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Preliminary Report

# ON NORMATIVE MATHEMATICAL MODELS OF INFORMATION SYSTEMS*#

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# PREFACE

The author would like to apologize for the delay in acknowledging requests for advance copies of this paper. In addition to a nominal clerical delay resulting from normal overloads at the beginning of an academic year, Section 3 of the original draft was substantially revised and extended at the suggestion of some colleagues at M.I.T.

The research reported herein is one part of a larger program currently in progress. Any comments or criticism relating to this research would be most welcome.

C. H. Kriebel

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The author [illegible] for the delay in acknowledging requests [illegible] copies of this paper. [illegible] clerical delay resulting [illegible] at the beginning [illegible] of the original draft [illegible] and extended at the [illegible] at M.I.T.

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ABSTRACT

Much of the existing normative research on management information systems has followed the historical course of "systems and procedures analysis" in the office. The development of electronic computers, requiring large capital investments, has stimulated a need for comprehensive analytical techniques to assess the economic potential of modern information systems in the firm.

Attempts to construct mathematical models of total information systems have consisted primarily of the following: (1) Shannon's "information theory" and constructs from systems engineering analysis; (2) flow diagrams and network analysis, including experimental research from group psychology on "communication patterns"; (3) characterizations of information system relationships in terms of matrices and matrix algebra; (4) macro-simulators of organization dynamics and of systems; and (5) decision theory models from economic and organization theories, including statistical decision and the Marschak-Radner "team theory" concepts.

This paper discusses some of the practical implications for business organizations of normative mathematical research on information systems, and includes an illustrative application of team decision theory in modeling the information system of an existing firm.

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## 1. Introduction

Since its practical beginning in the early 1950's, the industry of business computer systems has grown to exceed a sales market of $1.5 billion a year. Present day computer technology can provide an organization with equipment which can process, organize, and summarize large volumes of data at great speed. Potentially, the economic gains from use of this technology are substantial; on the other hand so are the costs. In day to day operations the firm requires some method for striking a balance between these economic gains and costs.

> "How and on what basis should management evaluate its present and planned investment in an information system?"

Much of the existing normative research on management information systems has followed the historical course of "systems and procedures analysis" in the office. While details of documentation, format, number of reports issued, and the like, may be negotiable points within departments, few in an organization will view objectively a suggestion that their functions are expendable. Increasing concern for the advance in capital investment required by computerized information systems has triggered a need for more comprehensive analytical techniques to evaluate the economic potential of such systems.

If one seeks precision in the evaluation of system alternatives, then the development of a normative mathematical model provides a sufficient basis for the

analysis. Typically, normative, vs. explanatory or descriptive, analysis is concerned with the assessment of economic "weights" for a presupposed set of known and/or given properties, characteristic of the system and the organization.[1] In many instances, the decision-making functions of an organization serve to isolate these properties as input requirements to the system for information. On the other hand, this need not be the case.

The purposes of this paper are: (1) to briefly review the current state of normative mathematical research on management information systems; and (2) to present an illustrative analysis from one of the "more promising" areas of research.

Quite apparently, within the limited space of this paper it is impossible to consider comprehensive coverage of all research directed toward modeling information systems. The lead-in survey of Section 2 is a "representative sample" of some of the research accomplished to date and the general tenor of emphasis; no attempt is made to suggest extensions in the application of these "models".[2] In Section 3 attention is focused on one area of model construction, team decision theory, and the discussion proceeds with more detailed considerations for an existing organization. Although empirical research has been limited, the application of a decision

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1 For example, see the paper by C. J. Hitch, p. 43-51, in Eckman (ed.), [9].

2 We also assume some familiarity with theoretical details within areas; the uninformed reader is referred to the references cited in the Bibliography.

theory framework to the analysis of information systems appears to hold considerable promise for management scientists. The paper concludes with some observations on the practical consequences of formal research on information systems for operating management.

## 2. A Survey of Information System Models.

One of the earliest "text books" to appear on the subject of business electronic data processing was the work resulting from research for the U.S. Army Ordnance Corps by Gregory and Van Horn [20]. Since that time many discussions of the topic have appeared, including AMA[1], Malcolm [35], and McKinsey [44] on applications; and Armer [2], Eckman [9], Greenberger [19], Hayes [22], Machol and Gray [34], and Porter [45] on new developments. Problems encountered in the normative analysis of business computer and information systems have been recognized and well documented in the literature; examples are Boyd and Kransnow [8], Gatto, et al, [15], Gerber [16], and Starr and Miller [53].

### 2.1 Information Theory.

Perhaps the earliest quantitative definition of the concept of "information" was that proposed by R. A. Fisher in 1925. As a statistician, Fisher in his work on the theory of estimation was interested in a measure of the amount of information supplied by data about an unknown

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1 R. A. Fisher, "Theory of Statistical Estimation", Proc. Cambridge Philosophical Soc., Vol. 22 (1925), p. 700-725: paper No. 11, in Fisher [12].

parameter, θ. He defined, I, the "amount of information" in a sample as the expected value of the square of the derivative with respect to θ of the log of the likelihood function for the sample. The reciprocal of I is commonly referred to in statistics as the "minimum variance bound" for an estimate of the parameter θ, i.e., a theoretical lower limit on the error variance associated with any estimator of θ. (Kullback [31] contains a well referenced summary of statistical research on Fisher's "information theory".)

Independent of Fisher, in 1948 C. E. Shannon published research[1], originating from his wartime investigation of secrecy codes, which described logarithmic measures of information for use in "communication theory".[2] For Shannon, the measure of information contained in a communicated message was a function of the number of messages that could have been sent, and the probability of the selection of each message. The negative equivalent of the information content was called the "entropy content" of the scheme which in a reasonable way was taken as a measure of the amount of uncertainty.

The basic contribution of information theory has been in providing a technical definition for the concept of <u>information</u>, separate and distinct from non-technical

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1 See Shannon [51] and [52].

2 Many have erroneously considered "information theory" and "communication theory" as synonomous. <u>Communication theory</u> is a field of study primarily within electrical engineering which has employed Shannon's technical measure to advantage. <u>Information theory</u> is a branch of the mathematical theories of probability and statistics which more generally has been implemented in both primary and applied fields.

usage, such as Bar-Hillel and Carnap [3] on "semantic information". Among the more rigorous of the many references on information theory is the work by the eminent Russian mathematician A. I. Khinchin [29] (translated).

## 2.2. Communication Theory.

Interest in the application of information theory was greatly stimulated by N. Wiener's work [60] which resulted in part from wartime research on automatic fire control.[1] In developing a formal theory of communication, both Wiener and Shannon presupposed that a signal (or message) as well as interfering noise could be represented in probabilistic terms only, as members of a suitably defined ensemble. Wiener's analysis was based on the assumption that a signal could be processed only after it had been transmitted through a noisy "channel". Shannon, on the other hand, assumed that the signal could be processed both before and after it had been corrupted by noise. While this distinction may appear minor, except for their common probabilistic approach there is little overlap between the work by Shannon and Wiener in communication theory. Both men studied the influence of delay on the faithfulness of signal reproduction; however, Wiener placed greater emphasis on "prediction", viz., negative delay. Thus, research in communication theory resulting from Wiener's work has been most useful in the analysis

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1 Wiener [61] has further suggested that Fisher's definition can be replaced by Shannon's measure; however, this opinion has not been shared by statisticians, e.g., Kullback [31].

of problems of filtering and prediction arising in automatic control. Whereas Shannon's efforts and the resulting research have been of particular interest in analyzing the efficient utilization of communication channels.[1] In this latter context, the purpose of a communication system is viewed as to reproduce the output from a source (or sender) at a convenient location according to a fidelity criterion specified by the user (or receiver). To date, nearly all of the research on communication theory has found its widest application in engineering design for man-machine and machine-machine systems. Related references from the bibliography include Jackson (ed.) [3], Fano [11], Goode and Machol [17], Shannon [52], and Wiener [60].[2] In view of the relative richness of the model framework (See Figure 2.1) it seems surprising that this research has received virtually no application to man-man or management information systems analysis.

2.3. Systems Engineering

Another branch of electrical engineering, and more recently inter-allied with other engineering specialties, noteworthy in the formal analysis of systems is the field of "control processes". The development of control process technology has resulted primarily from research by servomechanism engineers, and its most extensive application has

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1 A recent text based on Shannon's work is R. M. Fano [11]; one based on Wiener's work is Y. W. Lee, STATISTICAL THEORY OF COMMUNICATION (J. Wiley & Sons, 1960). The preceding remarks have been based in part on the lead-in discussions in both of these books.

2 See also F. L. Stumpers, A Bibliography of Information Theory, IRE Transactions on Information Theory, PGIT-2 (Nov. 1953), through (third supplement) IT-6 (March 1960); Fano [11], p. 20.

been in the analysis and synthesis of electro-mechanical control devices. More recently, and particularly as a consequence of the growth in operations research, the concepts of "feedback", "impulse response", "control loop", "transfer function", and the like have been applied to the analysis and synthesis of decision and information systems in business firms. Among the literature on this and other areas within systems engineering are included Bellman [6], Eckman (ed) [9], Howard [27], Lee [32], and Sakaguchi [50].

Much of control process analysis in systems engineering is concerned with obtaining transfer functions (and inverse transforms) for time functions of various physical systems. A variety of mathematical and related techniques (e.g. signal flow graphs) have been developed for the specific purpose of analyzing the behavior or synthesized systems. In the business context, an example might be an inventory control system, represented by considering the "sales rate of a product" as system input, the "production rate" as system output, and "inventory shortage" as the system response. Here the investigation might focus on obtaining transfer functions which relate production and inventory shortage to sales. The desirability of such a system could then be explored by subjecting it to various sales inputs and observing its response.[1]

Much of the "information" retained by large business organizations is for purposes of control -- accounting or otherwise. One major advantage in the above approach to

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1 For example, see Howard [27] and Chapter 19 in Holt, et. al. [25]. (An introduction to servo analysis in production decision systems is contained in H. A. Simon, "On the Application of Servo-Mechanism Theory in the Study of Production Control", Econometrica (1952), p. 247-268.)

information systems is that dynamic behavior of the decision-making functions within an organization can be analyzed as a control process. "Suboptimization" notwithstanding, large complex systems can be partitioned into less complex parts and later synthesized within the analysis. One drawback with this, and earlier approaches, is that within the analysis there is insufficient provision for an economic evaluation of system performance and for ascertaining the "information requirements" of decision-making functions within the organization. Among recent reports of systems engineering research in the context of performance evaluation is Eckman (ed), [9], particularly Chapters 3, 5, 12, and 13.

