ON THE USE OF ALTERNATE METHODOLOGIES IN DETECTING ABNORMAL PERFORMANCE WITH CALENDAR TIME CLUSTERING OF EVENTS

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1. Introduction

Clustering of events in calendar time occurs quite often in accounting studies, particularly in studies which assess the impact of regulatory events, SEC or FASB pronouncements.\(^1\) Even studies examining stock price reaction to accounting data [e.g., Ball and Brown (1968), Biddle and Lindahl (1982), Ricks (1982), Beaver and Landsman (1983)] can also have events clustered in calendar time. With calendar time clustering of events, the issue of cross-sectional dependence in returns becomes important, and if not properly accounted for, it can result in serious errors of inference. Researchers [e.g., Collins and Dent (1984), hereafter, CD] have recommended the use of estimated generalized least squares (EGLS) in such situations. The objective of this paper is to provide evidence on the performance of alternate methodologies employed in detecting abnormal performance when events are clustered in calendar time. I put particular emphasis on examining the performance of EGLS, since currently, there is a lack of both analytical results as well as experimental evidence on its performance.\(^2\)

I examine the issue of sampling error in detail in this paper. Two factors make sampling error important. First, in many empirical studies, especially if EGLS is to be employed, the number of parameters to be estimated is generally non-trivial compared to the number of available time-series observations. Second, actual returns are unlikely to conform to the assumptions of any model and departures from the assumptions (e.g., non-

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\(^1\)See, for example, studies such as Beaver, Christie, and Griffin (1980), Collins, Rozeff, and Dhaliwal (1981), Leftwich (1981), and Schipper and Thompson (1983).

\(^2\)Phillips (1985) derives the exact finite sample distribution of the EGLS estimator. His analysis however also does not lead to analytically tractable expressions for the problem at hand.
stationarities in the data) also contribute to sampling error. For this reason, I conduct simulations using actual returns.

Brown and Warner (1980, 1985), and Dyckman, Philbrick, and Stephan (1984), hereafter, BW'80, BW'85, and DPS respectively, also use actual returns data in their simulations to examine the performance of alternate event study methodologies. However, in these papers, since the primary focus is not on calendar time clustering, this issue is addressed only to a limited extent, e.g., potential methodologies like EGLS that can be used in the presence of calendar time clustering are not considered. I consider almost all the possible methodologies that can be used in the presence of calendar time clustering.

There is growing tendency toward using weekly data since it provides a reasonable balance between reducing measurement error in returns (a potential problem in using daily data) and still maintaining enough power (a potential problem in using monthly returns). Hence, in addition to daily data, I also use weekly data. The use of weekly returns also facilitates direct comparison of my results with those of CD.

I find that with EGLS sampling error is a very serious problem. If there are only about ten time-series observations available for each parameter to be estimated in EGLS then the method leads to type I error rates that are about twice the nominal rate. If there are only about four

3 CD also examine some of the same issues as in this paper. However, the issue of sampling error is not addressed in their paper. Sampling errors do not appear to be important in the CD paper for two reasons. First, in all their simulations, the number of time-series observations (100 on each firm) is considerably larger than the number of parameters being estimated (number of firms in each sample is 10). Second, they use artificially generated returns data, which do not suffer from non-stationarities that actual returns may possess. CD do note the limitations of using artificially generated returns data and do not imply that their results apply to actual returns data too.
to five times as many time-series observations as the number of parameters then an application of EGLS can lead to actual type I error of three to four times the expected rate.

The use of the CRSP equally-weighted index as the market index leads to almost negligible cross-correlations in residuals and for a random sample of firms, all methods other than EGLS result in type I error rates close to the nominal rate. However, if the value-weighted index is used, the performance of methods which do not control for cross-correlations gets worse as the sample size increases or as one goes from daily to weekly data. These increased rejection rates are consistent with the presence of non-zero cross-sectional correlations in (value-weighted) market model residuals, even when they are not from the same industry.

With simultaneous industry and calendar time clustering, methods ignoring cross-correlations in residuals result in higher than expected type I errors and even the use of an equally-weighted market index does not eliminate the problem. The portfolio return method generally works well in such situations. Also, there is evidence indicating that some improvement in the performance of EGLS can be achieved if one can reduce the number of parameters to be estimated by imposing reasonable restrictions on the parameters. The work here also suggests that if one is interested in cross sectional variation in abnormal returns when calendar time clustering is present, the use of portfolio method recommended by Sefcik and Thompson (1986), if feasible, may in general be preferable to EGLS. Alternatively, in using EGLS, one could use bootstrap (e.g., using bootstrap bias-adjusted

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4 This can, for example, be achieved by constraining the correlation coefficient to be the same for each firm-pair within an industry.
asymptotic standard errors as suggested by Marais (1986b)] or other computer intensive resampling techniques to estimate the standard errors correctly.

The rest of the paper is organized as follows. In section 2, I discuss the problem and the alternate methodologies that can be employed. In section 3 I discuss the approach adopted for simulations and the results of those simulations and in section 4, I present some concluding remarks.

2. THE PROBLEM AND THE CHOICE OF ALTERNATE METHODOLOGIES

The problem that I consider can be formulated as a special case of the Multivariate Regression Model (MVRM) of Schipper and Thompson (1983), that in which all sample firms share a common reaction (abnormal performance) \( \mu \) to a single event.\(^5\) Let \( \bar{R}_{ir} \) denote the return on security \( i \) in period \( r \) (the period \( r \) can be one day or one week):

\[
\bar{R}_{ir} = \alpha_i + \beta_i \bar{R}_{mr} + \mu \delta_r + \bar{\varepsilon}_{ir}
\]

where \( \bar{R}_{mr} \) is the (value-weighted or equally-weighted) return on the market in period \( r \), \( \bar{\varepsilon}_{ir} \) is the idiosyncratic component of the return \( \bar{R}_{ir} \), \( \delta_r \) equals 1 in the event period and 0, otherwise, and \( i \) ranges from 1 to \( N \). The variance of \( \bar{\varepsilon}_{ir} \) is \( \sigma_i^2 \) and \( \alpha_i \) and \( \beta_i \) are assumed to be firm-specific constants. The problem is that of estimating the abnormal performance \( \mu \) and the null hypothesis is that there is no abnormal performance, i.e., \( \mu = 0 \).

There are several different procedures available to estimate \( \mu \) and each one makes different assumptions about the variance-covariance matrix \( \Sigma \) of the

\(^5\)The MVRM methodology in general allows for firm reactions to vary cross-sectionally during the event period. If in the true model firm reactions do vary then EGLS would yield biased and inconsistent estimates. My purpose here is to examine the impact of sampling errors on the performance of EGLS and I want the basic assumptions underlying EGLS to hold. Hence, I assume that all firms share the same common reaction. The simulations design is also consistent with this assumption. However, if the true model is that the firm reactions vary cross-sectionally then one can examine joint hypotheses about these reactions by using tests developed in Binder (1985) and Schipper and Thompson (1985).
prediction errors $\bar{e}_{it}$. I consider the following five different alternate methodologies:

(1) Ordinary Least Squares in Cross-section (OLS): Under this approach, one assumes $\Sigma = \sigma^2 I$ and estimates $\mu$ by computing a simple cross-sectional mean of the prediction errors during the event period $t$.

\[
\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} e_{it} = \bar{e}_t \tag{2}
\]

\[
\text{var}(\hat{\mu}) = \frac{\hat{\sigma}^2}{N} \tag{3}
\]

where $\hat{\sigma}^2$ is the cross-sectional variance of estimated prediction errors:

\[
\hat{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (e_{it} - \bar{e}_t)^2. \tag{4}
\]

Under the null hypothesis the statistic $\hat{\mu}/[\text{var}(\hat{\mu})]^{1/2}$ follows the t-distribution with $N-1$ degrees of freedom.

