{Q,T,u} + E_{m_1,u} + R \cdot E_{m_1,u} \]

(7) \[ E_{q,u} = [I,R] \cdot [E_{Q,T} + E_{m_1}] \]

The optimality conditions are then obtained by substituting the elements on the right hand side of equation (7) for the elements of \( E_{q,u} \) in equation (2).
COROLLARIES

The optimization rules used in the empirical studies referred to in the introduction can be derived very easily from corollaries to the theorem. In deriving these corollaries, we will use both forms of the theorem as obtained in equations (6) and (7). When we use the phrase "no competitive reaction" we mean that the company in deciding on values for its decision variables assumes no change in the values of competitors' decision variables. That is the case of a follower or a Cournot-type oligopolist. "Competitive reaction" will refer to the case where competitive reactions are explicitly taken into account in determining values for the decision variables. This is the case of a leader, as defined by Stackelberg.

MONOPOLY AND MONOPOLISTIC COMPETITION

Corollary 1: In monopoly q = Q, and U and R do not exist. It follows that:

\[(8) \quad E_{q,u} = E_{Q_T,u} \]

In a monopolistic competition situation, each firm faces its own demand curve and no industry demand is defined. This is the case derived by Dorfman and Steiner [9], and applied by Palda [27], and Feldstein [13] to study cases of monopolistic competition in the drug industry, and by Lambin for a frequently purchased consumer product [20].
**Corollary 2**: No competitive reaction, i.e. the matrix of reaction
elasticities \( R = 0 \). In which case:

\[
E_{q,u} = E_{m_1,u}.
\]

With a multiplicative market share function of the following type:

\[
m = a \cdot \left( \frac{p}{p + \bar{p}} \right)^{n_{m_1,p^*}} \cdot \left( \frac{s}{s + \bar{s}} \right)^{n_{m_1,s^*}} \cdot \left( \frac{x}{x + \bar{x}} \right)^{n_{m_1,x^*}},
\]

it is easily seen that:

\[
E_{m_1,u} = E_{m_1,u} = E_{m_1,u}^*.
\]

Equation (7) then reduces to

\[
E_{q,u} = E_{m_1,u}^*
\]

This has been used by Lambin in a study of a low price consumer durable good [22].

Sometimes one uses market share functions with the decision variables in share form, e.g.

\[
m = a \cdot \left( \frac{p}{p + \bar{p}} \right)^{n_{m_1,p^0}} \cdot \left( \frac{s}{s + \bar{s}} \right)^{n_{m_1,s^0}} \cdot \left( \frac{x}{x + \bar{x}} \right)^{n_{m_1,x^0}}
\]
For this type of market share function it is easily shown that

\[ E_{m_1,u} = - E_{m_1,U} = D_{U_0} \cdot E_{m_1,u^0} \]

where \( D_{U_0} \) is a diagonal matrix of the elements of vector

\[
U^0 = \left( \frac{U_1}{u_1 + U_1}, \frac{U_2}{u_2 + U_2}, \ldots, \frac{U_n}{u_n + U_n} \right)
\]

Equation (9) can then be written as

\[ E_{q,u} = D_{U_0} \cdot E_{m_1,u^0} \]

Equation (13) is appropriate for Cowling and Cubbin's study of the car market in the United Kingdom [8], although they implicitly assumed \( E_{q,u} = E_{m_1,u^0} \).

**Corollary 3: Simple Competitive Reaction**

\[ R = R_d, \text{ where } R_d \text{ has the same diagonal elements as } R, \]

but has zero off-diagonal entries. Equation (7) becomes:

\[ E_{q,u} = [I, R_d] \cdot E_{m_1} \]

and in the special case of a market share function in relative form,
\( (15) \quad E_{q,u} = [I - R_d] \cdot E_{m_i,u^*} \),

since according to equation (10),

\[
E_{q,u} = [I,R_d] \cdot \begin{bmatrix} E_{m_i,u} \\ E_{m_i,u} \end{bmatrix} = [I,R_d] \cdot \begin{bmatrix} E_{m_i,u^*} \\ -E_{m_i,u^*} \end{bmatrix}
\]

an expression equivalent to equation (15). Telser, in his study of advertising in the cigarette industry [30], derived a relation between absolute advertising elasticity \( \eta_{q,s} \) and relative elasticity \( \eta_{m_i,s^*} \), which is the advertising equation in (15). For a comparison of Lambin's results [22], and Telser's findings, see Naert [25].

We may also observe that for all cases with stable industry demand:

\[
E_{q,u} = E_{m,u}
\]

Using this in equation (15) gives:

\( (16) \quad E_{m,u} = [I - R_d] \cdot E_{m_i,u^*} \),

which is the form actually used by Telser.

**Corollary 4 : Multiple Competitive Reaction**

Only \( E_{Q_T} = 0 \), and hence
(17) \[ E_{q,u} = [I,R] \cdot E_{m,1} \]

or with the market share function in relative form

(18) \[ E_{q,u} = [I - R] \cdot E_{m,1},u^* \]

The special case of equation (18) has been derived by Bultez [6]. We refer to section II for an empirical example.

**OLIGOPOLY AND EXPANSIBLE INDUSTRY DEMAND** \((E_{Q,T} \neq 0)\)

**Corollary 5:** No Competitive Reaction. \(R = 0\), and equation (6) becomes:

(19) \[ E_{q,u} = E_{Q,T},u^* + E_{m,1},u \]

With the market share function in relative form this becomes

(20) \[ E_{q,u} = E_{Q,T},u^* + E_{m,1},u^* \]

The study by Bass on advertising and cigarettes is of this kind [3].

Given a market share function with the decision variables in share form, equation (19) can be written as:

(21) \[ E_{q,u} = E_{Q,T},u^* + D_{u,0} \cdot E_{m,1},u^0 \]
Equation (21) is the relevant form of equation (6) for Schultz' study of competition between airlines in a two-city market [28].

**Corollary 6**: Simple Competitive Reaction. \( R = R_d \), and equation (5) becomes:

\[
E_{q,u} = [I,R_d] \cdot [E_{QT} + E_{mi}] 
\]

Telser's study of advertising and cigarettes contains a relation equivalent to (22) for advertising [30]. This equivalence is demonstrated in the appendix.