## 2.4 Group Psychology.

The field of group psychology has approached the problem of evaluating the performance of a system from an alternative path. Emphasis in this research has been in studying the effect of various communication networks on the completion of preassigned tasks by controlled groups of individuals through highly structures physical experiments. In this context the network of communication may be represented abstractly by means of a topological diagram or graph, where the nodes in the network represent the group members and the directed arcs which connect the nodes represent the allowed communication channels. For each experiment a particular network is selected, the group is assigned a simple task to accomplish, and the

method and type of communication is specified.[1] Group and network performance are evaluated on the basis of the elapsed time required for the group to complete the task. Various network structures, such as those designated "star", "pinwheel", "circle", "comcon", and so on, have been analyzed by psychologists through the use of such controlled experiments.[2] In addition, experiments have been performed which have attempted to evaluate the influence of message filtering processes, patterns of communication, leadership within a group, message redundancy, system noise, and the like on total group performance. Related references from the bibliography include Bavelas [4], Hare [21], Johannsen and Edmunds [28], and McCloskey and Coppinger (ed.) [41].

## 2.5 Structures Based on Matrix Algebra.

Requirements for information within a system and an organization have been analyzed with varied success through models based on (1) matrix algebra, and (2) statistical decision theory. In the former instance, mathematical models of information systems based on structures from matrix algebra have been concerned primarily with identifying information relationships between functional decision-making areas within an organization,

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1 For example, a group of five students is selected, arranged in a circle and isolated from one another, and each student is given a different colored marble. Communication between students is limited to written messages which may be passed in one direction only. The group assignment is for each student to find out the colors of the five marbles. The task is completed when all students in the group know the answer.

2 For specific definitions and examples see Bavelas [4], Hare [21], or McCloskey and Coppinger (ed) [41].

and with the generation of various common data aggregates in the formal system, notably, reports. This research has included the development of "decision tables", Grad [18]; data classification based on set theory and symbolic logic, Young [62]; and "integrated data system models" in terms of matrix relations, Lieberman [33] and Homer [26]; and as integer linear programming problems, Henderson [23].

For example, Lieberman's model takes the form of a series of matrices which are ordered in terms of "organizational levels". The model is concerned basically with sources of data origination (the row designations of the "lowest level matrix"), the channels through which documents flow (non-zero elements in the series of matrices), and the functional areas of data use in the organization (the column designations of the "highest level matrix"). The series of matrices can be operated upon to obtain various characterizations of the physical data system -- document flows, routing, and so on. Homer [26] has appended Lieberman's basic structure for the case where matrices over levels may not be compatible for multiplication. This and similarly based models from the area provide a concise descriptive framework within which to consider the information system and organization hierarchy. To date, however, they have not provided for explicit consideration of system costs, and, more importantly, the relative effectiveness of different kinds and amounts of information supplied to the decision-making functions as end users of the data.

## 2.6 Decision Theory.

The area called decision theory can be dichotomized for convenience into the formal analysis of decision-making: (1) by individuals, and (2) by groups. Noteworthy research in the first instance has been the development of Wald's concept of "statistical decision"[1], and in the second case the incidence of Marschak and Radner's "theory of teams".[2]

Statistical decision concepts deal with the mathematical analysis of decisions made by an individual when the outcome or state of the world is uncertain but further information can be obtained by experimentation. It is possible within this framework to analyze in an explicit fashion the cost and value of different kinds and amounts of information supplied to a decision-maker, once such a measure has been defined. The theory contains exact definitions of payoff function, information structure, decision rule, and outcome, which can be used to identify and evaluate the "information requirements" of an individual in an organization. Although this framework contains many useful economic guidelines for decision and information system analysis, to date its application in this area has been quite limited.[3]

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1 Wald [59], and more recently Raiffa and Schlaifer [49], among others.

2 Marschak [36], [37], [38], and [39]; and Radner [46], [47], and [48].

3 For example, the analogy between information retrieval and sequential sampling processes by Wadsworth and and Booth [57], and the survey of elementary considerations by Wagner [58].

One short-coming in the above framework as the theory exists today is the omission of so-called "N-person phenomena" which arise when considerations are extended to include interacting groups of decision-makers. That is, the complexities of systems which service and provide information for many decision-makers in an organization are typically outside the province of the basic statistical decision model. Pioneering research on the formal analysis of group decision and information processes has been performed by J. Marschak and R. Radner under their theory of "teams". Determination of the "most efficient" rules for selecting information structures and for making joint decisions, as in a sequential process or within a decision-making group, is the basic problem dealt with in the economic theory of teams.[1] Through the format of team theory groups of decision-makers can be categorized into classes as (i) coalitions, (ii) foundations, or (iii) teams. Membership in a particular class is determined by knowing which organizational factors of a given set are applicable to the group; such as, the nature of the group's reward structure, the preferences and expectations of the individual members, the degree of solidarity of interests among members, and so on.[2] Some discussions which report generally on the application of team decision models are

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1 The <u>information structure</u> in a decision problem is an explicit description of environmental information which is available to the decision-maker at each instant of time. (See the references by Marschak and Radner cited at the beginning of this discussion.)

2 See Marschak, p. 188 to 190, in Thrall, <u>et al</u>, [55].

Beckman [5], Marschak and Radner [40], McGuire [43], and Radner [46]. In Section 3 we present a simplified analysis of the management information system in an existing organization based on the team decision framework. It is important to note that within reasonable limits the firm analyzed in this section satisfies the assumed requirements for a team organization.[1] Although this framework is one of the "most promising", it appears that before team models can gain widespread application in information system analysis, further theoretical progress is required in the effective development of normative criteria for other organizational forms -- such as "foundations" and "coalitions".

## 2.7 Summary.

Other mathematical research on information systems has included the application of "industrial dynamics" and simulation, queueing theory, network flow theory, computer based analytical programs, and general systems analysis. To conveniently summarize this broad survey of system research we present the table in Figure 2.1. The system properties listed in the table are briefly defined in Figure 2.2. The selection criterion for these properties has been based, in part, on those aspects which have occupied the attention of past qualitative and quantitative research on the field of information systems. While it is apparent that alternative and/or more extensive lists of "properties" might be proposed, the listing indicated is

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1 Ibid.

## TABULAR SUMMARY OF FORMAL RESEARCH ON INFORMATION SYSTEMS.

| SOURCE AREA OF RESEARCH | SYSTEM PROPERTIES "EMPHASIZED" WITHIN MODEL FRAMEWORK* | | | | | | | | | | | | | BIBLIOGRAPHY REFERENCES | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (a) | (b) | (c) | (d) | (e) | (f) | (g) | (h) | (i) | (j) | (k) | (l) | (m) | Theoretical | Application |
| Information Theory | | | | | | | X | X | X | | | | | 12, 29, 31, 51, 61 | 30 |
| Communication Theory | X | | | X | | X | X | X | X | | X | X | X | 3, 11, 52, (60) | 11, 17 |
| Systems Engineering | X | X | X | X | | X | | | X | | | | X | 6, 9, 27, 50 | 9, 32 |
| Structures Based on Matrix Algebra | | X | X | X | X | X | X | | | | X | X | | 18, 23, 26, 33 | 23, 62 |
| Group Psychology | X | X | X | X | | X | X | X | X | | X | (X) | | 4, 21, 28, 41 | 28, 41 |
| Network Analysis | | X | | X | | X | | | | | | X | X | 12B | 14 |
| Simulation | X | X | X | X | X | X | | | X | X | | | X | 7, 8, 13 | 7, 8 |
| "Systems Analysis" | | | | | X | | X | X | | X | X | X | X | 9, 15, 53, 54 | 1, 35, 44, 53 |
| Decision Theory: Statistical Decision | | | X | | | | X | X | | | X | | | 49, 55, 59 | 57, 58 |
| Team Theory | X | X | X | X | | X | X | X | X | | X | | X | 36, 37, 39, 47, 48, 50 | 5, 40, 43, 46 |
| Others | X | X | | X | | | | | | X | X | X | X | 2, 10, 42, 53 | 24, 45, 62 |

*See Figure 2.2.

Figure 2.2

Definitions of System Properties for Model Consideration

(a) DYNAMICS - Changes in the response of organization and system characteristics over time, including adaptive contexts of "feedback", "reenforcement", and "learning".

(b) SIZE OF GROUP - Number of functional areas or members in the context of organization; also the number of stages ("time planning horizon") in a sequential process.

(c) DECISION STRUCTURE - Identification of the form of organization serviced by the system according to some mechanism which relates to the decision-making functions (e.g., preference structure, interaction, etc.); functional context of the use of information supplied by the system.

(d) PATTERN OF COMMUNICATION - Network indicating which members or decision-making areas have access to direct communication channels within the organization ("Who communicates to whom?").

(e) HIERARCHY - Recognition of managerial levels ("chains of command") of the organization; influence of organization "superstructure" and "authority relations" on information system.

(f) METHOD OF COMMUNICATION - The means by which communication takes place, e.g., written report, telephone, etc. (for convenience we include speed of transmission of information -- often the most distinguishing characteristic of various methods).

(g) MESSAGE CONTENT - Specific identification of that which is communicated, e.g., a descriptive statistic, source data, a binary number, an "information function", etc.

(h) ACCURACY OF MESSAGE - "Errors" introduced at the source or by the sender; errors of observation or summarization on the part of the sender; e.g., sample variance, misinterpretations of observations transmitted as message, etc.

(i) SYSTEM NOISE - "Errors" introduced into a message by the system; decrement in "fidelity"; any message not wanted by the user which is sent to him.

(j) INTERVAL and DELAY - (Gregory and Van Horn [20] context)

INTERVAL is the time period when observation is being recorded and "history" (data) is accumulated.

DELAY is the elapsed time period from the end of the INTERVAL until the data is available for use; within the context of DELAY we include the elapsed time required for system response to an inquiry.

(k) CODING AND RETRIEVAL -

CODING: transformation of information into input compatible with the system (includes "encoding" and "decoding" in communication context).

RETRIEVAL: means by which data is obtained from the system (in response to an inquiry), includes filing or method of data storage.

(l) PROCESSING AND VOLUME OF DATA - operations performed on data in transforming or transmitting, e.g., filtering information; also aggregative statistics of operations.

(m) CAPACITY OF SYSTEM - Size of system in context of transmission, filing or memory storage limitations; operating constraints on any functional operation (e.g., processing phase) of the system.

more than adequate to obtain a general comparison of the models discussed. It is also apparent that the basis for determining which properties are "emphasized" within a source area of research is somewhat more arbitrary and, perhaps, a matter of opinion. In this instance, the classification has been biased intentionally towards "errors of omission" rather than "errors of inclusion" to facilitate obtaining more restrictive selection across all areas.[1]

The tabular summary should be self-explanatory. In the final section we comment on some conclusions which the table suggests.

## 3. An Illustrative Analysis

To explore some of the preceding considerations in more detail, we present a simplified analysis of the decision and information system in an existing organization. This analysis is based on the framework of team decision theory developed by J. Marschak and R. Radner. In the interests of brevity, the current discussion has been limited only to some of the developments for the case study analyzed. A more comprehensive and detailed analysis of the firm in question is currently in progress and will be available in the near future.

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1 Note, the "bias" in this context is with regard to the degree of emphasis within an area. For example, all of the "properties" might be included generally within the area of simulation; however, in the author's opinion only those indicated give evidence of having been emphasized by past research on systems employing the technique of simulation.

