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# On the Volatility of Stock Market Prices

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# 1 Introduction

The majority of studies to date dealing with stock price volatility have been micro-studies.[1] This line of research has its origins in the important early work of Shiller (1981) and LeRoy and Porter (1981), which found evidence of excessive volatility of stock prices relative to the underlying dividend/earnings process. Using data for a hundred years, Shiller (1981) in particular, reported that, in his model, the volatility of actual stock prices exceeded the theoretical upper bound by a factor of 5.59. These studies use a constant interest rate, an assumption subsequently relaxed by Grossman and Shiller (1981) who addressed the issue of varying interest rates. They concluded that although this reduced the excess volatility, Shiller's conclusion could not be overturned for reasonable values of the coefficient of relative risk aversion.

The conclusions of the above cited studies have been challenged in recent years, most notably by Flavin (1983), Kleidon (1986) and Marsh and Merton (1986). These challenges appear to have merit. The essence of their criticism is that the tests are biased, the confidence intervals wide and sensitive to trend. They emphasize the importance of low frequency movements in dividends. Gilles and LeRoy (1990) in their critical review of the variance bound literature point out that Shiller's volatility tests are likely to be biased if the stochastic process generating dividends is such that the detrending procedure is inappropriate. The later variance bound tests of West (1986) and Mankiw, Romer and Shapiro (1985) are unbiased but essentially inconclusive because, like Shiller's tests, they leave open the question of sampling variability. The interested reader is referred to Gilles and LeRoy (1990) or Shiller (1989) for a detailed overview. Gilles and LeRoy conclude "... This finding of excess volatility is robust ...."

This paper shifts the focus of analysis from the firm to the aggregate level and complements the work by Grossman and Shiller (1981). Rather than studying individual securities, we choose to examine issues of volatility utilizing aggregate stock market values and aggregate after-tax net cash flows as a ratio to National Income. Our approach is in the tradition of

[1] A notable exception is Grossman and Shiller (1981).

the infinitely-lived classical growth model of Solow, where the behavior of capital, consumption and investment are studied as shares of output, bearing in mind the well-documented regularities of their ratios (Solow (1970)). This model is a central construct in contemporary finance, public finances and business cycle theory[2] and its variants have been used extensively by, among others, Abel et al (1989), Auerbach and Kotlikoff (1987), Barro and Becker (1988), Brock (1979), Cox, Ingersoll and Ross (1985), Lucas (1988), Kydland and Prescott (1982) and Merton (1971).

This paradigm has several advantages: The partial equilibrium micro studies cited earlier ignore the interaction of consumption growth and interest rates, implicitly assuming their independence. In contrast, the neoclassical growth model explicitly captures their interaction. Secondly, examining aggregate values relative to National Income is natural in this theoretical setting since detrending is not a problem as these series appear to be co-integrated.

The principal results of our study cover the U.S. economy for the post War years 1946–1987. During this period we observe that the value of equity in the U.S. as a ratio of National Income has moved by a factor of about three, from a low of 0.48 of National Income in 1948 to a high of 1.33 of National Income in 1968, dropping down to 0.53 of National Income in 1974. Furthermore, there is a fair amount of persistence in the plot of the ratio of market value of equity to National Income vs. time (see figure 1). During the same period, the share of claims to equity has been relatively stable (approximately 2.5%), ranging from 2.67% of National Income in 1948 to 2.91% in 1968 and 2.02% in 1974 (see figure 2).

In this paper, we analyze the behavior of equity as a ratio of National Income. We address the question as to whether this behavior is a) consistent with the standard neoclassical growth construct; b) confirms the challenges to Shiller (1981) and LeRoy and Porter (1981).

The study consists of two main parts. To build some intuition, we first address these issues in a deterministic steady state context. Next we extend our analysis to stochastic models with low frequency movements to gauge if this implies large movements in the ratio of Equity ($e$) to National Income ($y$). In addition, we will also discuss the implied relation

[2]Barro and Becker (1988) provide a justification for the infinitely lived family construct in their formulation of a dynastic utility function.

between growth rates and $e/y$. The paper is organized as follows: Section 2 summarizes the U.S. historical experience for the period 1946–87. Section 3 deals with deterministic steady state models. Section 4 examines the extension to stochastic economies and section 5 concludes the paper.

## 2 Data

The data used in this paper consists of a set of series for the period 1946–1987. These are individually described below.

(i) Series $y$: National Income data; obtained from *The Economic Report of the President.*

(ii) Series $ry$: Real per capita National Income. This is series $y$ divided by the population and the GNP deflator from the Economic Report of the President.

(iii) Series $e$: Market Value of Equity; obtained from the Board of Governors publication, *Flow of Funds Accounts Financial Assets and Liabilities Year-End Values.* The Board of Governors data for 1982–1987 is obtained by multiplying the value of equity traded on major exchanges by 1.25 to adjust for privately held corporations. For the period 1946–1981, the SEC supplied the estimates for the total value of equity including privately held stocks. After 1981 (when the SEC discontinued supplying the data), the Board of Governors used the ratio of total value of equity to the market value of traded equity (which was 1.25 in 1981) to adjust subsequent data.[3] For the purpose of the present study, which uses data only until 1987, the above may be an adequate approximation. However, with the proliferation of Leveraged Buy Out activity since 1988 and the subsequent change in the ratio of publicly traded equity and privately held stocks, it probably understates the true value. The ratio of $e/y$ is plotted in figure 1.

