QUOTAS AND THE STABILITY OF IMPLICIT COLLUSION

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I Introduction

This paper shows that the imposition of an import quota by one country can lead to increased international competitiveness: protection can reduce the price in the country that imposes the quota, the foreign country, or both. This somewhat paradoxical result emerges from a model of implicit collusion. In such a setting the firms in an industry sustain collusive prices by the threat that more competitive pricing will ensue if any firm deviates. The more powerful the threat, the more collusion that can be sustained. Since a quota reduces the ability of the foreign firms to punish a deviating domestic firm, the amount of collusion that can be sustained is correspondingly diminished.

We study both the case in which sales and the case in which prices are the strategic variables. Our results are strongest when prices are the strategic variables: quotas always make monopolization of the domestic market more difficult in that case. This is in sharp contrast to the results in static imperfect competition models in which quotas tend to make it easier for the domestic firm to act as a monopolist at home. It is also different from the impact of tariffs which, in a dynamic setting like the one studied here, do not necessarily make it more difficult for the firms to sustain collusive outcomes.

Whether tariffs make collusion more difficult to sustain or not depends on the severity of the punishments that the firms can reasonably be expected to inflict on their cheating rivals. The maximal punishments of the style developed by Abreu (1982) involve an outcome in which the domestic firm earns zero profits. This is because even with a very large tariff, the foreign firm can, if it is willing to tolerate the ensuing losses, charge a price so low that it makes it impossible for the domestic firm to earn profits at home. As a result, tariffs do not affect the ability of the duopoly to maintain
monopolistic outcomes.

Contrast this with a quota. There the maximum punishment the foreign firm can inflict on the domestic firm is to sell its entire quota. This generally still yields positive profits for the domestic firm. Thus since the domestic firm faces a lower punishment, it has a larger incentive to deviate from the monopolistic outcome. This general intuition must be qualified somewhat by the fact that, as pointed out by Davidson (1984), trade restrictions also make it less costly to the foreign country to let the domestic country have a larger share of the domestic market. Thus the domestic firm's incentive to abide by the collusive scheme can at least be partially restored by altering the market shares in its favor and thereby giving it more to lose if it deviates.

In this case of maximal punishments, therefore, our results have the opposite implication of those of Bhagwati (1965). In his classic paper he showed that a single domestic producer who faced a competitive foreign market would act more competitively with a tariff than with a quota. When we consider a single domestic producer and a single foreign producer, the opposite result emerges.

If the punishments are not as grim as this, but instead involve reverting to the equilibrium from the one-shot game in each period, then large tariffs (which are analogous to small quotas) make collusion more difficult to sustain. In that case, then, the consequences of tariffs and quotas are similar.

As mentioned previously, our results are strongest for the case in which prices are the strategic variables. When we analyze the case in which quantities are the strategic variables we find that only small quotas break the discipline of the oligopoly while large quotas, if anything, strengthen the incentives to go along with the monopolistic outcome. Thus our results for this case parallel closely those of Davidson (1984) for tariffs. He uses quantities
as the strategic variables in a model very similar to ours and shows that small tariffs promote collusion, while the opposite is true for large tariffs.

After considering the effects on monopolization in the country that imposes the quota we study the foreign repercussions of quotas. We notice immediately that the arguments we used to show that more competition emerges in the country that imposes the quota also imply that quotas lead to increased competition abroad if the domestic firm is capacity constrained. We model the capacity constraint in a simple way, assuming that the firm can produce at constant marginal cost up to some limit and cannot exceed that limit. Suppose that a quota is imposed when the domestic firm is capacity constrained in this way. If the monopoly output is to be sold in the domestic market, the presence of a quota requires that the domestic firm sell more at home. Therefore, it has only a limited capacity to supply the foreign market. In that case its capacity constraint has the same effect abroad as the quota does on the imports of the foreign firm. Thus, for the reasons discussed above, more competition results in the foreign market.

It is possible for quotas to lead to increased competition abroad even in the absence of capacity constraints, however. Suppose, for example, that the levels of demand fluctuate over time and are not perfectly correlated across the two markets. In that case, in either market taken alone, the firms may be unable to sustain the monopoly level of prices in all states. As in Rotemberg and Saloner (1986), when demand is sufficiently high, the incentive to cheat may outweigh the punishment which is meted out in later, "more normal", periods.

As Bernheim and Whinston (1986) show, in such settings the firms can sustain higher profits when they share two markets with imperfectly correlated demand. In the simplest case, (which is also the one that we study), the markets are of equal size and have perfectly negatively correlated demand. In
that case the two markets taken together are perfectly stable over time and hence, since the firms' incentive to cheat doesn't fluctuate over time, they may be able to sustain monopoly profits each period. Imposing a quota on one of the markets, however, means that the firms lose the countervailing force that smooths the incentive to cheat when demand is high in the foreign market. Instead, the foreign market is transformed into a market with fluctuating demand and monopoly profits may no longer be sustainable there.

Thus the imposition of a quota can increase competition in the foreign market or in the domestic market. The natural question is whether the separate analyses discussed above can be combined to show that the imposition of a quota can contemporaneously increase competition in both the domestic and foreign markets. We show that this is indeed the case.

The presence of implicit collusion is not the only reason why quotas may result in increased competition in both markets. Indeed we demonstrate that the model developed in Dixit and Kyle (1985) can be modified to show that potential entry can have the same effect.

The paper is organized as follows: We show that quotas can increase competition at home with prices as the strategic variables in Section II and with quantities as the strategic variables in Section III. In Section IV, we use the analysis developed in these sections to show that quotas can lead to increased competition abroad if the domestic firm is capacity constrained. We show that in the presence of fluctuating demand that this can occur even when the domestic firm is not capacity constrained in Section V. In Section VI we show that increased competition can result in both the domestic and the foreign market. Section VII concludes the paper with a discussion of the effect of potential entry.
II Quotas and price competition at home

There are two countries, domestic and foreign. We consider an oligopolistic industry with one domestic and one foreign firm. To start with the simplest case we assume that the domestic firm makes no sales abroad. This restriction is relaxed in Section IV. Marginal cost is constant and equal to $c$ in both countries. Foreign firms do not face any transportation costs from shipping the good to the domestic country. However, the markets are segmented so that consumers can only buy the good in their own country. Thus the foreign firm can charge a different price abroad than it does at home. Yet, and this slightly contradictory assumption is made mainly for tractability, the goods sold by the two firms are viewed as perfect substitutes in the domestic market. Demand is given by:

$$P = a - bQ$$

(1)

where $P$ is the industry price and $Q$ is the sum of the amounts sold by the domestic and foreign firms.

In this section price is the strategic variable. This means that if one firm quotes a price lower than the other, it supplies the entire market. If the two firms quote the same price they share the market. There are two possible assumptions about the way in which this sharing is done. The first is that each gets half the market. The second is that any possible market division is feasible and that market shares are also implicitly agreed upon. However, in the presence of quotas the foreign firm may be forced to have less than half of the market. Thus the latter assumption is the only one that makes sense in this case.
We start by analyzing equilibrium under free trade. This equilibrium is the standard duopoly equilibrium in a repeated setting. We assume that the firms try to sustain the monopoly price \((a+c)/2\) and that they each serve half the market. Then, if neither firm deviates, each earns \((a-c)^2/4b\) per period. If either firm deviates, it undercuts the price slightly and captures the entire market so that it earns twice this amount. However, after a deviation the firms are assumed to revert forever to the noncooperative equilibrium for the corresponding one-period game which has a price equal to the marginal cost c. This punishment which gives each firm profits of zero is also the strongest possible punishment that can be conceived if the firms are free to leave the industry. It is then impossible to make them earn negative profits.