## 3.1 Company Background

The industrial organization with which this analysis is concerned is a non-profit cooperative laundry association which services several large hospitals in metropolitan Boston. The organization of the association is partitioned broadly into three groups: a policy-making executive Board, a laundry plant, and the central linen supply facilities in each of the participating hospitals. Since the first few months of business, the executive board has not actively participated in the management of the laundry operations. The principal operating officer at the plant is the General Manager, who is responsible for plant decisions on production scheduling and the work force, such as, hiring and layoffs. An Executive Housekeeper is in charge of the linen control supply facilities at each hospital and makes the inventory decisions for his establishment.

Normal operation of the plant consists of an eight-hour work day, six days a week. The plant employs a total of 114 people, 96 of whom are direct labor, and on the average processes a total of 270,000 pounds of laundry each week for the participating hospitals. The direct labor work force is non-unionized, unskilled, and, due to the relatively low status grade of work required in an industrial laundry, experiences an unusually high rate of turnover -- even though relations between labor and management are quite congenial. As a consequence of high labor turnover, work force decisions by management can be considered to occur on a daily basis. Hospitals participating in the association contract for laundry services at an

estimated usage rate [average weekly poundage] over a specified period of time [usually one calendar year]. They are billed weekly for services rendered on the basis of actual laundry poundage processed. Whenever an operating surplus occurs, rebates are made to the hospitals in direct proportion to their physical participation during the period.

## 3.2 Operating Decision Structure.

For computational convenience we will assume that there are only three hospitals participating in the association.[1] At the end of each day the soiled linen used by the hospital during the day is collected by the laundry, processed, and returned as clean linen within twenty-four hours. Identifying each hospital by the subscript i, the inventory balancing equation for each hospital can be written as

$$I_{it} = I_{it-1} - S_{it} + P_{it} \quad \text{for } i=1,2,3;\ t=1,2,\ldots,T; \qquad [3.1]$$

where $I_{it}$ is the total clean linen poundage in inventory at the end of day t, $S_{it}$ is the total poundage of linen used during day t, and $P_{it}$ is the total poundage of linen processed as production at the laundry for hospital i during day t. Total production at the plant on day t is therefore

$$P_t = \sum_{i=1}^{3} P_{it} \quad , \quad t=1,2,\ldots,T. \qquad [3.2]$$

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1 In reality there are currently fourteen members; however, the three largest participants account for approximately 75% of the total volume of business. Consideration of all fourteen members at this time adds considerable computational detail in the analysis which follows.

Since total production at the plant on a particular day is the consequence of the previous day's usage at the hospitals, the production rate at the plant can be considered as a decision made by the respective Executive Housekeepers in each hospital. That is, referring to [3.1] above, we can consider $P_{it}$ as a terminal action taken by Executive Housekeeper i, where $S_{it}$ is a stochastic variable (for each i,t), and $\Delta I_{it-1}=I_{it}-I_{it-1}$, the change in the level of inventory at i, is allowed to vary.[1] Then, on any given day the Executive Housekeepers for all hospitals acting "together", in the context of [3.1] and [3.2] specify the aggregate production rate at the laundry plant.

Within the plant environment, the General Manager is concerned primarily with three operating decisions: the aggregate employment level of the direct labor work force, changes in the level of the direct labor work force, and scheduling overtime operations. Let $W_t$ represent the direct labor work force at the plant on day t, and let $\Delta W_{t-1} = W_t - W_{t-1}$, represent the change in the level of $W_t$; so that positive values of $\Delta W_{t-1}$ indicate "workers hired" and negative values of $\Delta W_{t-1}$ indicate "workers laid off".

Thus, for any time period (day) t, we can identify the actions taken by the management of the laundry association as a vector,

$$\underline{a}_t = (P_{1t}, P_{2t}, P_{3t}, W_t)' \quad , \text{ for } t=1,2,\ldots,T;$$

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[1] The beginning inventory, $I_{i0}$, is taken as a known initial condition for each i. A similar assumption applies regarding the beginning level of the work force, $W_0$.

where the first three components of $\underline{a}_t$ are the actions of the respective Executive Housekeepers on day t, and the last component is the action of the plant General Manager. In the context of the team theory framework, the team action vector is thus:

$$\underline{a} = \left(\underline{a}_1', \ldots, \underline{a}_t', \ldots, \underline{a}_T'\right)' \ .$$ [1]

### 3.3 Operating Decision Costs.

The total cost function in the firm investigated is generally quadratic in the action and state variables. The detailed analysis of the respective cost components has been performed using conventional estimation procedures from statistics and is available in the more detailed presentation (see beginning remarks). For present purposes we have omitted this discussion, and merely indicate (some of) the operating decision costs that were considered, assuming the appropriateness of the respective quadratics.[2]

Regular payroll, hiring, and layoff costs.

The regular payroll cost consists of a fixed term, $C_{11}$, which represents the minimum level work force cost required to initiate operations at the plant, and a variable cost component, where $C_{12}$ represents the average regular time wage per worker-day for direct labor.

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1 Alternatively, the team action vector might be partitioned according to the terminal actions over time rather than by time periods as indicated. For example, Van de Panne and Bosje, [56] in their discussion of the Carnegie Tech "linear decision rules" for scheduling production and work force, employ this alternative formulation.

2 The interested reader is referred to Chapters 2 and 3 in Holt, et. al., [25], and Van de Panne and Bosje, [56], for background and empirical considerations regarding cost estimation in this case.

$$\text{Regular payroll cost} = C_{11} + C_{12} W_t \qquad [3.3]$$

The cost of hiring and layoffs is assumed asymmetric in the decisions and includes office paper work considerations and accumulated vacation and sick pay in the instance of layoffs. A simple quadratic was used as a first approximation of this cost component:

$$\text{Cost of hiring and layoffs} = C_{21}(W_t - W_{t-1} + C_{22})^2 =$$

$$= C_{21}(\Delta W_{t-1} + C_{22})^2. \qquad [3.4]$$

Overtime costs.

Overtime labor costs for direct labor consist of one and one half times the regular hourly wage rate. The overtime cost component also includes cost considerations for undertime operations. Since overtime decisions involve both the level of production and the size of the work force, for the case at hand this cost component recognizes some of the interaction that is present between decisions taken by the General Manager and the individual Executive Housekeepers. A simple quadratic was used as a first approximation of this cost component:

$$\text{Cost of overtime anticipated} = C_{31}(P_t - C_{32}W_t + C_{33})^2, \qquad [3.5]$$

where $P_t$ is defined in equation [3.2].

Absenteeism in the Work Force.

Each day at the plant the total number of direct labor employees reporting for work, say $L_t$, is some amount less than the aggregate payroll, on the average 87% of

the total number employed. As a consequence the General Manager and his supervisors are required to devote some portion of their time at the beginning of each day in rescheduling and reassigning the available work force to accommodate absenteeism. The relative low labor skill requirements of plant operations in general, permit a range of flexibility in reassigning personnel. When the number of absentees increases above the expected level, rescheduling becomes more time consuming for plant management, and in many cases can result in a reduction in the quality of the production output, as well as a higher incidence of bottlenecks in the normal production schedule for the day. Assume that $L_t$ is a random variable with known probability density function, $g\ (\tilde{L}_t \mid \bar{L})$, which is identical for each working day, $(t=1,2,...)$, and where $\bar{L} = .87\ W_t$. A simple approximation to the cost of absenteeism in the plant work force is then[1]

$$\text{Cost of absenteeism} = C_{41}\ (W_t \quad \tilde{L}_t)^2 \qquad [3.6]$$

Inventory Connected Costs.

The inventory connected cost component includes the costs of holding surplus linen stocks at each hospital and the costs of incurring a shortage of linen at the hospital relative to the requirements for the day. The shortage

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1 For present convenience we ignore the fact that interaction between various levels of production and the absenteeism rate should be included within this cost component. (The more detailed analysis forthcoming recognizes this added complexity.)

component in this case was readily identified as the cost of a "crash program" at the plant for the particular hospital's linen in-process, plus the cost for a special delivery in addition to that normally scheduled. Both holding and shortage cost components are directly proportional to the number of pounds involved in each case, and, after discussion with the association management, were assumed reasonably to be the same for each hospital.[1] A simple quadratic was used as a first approximation of this cost component:

$$\text{Inventory connected costs} = \sum_{i=1}^{3} C_{51} \, (I_{it} - C_{52} - C_{53} \, \tilde{S}_{it})^2, \quad [3.7]$$

where $\tilde{S}_{it}$ is a random variable representing the total linen requirements for hospital i on day t.

Summarizing the above considerations, we can write the general equation for the total cost of operation over T periods (days), as

$$C_T = \sum_{t=1}^{T} [C_{11} + C_{12} W_t + C_{21} \, (\Delta W_{t-1} + C_{22})^2$$

$$+ C_{31} \, (\sum_{l=1}^{3} P_{it} - C_{32} W_t + C_{33})^2$$

$$+ C_{41} \, (W_t - L_t)^2 + \sum_{i=1}^{3} C_{51} (I_{it} - C_{52} - C_{53} \, S_{it})^2 \, ], \quad [3.8]$$

---

1 Note, absolute costs differ for each hospital as a function of the actual volume; however, the cost coefficients in [3.7] are assumed to be the same for all hospitals. For convenience we add the assumption that "excess demand' is back-ordered, so that shortage costs represent the added expense associated with back-ordered demand, such as, special delivery costs,etc.

where $I_{it}$ is defined by the inventory balancing equation in [3.1]. It is important to note that the general expression of [3.8], in reality, is three separate equations, since the costs in the first and last periods differ from the intervening periods by the initial and terminal conditions, respectively. That is, the initial conditions given are $I_{i0} = I^o_{i0}$ for i=1,2,3 and $W_0 = W^o_0$; and the assumed terminal conditions are $I_{iT+1} = I^o_{iT+1}$ for i=1,2,3 and $W_{T+1} = W^o_{T+1}$.

