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ON THE REASONABLENESS OF REGRESSIVE EXPECTATIONS

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CORRECTIONS

Page

- 12 In the line above equation (2.6), A_{t-5} should read A_{t-5}
- 21 The last sentence of the first paragraph should read: "Pondering this question, . . . , is it true that Y is non-negative for the series being forecasted by businessmen?"
- 22 In equation (3.2), g_t should read \bar{g}_t .
- 31 In the column headings of Table 6, the first g_{bt} should read \bar{g}_{bt} and the second g_{bt} should read \bar{g}_{kt} .
Also, the title of Part B should read "Regressions of $(SA_t/SA_{t-1})-1$ on a_{t-1} ."
- 38 In equation (3.17), g_t'' should read \bar{g}_t'' .
- 57 The following points on Chart 3 have not been labelled: "A" (with coordinates .20, .18), "B" (with coordinates .30, -.27), and "C" (with coordinates .20, 10).
- 76 Missing reference: 40a. Hurwicz, L. "Least-Squares Bias in Time Series," in Statistical Inference in Dynamic Economic Models (New York, Wiley, 1950), pp. 375-383.

FURTHER CORRECTIONS

Page

- 54 The following equation numbers are erroneous:
- | | | | |
|--------|------------|-------------|--------|
| (7.11) | on line 4 | should read | (5.87) |
| (7.7) | on line 14 | should read | (5.4) |
| (7.11) | on line 15 | should read | (5.8) |
- 55 Additional erroneous equation numbers:
- | | | | |
|--------|------------|-------------|-------|
| (7.7) | on line 10 | should read | (5.4) |
| (7.12) | on line 12 | should read | (5.9) |
| (7.12) | on line 13 | should read | (5.9) |
| (7.4) | on line 13 | should read | (5.5) |
- 66 Additional erroneous equation numbers:
- | | | | |
|--------|-------------------|-------------|--------|
| (7.7) | on line 4 | should read | (5.4) |
| (7.9) | on line 5 | should read | (5.5) |
| (7.10) | on line 10 | should read | (5.10) |
| (7.13) | on line 11 | should read | (5.10) |
| (7.7) | on line 11 | should read | (5.4) |
| (7.14) | on line 12 | should read | (5.11) |
| (7.6) | on line 16 | should read | (3.7) |
| (7.8) | on line 1 of fr 1 | should read | (5.5) |
- 67 The sentence beginning on line 19 should read "While our explanation does relate regressivity to . . ." rather than ". . . relate regressively to . . ."

John Bossons and Franco Modigliani

Most models of short-term economic fluctuations have assumed that businessmen's short-term anticipations about operating variables such as their companies' sales can, when aggregated, be regarded as an extrapolation of the current trend or level of those variables. Unfortunately, however, evidence from a number of surveys of such anticipations suggests that this assumption is inaccurate: that businessmen, rather than extrapolating the current trend, consistently tend to forecast a reversal of that trend. A number of analysts have attempted to find explanations for this evidence that would not conflict with the assumption that expectations are predominantly extrapolative. But basic to such attempts is a presumption that regressivity (as such persistent trend-reversal has come to be named) is not a "real" phenomenon, and so must be explained away if it occurs. Is this a valid presumption? The thesis of this paper is that it is not. We propose this thesis by first demonstrating the existence of regressivity in data on individual firms' sales, then constructing a model to explain the regressivity in actual data and hence in anticipations, and finally going on to resolve the apparent paradox posed by forecasts that continue to predict turning points when aggregated even though such aggregation causes regressivity to disappear in the actual data.

¹ Many of the ideas expressed in this paper have been in circulation for some time and have benefited from criticisms made by a number of colleagues. Parts of this paper were discussed in a paper [4] given at the 1960 Stanford meetings of the Econometric Society and at a Workshop at the University of Wisconsin. We are indebted to the discussants of that paper and particularly to Guy Orcutt for some stimulating suggestions. A number of the essential ideas of this paper were also included in an earlier draft monograph circulated in 1958 under the title "The Regressiveness of Short-Run Expectations as Reported to Surveys -- An Explanation and its Implications," and we are grateful to readers of that paper and particularly to Albert Hart and Marshall Kolin for their criticisms. The current version of this paper has been presented to seminars at Carnegie Institute of Technology and the University of Chicago, and has benefited particularly from comments by Kalman J. Cohen, Allan H. Meltzer, and Lester G. Telser. We should also like to thank Donald J. Daly, Donald E. Farrar, Edward Foster, Douglas Hartle, Marshall Kolin, John Lintner, and Thomas A. Wilson for suggestions made in discussing drafts of the current version of this paper with us. Computations underlying the empirical evidence presented in this paper were supported by a grant from the Sloan Research Fund of the M.I.T. School of Industrial Management, and were carried out on the School's IBM 1620 and on the IBM 7090 of the M.I.T. Computation Center. We are indebted to Alain Barbier, John Bauer, David Labson, James Stam, and William Steiger for their labors on our behalf in processing these computations.

1. Introduction

Twenty years ago, economics took a significant turn with the publication of Albert Hart's pathbreaking monograph on the role of anticipations in the theory of the firm [31]. While expectational variables are still introduced explicitly into few aggregate models, a number of economists have become concerned with incorporating businessmen's expectations into models based upon businessmen's decision processes. Interest in surveys of businessmen's anticipations has consequently mounted. Unfortunately the net predictive value of the short-run anticipations reported in these surveys has been discouragingly low. Indeed, not only have analysts [34] [39] [46] found the surveys to have less predictive value than alternative naive-model extrapolations of the variables predicted by the respondents, but they discovered [9] [23] [32] that the forecasts consistently tended even to predict the direction of change incorrectly.

Several attempts have been made to find explanations of such inaccuracy and, in particular, of such seemingly improbable regressivity in the forecasts. Since most analysts have presumed that expectations used in businessmen's operations and planning cannot actually be consistently regressive, they have found themselves in the position of having to explain away the observed evidence for regressiveness in the aggregated forecasts. Their explanations have consequently argued that such regressiveness is introduced into the expectations either by biases in the design or administration of the survey or by confusion among the respondents about the base from which they forecast change.¹ Thus Ferber [22]

¹ A significant exception is Robert Eisner who has suggested in [17] and [18] that regressiveness in individual forecasts may well be justified by a corresponding regressiveness in firms' actual sales. His hypothesis that businessmen view the determination of quarterly sales figures as analogous to drawing from a fixed sample without replacement can be fruitfully applied in terms of the model developed in Section 3 below.

and Hastay [58] have postulated the inclusion in survey samples of a large group of respondents who arbitrarily make "no change" forecasts of 4-quarter change, Hart [32] has hypothesized a "conservative" downward editing of the magnitude of 4-quarter change forecasts by both collectors and respondents, Modigliani and Sauerlender [46] have dwelt upon the possibility of confusion among respondents between 1-quarter and 4-quarter change, and Daly [15] has suggested an inadequate recognition by respondents of the extent of non-seasonal change.¹

All of these hypotheses undoubtedly explain a few of the regressive expectations reported in each survey. However, for any of them to have general validity, there must be some substance to the underlying presumption that businessmen's anticipations are not actually regressive. For if, contrary to this assumption, anticipations were "really" regressive, an attempt to explain the regressiveness away could obviously not be justified. Is this proposition valid?

We have shown elsewhere [7] [9] that the short-run anticipations which individual respondents report to surveys such as the Dun and Bradstreet and Canadian employers' surveys are quite strikingly regressive. Studies by Theil [55] and Anderson et al [2] present evidence indicating that regressiveness is also present to a lesser degree among the anticipations reported by individual firms to the Konjunkturtest of the IFO-Institut für Wirtschaftsforschung. Similar evidence has been presented by Ferber [24] for the anticipations collected by the

¹ All of these hypotheses rest to some extent on the fact that the surveys which exhibit the most striking evidence of regressivity ask businessmen to report their forecasts in terms of change over a period which substantially overlaps change that has already occurred. For instance, the surveys conducted by Dun and Bradstreet or the Railroad Shippers' Associations in a given quarter ask respondents to forecast their sales or shipments in the forthcoming quarter as a ratio of the level in the same quarter of the previous year. Regressiveness consequently shows up in the reported anticipations in the form of a forecast of 4-quarter change which is smaller in magnitude than the actual change which has already taken place before the forecast is made.

Illinois Labor Force survey.¹ Eisner [16] has indicated that even the yearly sales forecasts made by respondents to the McGraw-Hill survey exhibit similar regressiveness, and Pashigian [51] has suggested that this may also be true for the yearly forecasts reported to the SEC-Department of Commerce surveys.

This finding does not in itself show that businessmen's operating expectations are regressive. It does show that distortions introduced in the collection and aggregation of individual responses can account for little of the regressiveness in the published forecasts. But it may be that the forecasts made by respondents are unintentionally distorted because of confusion over the base from which the forecasts are supposed to be made. Or, failing that, it may conceivably be that the forecasts, though regressive, are not in fact operational.² While our findings in [9] may be suggestive, we shall

¹ In both the Konjunkturtest and the Illinois labor force survey, less than a majority of Classifiable Observations are generally regressive. Their significance as evidence for the prevalence of regressiveness in businessmen's forecasts results from the significant proportion of responses which are on the average regressive in spite of the shorter forecasting horizons represented by these surveys.

² Conceivably distortion could be introduced through the sort of editing proposed by Hart [32] if the forecasts reported to the surveys did not enter into businessmen's decision-making and were not proxies for other forecasts of operational variables, for in such a case the forecasts might represent little more than the results of a game played between businessmen and survey interviewers for public relations or other purposes. Undoubtedly this is the case in a few instances. It is, however, worth emphasizing that no current variable is likely to be more important than sales in businessmen's operational planning. Moreover, since the Dun and Bradstreet survey obtains forecasts of sales from executives responsible for such plans and forecasts, it is likely that at least in this survey most of the reported forecasts represents operating expectations.

thus have to turn to the presumption itself to examine its validity.

Why is it that businessmen's anticipations cannot actually be regressive even though businessmen do state them in the form of regressive forecasts?

Most analysts would argue that this presumption is valid for two reasons:

(1) that it is not reasonable for businessmen to have anticipations so consistently counter to prevailing trends, and (2) that if businessmen did have consistently regressive expectations (even though it were unreasonable for them to do so), such expectations could not then be of operational significance in their decision-making.¹

In this paper we shall not discuss the second claim, which is largely a reflection of the first. But let us turn to the first. Is it unreasonable for businessmen to have regressive anticipations?

We shall discuss different salients of this question in the following five sections. In Section 2 we first review the general rationals claimed for extrapolative (i.e., non-regressive) forecasts. We then proceed in Section 3 to examine the extent to which this rationalization of non-regressive forecasts is justified by the actual pattern of changes in individual firms' sales. In Section 4 we construct a forecasting model that is consistent with the cross-sectional evidence presented in Section 3 and then test some of its implications for forecasts of different variables. Since the explanations of individual firms' forecasts advanced in Sections 3 and 4 do not suffice in themselves to explain the behavior of aggregated forecasts, we then attempt in Section 5 to extend the anticipations model postulated in Section 4 in order to explain the regressiveness in aggregated forecasts.

¹ See for instance Hart and Kolin in [33].

2. The Rationale for Extrapolative Forecasts

In this section we shall both define more fully what is meant by "extrapolative" and "regressive" forecasts and review the usual justification made for assuming that forecasts are generally extrapolative. To do this we shall examine how we might expect rationally-formed forecasts to be related to other information, given different kinds of "other information." We shall follow the usage of most analysts of expectational surveys and define "rational" forecasts as forecasts which yield unbiased predictions.¹

The simplest case is undoubtedly that in which a close-to-minimal amount of information is available to the forecaster. Let us suppose that a forecaster knows merely the level of a variable in the period immediately prior to the period for which he wants to forecast. We shall use the symbol A_t to denote the actual level in quarter t of some variable which is of interest to a firm's management as of quarter $t-1$, and shall let ${}^s A_t$ denote the seasonally-adjusted value of that level.² If the difference between ${}^s A_t$ and ${}^s A_{t-1}$ is not related to the level of A in $t-1$, then clearly the best way to predict ${}^s A_t$ given only knowledge about

¹ Defining "rationality" in forecasting as unbiasedness corresponds to John Muth's definition [48] of rational expectations as forecasts which mirror the predictions of the economic model which is relevant in the context of the set of information available to a decision-maker.

² For convenience, we henceforth denote A in terms of a dating scheme which is strictly speaking relevant to the quarterly forecasts collected by the surveys investigated in [9]. The reasoning is, however, readily applicable to the forecasts collected by other surveys, such as the annual forecasts collected by the McGraw-Hill and SEC-Department of Commerce surveys -- in which the relevant comparison is of expected change over the forthcoming year, $(E_y/A_{y-1})-1$, to change over the previous year, $(A_{y-1}/A_{y-2})-1$ -- or the monthly forecasts collected by the IFO-Institut für Wirtschaftsforschung.

${}^sA_{t-1}$ is simply to extrapolate ${}^sA_{t-1}$. Businessmen presumably know more about influencing A than simply the information provided by their knowledge of ${}^sA_{t-1}$, and one would consequently expect their expectations to be more accurate than such simple extrapolations. Nevertheless, if one were to assume that both businessmen's expectations and simple extrapolations of current level are unbiased predictors of the forthcoming level of A, then it would obviously be reasonable to describe businessmen's anticipations as on the average extrapolative of level.

So long as we have information only on ${}^sA_{t-1}$ and on forecasts of sA_t we cannot say anything about whether forecasts are regressive or extrapolative (in the sense in which "extrapolative" is opposed to "regressive"). "Regressivity" is meant to denote reversal of trend, and is consequently opposed to an extrapolation of change. It thus can be defined only in terms of a relationship between forecast change and previous change. Specifically, let sE_t denote decision-makers' forecasts of sA_t , and let Δ_{t-1} denote change per period immediately prior to t-1. Then we can define regressive and extrapolative forecasts by specifying the conditionally expected value of $({}^sE_t - {}^sA_{t-1})$ given Δ_{t-1} as

$$(2.1) \quad \mathcal{E}({}^sE_t - {}^sA_{t-1} \mid \Delta_{t-1}) = h \Delta_{t-1}.$$

where h is some constant. We can describe forecasts as generally extrapolating past change if $h > 0$ or as generally regressive if $h < 0$. (If $h = 0$, the forecasts on the average merely extrapolate ${}^sA_{t-1}$, the level of the immediate past.)

We have up to this point assumed nothing which would enable us to put a priori limits on the range of values which h is likely to take on. Assuming, however, that decision-makers' expectations are "rational" in the sense of being unbiased predictors, it would seem reasonable to suppose that $\mathcal{E}({}^sE_t - {}^sA_{t-1} \mid \Delta_{t-1}) = \mathcal{E}({}^sA_t - {}^sA_{t-1} \mid \Delta_{t-1})$, so that we can as a result of this assumption predict h

from the relationship between $(^sA_t - ^sA_{t-1})$ and Δ_{t-1} . Let us suppose that

$$(2.2) \quad \mathcal{E} (^sA_t - ^sA_{t-1} \mid \Delta_{t-1}) = \gamma \Delta_{t-1}.$$

Then on the average $h = \gamma$ if decision-makers' expectations are truly "rational," so that if this assumption is valid we would expect observed values of h to be distributed around γ .

A cursory examination of any reasonably aggregated time series for sales, profits, or other operating variables indicates that intracyclical trends spanning periods which do not contain cyclical turning points are typical of the majority of forecasting situations. It would thus seem quite realistic to assume that generally $\gamma > 0$, so that "rational" forecasts would be extrapolative. Table 1 presents the results of a somewhat less cursory attempt to estimate γ for aggregate manufacturing sales. These results support the presumption that γ , where significant, is positive.¹ Many investigators, whether or not explicitly Bayesian in their methodological convictions, have consequently approached evidence on short-run expectations with a prior outlook heavily weighted in favor of the notion that h should be positive.