With this punishment, each firm will be deterred from deviating as long as:

\[
\frac{(a-c)^2}{4b} \leq \delta \frac{(a-c)^2}{4b(1-\delta)}
\]

where the RHS of this equation represents the future profits that are given up by cheating and \(\delta\) is the rate at which future profits are discounted. So as long as \(\delta\) equals at least 1/2, the monopoly price is sustainable.

We now consider the effect of a quota. Let the amount the foreign firm is allowed to import be \(\varepsilon(a-c)/b\) so that it is scaled by the amount that would be sold under perfect competition. A quota where \(\varepsilon\) is 1 would allow the foreign firm to supply the amount demanded at a price equal to marginal cost. Similarly, if \(\varepsilon\) is 1/2, it can sell the amount demanded at the monopoly price of \((a+c)/2\), and so on.

Notice as an aside that any quota which is binding at the original equilibrium, i.e. which reduces the amount imported, raises the standard measures of domestic welfare. This is so because, even if the price remains at
\[(a+c)/2\], the domestic firm having higher sales, now earns higher profits. Domestic welfare is only increased further if the price actually falls\(^6\). Since the national identity of firms is a slippery concept, however, we focus mainly on the competition-enhancing affects of quotas.

We begin studying the equilibrium with quotas by analyzing the punishments for deviating from the implicitly collusive understanding. As is standard practice in models of repeated oligopoly (see for example, Friedman (1971), Rotemberg and Saloner (1986) and Brock and Scheinkman (1986)) we assume that firms revert to the Nash equilibrium in the one-shot game if any firm deviates from the collusive understanding. This is in contrast to Abreu (1982) who focuses instead on the maximal feasible punishment for any deviating firm. While, as we show below, our results are not very sensitive to our assumption of one-shot Nash punishments, there are three reasons why these may be preferable even when they are lower than the maximal punishments. First, if the owners of the firm can sell the firm forward, the only equilibrium until the sale takes place is the one-shot Nash equilibrium.\(^7\) Second, it is in the interest of the managers of the firm to signal, once punishments begin, that they are unwilling to post prices other than those of the one-shot Nash equilibrium. If this signalling is credible maximal punishments as in Abreu (1982) become infeasible.\(^8\) Third, the one-shot Nash equilibrium is easier to characterize than the maximal punishments.

The static one-shot game to which firms revert when they are punishing each other has no pure strategy equilibrium. Kreps and Scheinkman (1983) analyze the mixed strategy equilibrium, and compute the expected profits of the firm with larger capacity. They also exhibit one such mixed strategy equilibrium (for which they compute the entire distribution of prices). It is straightforward to show that this equilibrium is the unique mixed strategy equilibrium and thus to
also compute the expected profits of the foreign firm at this equilibrium.

We now list the salient features of this equilibrium. The arguments behind these assertions are contained in the appendix:

(i) The highest price charged by both firms is \(\frac{a+c-\varepsilon(a-c)}{2}\).

(ii) The lowest price charged is
\[
\sigma = \frac{(a+c)/2 - (a-c)(2\varepsilon-\varepsilon^2)^{1/2}}{2}
\]
\[\text{(2)}\]
and it is charged by both firms with probability zero.

(iii) In equilibrium, the domestic firm has expected profits of \(\frac{(a-c)^2(1-\varepsilon)^2}{4b}\) per period while those of the foreign firm equal \(\varepsilon(a-c)(a-c)/b\).

Notice that this static equilibrium has the features of the differentiated products model of Krishna (1986). A higher quota (a higher \(\varepsilon\)) lowers both the highest and lowest price charged. Kreps and Scheinkman (1984) show that the entire distribution of prices is stochastically dominated by that with a lower quota.

There is one more feature of this punishment equilibrium that deserves comment. At this equilibrium the domestic firm earns a present value of \(\frac{(a-c)^2(1-\varepsilon)^2}{4b(1-\delta)}\). What must be noted is that the domestic firm can never earn less than this present discounted value of profits at any equilibrium; even the one involving maximal punishments. The reason for this is that the domestic firm can guarantee for itself that it will earn \(\frac{(a-c)^2(1-\varepsilon)^2}{4b}\) per period by posting a price equal to \(\frac{a+c-\varepsilon(a-c)}{2b}\). This limit on the punishability of the domestic firm is what gives quotas their ability to break the monopoly price.

With the value of the punishments in hand we now turn to the analysis of the repeated game. The price preferred by the domestic firm continues to be \(\frac{a+c}{2}\) while the foreign firm, which is subject to a quota, naturally prefers a higher price. Yet we concentrate on the question of whether the duopoly can
sustain the "monopoly" price of \((a+c)/2\). We do this for two related reasons. First, higher prices are more difficult to sustain so that if the firms cannot sustain \((a+c)/2\) they cannot sustain any higher price. Second, we are interested in whether the introduction of a quota lowers equilibrium prices from their free trade level of \((a+c)/2\).

The sustainability of the price \((a+c)/2\) depends on the amount the foreign firm is expected to sell at this price. If, for instance, the foreign firm is allowed to sell its entire quota, \(\varepsilon(a-c)/b\), at this price the foreign firm will have no incentive to deviate. In this case, the domestic firm will always deviate since it earns more in the static game. Similarly, if the foreign firm is expected to sell nothing, the domestic firm never deviates but the foreign firm always deviates. So, we consider the intermediate case in which the foreign firm is supposed to sell \(\mu(a-c)/b\) \((0<\mu<1)\) in the collusive arrangement.

Then, by going along with the collusive arrangement, its profits are \(\mu(a-c)^2/2b\). Instead, if it deviates by undercutting the price slightly, it sells either total demand or its entire quota at a price essentially identical to \((a+c)/2\). We analyze separately the case in which \(\varepsilon\) is smaller than or equal to 1/2, in which case it sells its entire quota, and the case in which it exceeds 1/2, in which case it sells \((a-c)/2b\).

Consider first the former case. By deviating, the foreign firm earns \(\varepsilon(a-c)^2/2b\). It will thus choose to deviate unless:

\[
(\varepsilon - \mu)/2 \leq \delta[\mu - \varepsilon + \varepsilon(2\varepsilon - \varepsilon^2)^{1/2}]/2(1-\delta)
\]  

(3)

or:

\[
\mu \geq \varepsilon - \delta\varepsilon(2\varepsilon - \varepsilon^2)^{1/2}.
\]  

(4)
Note that for small $\varepsilon$, $\mu$ must essentially equal $\varepsilon$. The foreign firm knows that the price will roughly equal $(a+c)/2$ whether it goes along or is being punished. Thus it deviates unless it is allowed to sell essentially its entire capacity. Note also that the higher is the firms' discount factor, $\delta$, the less the foreign firm is willing to sell without cheating.

