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PERSONAL SELLING DECISIONS*
David B. Montgomery** and Glen L. Urban

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PERSONAL SELLING DECISIONS

INTRODUCTION

In the previous three chapters of this book the marketing mix elements of advertising, price, and distribution have been considered. One other major controllable marketing variable remains. This is personal selling. In spite of the fact that it is the largest single item in the marketing budgets of most firms, personal selling continues to be an illusive and poorly understood element of the marketing program. Only a small number of analytical or management science efforts have been reported during the past fifteen years. However, developments in marketing information systems and in the technical aspects of management science can be expected to expand both the need for and potential of management science approaches in this important marketing decision area. Thus the time is ripe for an accelerated application and development of management science models in this rather neglected area of marketing management.

In this chapter attention will focus upon sales force decisions. The major decision areas are structured in Figure 7-1. The first step in the decision process is to recognize the role of personal selling in the firm's total marketing program and to establish goals or criteria for use in sales force decision making. After the criteria for the evaluation of decision alternatives have been specified, a resource commitment to the personal selling effort must be established. This total resource commitment involves setting the sales budget and determining the size of the sales force.

After a preliminary budget has been established the problem of allocating the sales resources must be attacked. The sales effort must be allocated along three dimensions: (1) customers, (2) sales territories, and (3) time (i.e., scheduling effort). The question of effort allocation
Establish Goals for Personal Selling Decisions

Determine the Magnitude of the Personal Selling Effort

Allocate the Selling Effort Over Customers, Geographic Areas and Time

Organize and Control the Sales Force

Figure 7-1

PERSONAL SELLING DECISIONS
over a product line is considered in Chapter 8, but it can be stated now that this is a complex problem and is a useful area for future research. The allocation decision very often has a significant interaction with the budget and size of sales force decisions. For example, at the allocation stage, the firm might discover that sales response is greater than expected so that profits can be enhanced by allocating more resources to personal selling. Conversely, a need to prune the personal selling effort might be identified. The commitment of resources to personal selling is generally made with some preliminary allocation in mind. This initial resource commitment may be updated in the light of the optimal or near optimal customer and territory allocations. It would be theoretically attractive to allocate the sales force resources simultaneously across customers, territories, and time, but in actual practice firms usually allocate sequentially across the dimensions.

After budget and allocation decisions have been considered, a number of organizational decisions must be made. For example, the number of levels in the sales organization must be determined and control units must be defined. Another organizational decision is related to motivation of the sales force. Although the average compensation of each salesman may have been estimated at the budget decision level, the details of the compensation plan must be specified. The details of selection, training, and assignment of salesmen to control units must also be developed. The final organizational aspect to be considered in this chapter is the control procedure that should be developed in support of the selling program. In order to improve its sales performance in the future and discover potential problem areas, the firm will want to provide for continuous evaluation and control of its personal selling effort. The information obtained by these activities should then be fed back into all the decision points in order to assist the firm in adapting
to changing market conditions.

Each of the decisions described in Figure 7-1 will be considered in this chapter. The goal hierarchy which is present in sales decisions will be outlined, but most of the attention will be directed at the budget and allocation decisions. The chapter closes with a discussion of some selected organizational aspects of personal selling management.

PERSONAL SELLING GOALS

In most firms, personal selling is a vital element in the firm's communications with its markets. As an element in the total marketing mix and more particularly in that subset of the program called the communication mix, personal selling would seem to play at least three distinct roles:

1. disseminating factual information,
2. presenting persuasive information,
3. rendering service.

The salesman's informational function operates in two directions. He communicates to the market information about the technical characteristics, prices, service aspects, and availability of his firm's product offerings. The opposite information flow is the transmittal of market information back to the firm. The salesman is often in a good position to detect problems developing in the firm's competitive position, to identify potential areas for new products, and to recommend service policies which would better fulfill customers' needs. The presentation of persuasive information is the personal selling role most often associated with the salesman. In his persuasive role he marshalls evidence in the social-psychological environment of the salesman-customer interaction in an effort to influence the customer to purchase the product and service offerings of his firm. Service is the third important function of the salesman in many selling situations. For example, the salesman may be able to assist the customer in solving some problem related to the firm's product line. The advice given by Xerox
salesmen concerning paper flow in the customer's office is a case in point. Expediting orders and setting up point-of-sale displays are further examples of service functions performed by salesmen.

The three communication functions of personal selling discussed in the previous paragraph closely parallel the functions of distribution discussed in the last chapter. In fact, personal selling may be viewed as one aspect of a firm's channels of distribution. The persuasive and service functions of the salesman are part of the demand creation aspects of the channel system. Likewise, the information function of channels may be carried out by the manufacturer's own salesmen. The sales force may even be called upon to carry out the availability function of distribution. For example, in the food industry the salesman often stocks the shelves for the grocer and removes any product which is too old to sell.

The use of a direct sales force by a manufacturer generally reflects the result of his analysis of the relative merits of alternative channels. In such a case, the use of a direct sales force may have been found to yield the most profitable methods to fulfill the availability, information, and demand creation functions of distribution.

Another interaction between personal selling and channels of distribution is the use of a sales force to convince middlemen to carry out the desired distribution functions. This selling within the channel may be necessary in order to implement the total distribution plan. Personal selling effort may convince a middleman to stock the product (availability function) and support the product with the desired service and promotional effort (demand creation function). The sales force may also provide information feedback to the manufacturer from the middlemen and the ultimate market.

Thus, the sales force functions as part of the firm's distribution system
and transmits appeals and product information as part of the firm's total communication mix. These two functions were discussed in greater detail in the distribution and advertising chapters, and they comprise the objectives of personal selling effort. The ultimate goal of the firm's personal selling effort is to fulfill these functions in the most profitable manner.

As in other decision areas, profit is the primary goal. This goal may be translated into lower level goals in a meaningful fashion. For example, a goal for the sales manager may be to maximize sales subject to a fixed sales force budget constraint. This is consistent with profit maximization if the size of the sales force and other constraints have been calculated to generate a maximum return for the firm. This return might be profit at the marketing manager level where the manager is faced with the problem of allocating a fixed budget to marketing mix elements and where the expenditure on personal selling is considered as a cost and deducted from revenues in each year. At the financial level the return may be measured by the rate of return on investment because the sales expenditure is considered an investment rather than a cost. For purposes of this development the sales force problem will be analyzed primarily at the level of the marketing manager and long run profit maximization will be the goal.

SIZE OF PERSONAL SELLING EFFORT

The Theory of Optimal Sales Force Size Determination

The budget for personal selling represents the financial resources the firm intends to commit to personal selling effort during the budget period. This budget will depend upon both the size of the sales force and the firm's salesman compensation and selling expense plan. If the budget were specified, the sales force size could be obtained by dividing the budget by the estimated average compensation per man. If the number of salesmen were
determined first, the budget would be specified by the total compensation each man receives plus the selling costs. These transformations are deceptively simple. The proposed compensation per salesman will affect the quality of salesmen the firm can attract. This quality will affect the sales response to selling effort and therefore will also affect the optimal sales budget and again the number of salesmen. Whether the budget or sales force size decision is made first, it will have to be made with an implicit compensation decision and salesman quality level in mind. It may be necessary to specify the budget and sales force size under a number of overall compensation-quality conditions until the decision converges to the best budget and number of salesmen.

The best sales force is simply the one that helps to maximize the firm's profits. If all other marketing variables (e.g., price, advertising, etc.) are specified, and if there are no carry-over or competitive effects due to personal selling expenditures, the problem is to maximize

\[
Pr = pQ(PS) - TC(0) - C(PS)
\]

where

- \( Pr \) = profit
- \( p \) = selling price per unit of product
- \( PS \) = the level of personal selling effort
- \( Q(PS) \) = number of units sold as a function of personal selling effort given that the effort is allocated optimally
- \( TC(Q) \) = total costs of producing and merchandising \( Q \) units, exclusive of personal selling costs
- \( C(PS) \) = total costs of personal selling effort as a function of \( PS \), the level of personal selling effort

In this equation the variable \( PS \) could be measured by the number of salesmen of a specified quality level. The quality level would be associated with some average compensation level which would allow budget estimates \([C(PS)]\) to
be generated. The details of the compensation plans for selling effort will be discussed in the final section of this chapter, but the compensation implication is clear at the resource commitment stage. In fact, in some cases it might be useful to make the sales response \([Q(PS)]\) a function of the number of salesmen and the level of compensation.

Equation 7-1 would be much more complex if the interaction effects between selling and the other marketing variables were considered. The equation would become a multivariate one and the modeling and solution procedures indicated in equations 5-9 to 5-21 of the pricing decision chapter would be appropriate. The budget determination would also be more difficult if competitive interdependencies were to be included in the decision. In this case, the competitive Bayesian and Game Theory models discussed in the price and advertising chapters would be useful. Since these modeling concepts have been outlined in previous sections of this book, this chapter will treat the size of sales effort decision as an independent decision. Under these conditions, finding the optimal size of sales force would be relatively simple if the sales response function to selling effort \([Q(PS)]\) were known, since then the one variable model could be optimized by calculus if \(O(PS)\) were differentiable. The cost functions \([C(PS)\text{ and } TC(Q)]\) would also have to be known and differentiable, but the primary difficulty is involved in identifying \(Q(PS)\).

The discussion of the determination of the optimal size of sales force will center upon the estimation of the sales response to selling effort. It is, of course, possible to specify the sales response by the subjective judgments of the firm's managers, but this should be relied upon only after empirical estimation procedures have been exhausted. Although the measurement techniques are presented in the context of the personal selling decision, it should be realized that they are also useful in estimating the
response to other marketing variables.

**Estimating Sales Response to Personal Selling Effort**

Three basic management science approaches to the estimation of sales response to personal selling can be identified: (1) analysis of historical data; (2) field experimentation; and (3) simulation. Each of these approaches will be discussed and examples of the application of the techniques to the size of sales effort decision will be presented.

**Analysis of Historical Data**

Perhaps the most accessible data are contained in the historical records of the firm's past sales effort. If the future responses to sales effort can be expected to be similar to those in the past, estimation based on this data base may shed some light on the sales effects of personal selling effort that may be expected in the future.