¹ We have not explicitly included a constant term in equations (2.1) and (2.2) which would incorporate the influence of longer-term trends not subsumed in $\gamma \Delta_{t-1}$. As Table 1 indicates, the decreasing significance of the estimated slope parameters as the interval over which Δ_{t-1} is defined is increased results in an increasingly large constant term, but only sufficiently so that the constant term is significantly non-zero at the 10 per cent level for Δ_{t-1} defined over four quarters. Given the level of the mean of $(^sA_t - ^sA_{t-1})$ -- 0.734 billion dollars for the fourth regression in Table 1 -- there would appear to be little potential for further increase in the constant term and so little evidence to substantiate an assumption of non-zero constant terms in these equations, at least for the range of definitions of Δ_{t-1} which we are considering.

TABLE I

PARAMETERS OF REGRESSIONS OF ($S_{A_t} - S_{A_{t-1}}$)
 ON DIFFERENT DEFINITIONS OF Δ_{t-1}^k
 FOR AGGREGATE QUARTERLY SHIPMENTS OF ALL
 MANUFACTURING INDUSTRY, 1948 To 1960

Definition of Δ_{t-1}	Slope Parameter	Constant Term	Correlation Coefficient	Number of Observations
$S_{A_{t-1}} - S_{A_{t-2}}$	0.302** (.137)	0.443 (.382)	0.298**	50
$S_{A_{t-1}} - S_{A_{t-3}}$.113 (.090)	.486 (.411)	.177	49
$S_{A_{t-1}} - S_{A_{t-4}}$.067 (.073)	.541 (.434)	.131	48
$S_{A_{t-1}} - S_{A_{t-5}}$	-.012 (.062)	.772* (.449)	-.028	47

Notes: Estimated standard errors associated with each regression parameter are listed in parentheses under the estimate of the parameter. The constant term has the dimension of billions of dollars. Data was obtained from revised monthly statistics published in the following sources: for 1948-50, Survey of Current Business, December 1953, p. 24; for 1951-60, Business Statistics for 1955, 1957, 1959, and 1961.

- * Significant at the 10 per cent level
- ** Significant at the 5 per cent level
- *** Significant at the 1 per cent level

In actual fact, most analysts of expectations' surveys have approached survey evidence with prior expectations for h that are more in the nature of a horizontal distribution over the interval $(0, \gamma)$ than in the form of a point estimate. In other words, their presumptions about h can be better stated in terms of a belief that

$$(2.3) \quad \mathcal{E} h = f \gamma, \Pr(f \leq F) = F, 0 \leq F \leq 1,$$

rather than in terms of a belief that $\mathcal{E} h = \gamma$. Such range estimates have sometimes arisen from a presumption that any collection of responses to a survey is made up of some unknown fraction (f) obtained from individuals who extrapolate change, with the remaining fraction $(1-f)$ obtained from individuals who merely extrapolate level. The Ferber-Hastay hypothesis - cf [22], [58], and [9, p. 250] - is an extension of this presumption. A second, perhaps more realistic source of such range estimates is the assumption that, aggregation problems aside, forecasters will consistently tend to underestimate the magnitude of change that actually occurs over forecasting horizons.¹ Both assumptions reflect a modification of the

¹ The reasons for such understatement have been amply delineated by Henri Theil [55, pp. 156-161]. It should be emphasized that such understatement is of the change that occurs after a forecast is made. In the context of the dating scheme used in the text, which is specifically relevant to the quarterly forecasts reported by surveys such as those conducted by the Railroad Shippers' Association, Dun and Bradstreet, or the Canadian Department of Labour, such understatement is thus of the change that occurs between $t-1$ and t . Because the forecasts are reported in terms of 4-quarter change, regressive (trend-reversing) forecasts can appear in the form of understatement of actual 4-quarter change. It is consequently important to differentiate clearly between understatement of the type discussed by Theil and this second (regressive) understatement, which is of change over a period including both the forecast horizon and several quarters prior to the making of the forecasts. Cf, however, the comments of Hart and Kolin in [33, p. 260].

"rationality" customarily associated with extrapolative forecasts, of course, for the rational expectations postulate would imply $f = 1$, not $0 \leq f \leq 1$. But "irrationality" should not necessarily be regarded as unreasonable.¹

In any case, even including this modification of the "rationality" postulate, the expectation of most analysts of survey responses is that h should be positive -- or, at the very least, non-negative. We shall proceed to derive the implications of this presumption for relationships which can be derived from data obtained from expectational surveys.

Because seasonally-adjusted figures are typically less familiar to businessmen than comparisons with the same period of the preceding year, most surveys ask businessmen to report expectations and past data in terms of 4-quarter change expressed in percentages. It is consequently necessary to derive the implications of equation (2.1) for 4-quarter change data.

¹ Edwin Mills [44] has remarked that the point of the rationality hypothesis advanced by Muth [48] is simply that it is not plausible to assume that forecasts made by decision-makers who are otherwise assumed to act rationally will continue to differ from experienced results in a mechanical and easily perceived fashion. It would thus seem irrational to find a persistence of the bias engendered by underestimation of change. Before adopting this conclusion, however, it may be useful to consider the pitfalls in relying excessively on a definition of "rationality" framed in terms of unbiasedness. Theil [55, section 3.3] has pointed out in analyzing postwar macroeconomic forecasts in Scandinavia and the Netherlands that a simple, mechanical correction for the systematic understatement of change in the forecasts leads to a considerably larger error variance for the "corrected" forecasts. As Theil [p. 76] has commented, "Although the elimination of a systematic forecasting error seems desirable in itself, it is far from sure whether we should advise the Governments of these countries to base their economic policies on the assumption that the actual changes are 50 to 100 per cent larger than the predictions of their forecasting agencies, even if we feel sure that the bias toward underestimation will remain unimpaired."

To do so, we shall set $\Delta_{t-1} = {}^s A_{t-1} - {}^s A_{t-4}$ and write equation (2.1)

in the form

$$(2.4) \quad {}^s E_t - {}^s A_{t-1} = h ({}^s A_{t-1} - {}^s A_{t-4}) + w_t,$$

where w_t is the error of the predictive equation (2.1). Adding $({}^s A_{t-1} - {}^s A_{t-4})$ to both sides of (2.4) and dividing both sides of the resultant equation by ${}^s A_{t-4}$, we obtain

$$(2.5) \quad \frac{{}^s E_t}{{}^s A_{t-4}} - 1 = (1 + h) \left[\frac{{}^s A_{t-1}}{{}^s A_{t-4}} - 1 \right] + u_t,$$

where $u_t = w_t / {}^s A_{t-4}$. Since ${}^s E_t / {}^s A_{t-4}$ can be written simply as E_t / A_{t-4} if the seasonal is stable, this transformation permits us to express forecasts in terms of non-seasonally-adjusted figures. Past change can likewise be expressed in terms of 4-quarter change indices by defining a constant α_t such that, denoting $[(A_{t-1} / A_{t-4}) - 1]$ by a_{t-1} ,

$$(2.6) \quad a_{t-1} = (1 + \alpha_t) \left[\frac{{}^s A_{t-1}}{{}^s A_{t-4}} - 1 \right]$$

If a trend exists in the series defined by successive values of A , then a_{t-1} will be greater than $[({}^s A_{t-1} / {}^s A_{t-4}) - 1]$. Indeed, if such a trend is linear, then because a_{t-1} spans one more quarter than does $[({}^s A_{t-1} / {}^s A_{t-4}) - 1]$ it will be roughly 4/3 as large. The average value of α_t would then be roughly

1/3.¹ Actually, of course, the fact that trends do not persist indefinitely will result in smaller values of α . As Table 2 indicates, estimates of α obtained from regressions of a_{t-1} on $[(^sA_{t-1}/^sA_{t-4}) - 1]$ for manufacturing sales between 1948 and 1960 are all well below 1/3, ranging in the neighborhood of 0.05, though the width of this neighborhood is rather wide.²

Denoting $[E_t / A_{t-4}) - 1]$ by e_t and substituting from (2.6), equation (2.5) can then be written as

$$(2.7) \quad e_t = \frac{1+h}{1+\alpha_t} a_{t-1} + u_t.$$

Using an estimate of the average value of α_t obtained from the regressions presented in Table 2, we can rewrite (2.7) as

$$(2.8) \quad e_t = B^* a_{t-1} + \epsilon_t^*,$$

where $B^* = (1+h)/(1+\alpha)$, and where $\epsilon_t^* = u_t$ plus any additional residual error resulting from constraining α_t to be a constant. (The reason for this particular choice of notation will be made clear later.) If $f = 1$ so that $h = \gamma$, then,

¹ The average value of α_t is actually somewhat greater than 1/3 in the presence of a linear trend. Rearranging (2.6), we obtain

$$\alpha_t = \frac{{}^sA_{t-4}}{{}^sA_{t-5}} \left[\frac{{}^sA_{t-1} - {}^sA_{t-5}}{{}^sA_{t-1} - {}^sA_{t-4}} \right] - 1$$

It is evident that the average ratio of $({}^sA_{t-1} - {}^sA_{t-5})$ to $({}^sA_{t-1} - {}^sA_{t-4})$ is 4/3 in the presence of a linear trend, but that $({}^sA_{t-4} / {}^sA_{t-5})$ is on the average equal to $(1 + q)$, where q is the average quarterly trend increment. The average value of α_t is hence $(4/3)(1 + q) - 1$. (We are indebted to Edward Foster for pointing out an error in our original analysis.)

² Only one-third of the 15 industry regression slope parameters are significantly different from unity even at the 10 per cent level; moreover, their standard deviation is .105, more than twice the average estimate of α .

TABLE 2

PARAMETERS OF REGRESSIONS OF a_{t-1} ON $[(^S A_{t-1} / ^S A_{t-4}) - 1]$
 FOR DATA ON AGGREGATE QUARTERLY SHIPMENTS IN MANUFACTURING
 INDUSTRIES, 1948 - 1960

	Slope Parameter	Estimate of \mathcal{L}	Constant Term	Correlation Coefficient	Number of Observations
A. SUMMARY OF DISTRIBUTION OF PARAMETERS OF REGRESSIONS COMPUTED FOR 15 INDUSTRIES					
Median	1.084	0.084	0.012	0.907	
Mean	1.041	.041	.012	.878	
Standard deviation	0.079		.005	.062	
Interquartile range	1.010-1.136		0.007-0.015	0.843-0.919	
Range	0.808-1.151		0.003-0.025	0.727-0.937	
B. PARAMETERS OF REGRESSION COMPUTED FOR AGGREGATE SHIPMENTS IN ALL INDUSTRIES					
ALL MANUFACTURING	1.075 (.071)	.075	.009 (.006)	.910	47

Notes: Estimated standard errors associated with each regression parameter are listed in parentheses under the estimate of the parameter. The constant term is, of course, dimensionless. Data sources are as in Table 1. All slope parameters are significantly different from zero. Significance of \mathcal{L} is measured from zero.

*Significant at 10 per cent level

**Significant at 5 per cent level

***Significant at 1 per cent level

since both α and γ represent extrapolations of $[(^S A_{t-1})-1]$ over bordering quarters, it is evident that α should equal γ in the presence of a recognizable linear trend extending over the five quarters between $t-5$ and t , thus making B^* unity. The slope of the regression of e_t on a_{t-1} should thus in this case equal unity.¹ If, on the other hand, $f = 0$ so that $h = 0$, the expected slope of the regression of e_t on a_{t-1} is $1/(1+\alpha)$, where α is the average value of α_t . Since the boundary discriminating between regressive and non-regressive forecasts is defined by the extrapolation of level corresponding to $h = 0$, the average slope of this boundary is thus somewhat less than unity.²

We have up to this point based our definition of alternative extrapolative hypotheses upon a direct estimate of α because of the intrinsic interest in α resulting from the close similarity of α and γ . However, in evaluating the parameters of a given regression of e_t on a_{t-1} , it may bias the results towards concluding that forecasts are regressive if we compare the estimated B^* to the value of $1/(1+\alpha)$ obtained from this direct estimate of α if there is

¹ It may be useful to specify the inclusion in (2.5) of a constant term representing the average net effects of longer-run trends not wholly subsumed in the short-run trends represented by h (a_{t-1}). A constant term in the regression of e_t on a_{t-1} may then be interpreted as an estimate of this net longer-run effect.

² Cf. [9, p. 248]. This discussion of the relationship between e_t and a_{t-1} is, of course, at a somewhat general level. As has been pointed out in [9, esp. fn. 16a], it will be useful in analyzing the regression of e_t on a_{t-1} in a particular sample to take explicitly into account the value of α_t determined by the average relationship between $^S A_{t-1}/^S A_{t-4}$ and $^S A_{t-4}/^S A_{t-5}$ in that sample.

attenuation in either estimate.¹ As an alternative, we can estimate the relationship between e_t and a_{t-1} for forecasts which extrapolate level by estimating $1/(1+\alpha)$ directly from regressions of $[(^S A_{t-1}/^S A_{t-4})-1]$ on a_{t-1} . By so doing, we can thus be sure that any attenuation in these estimates will bias comparison with slopes of regressions of e_t on a_{t-1} in favor of "verifying" that the forecasts are extrapolative.

¹ Attenuation resulting from covarying error terms and independent variables, perhaps partly because it is often called "regression bias," can be confused with the regressivity phenomenon. One specification of such bias is attributed to us by Muth [48, pp. 332-333] in the course of advancing an alternative explanation of underestimation to that advanced by Theil [55]. We should emphasize that the analysis there presented should not be construed as an explanation of regressivity. Indeed, it can be shown that Muth's explanation of understatement has no implications for the value of h and is hence a less general explanation of understatement than is Theil's. Let Δ_t^e be expected change, Δ_t be realized change, Δ_{t-1} be non-overlapping but contiguous previous change, and u_t , v_t , and w_t be random variables. Muth's "rational expectations" hypothesis is that $\Delta_t = g^{-1}\Delta_t^e + v_t$, $\mathcal{E} v_t = \mathcal{E} v_t \Delta_t^e = 0$, $g = 1$, which implies $\Delta_t^e = g \Delta_t + u_t$, where $u_t = -gv_t$, so that, using \hat{g} to denote the direct least squares estimate of g , we have $\text{plim } \hat{g} < g = 1$. Thus even though the forecasts Δ_t^e are unbiased, the observed relationship between Δ_t^e and Δ_t indicates underestimation of change. As in (2.2) above, let $\Delta_t = \gamma \Delta_{t-1} + w_t$, $\mathcal{E} w_t = \mathcal{E} w_t \Delta_{t-1} = 0$. Then $\Delta_t^e = g \gamma \Delta_{t-1} + gw_t + u_t$, so that, again using $\hat{g}\hat{\gamma}$ to denote the least squares estimate of the coefficient of Δ_{t-1} , we have $\text{plim } \hat{g}\hat{\gamma} = g \gamma + g[\text{cov}(w_t \Delta_{t-1})/\text{var } \Delta_{t-1}] + \text{cov}(u_t \Delta_{t-1})/\text{var } \Delta_{t-1}$. But the first covariance term is of course zero, and $\text{cov}(u_t \Delta_{t-1}) = -\text{cov}(v_t \Delta_{t-1}) = 0$ since Δ_{t-1} is known when the forecast Δ_t^e is made and since any systematic effect of any known variable on Δ_t must be incorporated in Δ_t^e under the Muth rational expectations hypothesis. Therefore $\text{plim } \hat{g}\hat{\gamma} = g \gamma = \gamma$. In other words, the probability limit of the slope parameter of the regression of Δ_t^e on Δ_{t-1} , which we have denoted by h , is equal to γ . Thus $f = 1$ if expectations are "rational" even though then $\text{plim } \hat{g} < 1$. Thus $\text{plim } \hat{g} < 1$ (implying underestimation of current change) is quite consistent with accurate extrapolation of past change (or in current terminology with $f = 1$). In other words, even though underextrapolation of past change (meaning $f < 1$) will imply understatement of current change, since then $\text{plim } \hat{g}\hat{\gamma} = h = f \gamma < \gamma$, the converse is not true. Underextrapolation of past change is a quite different phenomenon from understatement of current change.