Now consider the domestic firm. If it goes along it sells $(a-c)(1/2 - \mu)/b$ at the monopoly price of $(a+c)/2$, while if it cheats it can sell $(a-c)/2b$ at that price. On the other hand it earns only $(a-c)^2(1-\varepsilon)^2/4b$ per period after cheating. Thus the domestic firm is deterred from price-undercutting if:

$$\mu/2 \leq \delta[\varepsilon/2 - \mu/2 - \varepsilon^2/4]/(1-\delta)$$

or:

$$\mu \leq \delta\varepsilon - \delta\varepsilon^2/2.$$  \hspace{1cm} (5)

Equation (5), which is valid also when punishments are maximal, shows that $\mu$ must be relatively small if the domestic firm is to be deterred from cheating since higher levels of $\mu$ make cheating more attractive without increasing its cost to the firm.

If (4) and (5) contradict one another the monopoly price is not sustainable. To see under what conditions this occurs we combine the conditions and obtain:

$$\varepsilon[1 - \delta - \delta[(2\varepsilon - \varepsilon^2)^{1/2} - \varepsilon/2]] \leq 0.$$  \hspace{1cm} (6)
For ε small enough we can neglect the term in square brackets (which is of second order) and the condition is clearly violated. When ε is small enough we saw that the foreign firm must be allowed to sell essentially its entire quota. But the domestic firm always requires that μ be smaller than δε which is strictly smaller than ε; for μ equal to ε it has too strong an incentive to undercut the monopoly price.\(^1^0\)

For ε between 0 and 1 the term in brackets is positive. The term in square brackets is increasing in ε until ε reaches .553. Yet this analysis is only relevant for ε up to 1/2 since beyond this the foreign firm cannot sell its entire quota when it cheats. For this maximal applicable ε, δ must exceed about .62 for (6) to be satisfied.

Now consider the case in which ε exceeds 1/2 so that when the foreign firm cheats it earns \((a-c)^2/4b\). Then the foreign firm will cheat unless:

\[(1/2 - μ)/2 \leq δ[μ - ε + ε(2ε - ε^2)^{1/2}]/2(1-δ)\]

or:

\[μ \geq (1-δ)/2 + δε - δε(2ε - ε^2)^{1/2}. \quad (7)\]

which must now be satisfied together with (5) for monopolization to be feasible.

So we substitute (7) (holding with equality) in (5) and obtain:

\[(1-δ)/2 \leq δε(2ε - ε^2)^{1/2} - δε^2/2. \quad (8)\]

For ε equal one, that is when the foreign firm can sell the entire quantity demanded at the competitive price, (8) requires that δ exceed 1/2 as it did
under free trade. Since the RHS of (8) is strictly increasing in $\varepsilon$, the level of $\delta/(1-\delta)$ (and thus of $\delta$) required to make (8) hold, falls strictly when $\varepsilon$ rises.

To summarize the results of this section, more restrictive quotas (starting at a quota which allows the foreign firm to sell the entire amount demanded at the competitive price) monotonically reduce the ability to monopolize the market. Note that, for a given $\delta$, the quota that gives the minimum price in the domestic market is strictly smaller than the one which satisfies (6) (or (8)) with equality. When these equations hold with equality, monopoly is just sustainable. For lower values of $\varepsilon$, the price falls. However, if the quota is reduced significantly more, the price starts rising again as the prices charged even in the one-shot game rise. For $\varepsilon$ equal to zero, the monopoly price is reestablished.

It must be pointed out again that the increased competition brought about by quotas is not sensitive to the use of one-shot Nash punishments. For instance even if deviations by the foreign firm lead this firm to earn zero profits from then on (which for low $\varepsilon$ is a much harsher punishment than the maximal punishment) the foreign firm will require that $\mu$ equal at least $(1-\delta)\varepsilon$ so as to refrain from deviating. This is inconsistent with (5) for $\delta$ equal to 1/2 and any positive $\varepsilon$.

The increased competition brought about by quotas in this setting is in direct contrast to the conclusions of the static analysis reported by Krishna (1985). We now briefly compare the results to those one would obtain when analyzing tariffs in a similar setting. A tariff simply implies that the costs of the foreign firm are higher, by the tariff rate, than the costs of the domestic firm. The repeated game in which the two firms have different costs has been analyzed by Bernheim and Whinston (1986), who consider optimal
punishments in the style of Abreu (1982). Then, both the domestic and foreign firms still earn zero profits when they are being punished\textsuperscript{11}. This means that the incentives to participate in the implicit agreement that requires that the price \((a+c)/2\) be charged, do not change as a result of the tariff. Moreover, Bernheim and Whinston (1986) show that the equilibrium that involves the highest profits for the duopoly as a whole now has a price higher than \((a+c)/2\). This occurs because the profit maximizing price from the point of view of the foreign firm is now higher. So a tariff has the potential for increasing the domestic price above the monopoly price.

Thus, in the case of maximal punishments the classic results of Bhagwati (1965) about competition between a foreign and a domestic firm, are precisely reversed. A quota, because it makes it impossible for the domestic firm to be punished effectively, makes it difficult to collude, while a tariff has no such consequence. This raises the intriguing possibility that this is the reason governments seem to prefer quantitative restrictions to tariffs\textsuperscript{12}.

However, it must be pointed out that the robustness of the monopoly outcome with respect to a tariff is sensitive to the use of maximal punishments. If instead, reversions to the Bertrand outcome are used, a tariff which raises the foreign firm's costs barely below the monopoly price of \((a+c)/2\) makes the monopoly price unsustainable. The reason for this is that to maintain this price the domestic firm must give a sizeable fraction of the market \((1-\delta)\) to the foreign firm. Thus if the foreign firm's costs are near \((a+c)/2\) the domestic firm will actually earn more during the period of punishment (when it charges a price barely below the foreign firm's costs) than when it goes along with the monopoly price.

The results of this section are sufficiently striking that they deserve to be qualified somewhat further. The analysis hinges critically on the presence
of only one domestic firm. Since this means that only the foreign firm can punish the domestic firm, this punishment ability is curtailed in the presence of a quota. If there were more domestic firms in the market, they could all revert to a price of c if any of them deviated and thus keep the oligopoly at the monopoly outcome even in the presence of a quota. On the other hand, if the domestic firms are capable of concerted cheating, the approximation considered in this section of only one domestic firm may be a good one.

III Quotas and quantity competition at home

In this section we assume that quantities are the strategic variables. This has two consequences. First, neither firm eliminates the sales of its competitor when it deviates. These are maintained at the implicitly colluding amount. Second, punishments are characterized by reversion to the single period Nash equilibrium in quantities, which is a much milder form of punishment than when prices are the strategic variables.13

The results of this section are directly comparable to Davidson's (1984) analysis of tariffs, in which he also assumes that quantities are the strategic variables. We show that the result that quotas lead to less sustainability of collusion now only holds for very restrictive quotas that allow the foreign firm to import very little. For larger quotas, we find that collusion is, if anything, enhanced by the quota. Thus our results here are similar to Davidson's (1984) since he shows that small tariffs (which correspond to large quotas) enhance collusion while large tariffs do the opposite.

Again, the firms are assumed to be able to monopolize the domestic market in the absence of quotas. By going along with selling \((a-c)/4b\) each, each firm earns \((a-c)^2/8b\). If either firm were to deviate it would choose to sell \(3(a-\)
c)/8b which would give it profits of 9(a-c)^2/64b in the period of the deviation. From then on, the firms would revert to Cournot-Nash equilibrium so that each would sell (a-c)/3b and earn profits of (a-c)^2/9b per period. So, each firm will be deterred from cheating if:

\[ [9/64 - 1/8](a-c)^2/b \leq \delta [1/8 - 1/9](a-c)^2/b(1-\delta), \]

which requires only that \( \delta \) exceed 9/17.