(iv) Series $xe$: Extended Market value of Equity. For the period 1945–1987 the values were taken from the Board of Governors publication: *Flow of Funds Accounts Financial*

[3]We thank Judy Ziobro, an economist with the Board of Governors, for this information.

*Assets & Liabilities Year-End Values* and are identical to series $e$. For the period 1929–1944 the values were taken from Holland and Myers (1984) after an adjustment discussed below.

Holland and Myers report equity values from 1929–1981 for nonfinancial corporations. Since there is overlapping data from the period 1945–1981 (36 years), we calculated the mean value of the ratio of the Holland-Myers data to the Flow of Funds year end data. The value is 0.644, i.e., the Holland-Myers data is systematically biased downward with mean 0.644 and variance .00377. We used this value to adjust the Holland-Myers data from 1929–1944. The ratio $xe/y$ is plotted in figure 4.

(v) Series $d$: Dividend data; obtained from *The Economic Report of the President.*

(vi) Series $ne$: Net New Equity Issues; obtained from the Board of Governors publication, *Flow of Funds.*

(vii) Series $x$: After-Tax Cash Flow to Equity; computed as $x = d - ne$. The ratio of $x/y$ is plotted in figure 2.

(viii) Series $c$: Consumption of Nondurables and Services; obtained from *The Economic Report to the President.*

The study commences with information from 1946 since reliable data for the series '$ne$' and '$e$' is unavailable prior to that year.

## 3 Deterministic Steady State Analysis

In this paper we consider the neoclassical growth model with labor-augmenting technological progress.[4] The economy we consider has a single representative 'stand-in' household. This unit orders its preferences over consumption paths by

$$\sum_{t=0}^{\infty} \beta^t u(c_t) , \quad 0 < \beta < 1 \tag{3.1}$$

[4]In the Growth Literature this is often referred to as a Harrod neutral technical change.

where $c_t$ is the per capita consumption, $\beta$ is the subjective time discount factor and $u : R_+ \rightarrow R$ is the increasing, continuously differentiable concave utility function. We further restrict the utility function to be of the constant relative risk aversion class

$$u(c, \alpha) = \frac{c^{1-\alpha} - 1}{1 - \alpha}, \quad 0 < \alpha < \infty \tag{3.2}$$

where the parameter $\alpha$ measures the curvature of the utility function. When $\alpha = 1$, the utility function is defined to be logarithmic which is the limit of the above representation. In each period $t$, there is a single good, $y_t$, that is produced using two inputs, capital, $k_t$, and labor, $l_t$. We assume that technological change increases the effective supply of labor units each period by the factor $(1 + \eta) > 1$. In addition the production function $f(k, l) : R_+ \times R_+ \rightarrow R_+$ which has capital and effective labor units as arguments, is strictly increasing, strictly concave, continuously differentiable and exhibits constant returns to scale. Then, letting $i$ denote the gross investment, the optimal growth problem can be written as

(P)
$$\max_{\{(c_t, i_t, k_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{1-\alpha} - 1)}{(1 - \alpha)}$$

$$\begin{aligned} \text{s.t.} \quad c_t + i_t &\leq f(k_t, l_t) \quad \text{all } t \\ l_{t+1} &= (1 + \eta) l_t \quad \text{all } t \\ k_{t+1} &= i_t \quad \text{all } t \\ k_0, l_0 &> 0 \quad \text{given} \end{aligned}$$

If we renormalize all variables by $(1 + \eta)^t$ and assume that $\beta(1 + \eta)^{1-\alpha} < 1$, the above becomes a "standard" optimal growth problem. (See Stokey, Lucas and Prescott (1989).) The solution to (P) is thus equivalent to finding a value function $v(k_t, l_t)$ where

$$v(k_t, l_t) = \max_{0 \leq i_t \leq f(k_t, l_t)} \{u(f(k_t, l_t) - i_t) + \beta v(i_t, (1 + \eta) l_t)\} . \tag{3.3}$$

The solution to (3.3) is a pair of optimal policy functions $c(\cdot)$ and $i(\cdot)$ which determine optimal consumption and investment in every state. It is well known that by repeated application of these policy functions, the economy will converge to a steady state growth rate; that is, in the steady state, consumption and investment will each be growing at a rate $\eta$ every period.