To demonstrate a historical data-based approach to the size of sales force decision, the modeling work done by Semlow will be discussed. His estimation procedure requires that the firm possess a good measure of the sales potential of each territory as well as historical sales performance records of salesmen in territories of different sales potential. The performance criterion in each sales territory is taken as the dollar sales per one percent of the total market potential. It is generally found that territories having greater sales potential also have more, but not proportionately more, sales. A representative result is shown in Figure 7-2 where each point represents a territory. From the figure it is seen that sales per one percent of market potential decline as the size (measured in terms of market potential) of sales territories increases. This figure represents the historical pattern of sales response to selling effort in the firm's sales territories.
EXHIBIT II. RELATIONSHIP BETWEEN SALES POTENTIAL PER TERRITORY AND SALES VOLUME PER 1% OF POTENTIAL
This response relationship can be useful in determining the sales force size. If maintenance of salesmen in the field were cost free and if motivational considerations were ignored, the above analysis would suggest that the firm might do well to have many salesmen, each of whom has a territory having only a small fraction of the total market potential. However, consideration of the field maintenance costs led Semlow to suggest the following simple marginal rule for adding salesmen:

\[(7-2) \quad S\hat{p} - C > 0\]

where
\[S = \text{sales volume of each additional salesman}\]
\[\hat{p} = \text{expected profit margin per unit on this sales volume}\]
\[C = \text{cost of maintaining this salesman in the field}\]

This is essentially a restatement of the necessary condition for an optimum; the first differential of the profit function (7-1) must be zero. The data for \(\hat{p}\) and \(C\) should be fairly accessible from company records.

The validity of the results depends on the accuracy of the response estimation procedure. The dollar sales per one percent of market potential information discussed above may be used to estimate sales results assuming that:

1. All salesmen, including those to be added, are homogeneous in terms of their sales performance,
2. A good measure of territory potential is available,
3. Competitive conditions are relatively equal in all sales territories,
4. The firm has sufficient salesmen to provide a sound basis for analysis,
5. The firm is not dominant in the industry and that a substantial increase in sales will not lead to destructive competitive retaliation, and
6. The firm will assign territories of equal potential to its salesmen.

In addition, Semlow's historical analysis implicitly assumes that intensity of past sales effort accounts for the observed relationship. Other
factors, such as travel requirements and cumulative past territorial effort, are confounded with the intensity of coverage. Furthermore, the analysis implicitly assumes that the type of salesmen currently in the field is the correct one. Perhaps a higher salary would attract a better average quality of salesman and therefore change the optimal size of the sales force. The analysis could be extended by repeating the sales response estimation and sales force size determination for several compensation/salesman quality cases. If the desired compensation/salesman quality levels can be identified in the current sales force, this analysis may be based upon historical data. If not, judgmental inputs will be required. Using this approach, the firm would choose that quality/compensation, and size combination which generates the greatest profit. Under these assumptions and keeping these considerations in mind, the firm may use the analysis of historical data depicted in Figure 7-2 to estimate total sales for a given size of sales force. The rule given in (7-2) may then be used to determine the profit maximizing size of the sales force.

Another interesting historical estimation procedure and management science approach to personal selling decisions is the Waid, Clark, and Ackoff study of the General Electric Lamp Division. It presents an example of the usefulness of historical analysis. Before the study the division was about to undergo a reorganization which at first appeared to require a substantial increase in the size of the sales force. Before making this commitment, management decided to request an Operations Research study of the problem. Assuming the intuitively attractive "S"-shaped response function for cumulative sales volume versus sales time spent with a customer, the research team concluded that the division was presently over-allocating calls to customers. That is, the company was operating in the saturation region
of the response curve. Consequently, the G.E. Lamp Division was able to reduce the average number of calls per customer and thereby handle the increased sales call load entailed in the reorganization without having to hire additional salesmen. Sales results, taken eighteen months after the study recommendations were implemented, were substantially the same as results anticipated by the study. Furthermore, it was estimated that the savings in the first year were twenty-five times greater than the cost of the study.

The authors recognized in their recommendations that the sales response to a reduction in the number of sales calls might exhibit a considerable lag. Consequently, the research team recommended that G.E. monitor sales response via call reports in order to detect any lagged deterioration in market position due to the recommended reduction in calls. The coming era of marketing information systems should render this type of monitoring much more feasible in the future than it has been in the past and therefore improve the historically based response estimates.

In summary, there are several general limitations in the analysis of historical data. First, it is difficult, if not impossible, to establish causality. The confounding of a multitude of forces in the past data may lead to the assumption of spurious relationships or may altogether obscure the causal relations. Furthermore, the results of historical analysis are relevant only to the past operating range. In addition, the important factors and relations must remain stable over time. The researcher must be willing to make such additional assumptions in order to render extrapolation feasible. Fortunately, there quite often is a fair degree of inertia or autocorrelation in market factors and relationships so that historical data analysis may yield useful estimates of sales response.
Field Experimentation

The causal relation between number of salesmen and sales response often can be best determined by experimental procedures. Different sales intensities may be applied to different sales areas according to an experimental design. If the experiment is well designed and executed, it should yield information relevant to the optimal size of the sales force.

Sales effort estimations based on such a controlled field experimental design have been reported by Brown, Hulswitt, and Kettelle. The researchers specified that salesmen allocate various levels of effort -- high, medium, and low -- to three groups of accounts. The objective was to ascertain which level of effort had the greatest market impact.

Given the experimental results, the best levels of effort were designated and a model which indicated the best number of salesmen was developed on the basis of the recommended sales allocation to the groups. The modeling aspects of this study will be reviewed in the allocation section of this chapter, but the measurement aspects of the experimental approach are relevant to the current topic.

The Brown, Hulsitt, and Kettelle study is subject to certain limitations of because/the design of the experimental procedure. The largest accounts were assigned the greatest effort, while the salesmen were allowed to choose which accounts would receive the medium and low sales effort. Both of these factors violate the random assignment assumptions necessary for good experimental design and contribute to an overstatement of the market response to sales effort. Moreover, the procedure assumes that customers and salesmen are perfectly substitutable (i.e., it ignores heterogeneity).

Field experimentation, in general, has other limitations. It tends to
be a costly and time-consuming activity and, in many instances, historical data, field surveys, or subjective judgments may provide better data for decisions in terms of a costs/benefits tradeoff. In addition, changes in size of sales force for experimental purposes involves changing the number and perhaps location of people. Altering these control variable levels is not as easy as in other areas of marketing. In view of these limitations, it may be expected that field experimentation is likely to enter the sales force size decision only indirectly through the more readily controlled areas of allocation such as call frequency and scheduling.

Simulation

The third principal approach to estimating the sales response to selling effort is simulation. Simulation models do provide a framework within which the manager or researcher can identify improved levels of personal selling. Given a valid model of the market, simulation enables its user to ask "what if?" types of questions. For instance, he may interrogate the model concerning market response to X salesmen, X + 1 salesmen, etc. In this way, the manager or researcher can generate alternatives and choose the best one. This will probably represent a good solution to the size of effort problem. This analysis assumes that a good market model exists. For example, if the firm has developed a microanalytical simulation (such as those employed by Amstutz\(^5\)) which examines in detail the interaction between the salesman, the product, and the customer, information regarding the number and type of salesman the firm should have could be generated.

In spite of their considerable potential, few market simulation approaches to sales force decisions have been reported. Only two special purpose sales simulators seem to have been developed. One is the simulation developed by Stokes and Mintz which had as its objective the determination
of the number of clerks to assign to a floor in a department store. This Monte Carlo queuing model was discussed in the distribution decision chapter. The use of stochastic variables to represent the arrival of customers, the service time, the incremental value of sales, and the amount of time a customer is willing to wait for service, enabled profits to be imputed to alternate sales force sizes.

The second published report of a simulation approach to selling decisions deals with the service aspect of personal selling. Service is an important factor in the computer market, business equipment market (e.g., Xerox), and major household appliance markets (e.g., Sears service). A firm which assumes the responsibility to service all its machines as part of its sales agreement is faced with the need to develop a plan for its service function. An interesting example of a simulation approach to this problem is given by Hespos. A field survey indicated that customers differed considerably in their service expectations. Consequently, the use of a simple FIFO priority rule led to more customer dissatisfaction than was necessary. From data on the distribution of customer service expectations, the distribution of the occurrence of service calls, and the distribution of service time requirements, a simulation was performed which helped identify the best (or at least, a good) combination of call scheduling rules and size of the service staff. The model was also useful in identifying future service needs by enabling management to identify the best operating posture for the various future levels of machines in the field.

These model approaches indicate the potential of the simulation technique in personal selling. The technique is useful, but it requires a valid model of the market-salesman interaction. There is a need for a sound model of the customer-salesman interpersonal communication process. If this sort of
model could be developed and validated, it would become the essential element of a heterogeneous microanalytical simulation that could be used to estimate the sales responses to individual and aggregate personal selling effort.

**Summary of Size of Sales Force Determination**

This section began by examining a simple profit equation as a function of sales effort. Given this equation three measurement techniques associated with determining the sales response to sales effort were discussed. The three approaches were historical data analysis, experimentation, and simulation. Published examples of each of the techniques were presented. These techniques were associated with models oriented towards solving the size of sales force problems, but the models were very simple. They did not consider the multivariate effects of other marketing mix elements or the problems of competitive interdependencies. More complex management models were not reported because none exist in the sales force size determination area. This void could be easily filled by a transference of the models that exist in the price and advertising decision areas. Multivariate, dynamic, game theory, Bayesian, and adaptive modeling techniques would be very appropriate and valuable in the determination of the optimal size of the sales force.

**Allocation of Sales Effort**

After the sales force size and budget have been determined, this resource commitment must be allocated over the potential sources of sales. The sources of sales can be characterized by the type of customer, the geographic location of the customer, and time at which a sales stimulus is presented to the customer. It is conceptually useful to think of three kinds of allocation:

1. Allocation to customers or types of customers,
2. Allocation to sales territories,
3. Allocation over time.
Ideally the allocation should simultaneously be made across all three dimensions, but this problem has not yet been solved. In most firms decision sequences based on the firm's perception of the market are used. These perceptions may be expressed in the firm's definition of the decision or control unit it utilizes in its operation.

