OPTIMAL ALLOCATION OF COMPETITIVE MARKETING EFFORTS REVISITED

Philippe A. Naert*

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139
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*Assistant Professor of Management, Sloan School of Management, M.I.T.
ABSTRACT

In a recent article in this Journal, Lambin presented an extension of the Dorfman-Steiner theorem to the case of an oligopolistic market. It is demonstrated that the market share optimization rule derived by Lambin is incorrect. A correct formulation is presented. A comparison of the absolute sales and market share optimization rules yields a relationship between absolute and relative advertising elasticity, previously obtained by Telser and used in the Lambin article. Analogous results for price and quality elasticity are also reported. Finally, some problems associated with how Lambin interprets his results are discussed.
Introduction

In a recent article in this Journal, Jean-Jacques Lambin presents an interesting theoretical, but empirically verified, extension of the Dorfman-Steiner theorem to the case of an oligopolistic market\(^1\). The objective was to derive a market share optimization rule, that is, a rule where competitive effects are explicitly taken into account. In the first part of this paper we will rederive the market share optimization rule and we will thus demonstrate that the result obtained by Lambin is incorrect\(^2\).

Comparing the absolute sales and the market share optimization rules yields a relationship between absolute and relative advertising elasticity, previously obtained by Telser and used in the Lambin article\(^3\). Analogous results for price and for quality elasticity are also derived. In the second part of this paper we will comments on the economic interpretation of the results.

First we will point out a case of suboptimization, and finally we will prove that what Lambin calls a long-term optimization rule is really something else. The discussion in the second part is of a more general nature in that most of it would remain valid even if Lambin’s market share optimization rule were correct.
Notation

Let \( q = q(p,s,x) \) = Unit sales of brand \( i \) per time period.\(^4\)

\( p = \) \( i \)'s sales price

\( s = \) \( i \)'s advertising outlays

\( x = \) \( i \)'s index of quality

\( c = c(q(p,s,x),x) = c(q,x) \) = unit average cost function

\( m = \) \( i \)'s market share

\( Q = \) total industry sales, i.e. \( m = q/Q \)

\( P = \) average market price

\( S_i = \) competitors' advertising outlays excluding brand \( i \)'s

\( X = \) average product - quality index

\( p^* = \) \( i \)'s relative price, i.e. \( p^* = p/P \)

\( s^* = \) \( i \)'s relative advertising, i.e. \( s^* = s/S_i \)

\( x^* = \) \( i \)'s relative quality, i.e. \( x^* = x/X \)

\( \eta_p = -(\partial q/\partial p) (p/q) = \) \( i \)'s absolute price elasticity

\( \mu = p \partial q/\partial s = \) \( i \)'s absolute marginal revenue product of advertising

\( \eta_s = \) \( i \)'s absolute advertising - sales elasticity

\( \eta_x = \frac{(\partial q/\partial x) c}{(\partial c/\partial x) q} = \) \( i \)'s absolute product quality elasticity

\( \eta_{p^*} = -(\partial m/\partial p^*) (p^*/m) = \) \( i \)'s market share elasticity with respect to \( i \)'s relative price

\( \eta_{s^*} = (\partial m/\partial s^*) (s^*/m) = \) \( i \)'s market share elasticity with respect to \( i \)'s relative advertising outlay.

\( \eta_{x^*} = (\partial m/\partial x^*) (x^*/m) = \) \( i \)'s market share elasticity with respect to \( i \)'s relative quality

Other symbols will be defined when needed.
The market share optimization rule

Brand $i$'s profit function can be written as

$$\pi = pq - qc - s$$

(1)

or

$$\pi = q(p,s,x) \left[ p - c(q(p,s,x), x) \right] - s$$

(2)

The Dorfman-Steiner theorem is simply the optimization of equation (2) with respect to absolute price, advertising and quality. The following well known result obtains

$$\eta_p = \mu = \eta_x \frac{p}{c} = \frac{1}{w}$$

(3)

where, with MC equal to marginal cost,

$$w = (p - MC)/p = \text{the percentage of gross margin}$$

Using the relative values defined in the previous section, equation (2) can be rewritten as

$$\pi = Qm (p^*, s^*, x^*) \left[ p^{*P} - c(m (p^*, s^*, x^*), x^*) \right] - s^*S_i$$

