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A PARSIMONIOUS DESCRIPTION  
OF THE HENDRY SYSTEM

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(M.I.T.)

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WP 830-76

January 1976

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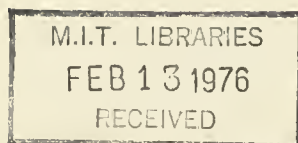
WP 830-76

January 1976

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## ABSTRACT

In this paper we will present some basic concepts of the Hendry System and derive the results claimed in the HendroDynamics Chapters [2] from two simple probabilistic assumptions; namely, zero order consumers and switching proportional to market share. We will develop the notion of partitioning which constitutes a major component of the Hendry System by briefly describing the procedure for identifying the partitioning structure in a given market. With this illustration we will point out various implications which can be drawn from a correctly identified market structure. All of our results are obtained without resorting to the entropy concept that has caused so much confusion in recent debates over the validity or usefulness of the Hendry Model.



A PARSIMONIOUS DESCRIPTION OF THE HENDRY SYSTEM\*

The Hendry Corporation has developed an innovative approach to consumer behavior -- HendroDynamics -- which has been receiving considerable attention recently. This paper focuses only on the description of some brand switching aspects of consumer behavior and the subsequent structuring of markets. The more involved HendroDynamics models which allegedly can be used to set optimal advertising and price levels are not discussed here. We do not have complete information on the Hendry Models and our understanding of HendroDynamics is based mostly on the material presented in the two privately printed chapters [2].

1. INTRODUCTION

While most of the results derived in this paper are presented in the two Hendry Chapters, the contribution of this paper lies in actually deriving those results from simple probabilistic assumptions. As we understand it, the Hendry Model is based on ideas which individually are simple, but in combination produce an interesting and useful "package." The first of these concepts is the "zero-order effect" assumption, which can be stated as follows:

Each consumer  $j$  has a probability  $p_{ij}$  of buying Brand  $i$ . On each purchase occasion, an individual consumer chooses among  $g$  brands on

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\* We wish to acknowledge Ben Butler, Sr., Ben Butler, Jr. and in particular David Butler of the Hendry Corporation for their helpful comments and general interest in the publication of this paper.



the basis of a constant probability vector  $(P_{1j}, P_{2j}, \dots, P_{gj})$ . However, each consumer is not assumed to have the same purchase probability  $p_{ij}$  of buying Brand  $i$ . Thus, the model assumes a heterogeneous population of zero order consumers.<sup>1</sup>

In view of this assumption the Hendry Model can be classified as a heterogeneous multinomial probability model. For a product class with only two brands this model becomes a heterogeneous Bernoulli model. Thus, the Hendry Model uses the same starting point as some of the models developed in Morrison's dissertation which appear in Chapter 3 [4]. However, it is only fair to say that the Butlers have produced a richer set of properties, with more marketing insights than anything in [4].

For clearer understanding and convenience let us define various symbols and terms which will be introduced during the course of subsequent derivations. We have used the same notation found in [2] in order to facilitate comparisons with material printed by the Hendry Corporation.

$N$  = Number of consumers in the product class.

$N_i$  = Number of consumers buying Brand  $i$  on any particular purchase occasion.

$g$  = Number of brands in the product class.

$p_i$  = Market share of Brand  $i$ .

$p_{ij}$  = Probability of consumer  $j$  choosing Brand  $i$ .

$p_{ws}$  = Unconditional probability of switching from Brand  $s$ .

= Proportion of consumers switching from Brand  $s$  on any particular purchase occasion.

$P_{(s,s)}$  = Unconditional probability of repeat buying Brands  $s$ .

= Proportion of consumers repeat buying Brands  $s$ .





$P_{(s,h)}$  = Unconditional probability of switching from Brand s to Brand h.

$P_{h|s}$  = Probability of switching to Brand h for a consumer whose previous purchase is Brand s and who actually has switched out of Brand s. Note that this is a double conditional probability. It is conditional on having previously purchased Brand s and conditional on the subsequent purchase not being Brand s. For clarity it should be pointed out that this is not a Markovian transition probability, i.e.  $P_{h|s} \neq \frac{P_{(s,h)}}{P_s}$ . The Markovian transition probability  $\frac{P_{(s,h)}}{P_s}$ , is defined as the probability of purchasing brand h given previous purchase is Brand s. This Markovian probability, however, does not exclude the possibility of repeat purchasing Brand s. In terms of our previously defined quantities

$$P_{h|s} = \frac{P_{(s,h)}}{P_{ws}}$$

For clarity, it will be useful at this point to anticipate the section on partitioning. A partition is assumed to contain what the Hendry Model calls "directly competing brands." For a consumer who switches out of Brand i, his expected probability of buying Brand h is proportional to the market share of Brand h. That is,