With the market share function in relative form one gets:

\[
E_{q,u} = [I,R_d] \cdot E_{QT} + [I - R_d] \cdot E_{mi}, u^* 
\]

**Corollary 7**: Oligopoly - Expansible Industry Demand - Multiple Competitive Reaction - Specific Functional Form for Market Share.

The theorem holds for any kind of market share function. In the special case of a market share function with the decision variables in relative form, we find:

\[
E_{q,u} = [I,R] \cdot E_{QT} + [I - R] \cdot E_{mi}, u^* 
\]

a result also derived by Bultez [6].
The theorem and corollaries 2 to 6 are succinctly summarized in Table II.

Corollaries 2 to 7 demonstrate how total sales elasticities relate to industry sales and market share elasticities for various kinds of competitive reaction in different types of oligopolies. In each case, we have presented this relationship in general, and also for specific forms of the market share function. For each corollary we have referred to at least one application, except for the multiple competitive reaction cases. Empirical evidence demonstrating the importance of considering multiple competitive reaction will be presented in the following section.

Figure 1 illustrates the explicit decomposition of $\eta_q$, for the cases considered in this section.
II. THE EMPIRICAL STUDY

The objective of this section is to present an empirical application of the multiple competitive reaction concept introduced in the first part of this paper. The product studied is a low price consumer durable good having reached the saturation phasis on its life cycle curve. The market is dominated by three brands which represent altogether approximately 90% of total industry sales. Firms A, B and C manufacture different brands of the same general product category. They do not offer, however, identical quality nor do they charge identical prices. Advertising expenditures represent in each case a significant but variable proportion of total sales revenue. Since primary demand is stable, all the increased sales of a given firm come necessarily at the expenses of its rivals and consequently the degree of recognition of mutual interdependence is very high. Thus, the market has the structural characteristics of a differentiated oligopoly.

The data available for this study are fairly reliable. Market shares, retail prices and distribution rates come from a dealer panel. These panel data, which are purchased by the three firms are available on a quarterly basis from 1960 to 1966 and over four geographic regions. Company sales data come from company sources and total industry sales can also be estimated through statistics published by governmental agencies. Media advertising expenditures (press, radio and television) per brand, month and region are provided by a commercial service from which monitors the media and values
time and space at the rate card list price. Point of sales promotion expenditures are not included in the sample. Media expenditures represent about 85% of total advertising outlays. Each brand quality is estimated by a synthetic index derived from a semantic differential scale [26]; empirically selected product attributes where then subjectively rated, period by period, by the marketing staff of one of the firms.

Bain [1] suggests that product differentiation inhibits changes in the percent of the market held by the oligopolistic firms. In contradiction with this view, a high degree of market share instability is observed in this market during the period under study. The estimated indices of market share instability are presented for each region in Table III. In 1960, the partition of the market among the three firms - the others not included - was as follows: 30.2%, 37.9% and 31.9% for firms A, B and C respectively. The dominant fact of the 1962-66 period is the aggressive and successful marketing strategy of firm B, whose market share increased from 32.8% in 1962 to 66.1% in 1966. As a result of firm B aggressive policy, drastic changes occurred in the degree of market concentration. The Herfindahl index of Table III, graphically represented on figure 2, illustrates this fact.

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In assuming the leadership of the market, firm 3 placed heavy emphasis on advertising and its share of total industry advertising expenditures increased from 18.9% in 1962 to 61.7% in 1965. Because of the ratchet nature of advertising and of reactions from the competing firms, total (deflated) industry advertising expenditures have dramatically escalated and were four times higher by the end of the period studied. The advertising-sales ratios of the firms are presented in Table IV. They show that advertising has been particularly instrumental in the growth of firm's B market position and present evidence of strong competition by means of advertising.

Selected ordinary least-squares estimates of market share elasticities with respect to price, quality and advertising are also presented in Table V. These results suggest that price and quality have also played a significant role since a significant part of the firms market share variance can be associated, not only with changes in advertising intensities as expected in oligopoly, but also with price and quality adjustments. Thus, the firms strategy has been that of the marketing mix. In view of this competitive situation it is of interest to see how the competing firms have reacted to the aggressive policy of firm B and, successfully or not, adjusted their own marketing variables to the new competitive situation created by the leader. The multiple reaction functions concept
should be an instructive analytical tool for that purpose.

ESTIMATION OF THE COMPETITIVE REACTION ELASTICITIES

As pointed out by Brems [5], expected rival response is uncertain and has a time dimension as well as an extent dimension. Of the two questions, how will the rival respond and how soon will be respond, the latter is not less relevant than the former. The timing of expected competitive reaction to price changes is radically different from the timing of expected rival reactions to changes in the non-price variables such as advertising and quality. Price variations can be put into action immediately, while variations in quality attributes often require technological research. Similarly, because of media space limitations and of the time required for the preparation of an advertising campaign, competitive reaction by means of advertising are generally slower than price reactions. Thus, the functions are as follows:

\[ P_t = K_p \cdot \frac{\partial P_t}{\partial P_t} \cdot \frac{\partial P_t}{\partial s_t} \cdot \frac{\partial P_t}{\partial x_t} \]

and similarly, but lagged, for advertising and quality. The double logarithmic form is adopted here to cope explicitly with the interacting pattern existing among the marketing variables. Thus, the matrix of reaction elasticities \([R]\), is constant.
The periodicity of the data used to estimate the key-parameters is annual because sales being very concentrated in one period of the year, it is practically impossible for competition to react within a shorter period of time. By pooling the data over time and regions we obtain a total of 28 yearly observations. However, since the quality of products and the prices charged to the consumers are identical across regions, we only have 7 degrees of freedom left for these variables.

On the basis of the leader-follower reaction theory we designed various simultaneous equation models and estimated them using the well-known two-stage least-squares method in order to test one major assumption, namely: interdependence between \( P \), \( S \) and \( X \), which would result from the followers full marketing mix orientation. Because of the postulated fundamental dependence of \( P \), \( S \) and \( X \) on the same three leader's decision variables \( p \), \( s \) and \( x \), we wondered whether the detected dependence between \( P \), \( S \) and \( X \) might not merely be due to a spurious correlation effect. The test we devised was unconclusive. Furthermore, the contemporaneous covariances of the residuals across equations were not sufficiently high to justify the application of the joint generalized least-squares technique [32, pp. 295-311].