(4)

Necessary conditions for optimality are

$$\frac{\partial \pi}{\partial p^*} = \frac{\partial \pi}{\partial s^*} = \frac{\partial \pi}{\partial x^*} = 0$$

(5)

$$\frac{\partial \pi}{\partial p^*} = Q \frac{\partial m}{\partial p^*} (p^{*P} - c) + Qm (P + p^* \frac{\partial p^*}{\partial p^*} - \frac{\partial c}{\partial m} \frac{\partial m}{\partial p^*}) = 0$$

(6)
\[
\frac{\partial \pi}{\partial s^*} = Q \frac{\partial m}{\partial s^*} (p^* p - c) + Q m \left( - \frac{\partial c}{\partial m} \frac{\partial m}{\partial s^*} \right) - S_i - s^* \frac{\partial S_i}{\partial s^*} = 0 \quad (7)
\]

\[
\frac{\partial \pi}{\partial x^*} = Q \frac{\partial m}{\partial x^*} (p^* p - c) + Q m \left[ - \frac{\partial c}{\partial m} \frac{\partial m}{\partial x^*} - \frac{\partial c}{\partial x^*} (x + x^* \frac{\partial x^*}{\partial x^*}) \right] = 0 \quad (8)
\]

We will concentrate here on equation (7) and refer to the appendix for equations (6) and (8). First, we want to rewrite \( S_i + s^* \frac{\partial S_i}{\partial s^*} \) as a function of \( \theta \)

where

\[
\theta = 1 - \frac{s}{S_i} \frac{\partial S_i}{\partial s}
\]

\[
\frac{\partial S_i}{\partial s^*} = \frac{\partial S_i}{\partial s} \frac{\partial s}{\partial s^*} = \frac{\partial S_i}{\partial s} (S_i + s^* \frac{\partial S_i}{\partial s^*})
\]

\[
\frac{\partial S_i}{\partial s^*} (1 - s^* \frac{\partial S_i}{\partial s}) = S_i \frac{\partial S_i}{\partial s}
\]

\[
\frac{\partial S_i}{\partial s^*} = \frac{S_i \frac{\partial S_i}{\partial s}}{1 - (s/S_i)/(\partial S_i/\partial s)} = \frac{S_i \partial S_i/\partial s}{\theta} \quad (9)
\]

Using (9), \( S_i + s^* \frac{\partial S_i}{\partial s^*} \) reduces to \( S_i / \theta \).

Replacing \( S_i + s^* \frac{\partial S_i}{\partial s^*} \) by \( S_i / \theta \) in equation (7) and dividing by \( Q \frac{\partial m}{\partial s^*} \) we obtain

\[
p^* p - c - \frac{\partial c}{\partial m} - \frac{S_i}{\partial Q} \frac{\partial m}{\partial s^*} = 0 \quad (10)
\]

Now \( p^* p = p \) and

\[
c + m \frac{\partial c}{\partial m} = MC
\]
Equation (10) then reduces to

\[ \frac{S_i}{\theta Q \delta m / \delta s^*} = p - MC \]  

(11)

Multiplying the numerator and denominator of the left hand side of equation (11) by \( s^*/m \) and using \( s^*S_i = s \) and \( m = q/Q \), we obtain

\[ \frac{s/q}{\delta n_{S^*}} = p - MC \]  

(12)

In order to derive a relationship between absolute and relative advertising elasticity, we return to equation (3)

\[ \frac{1}{\mu} = w = \frac{(p - MC)}{p} \]

or

\[ \frac{s/q}{p(s/q)(\delta q/\delta s)} = \frac{p - MC}{p} \]

or

\[ \frac{s/q}{n_S} = p - MC \]  

(13)

Comparing equations (12) and (13) we find

\[ n_S = n_{S^*} q \]  

(14)

a relation obtained by Telser and used in the Lambin paper.\(^8\)

The following results based on equations (6) and (8) are derived in the appendix.

\[ \frac{p}{\zeta n_{P^*}} = p - MC \]  