Below I provide only brief discussions of the second and third methods. The appendix provides more details.

(2) PATELL'S STANDARDIZED RESIDUAL METHOD (PATELL):

This method takes into account the cross-sectional differences in residual variances. Under this method each prediction error is first divided by its estimated standard deviation to yield a standardized prediction error $V_{it}$:

\[
V_{it} = e_{it}/(s_{it}C_{it}) \tag{5}
\]

where $s_{it}^2$ is an estimate of $\sigma_i^2$ obtained from the market model regression and $C_{it}$ is an adjustment factor used to correct the prediction error.

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6 The prediction error $e_{it}$ for firm $i$ during the event period $t$ is obtained as:

\[
e_{it} = R_{it} - (a_i + b_i R_{mt})
\]

where $a_i$ and $b_i$ are the estimates of $\alpha_i$ and $\beta_i$ from the market model regression over the estimation period.
variance because the prediction errors are outside the estimation period.
The standardized prediction errors are aggregated assuming cross-sectional
independence to form the normalized sum \( Z_{vt} \) which is normally distributed
for large \( N \). Patell's method is similar to estimating \( \mu \) through WLS.\(^7\)

(3) Estimated Generalized Least Squares (EGLS):

If one can estimate \( \Sigma \), the variance covariance matrix of the \( e_{it} \)s, then
one can apply EGLS to estimate \( \mu \). Let \( S \) represent the estimate of \( \Sigma \). \( S \)
will be non-singular if \( T \), the number of time series observations per firm,
is greater than \( N \), the number of firms in the sample. Non-singularity of \( S \)
is a requirement of EGLS.\(^8\) Then the EGLS estimator of \( \mu \) is:

\[
\hat{\mu} = l'S^{-1}e_t/(1'S^{-1}l)
\]
and its estimated variance is:

\[
\text{var}(\hat{\mu}) = (1'S^{-1}l)^{-1}.
\]

Here \( e_t \) is the \( Nx1 \) vector of the prediction errors \( e_{it} \)s and \( l \) is an \( Nx1 \)
vector of ones. Under the null hypothesis the statistic \( \hat{\mu}/[\text{var}(\hat{\mu})]^{1/2} \)
follows the \( t \)-distribution with \( N-1 \) degrees of freedom.

(4) Estimated Generalized Least Squares - INDUSTRY BASIS (EGLS-IND):

This method is same as method (3) with the only difference that the
contemporaneous variance-covariance matrix \( \Sigma \) is assumed to be block diagonal
instead of assuming it to be full as in EGLS. Firms are assumed to be
stacked according to industries, with each block representing one industry.
The cross-industry correlations are assumed to be zero and the within-

\(^7\)In fact, all the tests were conducted employing WLS. The results are
similar to those for Patell's method and are not reported.

\(^8\)Although \( S \) is non-singular for \( T>N \), for normal disturbances the
inverse of the estimated variance covariance matrix \( S^{-1} \) follows an inverted
Wishart distribution, which has undesirable properties if \( T \), the number of
time series observations in the estimation period, is less than twice the
number of firms \( N \) [See Press (1972)].
industry (block) elements of Σ are estimated as in method 3. This method can offer several potential advantages over plain EGLS (method 3). First, if most cross-industry correlations are zero then taking that fact into account will improve the efficiency of the estimators. Second, when Σ is assumed to be block-diagonal, one has to estimate fewer parameters than under plain EGLS and many times when EGLS is not feasible because of insufficient data, method 4 may be feasible.

The next method that I consider also accounts for the contemporaneous cross-firm correlations. This method was first proposed by Jaffe (1974) and Mandelker (1974).

(5) Jaffe-Mandelker's Portfolio Return Method (Portfolio):

This method uses a time series estimate of the variance of portfolio return as an estimator of var(μ). Specifically, during the estimation interval, for each period r, the residuals are aggregated across all the N firms to form the portfolio return in period r $\hat{\varepsilon}_p^r$:

$$\hat{\varepsilon}_p^r = \frac{1}{N} \sum_{i=1}^{N} \varepsilon_i^r$$

(8)

The sample variance of the portfolio return $\hat{\varepsilon}_p^r$ is:

$$\text{var}(\hat{\varepsilon}_p) = \frac{1}{T-1} \sum_{r=1}^{T} (\hat{\varepsilon}_p^r - \hat{\varepsilon}_p)^2$$

(9)

where $\hat{\varepsilon}_p$ (=$\frac{1}{T} \sum_{r=1}^{T} \hat{\varepsilon}_p^r$) is the mean portfolio return over the estimation interval. The estimator for the reaction μ during the test period is same as under method 1 [equation (2)] and the estimated var(μ) from equation (9) is used as an estimate of var(μ). Since the portfolio method (in the form

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9 Bernard (1987) provides evidence that the cross-industry correlations are small, although at the 5% level, one can reject the hypothesis that they are zero for both daily and weekly (value-weighted) market model residuals.
in which I use it) gives equal weight to all securities, it is likely to be inefficient relative to WLS and EGLS.\(^1\)

### 3. SIMULATIONS

#### 3.1 EXPERIMENTAL DESIGN

The experiments are conducted using both daily and weekly data. Four different sample sizes (10, 20, 50, and 100 firms) are considered for daily data and three different sample sizes (10, 20, and 50 firms) for weekly data. The experimental design for the initial experiments is as follows.

**Daily Data:** For a sample size of \(N\) firms, a day is selected at random from the time period July 1, 1963 to Dec. 24, 1985 and denoted day 0. Day 0 and the next four days comprise the five separate event days and the days -250 to -1 with respect to day 0 constitute the estimation interval for all the five event days. All the firms that have CRSP daily return data over the period -250 to +4 constitute the population of potential firms that can form the sample. Firms are selected at random without replacement from this population till there are \(N\) firms with no missing returns for each of the event days (days 0 through +4) and at least 150 non-missing returns over the estimation interval. For each of these firms, market model parameters as well as any other parameters, such as the variance-covariance matrix of the residuals, required for any of the five methods are estimated over the estimation interval. Firms in the same 2-digit SIC codes are assumed to

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\(^1\)It is, however, possible to refine this method further, for example in forming the portfolio return use inverse \(\sigma^2\) weights rather than equal weights.
belong to the same industry while applying the EGLS-IND method.\textsuperscript{11} Four different levels of abnormal performance (0\%, 0.5\%, 1.0\%, and 2\%) are introduced for each of the event days. The procedure for introducing a given level of abnormal performance on an event day is to add that level of abnormal performance to the observed return for each security on the event day. Each of the five methods is then used to detect abnormal performance for each of those five event days. The whole procedure is repeated for 100 independent trials.\textsuperscript{12}

**Weekly data:** The procedure followed is similar to that for daily data with the following changes. The time period from which event week 0 is drawn is from the beginning of July 1964 to the end of November, 1986. For this study, weekly returns are defined as continuously compounded returns over five (trading) day periods and are obtained by cumulating the CRSP daily returns. Week 0 and the following four weeks comprise the five separate event weeks while the estimation interval spans weeks -100 to -1. To be included in the sample, a firm has to have no missing weekly returns for weeks 0 through +4 and at least 50 non-missing weekly returns during the

\textsuperscript{11}Use of a 2-digit instead of 3-digit [Bernard (1987), CD] SIC codes for the definition of an industry implies broader industry categories. With a random sample of firms, using a narrower (3-digit) definition of an industry would imply that essentially the EGLS-IND method would be reduced to WLS.

