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ALFRED P. SLOAN SCHOOL OF MANAGEMENT

On Asset Prices in Exchange
and Production Economies

RAJNISH MEHRA*

Sloan School of Management
M.I.T.
and
University of California, Santa Barbara

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MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139
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Abstract

This note presents a formal proof that introducing production and capital accumulation in a pure exchange Arrow-Debreu economy will not increase the set of joint equilibrium processes on consumption and asset prices. A direct empirical implication is that if a model is rejected in an exchange paradigm introducing production will not change the result.
This note presents a simple proof to demonstrate that expanding the set of technologies in a pure exchange, Arrow-Debreu economy to admit capital accumulation and production does not increase the set of joint equilibrium processes on consumption and asset prices.

Modifying the technology in an exchange economy to incorporate production introduces restrictions which must be satisfied simultaneously with the equilibrium asset pricing relation such as the one in Lucas (1978).

Intuitively, the equilibrium stochastic process generated by any production economy can also be generated by an exchange economy that is endowed with the same stochastic process.

A direct empirical implication is that if a model is rejected in an exchange paradigm, introducing production and the concomitant restrictions will not change the result. We formalize this below.

Let \( \theta \) denote preferences, \( r \) technologies, \( E \) the set of exogenous processes on the aggregate consumption good, \( P \) the set of technologies with production opportunities, and \( m(\theta, r) \) the set of equilibria for economy \((\theta, r)\).

\[ m(\theta, r) \subseteq E \]

**Theorem**

\[ \bigcup_{r \in E} m(\theta, r) \supseteq \bigcup_{r \in P} m(\theta, r) \]

**Proof.** For \( \theta_0 \in \theta \) and \( r_0 \in P \) let \((a_0, c_0)\) be a joint equilibrium process on asset prices and consumption. A necessary condition for equilibrium is that the asset prices \( a_0 \) be consistent with \( c_0 \), the optimal consumption for the household with preferences \( \theta_0 \). Thus, if \((a_0, c_0)\) is an equilibrium then

\[ a_0 = g(c_0, \theta). \]
where $g$ is defined by the first-order necessary conditions for household maximization. This functional relation must hold for all equilibria, regardless of whether they are for a pure exchange or a production economy.

Let $(a_0, c_0)$ be an equilibrium for some economy $(\theta_0, r_0)$ with $r_0 \in \mathcal{P}$. Consider the pure exchange economy with $\theta_1 = \theta_0$ and $r_1 = c_0$. Our contention is that $(a_0, c_0)$ is a joint equilibrium process for asset prices and consumption for the pure exchange economy $(\theta_1, r_1)$. For all pure exchange economies, the equilibrium consumption process is $r$, so $c_1 = r_1 = c_0$, given more is preferred to less. If $c_0$ is the equilibrium process, the corresponding asset price must be $g(c_0, \theta_1)$. But $\theta_1 = \theta_0$ so $g(c_0, \theta_1) = g(c_0, \theta_0) = a_0$. Hence $a_0$ is the equilibrium for pure exchange economy $(\theta_1, r_1)$, proving the theorem. Since the set of equilibria in a production company is a subset of those in an exchange economy, it follows immediately that the equity premium in an exchange economy will not increase by modifying the technology to incorporate production.
Footnote

1. A related observation is made in Mehra and Prescott (1985) where they assert that incorporating production in their analysis along the lines of Brock (1982) or Donaldson and Mehra (1984) will not increase the equity premium.
References