To accommodate the analysis of the decision and information system for the firm it is convenient to rewrite equation [3.8] in terms of powers of the decision and state variables, by expanding the general quadratic and collecting terms. Given $(I^o_{i0}, I^o_{iT+1})$ for i=1,2,3 and $(W^o_0, W^o_{T+1})$ we obtain

$$
\begin{aligned}
C_T = \Bigg[ & \left\{ T\left(C_{11}+C_{21}C_{22}^2+C_{31}C_{33}^2+3C_{51}C_{52}^2\right)+C_{21}W_0^{o2}-2C_{21}C_{22}W_0^o \right\} + \\
& + \left(2C_{21}+C_{31}C_{32}^2+C_{41}\right) \sum_{t=1}^{T-1} W_t^2 + \left(C_{21}+C_{31}C_{32}^2+C_{41}\right) W_T^2 + \\
& + C_{31} \sum_{t=1}^{T} \left(\sum_{i=1}^{3} P_{it}^2 + 2\Sigma_{i<j} P_{it} P_{jt}\right) - 2C_{31}C_{32} \sum_{t=1}^{T} W_t \sum_{i=1}^{3} P_{it} + \\
& + \left(C_{12}-2C_{31}C_{32}C_{33}\right) \sum_{t=1}^{T} W_t - 2C_{41} \sum_{t=1}^{T} L_t W_t + 2C_{21}C_{22}W_T + \\
& - 2C_{21} \sum_{t=2}^{T} W_t W_{t-1} - 2C_{21} W_0^o W_1 + 2C_{31}C_{33} \sum_{t=1}^{T} \sum_{i=1}^{3} P_{it} \\
& + C_{41} \sum_{t=1}^{T} L^2_t + \sum_{t=1}^{T} \sum_{i=1}^{3} C_{51}\left(I_{it}^2 + C_{53}^2 S_{it}^2 - 2C_{52} I_{it} + \right. \\
& \left. - 2C_{53} S_{it} I_{it} + 2C_{52}C_{53} S_{it}\right) \Bigg] ,
\end{aligned}
\qquad [3.9]
$$

where $I_{it} = I_{it-1} + P_{it} - S_{it}$ for $i=1,2,3$ ; $t=1,2,\ldots,T$

$$\left.\begin{array}{l} \text{and } I_{i0} = I^{o}_{i0} \\ I_{iT+1} = I^{o}_{iT+1} \end{array}\right\} \text{ for } i = 1,2,3 \quad .$$

## 3.4 The Team Organization Structure.

In the above context, the overall objective of the firm is to obtain a system of decision rules for production, work force, and etc., which minimizes the total cost of operations, represented by the quadratic function in [3.9]. This problem may be characterized more generally as one of quadratic programming with linear equality constraints.[1] It is apparent that the composition and quality of any program of decision rules will depend directly on the information system which services the organization. "Better information" will facilitate "better" decisions, which in turn will result in improved performance over-all, viz., a reduction in total costs. Our concern in this regard is to determine what constitutes "better information", in order to prescribe normative criteria for the information system servicing the programmed decision-making organization. More specifically, within the formal analysis we would like to be able to

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[1] Inequality constraints if we include non-negativity restrictions on the variables. Discussions of the more general problem are readily available in the literature. For example, see Theil [54B], particularly p. 501-527; and more recently, J.C.G. Boot, "Binding Constraint Procedures of Quadratic Programming", Econometrica (July, 1963), p. 464-498.

answer such questions as: What programs of decision rules can be obtained under alternative information systems? What is the relative worth of one system versus another? Which information and decision system (combination) yields the <u>best</u> over-all performance?

As a prelude to considering these questions in detail, we identify the organization structure according to a framework which facilitates formal analysis. For the remainder of this discussion we will employ the following notation,

(i) define the <u>team decision function</u> (or rule):

$\underline{\alpha}' = [\underline{\alpha}_1, \ldots, \underline{\alpha}_t, \ldots, \underline{\alpha}_T]$, where $\underline{\alpha}'_t = (\alpha_{1_t}, \ldots, \alpha_{4_t})$,

for $t = 1,2,\ldots,T$;

(ii) define the <u>information structure</u> of the team:

$\underline{h}' = [\underline{h}_1, \ldots, \underline{h}_t, \ldots, \underline{h}_T]$, where $\underline{h}'_t = [h_{1t}(\underline{x}), \ldots, h_{4t}(\underline{x})]$,

for $\underline{x}$ the vector of states of the world, $\underline{x} \epsilon \underline{X}$, $x_j \epsilon \underline{x}$, and for $t = 1,2,\ldots,T$, and the <u>information function</u> for member i is $h_{it}(\underline{x})$, for $i=1,\ldots,4$; $t=1,\ldots,T$;

(iii) for all i, t, define an <u>observation</u> by member i, in period t:

$y_{it} = h_{it}(\underline{x})$, and

a <u>terminal action</u> by member i, in period t:

$a_{it} = \alpha_{it}[h_{it}(\underline{x})] = \alpha_{it}(y_{it})$ ;

(iv) define the <u>team action vector</u>:

$\underline{a}' = [\underline{a}_1, \ldots, \underline{a}_t, \ldots, \underline{a}_T]$, where $\underline{a}'_t = (a_{1t}, \ldots, a_{4t})$,

and $a_{it} = P_{it}$ for $i=1,2,3$ , $a_{4t} = W_t$, for $t=1,2,\ldots,T$.

The general problem we are considering, via [3.9], can now be reformulated as a matric equation by employing the above notation. In general, the payoff to the firm as a quadratic function of $(\underline{a},\underline{x})$ can be taken as the "inverse cost function", viz., negative costs. That is, for payoff $u(\underline{a},\underline{x})$ and cost $c(\underline{a},\underline{x})$:

$$\max_{\underline{a}} u(\underline{a},\underline{x}) = \max_{\underline{a}} [-c(\underline{a},\underline{x})] \Rightarrow \min_{\underline{a}} c(\underline{a},\underline{x}) .$$

Considering the general objective function of the form in [3.8-9] we have

$$c(\underline{a},\tilde{\underline{z}}) = k + 2\underline{\gamma}'\,\underline{a} + 2\underline{\beta}'\,\tilde{\underline{z}} + (\underline{a}'\,\underline{\underline{A}}\,\underline{a} + \tilde{\underline{z}}'\,\underline{\underline{B}}\,\tilde{\underline{z}} + \underline{a}'\,\underline{\underline{C}}\,\tilde{\underline{z}} + \tilde{\underline{z}}'\underline{\underline{C}}'\,\underline{a}) \qquad [3.10]$$

where $\tilde{\underline{z}} = \underline{\underline{R}}\,\underline{a} + \tilde{\underline{x}}$ .

This expression can be simplified by substituting for the vector $\tilde{\underline{z}}$, to obtain

$$c(\underline{a},\tilde{\underline{x}}) \equiv c[\underline{a},(\underline{\underline{R}}\underline{a} + \tilde{\underline{x}})] = \mu_o(\tilde{x}) + 2\underline{a}'\,\underline{\mu}(\tilde{x}) + \underline{a}'\,\underline{\underline{Q}}\,\underline{a} \qquad [3.11]$$

where $\mu_o(\tilde{\underline{x}}) = k + 2\,\underline{\beta}'\,\tilde{\underline{x}} + \tilde{\underline{x}}'\,\underline{\underline{B}}\,\tilde{\underline{x}}$

$$\underline{\mu}(\tilde{x}) = \underline{\gamma} + \underline{\underline{R}}'\,\underline{\beta} + (\underline{\underline{C}} + \underline{\underline{R}}'\,\underline{\underline{B}})\,\tilde{\underline{x}}$$

$$\underline{\underline{Q}} = \underline{\underline{A}} + \underline{\underline{R}}'\underline{\underline{B}}\underline{\underline{R}} + \underline{\underline{C}}\,\underline{\underline{R}} + \underline{\underline{R}}'\,\underline{\underline{C}}' .$$

We omit the algebraic detail associated with [3.10], and indicate the resulting expression for [3.11]. From [3.9], the components of $c(\underline{a},\tilde{\underline{x}})$ in [3.11] are identified as follows:

$$\mu_o(\tilde{x}) = \left[T(C_{11} + C_{\text{[illegible]}}C_{22}^2 + C_{31}C_{33}^2 + 3C_{51}C_{52}^2) + C_{21}W_0^{o2} - 2C_{21}C_{22}W_0^o\right] + \sum_{t=1}^{T}\sum_{i=1}^{3} C_{51}(C_{53}^2 S_{it} + 2C_{52}C_{53}\tilde{S}_{it}) + C_{51}\sum_{t=1}^{T}\sum_{i=1}^{3}\tilde{S}^{*2}_{it} +$$

$$+ 2C_{51}\sum_{t=1}^{T}\sum_{i=1}^{3}\tilde{S}^*_{it}(C_{52} + C_{53}\tilde{S}_{it}) + C_{41}\sum_{t=1}^{T}\tilde{L}_t^2 , \qquad [3.11a]$$

$$\text{where } \tilde{S}^*_{it} = \begin{cases} \tilde{S}_{i1} - I^o_{i0} & \text{for } t=1 \\ \tilde{S}_{it} & \text{for } t=2,\ldots,T-1 \\ \tilde{S}_{iT} + I^o_{iT+1} & \text{for } t=T \end{cases} \quad i=1,2,3;$$

$$\underline{\mu}(\tilde{\underline{x}}) = \begin{bmatrix} \underline{\mu}_1(\tilde{\underline{x}}) \\ \vdots \\ \underline{\mu}_t(\tilde{\underline{x}}) \\ \vdots \\ \underline{\mu}_T(\tilde{\underline{x}}) \end{bmatrix}, \quad \underline{\mu}_t(\tilde{\underline{x}}) = \underline{\gamma}_t + \sum_{\tau=t}^{T} \tilde{\underline{\beta}}_\tau - \sum_{\tau=t}^{T} \underline{\underline{b}}_{\tau\tau} \tilde{\underline{V}}_\tau \qquad [3.11b]$$

$$\text{for } t = 1,2,\ldots,T,$$

where

$$\underline{\gamma}_1 = \begin{bmatrix} C_{31}C_{33} \\ C_{31}C_{33} \\ C_{31}C_{33} \\ (1/2)C_{12} - C_{31}C_{32}C_{33} - C_{21}W^o_0 \end{bmatrix}, \quad \underline{\gamma}_T = \begin{bmatrix} C_{31}C_{33} \\ C_{31}C_{33} \\ C_{31}C_{33} \\ (1/2)C_{12} - C_{31}C_{32}C_{33} + C_{21}C_{22} \end{bmatrix},$$

$$\underline{\gamma}_t = \begin{bmatrix} C_{31}C_{33} \\ C_{31}C_{33} \\ C_{31}C_{33} \\ (1/2)C_{12} - C_{31}C_{32}C_{33} \end{bmatrix} \quad \text{for } t=2,\ldots,(T-1);$$

$$\tilde{\underline{\beta}}_t = -C_{51} \begin{bmatrix} C_{52} + C_{53}\tilde{S}_{1t} \\ C_{52} + C_{53}\tilde{S}_{2t} \\ C_{52} + C_{53}\tilde{S}_{3t} \\ 0 \end{bmatrix}, \quad \tilde{\underline{V}}_t = \begin{bmatrix} \tilde{S}^*_{1t} \\ \tilde{S}^*_{2t} \\ \tilde{S}^*_{3t} \\ -\tilde{L}_t \end{bmatrix} \quad \text{for } t=1,2,\ldots,T,$$

$$\text{and } \underline{\underline{b}}_{tt} = \begin{bmatrix} C_{51} & & & \underline{\underline{0}} \\ & C_{51} & & \\ & & C_{51} & \\ \underline{\underline{0}} & & & -C_{41} \end{bmatrix} \quad \text{for } t=1,2,\ldots,T;$$

nd

$$\underline{\underline{Q}} = \begin{bmatrix} \underline{\underline{Q}}_{11} & \underline{\underline{Q}}_{12} & \cdots & \underline{\underline{Q}}_{1T} \\ \underline{\underline{Q}}_{21} & \underline{\underline{Q}}_{22} & \cdots & \underline{\underline{Q}}_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{Q}}_{T1} & \underline{\underline{Q}}_{T2} & \cdots & \underline{\underline{Q}}_{TT} \end{bmatrix}, \qquad [3.11c]$$

here

$$\underline{\underline{Q}}_{tt} = \begin{bmatrix} (T-t+1)C_{51}+C_{31} & C_{31} & C_{31} & -C_{31}C_{32} \\ C_{31} & (T-t+1)C_{51}+C_{31} & C_{31} & -C_{31}C_{32} \\ C_{31} & C_{31} & (T-t+1)C_{51}+C_{31} & -C_{31}C_{32} \\ -C_{31}C_{32} & -C_{31}C_{32} & -C_{31}C_{32} & (2C_{31}+C_{31}C_{32}^{2}+C_{41}) \end{bmatrix}, \quad \text{for } t=1,2,\ldots,(T-1),$$

$$\underline{\underline{Q}}_{TT} = \underline{\underline{Q}}_{tt} - C_{21}\,\underline{\underline{U}}, \quad \text{for } \underline{\underline{U}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\underline{\underline{Q}}_{t\tau} = \begin{bmatrix} (T-t+1)C_{51} & & & \underline{\underline{0}} \\ & (T-t+1)C_{51} & & \\ & & (T-t+1)C_{51} & \\ \underline{\underline{0}} & & & -C_{21} \end{bmatrix} \quad \text{for } \begin{cases} t=2,3,\ldots,(T-1) \\ \tau=t-1 \end{cases}$$

$$\underline{\underline{Q}}_{t\tau} = (T-t+1)C_{51}\left[\underline{\underline{I}} - \underline{\underline{U}}\right] \quad \text{for } \begin{cases} t=3,4,\ldots,T \\ 0 < \tau \le t-2 \end{cases},$$

d $\underline{\underline{Q}}_{\tau t} = \underline{\underline{Q}}_{t\tau}$ for all $t,\tau$ .