Direct estimates of $1/(1+\alpha)$ are presented in Table 3. The slope parameters of the first regressions are estimates of $1/(1+\alpha)$. These estimates provide values of one boundary of the subjective prior range (for Bayesians) of B^* corresponding to the range of h defined in (2.3), namely, that for which $f = 0$. Table 3 also presents estimates of $(1+\gamma)/(1+\alpha)$, obtained from the slope parameters of regressions of a_t on a_{t-1} . This second group of slope parameters provides estimates of the value of B^* corresponding to that boundary of the range defined in (2.3) for which $f = 1$.

The mean estimate of $1/(1+\alpha)$ for the 15 industries in Table 3 is 0.741; the estimate obtained from a regression on aggregate shipments for all manufacturing industry is 0.770. Both these estimates are substantially below the value of $1/(1+\alpha)$ obtained from the direct estimates of α presented in Table 2; they imply a value of α on the order of 0.33 rather than 0.05. Nevertheless, even using this lower estimate of α to define the value of B^* corresponding to $f = 0$, all the available evidence obtained from surveys of businessmen's expectations generally yields substantially lower estimates of B^* . Estimates of B^* for several surveys are presented in Table 4. Since $B^* = (1+h)/(1+\alpha)$, we can substitute different estimates of α into this definition of B^* to obtain implied values of h . As Table 4 indicates, the implied values of h are negative, and substantially so. The forecasts are thus appreciably regressive.

Table 3
 PARAMETERS OF REGRESSIONS ON a_{t-1} OF TWO DEFINITIONS OF
 OVERLAPPING CHANGE FOR DATA ON AGGREGATE QUARTERLY
 SHIPMENTS IN MANUFACTURING INDUSTRIES, 1948 - 1960

Industry	Regression of $[(^s A_{t-1} / ^s A_{t-4}) - 1]$ on a_{t-1}			Regression of a_t on a_{t-1}				
	Slope Parameter	Constant Term	Correlation Coefficient	No. of obs.	Slope Parameter	Constant Term	Correlation Coefficient	No. of obs.
Primary metals	0.654 (.090)	0.010 (.025)	0.727	47	0.487 (.127)	0.031 (.036)	0.486	47
Fabricated metal products	.754 (.090)	.000 (.008)	.907	47	.790 (.088)	.014 (.014)	.791	47
Machinery (incl. electrical)	.768 (.041)	-.003 (.006)	.937	47	.846 (.077)	.012 (.012)	.845	47
Transportation equipment	.721 (.076)	.004 (.016)	.808	47	.614 (.114)	.020 (.024)	.615	47
Lumber and furniture	.770 (.042)	-.001 (.007)	.935	47	.836 (.080)	.004 (.013)	.836	47
Stone, clay, and glass	.756 (.047)	-.001 (.001)	.919	47	.823 (.083)	.011 (.012)	.822	47
Other durables	.767 (.048)	.000 (.007)	.917	47	.782 (.094)	.013 (.014)	.785	43
Food and beverage products	.704 (.065)	.001 (.004)	.843	47	.625 (.112)	.012 (.007)	.628	47
Tobacco	.698 (.086)	.000 (.005)	.763	47	.626 (.114)	.012 (.007)	.622	47
Textile and related products	.743 (.048)	.000 (.007)	.913	47	.776 (.088)	.007 (.013)	.787	47
Paper	.748 (.043)	.000 (.006)	.928	47	.830 (.075)	.014 (.010)	.847	47
Chemicals	.771 (.049)	-.002 (.006)	.914	47	.814 (.083)	.012 (.010)	.817	47
Petroleum and coal products	.743 (.058)	.000 (.006)	.879	47	.770 (.088)	.016 (.009)	.785	47
Rubber	.776 (.053)	-.002 (.010)	.903	47	.794 (.088)	.013 (.017)	.793	47
Other non-durables	.743 (.058)	.000 (.005)	.881	47	.787 (.090)	.005 (.007)	.785	47
ALL MANUFACTURING	.770 (.051)	-.001 (.005)	.910	47	.775 (.092)	.010 (.010)	.775	47

TABLE 4
ESTIMATES OF B* AND f OBTAINED
FROM DIFFERENT SURVEYS

Survey	Period	Estimate of B*	Implied estimate of f with	
			$\lambda = .05$	$\lambda = .33$
Advisory Board of Railroad Shippers	1927I-1939IV	.48	-.50	-.39
Canadian Employment Forecast Survey	1949I-1956III	.62	-.45	-.17
Dun and Bradstreet	August 1949	.46	-.52	-.41

Notes: The estimates of B* are slope parameters of regressions of e_t on a_{t-1} for aggregate data on railroad shipments and employment for all manufacturing industry in the first two surveys. The data is obtained from [32, Table A-1] and [35, Chart 2]. For the Dun and Bradstreet survey, the estimate of B* is the slope parameter of a regression of e_t on a_{t-1} for data on average changes in sales in each major manufacturing industry, and is obtained from [9, equation 9].

These results are far out of line with most analysts' a priori notions. Table 3 also presents slope parameters of regressions of a_t on a_{t-1} , which we shall denote by B. Since B corresponds to the value of B^* defined by $f = 1$, we have $B = (1+\gamma)/(1+\alpha)$. Using the value of α estimated in the first regression of Table 3 (i.e., $\alpha = 0.33$), we obtain for all manufacturing industry the implied estimate $\gamma = 0.034$. For the same value of α , Table 4 indicates that h is on the order of -0.40 , thus implying that f is on the order of -12 . Needless to say, this value of f is far outside the a priori range specified in (2.3).

For four industries -- primary metals, transportation equipment, food and beverage products, and tobacco -- the estimated value of $1/(1+\alpha)$ is less than the estimated value of B, thus implying a negative value of γ for these four industries. The tendency to regressivity present in aggregate sales of these four industries is clearly shown in Table 5, which presents parameters of two regressions analogous to those computed in Table 1. Aside from the primary metals industry, however, whose very low estimate of B is probably due to the recurring importance of strikes in the steel industry, all industries are characterized by values of B which are substantially larger than the values of B^* presented in Table 4. Moreover, Table 5 indicates that there is little difference between direct and indirect estimates of γ .

Notes to Tables 3 and 5:

Estimated standard errors associated with each regression parameter are listed in parentheses under the estimate of the parameter. The constant term is dimensionless. Data sources are as in Table 1. All slope parameters and correlation coefficients are significantly different from zero in Table 3.

- * Significant at 10 per cent level
- ** Significant at 5 per cent level
- *** Significant at 1 per cent level

Table 5
 PARAMETERS OF REGRESSIONS OF $[(^S A_{t-1}/^S A_{t-1})-1]$ ON TWO DEFINITIONS
 OF THE PAST RATE OF CHANGE FOR DATA ON AGGREGATE QUARTERLY
 SHIPMENTS IN MANUFACTURING INDUSTRIES, 1948 - 1960

Industry	Regression of $[(^S A_{t-1}/^S A_{t-5})-1]$ on $[(^S A_{t-1}/^S A_{t-1})-1]$			Regression of $[(^S A_{t-1}/^S A_{t-4})-1]$ on $[(^S A_{t-1}/^S A_{t-1})-1]$				
	Slope Parameter	Constant Term	Correlation Coefficient	Number of obs.	Slope Parameter	Constant Term	Correlation Coefficient	No. of obs.
Primary metals	-0.194** (.080)	0.031 (.023)	-.331**	47	-.145 (.091)	0.024 (.023)	-.223	48
Fabricated metal products	.012 (.056)	.014* (.008)	.032	47	.070 (.067)	.010 (.008)	.150	48
Machinery (incl.electrical)	.057 (.046)	.014* (.007)	.177	47	.122** (.056)	.009 (.007)	.296**	48
Transportation equipment	-.129* (.075)	.020 (.015)	-.244*	47	-.082 (.084)	.016 (.015)	-.139	48
Lumber and furniture	.051 (.050)	.003 (.008)	.146	47	.120* (.061)	.000 (.008)	.273*	48
Stone, clay, and glass	.046 (.054)	.012 (.008)	.124	47	.086 (.067)	.009 (.008)	.183	47
Other durables	-.004 (.056)	.013 (.008)	-.012	43	.054 (.065)	.011 (.008)	.125	44
Food and beverage products	-.070 (.074)	.010* (.005)	-.139	46	-.002 (.090)	.008 (.005)	-.004	47
Tobacco	-.072 (.093)	.012** (.005)	-.112	47	-.141 (.096)	.013*** (.004)	-.207	48
Textile and related products	.026 (.053)	.005 (.003)	.070	47	.103* (.062)	.003 (.007)	.232	48
Paper	.049 (.047)	.015** (.006)	.150	47	.107* (.060)	.010 (.006)	.247*	48
Chemicals	.024 (.052)	.014** (.006)	.067	47	.065 (.064)	.010 (.006)	.143	48
Petroleum and coal products	.009 (.056)	.017*** (.006)	.024	46	.013 (.077)	.014** (.006)	.024	47
Rubber	.002 (.056)	.015 (.010)	.006	47	.039 (.064)	.013 (.010)	.087	47
Other non-durables	.045 (.069)	.004 (.006)	.095	47	.019 (.081)	.006 (.005)	.034	48
ALL MANUFACTURING	-.005 (.057)	.011* (.006)	-.013	47	.065 (.067)	.007 (.006)	.139	48

The fact that responses to most surveys of businessmen's anticipations exhibit significant regressivity -- in the form of regression slope estimates of B^* which are both appreciably less than unity and imply substantially negative values of f -- is a fact which is obviously irreconcilable with the two presumptions typically held by analysts: namely, that businessmen's expectations should on the whole be "rational" and that "rational" forecasts should extrapolate past change since $\gamma > 0$. We will not discuss the presumption that businessmen's expectations must be "rational" if they are to be useful to the businessman in his operating decisions, even though it is evident that this presumption must be qualified if it is to be descriptive.¹ But even accepting this last presumption, is it true that expectations must be extrapolative in order to be "rational"? Rewording this question, even accepting the presumption that $E h = f \gamma$ and that $0 \leq f \leq 1$, is it true that - is non-negative for the series being forecasted by businessmen?

The italics in the last sentence point to a crucial fact: namely, that the analyses which have been presented in this section have all been concerned with aggregated data. We shall turn in the next section to an analysis of the series which are actually predicted by businessmen. As we shall see, the presumption that γ is positive for series being predicted does not hold up very well when applied to data on individual firms' sales.²

¹ Cf. however, Albert Hart's remarks in [32] and [33, pp. 260-261].

² Actually, as Table 5 has indicated, it is evident that this presumption is not always valid even when applied to aggregated data. However, the exceptions occur only at a somewhat disaggregated level, and suggest only that they are indeed exceptions. Peter Pashigian [51] has suggested that aggregate sales of all manufacturing industry are regressive, basing this hypothesis on a negative though non-significant correlation between the ratio of annual sales to sales in January of that year and the ratio of January sales to sales in the preceding November. However, this result is not very suggestive, both because of the bias toward negative correlation introduced by the common variable in the two ratios and because of the inevitable concentration of noise in monthly data relative to that in annual data.

3. Actual patterns of change in individual firms' sales

Reliance on aggregate data in forming judgements about businessmen's forecasts may be almost as misleading as it is convenient. For it can very quickly be shown that there is no necessary relationship between the pattern of changes experienced by individual firms and that evidenced by aggregate data except under quite special circumstances. Let us denote the relation between successive 4-quarter changes in an individual firm's sales by the following expression:¹

$$(3.1) \quad a_t^i = \beta_i a_{t-1}^i + K_i + \theta_{it}, \quad t = 1, \dots, T.$$

The residual term θ_{it} may be non-zero at any given time as a result of factors operating at that time. If these factors are identifiable, then $\sum \theta_{it}$ will be non-zero as a consequence. In the absence of such identification, we shall simply assume that $\sum \theta_{it} = 0$. Aggregating (3.1) to obtain the mean 4-quarter change in an industry or group of industries,² we obtain

$$(3.2) \quad \bar{a}_t = \frac{1}{N} \sum_i \beta_i a_{t-1}^i + \bar{K} + \bar{\theta}_t, \quad t = 1, \dots, T.$$

For aggregate data, we can estimate the regression

$$(3.3) \quad \bar{a}_t = B \bar{a}_{t-1} + K + \epsilon_t, \quad \sum \epsilon_t = 0,$$

as was done in Section 2. Given information about B, we can identify β_i only under the highly restrictive assumption that all β_i are identical. Clearly, if all β_i are the same for all firms in an industry or group of industries, then $\beta_i = B$, $K = \bar{K} = K_i$, and $\bar{\theta}_t = \epsilon_t$. In view of the evidence presented in the preceding section, there would thus be very little basis for businessmen

Footnotes 1 and 2: see next page

to have "rational" expectations as regressive as is indicated by the regression slopes presented in Table 4.

If this assumption of intra-industry homogeneity is not made, then it is not possible to place any limitations on either the mean and range of β_i

¹ We need to differentiate between a number of different types of regressions in this paper, and it will be convenient to adopt a reasonably precise notation in doing so. Unfortunately the price of precision is a certain amount of complexity. For ease of reference, we therefore append a list of the symbols used to differentiate the various types of regressions of a_t and e_t on a_{t-1} in the following table:

TYPE OF REGRESSION	DEPENDENT VARIABLE	SLOPE PARAMETER	CONSTANT TERM	ERROR OR RESIDUAL	RUNNING SUBSCRIPT
Temporal regressions for individual firms	a_t^i	β_i	K_i	θ_{it}	t
	e_t^i	β_i^*	K_i^*	θ_{it}^*	t
Cross-sectional regressions	a_t^i	b_t	k_t	η_{it}	i
	e_t^i	b_t^*	k_t^*	η_{it}^*	i
Aggregate regressions	\bar{a}_t	B	K	ϵ_t	t
	\bar{e}_t	B^*	K^*	ϵ_t^*	t

As can be seen from the table, upper case letters denote parameters of aggregate regressions, lower case Latin letters denote parameters of cross-sectional regressions, and lower case Greek letters denote parameters of temporal regressions for individual firms. The i subscript (superscript on equation variables) denotes the i th individual firm; the t subscript denotes quarter t . Asterisks denote parameters of regressions in which the dependent variable is a businessman's expectation.

² In what follows we assume for convenience that aggregate 4-quarter change can be identified with the mean change for firms in the industry. These two quantities are, of course, not strictly identical, since an unweighted average of ratios need not coincide with the ratio of the corresponding aggregates. Nevertheless, the effects of the failure of this identity can be expected to be both too small and too unsystematic to affect our argument.

for individual firms or on the relationship between a_t^i and a_{t-1}^i for a particular cross-sectional sample. Given that (3.1) describes the average relationship over time between a_t^i and a_{t-1}^i for each firm, then at any time we can specify a cross-sectional regression of the form

$$(3.4) \quad a_t^i = b_t a_{t-1}^i + k_t + \eta_{it}, \quad \sum_i \eta_{it} = 0,$$

where the values of both the slope parameter b_t and the constant term k_t are dependent on the range of β_i for the firms in the sample and on the particular realizations of the residual terms θ_{it} . Since it follows from (3.4) that

$$(3.5) \quad \bar{a}_t = b_t \bar{a}_{t-1} + k_t,$$

then (3.3) and (3.5) imply only that

$$(3.6) \quad b_t \bar{a}_{t-1} + k_t = B \bar{a}_{t-1} + K + \epsilon_t$$

so that

$$(3.7) \quad k_t = (B - b_t) \bar{a}_{t-1} + K + \epsilon_t.$$

The only implication for cross-sectional regressions of the temporal relation between successive mean changes in an industry or group of industries is thus that k_t , the constant term of the cross-sectional regression, be correlated with \bar{a}_{t-1} up to a constant term K and a random component ϵ_t . It is thus quite possible for both b_t and β_i to be consistently less than $1/(1+\alpha)$, the boundary between regressive and non-regressive values of slope parameters, and for B to be greater than $1/(1+\alpha)$.