$$P_{h|i} = k_i P_h \quad \text{for all } i \neq h. \quad (1)$$

The above equation does not describe the behavior of an individual consumer switching out of Brand i. The probability,  $P_{h|i}$ , is the expected



conditional probability of buying brand h for a random consumer who switched out of Brand i. The Hendry Model assumes that this equation holds true within a partition and thus, equation (1) becomes the definition of a partition. This assumption has caused some confusion. Some researchers state unequivocally that the above equation does not hold true in a product class. However, the Hendry Model partitions a product class until equation (1) holds within each partition, but not necessarily for pairs of brands in different partitions. Again we repeat that (1) is essentially the Hendry definition for a partition. Equation (1) is not an empirical fact and it does not necessarily hold for all pairs of brands within an entire product class.

## 2. LAW OF DETAILED BALANCING

We will show here that this "law" discussed in [2] is a direct consequence of the zero-order assumption.

Consumer j's preference for Brand i is defined as the probability  $p_{ij}$  that consumer j chooses Brand i, such that

$$\sum_{i=1}^g p_{ij} = 1.$$

This equation implies that each consumer's purchase probabilities over all the g brands in the market add up to 1. Also, we have

$$\sum_{j=1}^N p_{ij} = E[N_i].$$

That is, summing the preferences for Brand i over all the N consumers gives the expected number of consumers who will buy Brand i. The actual



number of customers choosing Brand  $i$ ,  $N_i$ , of course, is a random variable. The preference vectors,  $p_{ij}$ 's, are assumed to be stationary over time.

Now let us consider two purchase decisions in which a consumer switches brands. The probability of consumer  $j$  choosing Brand  $s$  the first time and Brand  $h$  the second time is (by the zero order assumption)

$$P_{(s,h)_j} = p_{sj} p_{hj}.$$

where  $p_{(s,h)_j}$  is the probability of consumer  $j$  switching from Brand  $s$  to Brand  $h$ . The probable number of buyers switching from Brand  $s$  to Brand  $h$  becomes

$$E[N_{(s,h)}] = \sum_{j=1}^N p_{sj} p_{hj}.$$

Similarly, the probable number of buyers switching from Brand  $h$  to Brand  $s$  is

$$E[N_{(h,s)}] = \sum_{j=1}^N p_{hj} p_{sj},$$

from which

$$E[N_{(s,h)}] = E[N_{(h,s)}], \quad (2)$$

i.e., the expected number of buyers switching from Brand  $s$  to Brand  $h$ , is equal to the expected number of buyers switching from Brand  $h$  to Brand  $s$  at equilibrium (i.e. when the probabilities in the preference vectors are stationary -- constant -- over time.).



Since Brands  $h$  and  $s$  were chosen arbitrarily, the expected number of buyers switching between each pair of brands is equal in both directions. It follows from this that the expected total number of consumers switching out of a brand is equal to the expected total number switching into the brand. The zero-order process assumption, as shown above, is crucial in the derivation of the balanced switching relationship. Also, the relationship holds good only under equilibrium conditions in the market since we require brand preferences to be stationary.

### 3. SWITCHING CONSTANT $K_w$

The switching constant,  $K_w$ , plays a central role in describing consumer behavior under a zero-order process. We will now derive its value.

The probability of a consumer switching from Brand  $s$  to Brand  $h$  may be expressed as

$$P_{(s,h)} = p_{ws} p_{h|s} \cdot \quad (3)$$

The first term on the right,  $p_{ws}$ , is the probability of switching out of Brand  $s$ . The second term,  $p_{h|s}$ , represents the probability of buying Brand  $h$  for a consumer switching from Brand  $s$ . The term,  $p_{h|s}$  can be understood by noting that

$$\sum_{\substack{i=1 \\ i \neq s}}^g p_{i|s} = 1 \cdot \quad (4)$$





That is, for a consumer switching out of Brand  $s$  his purchase probabilities sum up to 1 over the remaining brands. We know that the sum of market shares of all the  $g$  brands adds up to unity, that is

$$\sum_{i=1}^g p_i = 1.$$

Recall the Hendry definition of a particular stated earlier. Namely,

$$p_{h|s} = K_s p_h \quad (1)$$

That is, given that consumers switch from brand  $s$ , they switch to the remaining brands in proportion to the market shares of these other brands.