\textsuperscript{12}The experimental set-up thus yields 500 observations [100 trials x 5 event days] for each sample size. Alternatively, one could use only day 0 as an event day and repeat the experiment 500 times. The advantage of my set-up is lower computational costs. The disadvantage is that my procedure introduces some dependence within each set of five observations corresponding to the same estimation interval. The same values of market model parameter estimates are used in computing the prediction errors over the different event days, and the estimation errors in these parameters are likely to introduce some dependence in the observations.
estimation interval. The levels of abnormal performance introduced during the event weeks are 0%, 1%, 2% and 5%.

3.2 RESULTS FOR A RANDOM SAMPLE OF FIRMS

Table 1 presents the cross-sectional means and standard deviations of selected market model parameters (using the equally-weighted index) by sample size. The cross-sectional means and standard deviations are computed over the 100 sample-wide average estimates of the various parameters. For daily data, the grand average residual variance is about $8 \times 10^{-4}$, while for weekly data, it is about $29 \times 10^{-4}$, less than five times the daily residual variance. This is consistent with the presence of negative serial correlations in daily residuals.\(^{13}\) Table 1 also indicates that the cross-firm (irrespective of the firms' industries) correlations in the (equally-weighted) market model residuals are negligible ($\sim 0.1\text{-}0.2\%$, although many of them are significant at the 5% level). The magnitudes for within-industry correlations are about 1% in daily data and about 3% in weekly data. Most of these correlations are significant at the 1% level indicating that some within-industry cross-correlation in residuals remains when the equally-weighted market index is used.

Tables 2 and 3 respectively present the rejection frequencies for daily and weekly data for the various sample sizes and for the various levels of abnormal performance. The rejection frequencies are based on using one-tailed tests at the 0.05 level of significance. Only the average results over the five days (weeks) are reported. Assuming that the observations

\(^{13}\)French and Roll (1986) also report a similar finding on raw returns for the time period of 1963-1982. Bernard finds evidence of positive serial correlation in daily residuals but his result is likely to be sample-specific since he uses 1984 data for daily residuals and 1981-84 data for weekly residuals.
across the five days (weeks) are independent, the 95% confidence interval for the mean rejection rate over the five days (weeks) with 0% abnormal performance introduced, is \(5\% \pm 2\%\).\(^\text{14}\)

The major observations from these tables are following.

1. For all methods other than EGLS, with no abnormal performance introduced, either with daily or weekly data, the mean frequency of type I error is close to 5% and generally lies within the 95% interval of 3% to 7%. OLS is less powerful than Patell's or EGLS-IND. Since the firms in the sample are not constrained to come from any one industry, results from the latter two are very similar. The portfolio return method is also less powerful than the others, again reconfirming its inefficiency.

2. The use of EGLS has serious problems. Except for a sample size of 10 using daily data, the mean frequency of type I error exceeds the upper limit of the 95% interval around the expected rate of 5%, e.g., for daily data a sample size of 100 firms with 250 time-series observations leads to an empirical rejection frequency of 17.6%, almost four times as much the expected rate of 5%. The deterioration is directly related to \(N/T\) -- which is about twice the ratio of the number of parameters to be estimated in using EGLS to the number of available time-series observations.\(^\text{15}\)

The possible misspecification of the various methods, especially EGLS, is examined in detail as follows. For each method with no abnormal

\(^{14}\)Using a normal approximation to the binomial distribution, the general formula for the 95% confidence interval about the expected rejection rate can be written as: \(r \pm 1.96/\sqrt{p(1-p)/N}\) where \(r\) is the expected rejection rate, \(p\) is the probability of rejection and \(N\) is the number of trials. Assuming independence across the five days (weeks), \(N\) equals 500 in this calculation.

\(^{15}\)The ratio of the number of parameters to be estimated to the number of available observations is given by \(N(N+1)/2NT\).
performance introduced, a cross-sectional mean and standard deviation of \( \hat{\mu} \) and a cross-sectional average of the estimated standard error of \( \hat{\mu} \) are computed by sample size. These cross-sectional numbers are computed across the 500 trials [5 event days (weeks) x 100 trials].

To see if a method produces an unbiased estimate of the abnormal performance \( \mu \), t-statistics are computed using the cross-sectional mean and standard deviation of \( \hat{\mu} \). None of these t-statistics exceeds two in absolute value suggesting that all methods produce unbiased estimates of \( \mu \). The next question is whether the standard errors of \( \hat{\mu} \) generated by the various methods are also unbiased. For a given sample size, for any method, this question can be answered by comparing the estimated cross-sectional average of the standard error of \( \hat{\mu} \), denoted se-est., with the estimated cross-sectional standard deviation of \( \hat{\mu} \), denoted se-true. Table 4 provides this comparison. As an example, for a sample size of 100 and with daily data, for the EGLS method an unbiased estimate of the true standard error of \( \hat{\mu} \) (se-true) is 0.26% while the estimate that EGLS uses is only 0.151% on average (se-est.). Thus the estimate of the standard error used by EGLS is on average only 58.1% of the true standard error. Table 4 also provides the ratios of se-est. to se-true. If a method produces an unbiased estimate of the standard error of \( \hat{\mu} \) then the ratios in the table corresponding to that

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16 The separate estimates based on the 100 trials for each of event day (week) are similar across event days (weeks) and are close to those computed for all the 500 trials.

17 This procedure assumes that for any method the estimated cross-sectional standard deviations of \( \hat{\mu} \) is a reasonably precise estimate of the true standard error of \( \hat{\mu} \). As noted earlier, the separate estimates of the cross-sectional standard deviation of \( \hat{\mu} \) for the various event days (weeks) are quite similar across the event days (weeks) and also are close to those computed over all the 500 trials. Hence the comparison is likely to be meaningful.
method should be close to 1.0. The main observation from this table is that except for the EGLS method, all other methods appear to produce unbiased estimates of the standard error. EGLS on the other hand generally understates the standard error and the degree of understatement increases with N/T.\(^{18}\) This evidence is consistent with the higher than nominal type I errors reported in tables 2 and 3.