If the restrictive assumption of industry homogeneity holds, then of course much more precise statements can be made. Specifically, not only do all $\beta_i = \bar{\beta}_i = B$, but also all $b_t = \beta_i$ so that $(k_t + \eta_{it}) = \theta_{it}$ and $b_t = B$. It is for this reason that the homogeneity assumption is so convenient. For

if it were true -- as often implicitly assumed -- that typical distributions of changes in individual firms' sales around the industry mean were not very wide, so that most firms' sales moved in roughly the same pattern as that of total industry sales, then "rational" expectations would clearly mirror a series whose behavior over time would typically be non-regressive.

In actuality, typical distributions of changes in individual firms' sales are anything but narrow. As indicated in [7], the movement of individual firms' sales bears little relationship to changes in the industry aggregate. For one industry, for instance, -- the fabricated metal products industry, by no means the most heterogeneous industry listed in Table 3 -- the median value for 51 firms of the coefficient of determination of the regression of the quarterly rate of change in the individual firms' sales on the quarterly rate of change of total industry sales was but 0.07. The consequently wide scatter of changes in sales for individual firms in each industry is shown in Chart 1, which presents frequency distributions of 4-quarter changes in sales reported by individual firms in two typical Dun and Bradstreet survey industry samples.¹ Thus, for instance, actual changes in sales reported to the October 1949 Dun and Bradstreet survey by 41 respondents in the machinery industry ranged from a decrease of 50 per cent to an increase of 30 per cent, with an interquartile range of 17 per cent. For the other sample portrayed in Chart 1 the interquartile range is 15 per cent.

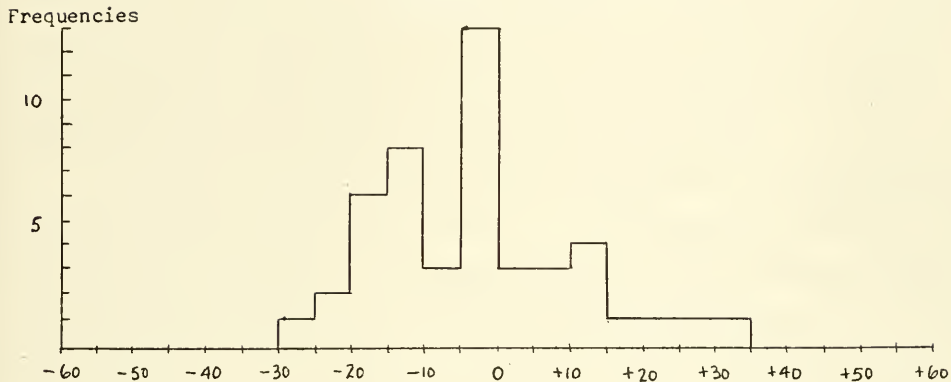
The mere fact that it is not accurate to postulate homogeneous industries

¹ That the dispersion of these two distributions is typical of Dun and Bradstreet industry samples is indicated by the fact that the range of standard deviations of a_{t-1} for all industry samples of August, September, October and November of 1949 is from 4.7 per cent to 47.4 per cent. The median of this distribution of standard deviations is 17.4 per cent, with an interquartile range of 10.9 per cent. The standard deviations of the two distributions presented in Chart 1 are 17.4 per cent and 15.9 per cent.

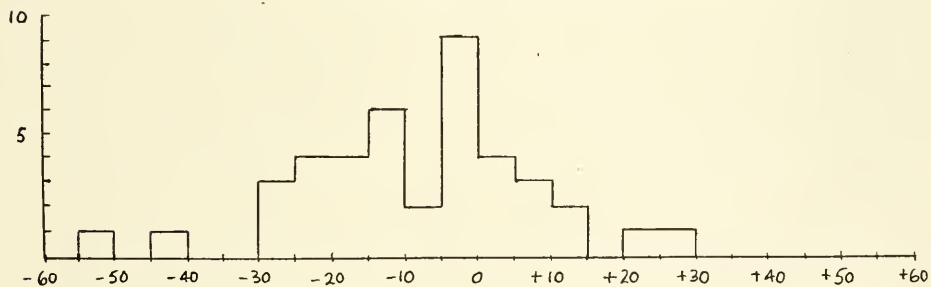
CHART 1

FREQUENCY DISTRIBUTIONS OF a_{t-1}
IN DIFFERENT SAMPLES

1. Chemicals (September 1949)



2. Machinery (October 1949)



Source: Dun and Bradstreet surveys

does not by itself mean that one should discard the assumption, of course. As Robert Solow [54] has pointed out, good theorizing lies not so much in choosing assumptions which are accurate as in choosing assumptions whose inaccuracy does not seriously affect the results of the analysis for which they are made. The importance of the homogeneity assumption lies not so much in the fact that it is substantially incorrect as in the fact that it can significantly bias an analysis of how businessmen form their operating expectations.

This can be seen by considering the implications of the wide dispersion in firms' experiences portrayed in Chart 1. It would seem most unlikely -- just to take the first example -- that almost a quarter of the respondents sampled in the machinery industry could long sustain annual sales decreases of more than 20 per cent, and just somewhat less likely that another quarter could continue to increase their own sales while total industry sales declined. Businesses do fail, of course, and some others are unusually successful. But the proportion of the business population which fails is not much more than 0.4 per cent in any year,¹

¹ This figure represents the postwar average business failure rate. Some change has occurred in this proportion; indeed, between 1946-1949 and 1956-1959 it has increased roughly by a factor of 4. In 1946-1949, the period spanning the data in our sample, commercial and industrial failures per year never ranged above 0.34 per cent of the total business population. See Economic Report of the President for 1960, Table D-64. Because businesses within the coverage of the SEC sample are by no means necessarily representative of all manufacturing enterprises, it is necessary to use the global indices of business failures with some caution in the present context. A priori, one would expect business failures to be less frequent among listed corporations. Hart and Prais [30, Table 7] have presented some evidence that is relevant to this matter, indicating that roughly 12 per cent of the firms listed on the London Stock Exchange in 1939 ceased to be listed on the exchange over the following eleven years. On the basis of a further analysis of a sample of 400 of the firms removed from the Stock Exchange list between 1885 and 1950, they concluded [30, fn. p. 165] that roughly 37 per cent of such "deaths" are due to voluntary or involuntary liquidation, 34 per cent to mergers, and 27 per cent merely to the loss of active quotation. It would consequently seem not inaccurate to say that not more than 4 per cent of the firms listed in 1939 were forced into liquidation during the ensuing decade.

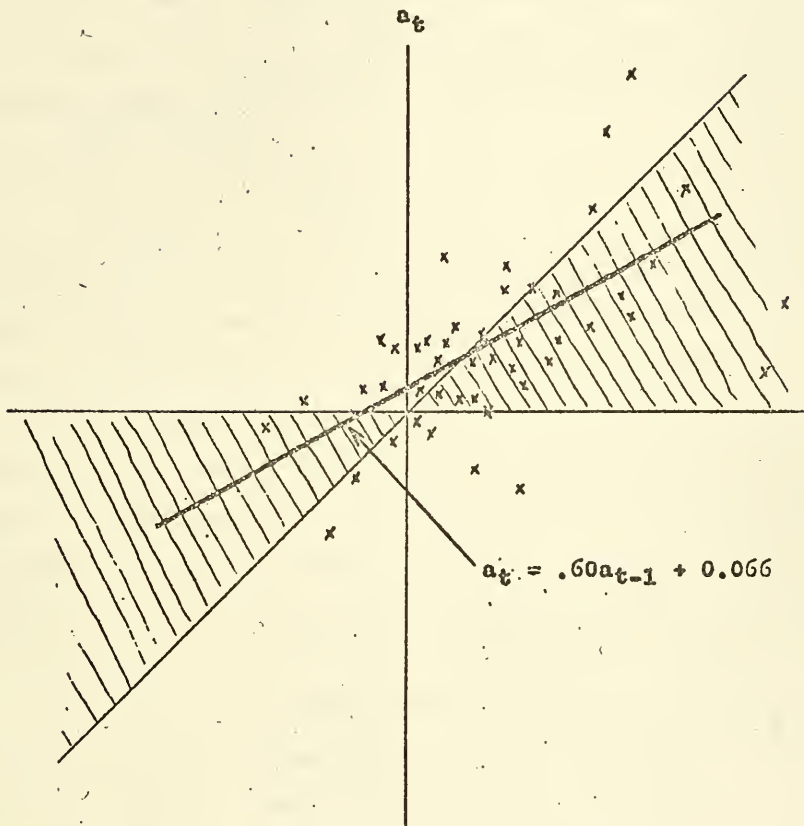
and the proportion of firms which consistently do appreciably better than average is probably not a very great deal larger. Rather than expecting such increases or decreases to continue, it would instead seem reasonable to expect that, for many of the firms outside the interquartile range, their relatively extreme changes in sales over the previous year resulted from unusual events such as a major interruption in production, a special sales promotion or innovation, or a cumulation of less spectacular developments for the firm or for its customers and competitors. To the extent that such circumstances were the result either of random, non-repetitive occurrences or simply of differences between the seasonal timing of firm and industry sales, their influence would tend to disappear in ensuing periods, thus causing sales to move back to a more "normal" level and so bringing about a reversal in the direction in which the firms' sales moved.¹ In other words, a substantial portion of changes in firms' sales may be transitory in nature, thus inducing subsequent actual movements of their sales to be regressive as the influence of temporary shocks declines.

An analysis of actual sales data seems to bear this out. Chart 2 presents a scatter diagram of observations of a_t and a_{t-1} obtained from SEC Quarterly Sales Reports for a group of firms classified in the apparel and textile mill products industries for a period when the quarter denoted by t is 1947III. As in [9, Charts 3 and 4], the 45° line is approximately the boundary between trend-continuing and trend-reversing sales changes, with the shaded area between the 45° line and the abscissa containing those trend-reversing forecasts

¹ The influence of random shocks is particularly important in a variable such as 4-quarter change, both because (since a quarter is a relatively short length of time) there will be less chance for offsetting random shocks to occur during the period and because the random variability of 4-quarter change will be greater than that of quarterly sales since both numerator and denominator are subject to such shocks.

Chart 2

RELATION BETWEEN a_t AND a_{t-1} FOR
INDIVIDUAL FIRMS IN THE APPAREL AND
TEXTILE MILL PRODUCTS INDUSTRIES
($t = 1947$ III)



Scale:

-1.00 -0.80 -0.60 -0.40 -0.20 0 .20 .40 .60 .80 1.00

Source: SEC Survey of American Listed Corporations --Quarterly
Sales Data

which are "classically" regressive in the sense of merely turning back towards a_{t-1} , and with the northwest and southeast quadrants containing those observations which are trend-reversing to the point of reversing the direction of 4-quarter change. Just under two-thirds of the observations plotted in Chart 2 can be classified as regressive; the impression thus engendered of a tendency for actual sales changes to be regressive is further confirmed by the regression of a_t on a_{t-1} for this sample:

$$(3.8) \quad \mathcal{E}(a_t | a_{t-1}) = \underset{(.20)}{0.60} (a_{t-1}) + 0.055 \quad (r = .65)$$

The slope parameter of this regression is substantially below the value of B presented in Table 3 for textiles and related products, even though the ratio of \bar{a}_t to \bar{a}_{t-1} for the sample is 0.85 and thus higher than that value of B.

Similar cross-sectional evidence is presented in Tables 6 and 7, for all of the sales data for individual manufacturing firms available from the SEC sample.¹ As Part A of Table 6 indicates, all but one of the slope parameters of regressions of a_t^i on a_{t-1}^i computed over all firms in each cross-section are appreciably less than the values of both B and $1/(1+\alpha)$ presented in Table 3 for all manufacturing. It is of course conceivable that the relatively low values of b_t obtained from the first regression arise in part from attenuation. For comparison, parameters of regressions of 1-quarter change on non-overlapping past change are presented in Parts B and C of Table 6. All but two of the slope parameters presented in Part C -- direct cross-sectional estimates of γ at the firm level, although for change expressed as ratios -- are significantly negative.

¹ The sample consists of data on quarterly sales for 1141 identified manufacturing firms, which we have here classified into 20 industry groups according with the 2-digit SIC classification. Unfortunately the data is available only from 1946 through the first quarter of 1949, a time span which is both close to World War II and short.

TABLE 6

PARAMETERS OF AGGREGATE REGRESSIONS COMPUTED
OVER ALL FIRMS IN EACH CROSS-SECTION

DATE (=t) OF CROSS-SECTION	SLOPE PARAMETER (b_t)	0_{b_t}	CONSTANT TERM(k_t)	0_{b_t}	CORRELATION COEFFICIENT	NUMBER OF FIRMS
A. REGRESSIONS OF a_t ON a_{t-1}						
1947 II	.311	.340	.104	.735	.034	721
1947 III	.019***	.001	.327***	.022	.505***	905
1947 IV	.379***	.022	.131***	.019	.491***	923
1948 I	.370***	.022	.081***	.015	.488***	914
1948 II	.478***	.026	.069***	.013	.527***	914
1948 III	.532***	.033	.096***	.015	.470***	916
1948 IV	.792***	.027	-.023*	.014	.697***	921
1949 I	.218**	.096	.045	.058	.075**	894
B. REGRESSIONS OF ($\frac{S}{A_t} / \frac{S}{A_{t-1}}$) -1 ON ($\frac{S}{A_{t-1}} / \frac{S}{A_{t-4}}$) -1						
1947 II	-.016***	.006	.113***	.012	-.106***	727
1947 III	.000	.001	.028**	.011	.011	909
1947 IV	-.064***	.018	.188***	.015	-.116***	926
1948 I	-.036**	.018	.007	.012	-.065**	919
1948 II	.020	.026	.076***	.013	.026	916
1948 III	-.076***	.028	.052***	.013	-.090***	917
1948 IV	-.021	.021	.098***	.011	-.034	925
1949 I	-.036	.072	.052	.044	.017	895
C. REGRESSIONS OF ($\frac{S}{A_t} / \frac{S}{A_{t-1}}$) -1 ON ($\frac{S}{A_{t-1}} / \frac{S}{A_{t-4}}$) -1						
1947 II	-.000	.001	.099	.011	-.016	914
1947 III	-.085***	.015	.054***	.012	-.183***	939
1947 IV	-.219***	.023	.199***	.014	-.300***	934
1948 I	-.092***	.013	.020*	.011	-.232***	931
1948 II	-.246***	.030	.106***	.012	-.263***	922
1948 III	-.196***	.022	.076***	.012	-.284***	924
1948 IV	-.146***	.017	.103***	.010	-.271***	932
1949 I	-.116*	.069	-.034	.044	-.055*	904

Notes: Data was obtained from Survey of American Listed Corporations: Quarterly Sales Data, Securities and Exchange Commission, multilith, 1947 through 1949.

- * Significant at 10 per cent level
- ** Significant at 5 per cent level
- *** Significant at 1 per cent level

Four of the slope parameters presented in Part B are likewise significantly negative. None are significantly positive. This evidence for cross-sectional regressivity is further substantiated by the distributions of slope parameters of regressions computed at the industry level which are presented in Table 7. More than 75 per cent of the 168 slope parameters of industry cross-sectional regressions of a_t^i on a_{t-1}^i are less than the average value of $1/(1+\alpha)$ for the industries tabulated in Table 3. Likewise, more than 50 per cent and more than 75 per cent of the slope parameters of the industry cross-sectional regressions for non-overlapping change are negative. There would thus seem to be ample evidence of cross-sectional regressivity among individual firms' sales.

Since it is possible (though unlikely, in view of scatters such as is presented in Chart 2) that this cross-sectional regressivity arises from the effect of outlying observations, it is of interest to examine the slope parameters of similar regressions computed from successive data obtained for each firm. Table 8 presents distributions of the slope parameters of each regression computed for each of 850 firms for which sales data was available for 12 or more of the 13 quarters covered by the SEC sample. The slope parameters provide estimates of β_1 and γ for each firm. As is evident from the table, the distribution of β_1 is wide, indicating substantial interfirm differences in the actual regressivity of sales. Given the small number of degrees of freedom in each regression (a maximum of 6 for the first two and of 7 for the third) there is little point in attempting to generalize very far from this evidence. In addition, the slope parameters of these regressions will be biased toward zero if the regression model specification is accurate.¹ Nevertheless, it is interesting to note that these firm regressions seem to exhibit more regressivity than is indicated by the slope of the cross-sectional regressions. The average estimated value of β_1

¹ Cf. Hurwicz [40a].