From (4) substituting for  $p_{i|s}$ , we have:

$$K_s \sum_{\substack{i=1 \\ i \neq s}}^N p_i = 1.$$

Hence, the proportionality constant  $K_s$  is

$$K_s = \frac{1}{(1-p_s)}.$$

Note that although the concept embodied in the key assumption (1) is the same for all brands in the partition, the proportionality constant differs for each brand. From equation (1) using  $K_s = \frac{1}{(1-p_s)}$ , we have:

$$p_{h|s} = \frac{p_h}{1-p_s}. \quad (5)$$



Substituting for  $p_{h|s}$  in equation (3) we find

$$p_{(s,h)} = p_{ws} \frac{p_h}{1-p_s} .$$

Similarly,

$$p_{(h,s)} = p_{wh} \frac{p_s}{1-p_h} .$$

From the Law of Detailed Balancing, we know that

$$p_{(s,h)} = p_{(h,s)} ,$$

and therefore

$$p_{ws} \frac{p_h}{1-p_s} = p_{wh} \frac{p_s}{1-p_h}$$

Dividing both sides of the equation above by  $p_s p_h$  , we have

$$\frac{p_{ws}}{p_s (1-p_s)} = \frac{p_{wh}}{p_h (1-p_h)} . \quad (6)$$

Since Brands  $s$  and  $h$  were chosen arbitrarily, we can generalize equation (6)

as

$$\frac{p_{wi}}{p_i (1-p_i)} = K_w , \quad (7)$$

for all brands  $i = 1, \dots, g$  in the partition.

$K_w$  in the equation above is termed the Switching Constant in the Hendry Model. The term,  $p_i$ , represents the market share of Brand  $i$ . If all the buyers of Brand  $i$  had identical probabilities of purchasing Brand  $i$ ,  $p_i$ , then the denominator in equation (7),  $p_i(1-p_i)$ , would represent



the switching proportion from Brand  $i$  to the competitors of Brand  $i$ . The numerator in equation (7),  $p_{wi}$ , represents the proportion of consumers switching from Brand  $i$  under the heterogeneous multinomial probability model assumption. The switching constant,  $K_w$ , is thus the ratio of total brand switching in the actual heterogeneous multinomial probability model to total brand switching that would result from a homogeneous multinomial probability model that produced the same market shares. At equilibrium, this ratio has the same numerical value for all brands, within a partition.  $K_w$  ranges between 0 and 1 and the smaller its value the more heterogeneous is the partition.

The switching constant,  $K_w$ , has several other implications in terms of the switching behavior of consumers in the market which become clearer upon algebraic manipulation of equations (3) through (7). Multiplying equation (6) by  $p_s p_h$ , we have

$$p_s p_h \frac{p_{ws}}{p_s (1-p_s)} = p_s p_h \frac{p_{wh}}{p_h (1-p_h)},$$

from which we obtain, by substitution from equation (7),

$$\frac{p_{ws} p_h}{(1-p_s)} = p_s p_h K_w.$$

Finally, using equation (7), we find

$$p_{(s,h)} = K_w p_s p_h, \tag{8}$$

where  $K_w$  is the switching constant.

Bass [1] has derived the brand switching relationship presented in equation (8) using a different approach from the one presented here.



He obtains:

$$P_{(i,j)} = (1-\rho) p_i p_j \quad (9)$$

where  $\rho$  represents the correlation between the purchases of any one of the  $g$  brands in a product class over two purchase occasions. Each product class is characterized by a single correlation measure,  $\rho$ , and it may be thought of as the "product class brand loyalty factor." In deriving this brand switching equation Bass [1] makes two assumptions: (i) each consumer is a zero-order process, and (ii)  $V_i$ 's, the utilities for brands are distributed gamma over the population with the property that  $\text{Var} [V_i]/E[V_i] = \text{constant}$ , for all brands  $i$ . The probability that a consumer chooses brand  $i$  is then his utility for brand  $i$  divided by the sum of the utilities for all brands, including brand  $i$ .

From equation (7), we have

$$p_{wi} = K_w p_i (1-p_i) \quad ,$$

from which the total switching for all brands becomes:

$$P_w = \sum_i p_{wi} = \sum_i K_w p_i (1-p_i) \quad .$$

Solving for  $K_w$ , we obtain:

$$K_w = \frac{P_w}{\sum_i p_i (1-p_i)} \quad . \quad (10)$$

Thus, the switching constant  $K_w$  is applicable to the total product class as well as to the individual brands within the product class. In the Hendry Model, the switching constant as calculated from equation (7) for individual brands within a partition should equal the





switching constant obtained from equation (10) using total switching within the partition. Thus, within a partition there is one value of the switching constant which describes switching behavior for individual brands as well as the aggregate switching behavior within the partition.