For example, the firm may perceive its market in terms of areas and define a geographic control unit. Counties might then be chosen as the smallest meaningful control unit for planning purposes in the firm because this is the finest level at which the firm is able to obtain measures of market potential and response. The definition of a control unit may affect the allocation sequence. If counties are control units, this specification may have strong implications about the importance of sales effort allocation to geographic areas. The use of a geographic control unit would lead directly to the territorial decision as the first dimension of sales allocation. The definition of consumer types as a control unit would lead to allocation of effort to customers first.

After allocating over geographic areas or customers the allocation over the other dimensions would be undertaken. In most cases the last dimension would probably be time. This allocation specifies the call sequence and route for a salesman to follow in completing the customer allocations in specific geographic areas.

In this chapter the dimensions will be examined sequentially: (1) customers, (2) territories, and (3) time. The discussion will indicate the interaction between these types of allocation and will close with a consideration of the multidimensional allocation problem. In each area examples will be cited to clarify the problems. During the analysis of the examples and allocations, the high level of interaction between the budget
and allocation decision will be obvious. If the allocation procedure yields results that are consistent with the response assumptions of the budget decision, there may be no feedback and no revision of the size of sales force decision. If allocation procedures produce unexpected responses, the budget will have to be reviewed. In fact, some firms possess so little information about the responses that might be expected from a good allocation that they give only the briefest consideration to the size of sales force before allocation. They then return to the budget decision after the allocation plan has been developed. In this fashion a firm can utilize the feedback loops indicated in Figure 7-1 to converge upon the best or at least good budget and allocation decisions.

Allocation to Customers

A number of management science modeling approaches are available for use in allocating selling effort across customers. The first considered in this section is the deterministic modeling approach. Then a stochastic formulation is presented. Finally, models that attack the allocation of effort between new and old accounts and questions of new account acquisition are discussed.

**Deterministic Modeling of the Customer Allocation Program**

The basic question in allocating effort is the sales response that various customer types will display. With an estimate of the response of various groups, the effort can be carried out so that the marginal returns to each customer type are equal and total returns are maximized. The most direct approach to this problem is to develop deterministic response functions for each of the customer groups and then to allocate effort on this basis.

Buzzell has reported a deterministic study in which the firm sought to specify the extent to which it should go directly to customers versus the
extent to which it should sell to wholesalers who would then sell to final customer. This application considered the question of allocation between direct and wholesale accounts. The criterion used in this analysis was the maximization of profit subject to the requirement of a 10 percent return on sales. The return on sales constraint represented a minimum profit expectation. In this analysis, all salesmen were assumed to be equal, all were assumed to sell the same mix of products, and no competitive effects were considered. Two allocation procedures were proposed. One ignores geographic aspects while the other treats them as given (i.e., sales territories are specified).

The first solution method is based on maximizing profits. The profits were dependent upon the quantities sold to wholesalers and those sold directly to customers. The deterministic sales responses were assumed to be of the form:

\[ Q_i = S(1 - e^{-a_i n_i}) \]

where

\( Q_i \) = quantity sold to customer type \( i \)

\( S \) = saturation level of sales to either customer

\( a_i \) = constant reflecting the sensitivity of sales to increase in \( n_i \) (Note that if \( a \) is large, sales will increase rapidly toward \( S \) as \( n_i \) is increased from zero. Conversely, the smaller the magnitude of \( a_i \), the slower will be the response of sales as \( n_i \) is increased from zero.) \( a_i > 0 \)

\( n_i \) = number of salesmen serving customer type \( i \)

\( i=D \) denotes direct customers

\( i=W \) denotes wholesale customers

Note that the saturation (asymptotic) sales level is assumed to be identical for both classes of customers. The sales response functions for \( i=D \) and \( i=W \) were based upon one empirical observation per customer class. The data point provided information for \( Q_D, Q_W, n_D, \) and \( n_W \). This left the researchers
with two equations and three unknown parameters: $S$, $a_D$, and $a_W$. In order to achieve a solution for $a_D$ and $a_W$, they used a subjective estimate of the saturation level of sales, $S$. Values of $a_D = 0.01725$ and $a_W = 0.065$ were found from this solution. As would have been expected from the extended sales reach achieved through the use of wholesalers, $a_W$ is greater than $a_D$. This indicates that sales respond more rapidly to the first few wholesale salesmen than to the first few direct salesmen. The empirical estimation procedure used for this model is subject to criticism. It is statistically unsatisfactory to estimate a two-parameter response function using only one data point. Such a statistically underidentified response function provides no information concerning the adequacy of the representation imbedded in the response function. A preferable approach would have been to examine historical records for other time periods in order to obtain more than one point for estimation purposes. To proceed with this approach, however, would require that the size of the respective sales forces had varied somewhat in the past.

With these parameter estimates the problem is to maximize profits subject to a profit return on sales constraint. The profit function in this analysis was:

$$(7-4) \quad Pr = m_D S (1 - e^{-a_D n_D}) + m_W S (1 - e^{-a_W n_W}) - c(n_D + n_W) - FC$$

where

$Pr = \text{profit}$

$m_D = \text{profit margin per unit of direct sales}$

$m_W = \text{profit margin per unit of wholesale sales}$

$c = \text{cost of maintaining a salesman in the field}$

$FC = \text{fixed cost}$

It should be noted that the marginal profit contribution is taken to be constant over the entire range of sales for both wholesale and direct sales.
This contains the implicit assumption that there are no cost economies as sales increase and further that there is no need to use price cutting as a weapon to increase sales. In addition, it is assumed that field maintenance costs are identical for both wholesale and direct salesmen. Equation (7-4) is a function of two variables, \( n_D \) and \( n_W \), but for any given size of sales force \( n \) the problem becomes a one variable problem since \( n_D = n - n_W \). The simplified one variable problem can be solved by differentiating the profit equation and using the usual calculus procedures.

The profit equation also allows the exploration of the profit effects of varying the sales force size. This information may lead to a re-evaluation of the size of commitment the firm is willing to make to personal selling. Thus the allocation decision has implications for the budget and size of sales force decisions which have been established.

The results of the analysis in Buzzell's example are shown in Figure 7-3. The figure indicates the profit maximizing allocation of salesmen to each of the two types of accounts for various sizes of sales forces. The optimum allocations were based on Equation (7-4).

**FIGURE 7-3**

Allocation of Sales Effort to Customers

<table>
<thead>
<tr>
<th>Number of Salesmen Assigned to Each Channel</th>
<th>Direct - ( n_D )</th>
<th>Wholesale - ( n_W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of Sales Force ( n = n_D + n_W )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The results of these analyses were some interesting recommendations for changes in sales strategy. The sales force of the company at the time of the study was composed of 42 salesmen, 40 direct and two indirect. The results suggested that for a sales force of 42 salesmen the allocation should be 19 salesmen to final customers and 23 salesmen to distributors. Thus the analysis indicated that a considerable shift in emphasis in sales effort toward distributors was in order. Further results indicated that if the sales force size could be changed the best size would be 156 men of which 110 would sell to final customers and 45 would sell to wholesalers. As indicated in Figure 7-3 when the total size of the sales force increases, the proportion of men assigned to direct sales should be increased.

Analysis of the allocation of effort between the two customer groups was based on two aggregate deterministic response estimates. This response model treated geographic territories as external to the model and therefore was at a relatively high level of abstraction. To include territory considerations the research team reformulated the allocation question. In the reformulation, territories were developed by applying a set of rules to the market. Each territory was to contain less than two million people, be smaller than 10,000 square miles, and be "reasonable" in the minds of the company executives. It was assumed that a direct salesman could cover one such territory and a wholesale salesman could cover five territories defined in this manner. This five to one ratio reflects the relative sales response of the two sales systems. With this given set of territories, the market potential for each region was calculated assuming the firm's market share would be twenty-five percent whether the territory was covered by a direct or indirect salesman. With these territories arranged by sales potentials in
a decreasing order, the lowest potential that could justify a direct salesman was found. In this lowest territory, the direct salesman would just break even, or the profit in the area would be zero. Customers in all territories above this would receive direct sales effort. Customers in territories below the break-even territory would receive no direct sales effort, rather the wholesalers in these areas would receive sales effort.

This analysis indicated that the best allocation of the current 42 salesmen would be 20 salesmen assigned to direct sales and 22 to distributor sales. This is reasonably consistent with the first model's results. In the second model geographic influences were considered, but the response functions were much simpler. The relative sales response was a simple ratio rather than the more sophisticated response function used in the first model. The second model assumed a constant 25 percent share of market regardless of the allocation between direct and indirect customers. Recognizing that the analysis was subject to errors in its market share and market potential assumptions, the research team explored the sensitivity of the policy recommendations to substantial changes in the assumptions. This sensitivity analysis indicated that the policy recommendations of the second model were basically sound even if there were substantial errors in the estimates of market share and potential.

Recommendations from such sales allocation analyses can at times be difficult to implement. For example, the recommended almost fourfold increase in sales force size is an action which can only be implemented slowly due to inherent problems in recruiting, selecting, and training such a substantial increase in the sales force. In addition, problems may occur when personnel must relocate or change their job patterns. It may also be difficult to implement the direct-indirect customer allocations in the specified
territories since it may be difficult or impossible to locate competent distributors in the appropriate areas.

The two deterministic models present good examples of the factors to be considered in sales force allocation. The old analyses began with a given sales force size, but proceeded to the size decision and recommended a new number of salesmen. This represents the feedback between the size of sales force and allocation decisions indicated in Figure 7-1. In attacking the allocation problem the researchers began by analyzing the customer dimension, but they were led to consider geographic effects. They did not directly attack territorial design, but rather used some simple rules to define territories and then considered them as given in the customer allocation analysis. Although this second analysis was made under some strict assumptions, the sensitivity of the results to those assumptions was examined. This is a good methodology for all management science models where strong assumptions are necessary.