TABLE 7

DISTRIBUTIONS OF SLOPE PARAMETERS OF
INDUSTRY CROSS-SECTIONAL REGRESSIONS
COMPUTED IN EACH PERIOD

DATE (=t) FOR CROSS-SECTION	NI	REGRESSIONS OF a_t ON a_{t-1}		REGRESSIONS OF $(\frac{S_{A_t}}{S_{A_{t-1}}})-1$ ON a_{t-1}		REGRESSIONS OF $(\frac{S_{A_t}}{S_{A_{t-1}}})-1$ ON $(\frac{S_{A_{t-1}}}{S_{A_{t-4}}})-1$	
		(1)	(2)	(1)	(2)	(1)	(2)
1947 II	20	.227		-.084		-.285	
		.383	13.1	-.016	-.040	-.092	-.201
		.734	(55.4)	.026	(.320)	.000	(.403)
1947 III	20	.176		-.337		-.292	
		.357	.358	-.093	-.165	-.137	-.184
		.590	(.244)	.000	(.273)	.002	(.390)
1947 IV	20	.242		-.234		-.764	
		.337	.502	-.096	-.182	-.294	-.440
		.583	(.675)	.006	(.309)	-.170	(.373)
1948 I	20	.370		-.067		-.396	
		.510	.532	.008	.020	-.193	-.253
		.716	(.305)	.060	(.257)	-.076	(.223)
1948 II	20	.426		-.102		-.603	
		.576	.676	-.027	.160	-.355	-.517
		.876	(.403)	.147	(.686)	-.110	(.730)
1948 III	20	.403		-.128		-.430	
		.542	.534	-.058	-.019	-.288	-.337
		.741	(.235)	.126	(.283)	-.090	(.385)
1948 IV	20	.342		-.333		-.561	
		.448	.509	-.153	-.191	-.443	-.404
		.714	(.249)	-.005	(.353)	-.061	(.374)
1949 I	20	.218		-.167		-.405	
		.482	.500	-.016	-.074	-.279	-.345
		.884	(.686)	.124	(.599)	(.099)	(.387)

Table 7 continued

	NI	(1)	(2)	(1)	(2)	(1)	(2)
All Periods	168	.253		-.165		-.438	
		.465	2.01	-.026	-.060	-.236	-.319
		.699	(19.5)	.054	(.424)	-.056	(.426)

Column headings: NI = Number of industry slope parameters in distribution.

(1) = First quartile, median, and third quartile of distribution of parameters.

(2) = Mean and standard deviation of distribution of parameters.

TABLE 8

DISTRIBUTIONS OF SLOPE PARAMETERS OF TIME-SERIES
REGRESSIONS COMPUTED FOR EACH FIRM

Coverage	Number of firms	Regressions of a_t on a_{t-1}		Regressions of $(\frac{S A_t}{S A_{t-1}})^{-1}$ on a_{t-1}		Regressions of $(\frac{S A_t}{S A_{t-1}})^{-1}$ on $(\frac{S A_{t-1}}{S A_{t-4}})^{-1}$	
		(1)	(2)	(1)	(2)	(1)	(2)
All		.109		-.283		-.510	
Industries	850	.398	.361	-.069	-.117	-.215	.286
		.628	(.445)	.073	(.481)	-.027	(.345)

Column headings: (1) First quartile, median, and third quartile of distribution.
(2) Mean and standard deviation of parameters.

for the 850 firms is 0.361; their median value is 0.398. Both are appreciably below even the mean values of b_t presented in Table 7. Indeed, more than 75 per cent of the estimates of β_i for each firm are less than the value of $1/(1+\alpha)$ presented in Table 3. Likewise, roughly 60 per cent and more than 75 per cent of the slope parameters of regressions for non-overlapping change for each firm are negative.

There thus seems to be substantial evidence for the proposition that individual firms' sales are themselves regressive. In its essence this proposition is not novel. Thirty years ago, Horace Secrist [52] presented voluminous evidence showing the existence of regressivity in rates of change of a number of business ratios. Indeed, were it not for the fact that this proposition is even now at variance with commonly-held assumptions about businessmen's expectations, the proposition could perhaps be considered trivial. As Harold Hotelling [38, p. 199] pointed out in commenting on Secrist's contribution, regressiveness can be deduced from a very simple model:

Consider a statistical variate x whose variance does not change from year to year, but for which there is a correlation r between successive values for the same individual. Let the individuals be grouped so that in a certain year all those in a group have values of x within a narrow range. Then among the mean values in those groups, the variance (calculated with the group frequencies as weights) will in the next year be less than that in the first year, in a ratio of which the mean value for linear regression and fine grouping is r^2 , but in any case is r^2 , less than unity.

In other words, if $(A_t - \bar{A}) = b(A_{t-1} - \bar{A})$, then $b = r$ if the variance of A is constant. Consequently $b < 1$ so long as $r < 1$, implying a tendency to "regress" toward the mean. As Hotelling rather harshly remarked [loc. cit.].

This theorem is proved by simple mathematics. It is illustrated by genetic, astronomical, physical, sociological, and other phenomena. To "prove" such a mathematical result by a costly and prolonged numerical study of many kinds of business profit and expense ratios is analogous to proving the multiplication table by arranging elephants in rows and columns, and then doing the same for numerous other kinds of animals. The performance, though perhaps entertaining, and having a certain pedagogic value, is not an important contribution either to zoology or to mathematics.

Actually, of course, the empirical verification of the relevance of a "true but trivial" theorem is not always trivial. The fact that regressiveness can be deduced from a simple model provides no empirical justification for the model itself, in spite of Hotelling's cavalier remarks. Seldom is the applicability self-evidence of any theorem which purports to be operational. In particular, it is not self-evident from Hotelling's "theorem" that regressivity can be effectively postulated without recourse to the sort of empirical analysis pioneered by Secrist, for the constancy of the variance of a variable is no more an a priori certainty than is the presence or absence of regressivity itself. Moreover, it is possible to obtain only one realization of a stochastic process at any given time.¹ Hotelling's theorem, though interesting as an example of the ease with which it is possible to derive an explanation of regressivity, does not save us from the necessity of obtaining further information on the nature of the stochastic process underlying successive observations of an individual firm's sales if we wish to explain regressivity in such observations.

As Hotelling observed, regressiveness has been noted in widely scattered phenomena.² Perhaps the most notable common feature of each of these phenomena is the fact that the level of the relevant variable -- be it height

¹ Evidence on temporal differences in cross-sectional variances may nevertheless provide some illumination of the temporal stability of firms' sales. Hart and Prais [30, esp. Table 8] present some evidence on longer-run differences, using London Stock Exchange data on market value as a rough measure of firm size. Such differences appear to be substantial, with the cross-sectional variances of sets of firms existing through several periods of more than a decade increasing over the periods by multiplicative factors ranging from 0.45 to 5.60. For the SEC data analyzed earlier, the ratio of the variance of a_t to the variance of a_{t-1} is significantly different from zero at the 2 per cent level on a two-tail test for 138 out of 166 industry cross-sections. Thus 83.1 per cent of these F-ratios are significant at the 2 per cent level -- which is scarcely encouraging for the applicability of the Hotelling theorem.

² Possibly its most well-known incidence was pointed out by Galton [27][28] in his finding that children of taller-than-average parents tend to be smaller than their parents, while children of smaller-than-average parents tend to be larger.

of individuals, bacterial resistance to antibiotic dosages, or the seasonally-adjusted level of a firm's sales or profits -- is partially ephemeral, suggesting that at least as a crude first approximation such phenomena can be described by a stochastic process defined as the sum of two components, one fixed and the other a random variate. Following the terminology introduced by Friedman and Kuznets [26, ch. 11][25], we shall call the fixed component "permanent" and the other component "transitory." We can easily show that unbiased forecasts of a variable described by such a process will be regressive. Let P denote the "permanent" component of A_t and ξ_t the "transitory" component. Then

$$(3.9) \quad A_t = P + \xi_t, \quad \sum \xi = 0,$$

so that

$$(3.10) \quad A_t - A_{t-1} = \xi_t - \xi_{t-1}$$

$$(3.11) \quad A_{t-1} - A_{t-k} = \xi_{t-1} - \xi_{t-k}$$

Since $\sum \xi = 0$, a linear regression of $(A_t - A_{t-1})$ on $(A_{t-1} - A_{t-5})$ will be homogeneous and hence of the form $E(A_t - A_{t-1} | A_{t-1} - A_{t-5}) = \gamma (A_{t-1} - A_{t-5})$, where the probability limit of the least-squares estimate of γ is

$$(3.12) \quad \gamma = \text{plim } \hat{\gamma} = \frac{E \xi_t \xi_{t-1} - E \xi_{t-1}^2 - E \xi_t \xi_{t-5} + E \xi_{t-1} \xi_{t-5}}{E \xi_{t-1}^2 - 2 E \xi_{t-1} \xi_{t-5} + E \xi_{t-5}^2}$$

If $\gamma < 0$, then of course A is regressive. If ξ_t is independently and identically distributed in each period, it is apparent that then $\gamma = -1/2$. If ξ is less conveniently distributed, the value of γ is not so readily apparent. If, for instance, ξ is identically distributed in each period but $E(\xi_t | \xi_{t-1}) = g \xi_{t-1}$, then

$\sum \xi_t \xi_{t-k} = g^k \sum \xi_{t-k}^2$. Substituting in (3.12),

$$(3.13) \quad \gamma = \frac{1}{2} (g - 1)$$

Likewise if ξ were heteroscedastic as well so that, say, $\sum \xi_t^2 = (1+f) \sum \xi_{t-1}^2$, it is evident from (3.12) that then

$$(3.14) \quad \gamma = \frac{[(1+f)^4 - g^4] (g-1)}{1 + (1+f)^4 - 2g^4}$$

It is apparent from (3.14) that $\gamma < 0$ if $g < 1$. If $f = 0$ that Hotelling's theorem applies to the relationship between successive ξ , it is clear that g must then be less than unity. More generally, since

$$(3.15) \quad \rho_{\xi_t \xi_{t-1}}^2 = \frac{g^2}{1+f}$$

g must be less than $k\sqrt{1+f}$ if $\rho < k \leq 1$. We can assume that ξ_t and ξ_{t-1} are less than perfectly correlated -- usually much less so -- and that, for most time series, values of f very much different from zero are unlikely. Consequently we may conclude that regressivity is likely to be a characteristic of most time series which can be described in terms of the stochastic process specified in (3.9).

This discussion of regressivity has been phrased in terms of the relationship between successive changes in A rather than in terms of the relationship between successive rates of change. We can, however, easily transform any relationship of the form

$$(3.16) \quad A_t - A_{t-1} = K' + \gamma (A_{t-1} - A_{t-5}) + \theta_t'$$

to one of the form

$$(3.17) \quad \frac{A_t}{A_{t-1}} - 1 = K'' + \gamma'' a_{t-1} + \theta_t''$$

by simply noting that $\gamma'' = \gamma (A_{t-5}/A_{t-1}) = \gamma/(1 + a_{t-1})$, that $K'' = K'/A_{t-1}$, and

that $\theta_t'' = \theta_t' / A_{t-1}$. Since γ'' will always have the same sign as γ , regressivity will show up in either relationship. The only difference is that the magnitude of the regression coefficients will differ, reflecting the relative dimensionless of the rates of change in the second relationship.¹

The stochastic process specified in (3.9) is of course a very simple model of a time series. As demonstrated in [6], this model is a special case of two more general models: one model specified as

$$(3.18) \quad A_t = \xi_t + \lambda \sum_{j < t} \xi_j, \quad 0 \leq \lambda \leq 1,$$

where the parameter λ denotes the proportion of each random shock which is "permanent," and another model specified as

$$(3.19) \quad A_t = (1 - \delta)^r \sum_{j \leq t} \binom{t+r-j-1}{t-j} \delta^{t-j} \xi_j, \quad 0 \leq \delta < 1,$$

in which the "permanent" component of each random shock eventually diminishes with the passage of time. The model specified in (3.9) is the special case of these two models defined by setting $\sum \xi = P$ and either λ or δ (as the case may be) equal to zero.

The extent to which regressivity is exhibited by the stochastic processes defined by (3.18) and (3.19) along with varying specifications of ξ_j has been evaluated in [6], and we shall not reproduce those derivations here. Suffice it to say that the analysis reported in [6] indicates that there are diverse specifications of stochastic processes which exhibit regressivity. Furthermore, an analysis reported in [5] of a fairly lengthy sample of observations of shipments to the U.S domestic market of a reasonably representative product of manufacturing firms indicates that regressivity can be exhibited over forecast horizons of up to several years. It would thus appear that there is ample basis for regarding regressivity as a quite natural phenomenon in the behavior of a time series. Indeed, as was suggested above in evaluating equation (4.6) and demonstrated at greater

length in [6], there is a direct relationship between the extent to which a time series exhibits regressivity and the extent to which the effects of random shocks are confined to the periods in which they occur. The more evanescent the shocks, the more regressiveness will be exhibited. Regressivity should thus be regarded as a not at all surprising property of many time series.

4. "Adaptive" expectations and "rational" regressivity

Given evidence indicating that actual data may in fact be regressive, the existence of regressive forecasts should not be particularly surprising. "Rational" expectations will be regressive if the actual data is itself regressive. Consequently, the evidence presented in the preceding section makes, we believe, a convincing case for the rationality of regressive forecasts made by individual firms.

Nevertheless, though the regressivity of actual data in itself is evidence for the reasonableness of regressive expectations, it provides no evidence that businessmen do in fact have such expectations. We therefore attempt in this section to show that the expectations reported in survey responses exhibit behavior consistent with the rational expectations hypothesis, and so to substantiate the representativeness of such survey evidence.

The businessman predicting the future movement of his firm's sales is typically faced with much the same forecasting problem as that analyzed for the consumer by Friedman [25] and Modigliani and Brumberg [45]. Every quarterly sales figure provides information which should be taken into account in forecasting sales in subsequent quarters. At the same time, any one observation is subject to random influences which would suggest against basing a forecast exclusively on that observation. Consequently it would appear reasonable for the businessman to forecast sales by forming some sort of weighted average of the most recent sales figure and his previous forecast of sales based on information available before the most recent sales figure became known. Let us assume that he does so, continually adapting his forecast each period so that

$$(4.1) \quad E_t = \lambda A_{t-1} + (1-\lambda) E_{t-1}, \quad 0 \leq \lambda \leq 1,$$

where λ represents the businessman's evaluation of the information content of the

most recent sales figure relative to that of his previous forecast. Rearranging (4.1), it is clear that such averaging is equivalent to revising the forecast each period in proportion to the size of the forecast error in the preceding period -- that in other words

$$(4.2) \quad E_t - E_{t-1} = \lambda [A_{t-1} - E_{t-1}].$$

It is no coincidence that such a model of adaptive expectations, originally proposed by Cagan [13] and Nerlove [49][50] to describe price anticipations, is equivalent to the stochastic learning models that have been developed by Bush and Mosteller [12], Estes [19][20], and others. Though valid objections can be made to the application of reinforcement theory in many learning situations,¹ an assumption of a relatively simple stimulus-response relationship would not seem out of place here.