(15)
and

\[ \eta_p = \eta_{p^*} \zeta \]  \hspace{1cm} (16)

where

\[ \zeta = 1 - \frac{(p/P)(\partial P/\partial p)}{(1 - (p/P)(\partial P/\partial p))} \]

\[ \frac{x \partial c/\partial x}{\rho \eta_{x^*}} = p - MC \]  \hspace{1cm} (17)

and

\[ \eta_x = \eta_{x^*} \frac{c_p}{x \partial c/\partial x} \]  \hspace{1cm} (18)

where

\[ \rho = 1 - \frac{(x/X)(\partial X/\partial x)}{1 - (x/X)(\partial X/\partial x)} \]

Combining equations (15), (12) and (17) the market share optimization rule is obtained

\[ \zeta \eta_{p^*} = \frac{pq \theta \eta_{s^*}/s}{\theta \eta_{s^*}/s} = \frac{pp \eta_{x^*}/(x \partial c/\partial x)}{(x \partial c/\partial x)} = \frac{1}{\mu} \] \hspace{1cm} (19)

The optimization rule obtained by Lambin was

\[ \frac{p}{\eta_{p^*}} = \alpha/\eta_{s^*} = \beta/\eta_{x^*} \] \hspace{1cm} (20)

where

\[ \alpha = \frac{s}{q} \]

\[ \beta = \frac{x \partial c/\partial x}{x \partial c/\partial x} \]

We should in fact complete the optimization rule by adding another equality to (20), namely "is equal to \( p - MC \)." In order to make (20) and (19) comparable we can rewrite (20) as

\[ \eta_{p^*} = \frac{pq \eta_{s^*}/s}{\theta \eta_{s^*}/s} = \frac{pp \eta_{x^*}/(x \partial c/\partial x)}{(x \partial c/\partial x)} = \frac{1}{\mu} \] \hspace{1cm} (21)
Let us compare the optimization rule derived here (19) with the result obtained by Lambin (21). If there were no competitive reaction \( \Theta \) would be equal to one. Not so for \( \zeta \) and \( \rho \), because the average market price \( P \) and the average product quality index \( X \) are affected by changes in \( p \) and \( x \). Let us assume however, that \( P \) and \( X \) are defined excluding brand \( i \). We can then say that with no competitive reaction \( \Theta, \zeta, \) and \( \rho \) are equal to one. Furthermore, if there is no competitive reaction, \( \eta_{p^*} = \eta_p, \eta_{s^*} = \eta_s \) and \( \eta_{x^*} = \eta_x \), where \( \eta_x' = (\partial q/\partial x)(x/q) \), and the market share optimization rule should reduce to the absolute sales optimization rule\(^9\). That (19) reduces to (3) when there is no competitive reaction is easily verified\(^10\). It appears then as if Lambin's market share optimization rule was derived for the non-competitive-reaction case. While the objective was to derive a rule that takes competition explicitly into account, this was not actually carried out. For example, in Lambin's derivation \( \partial(p^*P)/\partial p^* \) is assumed to be equal to \( P \). That is only true if there is no competitive reaction. With \( \eta_{p^*} = \eta_p, \eta_{s^*} = \eta_s \) and \( \eta_{x^*} = \eta_x \), we would then expect Lambin's optimization rule to reduce to the absolute sales optimization rule. This is verified for the price and advertising elasticities but not for the quality elasticity. This is explained by an error in the transformation of the unit cost function from absolute values to relative values. Indeed, \( c(q(p,s,x),x) \) can be written as \( c(Qm(p^*,s^*,x^*), x^*x) \). Since \( Q \) is a constant, and if we omit the argument of \( m \), the unit cost function becomes
c(m,x^*x) and not c(m,x^*) as was used by Lambin.

In summary then, equation (3) is the optimization rule when we consider absolute elasticities, equation (19) is the market share optimization rule, based on relative elasticities. Looking at the two equations simultaneously leads to a relationship between absolute and relative advertising elasticity that had also been obtained by Telser, and at the same time produces two analogous results for price elasticity and quality elasticity.

