On the Equity Premium in Exchange
and Production Economies

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Abstract

This note presents a formal proof that introducing production and capital accumulation in a pure exchange Arrow-Debreu economy will not increase the set of joint equilibrium processes on consumption and asset prices. This formalizes the observation in Mehra & Prescott (1985) that modifying the technology in an exchange economy to admit production and capital accumulation will not increase the equity premium.
This note presents a simple proof to demonstrate that expanding the set of technologies an a pure exchange, Arrow-Debreu economy to admit capital accumulation and production does not increase the set of joint equilibrium processes on consumption and asset prices. Hence, modifying the technology in an exchange economy to incorporate production will not increase the equity premium.¹

Let θ denote preferences, τ technologies, E the set of exogenous processes on the aggregate consumption good, P the set of technologies with production opportunities, and m(θ,τ) the set of equilibria for economy (θ,τ).

Theorem

\[ \bigcup_{\tau \in P} m(\theta, \tau) \supset \bigcup_{\tau \in E} m(\theta, \tau) \]

Proof. For θ₀ ∈ θ and τ₀ ∈ P let (a₀,c₀) be a joint equilibrium process on asset prices and consumption. A necessary condition for equilibrium is that the asset prices a₀ be consistent with c₀, the optimal consumption for the household with preferences θ₀. Thus, if (a₀,c₀) is an equilibrium then

\[ a₀ = g(c₀, θ₀), \]

where g is defined by the first-order necessary conditions for household maximization. This functional relation must hold for all equilibria, regardless of whether they are for a pure exchange or a production economy.

Let (a₀,c₀) be an equilibrium for some economy (θ₀,τ₀) with τ₀ ∈ P. Consider the pure exchange economy with θ₁ = θ₀ and τ₁ = c₀. Our contention is that (a₀,c₀) is a joint equilibrium process for asset prices and consumption for the pure exchange economy (θ₁,τ₁). For all pure exchange economies, the equilibrium consumption process is τ, so c₁ = τ₁ = c₀, given
more is preferred to less. If \( c_0 \) is the equilibrium process, the corresponding asset price must be \( g(c_0, \theta^*_1) \). But \( \theta^*_1 \neq \theta^*_0 \) so \( g(c_0, \theta^*_1) \neq g(c_0, \theta^*_0) = a_0 \). Hence \( a_0 \) is the equilibrium for pure exchange economy \((\theta^*_1, r^*_1)\), proving the theorem. Since the set of equilibria in a production company is a subset of those in an exchange economy, it follows immediately that the equity premium in an exchange economy will not increase by modifying the technology to incorporate production.
Footnote

1. A similar observation is made in Mehra and Prescott (1985) where they assert that incorporating production in their analysis along the lines of Brock (1982), Donaldson and Mehra (1984) and Prescott and Mehra (1980) will not increase the equity premium. The proof in this note formalizes their assertion.
References