The direct relation between the degree of understatement of the standard error of and N/T is a reflection of the role sampling errors play in affecting the performance of EGLS. The estimate of the standard error of \(\hat{\mu}\) that EGLS uses is the asymptotic standard error \((1'S^{-1}l)^{-1/2}\). It can be proved that in finite samples this estimator is downward biased assuming that the disturbances \(z_{it}\)'s in equation (1) are normally distributed. In fact, theorem 2 in Freedman and Peters (1984) implies that

\[ \text{Var}(\hat{\mu}_{\text{EGLS}}) > \text{Var}(\hat{\mu}_{\text{GLS}}) > \text{E}[(1'S^{-1}l)^{-1}], \]

i.e., the true variance of the EGLS estimator of \(\hat{\mu}\) is greater than the variance of the exact GLS estimator of \(\hat{\mu}\), while the expected value of the the asymptotic variance of the EGLS estimator is even smaller than the variance of the exact GLS estimator.

Intuitively, there are two factors that contribute to the downward bias in the variance estimator \((1'S^{-1}l)^{-1}\) that EGLS uses. The first one corresponds to the discrepancy between \(\text{Var}(\hat{\mu}_{\text{EGLS}})\) and \(\text{Var}(\hat{\mu}_{\text{GLS}})\). The true variance of the EGLS estimator is larger than that of the exact GLS estimator since the sampling error in \(S\) also contributes to the variance of the EGLS estimator of \(\hat{\mu}\). The larger is the number of parameters to be

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\(^{18}\) These results corroborate the evidence in Freedman and Peters (1984) and Marais (1986a,1986b) on the downward bias in the standard errors generated by EGLS.
estimated compared to the number of observations (i.e., higher is N/T) the greater will be the sampling error in the true standard error of the EGLS estimator. The second factor corresponds to the discrepancy between \( \text{Var}(\hat{\beta}_{\text{GLS}}) \) and \( E[(1' S^{-1} 1)^{-1}] \). Since \((1' S^{-1} 1)^{-1}\) is a concave function of \( S \), sampling errors in \( S \) contribute to the downward bias in the asymptotic variance estimator \((1' S^{-1} 1)^{-1}\) as an approximation to the variance of the exact GLS estimator. An increases in \( N/T \) leads to greater sampling error in \( S \), causing more downward bias in the asymptotic variance estimator.

To illustrate these points more succinctly, table 5 presents another set of simulation results on EGLS. These simulations are for weekly data and for sample sizes of 20 and 50 with the number of observations in the estimation period allowed to vary. Panel A of table 5 presents the average rejection frequencies (for \( \alpha=0.05 \)) and panel B presents the numbers for se-est., se-true, and the ratios of the two.

Panel A shows that the rejection frequencies decrease as \( N/T \) decreases, but even when \( N/T \) is about 0.2, which corresponds to having about 10 time-series observations per estimated parameter, the rejection frequencies are twice the nominal rate. Panel B confirms panel A's results and clearly illustrates the sampling error problem. For a given sample size, as \( T \) increases, the estimated cross-sectional standard deviation of the EGLS estimator declines. This is consistent with the true variance of the EGLS estimator approaching the variance of the exact GLS estimator as the number of observations increases. For large \( N/T \) (e.g. >0.5), sampling error can increase the true standard deviation of the EGLS estimator by a factor of two or more. For example, when \( N=20, T=30 \) results in an estimate of the cross-sectional standard deviation of 3.08% compared to 1.09% for \( T=100 \).
An examination of the row corresponding to the average estimate of the standard error illustrates the downward bias in using the asymptotic variance estimator \((I'S^{-1}l)^{-1}\) as an approximation to the variance of the exact GLS estimator. The average value of the asymptotic standard error decreases as \(N/T\) increases. An increase in \(N/T\) leads to more sampling error in \(S\) leading to more downward bias in the asymptotic variance estimator \((I'S^{-1}l)^{-1}\) as an approximation to the variance of the exact GLS estimator.

Thus, lower is the number of available time-series observations per estimated parameters, higher will be the discrepancies between \(\text{Var}(\hat{\mu}^{\text{EGLS}})\) and \(\text{Var}(\hat{\mu}^{\text{GLS}})\) and between \(\text{Var}(\hat{\mu}^{\text{GLS}})\) and \(E[(I'S^{-1}l)^{-1}]\). Both these factors contribute to the higher rejection frequencies for EGLS. When the number of available time-series observations per estimated parameters is about 10 (i.e., \(N/T = 0.2\)), the EGLS estimator of standard error is about 75% of the true standard error. If there are only four times as many time-series observations available as the estimated parameters (i.e., \(N/T = 0.5\)), the EGLS estimator of standard error is likely to be less than 50% of the true standard error.

3.3 ROBUSTNESS OF THE RESULTS TO THE CHOICE OF THE INDEX

To check the robustness of the results on the performance of alternate methodologies to the choice of market index, all the experiments are redone using the CRSP value-weighted index. For these simulations, most of the market model statistics are similar to those using an equally-weighted index, except that the average values of cross-correlation (irrespective of firms' industries) and within-industry correlation in residuals are about 1% and 2% respectively for daily data and 3% and 6% respectively for weekly data. Most of these correlations are significant at the 1% level. Thus the use of a value-weighted index increases the cross-correlation in residuals.
The results (not reported) from the value-weighted experiments indicate that except for a sample size of 10 with daily or weekly data or a sample size of 20 with daily data, all methods other than the portfolio return method, have rejection frequencies exceeding the upper limit of the 95% confidence interval. With an increase in sample size, the performance of all the methods with the exception of portfolio return method deteriorates.

The decline in the performance of methodologies such as OLS and Patell arises because they ignore cross-correlation in residuals. The residuals from the market model based on a value-weighted index have positive cross-correlation, which has a non-negligible adverse effect on the performance of such methods. The poor performance of EGLS arises again because sampling error is not properly accounted for.

These observations suggest that the use of an equally-weighted index is preferable to that of a value-weighted index when one is dealing with a random sample of firms and does not explicitly want to account for cross-correlation. However, this procedure is not likely to work, as is confirmed by the simulations of the following section, if there is industry clustering in the sample since the use of an equally-weighted index does not completely eliminate the within-industry cross-correlation in residuals.

3.4 RESULTS FOR INDUSTRY CLUSTERING

To examine the influence of simultaneous industry and calendar time clustering on the performance of various methodologies, another set of simulations is conducted. For these simulations firms are assumed to belong to the same industry if they have the same 3-digit SIC codes, since within-

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19 In fact, even in BW'80, when the market index is defined as the CRSP value-weighted index, the type I error becomes about 15% (for $\alpha = 0.05$) with calendar time clustering and if cross-correlation in residuals is ignored [See footnote 36, p.235, BW'80].
industry cross-correlation in residuals is likely to be higher for firms in the same 3-digit SIC codes than 2-digit SIC codes. For these simulations, the procedure followed is the same as that for earlier simulations except that firms included in one sample are constrained to come from the same 3-digit industry.

To minimize the non-randomness in the selection of industries, sample size is not constrained to a given number (e.g. 10) and up to 40 firms in the industry with data over the relevant period are included in the sample.20 Thus, to construct a sample, first an event day (week) is selected at random, then from the potential population a firm is selected at random and the sample constructed to include all firms in the same industry as this firm. If an industry has more than 40 firms then 40 firms are selected at random to comprise the sample. The procedure is repeated 100 times. Thus the 100 different samples do not all have the same number of firms. Tests are conducted using both equally-weighted and value-weighted indices. For the simulations with the equally-weighted index, the median sample sizes are 27 for daily data and 28 for weekly data and with the value-weighted index these figures are 24 and 27 respectively.