As a result of this preliminary and rather unsuccessful econometric investigation, we turned back to a much simpler set of independent equations to estimate \([R]\). We used then the Cochrane-Orcutt (CODC) iterative procedure [32, pp. 253-256]. The results are presented in Table VI.
The direct reaction elasticities are all three positive and statistically significant at the 97.5 per cent level or higher. Their order of magnitude imply fairly strong response from rivals to the leader's moves in price and quality. As expected in the phase of innovative maturity the high quality reaction elasticity suggests an almost explosive reaction pattern in terms of product quality adjustments.

Significant support is also given to the multiple reaction concept, since three out of six indirect reaction elasticities (off-diagonal cells) are significant. The most significant indirect reaction elasticities are observed in the quality reaction function (despite the number of degrees of freedom left). The signs of the price and advertising variables are as expected. They indicate that brand B's competitors not only have reacted in terms of product quality adjustments to the leader's move in quality, but also to the leader's move in price and advertising. In the advertising reaction function, the positive sign of the price variable is surprising at first sight. We may have expected, indeed, a reverse counteraction, i.e. an increase in competitor's advertising to annihilate the leader's aggressiveness in price. In fact, this non-intuitive result is simply the manifestation of brand B's competitors failure to counteract efficiently. During the period under study, brand B has consistently dominated his rivals not only in terms of his product-quality ratio, but also in terms of its advertising intensity. Thus, this positive
sign may simply reflect the passivity of brand B's competitors who never succeeded to fill up their gap vis-à-vis the leader. By dramatically increasing its share of total industry advertising expenditure (up to 65%), brand B has prevented its rivals from effective counteraction by means of advertising. In other words, competition was unable to follow the advertising escalation in order to compensate for this price-quality gap. A further confirmation of the interpretation is given by the modest order of magnitude of the direct advertising reaction elasticity.

Thus, those results seem to be consistent with the competing firms actual marketing behavior. They also support the proposition that the firm competitive behavior must be analyzed within the context of the whole marketing mix.

**ESTIMATION OF THE LEADER'S SALES ELASTICITIES**

We now report results obtained when we assume a simultaneous dependence between the level of advertising expenditures and the level of sales or market share since the industry demand is stable. A static and a dynamic version of the model were estimated using the instrumental variables method (equivalent to the two-stage least-squares in this case). Brand B's market share elasticities are reported in Table VII.

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*INSERT TABLE VII ABOUT HERE*
In both static and dynamic versions the signs and absolute values of the parameters are as expected and are comparable with the ordinary least-squares estimates of Table V. They are all significant at the 95 per cent level or higher. In the static model, the test for serial correlation is performed by calculating the Durbin statistic \( d \) based on the residuals from the multiple regression of \( [m_t - \hat{m}_t, s_t^* \cdot s_t] \) on all the predetermined variables of the system, whether or not they occur in the fitted equation with non-zero coefficients [10, p. 377]. Referring to Durbin and Watson table [32, pp. 724-725], we realize that \( d \) indicates a positive autocorrelation. In the dynamic version, the test for serial correlation is carried out by testing the significance of the lagged residuals in the regression of the residuals on the lagged residuals and the explanatory variables [33, pp. 420-421]. The \( d \) statistic of the dynamic market share model is in fact the \( t \) statistic corresponding to the lagged residuals coefficient. We can thus conclude that the residuals of the dynamic market share model are uncorrelated. From a statistical specification point of view, the dynamic version of the model seems preferable to the static version. However, given the characteristics of the market and the periodicity of the data used, the static version is subjectively perfectly acceptable. As both versions have their merits we will continue to take both of them into account for the rest of the analysis.

Having ascertained the validity of the empirical results we may now return to the theorem we proposed in the first sec-
ECONOMIC INTERPRETATION OF THE RESULTS

The $E_{q,u}$ vector contains the total sales elasticities taking into account the reactions of competitors while the elements of vector $E_{m_1,u^*}$ are the total sales elasticities when competitors do not react. This can easily be proven since if competitors do not react $[R] = [0]$ and $E_{q,u} = E_{m_1,u^*}$. This important feature enables us to visualize the demand for the leader's products as a kinked response curve, drawn on figure 3.

It is worth noting that these estimates of sales elasticities ($E_{q,u}$) are significantly different from the market share elasticities ($E_{m_1,u^*}$). Thus, any judgment on the firms' behavior ba-
sed on the $E_{m_1,u^*}$ vector would have been seriously biased. For example, the leader's monopoly power would have been underestimated.

If we assume profit-maximizing behavior, then our estimate of the potential degree of monopoly power, $100 \cdot \left( \frac{p - MC}{MC} \right)$, the percentage mark-up on marginal cost, is equal to $-100/(1 + \eta_{q,p})$. Our estimates of price elasticities are -1.873 in the case of the static version and -1.734 in the case of the dynamic version. This gives a percentage mark-up of price above marginal cost of 114.54% or 136.29%, respectively and the corresponding percentages of gross margin are 53.39% and 57.68%. To appreciate the welfare loss attributable to this monopoly power the ratio of competitive price (MC) to monopoly price (p) is also very instructive; it is a decreasing function of the price elasticity : $[1 + 1/\eta_{q,p}]$. The ratio estimates are 0.4661 and 0.4232 in this case. All these results confirm the high degree of market concentration observed by the end of 1966.

As far as the advertising policy of the leader's is concerned, we can compute the value of the advertising-sales ratio at optimum derived from the Dorfman-Steiner rule, according to equation (3):

$$\eta_{q,p} = \left( \frac{p \cdot q}{s} \right) \cdot \eta_{q,s}$$

thus at optimum:

$$\alpha = \frac{s}{p \cdot q} = \frac{\eta_{q,s}}{-\eta_{q,p}}$$
The static model gives an estimate of a equal to 0.063 and the dynamic form an estimate equal to 0.002. A comparison of these values with the observed advertising-sales ratios, the mean of which is 0.088, shows that although firm B had been overspending from 1963 to 1965, on the average, its advertising behavior had been rational. An additional confirmation of this fact is obtained when the firm's marginal return on advertising is computed:

\[
\text{MRA} = (p - MC) \cdot \frac{\partial q}{\partial s}
\]

on the average

\[
\text{MRA} = (p - MC) \cdot \eta_{q,s} \cdot (\bar{q}/\bar{s})
\]

Assuming a 53% gross margin and using the static version estimates, we get:

\[
\text{MRA} = (10.71) \cdot (0.118) \cdot (983,000/1,407,257) = \$ 0.883
\]

Thus the last dollar spent in advertising produces approximately \$0.88 cents.