Now consider a quota of \( \epsilon(a-c)/b \). If \( \epsilon \) is above 3/8, so that the quota exceeds the best response to the monopolizing level of output, it has no effect. So suppose that it is lower than this level but higher than the Nash equilibrium level of sales, i.e. that \( \epsilon \) is between 3/8 and 1/3. Then the quota does not affect the foreign firm's ability to punish the domestic firm from deviating (since each firm produces (a-c)/3b in Cournot-Nash equilibrium. However, it makes the foreign firm's own deviation less profitable since this deviation cannot now exceed \( \epsilon(a-c)/b \). So such a quota has no effect when the firms can sustain the monopolistic outcome without a quota. Furthermore, it actually may make it possible to sustain this monopolistic outcome even when this cannot be done in the absence of a quota. For suppose that \( \delta \) is just below 9/17 so that the collusive output is not sustainable in the absence of a quota. With \( \epsilon \) between 3/8 and 1/3 the foreign firm is not only willing to go along with selling (a-c)/4b, since it now earns less when it deviates than it does if it isn't constrained, it will still not be induced to cheat even if its sales are somewhat lower. This means that it may be possible to sustain the monopoly output by having the foreign firm produce less than half of the total, and the domestic firm produce more than half. In essence, the "excess" unwillingness to cheat on the part of the foreign firm can be used to free up the constraint on
the domestic firm.

Thus for a quota to promote competition it must be below the Nash equilibrium \((a-c)/3b\), \(\varepsilon\) must be less than 1/3. We now analyze such a quota in detail. We first look for the lowest level of sales on the part of the foreign firm which makes it willing to go along with an equilibrium whose price is \((a+c)/2\). Then we analyze whether the domestic firm is willing to go along with this price given these sales by the foreign firm.

Suppose that the foreign firm is supposed to sell \(\mu(a-c)/b\) in equilibrium and that the firms attempt to sustain the monopoly level of output. We now derive the minimum level of \(\mu\) for which the foreign firm will not want to deviate. By cooperating it sells its output at a price of \((a+c)/2\) so that it earns \((a-c)^2\mu/2b\). The domestic firm sells that portion of the monopoly output not sold by the foreign firm, or \((a-c)(1/2 - \mu)/b\). We show below that the requirement that \(\varepsilon\) be smaller than 1/3 implies that when the foreign firm deviates it sells \(\varepsilon(a-c)/b\). Thus when it deviates it earns \((a-c)^2\varepsilon(1/2 - \varepsilon + \mu)/b\). Note that the profits to the foreign firm when it deviates are increasing in \(\mu\). This is because the higher is \(\mu\) the lower are the domestic firm's sales when they both go along with the proposed equilibrium output.

After its deviation the market reverts to the equilibrium in the one-shot game. Since the quota is less than the Nash equilibrium value of \((a-c)/3b\), the foreign firm sells \(\varepsilon(a-c)/b\) while the domestic firm sells its best response to this level of output, or \((a-c)(1-\varepsilon)/2b\). Thus the foreign firm earns \((a-c)^2\varepsilon(1-\varepsilon)/2b\). Hence the foreign firm will be deterred from deviating if:

\[
(a-c)^2[\varepsilon(1/2 + \mu - \varepsilon) - \mu/2]/b \leq \delta(a-c)^2[\mu - \varepsilon(1-\varepsilon)]/2b(1-\delta)
\]

i.e. if:

\[
(a-c)^2[\varepsilon(1/2 + \mu - \varepsilon) - \mu/2]/b \leq \delta(a-c)^2[\mu - \varepsilon(1-\varepsilon)]/2b(1-\delta)
\]
\[ \mu \geq \varepsilon - \delta \varepsilon^2/[1 - 2\varepsilon(1-\delta)]^{14}. \] (9)

Note that for values of \( \varepsilon \) near zero, the foreign firm will deviate unless the implicit agreement allows it to sell nearly its entire quota. The reason for this is that when the quota is small, price does not differ appreciably from \((a+c)/2\), even during the punishment phase. Then it is always in the foreign firm's interest to sell as much as it can and it cannot be deterred from deviating unless it is allowed to sell essentially its entire quota. This means that for sufficiently small \( \varepsilon \) the foreign firm has no punishment power since it is already selling all that it is allowed to bring in. Given this, it is not surprising that we establish next that for sufficiently small \( \varepsilon \), the equilibrium cannot involve the monopolization of the domestic market.

Consider the domestic firm's incentive to deviate. Suppose that the foreign firm is selling \((a-c)\mu/b\) as above. The domestic firm would sell its best response to this, \((a-c)(1-\mu)/2b\), if it deviated. It would then earn \((a-c)^2(1-\mu)^2/4b\) in the period in which it deviated. From then on the foreign firm would sell \((a-c)\varepsilon/b\) so that, by an argument similar to that above, the domestic firm would earn \((a-c)^2(1-\varepsilon)^2/4b\) per period.

So the domestic firm would be deterred from deviating only if:

\[
(1 - \mu)^2/4 - (1/4 - \mu/2) \leq \\
\delta[1/4 - \mu/2 - (1 - \varepsilon)^2/4]/(1-\delta). \] (10)

It is easy to see that in spite of the fact that profits from deviating increase when \( u \) falls, the inequality in (10) is more likely to hold the smaller is \( \mu \), as long as \( \mu \) is smaller than \( \delta/(1-\delta) \) (which is true by assumption). The
reason for this is that the principal effect of a decrease in $y$ is that the
domestic firm earns more by going along with the collusive agreement. Thus to
make the domestic firm as willing to go along as possible we let the foreign
firm sell only as much as is required to make (9) hold with equality.
Substituting this expression for $y$ in (10), multiplying by $(1-\delta)[1-2\epsilon(1-\delta)]^2$ and
collecting terms, the inequality (10) becomes:

$$
\epsilon^2(1-2\delta^2) - \epsilon^3(1-\delta)[4(1-\delta^2) + 2\delta] + \epsilon^4(1-\delta)[4(1-\delta) + 4\delta(1-\delta) + \delta^2] \leq 0. \tag{11}
$$

If we divide this expression by $\epsilon^2$ and take the limit as $\epsilon$ goes to zero we obtain $(1-2\delta^2)\leq0$, which requires $\delta$ smaller than about .71. Furthermore, the LHS of (11) (after dividing by $\epsilon^2$) decreases monotonically with $\epsilon$ in the relevant range of $\epsilon$. This means that if $\delta$ exceeds .71 it is always possible to sustain the monopoly price.

If, instead, $\delta$ is between $9/17$ and .71, the monopolistic outcome is
sustainable with free trade but not with a sufficiently small quota. In this
case of linear demand, however, the quota must be very small for this result to
hold. If we take the smallest discount factor consistent with collusion under
free trade ($9/17$), one can demonstrate numerically that the monopolistic outcome
is not sustainable for $\epsilon$ equal to .01 but is sustainable for $\epsilon$ equal to .015.
These are such low values of $\epsilon$ that even the noncooperative solution looks
essentially identical to the monopolistic solution.

IV Quotas and competition abroad: Constant demand

We now turn to the implications of the existence of a quota at home for
competition abroad. In this section we adapt the arguments presented in Sections II and III to argue that under certain circumstances they can be interpreted as providing conditions under which competition increases abroad. We assume that the domestic and foreign markets are identical so that their demand curves are given by (1). If both firms have infinite capacity, there is a monopolistic equilibrium in which \((a+c)/2\) is charged in both markets as long as \(\delta\) equals at least 1/2 when prices are the strategic variables and \(\delta\) equals at least 9/17 when quantities are strategic variables.