$$c_{t+1}^s = (1 + \eta) c_t^s \tag{3.4}$$

and

$$i^s_{t+1} = (1+\eta) i^s_t \,. \tag{3.5}$$

Given (3.4) the problem being analyzed is isomorphic to the one studied in Mehra and Prescott (1985) and Mehra (1988). Using the results and analysis in these papers, we compute the steady state equilibrium risk free rate. To do this, first we price the risk free security paying one unit of consumption each period. The expression for the equilibrium time 0 price of this security is given by

$$p_0 = \sum_{t=1}^{\infty} \beta^t \frac{u'(c_{t+1})}{u'(c_t)} \tag{3.6}$$

or

$$p_0 = \sum_{t=1}^{\infty} \beta^t \left( \frac{c_1^\alpha}{c_1^\alpha (1+\eta)^{\alpha t}} \right)$$

or

$$p_0 = \frac{\beta(1+\eta)^{-\alpha}}{1-\beta(1+\eta)^{-\alpha}} \,. \tag{3.7}$$

Such a security can also be expressed relative to its implied rate of interest. Since for a perpetuity

$$p_0 = \frac{1}{r} \tag{3.8}$$

where $r$ is the interest rate in the economy, we can obtain

$$r = \frac{1}{\beta(1+\eta)^{-\alpha}} - 1 \tag{3.9}$$

by equating (3.7) and (3.8).

Expression (3.9) is the discrete time analogue of a well known relationship in continuous time. This result is easily derived since

$$\begin{aligned} 1 + r &= \beta^{-1}(1+\eta)^{\alpha} \\ \ln(1+r) &\equiv r_c = -\ln\beta + \alpha\eta_c \,. \end{aligned}$$

Replacing $\beta$ by $e^{-\rho}$ (continuous compounding) we obtain

$$r_c = \rho + \alpha\eta_c \,. \tag{3.10}$$

The subscript $c$ denotes continuous compounding. This result can be found in Arrow and Kurz (1970), Solow (1970) or Dixit (1976). For convenience we retain the continuous time interest rate expression.

What observable quantities best correspond to the theoretical valuation expressions developed here? At the firm level, the value of a stock is frequently represented as the discounted present value of future dividends. This representation has been used in the work of Shiller (1981) and others cited earlier. However, the value of the equity of a firm is *not* equal to the present value of all future dividends, i.e.

$$e_0 \neq \sum_{t=1}^{\infty} \frac{d_t}{(1+r)^t} \tag{3.11}$$

where $e_0$ is the current value of the equity of the firm and $d_t$ is the value of the aggregate dividend paid out at time $t$.

The correct expression is[5]

$$e_0 = \sum_{t=1}^{\infty} \frac{d_t - ne_t}{(1+r)^t} \tag{3.12}$$

where $ne_t$ is the net new equity financing between time $t-1$ and $t$. Only in the special case when a firm finances using only retained earnings and neither issues nor repurchases shares, does (3.11) hold with equality.[6]

Since data on stock issues and repurchases is available since 1946, we calculate the net cash flow to equity holders in the economy as

$$x = d - ne$$

where we now interpret $d$ as the aggregate dividend and $ne$ the value of the net new equity issues. Hence the aggregate value of equity in this economy satisfies

$$e_t = \sum_{s=1}^{\infty} \frac{x_{t+s}}{(1+r)^s} \,. \tag{3.13}$$

In the neoclassical growth model in steady state along a balanced growth path, capital will be growing at a constant rate $\eta$. Let capital $k$ be divided into two components, debt capital

[5]For a comprehensive discussion see Miller and Modigliani (1961).

[6]At the aggregate level, this implies no *net* stock issue or repurchase for the firms in the economy.

$b$ and equity capital $e$, so that $k = b + e$. If the debt equity ratio is specified, then equity and hence the claims to equity will be growing at a constant rate $\eta$, i.e. $x_{t+1} = x_t(1 + \eta)$. Substituting in equation (3.13)

$$e_t = \frac{x_t(1+\eta)}{r-\eta} \tag{3.14}$$

or

$$e/y = \frac{x/y(1+\eta)}{r-\eta}\,. \tag{3.15}$$

Can the large changes in $e/y$ be accounted for within this standard neoclassical growth model? In this model, by varying parameters that are exogenous to the model, we can have different values for $e/k$, $k/y$ and $r$. Let us examine the effects of each of these on $e/y$ as a possible explanation.

a) In steady state along a balanced growth path, if the debt/equity ratio ($b/e$) is high with capital ($k$) fixed, then $e/y$ is low. Since

$$\frac{b}{e} + 1 = \frac{b+e}{e} = \frac{k}{e}\,,$$

a high $b/e$ implies a low $e/k$. As $k$ is fixed and $k/y$ is a constant, this implies $e/y$ is low. (Figure 5 illustrates the effect of a change in $d/e$ ratios on $e/y$ for an economy in steady state.)

Historically for the U.S., the debt/equity ratio ($b/e$) has steadily increased since 1950. Taggart (1985) reports that while in 1945 the debt/equity ratio ($b/e$) was $\approx 10\%$, in 1980 it was $\approx 40\%$ (see figure 3).[7] Taggart (1985) reports, "...the use of debt financing has increased considerably in the post war period .... This trend emerges regardless of the method of measurement employed ...." Does historical evidence support the steady state result that $e/y$ and $b/e$ move inversely? Debt/Equity ratios in the 1980s were comparable to those in the late 20s, whereas the ratios of market-value of equity to National Income ($e/y$) were significantly different (see Figure 4). During the period

[7] In a private communication, Robert Taggart suggested that $\frac{\text{Debt}}{\text{Assets}}$ was a reasonable approximation for $\frac{\text{Debt}}{\text{Debt+Equity}}$, from which the debt/equity ratio can be easily calculated. Note that, in this study, we use the *qualitative* fact that this ratio has been increasing in the post war period.