The switching constant can be used in the calculation of repeat purchase and Markovian transition probabilities. The probability  $p_s$  of buying Brand  $s$ , equals the probability  $p_{(s,s)}$  of having repeat purchased Brand  $s$  plus the sum of the probabilities  $p_{(i,s)}$  of having switched from any other brand to Brand  $s$ . That is:

$$p_s = p_{(s,s)} + \sum_{i \neq s} p_{(i,s)} .$$

Substituting for  $p_{(i,s)}$  from equation (8) and with some simple algebraic manipulations, we find:

$$p_s = p_{(s,s)} + K_w p_s (1-p_s) .$$

From which, we have:

$$p_{(s,s)} = (1-K_w) p_s + K_w p_s^2 . \tag{11}$$

The Markovian repeat transition probabilities are given by:

$$\frac{p_{(s,s)}}{p_s} = (1-K_w) + K_w p_s , \tag{11a}$$

which increases as  $p_s$  increases.

Using equation (8)

$$\frac{p_{(s,h)}}{p_s} = K_w p_h , \tag{11b}$$

which is independent of  $s$ .



As an illustration, it would be useful to examine various consumer statistics for a typical value of  $K_w$ . Consider a three-brand product class with brand shares 0.5, 0.3 and 0.2. We have chosen a value of 0.4 for the switching constant. This value of  $K_w$  is arbitrary but it is approximately the value which would be obtained as a theoretical estimate in the Hendry System for this product class using notions of entropy [2]. Table 1 contains "Markovian" repeat purchase and switching probabilities for this three-brand case which are obtained using equations (11a) and (11b).

TABLE I

<u>Buyers of Brand #</u>	<u>PROPORTION WHO NEXT BUY</u>		
	<u>1</u>	<u>2</u>	<u>3</u>
1	.8	.12	.08
2	.2	.72	.08
3	.2	.12	.68
<u>All Buyers</u>	<u>.5</u>	<u>.3</u>	<u>.2</u>

Total Switching = 24.8% \*

We find in Table 1 that switching probabilities (conditional) from any brand (say, #2, or #3) to a particular brand (#1 in this case) to be identical (because of equation (11b)). The repeat purchase probability (conditional) is found to be higher for larger brands; this result is in general agreement with empirically observed data.

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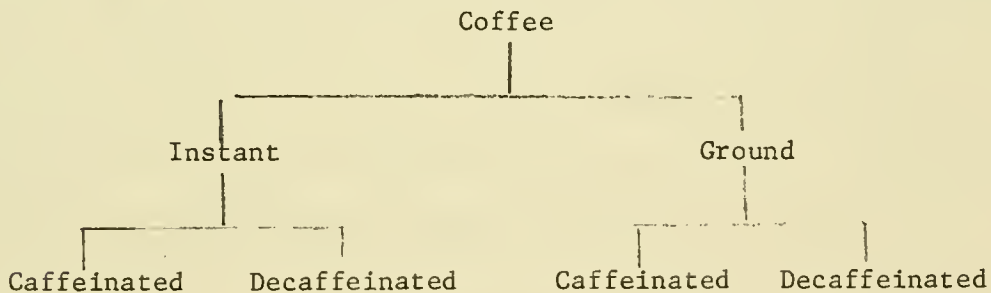
\* Total switching =  $p_w = K_w \sum_i p_i (1-p_i) = .248$



#### 4. PARTITIONING

In this section, we shall discuss the notion of partitioning -- the hierarchy of market structure relationships. From a marketing practitioner's view-point this is operationally one of the most important concepts underlying the Hendry approach. The Hendry Model posits that consumer preferences partition markets into a hierarchy of product set structures. For instance, the coffee market may be partitioned into instant vs. ground coffee and then within each category into caffeinated vs. decaffeinated brands. Consumer alternatives are substitutable within the smallest set to which they belong. (e.g. all brands within the set of instant-decaffeinated coffee).

##### A Hypothetical Illustration of Market Structure for the Coffee Market



A correctly identified preference structure provides the appropriate competitive frame for assessing the relative performance of brands in terms of market share, sales volume, profitability, etc.. The preference structure gives an idea of the relative influence of marketing strategies and thus, helps in the development of strategic marketing plans. In the Hendry System, the structure of partitioning within a product class is determined by analyzing actual consumer brand switching behavior. This