If we rely on the dynamic version estimates we should take the long-term impact of advertising into account. The long-term marginal cost marginal revenue equality rule in actualized form then becomes:

\[
(25) \quad \sum_{\beta=0}^{\infty} (p - MC) \cdot \left( \frac{\partial q_{t+\beta}}{\partial s_t} \right) = \sum_{\beta=0}^{\infty} (1 + r)^\beta
\]
where \( r \) is the cut-off rate. Assuming price and quality fixed,

\[
q_t + \beta = Q \cdot m_t + \beta = \bar{w} \cdot k_m \cdot s_t + \beta \cdot \sum_{\tau=0}^{\infty} \frac{\lambda^\tau \cdot [n_{q,s} - (1-\lambda) \cdot n_{m_i,s}] }{t + \beta - \tau - 1}
\]

where \( k_m = k \cdot K \cdot \frac{m_i \cdot p^*}{p} \cdot K \cdot \frac{m_i \cdot s^*}{s} \cdot K \cdot \frac{m_i \cdot x^*}{x} \cdot [p \cdot q, p^* \cdot x, q, x]
\]

\( \gamma = 1/(1 - \lambda) \)

Thus,

\[
\frac{\partial q_t + \beta}{\partial s_t} = \begin{cases} 
\lambda^\beta \cdot \frac{\partial q_t + \beta}{\partial s_t} = \begin{cases} 
\eta_{m_i,s^*} & \text{for } \beta = 0 \\
\eta_{q,s} - (1-\lambda) \cdot \eta_{m_i,s^*} & \text{when } \beta \neq 0 
\end{cases} \end{cases}
\]

Substituting the value of \( \frac{\partial q_t + \beta}{\partial s_t} \) extracted from (26) into equation (25) and solving for \( s_t \) gives:

\[
s_t = (p - MC) \cdot q_t \cdot \left( \frac{\eta_{q,s} - (1-\lambda) \cdot \eta_{m_i,s^*}}{\lambda} \right) \cdot \sum_{\beta=1}^{\infty} \left( \frac{\lambda}{1 + r} \right)^\beta \cdot q_t + \beta
\]

At equilibrium,

\[
s_e = \lim_{t \to \infty} [s_t] = (p - MC) \cdot q_e \cdot \left( \frac{\eta_{q,s} + r \cdot \eta_{m_i,s^*}}{1 + r - \lambda} \right)
\]

where \( q_e = Q \cdot \left( k_m \cdot s_e \right)^\gamma \),
and as a consequence,

\[
(29) \quad s_e = \left( (p - MC) \cdot Q \cdot k^\gamma_m \cdot \left( \frac{\eta_{q,s} \cdot r \cdot \eta_{m,t,s}^*}{1 + r - \lambda} \right) \delta \right)
\]

where \( \delta = \frac{1}{1 - \gamma \cdot \eta_{q,s}} \)

The long-run optimal equilibrium value of the advertising expenditures is thus equal to:

\[
(11.57) \cdot (2,100,000) \cdot (0.149)^{1.524} \cdot (\frac{0.093 + r \cdot 0.147}{1 + r - 0.344})^\delta
\]

where \( \delta = \frac{1}{1 - (1.524) \cdot (0.093)} \)

To discount the flow of returns produced by the advertising investment various interest rates may be considered. Yield on shares in the same industry would give an idea of profit opportunities offered to the firm by alternative investment decisions. However over the period studied, dividends were not high enough to compensate for the going down stocks so that yield on shares is not indicative of the external investment opportunities. As a result we turn to the interest borne by commercial bills over the same period. A 6.5% average rate was observed and the corresponding long-run optimal
level of the advertising expenditures amounts to $s_e = 1,424,079 which should be compared with firm B's real expenditures $s = 1,407,257. Although the average amount spent is below $s_e$ we are, once again, induced to conclude that firm B has been overspending over the 1964-1966 period. A sensitivity analysis of the long-run equilibrium level $s_e$ to the discount rate is shown in Table IX and we see that $s_e$ is rather insensitive to $r$. 

---

**INSERT TABLE IX ABOUT HERE**
CONCLUSION

In this paper we first generalized the well known Dorfman-Steiner theorem in considering explicitly the multiple competitive reaction case. We then reviewed the literature through a series of corollaries. On the basis of the theorem we developed, we attempted to shed light on an oligopolistic market over a period characterized by the rapid raise of a leader. To estimate the key-parameters we turned to a very simple model. The shortage of degrees of freedom prevented us from trying any more complex form. However weak the statistical properties of some of our estimates might be we are nevertheless confident in their descriptive as well as in their normative value. Indeed they seem to perfectly describe the firms' competitive behavior and reflect our subjective a priori feelings about the market. From a predictive point of view, the competitive reaction elasticities should probably be judgmentally guessed and adjusted 21 rather than econometrically derived. In our case, judgmental and econometric estimates were converging.
FIGURE 2
EVOLUTION OF MARKET CONCENTRATION
\[ q = f(p/s, x) \]

* Drawn from the dynamic version estimates
TABLE I - GLOSSARY OF SYMBOLS

COMPANY DECISION VECTOR
\[ u' = [u_1', u_2', \ldots, u'_i, \ldots, u'_n] \]

where typically,
\[ u'_1 = P = \text{average price of competitors} \]
\[ u'_2 = S = \text{advertising expenditures of competitors} \]
\[ u'_3 = X = \text{average product quality of competitors} \]

COMPETITORS' DECISION VECTOR
\[ U' = [U_1, U_2, \ldots, U_n] \]
\[ U_1 = P = \text{average price of competitors} \]
\[ U_2 = S = \text{advertising expenditures of competitors} \]
\[ U_3 = X = \text{average product quality of competitors} \]