1950–1970, when $b/e$ was monotonically increasing, $e/y$ was persistently high—in direct contradiction to our theoretical expectation.

Some caveats are in order.

(i) We are implicitly assuming a Miller (1977) model of capital structure.

(ii) Inflation tends to lower the value of equity since assets are depreciated on the basis of historical cost; on the other hand, the real value of a firm's long-term debt obligations declines, thereby raising the value of equity. We implicitly assume that these effects offset each other.

b) In steady state, if the capital/output ratio ($k/y$) is large, then $e/y$ is large (holding $b/e$ fixed). If $b/e$ is fixed, then

$$\frac{b}{e} + 1 = \frac{b+e}{e} = \frac{k}{e}$$

is fixed, which implies $e/y$ and $k/y$ are positively correlated.

Historically, the capital-output ratio ($k/y$) is trendless and constant and cannot, therefore, account for the movement in $e/y$.

c) A third implication of the deterministic neoclassical growth model concerns the interaction between real interest rates and consumption growth rates. Along a balanced growth path, $r = \rho + \eta\alpha$ implying that $r$ is high when the growth rate of consumption is high (given $\alpha > 0$). Hence a high growth rate ($\eta$) implies a low $e/y$ (given $x/y$ and $b/e$ are the same). This is not substantiated by our data. During the 1960's we observe a high $\eta$ as well as record high values of $e/y$.

To summarize, historical movements in $e/y$ cannot be systematically accounted for in the deterministic neoclassical growth model.

## 4 Stochastic Models

A deterministic, steady state analysis does not provide an adequate explanation of the observed large movements in the market value of equity as a share of National Income ($e/y$). In

an effort to achieve greater congruence between theory and observed data, we examine the behavior of $(e/y)$ in stochastic economies including those with persistent changes, generating random-walk type behavior of cashflows ("dividends").

We retain the recursive structure developed in section 3, where the economy is time invariant and economic agents solve a similar problem each period. All relevant information for individual decision making in such an economy can be characterized by a limited set of state variables. If $\{c, x, y\}$ is the equilibrium stochastic process of consumption, cash flow and output for such a homogenous consumer economy with specified preferences and technology then we can determine an equilibrium process $\{c/y, x/y, y'/y, e/y\}$. Consider an infinitely lived representative individual in such an economy. This individual orders his preferences over feasible consumption plans by

$$E_0\left\{\sum_{t=0}^{\infty} \beta^t u(c_t)\right\} \qquad 0 < \beta < 1$$

where $E_0\{\cdot\}$ is the expectation operator conditional upon information available at the present time. $c_t$ is the per capita consumption and $\beta$ the subjective time discount factor. The period utility function $u(\cdot)$ is assumed to be identical to the one considered earlier in the deterministic case. If we assume maximizing behavior on the part of the representative agent, the price of any asset $(e_t)$ with a stochastic process $\{x_t\}$ as its claims, satisfies the Euler equation

$$e_t = \beta E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}[e_{t+1} + x_{t+1}]\right\} . \tag{4.1}$$

Consequently the equilibrium process $\{c/y, x/y, y'/y$ and $e/y\}$ will satisfy

$$\frac{e_t}{y_t} = \beta E_t\left\{\left(\frac{c_{t+1}}{y_{t+1}}\Big/\frac{c_t}{y_t}\right)^{-\alpha}\left(\frac{y_{t+1}}{y_t}\right)^{1-\alpha}\left[\frac{e_{t+1}}{y_{t+1}} + \frac{x_{t+1}}{y_{t+1}}\right]\right\} \tag{4.2}$$

in addition to satisfying other restrictions imposed by technology.

If we *cannot* account for the variation in $e/y$ without imposing technological restrictions, then the addition of further restrictions will not change our results. Hence, in our initial formulation we do not explicitly model the technology. Instead, we assume that $\{c, x, y\}$ is the joint equilibrium process generated by an economy with specified preferences and a technology that incorporates capital accumulation.