**Stochastic Modeling of the Allocation of Sales Effort**

The deterministic approach to allocation is a useful one, but it assumes that consumer response can be described reasonably by a deterministic function. The discussion of the stochastic aspects of consumer behavior presented in the models of market response chapter of this book indicated that consumer behavior is so complex that probabilistic responses might be more valid and useful. This section will present an example of a stochastic sales effort allocation model that was developed by Magee.⁹

Magee attacked the problem of the allocation of missionary sales effort to retailers on the part of the manufacturer of a food product. While wholesalers served the inventory needs of the retailers, the objectives of the missionary salesmen were to obtain favorable shelf space and locations as well as to assist the retailers with displays and point-of-sale promotion for
the product. The question was, "What is the optimal level and allocation of this missionary sales effort?"

The firm's policy prior to the study was to make calls on the top 40% of the retailers as measured by the last two months sales. The present study was designed to answer the questions of:

"How good is this present allocation procedure?", and

"What is the optimum level of sales effort in terms of the proportion of retailers who should receive sales calls?"

In order to answer these questions, Magee first developed a probabilistic model of the market. He assumed that the distribution of the number of cases of the product sold to a given dealer in a unit of time (i.e., one month in this case) could be described by a Poisson distribution, given as:

\[
P(n) = \frac{e^{-c} \frac{c^n}{n!}}
\]

(7-5)

where \( P(n) \) = the probability that the dealer will order \( n \) cases in a month

\( c = \) the expected or average number of cases ordered per month by this dealer

Magee further assumed that different values of the Poisson parameter, \( c \), were distributed among the dealers, i.e., that dealers are heterogeneous with respect to their mean purchase rates \( c \). If dealers are ranked in order of decreasing \( c \), the distribution \( Y(c) \) will result. The distribution \( Y(c) \) is taken to be the probability density function for \( c \) in the population of dealers when all dealers receive normal promotion.

Experiments indicated the distribution \( Y(c) \) was of the form

\[
Y(c) = \frac{1}{s} e^{-c/s}
\]

(7-6)

where \( s = \) the average number of cases ordered per dealer per month in the entire population of dealers.

Using (7-5) and (7-6), the fraction of dealers ordering \( n \) cases in a given month is given by
Magee, however, was interested in evaluating the effect of the then current allocation rule. In order to make this evaluation, he needed to know the probability density function of the mean purchase rate \( c \) for groups of dealers when they received missionary sales effort. Experimental data were used to make this assessment. The total population of dealers in the experiment was divided into two groups -- those who were ordinarily eligible for promotion (the top 40% based upon the previous two month's sales) and those who ordinarily were not eligible for promotion (the lower 60%).

Trial and error curve fitting procedures, using the criterion of best prediction of \( f(n) \), were used to obtain the following distributions:

\[
(7-8) \quad Y_p(c) = \frac{(1-e^{-g(c/s)})e^{-c/s}}{s}
\]

\[
(7-9) \quad f_p(n) = s^n \left( \frac{1}{(s+1)^{n+1}} - \frac{1}{(s+g+1)^{n+1}} \right)
\]

\[
(7-10) \quad Y_{np}(c) = \frac{e^{-((g+1)/s)c}}{s}
\]

and

\[
(7-11) \quad f_{np}(n) = \frac{s^n}{(s+g+1)^{n+1}}
\]

\( Y(c) \) denotes the distribution of average monthly purchase rates and \( f(n) \) denotes the fraction of dealers purchasing \( n \) units in one month. \( S \) is defined in equation (7-6), \( a = \) the fraction of dealers promoted and \( g = a/(1-a) \). The subscript \( p \) denotes the group which would be promoted under the usual allocation rule \( (a) \) and \( np \) denotes the group which would ordinarily not have been promoted. These distributions were developed from experimental data in which all dealers received promotion.
The final estimate necessary to establish the stochastic sales response for the behavior of the lower 60 percent of the dealers was their response if they did not receive missionary sales effort. This will be denoted by $f_{np}(n)$. Data from a non-experimental period were used to estimate $f_{np}(n)$, the fraction of normally unpromoted dealers purchasing $n$ units, under conditions of no promotion. The result for this ordinarily non-promoted group was

$$(7-12) \quad f_{np}(n) = \frac{(0.7)(0.71s)^n}{(0.71s + g + 1)^n} \text{ for } n \geq 1$$

and

$$(7-13) \quad f_{np}(n=0) = (1-a) \left( 1 - \sum_{n=1}^{\infty} f_{np}(n) \right) \text{ for } n = 0$$

where $s$, $a$, and $g$ are as previously defined.

The impact of promotion on dealers who are ordinarily not promoted may be assessed by comparing equations (7-11), (7-12), and (7-13). Such a comparison yields Magee's conclusion that promotion has a dual impact on sales. In the first place, if a dealer is given no promotion, there is a probability of $1-0.7 = 0.3$ that he will act as though his average order size is zero. That is, there is at least a thirty percent chance he will not order. Moreover, in the remaining seventy percent of the time, when he does order, he will act as if his average order size ($c$) is only 0.71 of what it would be if he were promoted. The net effect is about a fifty percent reduction in business to an unpromoted dealer. In other words, the dealers who were normally bypassed would on average buy twice as much if they received promotional effort. This finding represents the sales effect of personal selling to the dealers not normally receiving promotion.

The empirical relations developed above may be used to link sales effort to sales and profit. The sales in the market are given by:
\[(7-14) \quad O(a) = N \{ a \int_{0}^{\infty} \frac{c^a}{(c)} \, dc + K(1-a) \int_{0}^{\infty} \frac{c^a}{np} \, dc \} \]
\[
= \frac{NS}{2} (1 + 2a - a^2) \quad \text{in this example}
\]
where \( Q(a) = \) quantity sold if "a" percent of the customers were promoted
\( K = \) proportionate loss in sales if a customer does not receive a sales call (Recall that in this case it was found to be 0.5.)
\( N = \) number of possible dealers with other notation as previously defined

The profit equation is then given by:
\[(7-15) \quad Pr = p \cdot O(a) - TC(Q) - C(a) - FC \]
where \( Pr = \) profit
\( p = \) unit price
\( TC(0) = \) total costs of selling quantity Q, except personal selling
\( C(a) = \) cost of extending personal selling effect to fraction "a" of the total number of dealers
\( FC = \) fixed costs

If equation (7-14) is substituted into (7-15), a single variable function will be produced and calculus procedures can be followed to obtain a solution. The solution will be the optimum proportion of dealers to receive missionary sales calls (a). This model may thus be used to answer the question of how to allocate sales effort over customer types when customers are characterized by size of order.

Magee has also discussed some of the limitations of this stochastic approach. First, it is a static analysis and does not incorporate the possibility of carryover effects of the promotional activity. However, in the present case Magee regarded this as minor, but in general this is a consideration in stochastic modeling. The dynamic stochastic models developed in the second chapter of this book could be utilized to overcome this
limitation. Secondly, the model does not explain the "why" of the response. It is more an empirical than a theoretical approach. If improvements in the quality of promotion are to be considered as a decision alternative, the "why" question will require examination. Magee also notes that this analysis is not directly applicable to completely new types of promotional activity where causality would have to be considered. A statistical limitation of Magee's model and many model applications should be emphasized. The distributions of the model were empirically determined by trial and error procedures. They were developed as a result of considerable "data massaging". This procedure limits the generalizability of the results and spurious relationships may be generated. One approach to overcoming this problem would be to save some data for validation of the empirically determined functions. Unfortunately, many practical applications do not have sufficient data available to enable the model builder to take this approach, but a good operating rule would be to save some data for validation whenever it is practically feasible.

The stochastic modeling approach to allocation appears to have great potential. This is especially true since the stochastic models developed to describe basic consumer response are waiting to be applied to the sales allocation problem. Magee's model seems to be an excellent starting point since it specifies a basic stochastic mechanism and indicates how to measure the factors needed in determining the best level of effort.

**Modeling the New Account Aspects of Allocation**

The allocation of sales effort between new and old accounts is separated from the two previous approaches because of the dynamic aspects of the new account acquisition process. Both of the examples in the deterministic and stochastic sections were static models. In this section both dynamic, deterministic, and stochastic modeling techniques will be utilized.
The study by Brown, Hulswit, and Kettelle\textsuperscript{10} cited earlier in this chapter represents an attempt to estimate the dynamic response functions of new and old customers and determines an optimal allocation between these two customer classes. The optimal allocation was then used to determine how many current and potential accounts should be assigned to each salesman. Once the sales program has been specified for the individual salesman and its effectiveness has been evaluated, the firm is then in a position to examine how many salesmen it should maintain in the field. This again represents a feedback between the allocation and size of sales force decision.

The application began with the determination of the sales response to selling effort for each type of account (current and potential). In contrast to the historical data approach employed in the case reported by Buzzell, the present study measured sales response to selling effort via field experimentation.

In order to provide a sound basis for their experimental design, a market survey was conducted. The survey indicated that over 88% of all customers tended to concentrate their business with a single supplier (i.e., gave over one-half of their business to one firm). This finding suggested a change in strategy. Previously, the firm had concentrated upon winning a particular job. This survey result suggested that a policy of striving to become the favored printer might be better. The survey also revealed that industry sales tended to be concentrated among the largest customers. Further analysis revealed that the 3,500 customers the firm was then calling upon (about 27% of the total number of customers) accounted for about 88% of total market sales. This suggested that the firm would probably be in a better position by striving for more effective sales results in the customers upon whom it presently called, than it would be in trying to reach the remaining 12% of the
market.

These two findings, concentration of industry sales in a small percentage of all customers and a tendency for customers to concentrate purchases with one supplier, provided the study team with a criteria for classifying customers as current or potential accounts. A current account was defined as any customer who already concentrated his purchases (i.e., purchased at least one-half of his total product need) with the firm. Potential accounts were defined as those which did not so concentrate their business with the firm.

An experiment was initiated to examine the increment in sales which would result from various increments of sales effort (measured in time per month spent with the customer). This experiment exemplified the difficulties of field experimentation in personal selling in particular, and marketing in general. In the first place, the experimental design was not satisfactory. Increases in effort were not randomly assigned to customers. In order to avoid lost sales opportunities during the experiment, the greatest increment in effort went to the largest customers. This, of course, violates the randomization assumptions necessary for good statistical results. Furthermore, the salesmen were allowed to choose which firms would receive what increment in selling effort. This introduces the chances of bias since the salesman's most favored accounts would get the incremental effort. The joint effect of these will be to overstate the average effectiveness of large increases in sales effort. Difficulties also occurred in maintaining appropriate levels of the control variable (sales time). Actual sales effort applied to a customer during the experiment often differed from what was called for in the experimental plan.