RELATIVE AND SHARE VARIABLES
\[ u^* = [u^*_1, u^*_2, \ldots, u^*_i, \ldots, u^*_n] \]
\[ u^*_i = u_i/U_i \text{ for } i=1,\ldots,n \]
\[ u^0 = [u^0_1, u^0_2, \ldots, u^0_i, \ldots, u^0_n] \]
\[ u^0_i = u_i/(u_i + U_i) \text{ for } i=1,\ldots,n \]

ENVIRONMENTAL VARIABLES
\[ Z' = [Z_1, Z_2, \ldots, Z_m] \]
\[ Z_1 = \text{per capita income} \]
\[ Z_2 = \text{population size} \]

DEMAND VARIABLES
\[ q = \text{company sales} \]
\[ Q = \text{industry sales} \]
\[ q_c = Q - q = \text{competitors sales} \]
\[ m = \text{company market share} \]

DEMAND EQUATIONS
\[ q = q_i(u, U, Z) \]
\[ Q = Q_i(u, U, Z) \]
\[ m = m_i(u, U) \text{ or,} \]
\[ m = m_i(u^*) \text{ or,} \]
\[ m = m_i(u^0) \]

COST AND PROFIT VARIABLES
\[ MC = \text{marginal cost} \]
\[ p = MC = w = \text{gross margin} \]
\[ (p - MC)/p = w^* = \text{percentage of gross margin} \]
\[ c = c_i(q, x), \text{cost equation} \]

VECTORS OF DEMAND ELASTICITIES
\[ E_{y,x} = [\eta_{y,x_1}, \eta_{y,x_2}, \ldots, \eta_{y,x_n}] \]

where \[ \eta_{y,x_i} = \frac{\partial y}{\partial x_i} \cdot \frac{x_i}{y} \]

and \[ y \in \{q, q_c, m, m_i, Q_i\} \]
\[ x \in \{u, U, u^*, u^0\} \]

\[ E_{m_1} = \begin{bmatrix} E_{m_1,u} & E_{m_1,U} \end{bmatrix} \]
\[ E_{Q_T} = \begin{bmatrix} E_{Q_T,u} & E_{Q_T,U} \end{bmatrix} \]

MATRIX OF REACTION ELASTICITIES
\[ R = \begin{bmatrix} \rho_{U_1, u_1} & \rho_{U_2, u_1} & \cdots & \rho_{U_n, u_1} \\ \rho_{U_1, u_2} & \rho_{U_2, u_2} & \cdots & \rho_{U_n, u_2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{U_1, u_n} & \rho_{U_2, u_n} & \cdots & \rho_{U_n, u_n} \end{bmatrix} \]

where \[ \rho_{U_j, u_i} = \frac{\partial u_j}{\partial u_i} \cdot \frac{u_i}{U_j} \]
**Table II**

**Decomposition of Total Sales Elasticities** \( E_{q,u} \)

**In Oligopoly**

<table>
<thead>
<tr>
<th></th>
<th>Stable Industry Demand</th>
<th>Expansible Industry Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>No competitive reaction</td>
<td>( E_{q,u} = E_{m_1,u} )</td>
<td>( E_{q,u} = E_{Q_T,u} + E_{m_1,u} )</td>
</tr>
<tr>
<td>Simple competitive reaction</td>
<td>( E_{q,u} = [I,R_d] \cdot E_{m_1} )</td>
<td>( E_{q,u} = [I,R_d] \cdot [E_{Q_T} + E_{m_1}] )</td>
</tr>
<tr>
<td>Multiple competitive reaction</td>
<td>( E_{q,u} = [I,R] \cdot E_{m_1} )</td>
<td>( E_{q,u} = [I,R] \cdot [E_{Q_T} + E_{m_1}] )</td>
</tr>
</tbody>
</table>
## Table III

**SELECTED INDICATORS OF MARKET STRUCTURE**

<table>
<thead>
<tr>
<th>INDICATORS</th>
<th>REGION I</th>
<th>REGION II</th>
<th>REGION III</th>
<th>REGION IV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Concentration measures</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Herfindahl</td>
<td>0.339</td>
<td>0.338</td>
<td>0.344</td>
<td>0.336</td>
</tr>
<tr>
<td>1960</td>
<td>0.458</td>
<td>0.529</td>
<td>0.497</td>
<td>0.536</td>
</tr>
<tr>
<td>- Entropy</td>
<td>1.090</td>
<td>1.092</td>
<td>1.083</td>
<td>1.095</td>
</tr>
<tr>
<td>1960</td>
<td>0.926</td>
<td>0.827</td>
<td>0.869</td>
<td>0.815</td>
</tr>
<tr>
<td><strong>2. Market share instability</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- $I_1$</td>
<td>17.5</td>
<td>20.4</td>
<td>16.3</td>
<td>22.3</td>
</tr>
<tr>
<td>- $I_2$</td>
<td>18.2</td>
<td>21.2</td>
<td>17.3</td>
<td>22.8</td>
</tr>
<tr>
<td>- $I_3$</td>
<td>16.8</td>
<td>16.2</td>
<td>16.9</td>
<td>19.4</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>PERIODS</th>
<th>FIRM A</th>
<th>FIRM B</th>
<th>FIRM C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>0.0579</td>
<td>0.0321</td>
<td>0.0226</td>
</tr>
<tr>
<td>1961</td>
<td>0.0828</td>
<td>0.0629</td>
<td>0.1370</td>
</tr>
<tr>
<td>1962</td>
<td>0.1229</td>
<td>0.0531</td>
<td>0.1009</td>
</tr>
<tr>
<td>1963</td>
<td>0.1435</td>
<td>0.0922</td>
<td>0.1083</td>
</tr>
<tr>
<td>1964</td>
<td>0.1833</td>
<td>0.1190</td>
<td>0.2034</td>
</tr>
<tr>
<td>1965</td>
<td>0.1643</td>
<td>0.1513</td>
<td>0.1253</td>
</tr>
<tr>
<td>1966</td>
<td>0.1972</td>
<td>0.1083</td>
<td>0.2662</td>
</tr>
<tr>
<td>MEAN</td>
<td>0.1290</td>
<td>0.0884</td>
<td>0.1377</td>
</tr>
</tbody>
</table>