The state of this economy $\{i,j,k\}$ follows an independent Markov process. Let $g_{ijk}$, $x_{ijk}$ and $c_{ijk}$ be the values of $y'/y$, $x/y$ and $c/y$, respectively, in state $\{i,j,k\}$. To capture the correlations between these variables let

$$g_{ijk} = \lambda_j \tag{4.3}$$

$$c_{ijk} = a_1\lambda_j + \theta_i \tag{4.4}$$

$$x_{ijk} = a_2\lambda_j + a_3\theta_i + \gamma_k \tag{4.5}$$

where $\lambda_j$, $\theta_i$ and $\gamma_k$ follow a Markov process and are iid. For notational simplicity let $z = \{i,j,k\}$ be the current state and $z'$ be the next period's state. Using this notation, equation (4.2) can be rewritten as

$$e_z = \beta \sum_{z'} \left\{ \pi_{z',z} \left( \frac{c_{z'}}{c_z} \right)^{-\alpha} (g_{z'})^{1-\alpha} [e_{z'} + x_{z'}] \right\} . \tag{4.6}$$

Calibration of the Economy

Step 1: Calculate the mean, variance and cross covariances of the Markov process $\{g_z, c_z, x_z\}$, with respect to their stationary distribution. Match these results to the corresponding sample moments of the U.S. Economy for the period 1946–1987. This is sufficient to calculate the values for $a_1$, $a_2$ and $a_3$. In addition, since $\lambda$, $\theta$ and $\gamma$ are represented by symmetric 2-state transition matrices, with these calculations we can also compute the *levels* of the two states.

Step 2: To calculate the persistence parameters (switching probabilities), for each of the transition matrices, compute the serial correlations for each of the variables and match them to the sample values for the U.S. Economy for the time period 1946–87.

*Sample Moments for the U.S. Economy: 1946–87*

| Variable | $g_z$ | $c_z$ | $x_z$ | $e_z$ |
|---|---|---|---|---|
| Mean | 1.016 | 67.00% | 2.52% | 83.80% |
| Standard Deviation | 3.33% | 2.34% | 8.40% | 24.37% |

$$\text{Cov}(g_z, c_z) = 1.48 \times 10^{-4} \qquad \text{Cov}(g_z, g_{z'}) = 4.56 \times 10^{-5}$$
$$\text{Cov}(g_z, x_z) = 1.08 \times 10^{-5} \qquad \text{Cov}(c_z, c_{z'}) = 3.94 \times 10^{-4}$$
$$\text{Cov}(c_z, x_z) = 6.26 \times 10^{-5} \qquad \text{Cov}(x_z, x_{z'}) = 5.31 \times 10^{-5}$$

*Calibration of the model economy yields the following results:*

$$a_1 = 0.13$$
$$a_2 = 0.01$$
$$a_3 = 0.12$$

| | | |
|---|---|---|
| $\bar{\theta} = 0.54$ | $\bar{\lambda} = 1.016$ | $\bar{\gamma} = -0.05$ |
| $\sigma(\theta) = 2.30 \times 10^{-2}$ | $\sigma(\lambda) = 3.33 \times 10^{-2}$ | $\sigma(\gamma) = 0.79 \times 10^{-2}$ |
| $\theta_1 = 0.56$ | $\lambda_1 = 1.050$ | $\gamma_1 = -0.042$ |
| $\theta_2 = 0.51$ | $\lambda_2 = 0.983$ | $\gamma_2 = -0.058$ |
| $q = 0.87$ | $p = 0.52$ | $r = 0.88$ |

where $p$, $q$ and $r$ are switching probabilities for the transition matrices for $\lambda$, $\theta$ and $\gamma$. That is.

$$\begin{array}{c} \\ \lambda_1 \\ \lambda_2 \end{array} \begin{array}{c} \begin{array}{cc} \lambda_1 & \lambda_2 \end{array} \\ \begin{pmatrix} p & 1-p \\ 1-p & p. \end{pmatrix} \end{array}$$

All the variables in equation (4.2) are in *real per capita terms.* However, since we are interested in the ratios of $e/y$, $c/y$ and $x/y$ we can use nominal aggregate values in both the numerator and denominator without affecting the results. The only exception is $y'/y$ where we must use the values of real per capita National Income (Series $ry$). Hence the mean and standard deviations of $g_z$ are for per capita real National Income. Established economic theory typically uses low values of $\alpha$. In this study we do not challenge this vast literature, but, based upon it, upper bound $\alpha$ by 10. For an alternative view, see Kandel and Stambaugh (1990).

Once the economy has been calibrated the state contingent values of $e/y$ ($e_{ijk}$) can be calculated from the following set of equations.

$$e_{ijk} = \beta \sum_{l=1}^{2} \sum_{m=1}^{2} \sum_{n=1}^{2} {}_{ijk}\phi_{lmn}\, g_{lmn}^{1-\alpha} \left( \frac{c_{lmn}}{c_{ijk}} \right)^{-\alpha} [e_{lmn} + x_{lmn}] \tag{4.7}$$

where ${}_{ijk}\phi_{lmn} = q_{il} \cdot p_{jm} \cdot r_{kn}$.

Note that equation (4.7) is linear in $e_{ijk}$, since we have assumed 2 state matrices for $\theta$, $\lambda$ and $\gamma$. There are eight such equations. These can be solved for the eight values of $e_{ijk}$.