Most of the market model statistics are similar for these simulations as for the earlier ones and hence are not reported. The average values of cross-correlation in residuals using the equally-weighted market index are 2.80% for daily data and 5.75% for weekly data. Correlations using the

20 An industry is only included if it has more than ten firms with available data. If an industry has more than 40 firms, 40 firms are selected at random from this industry. Sample size is constrained to lie between 10 and 40 so that meaningful comparisons can be made across observations. These constraints should not lead to any serious bias in the results from these experiments.
value-weighted index are 2.56% for daily data and 5.67% for weekly data.\footnote{All these correlations are significant at the 1% level.} As expected, these averages are higher than the within-industry cross-correlation averages for the previous simulations since those averages are based on 2-digit SIC codes.

Since all firms in a sample come from the same industry, the EGLS-IND method reduces to EGLS. However, I include another method -- restricted EGLS (EGLS-rest.), in which the cross-correlations for all firm-pairs are constrained to be equal. Since the problems with plain EGLS result from sampling error, which is a direct function of the number of parameters to be estimated compared to the number of available time-series observations, any reasonable restriction on the parameters that can reduce the number of the estimated parameters may help alleviate the sampling error problem. When all firms come from the same industry, it may be reasonable to assume that all firm-pairs have the same cross-correlation.

The results using the equally-weighted index, are reported in tables 6 and 7 for daily and weekly data respectively.\footnote{The corresponding results for the value-weighted index are not reported because they are similar although somewhat stronger.} The organization of these tables is similar to that of table 5: panel A presents the average rejection frequencies (for $\alpha=0.05$) at the various levels of abnormal performance and panel B presents the comparison of the average estimate of the standard error of $\hat{\mu}$ with the estimated cross-sectional standard deviation of $\hat{\mu}$.

For daily data with 0% abnormal performance the rejection frequencies are close to 5% for the various methods, although Panel B of table 6 indicates some underestimation of standard errors for the OLS, Patell, and the EGLS methods. As one goes to weekly data except for the portfolio
method, for all the methods the rejection frequencies with no abnormal performance exceed the upper limit of the 95% confidence interval. The results in panel B confirm that all methods other than the portfolio return method underestimate the standard error of the abnormal performance.

The use of EGLS-rest. appears to offer some improvement over plain EGLS. With weekly data the rejection frequency for EGLS-rest. is 7.0% compared to 10.8% for EGLS and the ratio of se-est./se-true is 0.740 compared to 0.634 for EGLS. However, EGLS-rest. also understates the standard error and thus sampling error still remains a problem, albeit to a lesser degree.

The degree of understatement for the OLS and Patell methods is consistent with that estimated using the point estimates of the average cross-correlation present in the data and using the median sample sizes. 23

The higher rejection frequencies and the understatement of standard error for the EGLS method are again attributable to the fact that the standard error that EGLS uses does not properly account for the sampling error.

The evidence from these experiments confirms that the performance of the methods that ignore cross-correlations in residuals gets worse with industry clustering. The evidence roughly indicates that in the presence of both industry and calendar time clustering, with weekly data, for a sample size of about 25, the standard errors of the abnormal performance computed under the OLS or the Patell methods would be about 60% of the true standard

\[
\frac{\text{var}(\mu: \text{cross-sectional independence})}{\text{var}(\mu: \text{dependence})} = \frac{(1-\rho)}{1 + (N-1)\rho}
\]

i.e., the ratio of \(\text{var}(\mu)\) without the dependence adjustment (as in OLS) is \((1-\rho)/(1+(N-1)\rho)\) times \(\text{var}(\mu)\) with dependence adjustment, where \(\rho\) is the magnitude of the cross-correlation in the residuals.

\[23\text{It is easily shown [see Sefcik and Thompson(1986) p.327-328] that:}\]

\[
\text{var}(\mu: \text{cross-sectional independence}) = (1-\rho) \quad \text{and} \quad \text{var}(\mu: \text{dependence}) = 1 + (N-1)\rho
\]
errors. Furthermore, the performance of these methods is likely to get worse as sample size increases.

If EGLS is applied in such situations the results from the earlier simulations indicate that even if the number of time-series observations is about ten times the number of parameters to be estimated, the standard error computed under EGLS is likely to be only 75% of the true standard error. Hence one must be very careful in choosing a method when events have simultaneous industry and calendar time clustering. If one chooses a method other than portfolio method, then an attempt should be made to get the correct standard errors.

4. CONCLUSIONS

The major conclusions of this research can be summarized as follows.

1. Sampling errors play a very important role when EGLS is used. If the number of time series observations available is about ten times the number of parameters to be estimated then EGLS is likely to lead to rejection frequencies of approximately twice the nominal rate and the performance of EGLS gets worse as N/T increases.

2. For both daily and weekly data, for a random sample of firms with calendar time clustering, the use of an equally-weighted index results in almost negligible cross-correlations in residuals, and hence methods such as OLS and Patell's that ignore cross-correlation work well and have more power than the portfolio method. However, the use of a value-weighted index in such a situation does leave significant cross-correlation in residuals and methods ignoring cross-correlation in residuals then do not work well and can lead to type I error much higher than the nominal rate. Thus, whenever possible, one should prefer the use of an equally-weighted index.
3. With both industry and calendar time clustering in the sample, methods such as OLS and Patell's that ignore cross-correlations in residuals result in understatement of the standard errors (even using an equally-weighted index). The portfolio method generally produces unbiased standard errors and either one should use that method or make appropriate corrections in the standard errors if other methods are used. The results in Marais (1986a, 1986b) suggest that bootstrap and other computer intensive resampling techniques have some promise in generating correct standard errors in such situations.
APPENDIX

SOME DETAILS ON PATELL'S METHOD AND THE EGLS METHOD

PATELL'S STANDARDIZED RESIDUAL METHOD (PATELL): 24

Under this method each prediction error is first divided by its estimated standard deviation to yield a standardized prediction error $V_{it}$:

$$V_{it} = e_{it}/(s_i C_{it})$$

where $s_i^2$ is an estimate of $\sigma_i^2$ obtained from the market model regression as follows:

$$s_i^2 = \frac{1}{T_i - 2} \sum_{r=1}^{T_i} (R_{ir} - a_i - b_i R_{ir})^2 .$$

Here $T_i$ is the number of days (weeks) in the estimation period. $C_{it}$ is an adjustment factor used to correct the prediction error variance because the prediction errors are outside the estimation period:

$$C_{it} = \left[ 1 + \frac{1}{T_i} + \frac{(R_{mt} - \bar{R}_m)^2}{\sum_{r=1}^{T_i} (R_{mr} - \bar{R}_m)^2} \right]^{1/2} .$$

Here $\bar{R}_m$ is the mean market return in the estimation period ($= \frac{1}{T_i} \sum_{r=1}^{T_i} R_{mr}$). The standardized prediction errors are aggregated assuming cross-sectional independence to yield the following normalized sum:

$$Z_{vt} = \frac{\sum_{i=1}^{N} V_{it}}{\left[ \sum_{i=1}^{N} \frac{T_i - 2}{T_i - 4} \right]^{1/2}} .$$

For large $N$, the variable $Z_{vt}$ is normally distributed.