* Industry sources
<table>
<thead>
<tr>
<th>Varpus</th>
<th>Vartaus</th>
<th>Vaparäkki</th>
<th>Vaparäkki</th>
<th>Vaparäkki</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.20</td>
<td>117.0</td>
<td>149.0</td>
<td>149.0</td>
<td>149.0</td>
</tr>
<tr>
<td>23.00</td>
<td>217.0</td>
<td>239.0</td>
<td>239.0</td>
<td>239.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>14.00</td>
<td>124.0</td>
<td>146.0</td>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>BRANDS</td>
<td>Relative Quality Elasticity $\eta_{m,x^*}$</td>
<td>Relative Price Elasticity $\eta_{m,p^*}$</td>
<td>Company Advertising Elasticity $\eta_{m,s}$</td>
<td>Competitive Advertising Elasticity $\eta_{m,S}$</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------</td>
<td>-------------------------------------</td>
<td>---------------------------------------</td>
<td>---------------------------------------</td>
</tr>
<tr>
<td>A</td>
<td>0.219$^d$</td>
<td>-3.818$^d$</td>
<td>0.256$^c$</td>
<td>-0.207$^b$</td>
</tr>
<tr>
<td>B</td>
<td>0.708$^a$</td>
<td>-4.929$^a$</td>
<td>0.159$^a$</td>
<td>-0.104$^b$</td>
</tr>
<tr>
<td>C</td>
<td>N.S.</td>
<td>-4.735$^a$</td>
<td>0.143$^a$</td>
<td>-0.301$^a$</td>
</tr>
</tbody>
</table>

* Short-term market share elasticities estimated in the regression:

$$m_{i,t} = k \lambda_{i,t-1} \cdot (p_{i,t}^*) \cdot (s_{i,t}^*) \cdot (x_{i,t}^*) \cdot (\eta_{m,p^*}) \cdot (\eta_{m,s^*}) \cdot (\eta_{m,x^*})$$

One-tailed t test: a, b, c and d mean significant at the 99%, 97.5%, 95% and 90% confidence level.
# Table VI

**Multiple Reaction Functions: Regression Results**

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>(1)</th>
<th>(\text{Log } p_t)</th>
<th>(\text{Log } p_{t-1})</th>
<th>(\text{Log } s_t)</th>
<th>(\text{Log } s_{t-1})</th>
<th>(\text{Log } x_t)</th>
<th>(\text{Log } x_{t-1})</th>
<th>(R^2)</th>
<th>(R^2)</th>
<th>(F)</th>
<th>((\nu_1, \nu_2))</th>
<th>D.W</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Log } P_t)</td>
<td>0.935</td>
<td>0.664</td>
<td>-</td>
<td>0.008</td>
<td>-</td>
<td>0.017</td>
<td>-</td>
<td>0.983</td>
<td>0.940</td>
<td>39.55</td>
<td>(3,2)</td>
<td>2.033</td>
</tr>
<tr>
<td>(0.119)</td>
<td>(0.030)</td>
<td>(0.005)</td>
<td>(0.053)</td>
<td>7.880</td>
<td>22.357</td>
<td>1.510</td>
<td>0.320</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Log } S_t)</td>
<td>-0.522</td>
<td>-</td>
<td>1.898</td>
<td>-</td>
<td>0.273</td>
<td>-</td>
<td>-0.774</td>
<td>0.670</td>
<td>0.608</td>
<td>10.83</td>
<td>(3,16)</td>
<td>1.879</td>
</tr>
<tr>
<td>(3.126)</td>
<td>(0.825)</td>
<td>(0.123)</td>
<td>(1.032)</td>
<td>-0.199</td>
<td>2.302</td>
<td>2.221</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Log } X_t)</td>
<td>1.506</td>
<td>-</td>
<td>-0.594</td>
<td>-</td>
<td>0.023</td>
<td>-</td>
<td>0.901</td>
<td>0.979</td>
<td>0.916</td>
<td>15.54</td>
<td>(3,1)</td>
<td>1.850</td>
</tr>
<tr>
<td>(0.176)</td>
<td>(0.046)</td>
<td>(0.007)</td>
<td>(0.058)</td>
<td>8.555</td>
<td>12.735</td>
<td>3.383</td>
<td>15.468</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

* Using the CORC iterative procedure.

- \(\nu_1\) and \(\nu_2\) are the degrees of freedom corresponding to the explained and unexplained sum of squares (in computing \(\nu_2\), the F-statistics and \(R^2\) we took into account the fact that price and quality decisions were identical across regions).

- D.W. is the usual Durbin-Watson statistic adjusted for gaps in the data.

- The standard errors (in parentheses) and t-statistics are reported below each coefficient.

- One-tailed t test: a, b, c and d mean significant at the 99 %, 97.5 %, 95 % and 90 % confidence-level, respectively.
<table>
<thead>
<tr>
<th>JOINTLY DEPENDENT VARIABLES</th>
<th>PREDETERMINED VARIABLES</th>
<th>STATISTICAL CRITERIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log $a_t$</td>
<td>Log $n_t$</td>
<td>1</td>
</tr>
<tr>
<td>Static version</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>2.068</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.219)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.439$^a$</td>
<td>6.260$^a$</td>
</tr>
<tr>
<td>0.178</td>
<td>-1</td>
<td>-0.801</td>
</tr>
<tr>
<td>(0.057)</td>
<td></td>
<td>(0.022)</td>
</tr>
<tr>
<td>3.121$^a$</td>
<td></td>
<td>-36.988$^a$</td>
</tr>
<tr>
<td>Dynamic version</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>2.043</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.216)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.444$^a$</td>
<td>6.238$^a$</td>
</tr>
<tr>
<td>0.147</td>
<td>-1</td>
<td>-0.512</td>
</tr>
<tr>
<td>(0.056)</td>
<td></td>
<td>(0.180)</td>
</tr>
<tr>
<td>2.593$^a$</td>
<td></td>
<td>-2.842$^a$</td>
</tr>
</tbody>
</table>
### TABLE VIII

**TOTAL SALES ELASTICITIES**

\[
R = \begin{bmatrix}
0.637^a & 1.898^b & -0.594^b \\
0 & 0.273^b & 0.023^d \\
0 & -0.774 & 0.901^b
\end{bmatrix}
\]