## 5 Results

Simulations on our calibrated economy yielded the following results:

**Table 1**

$\beta = .96$

| $\alpha =$ | 1 | 2 | 4 | 10 |
|---|---|---|---|---|
| Mean $(e/y)$ | 0.583 | 0.421 | 0.283 | 0.181 |
| $\sigma(e/y)$ | 0.035 | 0.040 | 0.045 | 0.058 |
| Range of $(e/y)$ | 0.53–0.64 | 0.36–0.48 | 0.29–0.35 | 0.12–0.27 |

**Table 2**

$\beta = .99$

| $\alpha =$ | 1 | 2 | 4 | 10 |
|---|---|---|---|---|
| Mean $(e/y)$ | 2.404 | 0.948 | 0.456 | 0.238 |
| $\sigma(e/y)$ | 0.100 | 0.078 | 0.070 | 0.077 |
| Range of $(e/y)$ | 2.27–2.54 | 0.84–1.06 | 0.36–0.56 | 0.14–0.35 |

Clearly, in the calibrated economy the standard deviation of $e/y$, in all cases, was much lower than that observed in the sample period (24.37%). For example in the case where $\alpha = 2$, the calibrated standard deviation was exceeded by a factor of 6. In addition, the level of $e/y$ moved in a narrow range in contrast to the wide range $[.48 - 1.33]$ observed in the period (1946–1987). For the period (1929–1987), the range of $e/y$ was $[0.48 - 1.9]$ almost a 4-fold difference between the upper and lower level. This is in spite of the fact that *the variation in $x/y$, $(x_z)$, in our calibrated economy matched that in the U.S. economy* in the post war period. Since the process on $c_z$ and $x_z$ displays a fair amount of persistence in the sample data, it is the additional increase in persistence in the growth rate of output that generates the wide range in $(e/y)$ (see Table 3). Some recurring trends are evident in

our results: In *all* the simulations, in states where consumption is low relative to output, i.e., $(c/y)$ is low, $(e/y)$ is correspondingly low (other variables held constant). A similar correspondence is observed between $(x/y)$ and $(e/y)$. For every scenario, in states where the cashflow to equity was low relative to output, $(e/y)$ was also low.

To test the robustness of our results we did a sensitivity analysis by varying various parameters. We report results of two polar cases of interest. First we consider the case where $p$, $q$ and $r = .99$, implying that $g_z$, $c_z$ and $x_z$ almost follow a random walk.

**Table 3**

$\beta = .96$

$p = q = r = .99$

| $\alpha =$ | 1 | 2 | 4 | 10 |
|---|---|---|---|---|
| Mean $(e/y)$ | 0.584 | 0.557 | Equilibrium does not exist | |
| $\sigma(e/y)$ | 0.139 | 0.274 | | |
| Range of $(e/y)$ | 0.40–0.77 | 0.21–1.04 | | |

Next we consider the case where $p$, $q$, $r = .50$, implying that successive changes in $g_z$, $c_z$ and $x_z$ are independent.

**Table 4**

$\beta = .96$

$p = q = r = 0.50$

| $\alpha =$ | 1 | 2 | 4 | 10 |
|---|---|---|---|---|
| Mean $(e/y)$ | 0.583 | 0.421 | 0.282 | 0.181 |
| $\sigma(e/y)$ | 0.022 | 0.032 | 0.043 | 0.066 |
| Range of $(e/y)$ | 0.56–0.61 | 0.39–0.46 | 0.23–0.33 | 0.11–0.26 |

Tables 3 and 4 clearly show that introducing low frequency movements in the growth rate of output greatly increases the range of values of $e/y$ and emphasizes the importance of low frequency movements in any study of volatility.

An interesting and intriguing relationship is observed between the growth rate of output ($g_z$) and $e/y$. For low levels of persistence, irrespective of the level of $\alpha$, a high growth rate resulted in a high value for $e/y$. *This is consistent with observations for the U.S. economy* which displayed (for the sample period) low levels of persistence of $g_z$. However, for sufficiently high levels of persistence and for $\alpha > 1$, the *relationship reversed.* In states where the growth rates were low, the ratio $(e/y)$ was high—consistent with the implications of the deterministic neoclassical growth model but *inconsistent with observations.* For the U.S. data, the range of $e/y$ was large *and* $e/y$ was positively correlated with the growth rate.

Intuitively, with high levels of persistence the economy behaves like a deterministic one, switching between two growth rates $\eta_1$ (high) and $\eta_2$ (low). In a deterministic economy along a balanced growth path,

$$e/y = \frac{x/y(1+\eta)}{\rho + (\alpha - 1)\eta}$$

where we have used equation (3.15) and the fact that $r = \rho + \alpha\eta$.

We see that $\frac{\partial(e/y)}{\partial\eta} < 0$ if $\alpha > 1 + \rho \approx 1$. Therefore, the $e/y$ ratio will be low when $\eta$ is high just as we observe in our simulations.

Our simulations indicate that it is the low frequency movements in the growth rate that are important in determining the volatility of stock prices. This is in sharp contrast to the determinants of the equity premium (see Mehra and Prescott (1985)) where high frequency movements in the growth rate were crucial. To further capture the implications of low frequency movements, consider the following thought experiment.

We retain all the parameters of the calibrated economy except that we consider the case where output grows at two rates, 3% and 1%, for an expected period of 10 years.[8] What will be the implications for our model?