Economic interpretation of the results

At this point we turn to the economic interpretation of the results. The following is one of the relations used by Lambin in determining whether the actual values of the decision variables are optimal or not

\[ \frac{\eta_{s^*}}{\eta_{p^*}} = \frac{s}{pq} \]  

If we assume that \( \zeta = 0 \), equation (22) also follows from the correct optimization rule. For the household durable product examined by Lambin, the following values were empirically determined: \( \eta_{s^*} = 0.283 \), \( \eta_{p^*} = 3.070 \), so that \( \eta_{s^*}/\eta_{p^*} = 0.092 \). The right hand side of equation (22), namely, unit advertising outlays expressed as a percentage of sales price had a mean value of 12.5 per cent for the last four years. Lambin then concludes that the firm seems to have overspent on advertising. However, this is not necessarily true. Indeed, concluding without
qualification that the firm seems to have overspent on advertising, implies an optimum (or at least a fixed) value for price. If the actual price were considerably below its optimal value, it would even be possible that the firm was underspending on advertising (when the actual price is made equal to the optimum price). So, for the actual price given and constant, Lambin's conclusion is correct. But if we do not know whether the actual price is optimal, then of course there exists an infinite number of pairs of values for p and s which satisfy (22). In other words, relationship (22) is a necessary but not sufficient condition for optimality. In order to find the optimum, we need to take into account the right hand side of the optimization rule, and in particular in this case

\[ \zeta \eta_p = 1/w \]  

(23)

In this case \( \eta_p = 3.070 \), \( \zeta \) is less than or equal to one, and we are given that \( w \) is approximately equal to 30 per cent. Thus, \( 1/w = 3.333 \), so that the actual price is below the optimum price. Solving equation (23) for \( p \) gives us the optimal price:

\[ p = \left( \frac{1}{1 - 1/\zeta \eta_p} \right) MC \]  

(24)

Given the optimal value for price, we could then use (22) to find the optimum value for unit advertising outlays.
So far we have concentrated on short-term considerations. Lambin also derives a long-term optimization rule, namely

\[
\frac{\eta_s^*}{\eta_p^*} = \frac{\eta_s^w}{1 - \lambda/(1+r)} \tag{25}
\]

which corresponds to the short-term optimization rule (22). Lambin assumed that the ratio of the long-term relative advertising and price elasticities \((\eta_{s^*}^{LT}/\eta_{p^*}^{LT})\) is equal to the ratio of the short-term relative advertising and price elasticities\(^{14}\). That is, he assumes

\[
\begin{align*}
\eta_{s^*}^{LT} &= \frac{\eta_{s^*}}{k} \\
\eta_{p^*}^{LT} &= \frac{\eta_{p^*}}{k}
\end{align*}
\]

where \(k\) stands for \(1 - \lambda/(1+r)\)

And similarly

\[
\begin{align*}
\eta_{s}^{LT} &= \frac{\eta_{s}}{k} \\
\eta_{p}^{LT} &= \frac{\eta_{p}}{k}
\end{align*}
\]

Now, let us examine (25) in more detail. We will show that (25) is not really the long-term optimization rule for determining optimum price and advertising. Indeed, \(\eta_{s^*}, \eta_{p^*}, \eta_{s}, \lambda, \text{ and } r\) are all given and constant. Therefore, if the two sides of (25) are not equal, only \(p\) can be changed to make them equal. So (25) does not tell us anything about the optimum value of \(s\). What does (25) actually mean then? From (14) and (16) we can write

\[
\frac{\theta \eta_{p^*}}{\eta_p} = \frac{\theta \eta_{s^*}}{\eta_s} \tag{26}
\]
Or assuming $\zeta = 0$ to make (26) comparable to Lambin's result we have

$$\frac{\eta_s^*}{\eta_p^*} = \frac{\eta_s}{\eta_p} \quad (27)$$

With $\eta_p,LT = \eta_p/k$, at optimality we have $\eta_p/k = 1/w$, so that (27) reduces to

$$\frac{\eta_s^*}{\eta_p^*} = \frac{\eta_s w}{1 - \lambda/(1+r)}$$

So, (25) is nothing more than an obscure way of writing $\eta_p/k = 1/w$. The actual long-term market share optimization rule is simply given by$^{15}$

$$\zeta \eta_p^*/k = pq\theta \eta_s^*/ks = 1/w \quad (28)$$

Similarly, the long-term absolute sales optimization rule is

$$\eta_p/k = pq\eta_s/ks = 1/w \quad (29)$$

Finding the optimal values goes along the same lines as in the short-term optimization problem discussed earlier in this section.