(3) Estimated Generalized Least Squares (EGLS):

Assuming stationarity, $\Sigma$ can be written as:

$$\Sigma = C_t \Omega C_t$$

24 For additional details, see Patell (1976).
where $C_t$ is a diagonal $N \times N$ matrix of the adjustment factors with the diagonal element for the $i$-th row $C_{it}$ given by equation (A3) and $\Omega$ is the $N \times N$ variance-covariance matrix of the $\hat{\epsilon}_{ir}$s, which can be estimated using the data from the market model regressions. Specifically, let $\hat{\epsilon}_{ir}$ denote the residual from the market model for period $r$ for firm $i$ (i.e., $\hat{\epsilon}_{ir} = R_{ir} - a_i - b_i R_{mr}$) then the diagonal elements of $\Omega$ can be estimated using equation (A2) and the off-diagonal elements can be estimated as:

$$\omega_{ij} = \frac{1}{T-2} \sum_{r=1}^{T} \hat{\epsilon}_{ir} \hat{\epsilon}_{jr}$$

(A6)

where $T$ is the number of periods during the estimation interval when both $\hat{\epsilon}_{ir}$ and $\hat{\epsilon}_{jr}$ can be computed. Let $S$ represent $C_t \hat{\Omega} C_t$. Then $S$ is an estimate of $\Sigma$. 
References


Table 1
Cross-Sectional Means and Standard Deviations of Selected Parameters from Market Model Regressions\textsuperscript{a,b}

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Average Beta</th>
<th>Avg. RES VAR x $10^4$</th>
<th>Avg. RSQ</th>
<th>Avg. CORR (%)</th>
<th>Avg. CORR-IND(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean  Std. Dev.</td>
<td>Mean  Std. Dev.</td>
<td>Mean  Std. Dev.</td>
<td>Mean  Std. Dev.</td>
<td>Mean  Std. Dev.</td>
</tr>
<tr>
<td>Daily Data:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.99  0.18</td>
<td>8.21  3.72</td>
<td>0.103  0.040</td>
<td>-0.06  1.15</td>
<td>0.44  5.22</td>
</tr>
<tr>
<td>20</td>
<td>1.02  0.12</td>
<td>7.76  2.99</td>
<td>0.113  0.035</td>
<td>0.02  0.58</td>
<td>1.44  3.67</td>
</tr>
<tr>
<td>50</td>
<td>1.01  0.09</td>
<td>7.77  2.68</td>
<td>0.103  0.030</td>
<td>0.12  0.22</td>
<td>1.38  1.73</td>
</tr>
<tr>
<td>100</td>
<td>0.99  0.06</td>
<td>8.02  3.10</td>
<td>0.103  0.031</td>
<td>0.04  0.12</td>
<td>1.16  0.87</td>
</tr>
<tr>
<td>Weekly Data:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.98  0.16</td>
<td>28.39  12.78</td>
<td>0.187  0.063</td>
<td>0.18  1.91</td>
<td>1.57  9.94</td>
</tr>
<tr>
<td>20</td>
<td>0.99  0.11</td>
<td>29.24  9.03</td>
<td>0.187  0.050</td>
<td>0.10  0.82</td>
<td>3.25  5.83</td>
</tr>
<tr>
<td>50</td>
<td>0.98  0.07</td>
<td>30.15  8.28</td>
<td>0.186  0.055</td>
<td>0.23  0.37</td>
<td>3.53  0.95</td>
</tr>
</tbody>
</table>

\textsuperscript{a} The Means and Std. Dev. are of the (sample-wide) average quantities and are computed across 100 independent trials.

The averages are equally-weighted averages over all the firms comprising a sample, e.g., avg. Beta is an equally-weighted average of betas of the firms comprising the sample.

\textsuperscript{b} Beta is the slope parameter from the market model regression. RES VAR and RSQ are respectively the residual variance and the R-squared from the same regression.

CORR refers to residual cross-correlation across a firm-pair (irrespective of whether the firms are from the same or different industries).

CORR-IND is within-industry cross-correlation of residuals.
Table 2

A Comparison of Different Procedures for Detecting Abnormal Performance for Daily Data:
Percentage of Trials When the Null Hypothesis is Rejected at a-level of 0.05.\(^a\)

\(H_0\): Abnormal Performance \(\mu = 0.0\) on event day \(t\).

<table>
<thead>
<tr>
<th>Actual level of Abnormal Performance</th>
<th>Method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Patell</td>
<td>EGLS</td>
<td>EGLS-IND</td>
<td>Portfolio</td>
</tr>
<tr>
<td></td>
<td>Sample Size: 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0% Avg. for days 0-4</td>
<td>4.8</td>
<td>5.8</td>
<td>4.6</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>0.5% Avg. for days 0-4</td>
<td>17.2</td>
<td>15.2</td>
<td>17.0</td>
<td>16.8</td>
<td>10.8</td>
</tr>
<tr>
<td>1.0% Avg. for days 0-4</td>
<td>37.8</td>
<td>38.6</td>
<td>41.6</td>
<td>41.4</td>
<td>27.8</td>
</tr>
<tr>
<td>2.0% Avg. for days 0-4</td>
<td>71.6</td>
<td>83.4</td>
<td>87.0</td>
<td>86.8</td>
<td>68.4</td>
</tr>
<tr>
<td></td>
<td>Sample Size: 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0% Avg. for days 0-4</td>
<td>5.4</td>
<td>5.6</td>
<td>8.2</td>
<td>7.0</td>
<td>5.0</td>
</tr>
<tr>
<td>0.5% Avg. for days 0-4</td>
<td>22.6</td>
<td>29.6</td>
<td>34.4</td>
<td>32.8</td>
<td>21.0</td>
</tr>
<tr>
<td>1.0% Avg. for days 0-4</td>
<td>55.6</td>
<td>69.8</td>
<td>76.8</td>
<td>75.2</td>
<td>50.4</td>
</tr>
<tr>
<td>2.0% Avg. for days 0-4</td>
<td>91.8</td>
<td>99.2</td>
<td>99.8</td>
<td>99.4</td>
<td>93.4</td>
</tr>
<tr>
<td></td>
<td>Sample Size: 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0% Avg. for days 0-4</td>
<td>3.8</td>
<td>5.0</td>
<td>8.4</td>
<td>6.0</td>
<td>3.6</td>
</tr>
<tr>
<td>0.5% Avg. for days 0-4</td>
<td>41.4</td>
<td>51.2</td>
<td>60.4</td>
<td>55.2</td>
<td>34.0</td>
</tr>
<tr>
<td>1.0% Avg. for days 0-4</td>
<td>82.2</td>
<td>95.2</td>
<td>96.2</td>
<td>96.8</td>
<td>80.6</td>
</tr>
<tr>
<td>2.0% Avg. for days 0-4</td>
<td>98.8</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>99.8</td>
</tr>
<tr>
<td></td>
<td>Sample Size: 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0% Avg. for days 0-4</td>
<td>4.8</td>
<td>4.6</td>
<td>17.6</td>
<td>5.8</td>
<td>5.2</td>
</tr>
<tr>
<td>0.5% Avg. for days 0-4</td>
<td>59.4</td>
<td>77.0</td>
<td>82.4</td>
<td>79.2</td>
<td>59.2</td>
</tr>
<tr>
<td>1.0% Avg. for days 0-4</td>
<td>96.2</td>
<td>99.4</td>
<td>99.8</td>
<td>99.4</td>
<td>96.8</td>
</tr>
<tr>
<td>2.0% Avg. for days 0-4</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

\(^a\)Event dates are same in calendar time for all securities in the sample. For each of the 100 replications, a day is selected at random from the 1963-1986 period and designated event day 0 and securities comprising the sample are also selected at random (without replacement). The average for days 0-4 is thus an average over 500 trials [5 days x 100 trials].
Table 3
A Comparison of Different Procedures for Detecting Abnormal Performance for Weekly Data: Percentage of Trials when the Null Hypothesis is Rejected at \( \alpha \)-level of 0.05.\(^a\)

H\(_0\): Abnormal Performance \( \mu = 0.0 \) in event week \( t \).