### 3.5 The "Routine" Case.

We introduce the discussion of information systems for the present organization by first considering the simplest alternative available. The case of "routine" is defined as the team payoff which results from employing the "best" team decision function under the null information structure, that is, where "no observations" are made by any of the members -- so that each member's information function $h_{it}(x)$ = a constant (independent of x). The routine case is of particular interest in that it provides a convenient reference origin of measurement for comparing other information structures. More specifically, the payoff for routine indicates the return to the team that would be realized using the "best" team decision function under minimum information, viz., the expected values of the random variables, $\tilde{\underline{x}}$.

Referring to equation [3.10], and taking expectations we obtain

$$E_{\underline{x}}\, c(\underline{a},\tilde{\underline{x}}) = \mu_o(\bar{x}) + 2\underline{a}'\,\underline{\mu}(\bar{x}) + \underline{a}'\underline{\underline{Q}}\,\underline{a} \quad , \qquad [3.12]$$

where $E_{\underline{x}}\, \mu_o(\tilde{x}) \equiv \mu_o(\bar{x})$, and $E_{\underline{x}}\, \underline{\mu}(\tilde{x}) \equiv \underline{\mu}(\bar{x})$. To optimize[1] equation [3.12] with respect to the team action vector, $\underline{a}$, first order conditions direct that we compute partial derivatives of the expression with respect to the components of $\underline{a}$ and equate to zero. Let $\partial/\partial\underline{a}$ denote taking partial derivatives with respect

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[1] Recalling the earlier discussion, since $u(\underline{a},\underline{x}) = -c(\underline{a},\underline{x})$, we can either minimize $c(\underline{a},\underline{x})$ or maximize $-c(\underline{a},\underline{x})$; the optimal vector $\underline{a}^*$ will be the same in each case.

to each component of $\underline{a}$, and arranging the results in a column vector.[1] Then

$$\frac{\partial}{\partial \underline{a}} E_{\underline{X}}\, c(\underline{a},\underline{\tilde{x}}) = 2\underline{\mu}\,(\bar{x}) + 2\underline{\underline{Q}}\,\underline{a} = \underline{0}. \qquad [3.13]$$

Solving [3.13] for $\underline{a}^*$, we obtain

$$\min_{\underline{a}} E_{\underline{X}}\ c(\underline{a},\underline{\tilde{x}}) \equiv \underline{a}^* = -\underline{\underline{Q}}^{-1}\ \underline{\mu}\,(\bar{x})\ . \qquad [3.14]$$

If we partition $\underline{\underline{Q}}^{-1}$ in the same manner as $\underline{\underline{Q}}$ in [3.11c], we can rewrite [3.14] as $(\underline{a}^*)' = (\underline{a}^*_1, \ldots, \underline{a}^*_t, \ldots, \underline{a}^*_T)$ where

$$\underline{a}^*_t = -\sum_{\tau=1}^{T} \underline{\underline{Q}}^{-1}_{t\tau}\ \underline{\mu}_{\tau}(\bar{x}) \quad ,$$

or

$$\underline{a}^*_t = -\sum_{\tau=1}^{T} \underline{\underline{Q}}^{-1}_{t\tau} \left[\underline{\gamma}_{\tau} + \sum_{\tau'=\tau}^{T} \underline{\xi}_{\tau'}(\bar{x})\right] , \qquad [3.15]$$

where

$$\underline{\xi}_t(\bar{x}) = -C_{51} \begin{bmatrix} C_{52}+C_{53}\ \bar{S}_{1t} + \bar{S}^*_{1t} \\ C_{52}+C_{53}\ \bar{S}_{2t} + \bar{S}^*_{2t} \\ C_{52}+C_{53}\ \bar{S}_{3t} + \bar{S}^*_{3t} \\ (C_{41}/C_{51})\ \bar{L}_t \end{bmatrix} \quad \text{for } t=1,2,\ldots,T.$$

Since our primary concern in this paper is with general results (i.e. parametrizations), we omit detailed

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[1] The reader is referred to any standard reference on vector and matrix calculus. (For example, see P. S. Dwyer and M. S. Macphail, "Symbolic Matrix Derivatives", Annals of Math. Stat. (1948), p. 519-534.)

considerations regarding computation of the inverse matrix in [3.14] and [3.15].[1]

The "best" team action vector $\underline{a}^*$ specifies the terminal actions of the team members for T periods into the future, under the null information structure. While the numerical result in [3.14] is of theoretical interest, it is apparent that even in the routine case the team may wish to reserve judgment on future actions until the particular period in which these actions are required. For example, this latter strategy would be more realistic if the probability distributions of $\underline{x}$ (i.e., for sales and reporting work force) were non-stationary over time. Thus, more practically, the team is interested in a strategy or program which indicates a team decision rule, $\underline{\alpha}$, by which subsequent terminal actions are derived, given the information available to the team at the time a group decision is required. For the case of routine we have restricted the team's "available information" to the expected values of the random variables. Incorporating these considerations into the formal analysis, we can modify the result in [3.14], to obtain [3.15] for t=1, by pre-multiplying both sides of equation [3.14] by the row vector $\underline{e}=[\underline{e}_1, \underline{e}_2, \ldots, \underline{e}_T]$, where $\underline{e}_1=(1, 1, 1, 1)$, and $\underline{e}_2=\ldots=\underline{e}_T=\underline{0}$. Thus, the "best" decision rule under the null information structure is given by

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1 Among the many references on matrix inversion are: D. Greenspan, "Methods of Matrix Inversion", American Mathematical Monthly (May 1955), p. 303-318; and F. Hildebrand, METHODS OF APPLIED MATHEMATICS (Prentice-Hall, 1952-1961), cf. p. 68-80 which lists some of the available numerical procedures.

Determination of which numerical procedures are computatationally efficient will depend, by and large, on the specific size and form of the matrix $\underline{\underline{Q}}$.

equation [3.15], where, in particular,[1]

$$E_{\underline{X}}(\alpha_1^*) = \underline{a}_1^* = -\sum_{t=1}^{T} \underline{\underline{Q}}_{1t}^{-1} \left[ \underline{\alpha}_t + \sum_{\tau=t}^{T} \underline{\xi}_{\tau} (\bar{x}) \right]. \qquad [3.16]$$

### 3.6 The Case of "Complete Information".

From the routine case we obtained a lower bound on the team payoff in the sense that substitution of the computed $\underline{a}^*$ into [3.11] yields the lowest cost under minimal information, i.e., the null information structure. It is apparent that the cost which results from implementing $\underline{a}^*$ should be greater than that which would obtain from the best $\underline{a}$ under information structures which provide additional information about $\underline{x}$. We can evaluate the relative worth of implementing alternative information structures for the team by computing the value of the information structure, $V(h^k)$, where

$$V(h^k) = \min_{\underline{\alpha}} E_{\underline{X}}\, c(\underline{\alpha}\,[\underline{h}^o(x)],\, \tilde{\underline{x}}) - \min_{\underline{\alpha}} E_{\underline{X}}\, c(\underline{\alpha}\,[\underline{h}^k\,(x)],\, \tilde{\underline{x}}), \qquad [3.17]$$

for $\underline{h}^k$ the information structure k, and $\underline{h}^o$ the null information structure.[2] From the discussion of the routine case above,

$$\min_{\underline{\alpha}} E_{\underline{X}}\, c(\underline{\alpha}\,[\underline{h}^o\,(x)],\, \tilde{\underline{x}}) = \min_{\underline{\alpha}} c(\underline{a}, \bar{\underline{x}}).$$

The return of an information structure is defined as the value

---

1 The results in this section can be obtained alternatively, albeit more computational detail, by operating on the original expression in [3.9], ignoring the reformulation in [3.11]. An example of this approach in a slightly different context (viz., different costs and T infinite) is given in Chapter 4 of Holt, et al, [25]. Van de Panne and Bosje [56] deal with this same case, reformulating the problem in matrix format, partitioned by decision instruments, i.e., $\underline{a}' = (\underline{W}\ \underline{P})$, where $\underline{W}' = (W_1, \ldots, W_T)$ and $P' = (P_1, \ldots, P_T)$; cf. Ch. 18 in [25].