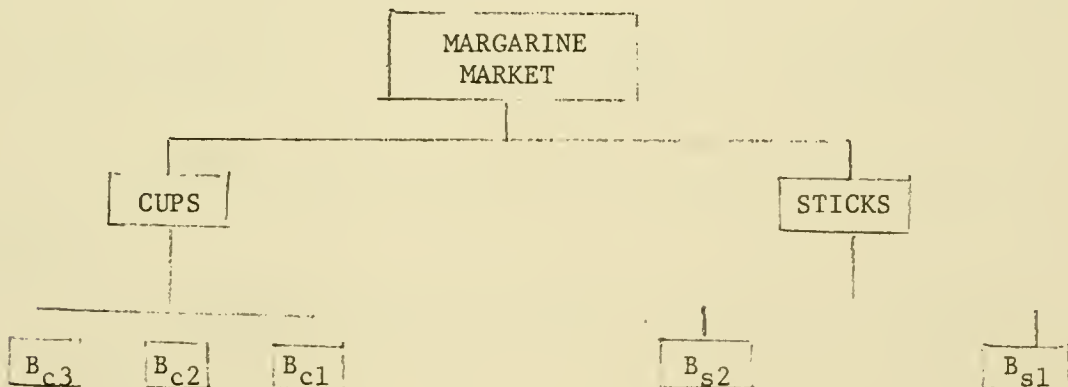


is in opposition to the general practice of discovering the structure of partitioning through the judgment of marketing managers and/or consumers' perceptions of the brands. The relative merits of these different approaches are not discussed in this paper.

Following the Hendry approach, a hypothetical partitioning structure based on "expert judgment" is set up and switching patterns are examined given this structure. With some experience in identifying "true" partitioning structures in markets, a certain amount of knowledge or learning is acquired from every partitioning hypothesis that is tested for the market under consideration. Upon one or more of these iterative attempts a market structure is identified which "fits" the empirical data reasonably well.

For instance, in the margarine market, the form of margarine (stick, cup, etc.) might represent the primary level of decision making. That is, a consumer decides first on the form of margarine she wishes to buy, and only then chooses a particular brand within that form. The structure of the market upon analysis may look as follows:

A Form-Primary Market







As shown in the figure above, in this form-primary margarine market we assume there are two forms of margarine (cups and sticks) and three brands (1, 2 and 3). Brand labels 1 and 2 are sold in both forms whereas Brand 3 margarine is sold only in cups. In order to describe some of the switching patterns which will operate in a form-primary margarine market, let us define some terms:

- $p_{\text{cups}}$  = Proportion of margarine sold in cup form.
- $p_{\text{sticks}}$  = Proportion of margarine sold in stick form.
- $P_{\text{ci}}$  = Market share of Brand  $B_{\text{ci}}$  within "cups" partition,  $i = 1, 2, 3$ .
- $P_{\text{si}}$  = Market share of Brand  $B_{\text{si}}$  within "sticks" partition,  $i = 1, 2$ .
- $K_{\text{wf}}$  = Switching constant describing switching across forms of margarine .
- $K_{\text{wc}}$  = Switching constant within the partition of cups form.
- $K_{\text{ws}}$  = Switching constant within the partition of stick form.

Following the derivations in earlier sections, the expected proportion of buyers switching between forms of margarine is given by:

$$\text{Total switching across forms} = K_{\text{wf}} p_{\text{cups}} p_{\text{sticks}} \quad (12a)$$

Note that the Hendry Model allows a consumer to purchase a set of Brands that are not all necessarily in the same partition. The switching level from a Brand  $B_{\text{ci}}$  to a stick form of margarine would be given by:

$$\text{Switching from } B_{\text{ci}} \text{ to sticks} = p_{\text{ci}} (K_{\text{wf}} p_{\text{cups}} p_{\text{sticks}}) \cdot \quad (12b)$$

Similarly,

$$\text{Switching from } B_{\text{si}} \text{ to cups} = p_{\text{si}} (K_{\text{wf}} p_{\text{cups}} p_{\text{sticks}}) \cdot \quad (12c)$$



For switching patterns within a partition we have:

$$\text{Switching from } B_{ci} \text{ to } B_{cj} = K_{wc} P_{ci} P_{cj} . \quad (12d)$$

Likewise

$$\text{Switching from } B_{si} \text{ to } B_{sj} = K_{ws} P_{si} P_{sj} . \quad (12e)$$

The switching constant for each brand within a partition should be identical and it should be equal to the switching constant describing aggregate switching behavior in that partition.

Loosely stated the Hendry partitioning algorithm would try various partitioning schemes until one was found where all of the numerous relations (12) are "satisfied." Obviously, some apriori notions of reasonable partitions are needed since the total number of possible partitions in any real market would be astronomically large. Admittedly, there is a certain ad hoc nature to this approach, but there is a clear cut, well defined principle being utilized. Namely, each partition consists of a set of heterogeneous multinomial consumers in which each brand in the partition has the same switching constant and this switching constant also applies to the partition as a whole.

