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in New Product Decisions

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INTRODUCTION

The role of new products in the growth and continuing profitability of modern businesses has been universally recognized by marketing practitioners and educators. This recognition has prompted the development of a considerable literature concerning new product decision making procedures. Throughout the literature one significant factor has been largely neglected or considered only on a subjective verbal basis. This factor is product interdependency. In today's multiproduct firm, interdependencies between new and old products are almost always present and should not be neglected when the new product decision is made.

"DIFFERENTIAL" CONSIDERATIONS

The new product will become part of the firm's product line. It may be complementary to the line and aid the other product offerings or it may have substitution effects and reduce the marketing effectiveness of existing products. The performance of the new product itself is not a satisfactory criterion for the decision to add the product when interdependencies are present. The decision should be based on the change in the total line's performance. Rather than basing the acceptance of the product on the profit of the new product alone, the change in the total line profit should be considered. The change in the total line profit is called the "differential profit."¹ The use of "differential" parameters allows the explicit consideration of
product interdependency effects.

In actual business practice two basic errors may be made regarding the place of interdependency in decisions. In some situations interactions are neglected and in others they are given too much emphasis. An example of the second error is the situation where a new product idea is rejected because it will "steal sales from another product." Both of these errors can be eliminated by explicitly considering the effects of interactions and specifying the best marketing policy for the line. If a product is known to have demand substitution effects with other products, this may not make it a poor product; other interactions may outweigh the demand consideration. Changes in the design and marketing characteristics of new or old line products might also make the new product a good investment on the basis of the differential profit. Likewise, if the old line profits were expected to decline, the new product may be a wise investment even if it does not increase total profits above their current level, since it is the differential line profit and not the absolute level of profit that is important.

With the concept of differential profit in mind, the notions of complementarity and substitutability can be defined. If the differential profit is greater than the profit of the new product, the product is complementary to the line. If the differential profit is less than the new product profit, the product has substitution effects with the line. Only when the differential profit is equal to the new product profit is it justifiable to consider the product independently.
AREAS OF INTERDEPENDENCY

The interaction effects measured by the differential profit are actually the resultant effects of component interactions in the areas of demand, cost, allocation, and uncertainty.

Demand Interaction. The importance of demand interaction has long been recognized by economists. Economic theory first considered the relationship between products to be designated by the sign of the cross partial of the utility function. Since the cross partial is not invariant in sign under a change in the index of utility, the theoretical measure most accepted by economists today is the sign of the substitution term of the Slutsky equation. These theoretical considerations can be made operational by the use of the concept of cross elasticities. The price cross elasticity measures the proportionate change in the quantity of one product sold as the result of the proportionate change in the price of another product. The cross price elasticity could be supplemented by at least two non-price cross elasticities that measure the effects of advertising and sales effort. These cross elasticities would provide the framework to consider interaction effects between the marketing variables of one product on the sales of another.

These cross elasticities can be utilized in specifying a demand function for a new product. Given the basic demand format proposed by Philip Kotler, the interaction effects can be added in the following form for a two product line:
\[ x_1 = f(\text{life cycle, industry, competitive effects}) \times (\text{INTERACTIONS}) \]

where \( (\text{INTERACTIONS}) = \frac{\text{CPE}}{P_2} \frac{\text{CAE}}{A_2} \frac{\text{CDE}}{D_2} \)

\[ x_1 = \text{demand for product one} \]
\[ P_2 = \text{price of product two} \]
\[ \text{CPE} = \text{cross price elasticity} \]
\[ A_2 = \text{advertising for product two} \]
\[ \text{CAE} = \text{cross advertising elasticity} \]
\[ D_2 = \text{distribution effort on product two} \]
\[ \text{CDE} = \text{cross distribution elasticity} \]