2 If the original problem is phrased in terms of team payoff rather than costs, [3.17] would be rewritten

$$V(h^k) = \max_{\alpha} E_{\underline{X}}\, u(\alpha[\underline{h}^k(x)], \tilde{\underline{x}}) - \max_{\alpha} E_{\underline{X}}\, u(\alpha[\underline{h}^o(x)], \tilde{\underline{x}}).$$

of the structure less the cost of providing it,[1]

$$R(\underline{h}^k) \equiv V(\underline{h}^k) - E_{\underline{X}}\, c(\underline{h}^k, \tilde{\underline{x}}). \qquad [3.18]$$

To limit the evaluation of alternative information structures, we can determine an upper bound on the team payoff by finding the best possible team decision rule under the case of "perfect information" about the real world outcomes. That is, suppose the state of the world $\underline{x}$ is such that each team member observes a random variable $\tilde{y}_{it} = \theta_{it}(\tilde{x})$ on $\underline{X}$. "Perfect information" is the situation which obtains when $\underline{\theta}_t(\tilde{x}) = \underline{x}_t$, so that $\underline{h}_i\ (\tilde{x}) = \underline{x}_i$ for all i. The case called complete information indicates that team decision rule, say $\underline{\alpha}^{**}$, which minimizes total cost to the team when the state of the world is known "with certainty". From equation [3.11],

$$c(\underline{a}, \underline{x}) = \mu_o(x) + 2\ \underline{a}'\ \underline{\mu}(x) + \underline{a}'\ \underline{\underline{Q}}\ \underline{a}$$

and proceeding as before, we obtain

$$\underline{a}^{**} = -\underline{\underline{Q}}^{-1}\ \mu(x) \qquad [3.19]$$

or

$$\underline{a}^{**}_t = -\sum_{\tau=1}^{T} \underline{\underline{Q}}^{-1}_{t\tau} \left[ \underline{\gamma}_{\tau} + \sum_{\tau'=\tau}^{T} \underline{\xi}_{\tau'}(x) \right], \text{ for } t=1,2,\ldots,T. \qquad [3.20]$$

The vectors $\underline{\mu}(x)$ in [3.19] and $\underline{\xi}_{\tau'}(x)$ in [3.20] are not stochastic vectors, but rather vectors of the specific states of the world which occur in each time period t, for t=1,2,...,T. Note the similarity between the form of the results obtained in [3.14]- [3.15] and [3.19]- [3.20]. It is also noteworthy

---

[1] The cost of providing the null information structure, $\underline{h}^o$, is identically zero. If the cost of providing $\underline{h}^k$ does not depend on the states of the world, the second term on the right in [3.18] is independent of $\tilde{\underline{x}}$.

to call attention to the fact that $\alpha^{**}$ depends on $\underline{x}$ through $\underline{\mu}(x)$ only in [3.19], so that complete knowledge of $\underline{\mu}_i(x)$ is sufficient for each member to use $\underline{\alpha}_i^{**}$.[1]

Marschak [36] has distinguished a concept in team organization called "<u>cospecialization</u> of action and information", wherein it is more economical for each member to specialize in observing the individual $\theta_{it}(\tilde{x})$ over t, rather than pooling team resources for observations. In the present context <u>cospecialization</u> is equivalent to the instance where each member observes the corresponding first-order effect, $\underline{\mu}_i(x)$, of his own action on the team payoff in [3.11], where

$$\underline{\mu}_i(x) = \begin{bmatrix} \mu_{i1}(x) \\ \vdots \\ \mu_{it}(x) \\ \vdots \\ \mu_{iT}(x) \end{bmatrix}, \text{ and } \mu_{it}(x) = \gamma_{it} + \sum_{\tau=t}^{T} [\beta_{i\tau}(x) - b_{ii} v_{i\tau}(x)],$$

$$\text{for } t=1,2,\ldots,T; \text{ and all i.}$$

A glance at $\underline{\mu}_i(x)$ indicates that strict cospecialization is not appropriate in this case, since forecasts on $\tilde{\underline{x}}_\tau$ for $\tau > t$ are required with $\underline{\mu}_i(x)$ for each i. In this regard, we distinguish between observations made without "observation error" and the converse, which includes "observation forecasts" as a sub-category. If we admit "observation forecasts" (i.e., observations with observation errors) within the meaning assigned to <u>cospecialization</u>, the assumption of cospecialization of action and information is appropriate for the organization we are investigating.

The concept of cospecialization becomes important in the

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[1] Radner [47], p. 497 ff.

case called complete communication, that is, for the information structure wherein all members have the same information on which to base their decisions:

$$h_{it}(x) = \underline{\theta}_t(x) \equiv [\theta_{1t}(x), \ldots, \theta_{4t}(x)] \text{ , for all } i,t.$$

Radner[1] has shown that complete communication under cospecialization is equivalent to complete information, so that complete communication under cospecialization is sufficient to obtain $\underline{\alpha}^{**}$ in [3.19]. It is worthwhile to note that the information structure of complete communication can be realized alternatively by having each team member communicate his observations to a centralized authority which computes the best actions on the basis of the pooled observations, and then transmits these results to the corresponding members. Variations on this centralized organization are generated as information structures if processing errors or "system noise" are introduced during either phase of the two-way communication.

3.7 Some Examples of "Decentralization".

Referring to [3.19] above, it is apparent that $\underline{\alpha}^{**}$ cannot be realized in practice because it involves knowledge of future requirements for each hospital and the reporting work force, which in general is not available. One alternative to this rule might be to establish a "best" team decision rule under existing information, and then to provide for periodic adjustment of the established rule at regular intervals on the basis of improved subsequent knowledge. A

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1 Ibid.

common form of this alternative is the procedure called "dynamic programming".[1] We now wish to briefly consider some similar procedures in more detail.

The case called "complete communication" has been indicated as one form of centralization within an organization. In like manner the case of "no communication", as under routine, can be viewed as one form of a completely decentralized organization. In the formal evaluation of decentralized system alternatives we will employ some of Radner's theoretical results, which are stated below without the accompanying proofs[2]:

"<u>Theorem 1</u>. Suppose $E\,\mu_r^2 < \infty$, for r=1,...,N; then for any information structure $\underline{h}$ the unique (almost everywhere) best team decision function is the solution of $q_{rr}\,\alpha_r + \sum_{s\neq r} q_{rs}\,E(\alpha_s | h_r) = E(\mu_r | h_r)$, r=1,...,N. [3.21]

"<u>Corollary 1</u>. If $\hat{\underline{\alpha}}$ is an optimal team decision function with respect to an information structure $\underline{h}$, then

$$V(\underline{h}) = E\,\hat{\underline{\alpha}}'\,\underline{\mu} - (E\,\hat{\underline{\alpha}})'\,(E\,\underline{\mu}). \qquad [3.22]$$

"<u>Corollary 2</u>. If $\hat{\underline{\alpha}}$ is optimal for some $\underline{h}$, then

$$E\,\hat{\underline{\alpha}} = \underline{\underline{Q}}^{-1}\,E\,\underline{\mu}." \qquad [3.23]$$

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1 Cf., Bellman [6].

2 These results are given in Radner [47] on p. 493-4. The proofs for the more general case where $\underline{\underline{Q}}=\underline{\underline{Q}}(x)$ are contained in Radner [48], p. 863-868. The quadratic <u>payoff</u> considered by Radner in the former instance is $w(\underline{a},\underline{x})=\mu_o+2\underline{a}'\underline{\mu} - \underline{a}'\underline{\underline{Q}}\,\underline{a}$. Referring to [3.11] above, this requires multiplication of $\mu_o(x)$ and and $\underline{\mu}(x)$ by (-1) to make Radner's results directly appropriate. For example, if we take $\underline{\mu}(\bar{x}) = -E\underline{\mu}$ in our [3.14], then Radner's result in [3.23] applies directly. For the remainder of the discussion in this section we will assume $\mu_o \equiv -\mu_o(x)$, and $\underline{\mu} \equiv -\underline{\mu}(x)$. In Radner's terminology, the team decision rules that we are considering in this paper are <u>Bayes team decision functions</u> (cf., p. 860 in [48]).

For the organization at hand, the physical structure is such that in all instances it is reasonable to assume that the observational functions $\underline{\theta}_i$, are statistically independent for i=1,2,3,4. As Radner observes, this assumption does not mean that the information functions $h_{it}$ are also statistically independent; in fact, as soon as communication takes place the converse applies generally. Suppose, however, we initially assume that no communication takes place between any of the team members. Then $\underline{h}_i = \underline{\theta}_i$ for all i, and thus the (vector) information functions are also statistically independent. Referring to [3.21], the independence assumption implies that

$$E_{\underline{X}} (\underline{\alpha}_j \mid \underline{\theta}_i) = E_{\underline{X}} (\underline{\alpha}_j \quad , \quad \text{for } j \neq i. \qquad [3.24]$$

That is, person i's information does not help him to predict person j's action, in any given time period t, for t=1,...,T. Now consider Radner's theorem for time period one.[1] Noting the form of $\underline{\underline{Q}}$ in [3.11c] and identifying the elements of $\underline{\underline{Q}}_t$ as $q_{(ij)_{t\tau}}$, [3.21] reduces to

$$\hat{\alpha}_{i1} = \frac{1}{q_{(ii)_{11}}} \left[ E_{\underline{X}}(\mu_{i1} \mid \theta_{i1}) - \sum_{j \neq i} q_{(ij)_{11}} E_{\underline{X}} (\alpha_{j1}) - \right.$$

$$\left. - \sum_{\tau=2} q_{(ii)_{1\tau}} E_{\underline{X}} (\alpha_{i\tau} \mid \theta_{i1}) \right]$$

$$\text{for } i = 1,2,3,4. \qquad [3.25]$$

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1 See discussion under routine case, 3.5, p. 33-4.

The second term in the bracketed expression of [3.25] evaluates to some constant, say $M_i$ for each i, and is obtained directly from [3.15] for t=1, since $E_{\underline{X}}(\alpha^*_{jt}) = a^*_{jt}$. The third term, $E_{\underline{X}}(\alpha_i \mid \theta_{i1})$, also evaluates to a constant, say $N_i$ for each i, and is obtained from the components of [3.14] where the expectation operator over $\underline{X}$ is conditioned on the first period's observation, $\theta_{i1}$. Note in this last instance that the information functions $h_{it}$ are not independent over t for i fixed, since, "by definition", complete communication is present between a member and himself. The "best" team decision rule under the information structure of "no communication" and independent observations is therefore

$$\hat{\underline{\alpha}} = \begin{bmatrix} \hat{\underline{\alpha}}_1 \\ \vdots \\ \hat{\underline{\alpha}}_t \\ \vdots \\ \hat{\underline{\alpha}}_T \end{bmatrix}, \quad \text{for } \hat{\underline{\alpha}}_t = \begin{bmatrix} \hat{\alpha}_{1t} \\ \cdot \\ \cdot \\ \cdot \\ \hat{\underline{\alpha}}_{4t} \end{bmatrix}, \text{ and}$$

$$\hat{\alpha}_{it} = \frac{1}{q_{(ii)_{tt}}} \left[ E_{\underline{X}} (\mu_{it} \mid \underline{\theta}_{it}) - (M_{it} + N_{it}) \right], \quad i=1,2,3,4; \; t=1,2,\ldots,T \qquad [3.26]$$

where $\underline{\theta}_{it} = \left\{ \theta_{i1}, \ldots, \theta_{it} \right\}$, $M_{it} = \sum_{j \neq i} q_{(ij)_{tt}} E_{\underline{X}}(\alpha_{jt})$,

and $N_{it} = \sum_{\tau \neq t} q_{(ii)_{t\tau}} E_{\underline{X}} (\alpha_{i\tau} \mid \underline{\theta}_{it})$. The result in [3.26] indicates that the best terminal actions are simple linear functions of the respective observations, $\theta_{it}$.