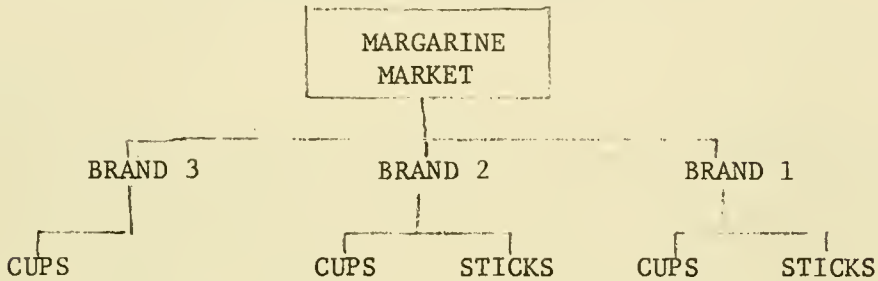
Thus, in the hypothetical illustration above, each form of margarine would represent a separate partition with its own switching characteristics. The margarine market can, in such a case, be classified as form primary. A consumer in such a form primary market will have a very small preference for a brand which does not carry her margarine form. That is, the form of margarine would be the primary structural component because the consumer would first identify margarine as having a particular form before choosing a brand within that form.

The identification of the partitioning structure in a market has several practical implications. These implications can be best described



by comparing the above structure with a brand-primary margarine market. Using the assumptions above, a brand-primary margarine market can be represented in a figure as shown below.

A Brand Primary Market



Firstly, in the form-primary market, the consumers exhibit lower brand-name loyalties than in the brand-primary market. Secondly, five separate brands are perceived by consumers in the form-primary margarine market. The firm manufacturing Brand 1 (say, Firm 1) has to promote its two forms of margarine separately. On the other hand, in the brand-primary case, Firm 1 could promote its two product types together. Finally, if Firm 3 wanted to introduce margarine in stick form in the brand primary case it would experience a certain amount of "cannibalism." On the other hand, in the form-primary case, if Firm 3 introduced margarine in stick form, it would end up getting a share in the "sticks" partition which would be independent of its share in the "cups" partition. These conclusions are of course overstated, but the general tone of these implications would be helpful in practice.

In concluding this section on partitioning, it should be pointed out that the Hendry Model identifies a partitioning structure which is con-



sistent with numerous relations (e.g. equations (12) defined by the model.) Whether this structure is the true partitioning structure would of course need further verification. The Hendry partitioning might be consistent over time (i.e. reliable), but the validity of this structure would still require other supporting data.

## 5. MODEL ANALYSIS AND ESTIMATION OF $K_w$

### Model Analysis:

A heterogeneous multinomial probability model is completely determined by the knowledge of preference distributions for individual brands in the product class. Various consumer statistics like repeat purchase rates, switching rates and the switching constant can be easily deduced from knowledge of the preference distribution. In this section, we shall estimate  $K_w$  from a knowledge of the preference distribution for two different cases of heterogeneity in the consumer population.

### Arbitrary Heterogeneity:

Let us suppose we are interested in analyzing a particular brand. Then, at each trial, or purchase occasion, the consumer can purchase either the brand under consideration (Brand 1) or some other brand in the aggregate "all other" class (Brand 0). Under the "zero-order process" assumption, on every trial the process has a probability  $p$  of being in State 1 (purchase of Brand 1) regardless of the past history of the process. The purchase probability  $p$  may have an arbitrary distribution  $f(p)$  over individuals in the population. Let  $\mu_1$  and  $\mu_2$  denote the first and second moments of this distribution about origin. We have:

$$\mu_1 = \int_0^1 p f(p) dp .$$





The first moment,  $\mu_1$ , above represents the market share of Brand 1,  $p_1$ . The second moment about the origin is a measure of the spread of the preference distribution of Brand 1 and is given by:

$$\mu_2 = \int_0^1 p^2 f(p) dp.$$

The quantity  $\mu_2$  in the equation above represents the second raw moment and not the variance of the preference distribution  $f(p)$ . We assume these values of the first two moments about the origin are known to us. From equation (18) we have for Brand 1

$$K_w = \frac{p_{w1}}{p_1(1-p_1)}.$$

The numerator,  $p_{w1}$ , in the equation above represents the level of switching between Brand 1 and all other Brands in the product class. We have

$$p_{w1} = \int_0^1 p(1-p) f(p) dp = (\mu_1 - \mu_2).$$

The switching constant,  $K_w$ , can therefore be obtained from the knowledge of the first and second moments of the preference distribution  $f(p)$ . This yields