This formulation does have the disadvantage of requiring the cross elasticities to be constant. A general formulation which allows non-linear and discontinuous interaction relationships can be developed by the use of "cross response functions." A cross response function measures the proportionate change in the sales of one product as the result of an absolute change in a parameter of another product. The \( \frac{\text{CPE}}{P_2} \) term in the equation can be replaced by the cross response function to provide a generalized statement of demand interaction effects. If "n" products are in the line, the interaction effects can be expressed as:

\[ (\text{INTERACTIONS}) = \frac{\text{CPR}_{12}}{\text{CAR}_{12}} \frac{\text{CDR}_{12}}{\text{CPR}_{13}} \frac{\text{CAR}_{13}}{\text{CDR}_{13}} \cdots \frac{\text{CPR}_{1n}}{\text{CAR}_{1n}} \frac{\text{CDR}_{1n}}{\text{CDE}} \]

\[ \text{CPR}_{1j} = \text{cross price response function between product one and "j"} \]
\[ \text{CAR}_{1j} = \text{cross advertising response function between product one and "j"} \]
\[ \text{CDR}_{1j} = \text{cross distribution response function between product one and "j"} \]
Cost Interactions. Cost interactions may be just as important as demand interactions. The technical production function may favor the production of two products together so that the total variable costs of production would be less than the sum of each produced separately. If the marginal cost of product one decreases as more of product two is produced, the products are complements. Conversely, if the marginal cost increases, the products are technical substitutes. The cost interactions can be encompassed in the framework of a linear programming model to minimize the total cost of producing a given set of product line quantity requirements. The programming problem is to:

\[
\begin{align*}
\text{minimize: } & \sum_{j} c_j I_j \\
\text{subject to: } & \sum_{j} a_{ij} I_j \geq b_i \text{ and } I_j \geq 0
\end{align*}
\]

where

\[
\begin{align*}
c_j &= \text{cost of input factor "j"} \\
I_j &= \text{amount of input factor "j" utilized} \\
a_{ij} &= \text{technical production and marketing relationships} \\
b_i &= \text{constraint on input values and quantities of goods to be produced}
\end{align*}
\]

Allocation Interactions. Interactions between products are automatically introduced when fixed resources are allocated between products. This is true since if one product gets more of a fixed resource, another must get less. The resource constraints may be in
the form of a fixed productive capability, fixed distribution system, limited management skill, or a specified budget restriction. These interactions are important since adding a new product usually will change the optimum allocation of resources. Considering the product alone may result in a sub-optimum for the product, but not an optimum for the complete line. The total optimization is achieved by maximizing the total differential profit subject to the constraints on the firm and the product. The problem is to:

maximize: Differential Profit = f(P_1, A_1, D_1, ..., P_i, A_i, D_i, ..., P_n, A_n, D_n)
subject to: \[ \sum_i A_i \leq A_T, \sum_i D_i \leq D_T \text{, and } x_i \leq x_{Ti} \]

where: \[ P_i = \text{price of product "i"} \]
\[ A_i = \text{advertising expenditure for product "i"} \]
\[ D_i = \text{distribution expenditure for product "i"} \]
\[ A_T = \text{advertising budget for the line} \]
\[ D_T = \text{distribution capacity for line} \]
\[ x_{Ti} = \text{productive capacity level for product "i"} \]

The objective function of this constrained maximization is very complex and the analytic methods of mathematical programming and Lagrangian analysis are not powerful enough to cope with the complications interjected by the consideration of interdependency. The optimization must be carried out by a systematic trial and error search routine which maximizes the differential profit generated by the product.
**Uncertainty Interactions.** The maximum differential profits must be balanced against the uncertainty associated with the new product before a decision can be reached. The uncertainty associated with a product line is not generally the sum of the uncertainty of each product. Fluctuations in one product may amplify or compensate for fluctuations in other products. The change in the total product line uncertainty must be considered when adding a new product. This may be conceived of as "differential uncertainty" and it is mathematically expressed as:

$$DU^2 = V' + V - 2 \text{COV}(Pr, Pr')$$

where:

- $DU$ = differential uncertainty
- $V'$ = variance of the new line profits
- $V$ = variance of old line profits
- $\text{COV}(Pr, Pr') = \text{covariance of new line profits (Pr') and old line profits (Pr)}$ or $E[(Pr-E(Pr))(Pr'-E(Pr'))]$ 

The decision to either add, reject, or study more extensively the new product can be determined by combining the differential profit, differential uncertainty, and investment. If the probability of making a minimum rate of return on investment is above a specified acceptance level, a commitment is made to market the product; if the probability is below a specified rejection level, the product idea is not accepted; and if the probability is intermediate, the product will receive further study.
CASE STUDY IN NEW PRODUCT DECISION MAKING

A proposed theoretical decision model, such as the one suggested in this paper, does not become useful in the solution of applied marketing problems until it is established that the model has practical value. A case study has been carried out to demonstrate how the proposed decision model can be implemented in an actual business decision situation.