Finally, we may have reason to believe that the ratio of short-term and long-term elasticities is not equal to $k$ for both the advertising and the price variables. That $\eta_s,LT = \eta_s/k$ followed from looking at advertising as an investment that has a return in the first period but also returns, although progressively smaller, in later periods. An analogous analysis for the price variables cannot be made. Sure, there may and will
be lags in the adjustment of changes in quantity to changes in prices, but there is no reason to believe that we have exactly the same effect as with the advertising variable. With the period of observation being one year, we may argue that $\eta_{p,LT}$ and $\eta_p$ are not very different. When this is true, and because including different lags for different variables may be hard, one will usually assume $\eta_{p,LT}$ and $\eta_p$ to be the same, and the optimization rules (28) and (29) can be adjusted accordingly.¹⁶
Appendix

Equation (6) is reproduced below as equation (A.1)

\[
\frac{\partial \pi}{\partial p^*} = Q \frac{\partial m}{\partial p^*} (p^*p - c) + Qm (p + p^* \frac{\partial p}{\partial p^*} - \frac{3c}{\partial m} \frac{\partial m}{\partial p^*}) = 0 \quad (A.1)
\]

\[
\frac{\partial P}{\partial p^*} = \frac{\partial P}{\partial p} \frac{\partial p}{\partial p^*} = \frac{3P}{\partial p} (P + p^* \frac{\partial p}{\partial p^*})
\]

\[
\frac{\partial P}{\partial p^*} = \frac{P}{1 - (p/P)(\partial P/\partial p)}
\]

with \( \xi = 1 - (p/P)(\partial P/\partial p) \) we obtain

\[
P + p^* \frac{\partial p}{\partial p^*} = P/\xi \quad (A.2)
\]

Using equation (A.2) in (A.1) and dividing through by \( Q3m/\partial p^* \) we obtain

\[
- \frac{mP}{\xi \partial m/\partial p^*} = p - MC \quad (A.3)
\]

Multiplying the numerator and denominator of (A.3) by \( p^*/m \), (A.3) reduces to

\[
\frac{p}{\xi \eta_{p^*}} = p - MC \quad (A.4)
\]

To find the relationship between \( \eta_p \) and \( \eta_{p^*} \) we write from equation (3)

\[
\eta_p = 1/w
\]

or

\[
P/\eta_p = p - MC \quad (A.5)
\]
Comparing (A.4) and (A.5) we find

\[ \eta_p = \eta_p \zeta \]  

(A.6)

Equation (8) is reproduced below as equation (A.7)

\[ \frac{\partial \pi}{\partial x^*} = Q \frac{\partial m}{\partial x^*} (p*P - c) + Qm \left[ - \frac{\partial c}{\partial m} \frac{\partial m}{\partial x^*} - \frac{\partial c}{\partial x^*} (X + x^* \frac{\partial X}{\partial x^*}) \right] = 0 \]

(A.7)

\[ \frac{\partial X}{\partial x^*} = \frac{\partial X}{\partial x^*} \frac{\partial X}{\partial x^*} = \frac{\partial X}{\partial x} (X + x^* \frac{\partial X}{\partial x^*}) \]

\[ \frac{\partial X}{\partial x^*} = \frac{X \partial X}{\partial x} \frac{\partial X}{\partial x} \]

\[ \frac{\partial X}{\partial x^*} = \frac{X \partial X}{\partial x} \frac{\partial X}{\partial x} \]

with

\[ \rho = 1 - (x/X)(\partial X/\partial x) \]

we obtain

\[ X + x^* \frac{\partial X}{\partial x^*} = X/\rho \]  

(A.8)

Using equation (A.8) in equation (A.7) and dividing through by

\[ Q \frac{\partial m}{\partial x^*} \]

we obtain

\[ \frac{m X \partial c}{\rho \partial m} = p - MC \]  