In this case, $\lambda_1 = 1.03$ and $\lambda_2 = 1.01$ and $p$, the transition probability satisfies the relation

[8]Of course, when the process on $g_z$ changes, the process on consumption $c_z$ and cash flows $x_z$ will also change; see equations (4.3)–(4.5).

$$\sum_{k=1}^{\infty} k(1-p)p^{k-1} = 10$$

or

$$p = \frac{10-1}{10} = .9\,.$$

Table 5

$\beta = .96$

$\lambda_1 = 1.03,\ \lambda_2 = 1.01,\ p = .90$

| $\alpha =$ | 1 | 2 | 4 | 10 |
|---|---|---|---|---|
| Mean | .584 | .39 | .24 | .117 |
| $\sigma$ | .035 | .039 | .04 | .04 |
| Range | [.53–.63] | [.33–.46] | [.17–.32] | [.06–.20] |

As expected, the range of $e/y$ has increased (the effects being more pronounced by higher values of $\alpha$), but, once again, high values of $e/y$ correspond to states where the growth rate is low.

To summarize, the stochastic model results do not match the values of the standard deviation and range of $(e/y)$ observed in the U.S. sample data. Our simulations do delineate the importance of low frequency movements (persistence) in the growth rate for any study of volatility. Consistent with actual observations, in periods of low persistence in growth rates, equity as a share of output will be positively correlated with growth rates. However, an increase in persistence, while increasing the range of $e/y$, reverses the correlation. Hence, in the stochastic economy studied it was not possible to match *both* a large range for $e/y$ and a positive correlation between growth rates and $e/y$.

Our results are in concurrence with the conclusions of Grossman and Shiller (1981), Shiller (1989) and Gilles and LeRoy (1990) regarding excessive volatility in the U.S. Economy. For the period 1946–87 both the cash flows to equity and consumption as a share of National Income were fairly constant. Yet there was significant movement in the value of the stock

market as a share of National Income. This suggests that the stock market may diverge significantly from the levels implied by the neoclassical growth model.

It may be the case that an overlapping generation structure will prove more useful in understanding why there are such large movements in the stock market relative to National Income and only small, relatively transitory movements in earnings as a share of National Income. This may be a fruitful area for future research.

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# STOCK MARKET VALUE/NATIONAL INCOME

FIGURE 1

# AFTER TAX CASH FLOW/NATIONAL INCOME

FIGURE 2

Ratios of market value of debt to replacement cost of assets. Source Taggart (1985)