Now assume that the domestic firm actually has limited capacity given by \(k(a-c)/b\). So marginal cost equals \(c\) until the firm produces \(k(a-c)/b\) and then becomes infinite. If \(k\) equals 2, for instance, the domestic firm can supply the quantity demanded at the competitive price in both markets. The idea of this section is that this capacity is sufficient to maintain the monopolistic outcome in both countries with free trade. However, with a quota that forbids all imports into the domestic country, the domestic firm must devote a larger fraction of its capacity to producing for the protected domestic market. It then effectively has a very small capacity for selling abroad. This has the same effect abroad as would a quota imposed by the foreign country.

We first treat the case in which prices are the strategic variables. If \(\delta\) is equal to 1/2 (the benchmark case) and \(k\) is less than 2, the two firms cannot maintain the monopolistic outcome in both markets even in the absence of a quota. This results because the domestic firm does not have sufficient punishment power to keep the foreign firm from cheating. Thus the monopoly outcome can at best be sustained for values of \(\delta\) that are greater than that required to sustain monopoly under free trade without capacity constraints.

We consider how large the "critical" value of \(\delta\) must be for different values of \(k\). Note first that \(k\) must be at least equal to 1/2 for the problem to
be interesting since otherwise the domestic country is unable to meet demand in its own market when that market becomes closed to foreign firms. So first suppose that \( k \) is barely above \( 1/2 \). Then we can use (6) with \( \epsilon \) equal to \( k/2 \) (the world market is equivalent to twice the domestic market) to obtain the value of \( \delta \) for which the monopoly outcome is sustainable in the world as a whole. This value of \( \delta \) is about .65. If instead \( k \) is equal to one, the critical value of \( \delta \) is about .62.

Now consider the effect of a quota. The domestic firm sells \( (\text{a-c})/2b \) at home and has \( (k-1/2)(\text{a-c})/b \) units left over to sell in the foreign country. What is important to notice is that the domestic firm is never willing to sell more than \( (k-1/2)(\text{a-c})/b \) units abroad, even when it is punishing the foreign firm for deviating. This is so because the marginal revenue from selling a unit at home always exceeds the marginal revenue from selling it abroad if less than \( (\text{a-c})/2b \) is sold at home. This means that the domestic firm is effectively capacity constrained abroad with a capacity of \( (k-1/2)(\text{a-c})/b \). Letting \( k-1/2 \) be denoted by \( \epsilon \) we can use (6) (or (8) for \( k \) bigger than 1) to discover the minimum value of \( \delta \) that sustains the monopoly outcome abroad. For \( k \) barely above \( 1/2 \) no value of \( \delta \) strictly below 1 is sufficient to maintain that outcome (in contrast to a value of .65 in the absence of a quota). For \( k \) equal to 1, a value of .65 is required when only .62 was required under free trade.

We now turn to the case in which quantities are the strategic variables. Here the monopoly outcome only broke down for tiny quotas. Hence if the domestic firm has a capacity of \( (1/2 + \epsilon) \) where \( \epsilon \) is negligible, the monopoly outcome can be maintained with free trade as long as \( \delta \) exceeds 9/17. On the other hand, once the domestic market is closed to the foreign firm, the domestic firm is willing to sell at most \( \epsilon \) abroad. This means that for \( \epsilon \) small enough if \( \delta \) is below .71, the monopoly outcome won't be sustainable abroad.
V Quotas and competition abroad: Fluctuating demand

This section considers a somewhat different model in which a quota in one country tends to increase competition in the other country. The idea is that collusion is maintained under free trade only because whenever the net benefits from cheating are big in one market, they are small in the other market and viceversa. In this setting, the overall incentive to cheat (taking the two markets together) are fairly constant over time. If one looks at either market separately, however, the incentive to cheat fluctuates over time. Then if, as in Rotemberg and Saloner (1986), future punishments are largely independent of the current period's incentive to cheat, then those punishments may be insufficient to deter cheating if the current incentive to cheat is particularly high. It thus becomes more difficult to prevent them from cheating. Since the imposition of a quota has the effect of separating the two markets, it thus makes implicit collusion more difficult.

Our specific model assumes that demand is random in both countries, as in Rotemberg and Saloner (1986), and negatively correlated across markets as in Bernheim and Whinston (1986). To simplify the analysis we assume that there are only two states of demand (high and low) whose realizations are independently distributed over time in any one country and perfectly negatively correlated across markets. So when demand is high in one market it is low in the other.\footnote{15}

In this section we assume that there are N/2 firms in each market while we do not restrict attention to linear demand curves. Since the countries are symmetric, the monopoly price in the high state, $p^u$, is the same in both countries and similarly for the monopoly price in the low state, $p^l$. We start the analysis by ensuring that these two prices $p^u$ and $p^l$ are sustainable with
free trade. That is we consider a situation in which all N firms sell in both markets and derive the condition under which no single firm wants to deviate from charging \( p^u \) in the country in which demand is high and \( p^1 \) in the other. Prices are again the strategic variable so that any deviation triggers a permanent reversion to the static equilibrium in which price equals marginal cost.

Let \( \pi^u \) be the total monopoly profits in the market with high demand and \( \pi^1 \) those in the market with low demand. As long as \( p^1 \) exceeds marginal cost, the former is bigger than the latter since the monopoly always has the option of charging \( p^1 \) even when demand is high. A firm that deviates from the monopoly outcome undercuts the price charged by the others slightly. It thus earns \( \pi^u \) in the market in which demand is high and \( \pi^1 \) in the other. On the other hand, by deviating it gives up both now and in all future periods its share \((1/N)\) of these profits. Thus each firm is deterred from deviating as long as:

\[
(\pi^u + \pi^1)(1 - 1/N) \leq \delta(\pi^u + \pi^1)/N(1 - \delta) \tag{12}
\]

or:

\[
N - 1 \leq \delta/(1 - \delta). \tag{13}
\]

We assume that either the number of firms is small enough or the discount factor big enough that (12) holds so that with free trade the two markets can be completely monopolized.

We now consider what happens when no imports are allowed into the domestic country. This drastic quota has two effects. First, it means that there are fewer \((N/2)\) active firms in the domestic market while \( N \) firms continue to sell
in the foreign market. As a result, the conditions under which monopoly will be sustainable will be different depending on which market has high demand. As Rotemberg and Saloner (1986) show, in this setting it is always easier to sustain collusion when demand is low. Thus to see whether complete monopolization is possible, we study the conditions under which such monopolization is possible in the high demand state.

Consider first the foreign country when demand is high. Suppose that the firms try to charge $p^u$. Any firm that undercuts this price earns $\pi^u$ instead of $\pi^u/N$ abroad. The losses from the ensuing competition are smaller for foreign countries since domestic countries may also lose any profits they make at home. So we concentrate on the incentive to deviate of foreign firms. If they deviate they lose the expected value of profits in the future. Since demand is independently distributed over time, these expected profits equal $(\pi^u + \pi^l)/2$. So, a foreign firm will deviate if:

$$\pi^u(1 - 1/N) > \delta(\pi^u + \pi^l)/2N(1-\delta).$$

Note that the RHS of (14) equals half the RHS of (12). The costs in terms of foregone future profits from deviating are half what they would be if foreign firms had access to both markets. Yet, insofar as $\pi^u$ is bigger than $\pi^l$, the LHS of (14) is bigger than half of the LHS of (12) so that it is possible for (12) and (14) to hold simultaneously. In particular, if (12) holds as an equality (so that monopolization is just sustainable with free trade) then if the profits in the high and low demand states are different, the inequality in (14) is always satisfied. When (14) holds, it is impossible to sustain monopoly abroad in the high state of demand so the quota increases competition.