The model of expectations presented in (4.1) has a long pedigree of applications, though in somewhat different form. Successively substituting the expressions defined by (4.1) for E_{t-1} and prior forecasts in the right-hand side of (4.1) itself, it is clear that

$$(4.3) \quad E_t = \lambda \sum_{j < t} (1 - \lambda)^{t-j} A_j.$$

Such exponentially-weighted averages, originally proposed by Koyck [42], have been widely used as models of expectations.² They have also been proposed as

¹ See for instance the dicta of Bruner, Goodnow, and Austin [11] on the analysis of concept attainment, generalization, abstraction, and other features of complex problem-solving situations. Their remarks are applicable with much force to the forecasting problem faced by businessmen; indeed, as we shall show later in section 5, the discrepancy between their learning model and the simpler stimulus-response model is essentially the "missing factor" which we need in order to explain the regressiveness of the aggregated forecasts. However, at the level of the individual firm the stimulus-response model would seem to be a good first approximation.

² See the references cited in [6, p. 18].

a means of forecasting sales in normative applications by Brown [10], Holt [36], Magee [43], and Winters [57], and empirical tests in [37, pp. 267-71] of this forecasting technique (modified to take account of seasonal and trend factors) have indicated that it performs better than reasonably sophisticated alternative naive-model extrapolations. Indeed, it can be shown that exponentially-weighted averages will provide optimal forecasts for certain types of stochastic processes. If it can be specified that the variate being predicted is a realization of the stochastic process defined by (3.18) and that the random shocks ξ_j are identically and independently distributed with zero mean, then, as John Muth [47] has shown, exponentially-weighted averages will in such cases be optimal since, from (4.10),

$$(4.4) \quad A_t - \xi_t = (A_{t-1} - \xi_{t-1}) + \lambda \xi_{t-1}.$$

so that, since $\sum \xi = \xi_t \xi_{t-1} = 0$, the best¹ estimate of A_t given only information about previous A is $E_t = A_t - \xi_t$. Substituting this definition of E_t in (4.4),

$$(4.5) \quad E_t = E_{t-1} + \lambda (A_{t-1} - E_{t-1}),$$

which is of course merely a rearrangement of (4.2).

That exponentially-weighted averages provide regressive forecasts is perhaps less evident. Nevertheless, as noted in section 3, a series generated by the process defined in (3.18) is regressive for a wide range of specifications of the process generating ξ_j , and it can be quickly shown that this is so for the particular specification for which exponentially-weighted averages provide optimal forecasts.² This can be done by noting from (4.4) that

$$(4.6) \quad A_t - A_{t-1} = \xi_t + (\lambda-1) \xi_{t-1}$$

¹ Defining "best" as minimum-variance unbiased.

² This is shown in more detail in [6, pp. 19-21].

and likewise that

$$(4.7) \quad A_{t-1} - A_{t-5} = \xi_{t-1} + \lambda \sum_{j=2}^4 \xi_j + (\lambda-1) \xi_{t-5}$$

so that, since $\Sigma (E_t - A_{t-1} | A_{t-1} - A_{t-5}) = \Sigma (A_t - A_{t-1} | A_{t-1} - A_{t-5})$ by virtue of the unbiasedness of E_t , then, specifying that $\Sigma (A_t - A_{t-1} | A_{t-1} - A_{t-5}) = \gamma (A_{t-1} - A_{t-5})$ as before, the least squares estimate of γ is $\hat{\gamma}$, with

$$(4.8) \quad \text{plim } \hat{\gamma} = \frac{\lambda - 1}{2(1-\lambda) + 4\lambda^2}$$

Since $0 \leq \lambda \leq 1$, it is apparent from (4.8) that both forecasts and actual data are in this case regressive so long as λ is not equal to zero (in which case change in A is completely unpredictable), and that moreover this regressiveness increases as the transience of the random shocks increases.

The predictive optimality of exponentially-weighted averages is of course limited to stochastic processes such as those defined above.¹ If for

¹ We should note explicitly that the phrase "such as" specifically includes processes with independent permanent and transitory components as well as those with proportional components as specified in (3.18). Specifically, let

$$A_t = \sum_{j \leq t} \xi_j + \xi_j^*$$

where $\xi_j = a + b \zeta_j$, $\xi_j^* = \beta \zeta_j^*$, and where ζ_j and ζ_j^* are independent realizations of pure Gaussian processes with zero mean and unit variance. Muth [47] has shown that exponentially-weighted averages provide optimal forecasts for such processes. We can show that they are regressive by the same procedure as used above. Specifically, setting $\Sigma (A_t - A_{t-1} | A_{t-1} - A_{t-5}) = \gamma (A_{t-1} - A_{t-5})$, the least-squares estimate of γ is $\hat{\gamma}$, with

$$\text{plim } \hat{\gamma} = \frac{4a^2 - \beta^2}{16a^2 + 4b^2 + 2\beta^2}$$

so that $\gamma < 0$ (and hence both forecasts and actual data are regressive) if $2|a| < \beta$. If, as before, ξ_j is specified to have zero expected value, then of course this condition is satisfied for all non-zero β , which simply means that the condition is satisfied if a transitory component does in fact exist.

instance other processes such as those defined in (3.19) are specified or it cannot be specified that the random shocks ξ_j in (3.18) are indeed independently and identically distributed, then exponentially-weighted averages will not be unbiased. In such cases, they consequently do not provide accurate models of (Muthian) "rational" expectations.

Nevertheless, it would not seem unreasonable to use exponentially-weighted averages as a model of the way in which businessmen's expectations are generated, either as a first approximation or modified à la Holt [36][37] to incorporate adjustments for underlying serial dependencies in ξ_j . Such a model is consistent with what would seem to be reasonable reactions by businessmen to uncertainty defined in terms of the transitory variability of a variable being forecast. If "permanent" changes in a series are small relative to the total variability of that series, the weight attached to the previous forecast will be relatively close to unity, giving relatively large weight to all past observations in order that transitory fluctuations may cancel each other. On the other hand, if "permanent" changes are large relative to transitory "noise," the weight attached to data no longer current will be small, giving credit to the greater amount of information about the current rate and direction of "permanent" change contained in current data.

Whether such a model (plausible though it be) is empirically valid is of course another question. Actually, the answer to this question is not crucial to the question we are attempting to answer in this paper, for even if exponentially-weighted averages are not themselves appropriate as models of "rational" expectations, it is apparent from the analysis of stochastic processes reported in [6] and discussed above that there are numerous processes for which unbiased anticipations will be regressive. Furthermore, the investigations reported in [5] and in section 3 above suggest that such regressive processes are not

unrepresentative of sales of manufacturing firms. Nevertheless, even if regressive anticipations are reasonable in the sense of providing unbiased forecasts, it does not necessarily follow that businessmen deliberately construct regressive forecasts. An affirmation of the empirical validity of adaptive expectations models would consequently provide an additional buttress for the hypothesis that businessmen's anticipations are generally both "rational" and regressive, quite beyond that already furnished by the evidence for cross-sections regressivity reported in [7] and [9].

It will not be possible here to test conclusively the validity of adaptive-expectations models. Nevertheless, some indication of their validity may be obtained by examining the extent to which differences in the degree of regressivity of responses to the Dun and Bradstreet survey for firms in different industries and for different variables is related to differences in the extent to which random shocks that occur affect subsequent values of those variables. As noted above, we should expect regressivity in businessmen's anticipations to vary with the amount of transience associated with the variables being forecast, if their expectations are "rational". Consequently we should expect differences in the average degree of transience associated with difference in the degree to which the reported forecasts are regressive.

The test which we will perform consists simply of comparing the regressivity of forecasts of different variables which a priori would seem to differ in the extent to which random shocks have only transitory effects. We shall confine ourselves to examining the differences in the regressivity of forecasts of three variables -- sales, profits, and selling prices -- for which comparable responses can be obtained from four Dun and Bradstreet surveys.

A priori, little can be said about the relative transience of the random shocks which affect profits compared with that of the shocks which affect sales.

The uncertainty which a businessman associates with gross profits as a result of this transience may be more or less than that which he associates with sales, depending upon whether changes in gross margins tend to directly or inversely correlate with changes in his firm's sales. Moreover, the relation between gross and net profits may vary considerably among firms. For many firms, profits may be a function of a "smoothed" average of recent sales rather than of current sales, thus cutting down the variability of net income for such firms. For others, the variability of net profits may be enhanced by relatively large or relatively fixed overhead expenses.¹ We should consequently not expect differences between the regressivity of sales forecasts and that of profit forecasts to be highly systematic. If anything, it would seem logical to expect transience to be more characteristic of profits than of sales. But this expectation has to be hedged for quarterly data because of the undoubted influence of smoothing effects on contiguous profit figures.

¹ These statements implicitly assume that the transience of the effect of shocks influencing the ratio of net to gross income is related to the variability of net income. This assumption is of course not necessarily valid. However, given the seemingly realistic presumption that the permanence of the effect of shocks on gross profits declines with time, as for the stochastic process specified in (3.19), any smoothing operator which reduces the variability of net income will also increase the time duration over which shocks have significant effect and so decrease their transience. (An example of firm characteristics which generate profit-smoothing would be policies that attempt to guard against being "caught" by shifts in demand, such as marketing policies which allocate advertising allowances to products on the basis of current product sales.) Accounting procedures such as standard costing may also smooth profits and reduce transience to that of sales by making sales and profits changes more directly proportional. Contrariwise, "stabilizing" policies which increase the relative variability of net income by smoothing non-profit variables such as work-force will result in an increase in the transient component of net income through decreasing the responsiveness of overhead costs to changes in gross income.

The difference between the regressivity of selling price forecasts and that of either sales or profits forecasts should provide a much sharper test of our hypothesis. We should expect to find significantly less transience associated with selling price changes by businessmen, both because such changes are likely to be less frequent and because they are more often likely to be controlled. We should consequently expect to find selling price forecasts less regressive than either sales or profits forecasts if our hypothesis is correct.¹

Table 9, a list of the parameters of the regressions of e_t on a_{t-1}

¹ The relative transience of random shocks for different series is difficult to estimate from cross-sectional data except under quite restrictive assumptions. Nevertheless, it is interesting to examine the extent to which our estimates of relative transience correspond to the relative cross-sectional diversity of past change experienced by each firm for each variable. The correspondence is indicated by the following tabulation of parameters of the distributions of the values of $\sigma_{a_{t-1}}$ computed for each variable in each industry:

SURVEY DATE	NUMBER OF VALUES OF $\sigma_{a_{t-1}}$	SALES		PROFITS		SELLING PRICE	
		(1)	(2)	(1)	(2)	(1)	(2)
August 1949	18	0.17	.19	.20	.21	.06	.08
September 1949	18	.12-.25	(.09)	.15-.29	(.09)	.05-.09	(.04)
		.21	.22	.19	.20	.07	.07
October 1949	20	.14-.28	(.10)	.14-.25	(.09)	.05-.08	(.02)
		.14	.16	.21	.17	.06	.06
November 1949	20	.10-.21	(.07)	.10-.23	(.08)	.04-.06	(.03)
		.17	.18	.18	.17	.06	.07
All 4 surveys	75	.13-.20	(.06)	.12-.22	(.07)	.05-.08	(.03)
		.17	.19	.18	.19	.06	.07
		.12-.23	(.08)	.13-.25	(.08)	.05-.08	(.03)

Column headings are as follows: (1) presents the median and under it the inter-quartile range, and (2) presents the mean and under it the standard deviation. The correspondence between the cross-sectional diversity indicated by these figures and our a priori estimates of relative transience is surprisingly good. Nevertheless, one should not put much faith in this correspondence in view of the evidence on intra-industry heterogeneity in time series regression parameters presented in [8] and in Table 8 above.

Table 9

REGRESSIONS OF e_t ON a_{t-1} FOR ALL SALES, PROFITS,
AND SELLING PRICE RESPONSES REPORTED TO THE DUN AND
BRADSTREET SURVEYS OF AUGUST, SEPTEMBER, OCTOBER,
AND NOVEMBER OF 1949

VARIABLE	SLOPE PARAMETER	CONSTANT TERM	CORRELATION COEFFICIENT	NUMBER OF OBSERVATIONS
Sales	0.604 ^{***} (.017)	0.026 ^{***} (.004)	0.635 ^{***}	1809
Profits	.642 ^{***} (.015)	.008 ^{***} (.003)	.734 ^{***}	1612
Selling price	.800 ^{***} (.023)	-.002 (.002)	.616 ^{***}	1999

Note: Estimated standard errors of regression parameter estimates are listed in parentheses under each parameter estimate. The number of observations corresponds to the number of firms giving quantitative replies to requests for e_t and a_{t-1} for each variable.

- * Significant at the 10 per cent level
- ** Significant at the 5 per cent level
- *** Significant at the 1 per cent level

for all individual sales, profits and selling price responses in the four Dun and Bradstreet surveys which are analyzed, indicates substantial agreement with the predictions of slope parameters that can be made a priori from our hypothesis. If anything, sales would seem to be more regressive (and hence implicitly less certain) than profits. But the difference between the slopes of the sales and profit regressions is relatively minor; using the estimated standard errors of estimate as valid estimates of the dispersion of such parameters, the sales and profit regressions differ significantly from one another only at a significance level greater than 8 per cent. By contrast, both sales and profits are, as predicted, substantially more regressive than selling prices. The difference between the profit and price regression slopes differs significantly from zero at virtually any significance level one might care to choose;¹ a fortiori this is the case for the difference between the sales and price regression slopes.

Table 10 indicates that these differences are maintained both for regressions computed at the industry level and for regressions computed for firms in all industries included in each survey. In only one survey -- August 1949 -- does the price regression computed from all responses to the survey exhibit more regressivity than either of the corresponding regressions for sales and profits, and in that survey the slope of the "aggregate" cross-sectional price regression over all industries is not representative of the slopes of the 18 regressions computed at the industry level. In all but the November 1949 survey,

¹ Even using the larger of the two estimated standard errors associated with the slope parameters of the price and profit regressions as an estimate of the "common" probabilistic dispersion of each of the two parameters yields a "t"-ratio for the difference between the two parameters of 4.79. The odds against obtaining such a "t"-ratio if both parameters deviated only randomly from some common value are more than 100,000 to 1.

Table 10

COMPARISON OF DISTRIBUTIONS OF PARAMETERS OF INDUSTRY REGRESSIONS OF e_t ON a_{t-1} OVER FIRMS IN EACH INDUSTRY IN A SURVEY WITH PARAMETERS OF THE SAME REGRESSION OVER ALL FIRMS IN THE SURVEY, FOR SALES, PROFITS, AND SELLING PRICE RESPONSES REPORTED TO THE DUN AND BRADSTREET SURVEYS OF AUGUST, SEPTEMBER, OCTOBER, AND NOVEMBER OF 1949

SURVEY DATE	NI	SALES			PROFITS			SELLING PRICE		
		(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
1. SLOPE PARAMETERS										
August 1949	18	.396 .476 .797	.631 (.251)	.571 (.028)	.402 .745 .981	.739 (.287)	.742 (.024)	.825 .950 .991	.890 (.235)	.683 (.024)
September 1949	18	.667 .742 .787	.773 (.151)	.797 (.031)	.571 .785 .909	.923 (.779)	.786 (.020)	.920 .980 1.017	1.034 (.327)	1.053 (.088)
October 1949	20	.185 .426 .515	.364 (.433)	.409 (.041)	.013 .194 .556	.217 (.645)	.418 (.037)	.405 .633 .835	.648 (.320)	.637 (.038)
November 1949	20	.174 .402 .515	.382 (.352)	.456 (.352)	.185 .390 .624	.422 (.406)	.432 (.038)	.114 .483 .598	.468 (.339)	.663 (.027)
2. CONSTANT TERMS										
August 1949	18	-.011 .009 .015	.002 (.028)	-.000 (.006)	-.019 -.005 .012	-.005 (.028)	-.006 (.006)	-.009 -.004 .001	-.008 (.014)	-.014 (.002)
September 1949	18	.002 .019 .039	.024 (.028)	.031 (.007)	-.018 -.001 .002	-.003 (.015)	.006 (.005)	-.003 .002 .005	.007 (.028)	.008 (.005)
October 1949	20	.007 .035 .048	.033 (.028)	.026 (.008)	-.010 .002 .017	.002 (.062)	.013 (.008)	-.014 -.005 .001	-.003 (.012)	-.004 (.003)
November 1949	20	.007 .034 .068	.036 (.033)	.043 (.008)	-.012 .003 .021	.017 (.055)	.021 (.008)	-.007 -.004 -.002	-.003 (.007)	.001 (.002)

COLUMN HEADINGS: NI = Number of industry regressions computed in each survey
 (1) = First quartile, median, and third quartile of distribution of parameters of industry regressions
 (2) = Mean and standard deviation of industry regression parameters
 (3) = Parameter of aggregate regression over all firms in survey, with estimated standard error of estimate

more than 75 per cent of the slope parameters of the sales regressions computed at the industry level are less than the median slope of the corresponding price regressions; the same is true for profit regressions in the September and October surveys. Taking all surveys together, almost 50 per cent of the slopes of all regressions at the industry level for both sales and profits are less than 75 per cent of the slopes of the corresponding regressions for prices; more than 85 per cent of the sales and profit regression slopes are smaller than 50 per cent of the slopes of the price regressions. This is shown in more detail in Table 11.