The experimental results, although not unbiased, indicated that the most productive sales calls were those made on the largest customers. Although
large customers were somewhat less likely to respond, this was more than compensated by the magnitude of the response when they did respond.

The classification of customers into new and old accounts led the research team to consider two types of effort -- conversion and holding. The former represents effort applied to new accounts, while the latter represents effort allocated to old accounts. Conversion effort was described by a conversion curve derived from the experimental data which related the sales effort per month \( (x) \) expended upon a customer to his probability of exhibiting a substantial increase in purchases \( C(x) \). The criterion used for specifying the best level of sales effort to expend upon new customers \( (X^*_C) \) was to maximize the expected conversions per hour of sales effort, which is given by the point \( x \) for which \( C(x)/x \) is a maximum.

Holding effort was described by a holding function relating the sales effort per month \( (x) \) expended upon a customer to his probability of not decreasing his purchases significantly during the month \( H(x) \). The best rate of holding effort \( (X^*_H) \) was determined by the principle of expending holding effort up to the point of equal incremental profitability for conversation and holding effort. Recall that the best level of conversion effort has already been determined, so that the holding effort rate is based on the outcome of that analysis.

The analysis then proceeded to the assignment of accounts to salesmen. The first relation in this analysis was established using the effort rates determined above as:

\[
(7-16) \quad X^*_C N_C + X^*_H N_H = T
\]

where \( T = \) total monthly sales time available to a salesman

\( X^*_C = \) optimal monthly sales effort to allocate per conversion customer

\( X^*_H = \) optimal monthly sales effort to allocate per holding customer
\[ N_C = \text{number of new (conversion) accounts assigned per salesman} \]
\[ N_H = \text{number of old (holding) accounts assigned per salesman} \]

Equation (7-16) is an equation in two unknowns, \( N_C \) and \( N_H \). Another relation is required for solution. The system will be in equilibrium whenever the expected conversions per month balance the expected relapses per month. This second relation is given by

\[(7-17) \quad N_C \Pr(C) = \Pr(H)N_H \]

where
\[ \Pr(C) = \text{probability of conversion with optimal sales effort } X^*_C \]
( obtained from conversion response function )
\[ \Pr(H) = \text{probability of holding with optimal sales effort } X^*_H \]
( obtained from holding response function )

The equations (7-16) and (7-17) may be solved to yield \( N_H \) and \( N_C \), the number of old and new accounts to assign per salesman. The use of (7-16) and (7-17) along with the costs of adding additional salesmen could be used to make calculations of the profit implications of adding an additional salesman. The results of the allocation analysis therefore yield response information that can be used in a review of the total sales force resource commitment. The analysis assumes that the criterion is to establish an equilibrium sales level. It would be useful to extend (7-17) to allow for growth. Then the allocation necessary for increasing sales could be examined. This would yield valuable information for determining the growth pattern for the sales force.

Once again, this illustrates the considerable interaction which often occurs between the size and allocation decisions in personal selling.

The modeling of the allocation between new and old accounts is sensitive to dynamic process by which new accounts are developed. The new account acquisition is worthwhile to model separately so that the allocation of effort over classes of potential accounts can be determined. Two new account models will be presented. The first classifies new accounts by the number of sales
calls they have received while the second groups potential new accounts by the level of interest they display in the firm's product offerings. Both models are stochastic models that utilize absorbing state Markov chain theory.

The first new account effort allocation model has been developed by Shuchman. In his model, effort is defined as the number of calls made on a prospect. The model yields information for controlling sales effort as well as for the determination of the number of times to call upon a prospect before dropping the prospect. This is termed the call frequency policy.

Each prospect is classified by the number of calls which the salesman has made upon him in the past. This description is the state of the prospect in a Markov model. There are two mechanisms whereby a prospect may leave the prospect class: (1) he purchases from the salesman and thereby enters the established customer class, and (2) he is dropped as a prospect by the salesman. The Markov transition matrix for prospects is given in Figure 7-4. The elements in this matrix represent the probability of the prospect going from state $i$ to state $j$ between times $t$ and $t+1$. Implicit in this representation is the assumption that each prospect is called upon once and only once during each interval of time. Under Shuchman's assumptions one of three things must happen to a prospect in state $i$ who is called upon between times $t$ and $t+1$. He either enters the prospect state $i+1$ with a probability $P_{i,i+1}$, is sold with probability $P_{i,s}$, or is dropped with probability $P_{i,d}$. In this model, $n$ represents the maximum number of calls that will be made upon a prospect before that prospect is either sold or dropped. This value is a policy value which can be studied via the model.

Results from the theory of absorbing state Markov chains can now be used to yield analytic estimates of:
1. The expectation and variance of the number of calls which will be made upon a prospect starting in any non-absorbing state before it is either sold or dropped.

2. The expectation and variance of the proportion of prospects sold and dropped for any initial state distribution of prospects.

**FIGURE 7-4**

Absorbing State Markov Model for Determining Prospect Call Policy

<table>
<thead>
<tr>
<th>Time t+1</th>
<th>s</th>
<th>d</th>
<th>0</th>
<th>1</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

| Time t          | 0    | $P_{os}$ | $P_{o,d}$ | 0    | $P_{os}$ | ... | 0    |
| i              | $P_{is}$ | $P_{i,d}$ | 0    | 0    |     | 0    |

$P_{n-l,s}$  $P_{n-l,d}$  0    0    ...  $P_{n-l,n}$

| n    | $P_{ns}$ | $P_{nd}$ | 0    | 0    | ... | 0    |

**State Definitions**

State $s$ = the prospect is sold by the salesman and leaves the prospect class for a customer class

State $d$ = the prospect is dropped by the salesman

State $i = 0, 1, ..., n$ denotes that the prospect has been called upon $0, 1, ..., n$ times where $n$ is the maximum number of times a prospect may be called upon

If it is further assumed that the salesmen add to their prospect lists at each time period a number of prospects equal to those which have been absorbed into either state $s$ or state $d$, a steady-state age distribution of prospects will result. The mean and variance of this distribution may be obtained as may the mean and variance of the number of prospects sold and dropped once the steady-state is reached.
The means and variances are associated with some specified policy of the maximum number of sales calls (n) per prospect. The Markov analysis can then be used to generate the expectation and variance of the number of sales per period which will result in the steady state for alternative values of n. If the firm then relates the profitability of these sales to the cost implications of the alternative n, it has sufficient information to establish the best maximum call policy (n).

The results of the model may also be used for control purposes. The model yields the mean and variance of several statistics of interest, such as the total number of prospects who will be sold in a given period. This type of information may serve as a base line in an exception reporting system. Salesmen's performance can be monitored and compared to these base line results. Since both the mean and variance are available, control chart procedures may be used to indicate cases of exceptionally good or very poor results. This information may then serve as the impetus for further study of these exceptional situations and perhaps ultimately may serve as the basis for reward or corrective action, depending upon the direction of the deviation.

This model incorporates a number of assumptions and these are pointed out by Shuchman. For instance, it assumes that the transition probabilities are stationary (or constant) in time and that they are independent of the initial age distribution of prospects. The latter assumption seems reasonable. In view of the dynamic characteristics of most markets, the description of stationarity of the transition probabilities will not hold except perhaps in the short run. Seasonal and cyclical fluctuations also pose problems. Nevertheless, if short run stationarity holds, the model can be estimated and subsequently used to make conditional predictions concerning what will occur if no change takes place. A further assumption is that prospects are of
equal value. This, of course, is unrealistic. However, prospects may be
stratified into relatively homogeneous value groups and the analysis run
separately for each of these groups. An implicit assumption is that all
prospects in the analysis are identical in terms of their transition probabilities.

An interesting research topic would be the incorporation of heterogeneity
of transition probabilities for a class of prospects.  

An alternative formulation of an absorbing state Markov chain model for
new account sales allocation has been presented by Thompson and McNeal.  

Their model involved two absorbing states and four transient or non-
absorbing states. These states were defined as:

$$S_1 = \text{an absorbing state indicating that a sale was made on the most recent call},$$

$$S_2 = \text{an absorbing state which indicates that the customer was deleted from the prospect list as of the most recent call},$$

$$S_3 = \text{a transient state indicating a new prospect had no history of sales calls},$$

$$S_4 = \text{a transient state which indicates that the prospect expressed a low degree of interest on the most recent sales call},$$

$$S_5 = \text{a transient state indicating that the prospect expressed a medium degree of interest during the most recent sales call, and}$$

$$S_6 = \text{a transient state indicating that the prospect expressed high interest during the most recent sales call.}$$

The state definitions are related to interest level and are not related to real time effects as were the Shuchman state definitions. The absorbing chain
transition matrix appears in Figure 7-5. Note that the entire column of transition probabilities under $S_3$ (the new prospect state) is zero. This reflects that a new prospect must enter one of the other five states once one sales call has been made.

Absorbing state Markov chain theory will yield the expected number of sales calls which will be made upon a prospect before he enters one of the two
An Alternative Absorbing Chain Call Policy Model

Prospect's state after new sales call

<table>
<thead>
<tr>
<th>Prospect's State</th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
<th>S₄</th>
<th>S₅</th>
<th>S₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₂</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₃</td>
<td>P₃₁</td>
<td>P₃₂</td>
<td>0</td>
<td>P₃₄</td>
<td>P₃₅</td>
<td>P₃₆</td>
</tr>
<tr>
<td>S₄</td>
<td>P₄₁</td>
<td>P₄₂</td>
<td>0</td>
<td>P₄₄</td>
<td>P₄₅</td>
<td>P₄₆</td>
</tr>
<tr>
<td>S₅</td>
<td>P₅₁</td>
<td>P₅₂</td>
<td>0</td>
<td>P₅₄</td>
<td>P₅₅</td>
<td>P₅₆</td>
</tr>
<tr>
<td>S₆</td>
<td>P₆₁</td>
<td>P₆₂</td>
<td>0</td>
<td>P₆₄</td>
<td>P₆₅</td>
<td>P₆₆</td>
</tr>
</tbody>
</table>

Notation: \( P_{ij} \) = the probability that a prospect that was in state i after the last sales call will be in state j after the next sales call.