Institutional Background. The case was conducted in an industrial market. The firm concerned, called "Chemi" in this study, produced basic chemical products which were processed by other manufacturing concerns into finished goods. Chemi had developed a new nylon compound which had both cost and performance advantages in several significant markets. It was estimated that the new product would experience demand interactions with two other products currently being marketed by the firm. The most important interaction was with plastic compound used for small gears. The new product would compete with this product directly on the basis of price and performance advantages. The second market affected by the new product was a small specialty market for bearing linings. The new nylon would have performance advantages in this market, but its major advantage would be that its costs were much lower. The remainder of the sales for the new product would come from markets in which Chemi had no product offerings. The firm wanted to know if the product should be introduced, should be rejected, or if more studies should be done on the product.
In this case study the proposed decision model was called "SPRINTER." This stands for Specification of PRofits with INteraction under Trial and Error Response.

**Input to Model.** The starting point for the data gathering for SPRINTER was a description of some specific marketing program that the firm could visualize for the product over the planning period of ten years. This program included the price, advertising, and distribution plans for the new and old products and the competitive strategies for these products over the ten year period. The execution of the marketing program would result in the generation of sales for the new product in each year. The estimates of these quantities for each year in the planning period comprised the reference life cycle estimate for the planning period. The input estimates for the life cycle were closely described by the equation:

\[
H(4-t) \left(100 e^{1.23(t-1)}\right) + H(t-5) \left(1000(10.07)^{1.08(t-5)}\right)
\]

\[
H(t-4) = \begin{cases} 
1 & \text{if } t \leq 4 \\
0 & \text{if } t > 4 
\end{cases} \quad \text{and} \quad H(t-5) = \begin{cases} 
1 & \text{if } t \geq 5 \\
0 & \text{if } t < 5 
\end{cases}
\]

\[
t = \text{time period of analysis}
\]

The first four years were best described by an exponential function and the last six years by a Gompertz function.

This life cycle estimate was considered the reference point for the determination of the parameter response functions. All response
functions measured the deviations from the reference life cycle estimates produced by changes in price, advertising, or distribution. For example, questioning revealed the new product price response function to be:

$$ PR_1 = \frac{268}{P_1} - 104.5 - 0.844 \quad \text{where} \quad P_1 = \text{new product price}. $$

This function implies that if prices were decreased to $200/carton, the sales would be almost twice the reference level in that period.

The equation of this response function changed during the planning period to indicate the changing price sensitivity of the new product. Similar functions were determined for advertising and distribution response.

Demand interdependencies were considered by the development of interaction response functions. These functions were expressed in terms of the penetration the new product would make into existing product markets. For example, the price cross penetration response function for the new nylon (product X) and old gear (product Y) markets was:

$$ PEN_{XY} = PEN_{XY} \cdot \left(\frac{x}{x_R}\right) \cdot (0.1) \left(\frac{P_{xy}}{P_y} - P_y\right) $$

$$ PEN_{XY} = \text{reference penetration of product X into market Y} $$

$$ P_y = \text{price of product Y} $$

$$ P_{xy} = \text{price of product X in market Y} $$

The term $x/x_R$ reflects the belief that the penetration into market Y will be proportionate to the total sales of product X. This function measures the demand interactions between the two products and reflects
the changes in penetration from the reference levels as the price of the new or old products is changed.

The competitive effects in the new product market were described on the premise that the competition's strategy would be to enter with a similar product at the same price, but with twice as much advertising and distribution effort. The market share of the introductory firm was defined on the basis of the relative cumulative marketing effectiveness of the firm.5

The best estimate of the entrance time of competition was five years after the introduction of the new product. As new firms entered the market, aggregate industry effects were produced and were functionally analyzed. The effect of the combined marketing effort was to increase the industry sales by twenty-five percent at the reference marketing program for the new product.

The total life cycle, industry, competitive, and interaction effects were combined to define the total demand equations for each product in the line (as indicated on page four).6 When the total minimum cost of producing the new line was subtracted from the total new line revenue (price times quantity demanded for each product), the new line profits were determined. The differential profit was specified when the estimated old line profits were deducted from the new line profits. The differential profit was optimized subject to the constraints on the decision. Several important constraints limited the number of feasible solutions to be tested by SPRINTER. The constraints
upon the new product allocation model were in the form of constraints on the advertising budget, plant output capacity, technical service, and pricing policy.