(A.9)
\[ \mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \]

\[ \frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} \quad \frac{\partial u}{\partial y} = \frac{\partial \phi}{\partial y} \]

\[ \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{\kappa^2} \]

\[ \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial y^2} \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \phi}{\partial x} \]

\[ \frac{\partial \phi}{\partial y} \bigg|_{y=0} = 0 \quad \frac{\partial \phi}{\partial y} \bigg|_{y=\eta} = 1 \]

\[ \phi = \frac{2 \kappa^2}{\kappa^2 + 1} x y + \chi \]

\[ \chi \]

\[ \chi = \frac{2 \kappa^2}{\kappa^2 + 1} x y + \phi \]

\[ \phi = \frac{2 \kappa^2}{\kappa^2 + 1} x y + \phi \]

\[ \phi = \frac{2 \kappa^2}{\kappa^2 + 1} x y + \phi \]
Multiplying both the numerator and denominator of (A.9) by \( x^*/m \), (A.9) reduces to

\[
\frac{x \partial c/\partial x}{\rho \eta_{x^*}} = p - MC
\]  
(A.10)

To find the relationship between \( \eta_x \) and \( \eta_{x^*} \) we use equation (3) again

\[
\eta_x \frac{p}{c} = \frac{1}{\omega}
\]

or

\[
\frac{c}{\eta_x} = p - MC
\]  
(A.11)

Equating (A.10) and (A.11) we obtain

\[
\eta_x = \eta_{x^*} \frac{c \rho}{x \partial c/\partial x}
\]  
(A.12)

The relationship between relative and absolute quality elasticity seems slightly more complicated than the analogous results for price elasticity and advertising elasticity. The reason is that \( \eta_x \) and \( \eta_{x^*} \) were defined in different ways. Indeed, if we had defined absolute quality elasticity as \( \eta'_x = \frac{\partial q}{\partial x} \frac{x}{q} \) in analogy to the definition of relative quality elasticity \( \eta_{x^*} \), the absolute sales Dorfman-Steiner rule would be (also using \( \eta_s \) and not \( \mu \))

\[
\eta_p = \frac{pq}{s} \eta_s = \frac{\eta_{x^*} p}{x \partial c/\partial x} = \frac{1}{\omega}
\]  
(A.13)
from (A.13), it follows that

$$\frac{x \frac{\partial c}{\partial x}}{\eta_x} = p - MC \quad \text{(A.14)}$$

Comparing (A.14) and (A.10) we now find

$$\eta_{x'} = \eta_{x*} \rho \quad \text{(A.15)}$$
Footnotes


2I should mention here that I read a draft of this paper in March of 1969. At that time I did not catch any major errors. We may point out in passing several printing errors. In particular, equations (A.4), (A.5), (A.6), (A.8), (A.9) and (A.10) all have $c_i$ (argument) replaced by $c_i$ times that argument. Also in (A.10),

$$\left( - \frac{\partial c_i}{\partial x_i^*} \frac{\partial c_i}{\partial m_i} \frac{\partial m_i}{\partial x_i^*} \right)$$

was intended to read

$$\left( - \frac{\partial c_i}{\partial x_i^*} - \frac{\partial c_i}{\partial m_i} \frac{\partial m_i}{\partial x_i^*} \right).$$

The last term in equation (14) should be $m \lambda n/(1 + r)^n$, and in introducing estimates in equation (47), 0.07 was substituted for 1.07.


4Since we will discuss one brand versus the industry, we can make the notation less cumbersome by not introducing a subscript $i$. We will use lower case letters for the brand and capital letters for the industry.


6With Lambin, we assume throughout that industry sales $Q$ is a constant.

7Throughout we will assume that the second order conditions for a maximum are satisfied.

8Telser, "Advertising and Cigarettes", p. 486.

9Why $\eta_{x^*}$ does not reduce to $\eta_x$ is explained in the appendix.

10Note that it is easy to verify that (19) reduces to (A.13) in the non-competitive-reaction case, and (A.13) is just another way of writing (3).
Assuming of course that the estimated values are treated as true values. Lambin has always issued warnings regarding this point by considering the results derived from these estimates as indicators of direction, and not as "the" optimum values.

Finding the optimal price from equation (24) is not as trivial as it looks, except when MC is constant.


See Lambin's paper, Footnote 27, page 478.

We concentrate here on the price and advertising variables, and assume quality to be fixed.

For an example where ηₚ, LT and ηₚ are implicitly assumed to be equal, see Jean-Jacques Lambin, "Measuring the Profitability of Advertising: An Empirical Study", Journal of Industrial Economics, Vol. 17, No. 2, April 1969, pp. 86-103.