Absorbing states. This value is available for a customer presently in any of the four transient states. If it is assumed that there is a constant cost per sales call, the expected cost of calling upon a prospect presently in state i (i = 3, 4, 5, 6) may be computed. The theory also yields an analytic expression for the probability that a prospect currently in state i (i = 3, 4, 5, 6) will ultimately be sold (i.e., be absorbed into state 1). If a constant (or expected) profit contribution per sale (excluding the cost of sales calls) can be ascertained, the expected revenue which will result from a prospect currently in state i (i = 3, 4, 5, 6) may be obtained. The value of a customer in each state i may then be determined by the difference between the expected profit contribution and the expected sales call costs. The relative values of prospects in each of these states then provides useful information for allocating sales time. A salesman should first call upon the accounts which are in the state having the highest expected profit. He should
continue to allocate one call per account in this state until one of two conditions occurs:

1. He exhausts the time he has available during the call planning period, or

2. He exhausts the set of prospects available in this class or state.

When condition 1 occurs, he is finished for the period. When 2 occurs, he should then go to the next most attractive set of prospects (classified by their current state). Naturally, a class of accounts (i.e., accounts in a given current state) will only receive calls if the expected profit from calling upon them is positive.

As was true in Shuchman's model, the present model can be used to establish certain base lines for sales performance. Exception reporting schemes may then be implemented. In order to accomplish this, however, it will be helpful to utilize the variance of certain measures which are available from Markov theory, as Shuchman recommended.

Summary

In summary, the modeling approaches of Buzzell, Brown et al, Magee, Shuchman, and Thompson and McNeal described in this section indicate a number of interesting and rather sophisticated attacks on the problem of the allocation of sales efforts to consumer types. These applications represent the starting point for a surge of management science modeling in the area. It is evident that the models developed to describe basic consumer response and to solve advertising problems have a high degree of applicability to the sales allocation problem. The models discussed in this section have dealt with one dimension of the total allocation problem. The existing models attack the allocation of effort between customers, given a geographic plan or treating territories as exogeneous and without consideration of scheduling effects.
Allocation to Geographic Areas

While units of sales potential can be identified by the type of customers in that unit, it is also possible to characterize these units by geographic coordinates. This is a useful dimension for allocation and may take place before or after customer allocation. In this section a number of efforts to analyze the geographic allocation of sales effort will be outlined. These examples analyze area effects assuming a given customer and time allocation or assuming customer class and time effects to be exogeneous. Two approaches to geographic allocation will be considered. The first is the allocation of sales effort to given territories and the second is the use of territorial design to allocate sales effort.

Allocation to Given Territories. The simplest approach to area allocation is to maximize the total returns of allocating a given sales effort to a specified set of territories. In general, the necessary condition for an optimum will be that the marginal returns in each geographic area are equal. J. A. Nordin has described this marginal approach for allocating sales effort between two geographic areas.

Nordin's objective was to maximize total sales subject to a budget constraint on the total cost of the sales effort. His problem may be stated as:

\[
\text{(7-18) Maximize: Total Sales} = X_1' + X_2'
\]

\[
\text{Subject to: } TC_1 + TC_2 = K
\]

where

- \(X_1'\) = unit sales in area 1
- \(X_2\) = unit sales in area 2
- \(K\) = the available sales expense budget
- \(TC_1\) = the total sales costs of selling \(X_1\) units in area 1
- \(TC_2\) = the total sales costs of selling \(X_2\) units in area 2
Sales expense in each territory is taken to be a function of the number of units sold in that territory. In particular, Nordin assumed that the incremental cost \((y)\) of selling the \(X\)th unit in an area was of the form:

\[
y_1 = a_1 X_1^\alpha, \quad i = 1, 2
\]

Thus the total cost of selling \(X_1\) units in area \(i\) will be given by:

\[
TC_i = a_1 \int_0^{X_1} X_1^\alpha \, dX_1 = \frac{a_1(X_1)^{\alpha+1}}{\alpha + 1}, \quad i = 1, 2
\]

The maximization problem in equation (7-18) can be restated as a Lagrangean expression, and the necessary condition for a sales maximizing allocation is given by the simultaneous set of partial derivatives which result from the Lagrangean form. The necessary condition for an optimal allocation occurs when effort is allocated such that the incremental cost of selling the \(X_1\) th unit in each area is equal for all areas. That is,

\[
y_1 = a_1(X_1)^\alpha = a_2(X_2)^\alpha = y_2
\]

This marginal condition for sales maximization subject to a budget constraint also holds for any number of territories. The generalization of the sales maximizing condition for \(n\) territories is given by

\[
y_1 = y_2 = \ldots = y_n
\]

A more direct approach to allocation problems is to treat personal selling as a demand creation element rather than as a cost. This is more logically consistent with the functions visualized for personal selling. A. P. Zentler and Dorothy Ryde have proposed a model using this approach.

The problem is to maximize:

\[
\sum_{r=1}^{n} N_r R_r(X_r) \quad \text{subject to} \quad \sum_{r=1}^{n} N_r X_r = S
\]

where

\[
X_r = \text{sales effort to expand in area } r \text{ (i.e., the control variable)}
\]

\[
N_r = \text{sales potential of area } r
\]
\[ P \text{ } X_r = \text{sales response function of area } r \text{ to selling effort } X_r \]

\[ S = \text{sales effort budget} \]

Zentler and Ryde assume that \( R_r(X_r) \) varies between territories, but that the general form of response will be an "S" curve from all areas. The response is also made dependent upon past sales effort. The lagged relationships and the assumed "S" response form produce an \( R_r(X_r) \) that is very complex. This complexity precludes the direct use of calculus, since the partial equations describing the necessary conditions for a maximum response are not directly solvable. Zentler and Ryde propose a graphical heuristic approach to solve this maximizing problem. The heuristic is based on equating the marginal returns up to the budget constraint. Although the model is not analytically tractable, the consideration of the dynamic lagged effects compensates for this inconvenience. In general, it is better to compromise the technique rather than the essential characteristics of management's definition of the problem.

The competitive effects of sales effort could be encompassed in the game theory promotional allocation model developed by Friedman.\(^1^9\) Friedman's model is directed at the geographic allocation of advertising expenditure, but the promotional variable could be construed as sales effort. The model is a two person zero sum game where the decision alternatives are allocation plans which specify the amount of promotional effort each area is to receive. The pay-offs for the strategies are proportional to the firm's share of total industry promotional expenditure in the areas and the potential of the areas. Under the assumption of a zero-sum game, the maximin equilibrium is achieved when each firm allocates its promotional expenditures in proportion to the sales potential of each area. The greatest limitation of this formulation is the zero-sum assumption of the game, but it does represent an attempt to
build competitive effects into geographic allocation. The non-zero sum game concepts developed by Shakun, Krishman and Gupta, and Baligh and Richart have been discussed in earlier chapters would make this approach more feasible.

**Territorial Design to Allocate Sales Effort.** A second approach to area allocation is to treat territorial definitions as a variable and assume one salesman will be assigned to a given area. This is the converse of the approach outlined in the last section which treated territories as given and assigned an amount of sales effort (implicitly a number of salesmen) to each area.

The most elementary approach to territorial design is to divide the market into geographic areas of equal sales potential. By forming N such areas, where \( N = \text{number of salesmen} \), each salesman would be assigned to an area of equal potential. The disadvantage of this approach is that it does not include consideration of customer allocation or the routing and scheduling aspects of sales allocation.

A more sophisticated territorial design method is to design territories so each salesman has an equal workload. This design procedure is based on a given allocation of sales effort to customer types. The workload equalization procedure accepts the customer allocation in the form of the call frequency on each class of customers. With this desired level of effort, each customer territory is defined so that each salesman has a full workload based on the assigned number of calls to each account in his territory.

Walter J. Talley has outlined this approach. Given call frequencies for each account, territories are assigned in a heuristic manner so that the salesman's travel and sales call time provides him with a full workload. Talley has indicated that territories might be assigned not only on the basis of existing customers, but also on the basis of consideration of
potential calls. The potential calls would be an estimate of the number of customers and calling frequencies in the future. These future call potentials may be combined with existing requirements to design workloads and territories. The number of salesmen needed would be estimated by dividing the total number of calls required to accomplish the given call frequency requirements by the average number of calls a salesman could make.

\[
N = \frac{\sum_{i=1}^{n} f_i C_i}{F}
\]

where

- \( N \) = number of salesmen
- \( f_i \) = given call frequency on customer type \( i \)
- \( C_i \) = number of customers of type \( i \)
- \( F \) = average number of calls an average salesman can make

These considerations of sales force size may lead to re-evaluation of the sales force size decision.

The most detailed statement of the work study approach is given by J. O'Shaughnessy. O'Shaughnessy establishes the time requirements for selling, travel, and administrative activities for each customer type. Then, given a call frequency requirement for each class of customer, he groups accounts so as to give each salesman a full workload. This grouping is done on a trial and error basis. The groupings that produce the best workload patterns of the alternatives explored are specified as the respective salesman's territory.

**Summary**

In summary, the allocation of sales effort to geographic areas has centered around two approaches. The first is to allocate effort to a given set of territories. This approach is implemented by defining response functions for each area and resorting to calculus or other procedures to
obtain solutions. The second approach is to assume that customer sales effort allocation is given by call frequency requirements and to design territories to give each salesman a full work load. Both approaches attach the area allocation assuming a given customer allocation or treating customer classes as outside the analysis and assuming the calling schedule and routes are given.

**Allocation of Sales Effort Over Time: Scheduling and Routing**

The final dimension of sales allocation is the specification of the manner in which sales effort will be applied within a given unit of time. The dimension is measured in time or by the geographic position of a salesman at a specified time. Given effort allocations by customers in each geographic area, the solution to the time allocation problem would be the optimum sequence for calling on the customer. The sequence would include specification of the best route to follow and the schedule of calls to be made.