The value of this information structure for the computed [3.26] can be obtained directly by substitution into [3.22]. To clarify the expectation operators in Radner's corollary for purposes of this discussion, we will distinguish the domain

of the random variables $\theta_{it}$ as $\underline{Z}$, the observation (or sample) space, where $z \epsilon \underline{Z}$ and $\underline{Z} \subseteq \underline{X}$, the space of real world outcomes.[1] Incorporating this distinction into our results above, [3.26] can be restated as

$$\hat{\alpha}_{it} = \frac{1}{q_{(ii)_{tt}}} \left\{ E_{\underline{X}|\underline{Z}} \left[ \mu_{it}(\tilde{x}) \mid \underline{\theta}_{it}(\tilde{z}) \right] - M_{it} - \right.$$

$$\left. - \sum_{\tau \neq t} q_{(ii)_{t\tau}} E_{\underline{X}|\underline{Z}} \left[ \alpha_{i\tau}(\tilde{x}) \mid \underline{\theta}_{it}(\tilde{z}) \right] \right\},$$

where the last term within the brackets represents the modified $N_{it}$, $\underline{\theta}_{it}$ and $M_{it}$ are defined as before, and $\mu_{it}(\tilde{x}) = \gamma_{it} + \sum_{\tau=t}^{T} [\beta_{i\tau}(\tilde{x}) - b_{ii} V_{i\tau}(\tilde{x})]$. The components within [3.22] for this information structure, say $\underline{h}^{(1)}$, are then readily identified as

$$E\, \hat{\alpha}_{it}\, \mu_{it} = \frac{1}{q_{(ii)_{tt}}} \left\{ E_{\underline{Z}} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it})^2 - \right.$$

$$\left. - M_{it} E_{\underline{Z}} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it}) - E_{\underline{Z}} N_{it} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it}) \right\},$$

and

$$(E\hat{\alpha}_{it})\, (E\mu_{it}) = \frac{1}{q_{(ii)_{tt}}} \left\{ \left[ E_{\underline{Z}} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it}) - M_{it} - E_{\underline{Z}} N_{it} \right] \right.$$

$$\left. \left[ E_{\underline{Z}} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it}) \right] \right\},$$

for $i = 1,2,3,4$; $t = 1,2,\ldots,T$. Thus, the value of the information structure $\underline{h}^{(1)}$, which corresponds to "no communication and statistically independent observations", is

$$V(\underline{h}^{(1)}) = \sum_{t=1}^{T} \sum_{i=1}^{4} \frac{1}{q_{(ii)_{tt}}} \left\{ E_{\underline{Z}} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it})^2 - \right.$$

$$\left. - \left[ E_{\underline{Z}} E_{\underline{X}|\underline{Z}} (\mu_{it} \mid \underline{\theta}_{it}) \right]^2 \right\}. \qquad [3.27]$$

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1 Recall, in general that $\underline{\mu}_i = \underline{\mu}_i(\tilde{x})$ and $\underline{\theta}_i = \underline{\theta}_i(\tilde{x})$ in previous discussions. In this instance we are specializing $\underline{\theta}_i = \underline{\theta}_i(\tilde{z})$, for $\tilde{z} \epsilon \underline{Z} \subseteq \underline{X}$, $\tilde{x} \epsilon \underline{X}$.

The addition of the assumption of cospecialization of action and information to the above case provides the information structure called complete informational decentralization. That is, if we assume that "forecast" errors are negligible in the sense that $\theta_{it}(\tilde{z}) \sim \mu_{it}(\tilde{x})$, for $\underline{Z} \longrightarrow \underline{X}$ and all i,t, then $\underline{\theta}_i \sim \underline{\mu}_i$. Computation of the value of the structure complete informational decentralization, say $\underline{h}^{(2)}$, is obtained by solving [3.22] with the aid of the result in [3.27], which simplifies to

$$v\left(\underline{h}^{(2)}\right) = \sum_{t=1}^{T} \sum_{i=1}^{4} \frac{1}{q_{(ii)_{tt}}} \left\{ E_{\underline{X}}[\mu_{it}(\tilde{x})^2] - \mu_{it}(\bar{x})^2 \right\}. \qquad [3.28]$$

This last result indicates that the value of the structure is a simple linear function of the variance in the first order component, $\underline{\mu}$ $(\tilde{x})$, of the team payoff, and exhibits "constant returns to scale".[1]

To determine the return from each of the above information structures we require only the cost of observation (forecast) for each of the four members over the range of t, since communication is not present under either structure. For example, in the case where we assume cospecialization, the cost of "observation" (forecast) for each member would consist of the cost of maintaining records of sales (and forecasts) at each of the three hospitals, and a record of the reporting work force at the plant, for each time period. Thus, the return for the structure "complete informational decentralization" can be represented as

$$R\left(\underline{h}^{(2)}\right) = v\left(\underline{h}^{(2)}\right) - \sum_{t=1}^{T} \sum_{i=1}^{4} c(h_{it}^{(2)}), \qquad [3.29]$$

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[1] The equation [3.28] is consistent with Radner's theoretical discussion, cf., p. 499-501, in [47].

where $V(\underline{h}^{(2)})$ is given by [3.28], and $c(h^{(2)}_{it})$ is the individual cost of maintaining a record system[1] The normative implications for physical equipment in this instance are of little consequence.

Between the extreme cases of "complete communication" and "no communication" on a continuum are a wide variety of information structures which more typically represent those found in existing organizations. We conclude this discussion by illustrating one such structure from within the sub-classification of structures called <u>partial decentralization</u>. Both Radner and Marschak have dealt with various theoretical structures within this category, such as those called "partitioned communication", "reporting exceptions" (or management by exception), and "emergency conference". For present convenience, we only consider one of the simplest of these structures at this time, for aggregate production planning in the reference firm.

In any organization one of the most common forms of communication is the dissemination of "summary information" through published reports. We will identify this form of communication as the information structure $\underline{h}^{(3)}$, called "<u>dissemination of information</u> under partial decentralization."

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[1] Note the $c(h^{(2)}_{it})$ in [3.29] represent a near minimum cost of information "processing", viz., that of a concurrent filing system, and are independent of the state space $\underline{X}$. If it were appropriate to include forecasting costs, as well, then $c(h^{(2)}_{it})$ could be rewritten as the sum of the component costs of forecasting and filing, where the former might be dependent on $\underline{X}$.

To make $\underline{h}^{(3)}$ explicit in a formal sense, we will suppose that each member communicates some function of his observations to a <u>central</u> <u>staff</u>[1] in the organization, which compiles all such information received and then periodically disseminates this compilation to each member as a report.[2] Let $\delta_i$ represent a functional transformation of the observations by member i on $\underline{Z}_i$, where $\theta_{it}(\tilde{z})$ is over $\underline{Z}_i$; and let $d_{it} = \delta_i(z_{it})$ be the i-th member's communicated message to the central staff in period t, for all i, t.[3] (Note, alternatively, $d_{it} = \delta_i[\theta_{it}(z)]$.) Then the information function for each member is given as

$$h_{it}(x) = \left\{ \underline{\theta}_{it}(z), \underline{\delta}(x) \right\} , \text{ for } i=1,\ldots,4 \ ; \ t=1,\ldots,T \ ;$$

where $\underline{\delta}(x) = [\delta_1(x), \ldots, \delta_4(x)]$; $\underline{d}_t = [(d_{11},\ldots,d_{41}),\ldots,(d_{1t},\ldots,d_{4t})]$,

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1 We distinguish <u>staff</u>, within an organization, as individuals who possess no decision-making authority and, hence, have no formal control over terminal actions (or decision instruments).

2 For example, each member might communicate a "weighted average" of his past observations -- or a contraction of his present observation (or forecast) -- and receive, in turn, a similarly transformed set of observations from all other members.

3 For convenience, we assume that the periodicity of the staff report corresponds to that of the team decision, although this need not be the case. We also will assume that each report is available at the beginning of the time period, prior to the determination of decisions for the period, and that the report includes information regarding observations (or state variables) in all preceding time periods.

the collection of reports by the central staff to each member; and $\theta_{it}=(\theta_{i1},\ldots,\theta_{it})$, as above. Recalling our previous specification, we again note that $\mu_{it}$ is independent of $\{\theta_{jt}\}$ for $i\neq j$, and all t. In this regard, if $\theta_{it}$ represented a forecast of $\mu_{it}$, i.e., such that $\theta_{it}(z)=\mu_{it}(x)+e_{it}$, for $e_{it}$ a random error, the present simplification implies that the individual $\theta_{it}$ would be independent of both $\mu_{jt}$ and $e_{jt}$, for all $i\neq j$, $(t=1,2,\ldots,T)$.

Before proceeding to the derivation of the best team decision rule under $\underline{h}^{(3)}$, we note the following lemma due to Radner[1]:

<u>Lemma</u>. "Let A, C, and G be independent random variables; let B be a contraction of A, and D be a contraction of C; and let F be a real random variable defined by F=f(A,D,G), where f is some given measurable function; then

$$E\ [F|B,C,G,] = E[F\ |\ B,D,G,]." \qquad [3.30]$$

Applying the lemma to the considerations within [3.21], we can make the following simplifications[2]:

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1 R. Radner, [47], p. 504.

2 These results follow directly by substituting into [3.30] accordingly:

for [3.31]: $f=\alpha_{jt}, A=\underline{\theta}_{jt}, B=\underline{\delta}_j, C=\theta_{it}, D=\underline{\delta}_i, G=\{\underline{\delta}_k\}$ for $k\neq i,j$;

for [3.32]: $f=\alpha_{i\tau}, A=(\underline{\theta}_{i\tau},\underline{\theta}_{it}), B=\underline{\delta}_i, C=\underline{\theta}_{jt}, D=\underline{\delta}_j, G=\{\underline{\delta}_k\}$ for $k\neq i,j$;

and for [3.33[: $f=\mu_{it}, A=(\mu_{it},\underline{\theta}_{it}), B=\theta_{it}, C=\{\underline{\delta}_j\}$ for $j\neq i$, G,D = (a constant).

$$E_{\underline{x}|\underline{z}}\ (\alpha_{jt}|h_{it}) = E_{\underline{x}|\underline{z}}\ (\alpha_{jt}|\underline{\delta}) \quad , \ i \neq j,\ t=1,\ldots,T; \qquad [3.31]$$

$$E_{\underline{x}|\underline{z}}\ (\alpha_{i\tau}|h_{it}) = E_{\underline{x}|\underline{z}}\ (\alpha_{it}|\underline{\delta}) \quad , \ \text{all } i,\ \tau \neq t; \qquad [3.32]$$

and $$E_{\underline{x}|\underline{z}}\ (\mu_{it}|h_{it}) = E_{\underline{x}|\underline{z}}\ (\mu_{it}|\underline{\theta}_{it}) \quad , \ \text{all } i,\ t. \qquad [3.33]$$

From the above, it follows for the case at hand that [3.21] can be written as[1]

$$q(ii)_{tt}\alpha_{it} + \sum_{j\neq i} q(ij)_{tt}\, E_{\underline{x}|\underline{z}}(\alpha_{jt}|\underline{\delta}) + \sum_{\tau\neq t} q(ii)_{t\tau}\, E_{\underline{x}|\underline{z}}(\alpha_{i\tau}|\underline{\delta}) =$$

$$= E_{\underline{x}|\underline{z}}(\mu_{it}|\underline{\theta}_{it}), \quad \text{for } i=1,\ldots,4\ ;\ t=1,\ldots,T. \qquad [3.34]$$