It is easy to show that when (12) holds it is always possible to sustain
monopoly abroad in the low state of demand. This results from the fact that (12) ensures that deviations will not take place even when demand is constant. Yet, when demand is low, expected future losses from deviating exceed those that would prevail if demand were expected to stay low forever. This in turn results from the fact that even when (14) holds, it is still possible to sustain substantial profits in the high state. This can be seen as follows: Let the sustainable price (the highest price that can be charged without inducing any firm to deviate) in the high state be \( p^s \) and the ensuing profits \( \pi^s \). Then by an argument analogous to the one used to derive (14), a foreign firm will just refrain from undercutting \( p^s \) if

\[
\pi^s (1 - 1/N) = \delta (\pi^s + \pi^l)/2N(1-\delta)
\]

or:

\[
\pi^s = \pi^l/[2(N-1)(1-\delta)/\delta - 1]\]

so that \( \pi^s \) equals \( \pi^l \) when (12) holds as an equality and exceeds it when it holds as a strict inequality.

So far, we have shown that under plausible conditions the foreign market becomes more competitive while still retaining substantial profits. This result does not hinge on the fact that prices are used as the strategic variables. As in Rotemberg and Saloner (1986), it only requires that collusion be difficult to sustain when demand is high. As we showed there, this is also true when quantities are the strategic variables when demand is linear.

We now turn briefly to an analysis of the domestic market. If domestic firms were to deviate from the collusive arrangement they could be punished in
both markets. In this case, deviating firms would deviate in both markets at once. On the other hand we have already derived the net benefits from deviating abroad under the assumption that punishments would only be meted out abroad. To complete the analysis we thus look at the net benefits from deviating in the domestic market alone under the assumption that firms will only be punished in the domestic market for doing so. Then, the total net benefits to deviating are the sum of these two net benefits.

Suppose that the domestic market is fully monopolized. When demand is high, a deviating firm would earn \( \pi^u \) instead of its share \( (2/N) \) of \( \pi^u \). It would then lose its share of total expected profits in its own market from then on. It would thus be deterred from deviating if:

\[
\pi^u (1 - 2/N) \leq \delta (\pi^u + \pi^l)/N(1-\delta).
\]

(16)

The RHS of (16) is the same as the RHS of (12). While the punishment is meted out only in one market, the share of each firm in this market is double the share we previously considered. On the other hand the LHS of (16) is substantially smaller than the LHS of (12) since deviations are taking place in one market alone while, in addition, the net benefit is smaller because there are fewer firms active in the market and so the firm's share was larger to start with. So if (12) is satisfied (even with equality) then (16) is satisfied as a strict inequality. This applies to the case in which demand is high at home. In this case demand is low in the foreign market and our previous analysis showing that there is a net loss from deviating there applies. Thus if monopoly is sustainable in the absence of a quota when demand is high in the home market, then it is also sustainable with the quota.

When demand is low at home the corresponding condition at home has the same
RHS as (16) but an even smaller LHS. So the net benefits to deviating are negative here as well.

In summary, when (12) holds, monopolization is always possible in the protected domestic market. Indeed, since protected domestic firms strictly prefer to go along with monopolization than to deviate when (12) holds as an equality, we can conclude that protection may make monopolization feasible at home when it was not feasible with free trade.

The fact that collusion becomes easier for the domestic firms has the potential, in a richer model, to make monopolization feasible abroad as well. To do this we must allow the set of firms that charge the lowest price to split the market unevenly as is done in Bernheim and Whinston (1986) and as we do in Section II. Bernheim and Whinston (1986) show that the possibility of such arbitrary divisions allows "punishment power" to be shifted from a market in which there is more than enough to one in which there isn't enough to sustain monopoly outcomes. In our example, the domestic firms can offer the foreign firms a share higher than $1/N$ of the foreign market. Domestic firms would tolerate this for fear of losing their domestic profits if they deviate abroad. Foreign firms would earn more when they go along with the collusive arrangement and thus would be less willing to deviate. In this way monopolization is potentially achievable when demand fluctuates relatively little. This approach does not work, however, when $N$ is equal to 2 so that as in Sections II and III there is only one firm in the domestic market. Then this firm does not need to fear retaliation at home when it deviates abroad. Its incentives to deviate abroad are the same as those of foreign firms and giving the foreign firm a disproportionately large market share only increases the domestic firm's incentive to deviate abroad.
VI Quotas and Competition in Both Markets

In this section we consider the possibility that a quota will increase competition at home (as in Sections II and III) as well as abroad (as in Sections IV and V). The model we use to illustrate this possibility is the fluctuating demand model of Section V except that, to permit competition to increase at home, we consider a strictly positive quota. Also, to simplify the analysis we restrict attention to the case in which $N$ equals 2, so that there is one firm in each country, and $\delta$ is equal to 1/2, so that monopoly is just sustainable in both markets with free trade.

Since any firm that deviates in one market will be punished in both, any deviating firm will undercut the implicitly agreed upon price in both countries. So the incentive to deviate in any one country can be thought of as the sum of the net profits from deviating at home (i.e. the benefits from deviating at home minus the present value of the losses at home) plus the net profits from deviating abroad.

Suppose that the foreign firm has a share $\alpha$ of the foreign market. Then its net profits from cheating when demand is low are $(1-3\alpha/2)\pi^{1-\alpha}\pi^S/2$ while those when demand is high are $(1-3\alpha/2)\pi^S-\alpha\pi^1/2$. Similarly the domestic firm's net profits from cheating when demand is low are $(3\alpha/2-1/2)\pi^{1-(1-\alpha)}\pi^S/2$ and those when demand is high are $(3\alpha/2-1/2)\pi^{1-(1-\alpha)}\pi^S/2$. The important thing to notice is that, as we increase $\alpha$, the foreign firm's net profits from deviating fall by the same amount as the domestic profits from deviating increase. Thus, as long as in each state the total net profits from deviating (the sum of each firm's net profits from deviating) are zero, there will always be a redistribution of shares that will make both firms willing to go along.

We therefore now concentrate only on the total incentive to deviate. In
the foreign country this is \((\pi^1 - \pi^s)/2\) in the low state and \((\pi^s - \pi^1)/2\) in the high state. So unless \(\pi^s\) is equal to \(\pi^1\) the total incentive to deviate is positive abroad in one of the two states.

Suppose demand at home is given by \(P = a^i - bQ\) (where \(i\) equals either \(u\) or \(l\)). We can then write the quota in state \(i\) as equal to \(\varepsilon^i (a^i - c)/b\) where if the quota is small enough \(\varepsilon^i\) is less than \(1/2\). Suppose that at the monopoly price the foreign firm is allowed to sell \(\mu^i (a^i - c)/b\). Then we can obtain from (4) the foreign firm's net profits from deviating while from (5) we can obtain the domestic firm's net profits from deviating. Once again by varying \(\mu^i\) one firm's decreased incentive to deviate is the other's increased incentive so we can focus on the total incentive to deviate. This is given by:

\[
(a^i - c)^2 \left[ \varepsilon^i (1 - \delta) + \delta \varepsilon^{i2}/2 - \delta \varepsilon^i (2 \varepsilon^i - \varepsilon^{i2})^{1/2} \right]/b. \tag{17}
\]

Note that the condition that (17) be positive is the same as that (6) be violated which we proved for the case in which \(\delta\) is equal to \(1/2\) in Section II. There is thus a net incentive to deviate in the domestic market.