**Static Model**

\[
\begin{align*}
E_{m_i,u^*} &= \begin{bmatrix}
-5.543^a \\
0.178^a \\
0.503^c
\end{bmatrix} \\
E_{q,u} &= \begin{bmatrix}
-1.873 \\
0.118 \\
0.188
\end{bmatrix}
\end{align*}
\]

**Dynamic Model**

\[
\begin{align*}
E_{m_i,u^*} &= \begin{bmatrix}
-3.726^b \\
0.147^a \\
0.583^b
\end{bmatrix} \\
E_{q,u} &= \begin{bmatrix}
-1.138 \\
0.093 \\
0.171
\end{bmatrix}
\end{align*}
\]

**Short-run**

\[
\begin{align*}
E_{m_i,u^*} &= 0.147^a \\
E_{q,u} &= 0.093
\end{align*}
\]

**Long-run**

\[
\begin{align*}
E_{m_i,u^*} &= 0.223 \\
E_{q,u} &= 0.142
\end{align*}
\]
TABLE IX

SENSITIVITY ANALYSIS OF THE LONG-RUN OPTIMAL EQUILIBRIUM LEVEL OF ADVERTISING EXPENDITURES

<table>
<thead>
<tr>
<th>r</th>
<th>( s_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 %</td>
<td>$1,421,246</td>
</tr>
<tr>
<td>4 %</td>
<td>$1,422,083</td>
</tr>
<tr>
<td>5 %</td>
<td>$1,422,899</td>
</tr>
<tr>
<td>6 %</td>
<td>$1,423,689</td>
</tr>
<tr>
<td>7 %</td>
<td>$1,424,461</td>
</tr>
<tr>
<td>8 %</td>
<td>$1,425,209</td>
</tr>
<tr>
<td>9 %</td>
<td>$1,425,940</td>
</tr>
<tr>
<td>10 %</td>
<td>$1,426,650</td>
</tr>
</tbody>
</table>
Telser derived the following relation (translated in our notation) between the various elasticities with respect to advertising:

\[ (A.1) \quad \eta_{m,s} = (1 - m) \cdot (\eta_{q,s} - \eta_{Q,c,s}) = \eta_{m_i,s^*} \cdot (1 - \rho_{S,s}) \]

In fact, this should have been written as two separate equalities because they result from different assumptions. First, let us take

\[ (A.2) \quad \eta_{m,s} = \eta_{m_i,s^*} \cdot (1 - \rho_{S,s}) \]

This holds in the case of an oligopoly with stable industry sales, simple competitive reaction and the market share function in relative form (see equation (15)).

On the other hand,

\[ (A.3) \quad \eta_{m,s} = (1 - m) \cdot (\eta_{q,s} - \eta_{Q,c,s}) \]

is the advertising equivalent of equation (22), that is an oligopoly with expansible industry demand, simple competitive reaction and the market share function in general form. This equivalence will now be demonstrated.
Equation (A.3) can be written as

\[
\frac{1}{1 - m} \cdot \eta_{m,s} = \eta_{q,s} - \eta_{Q_c,s}, \quad \text{or}
\]

\[
(A.4) \quad \eta_{q,s} = \eta_{Q_c,s} + \frac{Q}{Q_c} \cdot \eta_{m,s}
\]

with \( Q = q + Q_c \),

\[
(A.5) \quad \eta_{q,s} = \frac{s}{Q_c} \cdot \frac{\partial Q_c}{\partial s} + \frac{q}{Q_c} \cdot \frac{s}{m} \cdot \frac{\partial m}{\partial s} + \frac{s}{m} \cdot \frac{\partial m}{\partial s}
\]

\[
(A.6) \quad \frac{\partial m}{\partial s} = \frac{\partial \left[ q/(q+Q_c) \right]}{\partial s} = \frac{Q}{Q} \cdot \frac{\partial q}{\partial s} - \frac{q}{Q} \cdot \frac{\partial Q_c}{\partial s} + \frac{1}{Q^2} \left( \frac{\partial q}{\partial s} + \frac{\partial Q_c}{\partial s} \right) \frac{Q_c}{Q} \cdot \frac{\partial q}{\partial s} - \frac{q}{Q} \cdot \frac{\partial Q_c}{\partial s}
\]

Substituting (A.6) in the second term of (A.5),

\[
\eta_{q,s} = \frac{s}{Q_c} \cdot \frac{\partial Q_c}{\partial s} + \frac{s}{Q} \cdot \frac{\partial q}{\partial s} - \frac{q}{Q} \cdot \frac{\partial Q_c}{\partial s} + \frac{s}{m} \cdot \frac{\partial m}{\partial s}
\]

\[
= \frac{s}{Q_c} \cdot \frac{Q_c}{\partial s} \cdot \left[ 1 - \frac{q}{Q} \right] + \frac{s}{Q} \cdot \frac{\partial q}{\partial s} + \frac{s}{m} \cdot \frac{\partial m}{\partial s}
\]

and since \( 1 - \frac{q}{Q} = \frac{Q_c}{Q} \),

\[
\eta_{q,s} = \frac{s}{Q} \cdot \frac{\partial Q}{\partial s} + \frac{s}{m} \cdot \frac{\partial m}{\partial s}
\]

\[
\eta_{q,s} = \frac{s}{Q} \cdot \frac{\partial Q}{\partial s} + \frac{s}{Q} \cdot \frac{\partial Q}{\partial s} \cdot \frac{\partial s}{\partial s} + \frac{s}{m} \cdot \frac{\partial m}{\partial s} + \frac{s}{m} \cdot \frac{\partial m}{\partial s}
\]
(A.7) \[ \eta_{q,s} = \eta_{Q_T,s} + \rho_{S,s} \cdot \eta_{Q_T,S} + \eta_{m_1,s} + \rho_{S,s} \cdot \eta_{m_1,S} \]

which is the advertising equation from vector equation (22).
For a presentation of the marketing mix concept, see Borden [4].

Some reactions may be virtually instantaneous, while others may occur after a considerable time lag. For example, price reactions are often almost immediate. Advertising reactions, however, usually involve substantial time lags. So, \( \rho_{U_k, U_k} \) could be defined as \( \frac{\delta U_{k, t}}{\delta u_{k, t-1}} \cdot \frac{u_{k, t-1}}{u_{k, t}} \), whereas \( \rho_{U_1, U_1} \) might be \( \frac{\delta U_{1, t}}{\delta u_{1, t}} \cdot \frac{u_{1, t}}{U_{1, t}} \).