<table>
<thead>
<tr>
<th>Actual level of Abnormal Performance</th>
<th>Method</th>
<th>OLS</th>
<th>Patell</th>
<th>EGLS</th>
<th>EGLS-IND</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rejection Percentage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sample Size: 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>Avg. for weeks 0-4</td>
<td>4.2</td>
<td>6.0</td>
<td>8.0</td>
<td>7.4</td>
<td>5.2</td>
</tr>
<tr>
<td>1.0%</td>
<td>Avg. for weeks 0-4</td>
<td>12.2</td>
<td>16.4</td>
<td>20.0</td>
<td>19.4</td>
<td>13.6</td>
</tr>
<tr>
<td>2.0%</td>
<td>Avg. for weeks 0-4</td>
<td>31.4</td>
<td>35.0</td>
<td>42.4</td>
<td>38.8</td>
<td>29.2</td>
</tr>
<tr>
<td>5.0%</td>
<td>Avg. for weeks 0-4</td>
<td>86.8</td>
<td>94.8</td>
<td>96.2</td>
<td>95.4</td>
<td>87.2</td>
</tr>
</tbody>
</table>

|                                     | Sample Size: 20 |
| 0%                                 | Avg. for weeks 0-4 | 3.4   | 4.6    | 10.0  | 4.2      | 4.4       |
| 1.0%                               | Avg. for weeks 0-4 | 25.2  | 27.4   | 34.8  | 29.8     | 22.4      |
| 2.0%                               | Avg. for weeks 0-4 | 57.8  | 63.6   | 71.8  | 69.6     | 53.2      |
| 5.0%                               | Avg. for weeks 0-4 | 99.2  | 99.4   | 99.6  | 99.8     | 99.0      |

|                                     | Sample Size: 50 |
| 0%                                 | Avg. for weeks 0-4 | 3.8   | 6.8    | 18.2  | 7.2      | 4.6       |
| 1.0%                               | Avg. for weeks 0-4 | 35.8  | 47.6   | 58.8  | 51.2     | 33.4      |
| 2.0%                               | Avg. for weeks 0-4 | 82.2  | 92.6   | 92.6  | 92.2     | 81.6      |
| 5.0%                               | Avg. for weeks 0-4 | 100.0 | 100.0  | 100.0 | 100.0    | 100.0     |

\(^a\)Event weeks are same in calendar time for all securities in the sample. For each of the 100 replications, a week is selected at random from the 1964-1986 period and designated event week 0 and securities comprising the sample are also selected at random (without replacement). The average for weeks 0-4 is thus computed over 500 trials [5 weeks x 100 trials].
Table 4

A Comparison of the Estimated Average of the Standard Error of the Abnormal Performance $\hat{\mu}$ with the Estimated Cross-sectional Standard Deviation of $\hat{\mu}$ for the Various Methods$^a,b$

<table>
<thead>
<tr>
<th>Method</th>
<th>OLS</th>
<th>Patell$^c$</th>
<th>EGLS</th>
<th>EGLS-IND</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Size</td>
<td>se-est., se-true</td>
<td>se-est., se-true</td>
<td>se-est., se-true</td>
<td>se-est., se-true</td>
<td>se-est., se-true</td>
</tr>
<tr>
<td>Daily Data</td>
<td>Ratio</td>
<td>Ratio</td>
<td>Ratio</td>
<td>Ratio</td>
<td>Ratio</td>
</tr>
<tr>
<td>10</td>
<td>0.821, 0.961</td>
<td>0.854, 0.726</td>
<td>0.861, 0.728</td>
<td>0.846, 0.723</td>
<td>0.866, 0.961</td>
</tr>
<tr>
<td>20</td>
<td>0.588, 0.626</td>
<td>0.939, 0.451</td>
<td>0.931, 0.468</td>
<td>0.872, 0.450</td>
<td>0.938, 0.626</td>
</tr>
<tr>
<td>50</td>
<td>0.380, 0.403</td>
<td>0.943, 0.296</td>
<td>0.885, 0.329</td>
<td>0.745, 0.297</td>
<td>0.892, 0.389</td>
</tr>
<tr>
<td>100</td>
<td>0.279, 0.284</td>
<td>0.982, 0.210</td>
<td>0.890, 0.260</td>
<td>0.581, 0.211</td>
<td>0.900, 0.273</td>
</tr>
<tr>
<td>Weekly Data</td>
<td>Ratio</td>
<td>Ratio</td>
<td>Ratio</td>
<td>Ratio</td>
<td>Ratio</td>
</tr>
<tr>
<td>10</td>
<td>1.616, 1.781</td>
<td>0.907, 1.293</td>
<td>1.606, 1.623</td>
<td>0.805, 0.773</td>
<td>1.299, 1.596</td>
</tr>
<tr>
<td>20</td>
<td>1.120, 1.204</td>
<td>0.930, 0.894</td>
<td>0.978, 1.091</td>
<td>0.750, 0.905</td>
<td>0.923, 1.171</td>
</tr>
<tr>
<td>50</td>
<td>0.766, 0.751</td>
<td>1.020, 0.570</td>
<td>0.684, 0.424</td>
<td>0.633, 0.916</td>
<td>0.463, 0.702</td>
</tr>
</tbody>
</table>

$^a$ se-est. is the estimated cross-sectional average of the standard error of $\hat{\mu}$ and se-true is the estimated cross-sectional standard deviation of $\hat{\mu}$ and both are reported in percent. Ratio is the ratio of the two, i.e., se-est./se-true.

$^b$ Both se-est. and se-true are computed over the 500 trials [5 event days (weeks) x 100 trials].

$^c$ Under the Patell method, a normalized statistic is directly calculated which is compared to a standard normal distribution. Since Patell's method is similar to WLS, se-est. reported under Patell's method is that for the WLS method (See also CD, Table 10).
Table 5
An Examination of the EGLS Procedure Used for Detecting Abnormal Performance with Weekly Data. Actual Level of Abnormal Performance Introduced = 0.0%.

### Panel A
Percentage of Trials When the Null Hypothesis is Rejected at \( \alpha \)-level of 0.05.a

\( H_0: \) Abnormal Performance = 0.0 in event week \( t \).