Some interesting further insights may be gained by examining the relation between the three sets of industry regression slopes. As Table 12 indicates, there is an appreciably greater amount of covariation among the slope parameters of the sales and profits regressions than among the slopes of the sales and price or profits and price regressions. This is consistent with what one would expect, in that fewer factors which determine transience are common to sales and prices or to profits and to prices than are common to sales and profits. In view of this, it is surprising that significantly greater covariation shows up only in the November survey. Comparing the means of the distributions of parameters in each survey presented in Table 10, it is evident that a substantial amount of the greater covariation between sales and profit parameters than between parameters for other pairs of regressions is due to the covariation of survey means. Since one would expect few intertemporal differences in perceived transience for each variable for a given forecaster, it is likely that the substantial intertemporal differences indicated in Table 10 result in large part from the fact that the firms samples are different in each survey.

Table 11

COMPARISON OF DISTRIBUTIONS OF SLOPE PARAMETERS OF INDUSTRY REGRESSIONS OF e_t ON a_{t-1} FOR SALES, PROFITS, AND SELLING PRICES IN THE DUN AND BRADSTREET SURVEYS OF AUGUST, SEPTEMBER, OCTOBER AND NOVEMBER OF 1949.

NUMBER OF SLOPE PARAMETERS OF VARIABLE 1 REGRESSIONS WHICH ARE LESS THAN THE FOLLOWING PARAMETERS OF THE DISTRIBUTION OF SLOPE PARAMETERS OF VARIABLE 2 REGRESSIONS:

SURVEY DATE	MINIMUM VALUE	FIRST QUARTILE	MEAN	MEDIAN	THIRD QUARTILE	MAXIMUM VALUE	NUMBER OF PARAMETERS
A. VARIABLE 1 = SALES, VARIABLE 2 = PROFITS							
August 1949	0	4	11	11	17	18	18
September 1949	0	2	15	12	14	18	18
October 1949	0	2	6	6	15	17	19
November 1949	0	6	9	9	15	20	20
All four surveys	0	12	40	40	62	75	75
B. VARIABLE 1 = SALES, VARIABLE 2 = SELLING PRICES							
August 1949	0	12	14	16	17	18	18
September 1949	3	15	17	16	17	18	18
October 1949	2	9	15	15	16	18	19
November 1949	1	3	12	12	15	20	20
All four surveys	2	38	52	63	69	75	75
C. VARIABLE 1 = PROFITS, VARIABLE 2 = SELLING PRICES							
August 1949	0	9	11	11	12	18	18
September 1949	5	11	17	14	17	18	19
October 1949	6	11	14	14	16	19	19
November 1949	1	3	10	10	11	19	19
All four surveys	3	32	46	57	62	74	75

Table 12

COEFFICIENTS OF DETERMINATION OF REGRESSIONS OF THE SLOPE PARAMETERS OF INDUSTRY REGRESSIONS OF e_t ON a_{t-1} FOR RESPONSES OF ONE VARIABLE ON THE SLOPE PARAMETERS OF THE CORRESPONDING INDUSTRY REGRESSIONS FOR RESPONSES OF ANOTHER VARIABLE, FOR SALES, PROFITS, AND SELLING PRICE REGRESSIONS FOR INDUSTRIES SURVEYED IN THE DUN AND BRADSTREET SURVEYS OF AUGUST, SEPTEMBER, OCTOBER, AND NOVEMBER OF 1949

COEFFICIENTS OF DETERMINATION OF REGRESSIONS BETWEEN SLOPE PARAMETERS OF INDUSTRY REGRESSIONS OF			
SURVEY DATE	SALES AND PROFITS	SALES AND PRICES	PROFITS AND PRICES
August 1949	.164*	.155	.068
September 1949	.069	.008	.100
October 1949	.059	.162	.169*
November 1949	.701***	.036	.156
All four surveys	.235***	.131***	.052*

* Significant at the 10 per cent level

** Significant at the 5 per cent level

*** Significant at the 1 per cent level

Equally interesting, as Tables 10 and 11 indicate, the variation of the slopes of profit regressions is substantially wider than that of the slopes of either sales or selling price regressions. For all surveys taken together, profit regression slopes range from -1.905 to 4.000 compared with ranges of from -.966 to 1.175 for sales and from -.300 to 2.317 for prices. The standard deviation of all industry regression slopes is .629 for profits, compared with .363 for sales and .379 for selling prices. Viewed in terms of our hypothesis that businessmen's expectations are on the whole rational, such substantially greater dispersion in regression slopes for profit responses suggests substantially greater variation in the degree of transience associated with the profit variable than in that associated with sales or selling prices. Though not obvious a priori, such a finding is quite consistent with our hypothesis. It would seem, on the whole, that there is substantial, if not conclusive, evidence indicating that the businessmen's expectations reported by surveys such as Dun and Bradstreet are not only regressive but also behave as if they were generated by businessmen in a rational fashion.

5. An explanation of "non-rational" regressiveness

If regressivity should not be surprising, why then does the very fact of regressivity make the aggregated survey forecasts inaccurate? Such a contradiction makes it quite evident that the explanations of regressivity advanced in the preceding sections, while good as a first approximation, are not sufficient to explain the regressiveness in the aggregated forecasts. For if regressiveness in individual businessmen's forecasts merely reflects a regressiveness that exists in their sales, as the hypotheses advanced in the preceding two sections have suggested, why does it not disappear on aggregation as does the regressiveness of actual data? Why do aggregated anticipations not parallel aggregate sales?

To answer this question, it will first be useful to show in fact how the marked regressiveness of individual firms' sales noted in Section 3 does disappear on aggregation. As before,¹ let us denote the relation between successive 4-quarter change ratios in an individual firm's sales as

$$(5.1) \quad a_t^i = \beta_i a_{t-1}^i + K_i + \theta_{it}, \quad \sum \theta = 0, \quad t = 1, \dots, T,$$

the cross-sectional relationship between a_t and a_{t-1} for a given sample of N firms at a particular time as

$$(5.2) \quad a_t^i = b_t a_{t-1}^i + k_t + h_{it}, \quad \sum_i h_{it} = 0, \quad i = 1, \dots, N$$

and the relationship between successive industry mean 4-quarter change ratios as

$$(5.3) \quad \bar{a}_t = B \bar{a}_{t-1} + K + \epsilon_t, \quad \sum \epsilon = 0, \quad t = 1, \dots, T.$$

As was shown above in Section 3, the only implication for cross-sectional relationships of the temporal relation between successive mean changes in an industry is that k_t , the constant term of the cross-sectional regression, be correlated

¹ See the exposition of our notation in footnote 1, p. 22.

with \bar{a}_{t-1} . (Even this implication doesn't hold unless b_t is either more or less constant or at least uncorrelated with \bar{a}_{t-1} .) In effect, then, the regressivity for individual firms' sales denoted by values of β_i and b_t which are less than $1/(1+\alpha)$ disappears on aggregation by virtue of the constant terms k_t being related to \bar{a}_{t-1} so as to force \bar{a}_t to lie on or around a line through the mean values of \bar{a}_t and \bar{a}_{t-1} with a slope equal to the value of B.

The nature of the relationship thus posited between cross-sectional and temporal regressions is illustrated in Chart 3 under the assumptions that $b_t = b = 0.5$, that $B = 0.9$, and that $K = \text{zero}$. The solid line through the origin of Chart 3 represents (5.3) under these assumptions. (The dashed line will be explained later.) The upper of the two parallel solid lines represents (5.2) at some specific date at which the mean value of a_{t-1} happens to be 0.20 and at which the corresponding value of \bar{a}_t falls precisely at the point "A" on the regression line defined by (5.3). Since ϵ_t is thus by chosen happenstance equal to zero, it is no surprise to find that the graph of (5.2) through "A" has a constant term of 0.08, precisely the value given by (3.7). Similarly the lower of the two parallel lines is the graph of (5.2) for some other point of time at which \bar{a}_{t-1} happens to be -0.325 and \bar{a}_t falls on the time-series regression at "B", thus determining a negative intercept $k_t = -0.13$.

The restrictions postulated in presenting Chart 3 provide a rough approximation to the actual cross-sectional and temporal regressions which have already been presented in Sections 2 and 3. While in reality the cross-section regression slopes will not all be equal and the means will be scattered around the temporal regression of aggregates, Chart 3 nevertheless provides a reasonably accurate picture of how the cross-section and temporal regression slopes are only seemingly inconsistent.

CHART 3

THE RELATIONSHIP BETWEEN CROSS-SECTIONAL
AND AGGREGATE REGRESSIONS OF a_t ON a_{t-1}

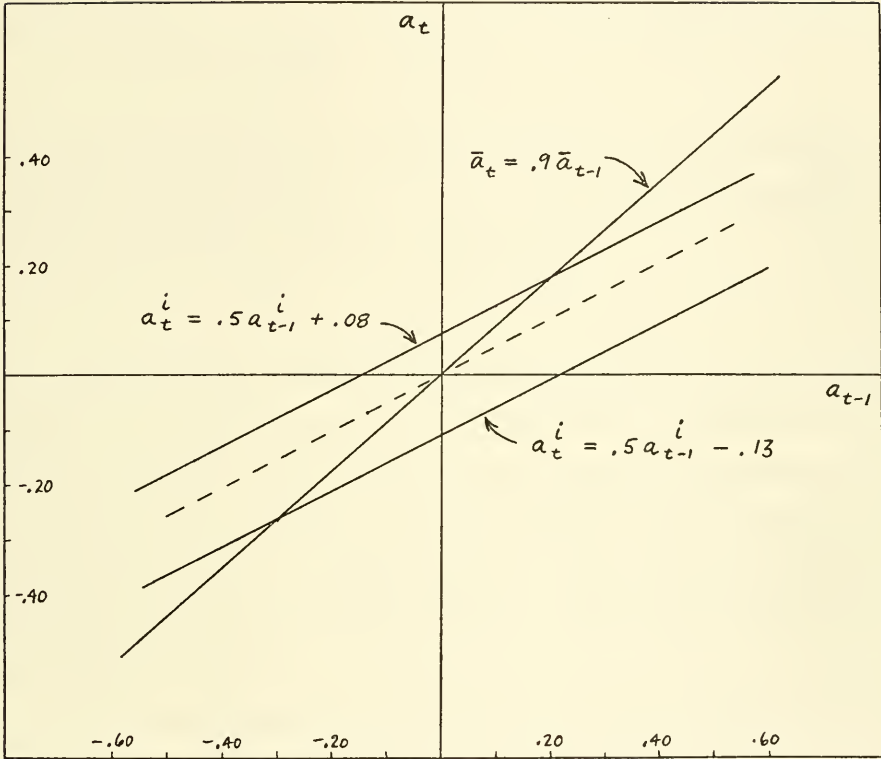


Chart 3 also highlights the difference between the relation between temporal and cross-sectional regression for actual data and that for anticipations.

At the individual firm level, the relation between e_t and a_{t-1} is roughly the same as that between a_t and a_{t-1} so that in the cross-sectional regression

$$(5.4) \quad e_t^i = b_t^* a_{t-1}^i = k_t^* + n_{it}^*, \quad \sum_i n_{it}^* = 0$$

the regression coefficient b_t^* is reasonably stable and of roughly the same order of magnitude as the coefficient b_t in (5.2). On the other hand, denoting the relation between successive industry aggregates by

$$(5.5) \quad \bar{e}_t = B^* \bar{a}_{t-1} + k_t^* + \epsilon_t^*, \quad \sum \epsilon_t^* = 0,$$

all the empirical evidence available indicates not only that $(K^* + \epsilon_t^*)$ is generally small, as in (5.3), but also that B^* is typically much smaller than B and, indeed, is of the same order of magnitude as the cross-section slopes b^* and b , so that (5.8) can be represented in Chart 3 by the dashed line through the origin. The aggregated forecasts are characterized by pronounced temporal regressiveness and so tend consistently to predict inaccurately turning points which do not occur in the actual series.

It is evident that any explanation of the size of B^* must rest on factors other than those which influence only b_t^* , for, as with actual data, there is no necessary relation between B^* and b_t^* so long as k_t^* is unconstrained. Aggregating (5.4), we obtain

$$(5.6) \quad \bar{e}_t = b_t^* \bar{a}_{t-1} + k_t^*,$$

so that, as with actual data, the only requirement for the consistency of equations (5.5) and (5.6) is that

$$(5.7) \quad k_t^* = (B^* - b_t^*) \bar{a}_{t-1} + K^* + \epsilon_t^*$$

We can, however, isolate the nature of the explanation that must be made in order

to obtain $B^* = b_t^*$. If in fact $B^* = b_t^*$, then, from (5.7), k_t^* must be uncorrelated with \bar{a}_{t-1} . In addition, since $(K^* + \epsilon_t^*)$, like $(K + \epsilon_t)$, is generally small, k_t^* should itself tend to be small. To explain the persistence of regressivity on aggregation, we thus need to explain why k_t^* tends to be constrained to be zero.

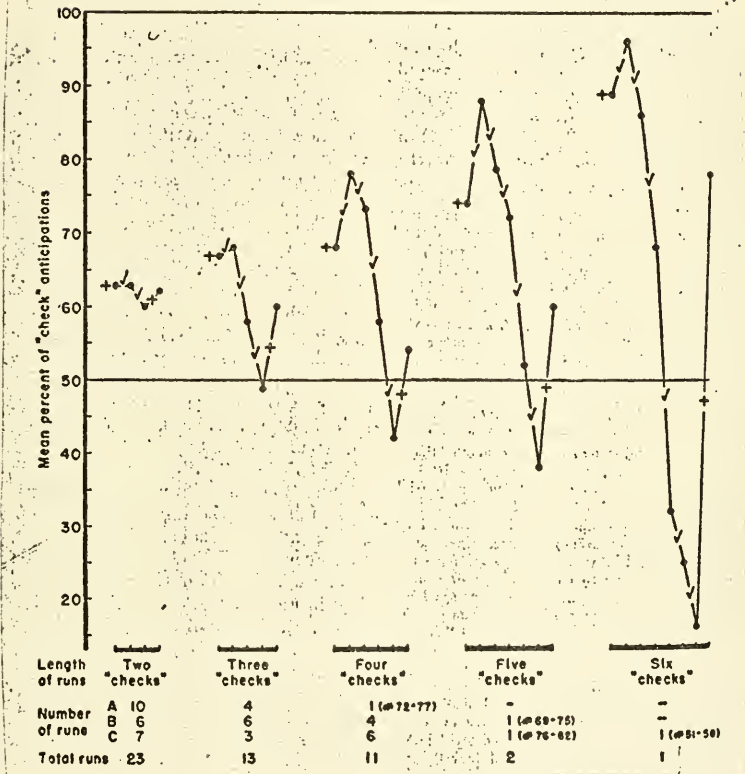
We shall do so in an indirect fashion by first discussing the result of this constraint on k_t^* -- namely, the resultant excessive prediction of turning points in the aggregated series. Such excessive prediction by individuals of turning points in macroeconomic or other variables is not an uncommon phenomenon, as is attested by the lengthy literature on the subject of the so-called "gambler's fallacy." It is, moreover, a phenomenon for which a fairly convincing explanation is available. While it is possible to overstate the value of purely stochastic models of macroeconomic variables, it has long been recognized that an important proportion of the variation of many such variables can best be regarded as the result of transitory random shocks.¹ As was noted above in Section 3, the degree of regressivity present in short-term fluctuations in individual firms' sales is evidence of a substantial transitory component in such sales. Such "randomness" in itself merely justifies the existence of regressivity in actual data and hence in unbiased forecasts. But, as Bruner, Goodnow, and Austin [11, p. 189] have observed, "It is a very general human tendency to deny the independence of temporally related events." Chart 4, adopted from Jarvik [41], presents some particularly graphic evidence of such a tendency.²

¹ See the historical references in [6, p. 14].