The routing aspects of time allocation have received a large amount of attention from management scientists. The work has been concerned with the special case of routing when one call is to be made on each of a given number of customers in various geographic locations. This has been called the "traveling salesman problem". Explicitly, the problem is to find the best route to follow in visiting each customer location once and returning to the starting point. The best route has alternately been defined as the shortest length route, the route requiring the least travel time, or the lowest cost route. Although this problem has been called the traveling salesman problem, it should be pointed out that it is not a general formulation of the intra-period time allocation problem of selling.

In fact, technical interest has centered upon the traveling salesman
problem as a general formulation of a certain class of scheduling problems. For example, it has been applicable to production and traffic flow problems. Thus the "traveling salesman problem" is not really an attempt to represent a sales problem in a realistic and meaningful fashion. It abstracts from the scheduling aspects of the calling sequence by assuming that one call will be made to each account each period. Therefore a one period cycle is specified. If some accounts were to receive more than one call per period, the time between these calls would affect the best route.

Solving the traveling salesman problem is a difficult task. The problem could be solved by examining each route, but since there are \((n-1)!\) possible routes, this is usually impossible. Many management science techniques have been indicated to solve the problem. The techniques of linear programming, integer programming, dynamic programming, heuristic programming, and branch and bound techniques have been proposed. It is not within the scope of this book to discuss these techniques in detail. All of them are capable of yielding solutions to the problem. The choice between them depends on the size of the problem, the degree of approach to optimality desired, and the computation costs. The use of any of these techniques to solve the sales call scheduling problem is valid only if each customer is to be called upon once during the travel period. This may be a reasonable approach if the list of customers to be called upon is specified for each routing period and is modified each period to reflect a given allocation of effort among specific customer types in each geographic area.

**Multi-Dimensional Allocation**

Each of the modeling attempts outlined in the last three sections have attacked one of the dimensions of sales effort allocation. Each analysis has examined either customer, area, or time allocation and treated the remaining dimensions as given or exogeneous to the analysis. It would be desirable to
have practical management science models that would simultaneously attack
the allocation along more than one dimension.

Only one reported management science research effort has been directed
towards this end. This is based on an interesting modification of the travelling
salesman problem that simultaneously considers allocation to consumer types.
It has been developed by James B. Cloonan. Cloonan attacks the sequencing
of sales calls and the development of a call policy. Beginning with a
traveling salesman problem solution (a routing to minimize the travel time to
visit each account and return home), he examines the effects of deviating
from this minimum cost route to increase the effectiveness of the sales effort.
The effectiveness of the effort is related to the time at which the call is
made. Cloonan proposes that the value of a call is parabolically related to
the time since the last call:

\[ V_{iT} = (at - bt^2 + c) V_{im} \]

where

- \( V_{iT} \) = value of call on customer \( i \) at time \( T \)
- \( t \) = time since last call on customer \( i \)
- \( V_{im} \) = maximum value of a call on customer \( i \)
- \( a, b \) = positive constants

This relationship indicates that scheduling of the calls on customers as well
as the route of salesmen are important to the value and return to sales
effort. The minimum cost route could be departed from whenever the value of
a call is greater than the costs of deviating from the minimum cost route.
To find the best schedule of calls, Cloonan proposes a heuristic programming
model. The model utilizes the concept of opportunity cost in calculating
the cost of making a call other than the one specified in the minimum cost
route. The opportunity cost of leaving the route is the difference between
the total time required to leave the network, to make a call, and to return
to the next customer, less the time necessary to transverse the next link
in the lowest cost route. For example, if the time to travel the next link of the minimum cost route is five hours (Point Z), the time to the alternate prospect is four hours (Point Y), and the time from the alternate customer to point Z is four hours, the opportunity cost is \(3 = (4 + 4) - 5\). The total time for the deviation from the route is the opportunity cost plus the time spent in the actual sales call. If the value of the call prescribed by equation (7-24) is greater than the cost of deviating from the route by a prescribed margin, the deviation is taken. By analyzing the minimum cost route in this way, the heuristic procedure attempts to generate a good sequence of calls which considers both the costs of routing and the increases in sales value associated with changing the calling schedule. In this heuristic program, customers could be visited more than once and therefore the program could be used to generate a call policy. The call policy would reflect the allocation of sales effort to various customers. The heuristic has not been tested to determine how close to optimal the generated schedule would be, but this preliminary model indicates a productive avenue for future research in the area of multi-dimensional sales force allocation.

**Summary of Allocation of Sales Effort**

The discussion of the allocation of sales effort has been developed by examining the three dimensions of allocation: customers, areas, and time. In each area the allocation problems were defined and existing management science applications in these areas were described. Deterministic, stochastic, and dynamic modeling techniques have been applied to allocation between classes of customers, but the potential for transferring other models developed for use in advertising and pricing decisions was emphasized. Geographic allocation has been attacked by allotting sales effort to given areas and by designing geographic sales territories so that each will represent a reasonable workload.
for one salesman. Friedman's game theory allocation model was cited as an example of the potential for the utilization of existing models in geographic allocation of sales effort. Other techniques such as mathematical programming would appear to be applicable to this problem. The last dimension of allocation discussed was the problem of routing or what has become called "the traveling salesman problem". It was emphasized that the traveling salesman problem is of interest in management science because it represents a class of theoretically interesting combinatorial problems and that it only represents a small portion of the total sales allocation problem. The final comments on the allocation of sales effort were directed at the desirability of developing multi-dimension allocation models.

A heuristic programming model that represents an attempt to simultaneous allocate across more than one dimension was presented to indicate the research potential in this area.

The allocation decision represents an attractive area for future model building effort. In this research, the usage of models developed in the pricing and advertising area will probably be feasible, but the researcher must not lose sight of the basic human aspects of selling. Salesmen are men and are therefore heterogeneous with respect to their behavior patterns. Good models should encompass this heterogeneity in their analyses of the problem of allocating sales effort.

ORGANIZATIONAL ASPECTS OF PERSONAL SELLING

After the size of the sales force has been determined and the available sales effort has been allocated to sales units, a number of organizational questions must be analyzed. It is not within the scope of this book to analyze all of the organizational aspects of personal selling. Only those aspects which seem most amenable to management science modeling will be
discussed in this section. First, the question of span of control will be considered. Specifically, the definition of control units and the question of how many sales offices to establish will be considered. The next topic will be salesman compensation. After considering this motivational question, attention will be directed toward the assignment of salesmen to territories and the implications of such assignment for the hiring of salesmen. Finally, sales control procedures and their interaction with information system design concepts will be outlined.

Span of Control in the Sales Organization

The definition of the control unit to be used in sales force decisions is necessary if organizational control procedures are to be efficiently implemented. The definition of the micro control unit is the beginning of a description of a hierarchy of control units characterized by an increasing level of aggregation. For example, counties may be the micro control units, but these micro units may be combined into states for use as district control units, and these district control units may then be combined to specify regional units. Thus there generally is a hierarchy of control units representing different levels of aggregation of primary sales units.

The design of the span of control of this hierarchy can be modeled. The best span of control is the one that would generate the best overall profits for the firm. If the revenue of the firm is not affected by the span of control, the best design is the one that minimizes costs. Stern has developed a model that attacks a part of this span of control problem. His model determines the optimal number of sales offices. The approach was to find the minimum cost number of brand offices for a given level of sales. Three major categories of costs were distinguished: overhead, travel, and direct customer costs. Overhead costs were composed of the base salary of the salesmen, the
branch manager's salary, and the capital and administrative costs associated with a branch office. Travel costs included direct transportation related costs, salesmen's personal expenses while traveling, and communication with the branch office while on a business trip. Direct costs involved salesmen's commissions, entertainment of customers, and extra discounts and services. For a fixed level of sales, the postulated relation between the cost components and the number of branch offices is given in Figure 7-6.

FIGURE 7-6

Cost versus Number of Branch Offices

Direct selling expense is unrelated to the number of offices and therefore does not affect the cost minimizing number of branch offices. Thus it could have been omitted entirely from this analysis. Overhead costs increase with the number of offices since there will be more managers and administrative expenses involved for a fixed level of sales. The shape of the curve reflects an assumption of economies of scale in the size of branch offices. Travel costs to customers from the branch offices will tend to decline as the number increases since the salesmen will have shorter distances to travel as well as fewer nights to spend on the road at company expense. In Figure 7-6, the cost minimizing number of branch offices is denoted by \( n^* \). Notice the relative insensitivity
of total cost to moderate departures from \( n^* \) in this example.

Stern's formulation would seem to have three major types of value to management. In the first place, the model forces the firm to consider how the various elements of sales costs vary with the level of sales. This is important input to the planning of the span of control. Secondly, it provides a measure of the adequacy of the present sales organization. For the present level of sales, the model will yield the optimal number of branch offices. This number may then be compared to the present number of branch offices in order to evaluate how far from the best span of control the present organization is. The model may also be used to evaluate the increased costs due to a current non-optimal number of offices. Finally, the model can be used for planning purposes. It can be used to explore the optimal number of branch offices \((n^*)\) for various anticipated future sales levels. If a probability distribution can be ascribed to these future sales levels, the expected optimal number of branches called for in the future may be determined. The potential of management science in organizational problems displayed by this elementary model should be exploited by further research.

**Compensation of Salesmen**

The motivational considerations of the method of compensation represent an area of organization decision-making where management science can be of use. A model that would include the motivational effects of compensation plans and the profit implications of the costs and revenues generated by such a plan could be an aid in finding the best level and method of rewarding salesmen.

A modeling effort has been expended in this direction by Farley. He has shown that under certain conditions an optimal policy is one of paying equal commission rates for each product, where the commissions are based on the gross margins of each product in a firm's line. It is
optimal in the sense that if a salesman maximizes his total income under this scheme, this behavior will result in a maximum profit contribution to the firm. The motivational assumption here is that salesmen attempt to maximize their dollar earnings. This is a simple mechanism and modeling efforts should be directed at including social and psychological rewards and motivations. These attempts should also be directed at evaluating the implications of salary, salary plus bonus, or straight commission plans in terms of the motivational effects on the salesman and their resultant effects on company sales and profits.