$$K_w = \frac{(\mu_1 - \mu_2)}{\mu_1(1 - \mu_1)}. \quad (13)$$

#### Beta Heterogeneity:

Once again, we assume that each consumer has a probability  $p$  of buying Brand 1 and the complementary probability  $(1-p)$  of buying Brand 0 (usually an aggregated "all other" brand). However, the purchase probability  $p$  is assumed to have a specific distribution; namely, beta. The beta distribution



has the following form:

$$b(p) = \frac{\Gamma(m+n)}{\Gamma(m) \cdot \Gamma(n)} p^{m-1} (1-p)^{n-1}, \quad \text{for } 0 < p < 1$$
$$= 0, \text{ otherwise.}$$

where  $\Gamma(\cdot)$  is the gamma function and the parameters  $m, n$  are  $> 0$ . The first and second moments ( $\mu_1, \mu_2$ ) about the origin, for this distribution, are:

$$\mu_1 = \frac{m}{m+n},$$
$$\mu_2 = \frac{m(m+1)}{(m+n)(m+n+1)}.$$

It should be noted that selection of a beta distribution for analysis here is based on its flexibility, tractability and successful applicability in various stochastic buyer behavior models. We can get a feel for the flexibility of the Beta distribution by noting the various shapes it can take. When  $m$  and  $n$  are less than one, it is "U shaped"; the bulk of population is concentrated near the extremes,  $p = 0$  and  $p = 1$ . For  $m$  and  $n$  less than 1, it takes a J-shape or a reversed J-shape accordingly as  $m$  is greater than or less than  $n$ . When  $m$  and  $n$  both exceed 1, the beta distribution is bell-shaped, and skewed left or right accordingly as  $m$  is greater or less than  $n$ . As  $m$  and  $n$  grow larger while  $m \approx n$ , the beta distribution approaches the familiar normal distribution. Finally, when  $m = n = 1$ , the beta distribution becomes a uniform distribution.

The switching constant,  $K_w$ , can be estimated for this case in two ways. Firstly,  $K_w$  can be estimated by substituting for the first and second moments. We have:



$$K_w = \frac{(\mu_1 - \mu_2)}{\mu_1(1 - \mu_1)} .$$

Substituting the expressions for the two moments yields

$$K_w = \frac{m+n}{m+n+1} .$$

A second way of estimating  $K_w$  is by deducing the value of switching level  $P_{w1}$  for Brand 1 given beta heterogeneity.<sup>2</sup>

$$P_{w1} = P(10) = P(0|1) P(1) = \left( \frac{n}{m+n+1} \right) \left( \frac{m}{m+n} \right) .$$

Using this expression for the numerator of the switching constant we find once again that

$$K_w = \frac{m+n}{m+n+1} . \tag{14}$$

In deriving the value of the switching constant and other results presented in Section 3 we made two assumptions -- namely, the zero order assumption and that switching is proportional to market share (equation 5). It is useful to note here that the relationship of switching proportional to market share

$$P_h|s = \frac{P_h}{1-p_s} \tag{5}$$

holds true for a dirichlet distribution which is a multivariate analogue of the beta distribution.

Estimation of  $K_w$ :

Empirically, it is possible to estimate  $K_w$  from equation (10). We find that estimation of  $K_w$  requires knowledge of market shares of various brands



and the total switching in the product class. This information can be obtained without much difficulty from consumer panel data.<sup>3</sup> Thus, this method of estimating  $K_w$  does not require the knowledge of the distribution of probabilities across the population of consumers.

If we assume a functional form for the distribution of probabilities, we could estimate the parameters of this distribution. Then, the switching constant will be a function of these estimated parameters (e.g., equation (14)).

#### An Algorithm for Applying Hendry Approach

The Hendry approach in so far as it deals with description of the brand switching aspects of consumer behavior and structuring of markets can be employed in a practical situation by following the sequence of steps given below.

1. Set up a hypothesis about the nature of partitioning in the markets. Such a hypothesis is typically developed on the basis of managerial judgment.
2. Analyze the actual switching behavior by calculating empirically the values of the switching constant both across and within partitions given the hypothesised partitioning structure of Step 1. If the switching patterns within and across partitions are found to be consistent with the partitioning hypothesis, as illustrated in the section on partitioning, the "true" Hendry partitioning structure has been identified. (As we can recall from the section on partitioning, within each partition every individual brand should have approximately the same value for switching constant.) If, however, the switching patterns are found to be inconsistent, return to Step 1 and set up a new hypothesized partitioning structure.