The input data used to calculate the differential profit were not known with certainty. Distributions of each of the parameters about the best estimates were obtained and combined to produce aggregate confidence estimates. In calculating the total uncertainty, the same underlying factors affected the demand for the new nylon and the old gear and bearing products. These interactions were analyzed by using the format proposed by Harry Markowitz. This procedure requires that some underlying index be specified which would reflect the common aspects of the interacting products. The variance of this index was specified and then combined with the relationships of each of the products to the index to yield an estimate of the covariance between each product. The uncertainty and differential profit data formed the basic input for the SPRINTER model.

Output of Model. To maximize profits SPRINTER utilized a simulation approach which began by evaluating Chemi's proposed marketing plan. Chemi's proposed pricing policy was to sell the new product at $350/carton for the first three years and $250/carton for the remaining seven years. One percent of the sales force would be allocated to the new product and $10,000 per year of advertising would be purchased for the product. This marketing program resulted in the generation of a total new product profit of $8,350,000 when the product was evaluated
independently. Since the total investment for the new product was $8,000,000 the product might have been accepted if certainty was assumed, and if the product was considered independent of the existing products. But the product was not independent; significant interdependencies were present. The total discounted differential profit for the new product was only $5,999,000. The loss in the profits of existing gear and bearing products accounted for the reduction of the profit of the new product from 8.4 million dollars, when viewed independently, to six million dollars, when viewed as an integral part of the product line. The decision at the initially proposed marketing mix would have been to reject the product. The level of discounted differential profit was below the eight million dollars of required investment and there was less than the specified probability (fifty percent for a rejection) of making the minimum rate of return (fifteen percent).

Although the project would have been rejected by SPRINTER at the reference price level, this did not have to be true for all marketing programs. By the application of a trial and error search routine, SPRINTER suggested a better marketing mix over the life cycle. SPRINTER recommended a price of $250/carton for the first three years and prices near $200/carton for the remaining seven years. The use of this pricing policy and the original advertising and distribution allocations improved profits greatly. The total discounted differential
profits attributable to the new product were increased to $10,830,000. The price determination was very sensitive in this case and SPRINTER was able to improve profits 4.8 million dollars by an optimal price determination routine. Even at this profit, however, the product could not be adopted since there was less than the specified probability (ninety percent for acceptance) of making the minimum rate of return (fifteen percent) on the investment.

The full power of the program was not exhausted at this point. SPRINTER proceeded to analyze the effects of varying the constraint levels. Enlarging the plant size increased profits, but would not produce a decision to accept the product. Increasing the sales force by one man did produce a decision to accept the new product. With a larger sales force the total profit was 12.2 million dollars and based on a differential uncertainty of 2.77 million dollars there was a ninety-one percent probability of making fifteen percent rate of return on investment.

Summary. The output of SPRINTER was the decision to add the new product to the line and to use a marketing mix over the life cycle that represented a simultaneous specification of price, advertising, and distribution at their best combination of values, given the best level of constraints on the firm's operations. The specification took full consideration of demand, allocation, and uncertainty interdependencies.
The optimum price, advertising, and distribution mix occurred at a point where the differential profit was six million dollars more than the profit level which would have occurred at the marketing mix proposed by Chemi Corporation.

The preliminary case study was a success. Real input data was processed to yield a meaningful decision and a significant increase in profits. All the essential elements of the decision were incorporated in the model and the important factor of product interdependency was explicitly analyzed. This case study could not be construed as a complete validation of SPRINTER, but the model represents a first step towards a meaningful mathematical consideration of product interdependency in new product decisions.
FOOTNOTES

1 This term is suggested because of the analogy to the total differential concept of mathematics. Professor Fritz Machlup first proposed the term with regard to "differential revenue".


3 Philip Kotler, "Competitive Strategies for New Product Marketing Over the Life Cycle," Management Science, XII (December 1965), B-104 to B-119, presents the mathematics of the first three terms of the functional equation.


5 See Urban, pp. 98-99 for the mathematical equation.

6 See Urban, pp. 117-120 for the detailed mathematical equation.


8 SPINNER systematically evaluated a range of two million marketing programs in optimizing the differential profit. The running time for SPINNER was one and one-half hours on a CDC3400 computer (about $650 of computing time).
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