Now consider the net incentive to deviate in the two markets taken together if we try to maintain monopoly at home. This is given by \((\pi^s - \pi^1)\) plus the expression in (17) with \(i\) replaced by \(1\) when demand is high abroad. It is given by \((\pi^1 - \pi^s)\) plus the expression in (17) with \(i\) replaced by \(u\) when demand is low abroad. At least one of these net incentives is positive so that it is impossible to sustain monopoly at home.

Now we consider whether it is possible to have an outcome different from the monopoly outcome at home such that the net incentive to deviate at home is so negative that the monopoly outcome can be maintained abroad even in the high state. That this is unlikely for small quotas should be apparent since, then,
the static game gives about as many profits as the monopoly outcome. This suggests that there are no prices between the monopoly price and those that prevail under static competition for which the costs of reverting to competition are high.

We now consider this issue more formally. We consider a price \( p \) at home and study the overall profits from deviating at home. The amount demanded at this price is \( (a-p)/b \) and, since the distribution of this demand across firms is immaterial, we let the foreign firm sell \( \epsilon^i(a^i-c)/b \) while the domestic firm sells the difference between the quantity demanded and the quota. The domestic firm's net profits from deviating are then given by:

\[
\epsilon^i(a^i-c)(p-c)/b - (p-c)[(a^i-p) - \epsilon^i(a^i-c)]/b + (a^i-c)^2(1-\epsilon^i)^2/4b.
\]

The foreign firm gains nothing by cheating but its cost from reversion to the static game is:

\[
\epsilon^i(a^i-c)(p - c)/b
\]

so that the overall benefits from deviating are:

\[
(p-c)(\epsilon^i(a^i-c) - (a^i-p))/b + (a^i-c)^2(1-\epsilon^i)^2/4b + \epsilon^i(a^i-c)(c-c)/b
\]

whose minimum is reached for a \( p \) equal \( [a^i+c-\epsilon^i(a^i-c)]/2 \). There, this expression equals:

\[
\{-\epsilon^2/2 + \epsilon[1 - (2\epsilon-\epsilon^2)^{1/2} + \epsilon^2]}(a-c)^2/2b
\]  

(18)
which is negligible for small $\epsilon^1$. The fact that (18) is small means that the maximum cost from deviating at home, $\phi$, is small. Yet, maximum profits in the high state abroad must satisfy:

$$(\pi^s - \pi^1)/2 - \phi \leq 0$$

so that for small $\phi$ it is impossible to make $\pi^s$ equal to $\pi^m$ as would be required to monopolize the foreign market.

VII. Conclusions

We have shown in a variety of models in which implicit collusion is important that quotas have the potential of increasing competition, not just in the domestic market, but also abroad. These results stand in sharp contrast to those obtained when there is a single domestic firm and goods from abroad are supplied competitively (Bhagwati (1965)) as well as those obtained when domestic and foreign firms act as in a one-shot game.

The intuition underlying our seemingly paradoxical results are not restricted to quotas. Indeed, any action by the government which makes competition when collusion breaks down more profitable makes collusion itself less viable. Thus Government imposed minimum price floors could have precisely the same effect.

One natural question to ask is whether our results depend critically on the existence of implicit collusion or whether they can also be generated in other models of strategic interaction. One particular concern is how our results would be modified in the presence of entry.

In this conclusion we sketch an argument that shows that quotas also tend
to increase competition by encouraging the entry of new (domestic) firms into the industry. The setting we consider is very closely related to that developed in Dixit and Kyle (1985). They analyze an industry with an established foreign firm and a domestic potential entrant. They show that it is possible that if the domestic country prohibits imports that there will be more competition in the foreign market. This occurs because the potential domestic firm may find it unprofitable to enter if it must compete with the foreign firm in both markets. By protecting it in the domestic market (while it competes in the foreign market), the domestic government provides it with sufficient revenues to cover its fixed costs. It then finds entry attractive, enters, and increases competition abroad. Since the domestic market was monopolized in any event, the domestic country is made better off since consumer surplus is unaffected and rents have been shifted towards the domestic country.

This analysis can easily be extended to include the case of a nonzero quota. Moreover, in that case, not only does competition increase abroad, but it increases in the domestic country as well. Consider the case where the firms compete with outputs as their strategic variables if entry occurs. Suppose further that the domestic potential entrant will find entry worthwhile if it earns its competitive revenues abroad plus some amount, K, of revenues in the domestic market, where K is greater than its domestic Cournot revenues and less than the domestic monopoly revenues. Now let the domestic government impose a quota which is such that the domestic firm earns revenues of K when it produces its best-response to the quota. In that case the potential entrant is willing to enter and does so. Once it has entered the firms produce their Cournot outputs in the foreign market, and in the domestic market the foreign firm sells its quota while the domestic firm sells its best-response to that. Competition increases in both countries.16
The Dixit and Kyle (1985) insight is that the domestic country may have to protect its domestic firm in order to encourage it to enter. However, what the above analysis points out, is that the domestic firm may not need complete insulation in its own market in which case domestic welfare can be improved even further by allowing some foreign competition.

In contrast to our analysis in Section II, this result holds whether the instrument used by the domestic country is a quota or a tariff. All that is required is that the domestic government increase the domestic profits of the domestic potential entrant to the point where entry is attractive. Since a tariff raises the costs of the foreign firm, they make it "less aggressive" in the domestic market which in turn makes that market more attractive to the potential entrant.

Thus when considerations of implicit collusion or of potential entry are important, quotas can lead to greater competitiveness.
In this appendix we consider the one-shot game in which prices are the strategic variables and in which one of the firms is capacity constrained. We establish the claims made in the text, namely that:

(i) The highest price charged by both firms is:
\[ s = \frac{a + c - \epsilon(a-c)}{2} \]

(ii) The lowest price charged is:
\[ \sigma = \frac{(a+c)}{2} - (a-c)(2\epsilon-\epsilon^2)^{1/2}/2 \]
and this lowest price is charged by both firms with probability zero

(iii) In equilibrium, the foreign firm earns \( \epsilon(\sigma-c)(a-c)/b \).

We discuss each of these in turn:

(i) The highest price charged by both firms is \( s \)

First note that, given that the foreign firm can only sell less than the domestic firm, it will never choose to charge a price higher than the highest price charged by the domestic firm. Hence the domestic firm knows that when it charges its highest price, it will be undercut with probability one. But the price that maximizes its profits when it is undercut for sure is \( s \), which is therefore the highest price charged by the domestic firm.

(ii) The lowest price charged is \( \sigma = \frac{(a+c)}{2} - (a-c)(2\epsilon-\epsilon^2)^{1/2}/2 \) and (iii) In equilibrium, the foreign firm earns \( \epsilon(\sigma-c)(a-c)/b \).