Assignment of Salesmen

All the models presented in this chapter have assumed that salesmen are identical. Salesmen are not homogeneous. The heterogeneity of sales ability can be considered in models designed to assign salesmen to territories. It is presumed that the number of salesmen have been determined and sales territories have been defined. The problem is to assign "n" heterogeneous salesmen to "n" heterogeneous sales territories in such a manner that profits will be maximized. The problem can be stated verbally as: Maximize the total profit subject to the constraint that each area is covered by one and only one salesman. This is a linear programming problem. In fact it is a special type of linear programming problem that occurs in so many fields that it has been called the "assignment" problem and special computationally efficient algorithms have been devised to solve it. Given a matrix that describes the profit resulting from assigning a particular salesman i to a particular territory j, the determination of the optimal assignment is a routine process of applying the algorithm. The marketing aspects of the problem are contained in finding the profit matrix. An example will clarify the issues involved.

Let us assume that not only do salesmen differ in their effectiveness with
different types of customers or territories, but that sales territories also differ with respect to the distribution of the customer characteristics which are relevant to the salesman/customer interaction. Suppose, for example, that there is only one relevant variable, customer size (profit potential) and that it is measured at three levels. Then the distribution of customer size for two territories, I and II, might be given as in Table 7-1.

### TABLE 7-1

<table>
<thead>
<tr>
<th>Customer Size Distribution by Sales Territory</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_{jk} Customer Size Group</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Territory</td>
</tr>
<tr>
<td>j = I</td>
</tr>
<tr>
<td>j = II</td>
</tr>
</tbody>
</table>

*Measured as dollar profit potential

**Number of accounts in this cell

The firm must now specify the profit potential of an account in each size group. These will be denoted as $P_s$, $P_m$, and $P_L$ for small, medium, and large accounts, respectively. Note that territory I has a concentration of small accounts while territory II has fewer total accounts but a preponderance of large accounts.

The next input required is a description of the salesmen in terms of their effectiveness with accounts of various sizes. The measure of effectiveness to be used will be the proportion of potential profit the salesman has been able to obtain for each class of accounts. If historical experience is not available, judgmental input would be required. A set of sample values appears in Table 7-2. Notice that salesman 1 is especially effective with small accounts, whereas salesman 2 is relatively more effective with large accounts.
TABLE 7-2

Sales Effectiveness by Customer Size*

<table>
<thead>
<tr>
<th>Customer size group</th>
<th>k=1 Small</th>
<th>k=2 Medium</th>
<th>k=3 Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=1</td>
<td>0.8**</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>i=2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.6</td>
</tr>
</tbody>
</table>

*Measured as dollar profit potential

**The proportion of total potential profits realized in this cell

Using the data from Table 7-1 and Table 7-2 along with $P_s$, $P_m$ and $P_L$, one may obtain the expected profitability of each salesman in each territory by the formula

\[(7-25) \quad V_{ij} = \sum_k D_{jk} \cdot SE_{ik} \cdot P_k\]

where $D_{jk} = \text{Number of customers of size k in territory j (see Table 7-1)}$

$SE_{ik} = \text{Sales effectiveness for salesman i for each customer size group k (see Table 7-2)}$

$P_k = \text{Profit potential of each customer in group k}$

$V_{ij} = \text{Expected profitability of assigning salesman i to territory j.}$

TABLE 7-3

Assignment of Salesmen to Territories

<table>
<thead>
<tr>
<th>Profit</th>
<th>Territory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>j=I</td>
</tr>
<tr>
<td>Salesmen</td>
<td></td>
</tr>
<tr>
<td>i=1</td>
<td>$V_{1,I}$</td>
</tr>
<tr>
<td>i=2</td>
<td>$V_{2,I}$</td>
</tr>
</tbody>
</table>
This profitability data can then be arrayed in a table such as Table 7-3. While the solution of the two salesman-two territory problem is obvious, the generalized n salesmen and n territories problem can be solved by the straightforward application of the assignment problem algorithm.

This simple model has been extended by King so that the selection of salesmen from a potential set of employees can be examined. His extension is based upon the use of probabilities of sales success given that a salesman is assigned to a particular territory. This is an alternate interpretation of the entries in table 7-2.

When the $SE_{ik}$ are interpreted as the probabilities of success, the $P_k$ used in equation 7-25 are the sales that a successful salesman would obtain in area k. King proposes the use of discriminant analysis to estimate these probabilities of successful performance. The discriminant analysis would be performed on past salesman-territory data. Each past salesman would be represented by a set of characteristics such as personality or ability test scores. The output of the analysis would be the probability of success in each territory for a salesman with a particular set of characteristics.

This information and a set of scores for a group of potential salesmen along with an assignment model can be used to select the men to hire. With the new scores and the discriminant analysis output, the probability of each new salesman succeeding in each area could be determined. Since the degree of sales difficulty may vary in each area, management probably would assign different prior probabilities of success to each of the territories. These two probabilities can be combined by Bayes Theorem to yield an estimate of the conditional probabilities of success for each proposed salesman given that he is assigned to a particular territory.
These are the desired $SE_k$ values and can be used with $P_k$ in equation 7-25 to generate a reward matrix.

This revised matrix can be used to make hiring decisions if all the potential candidates are represented in the matrix. Since the assignment algorithm requires a square matrix, column vectors representing fictitious territories could be added to the matrix until the number of rows and columns were equal. All the values in these vectors would be zero. The straightforward application of the assignment algorithm to this profit matrix would yield the best specification of salesmen to territories. Salesmen that were assigned to fictitious territories would not be hired, while all those assigned to real territories would be employed.

King's extensions to the simple model represent a healthy type of modeling. He combined the sales recruitment and selection problem with the assignment problem in an integrated model while still maintaining the essential behavioral heterogeneity of selection and assignment.

Control of the Selling Process

After salesmen have been assigned to territories or sales tasks, the selling process should be monitored to ascertain whether the plans were well designed, to identify any underlying changes that are taking place in the market which will require strategy changes, and to control the performance of salesmen in their many tasks. This control procedure should monitor sales information and submit this information to the data bank section of the firm's information system.
For example, detailed call reports containing the name of the party called upon, the sales presentation appeals, and aids used, a report of competitive activities, and the results of the encounter would be useful. In particular, this information would provide an effective historical database for analyzing various aspects of the firm's personal selling effort. In situations where sales are large and infrequent it would often appear desirable to establish a system that will monitor the effectiveness of sales calls in terms of awareness and attitudes.

At this level of control the sales data should be stored in disaggregated form. That is, if a sales report is submitted, all the information on the report should be stored under separate file labels. With this disaggregated data the sales response to selling effort could be monitored. The new estimates could be used to update the sales budget or allocation decisions. The firm might give consideration to implementing an adaptive model which includes continuous experimentation and updating of decisions. This type of model has been proposed in the advertising area and may be applicable to sales analysis as well.

The use of an information system in the selling area must be carefully instituted since second order motivational effects may result. Asking a salesman to fill out a lengthy call report so that a control procedure will be efficient may result in lost sales because of a loss of selling time and a reduction in the salesman's morale. The system input requirements must be carefully designed to balance the total costs of obtaining the information and the profit improvements this information would allow.
SUMMARY OF MANAGEMENT SCIENCE AND PERSONAL SELLING DECISIONS

In this chapter personal selling decisions in the areas of size of sales force, allocation of sales effort, and organization have been considered. Although these decisions were discussed sequentially, the feedbacks between them were emphasized. The discussion of the models in these areas was more detailed than in previous chapters so that the peculiar nature of selling decisions could be understood and so that the potential for additional modeling would be apparent.

The discussion of sales budgeting models concentrated upon the advantages and disadvantages of using historical data, experimentation, and simulation in estimating sales response. This emphasis was utilized since the modeling techniques applicable to the problem had previously been indicated in this book. The lack of complex models in the area of size of sales force determination could be overcome if the management science models developed in the price and advertising areas were transferred to the selling area and used as the basis of the development of comprehensive sales models.

The allocation of sales effort was discussed in each of the dimensions of customer type, geographic areas, and scheduling-routing. Examples of deterministic, stochastic, dynamic, and game theory models were presented to clarify the allocation problem and the potential of management science in improving allocation. Although each dimension was individually analyzed, the need for multidimensional allocation models that include the effects of salesmen's heterogeneity was indicated.

The human basis of sales operations was emphasized in a discussion of some selected organizational aspects of selling decisions. After describing the need for defining control units and a span of control for the selling
organization, the motivational aspects of compensation were examined. The need for the development of mathematical models that include the behavioral science aspects of motivation was suggested. The behavioral heterogeneity of salesmen was examined in the context of assigning salesmen to territories. The extension of the basic assignment model to salesman selection was cited as a good example of the potential of the modeling approach in organizational decision making. The final organizational topic discussed was that of control. The use of information systems carefully designed to consider the human aspects of control was suggested as a productive approach.

The discussion of sales decisions was intended to demonstrate that personal selling is an important, interesting, and challenging area for management science. Although it has been neglected in the past, the potential rewards for research in this area should result in a surge of model building and an improvement in the quality of sales management decisions.
FOOTNOTES

1 See "Pricing Decisions" pages 7 to 9 of Chapter 5 of this book for a discussion of the simple calculus model.


4 For an interesting discussion of the development of causal inferences outside of experiments, see H.M. Blalock, Causal Inferences in Nonexperimental Research (Chapel Hill, N.C.; University of North Carolina Press, 1961).


8 Robert D. Buzzel Mathematical Models & Marketing Management, (Division of Research, Graduate School of Business Administration, Harvard University, Boston, 1964), pp. 136-156.


12 See the discussion of conditional prediction in Chapter 2 of this book, "Models of Market Response".

13 Recall the discussion of Morriison's work with heterogeneous Markov models of consumer in Chapter 2. His results, however, were for ergodic rather than absorbing Markov chains.


16. See the "Pricing Decision" Chapter of this book for a discussion of the use of Langrangean expressions. The sufficient or second order conditions for a maximizing allocation will pose no problem so long as incremental selling expenses always are positive (i.e., as long as $y_1 > 0$ for all territories).


22. The name is perhaps unfortunate in that it has served skeptics of the management science approach as an example of the utter artificiality of analytic approaches and thus has contributed to a premature rejection of this developing field in certain circles.


32 This application follows M. Stern, op. cit., pp. 69-74.


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