<table>
<thead>
<tr>
<th>Number of Weeks in Estimation Period</th>
<th>Avg. Rejection Percentage for Weeks 0-4</th>
<th>Sample Size</th>
<th>Number of Weeks in Estimation Period</th>
<th>Avg. Rejection Percentage for Weeks 0-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>32.2</td>
<td></td>
<td>100</td>
<td>18.2</td>
</tr>
<tr>
<td>50</td>
<td>14.0</td>
<td></td>
<td>150</td>
<td>15.0</td>
</tr>
<tr>
<td>100</td>
<td>10.0</td>
<td></td>
<td>200</td>
<td>13.4</td>
</tr>
</tbody>
</table>

### Panel B
A Comparison of the Average Estimate of the Standard Error of the Abnormal Performance \( \hat{\mu} \) with the Estimated Cross-sectional Standard Deviation of \( \hat{\mu} \) b

<table>
<thead>
<tr>
<th>Estimation Weeks:</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. est. of Std. Error(%) : se-est.</td>
<td>0.509</td>
<td>0.730</td>
<td>0.819</td>
<td>0.424</td>
<td>0.492</td>
<td>0.534</td>
</tr>
<tr>
<td>Cross-sectional Std. Dev.(%) : se-true</td>
<td>3.080</td>
<td>1.231</td>
<td>1.091</td>
<td>0.916</td>
<td>0.833</td>
<td>0.771</td>
</tr>
<tr>
<td>Ratio (=se-est./se-true)</td>
<td>0.165</td>
<td>0.593</td>
<td>0.751</td>
<td>0.463</td>
<td>0.591</td>
<td>0.693</td>
</tr>
</tbody>
</table>

aEvent dates are same in calendar time for all securities in the sample. For each of the 100 replications, a week is selected at random and designated event week 0 and securities comprising the sample are also selected at random (without replacement). The avg. for weeks 0-4 is thus an average over 500 trials [5 weeks x 100 trials].

bThe average of the standard error of \( \hat{\mu} \) and the cross-sectional standard deviation of \( \hat{\mu} \) are both computed over 500 trials [5 weeks x 100 trials].
Table 6
A Comparison of Different Procedures for Detecting Abnormal Performance With Industry Clustering in Daily Data, Using Equally-Weighted Index as the Market Index.

Panel A
Percentage of Trials When the Null Hypothesis is Rejected at α-level of 0.05.a
H₀: Abnormal Performance = 0.0 on event day t.

<table>
<thead>
<tr>
<th>Actual level of Abnormal Performance</th>
<th>Method</th>
<th>OLS</th>
<th>Patell</th>
<th>EGLS</th>
<th>EGLS-Rest. Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% Avg. for days 0-4</td>
<td>Rejection Percentage</td>
<td>6.0</td>
<td>7.4</td>
<td>5.6</td>
<td>5.0</td>
</tr>
<tr>
<td>0.5% Avg. for days 0-4</td>
<td></td>
<td>32.6</td>
<td>35.2</td>
<td>31.6</td>
<td>27.2</td>
</tr>
<tr>
<td>1.0% Avg. for days 0-4</td>
<td></td>
<td>62.6</td>
<td>67.4</td>
<td>66.8</td>
<td>62.8</td>
</tr>
<tr>
<td>2.0% Avg. for days 0-4</td>
<td></td>
<td>91.2</td>
<td>95.4</td>
<td>95.8</td>
<td>94.8</td>
</tr>
</tbody>
</table>

Panel B
A Comparison of the Average Estimate of Standard Error of the Abnormal Performance \( \hat{\mu} \) with the Estimated Cross-sectional Standard Deviation of \( \hat{\mu} \)

<table>
<thead>
<tr>
<th>Avg. est. of Std. Error (%) : se-est.</th>
<th>Method</th>
<th>OLS</th>
<th>Patell</th>
<th>EGLS</th>
<th>EGLS-Rest. Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (%)</td>
<td></td>
<td>0.551</td>
<td>0.433</td>
<td>0.483</td>
<td>0.516</td>
</tr>
<tr>
<td>Cross-sectional Std. Dev. (%) : se-true</td>
<td></td>
<td>0.708</td>
<td>0.550</td>
<td>0.558</td>
<td>0.561</td>
</tr>
<tr>
<td>Ratio (=se-est./se-true)</td>
<td></td>
<td>0.778</td>
<td>0.787</td>
<td>0.866</td>
<td>0.920</td>
</tr>
</tbody>
</table>

aEvent dates are same in calendar time for all securities in the sample. For each of the 100 replications, a day is selected at random from the 1963-1986 period and designated event day 0 and the sample consists of all securities belonging to a randomly selected 3-digit SIC code. The avg. for days 0-4 is thus an average over 500 trials [5 days \( \times \) 100 trials].

bThe average of the standard error and the cross-sectional standard deviation are both computed over 500 trials [5 days \( \times \) 100 trials].
Table 7
A Comparison of Different Procedures for Detecting Abnormal Performance With Industry Clustering in Weekly Data, Using Equally-Weighted Index as the Market Index.

Panel A
Percentage of Trials When the Null Hypothesis is Rejected at $\alpha$-level of 0.05.$^a$
$H_0$: Abnormal Performance = 0.0 on event day $t$.

<table>
<thead>
<tr>
<th>Actual level of Abnormal Performance</th>
<th>Method</th>
<th>OLS</th>
<th>Patell</th>
<th>EGLS</th>
<th>EGLS-Rest. Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>Rejection Percentage</td>
<td>10.8</td>
<td>10.4</td>
<td>10.8</td>
<td>7.0</td>
</tr>
<tr>
<td>0.5%</td>
<td></td>
<td>32.0</td>
<td>33.4</td>
<td>33.0</td>
<td>24.6</td>
</tr>
<tr>
<td>1.0%</td>
<td></td>
<td>60.6</td>
<td>65.4</td>
<td>61.6</td>
<td>55.0</td>
</tr>
<tr>
<td>2.0%</td>
<td></td>
<td>96.6</td>
<td>97.4</td>
<td>96.6</td>
<td>96.2</td>
</tr>
</tbody>
</table>

Panel B
A Comparison of the Average Estimate of Standard Error of the Abnormal Performance $\hat{\mu}$ with the Estimated Cross-sectional Standard Deviation of $\hat{\mu}$$^b$

<table>
<thead>
<tr>
<th>Avg. est. of Std. Error (%)</th>
<th>se-est.</th>
<th>OLS</th>
<th>Patell</th>
<th>Method</th>
<th>EGLS</th>
<th>EGLS-Rest. Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional Std. Dev. (%)</td>
<td>se-true</td>
<td>1.619</td>
<td>1.495</td>
<td>1.584</td>
<td>1.605</td>
<td>1.619</td>
</tr>
<tr>
<td>Ratio (=se-est./se-true)</td>
<td></td>
<td>0.632</td>
<td>0.583</td>
<td>0.634</td>
<td>0.740</td>
<td>0.894</td>
</tr>
</tbody>
</table>

d Event dates are same in calendar time for all securities in the sample. For each of the 100 replications, a week is selected at random from the 1964-1986 period and designated event week 0 and the sample consists of all securities belonging to a randomly selected 3-digit SIC code. The avg. for weeks 0-4 is thus an average over 500 trials [5 weeks x 100 trials].

$^b$ The average of the standard error and the cross-sectional standard deviation are both computed over 500 trials [5 weeks x 100 trials].