Consider the lowest price \( \sigma \) which the domestic firm would be willing to charge if it could thereby be sure to capture the entire market (i.e. undercut the foreign firm). This price \( \sigma \) has a number of interesting properties. First, it cannot be the case in an equilibrium that the foreign firm always charges a price strictly above \( \sigma \). If it did so the domestic firm would always make more than its profits at \( s \) by charging a price between \( \sigma \) and
the lowest price charged by the foreign firm, which contradicts the fact that \( s \) is its highest price. Secondly, if \( \sigma \) is the lowest price charged by the domestic firm, it cannot be charged with positive probability since that would mean that the foreign firm would benefit from undercutting it with positive probability. This would imply that the domestic firm couldn't expect to capture all of demand at \( \sigma \) so that it would be unwilling to charge \( \sigma \). On the other hand unless \( \sigma \) is the lowest price charged by the domestic firm, the foreign firm (who seeks only to undercut the domestic firm) would never charge a price of \( \sigma \). So \( \sigma \) must be lowest price charged by the domestic firm who charges it with zero probability.

Since the foreign firm must make equal expected profits at all the prices it charges in equilibrium we can compute its expected profits simply by computing its profits when it charges \( \sigma \). When it does this it is assured of being able to sell its entire quota so these profits equal \( \varepsilon (\sigma - c)(a - c)/b \).

If the domestic firm is sure to sell the entire demand given by (1), its profits from charging a price \( p \) are given by:

\[
(p - c)(a - p)/b = (a - c)^2/4b - [p - (a + c)/2]^2/b
\]

which must equal \( (a - c)^2(1 - \varepsilon)^2/4b \) at a price of \( \sigma \). So \( \sigma \) equals\(^{17}\):

\[
\sigma = \frac{a + c}{2} - (a - c)(2\varepsilon - \varepsilon^2)^{1/2}/2.
\]
FOOTNOTES

1 This is proven by Mookerjee and Ray (1986) and Bernheim and Whinston (1986).
2 The notion of segmented markets is discussed, for the case of static imperfectly competitive models, in Helpman (1982).
3 For the specific demand and cost functions used here it is presented, for instance, in Rotemberg and Saloner (1986).
4 The first of these assumptions is not restrictive since, with constant demand, if the firms can sustain any collusive outcome, they can also sustain the monopoly outcome. The second assumption is the division of the spoils that makes it easiest to collude.
5 Each individual firm is able to resist a larger temptation to deviate if the punishment is large. Thus collusion is easier the larger is the punishment. This is what leads Abreu (1982) to focus on the maximal possible punishments.
6 The ability of tariffs to shift rents from foreign to domestic firms is considered in a static model by Brander and Spencer (1985).
7 In other words, following a deviation, the owner of the deviating firm signs a contract to pass ownership of the firm to a third party for a nominal fee (which can even be less than the scrap value of the plant). The contract specifies that the current owner will receive the profits that accrue until the date at which ownership passes. Then the current owner and the rival are involved in a finite-horizon game which has as its unique equilibrium the outcome from the one-shot game.
8 Such signalling might be credible, for example, if there are some types of managers who are "old-fashioned" in the sense that they have never conceived of maximal punishments, and who therefore revert to the one-shot Nash equilibrium.
in any reversionary period.

9 This result does not depend on the use of one-shot Nash punishments and can be derived also with maximal punishments. The reason for this is that, as mentioned above, the domestic firm can be sure to earn at least \((a-c)^2(1-\varepsilon)^2/4b(1-\delta)\) at any equilibrium. To make the foreign firm earn less than it does at the one-shot Nash equilibrium for at least one period it must charge a price \(v\) which is below \(c\). It must then be compensated in later periods for taking this loss. To obtain a lower bound on this price \(v\) we assume that, after taking this loss, the domestic firm earns the entire monopoly profits \((a-c)^2/4b\). Then \(v\) must equal at least \((a+c)/2 - [(a-c)/b][\varepsilon/2 + [\delta(2\varepsilon-\varepsilon^2)/(1-\delta)]^{1/2}]\) so that, to first order the foreign firm earns \(\varepsilon(a+c)/2\) even when it is being punished.

10 Footnote 7 establishes that for \(\varepsilon\) small, \(u\) must be essentially equal to \(\varepsilon\) for the foreign firm not to cheat even with maximal punishments. Since (5) is valid in this case as well, monopoly is unsustainable for small \(\varepsilon\) even with maximal punishments.

11 This conclusion, which is also derived by Mookerjee and Ray (1986) for a slightly different game, is somewhat striking since the domestic firm makes positive profits if there is static competition by simply charging a price just less than the foreign firm's marginal cost inclusive of tariff. The equilibrium that yields zero profits for the domestic firm is constructed as follows: In the first period the domestic firm charges a price, \(u\), so low that its losses (in that period) are exactly equal to the present value of the profits from the one-shot game from the second period on. The foreign firm charges a price just slightly higher than \(u\). The foreign firm charges this price every period until the domestic firm charges \(u\) for one period; thereafter both firms revert to the one-shot outcome each period.
12 For evidence on this fact and some alternative explanations see Deardorff (1986).

13 This hinges critically on the fact that we do not use Abreu's (1982) severe punishments. These are however difficult to characterize in this case.

14 This inequality can be used to demonstrate that the foreign firm always sells the entire quota when it deviates. When the firms are monopolizing the market, the domestic firm is selling \((a-c)(1/2-\mu)/b\) so that the best response to this amount is \((a-c)(1/2+\mu)/2b\). To show that this latter amount is larger than the quota it suffices to show that \(\epsilon\) is less than \((1/4+\mu/2)\). Suppose it were greater. Then by this inequality it would also have to be greater than \(1/4\) plus half the RHS of the inequality. This means that \(\epsilon\) must exceed \(1/2\) minus an expression whose maximum, reached when \(\epsilon\) is \(1/3\) and \(\delta\) is 1, equals \(1/18\). This is impossible if \(\epsilon\) is less than \(1/3\).

15 Bernheim and Whinston (1986) consider the more general case of imperfect correlation. They show that the sustainability of collusion is a monotonically decreasing function of the degree of correlation across markets, so that we are considering the extreme that makes collusion most sustainable.

16 This result is in contrast to that of Brander and Spencer (1981). They consider a model in which the incumbent is committed to selling its pre-entry output once entry occurs. In that setting, the incumbent can deter entry by producing a large enough output. Imposing a tariff on the incumbent may lead it to prefer to produce a lower output even though that will result in entry. However, since the incumbent still has the option of deterring entry, if it chooses instead to allow entry it must be the case that it receives a higher price when it does so. So, in their analysis, any tariff big enough to induce the domestic firm to enter necessarily raises the domestic price above what it
would have been if the domestic firm was deterred from entering. This result, however, depends critically on the absence of perfecteness of their equilibrium. Since there is no logical link between the incumbent's pre-entry output and its post-entry output, the incumbent should be expected to respond optimally to the entrant's post-entry output. This leads to the Cournot outcome in both markets post-entry. In that case the above analysis applies.

17 Note that this expression is strictly declining in $\varepsilon$ as are the expressions for $s$ (the highest price charged by the domestic firm) and $(a-c)^2(1-\varepsilon)^2/b$, the expected profits of the domestic firm. So our analysis of competition is consistent with that of Krishna (1985). She analyzes a duopoly producing differentiated products. She shows that, in the single shot static game, reducing a quota raises domestic prices as the domestic firm need not be so concerned with foreign competition. Our analysis shows that her result extends to the case in which the duopoly produces a homogeneous good.
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