The conditional expectation of [3.34], for $\underline{\delta}$ given, is then

$$\sum_{j\neq i} q(ij)_{tt}\, E_{\underline{x}|\underline{z}}\ (\alpha_{jt}|\underline{\delta}) + \sum_{\tau} q(ii)_{t\tau}\, E_{\underline{x}|\underline{z}}(\alpha_{i\tau}|\underline{\delta}) = E_{\underline{x}|\underline{z}}(\mu_{it}|\underline{d}_t),$$

$$\text{for } i=1,\ldots,4;\ t=1,\ldots,T. \qquad [3.35]$$

Subtracting [3.35] from [3.34], we obtain that for all i,t

$$\hat{\hat{\alpha}}_{it} = E_{\underline{x}|\underline{z}}(\alpha_{it}|\underline{\delta}) + \frac{1}{q_{(ii)_{tt}}}\left[E_{\underline{x}|\underline{z}}(\mu_{it}|\underline{\theta}_{it}) - E_{\underline{x}|\underline{z}}\ (\mu_{it}|\underline{d}_t)\right] \qquad [3.36]$$

As note earlier[2], [3.35] can also be rewritten as

$$\underline{\underline{Q}}\ E_{\underline{x}|\underline{z}}\ (\underline{\alpha}|\underline{\delta}) = E_{\underline{x}|\underline{z}}\ (\underline{\mu}|\underline{d})\ ;$$

which gives

$$E_{\underline{x}|\underline{z}}\ (\underline{\alpha}\,|\,\underline{\delta}) = \underline{\underline{Q}}^{-1}\ E_{\underline{x}|\underline{z}}\ (\underline{\mu}|\underline{d}). \qquad [3.37]$$

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1 Note that from [3.30], [3.31], and [3.32], $E_{\underline{x}|\underline{z}}\ (\alpha_{j\tau}/h_{it}) =$ $= E_{\underline{x}|\underline{z}}\ (\alpha_{j\tau}|\underline{\delta})$, however, from the specification of $\underline{\underline{Q}}$ in [3.11c], $q_{(ij)_{t\tau}} = 0$ for $t \neq \tau$, and all i,j. Hence, [3.21] further simplifies, viz., [3.34].

2 Ibid.

Substitution, component-wise, from [3.37] into [3.36] yields the best team decision function under $\underline{h}^{(3)}$; that is,

$$\hat{\hat{\underline{\alpha}}} = \begin{bmatrix} \hat{\hat{\underline{\alpha}}}_1 \\ \vdots \\ \hat{\hat{\underline{\alpha}}}_t \\ \vdots \\ \hat{\hat{\underline{\alpha}}}_T \end{bmatrix}, \text{ for } \hat{\hat{\underline{\alpha}}}_t = \begin{bmatrix} \hat{\hat{\alpha}}_{1t} \\ \vdots \\ \hat{\hat{\alpha}}_{4t} \end{bmatrix}, \text{ and}$$

$$\hat{\hat{\alpha}}_{it} = \sum_{\tau} \sum_{j} q^{(ij)}{}_{t\tau}\, E_{\underline{X}|\underline{Z}}(\mu_{j\tau}|\underline{d}_t) + \frac{1}{q_{(ii)}{}_{tt}} \Big[ E_{\underline{X}|\underline{Z}}(\mu_{it}|\underline{\theta}_{it}) - E_{\underline{X}|\underline{Z}}(\mu_{it}|\underline{d}_t) \Big] \quad \text{for } i=1,\ldots,4 \;, \; t=1,2,\ldots,T; \qquad [3.38]$$

where $q^{(ij)}{}_{t\tau}$ represent elements in the partitioned inverse of $\underline{\underline{Q}}^{-1}$. Recalling our earlier observation on the specification of $\underline{\underline{Q}}$ in [3.11c], the result in [3.38] can be simplified in this case to

$$\hat{\hat{\alpha}}_{it} = \sum_{j\neq i} q^{(ij)}{}_{tt} E_{\underline{X}|\underline{Z}}(\mu_{jt}|\underline{d}_t) + \sum_{\tau} q^{(ii)}{}_{t\tau} E_{\underline{X}|\underline{Z}}(\mu_{i\tau}|\underline{d}_t) + \frac{1}{q_{(ii)}{}_{tt}} \Big[ E_{\underline{X}/\underline{Z}}(\mu_{it}|\underline{\theta}_{it}) - E_{\underline{X}|\underline{Z}}(\mu_{it}|\underline{d}_t) \Big], \text{ all } i,t. \qquad [3.39]$$

The value of the information structure "dissemination of information under partial decentralization", $\underline{h}^{(3)}$, is determined by evaluating [3.22] for the team decision function obtained in [3.39]. Under the assumption of independence for $i\neq j$, this result can be simplified to

$$V(\underline{h}^{(3)}) = V(\underline{h}^{(1)}) + \sum_{t,i} \Big[ \sum_{\tau\neq t} q^{(ii)}{}_{t\tau} E_{\underline{Z}} \operatorname{Cov}(\mu_{it}\mu_{i\tau}|\underline{d}_t) + \Big( q^{(ii)}{}_{tt} - \frac{1}{q_{(ii)}{}_{tt}} \Big) E_{\underline{Z}} \operatorname{Var}(\mu_{it}|\underline{d}_t) \Big], \qquad [3.40]$$

where $V(\underline{h}^{(1)})$ is given in [3.27], above. The first term in

[3.40] follows our previous interpretation as the expected payoff to the team of each member's individual observations over time, whereas the second term in [3.40] can be interpreted as the expected additional payoff which results from having the members share their information through a common reporting network -- viz., the dissemination of $\underline{d}_1, \ldots, \underline{d}_T$.

To compute the return for the information structure $\underline{h}^{(3)}$ we must recognize that the cost of providing the structure, $c(\underline{h}^{(3)})$, includes both the fixed cost of establishing the communication network (the means by which the members communicate messages to the central staff and the staff disseminates the compiled report to each member) and the variable cost of observation by each member and report compilation by a central staff.[1] Although the normative implications for physical equipment and facilities are still of relative minor consequence, it is perhaps more apparent in this instance than in earlier cases, how these implications can enter explicitly into the analysis. The specification of a particular information structure details the communication network, the observation functions, the information processing, system noise, and so on, which are required within the organization to effect management decisions.[2] Once such properties have been

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1 Cf. Marschak [37] and [39], or "Theory of Teams: Introduction", Cowles Foundation Discussion Paper, Economics No. 31 [hectographed], (May, 1957).

2 For example, in the above case "system noise" could be identified as the dissemination of reports given by $\underline{d}_t + \underline{\tilde{e}}_t$, for $\underline{\tilde{e}}_t$ a vector or random errors.

introduced for consideration we possess a formal mechanism through which we can evaluate their relative worth and the best program of decision rules they can provide. That is, once the physical requirements of the system are identified and made explicit in the context above, the "return" to the organization for a particular information data-processing system can be evaluated directly by computing the value of the defining information structure in accordance with the relation in [3.22]. Thus, although the formal concepts of "team action vector", "information structure", "cospecialization", and the like, are unfamiliar to management, the preceding limited analysis indicates that these abstractions can be translated readily into such operating concepts as "sales forecast", "production rate", "available direct labor work force", etc., which are easily identified by managerial and staff personnel.

## 4. Some Conclusions: Practical Results

For emphasis, it appears worthwhile to restate the distinctions between aggregate production planning as considered herein using Marschak and Radner's framework of teams, and as considered by Holt (et al), and others in the context of "linear decision rules". In the former instance, the team construct recognizes that the planning process takes place within an organization composed of many decision makers which interact with one another in the formal determination of an aggregate planning schedule. This interaction is made explicit by the concept of information structure which incorporates the realization that information must be processed and communicated within the organization before it becomes available as

input to a programmed decision sequence. In introducing the notion that an organization rather than one individual is responsible for aggregate planning, limited information and "certainty equivalence" in the derivation of "best" linear decision rules obtain as a special case, viz., that of routine[1] As such, the routine case provides a convenient lower limit on payoff which can be used as a comparative reference for alternative information structures. Thus, optimal decision rules derived within the team framework can never be less efficient than those obtained within the "linear decision rule" analysis. Furthermore, once an optimal team has been identified with respect to a team decision rule and information structure, the analysis can realistically proceed to the determination of normative criteria for the physical system required to provide the desired information structure.

Perhaps the easiest way to synthesize this general discussion on normative mathematical models of information systems is to draw some comparisons of model areas on the basis of the summary presented earlier in Figure 2.1. Recalling our introductory remarks, we might think of a normative analysis as one which assigns economic weights to the properties identified and included within a descriptive model, so that the latter is a sufficient condition for the former. It appears appropriate, therefore, to initially dichotomize the model areas of research into (1) those which are relatively all-inclusive, and (2) those which include relatively few descriptive properties of the system and,

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1 For example, see the discussion in Ch. 6 of Holt, et al [25], particularly p. 126-130.

hence, are more specialized. For the model areas and properties listed in Figure 2.1 (and defined in 2.2) the former category would include the areas of communication theory, structures based on modern algebra, simulation, team decision theory, and (to a lesser degree) group psychology. These model areas consider "most of" the system's characteristics within their analytical framework and, hence, are generally broad in their scope of descriptive capability. In the second category of our dichotomy we would include the remaining areas, viz., information theory, control process analysis in systems engineering, network flow analysis and graph theory, "general systems analysis",[1] statistical decision theory, and those included under "others".

Quite naturally, the above dichotomy is relative, and therefore one should exert caution in attempting to generalize from it. Nevertheless, it is apparent that those models which possess general descriptive capability are more appropriate than specialized models in instances where a formal analysis is "in its infancy" or where a characterization of the total system is required. On the other hand, if an analysis is primarily concerned with isolated characteristics of a system (e.g., "system noise" or "message content"), then one of the more specialized structures would be the appropriate framework for the analyst to consider. Similar conclusions can be drawn regarding the relative effectiveness of individual models as "descriptors" within problem areas. In this context a further comparison can be made on the basis

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[1] The area we have called "general systems analysis" postulates a broad scope of consideration, however, relative to the other areas few of the properties listed in the table are formally included within this framework.

of the normative capabilities of the models surveyed. Of the models considered only four areas provided formal mechanisms for the direct evaluation of system performance according to "some" given criterion -- although similar criteria may be derived within other areas. Communication theory employs a fidelity criterion in evaluating channel transmission; psychological research on groups uses task completion time as a measure of performance; "general systems analysis" employs effectiveness criteria in the context of costs or some stipulated objective for the system; and team decision theory incorporates the concept of the value of an information structure for evaluating information and systems within organizations. Among these four, the last two appear to be the most meaningful for the analysis of management information systems, primarily because they can be more easily related to the decision making functions within business (or military) establishments.

Mathematical research on management information systems has been quite limited in comparison to the vast number of volumes that have appeared offering qualitative advice for systems managers. Although nearly all systems research is worthwhile, one shortcoming of much of the non-quantitative effort has been that it often becomes extremely difficult for a systems manager to translate the generalized counsel offered by this literature into specific operating rules and procedures for his organization. Broad connotations of systems terminology and concepts have also impeded the successful implementation of many management information systems. Perhaps the slow but eventual development of normative mathematical models in the systems area will overcome these existing handicaps.

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