## 6. CONCLUSION

In this paper we have presented what we believe to be the basic framework of the Hendry Model. As described in this paper the Hendry approach leads to several interesting conclusions about consumer markets.

Following the Hendry approach, a consumer market can be divided into a set of mutually exclusive and exhaustive partitions. Each of these partitions is treated as a distinct market and can be modeled according to a zero-order process. That is, within each partition consumer behavior is described by a heterogeneous multinomial probability model. The level of switching within a heterogeneous multinomial probability model is less than that for a homogeneous multinomial probability model. Hence, the value of the switching constant, within a partition, is less than unity. Within a partition, each brand is required to have the same value of the switching constant. As this value of  $K_w$  decreases it implies a higher degree of heterogeneity in consumer brand preferences.

From a practical viewpoint partitioning is one of the most significant concepts of the Hendry System. A knowledge of partitioning in a given market can be instrumental in the development of a sound promotional strategy. Further, it provides valuable information for new brand introduction in the given market -- particularly with respect to cannibalization.

The brand switching behavior results derived in this paper are based on two assumptions -- the zero order assumption and switching proportional to market share. Chapter 3 of Massy et. al. [4] provides three different procedures to test this zero order assumption.



The testing of the second assumption of switching proportional to market share requires that we have a prior description of the market structure relationships (e.g., from managerial judgment, or from consumers' perceptions of the brands). We can then test if the second assumption holds true within a partition. It would be meaningless to test the switching proportional to market share assumption in the Hendry Model framework since this relationship is the very definition used to identify a partition.

As shown in the procedural algorithm of the last section, application of the Hendry approach does not require the use of maximum entropy criterion. HendroDynamics does have a procedure to obtain a theoretical estimate of  $K_w$  which is based on the use of maximum entropy criterion. According to the Butlers, it is necessary to compute the theoretical value of the switching constant in consumer markets which are not in equilibrium. Also, the entropy concept is used as a system property of a consumer market for the advertising model of the Hendry System. However, we have avoided any discussion of entropy in this paper for two reasons: i) this concept of entropy in the Hendry System has caused a great deal of needless controversy, and ii) entropy is not needed to derive anything we have done in this paper. Chapters 5 and 6 of Kalwani's dissertation [3] go into considerable detail on the topic of entropy in the context of brand switching models.

One final comment is in order. We believe that the Hendry (and other similar) approaches are nothing more than parsimonious descriptions of consumer brand switching behavior; albeit, very useful and insightful descriptions. Others argue strongly that these approaches constitute a theory of switching behavior. We have only focused on the descriptive aspects in this paper, electing to leave the theorizing to some of our stochastic model building colleagues.



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3. Kalwani, M. U., "Zero-Order Models: Specification, Data Requirements and Estimation," Unpublished Ph.D. Dissertation, Graduate School of Business, Columbia University, (1975).
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FOOTNOTES

1. We should emphasize here that the zero-order effect assumption implies:  
(i) Any possible learning effects upon reaching equilibrium are of no consequence subsequently. (ii) Virtually all the brand switching data come from consumer panels where the unit is the household and not a particular individual. Hence, our zero-order model does not provide for different vectors for a household if purchasing is done by more than one family member. (iii) A zero-order model implies that consumers have stable attitudes towards various brands and thus constant purchase probabilities. It does not consider consumers to be in an exploratory or search process prior to their forming stable brand preferences.  
  
In actual practice, however, it can be argued that the zero-order effect assumption does not imply these severe restrictions but only that these three effects "wash-out" on aggregation of buyer behavior.
2. See Massy et.al. [3], Chapter 3, for a complete discussion of  $p(0|1)$  and similar conditional probabilities for heterogeneous Bernoulli models. Essentially an unconditional beta prior is being updated by a binomial sample outcome to a conditional beta posterior distribution. In the specific case here for  $p(0|1)$  the mean of this conditional posterior distribution  $n/(m+n+1)$ .
3. Consumer panel data is based on the purchase diaries of a representative panel of a large number of households scattered throughout the country. Each household reports in its diary the date the product was purchased, brand chosen, store visited, retail price, package size, number of units bought and if it was a deal purchase.















**BASEMENT**

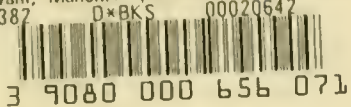
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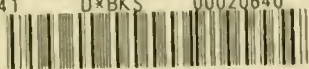
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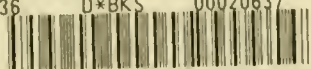
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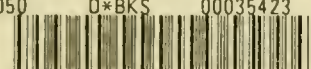
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