² The results presented in Chart 4 are an aggregation of the results of experiments involving 87 binary choices administered to three different groups (labelled A, B, and C). In all groups the task was to predict whether the next event would be "plus" or "check;" the correct responses (generated as a series of independent random trials with the probability of "check" equal to 0.60 in group A, 0.67 in group B, and 0.75 in group C) were announced after each prediction had been made. Apart from the "negative regency effect," the predictions in each group tended to move from an initial "no information" average of 50 per cent checks to an average roughly matching the probability of checks after something like 50 choices had been made.

CHART 4

JARVIK'S "NEGATIVE REGENCY EFFECT": DECLINE IN THE PROPORTION OF "CHECK" ANTICIPATIONS IN RUNS OF FROM TWO TO SIX CONSECUTIVE "CHECK" REINFORCEMENTS



Source: Jarvik [41]

Where the degree of randomness in a variate is substantial, we can expect many forecasters to exaggerate the significance of minimally-valid cues such as the length of immediately-previous runs in the direction of change in the variate. Moreover, as Whitfield [56] and Cohen [14] have demonstrated, there is some evidence that the gambler's fallacy is heightened by a further tendency for individuals to assume runs of events to be shorter than they are in actuality.

The "irrationality" of the gambler's fallacy has been pointed out since the time of Laplace. Nevertheless, the presence of such overvaluation of cues in uncertain situations and consequent event-matching or analogous behavior has been well-documented.¹ Excessive prediction of turning points should as a consequence not seem particularly surprising. This will be all the more true if, as is often the case, more credit is given to the forecaster who is accurate in predicting turning points than to a forecaster whose accuracy is confined to "easier" periods in which turning points do not occur.

¹ See the references cited in [11, chapter 7] and [40]. Such supposedly "irrational" behaviour seems to arise in most circumstances in which a wide variety of cues exist. As Bruner, Goodnow, and Austin [11, pp. 189-192] have pointed out, the tendency for individuals to seek patterns in data -- an attribute generally rewarded in management -- will lead to hypothesis-testing behaviour which will result in event-matching in most inductive situations. "Patterns" were for instance seen by 64 per cent of the group asked by Jarvik [41] to predict independent binary choices with a 75 percent likelihood for one of the choices. (An illuminating parlor game example of such behaviour can be obtained by watching the behaviour of participants playing ELEUSIS, an inductive game described in the June, 1959 issue of Scientific American.) Such pattern-seeking behaviour is also the basis of a simulation of binary-choice-making reported by Feldman [21]. Perhaps the most striking empirical corroboration of this explanation for event-matching behaviour is some evidence described in [11, pp. 215-216] indicating a tendency for event-matching behaviour to disappear only when there is little or no opportunity for decision-makers to validate hypotheses. Such a tendency might seem surprising in the light of conventional definitions of rational behaviour for a forecaster. But perhaps the fact which should be surprising is the conventional definition of rationality, for pattern-seeking behaviour can quite easily be viewed as rational. Indeed, as some recent experiments by Siegel [53] have demonstrated, one of the most important parameters defining the opportunity for subjects to indulge in hypothesis-testing is the perceived monetary cost of so doing.

While excessive prediction of turning points may thus appear "reasonable" on the basis of a tendency toward the "gambler's fallacy" among forecasters, such an explanation is in itself no more than suggestive. Even if b_t^* were as a result smaller than b_t , this in itself need not generate regressivity in the temporal regressions for the aggregated responses. As before, the only necessary implication of the relationship between temporal and cross-sectional regressions is that the regression of k_t^* on \bar{a}_{t-1} be consistent with it.

Nevertheless, the overvaluation of cues leading to excessive turning-point prediction has additional implications for the relationship of k_t^* on \bar{a}_{t-1} which are more conclusive. These additional implications derive simply from the fact that along with any overvaluation of some cues must go an undervaluation of others. Bruner, Goodnow, and Austin [11, p. 201] have noted a tendency for hypothesis-generation to be accompanied in many situations by an arbitrary collapsing of the range of possibilities in order to reduce the dimensionality of a problem and so (in their words) reduce "cognitive strain." Such dimensionality-reduction -- analogous to that performed in stock-price forecasting by Alexander's phase-space filter¹ -- could very possibly lead to undervaluation of the extent to which change in a firm's sales is likely to be not merely transient, particularly when reinforced by the typically large dispersion of actual changes in sales compared to average "permanent" change. A tendency to regard a non-zero mean value of a variate as effectively zero when the mean is small relative to the variate's variance is after all not limited to individuals who are unfamiliar with statistical theory. We should consequently expect businessmen in many cases

¹ See Alexander [1]. A more extreme example of dimensionality-reduction along the lines of Alexander's filter is provided by point-and-figure analysis. It should be noted that "easing cognitive strain" is often much more than a luxury if a solution is to be obtained; Bellman [3] has aptly termed the problems associated with possibility-enumeration the "curse of dimensionality."

to tend in effect to constrain the relationship between forthcoming and previous change to be homogeneous. We may consequently hypothesize that, as a general rule,

$$(7.11) \quad e_t^i = \beta_i^* a_{t-1}^i + \theta_{it}^*, \quad t = 1, \dots, T$$

where on the average θ_{it}^* is roughly zero, even though at any particular time θ_{it}^* will reflect particular cues which the forecaster regards as significant.

For each firm, β_i^* will depend on particular attributes of the firm's decision-makers and might, moreover, vary somewhat over time as a result of whatever changes in cue-utilizing hypotheses may result from reinforcement or non-reinforcement of the decision-makers' forecasts. To the extent that β_i^* varies with β_i , we may expect β_i^* to vary with the relative size of the transitory component associated with sales of the i th firm. The resulting variety in values of β_i^* may lead to a non-zero value of k_t^* in the cross-sectional relationship (7.7) existing at a given time for a given sample of firms, even assuming that (7.11) holds strictly for all firms and even if in addition $\theta_{it}^* = 0$ for all the firms in the particular cross-sectional sample. For since past 4-quarter changes of firms whose sales contain a relatively small transitory component will tend to cluster around \bar{a}_{t-1} , we should expect β_i^* to be inversely related to $(a_{t-1}^i - \bar{a}_{t-1})$ so that a regression line fitted to the cross-section of observations of e_t^i and a_{t-1}^i will tend to have a constant term k_t^* which varies with \bar{a}_{t-1} .

It would seem at first glance that relaxing the assumption that $\theta_{it}^* = 0$ for all firms would merely increase the likelihood of non-zero values of k_t^* . In reality, however, relaxing this assumption leads to the opposite result. In spite of interfirm differences in reactions to various cues, the fact that a number of cues will at any time be common to most firms means that the θ_{it}^* will

realistic. It can be readily seen that our conclusions are little affected if k_t^* is not identically zero, provided it is uncorrelated with \bar{a}_{t-1} and that its average value is close to zero. The only difference is that the cross-sectional regression (7.7) will then fluctuate around the dashed line in Chart 3, though remaining parallel to it. The aggregate relationship (7.8) will still be represented by the dashed line.¹ Moreover, our conclusion remains valid even if k_t^* (like k_t) is positively correlated with \bar{a}_{t-1} , so long as k_t^* changes less than k_t in response to variations in \bar{a}_{t-1} . Denoting the regression of k_t^* on \bar{a}_{t-1} by

$$(7.13) \quad \mathcal{E}(k_t^* \mid \bar{a}_{t-1}) = \delta \bar{a}_{t-1} + k^*$$

it is evident, substituting (7.13) into (7.7) and aggregating, that

$$(7.14) \quad (\bar{e}_t \mid \bar{a}_{t-1}) = (b^* + \delta) \bar{a}_{t-1} + k^*$$

so that $B^* = b^* + \delta$. The slope of the temporal regression between \bar{e}_t and \bar{a}_{t-1} will thus be larger than the slope of the cross-sectional regression of e_t^1 on a_{t-1}^1 . However, so long as δ is smaller than the slope of the regression of k_t on \bar{a}_{t-1} , which by (7.6) is equal to $B-b$, then $B^* = (b^* + \delta) < (b^* + B-b)$, so that B^* will still be less than B if $b^* = b$.

In summary, for our "modified rationality" explanation of regressivity to hold it is sufficient that k_t^* should generally be close to zero and not vary greatly with \bar{a}_{t-1} . The available cross-section evidence appears to support this

¹ If the mean value of k^* is not zero, then the aggregate regression (7.8) will have an intercept K^* equal to the mean value of k^* and will be represented in Chart 3 by a line through K^* parallel to the dashed line. So long as K^* is small relative to the temporal dispersion of \bar{a}_{t-1} , most points on the aggregate regression will still fall in the region of temporal regressiveness; \bar{a}_t will fail to understate \bar{a}_t only for a range of very small values of \bar{a}_{t-1} .

conclusion. As Table 10 indicated, the mean value of k_t^* is generally small for the industry regressions of e_t^i on a_{t-1}^i for sales responses in the four Dun and Bradstreet surveys which were investigated in Section 5. Moreover, as shown in Table 13, its cross-sectional dispersion is less than half that of \bar{a}_{t-1} for all but the August 1949 survey.¹ For all four surveys, values of k_t^* range from a minimum of $-.068$ to a maximum of $.082$ (a range of $.150$) compared with a range of $.597$ (likewise rough centered on zero) for values of \bar{a}_{t-1} . Not only is k_t^* generally closer to zero than \bar{a}_{t-1} ; it also varies substantially less with \bar{a}_{t-1} than does k_t . The slope of the regression of k_t^* on \bar{a}_{t-1} differs significantly from zero only for one of the four surveys, and even for that survey it is less than two-thirds of the typical difference between B and b_t . The empirical evidence would thus seem to support our hypothesis that so-called "irrational" regressivity can be explained in terms of widely-prevailing psychological reactions to that component of uncertainty which is defined by the existence of a transitory component in a variate.

At the same time, it should be emphasized that the evidence presented in Table 13 supports our hypothesis only in verifying that k_t^* is in fact roughly zero and little correlated with \bar{a}_{t-1} . It does not validate our explanation of why k_t^* is roughly zero. While our explanation does relate regressively to a substantial volume of experimental evidence, it should be noted that all of this evidence is of behavior on artificial situations rather than evidence on businessmen's actual forecasting behavior.

¹ Measuring cross-sectional dispersion in terms of the cross-sectional standard deviation of the variate. Measured in terms of the range of the variate, the cross-sectional dispersion of k_t^* is less than that of \bar{a}_{t-1} in August 1949 as well: k_t^* ranges from $-.068$ to $.040$ while \bar{a}_{t-1} ranges from $-.132$ to $.020$. The highest range of k_t^* in any survey is $.115$ (in September 1949); the lowest range of \bar{a}_{t-1} in a survey is $.152$ (in August 1949).

TABLE 13

PARAMETERS OF REGRESSIONS OF CONSTANT TERMS ($k_t^{\#}$) OF
 INDUSTRY REGRESSIONS OF e_t ON a_{t-1} ON MEAN PAST
 CHANGE (\bar{a}_{t-1}) OF THE OBSERVATIONS IN EACH INDUSTRY
 REGRESSION FOR SALES RESPONSES IN THE DUN AND BRADSTREET
 SURVEYS OF AUGUST, SEPTEMBER, OCTOBER, AND NOVEMBER
 OF 1949

SURVEY DATE	MEANS AND STANDARD DEVIATIONS OF		SLOPE PARAMETER	CONSTANT TERM	CORRELATION COEFFICIENT	NUMBER OF INDUSTRY REGRESSIONS IN EACH SURVEY
	$k_t^{\#}$	\bar{a}_{t-1}				
August 1949	.002 (.028)	-.031 (.036)	.298* (.165)	.012 (.008)	.392	18
September 1949	.024 (.028)	.014 (.109)	.080 (.058)	.023*** (.006)	.310	18
October 1949	.032 (.028)	-.024 (.082)	-.054 (.080)	.031*** (.007)	.151	19
November 1949	.036 (.033)	-.006 (.082)	.228*** (.074)	.037*** (.006)	.567***	20
All four surveys	.024 (.032)	-.012 (.083)	.108** (.043)	.025*** (.004)	.278**	75

Note: Standard errors of estimate of parameters are listed in parentheses under each parameter estimate; standard deviations are listed in parentheses below means.

* Significant at the 10 per cent level

** Significant at the 5 per cent level

*** Significant at the 1 per cent level

In addition, the static nature of our analysis should also be emphasized. Because of the relative non-significance of "permanent" change, recognition of any gap between perceived stationarity and actual non-stationarity will take place only slowly. However, we should expect some learning (i.e., gap recognition) to take place. We have not attempted to test whether such learning does in fact occur. If it does, it will have the effect of increasing the apparent inaccuracy of the surveys, since the correction lag should tend to result in a decrease in the degree of regressivity as time elapses between turning points, so that the least regressivity should thus occur at turning points. This seemingly paradoxical result only emphasizes the importance of distinguishing the greater value ex ante and consequent greater true rationality to homo sapiens, a pattern seeker, of adopting an "adaptive rationality" as opposed to attempting to "correct" forecasts in order to make them unbiased.

6. Conclusion

In this paper, we have attempted to show that regressivity in businessmen's expectations is a reasonable and easily-explicable phenomenon. We have done so by first showing that regressive anticipations reflect rational forecasts of individual firms, and then proceeding to explain how regressivity persists when such forecasts are aggregated. We demonstrated the regressive anticipations reflect rational forecasts by showing first that many businesses exhibit a substantial degree of regressivity in their sales and second that the businessmen's expectations reported to surveys seem to behave in accordance with implications of the rational expectations hypothesis. Not only do regressive forecasts seem to be rational, in other words, but they also seem to be made rationally by businessmen. Finally, we have suggested that the persistence of regressivity upon aggregation is due to an undervaluation of the short-term stationarity in the predicted time series resulting primarily from a pattern-seeking behavior that would seem to be sensible from the point of the individual businessman.

The result of the persistence of regressivity upon aggregation is that businessmen's expectations make poor predictors when aggregated. This in itself should not be particularly surprising, for, as Grunfeld and Griliches [29] have pointed out, it is often easier to predict change in an aggregate variable from an equation expressed in terms of aggregate variables than from the aggregated composite prediction of a set of micro-equations. The wide diversity among individual firms' experiences and consequently between those experiences and the movement of data aggregated over firms leaves substantial room for the operation of an aggregation bias.

This should not be taken to mean that data collected from businessmen by anticipations surveys are not useful in making macroeconomic forecasts. On the contrary, we have shown that there is substantial evidence indicating that such data does reflect businessmen's expectations with a reasonably high degree of accuracy. Consequently, such data can be used as evidence on businessmen's anticipations which can be incorporated into models of business planning and decision-making that can then be used to derive macropredictions. Perhaps the most suggestive example of such indirect use of businessmen's expectations is that developed by the Economics Branch of the Department of Trade and Commerce of the Canadian government, in which forecasts of employment obtained from the survey of employers conducted by the Canadian Department of Labor are compared with the Economics Branch's own expectations in order to derive estimates of the extent of regressiveness in businessmen's expectations which are then used to adjust the aggregated statistics on planned business investment obtained from the Canadian "Investment Intentions" survey. It is in such indirect applications that the most fruitful uses of data on businessmen's anticipations undoubtedly lie.

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