FIGURE 3

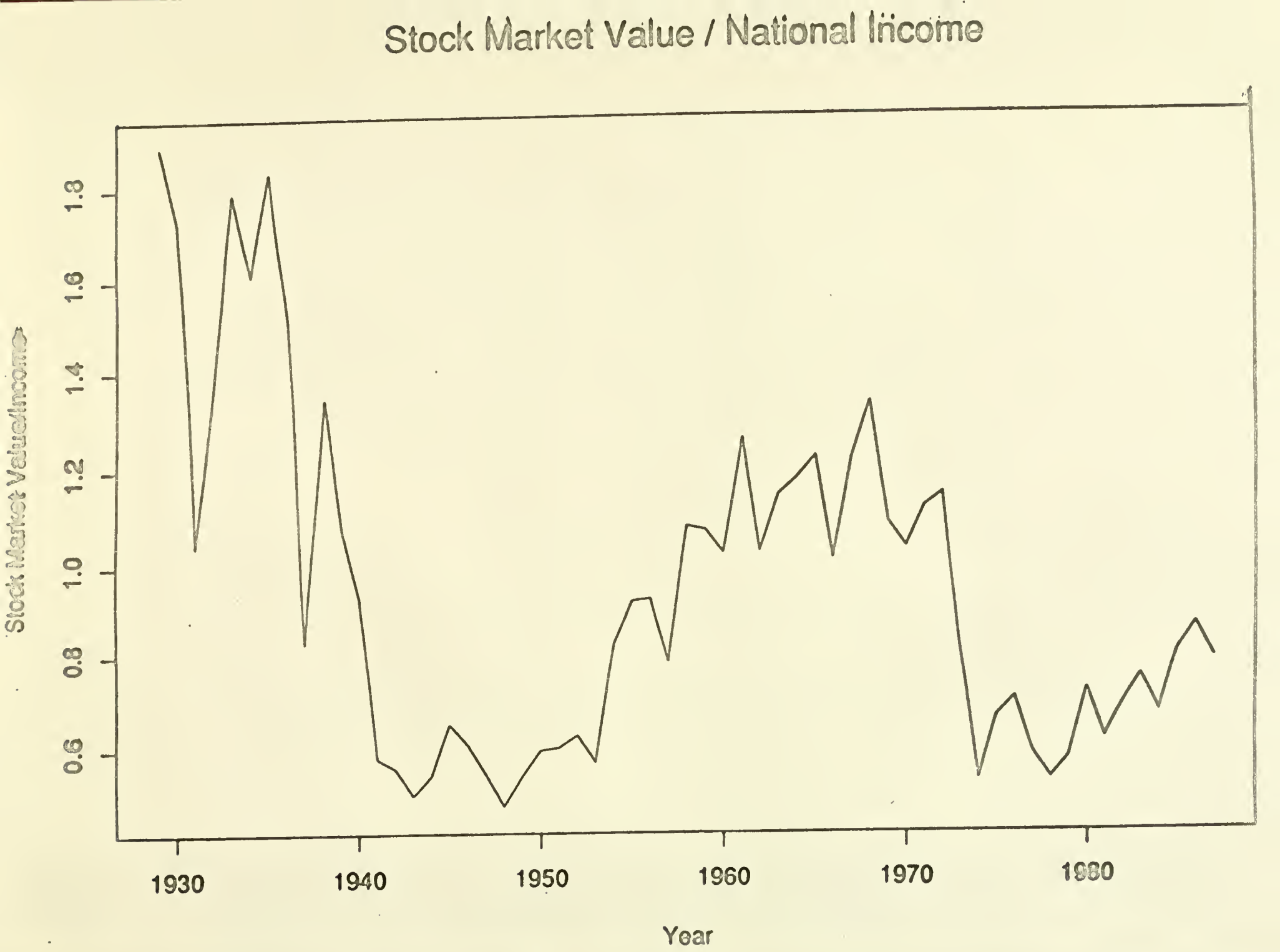
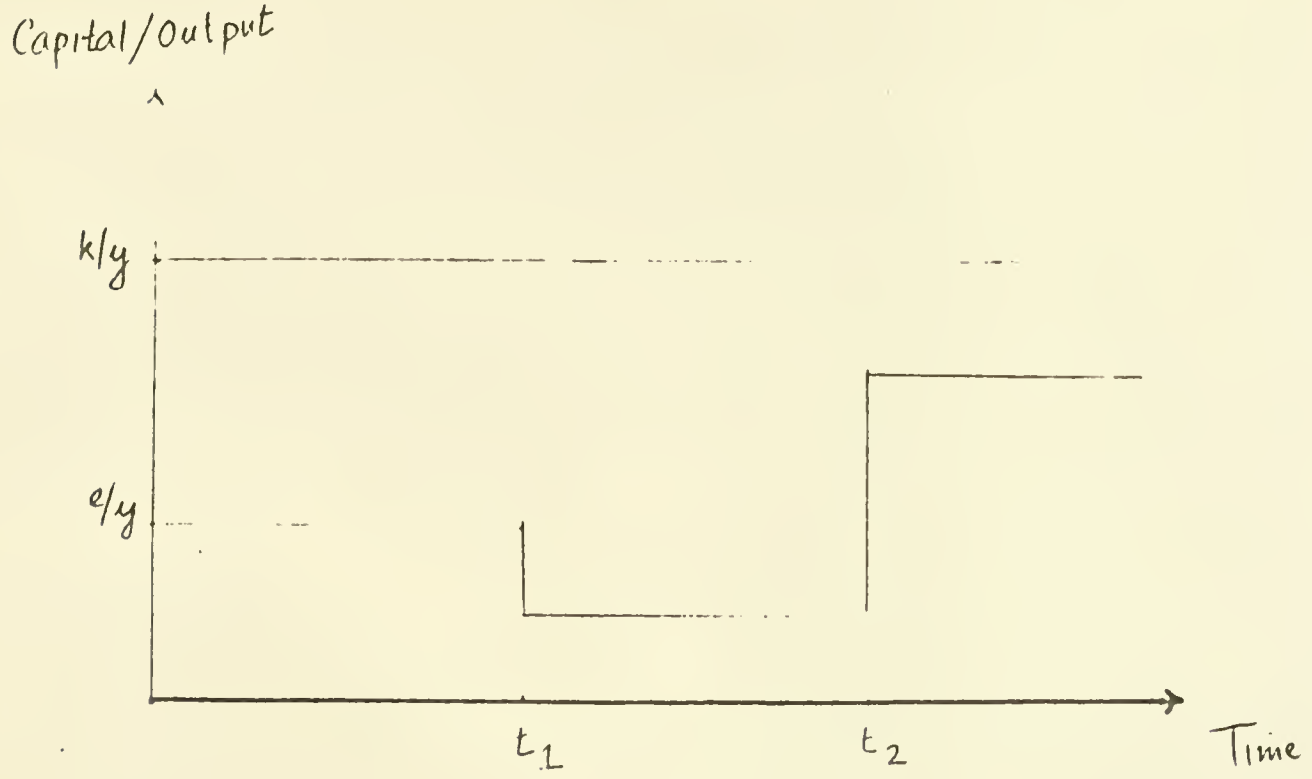

Stock Market Value / National Income
Stock Market Value/Income
0.6
0.8
1.0
1.2
1.4
1.6
1.8
1930
1940
1950
1960
1970
1980
Year

Figure 4




The effect of an increase in the b/e ratio at time t/1 and a decrease at time t/2. The economy is in a steady state.

Figure 5

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