A more general profit function could be considered including distribution, markup, etc. This simple form is used here to establish a link with Dorfman and Steiner's work.

Throughout we will assume that the second order conditions are satisfied.

For a comparison of alternative models of determinants of marketing expenditures in the firm, see Elliott [12].

\( \frac{\partial Q}{\partial U} \) is an \((n \times 1)\) vector, and \( \frac{\partial U}{\partial u} \) is an \((n \times n)\) matrix. To be conformable for multiplication we should write \( \frac{\partial U}{\partial u} \cdot \frac{\partial Q^T}{\partial U} \).

For a basic discussion of the Cournot and Stackelberg competitive reaction models, see for example Intriligator [17, pp. 205-19].
The direct elasticity (i.e., not taking into account possible competitive reaction) of market share with respect to price has been defined as \( \frac{\partial m_i}{\partial p} \cdot \frac{p}{m} \), i.e.,

\[
\frac{p}{m} \cdot \frac{\partial m_i}{\partial p} = \frac{p \cdot \left( a \cdot n_{m_1, p} \cdot \left( \frac{p}{(p + P)} \right)^{n_{m_1, p} - 1} \right) \cdot \left( \frac{1}{p} \right) \cdot \left( s^* \right) \cdot \left( x^* \right)}{a \cdot \left( p^* \right) \cdot \left( s^* \right) \cdot \left( x^* \right)}
\]

which simplifies to,

\[
\eta_{m_1, p} = \eta_{m_1, p^*}
\]

Similarly one obtains

\[
\eta_{m_1, p} = -\eta_{m_1, p^*}
\]

Analogous results obtain for the other decision variables and in vector notation we can write

\[
E_{m_i, u} = -E_{m_i, U} = E_{m_i, u^*}
\]

The direct elasticity of market share with respect to price is then

\[
\frac{p}{m} \cdot \frac{\partial m_i}{\partial p} = \frac{p \cdot \left( a \cdot n_{m_1, p} \cdot \left( \frac{p}{(p + P)} \right)^{n_{m_1, p} - 1} \right) \cdot \left( \frac{p}{(p + P)} \right) \cdot \left( s^o \right) \cdot \left( x^o \right) \cdot \left( x^o \right)}{a \cdot \left( \frac{p}{(p + P)} \right) \cdot \left( s^o \right) \cdot \left( s^o \right) \cdot \left( x^o \right) \cdot \left( x^o \right)}
\]
which reduces to,

\[ \eta_{m_i, p} = \eta_{m_i, p^0} \cdot \left( \frac{p^0}{p + \bar{p}} \right) \]

Similarly one obtains

\[ \eta_{m_i, u} = \eta_{m_i, u^0} \cdot p^0 \]

More generally, we can write

\[ E_{m_i, u} = E_{m_i, u^0} = D_{u^0} \cdot E_{m_i, u^0} \]

An index devised by Hymer and Pashigian [16]. \( I_1 \) and \( I_2 \) are respectively the unweighted and weighted sum of absolute changes in market shares:

\[ I_1 = \frac{1}{n} \cdot \sum_{i=1}^{n} |m_{i,T} - m_{i,1}| \]

\[ I_2 = \sum_{i=1}^{n} |m_{i,T} - m_{i,1}| \cdot m_{i,1} \]

where \( m_{i,t} \) is the market share of brand \( i \), at time \( t \),

\( n \) is the number of competing brands,
I is the extent of the observation period.

$I_3$ is an average instability index computed over the whole period of observation, i.e. $I_3 = \frac{1}{T-1} \cdot \frac{1}{n} \cdot \sum_{t=2}^{T} \sum_{i=1}^{n} |m_{i,t} - m_{i,t-1}|$.

Note that $I_1$ and $I_3$ are not exactly equivalent to the proposed Hymer and Pashigian's indices. However, they were computed as indicated above to facilitate the comparison with $I_2$.

For a comparison of the merits of the Herfindahl index with those of the entropy measure - also computed and reported in Table III, see Theil [31, pp. 316-18].

These simultaneous equation models are discussed in Bultez [7]. $x_t$, $p_t$, $s_t$, $s_t$ and $m_t$ were the jointly dependent variables, $x_t$, $p_t$, $p_{t-1}$, $x_{t-1}$, $s_{t-1}$, $p_{t-1}$, $s_{t-1}$ and $m_{t-1}$ the predetermined variables.

So few degrees of freedom were left in the estimation that the results are not worth reporting.

If we are willing to accept that the $t$ statistic computed here is distributed according to a Student's density function, an assumption, which may be asymptotically valid in the case of simultaneous equation model.
Note that the new Durbin statistic tests the non-autocorrelation against the alternative hypothesis of a first-order Markov dependence scheme. In the case of a Koyck model where the original disturbances are independent, when we apply Koyck's transformation the resulting error terms get correlated according to a first-order moving average scheme and thus the Durbin statistic is irrelevant.

Moreover, this asymptotically valid test has been designed for single equation regression model. Its performance is unknown when we apply it to residuals from one equation which is part of a system.

The long term market share elasticities are the elements of the vector \( \left( \frac{1}{1 - \lambda} \right) \cdot E_{m_i} u^* \), where \( \lambda \) is the coefficient of the \( \log m_{t-1} \) variable, i.e. 0.344.

The significance of the \( E_{q,u} \) estimator is unknown and depends critically on the statistical properties of the \( [R] \) and \( E_{m_i} u^* \) estimators. However the distribution of such an estimator may be empirically approximated by Monte Carlo experiments with various sample sizes.

Assuming a 58 % gross margin and using the dynamic version short-run estimates, we obtain:

\[
MRA_{S-R} = (11.57) \cdot (0.093) \cdot \left( \frac{983,000}{1,407,257} \right) = \$ 0.752
\]
A detailed demonstration of the long-run result may be found in BULTEZ [7].

Since we expect $0 \leq \lambda < 1$, \[ \sum_{t=0}^{\infty} \lambda^t = \frac{1}{1 - \lambda} \]

Since we need a long period of stable competitive behavior to estimate them.
REFERENCES


