THE RESOLUTION OF INVESTMENT UNCERTAINTY THROUGH TIME*

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ABSTRACT

Investments in operating assets with identical expected discounted return and identical risk characteristics (i.e., variances and higher moments) when measured at the outset may have significantly different patterns of uncertainty resolution over their lives. The concept of uncertainty resolution, although ambiguous, is a potentially important characteristic of an investment alternative. This paper explores the usefulness and limitations of the concept of uncertainty resolution in the evaluation of both single risky investments and in portfolios of risky investments. In cases where future investment opportunities are completely known the concept does not seem useful; however, in a more realistic setting where future investment alternatives are ill-defined at present, the concept may prove useful. Further research is needed to explore fully the questions raised here.

I. INTRODUCTION

This paper considers the resolution of uncertainty through time of a risky investment and a set of risky investments. By the term uncertainty resolution we refer to a specified probability tree involving stated probabilities of cash flows at different points in time over the life of the investment. If two mutually-exclusive investments in real assets have probability distributions of discounted returns with identical expected discounted returns, identical variances, and identical higher moments, it is reasonable to presume that the pattern of uncertainty resolution over the life of each investment will affect the choice between the investments. This paper will explore ways of dealing with this characteristic of investments in real assets under alternative assumptions regarding knowledge of future investment opportunities. The coverage presented here is not full and complete; a major goal has been to raise issues and stimulate additional research.

The potential importance of the problem being discussed can be illustrated as follows. Consider two investments, both with the same expected net present value and the same risk characteristics at the outset (all moments of the probability distribution of the present values of the potential outcomes are identical). Assume that in both investments, one of the outcomes in year 10 involves a positive cash flow of $1,000,000. With the first investment the outcome of year 10 is learned immediately, while with the second investment the corresponding outcome is learned ten years from now. These two investments will have different values to most individuals.

Previous Research

There has been relatively little work done on the problem of analyzing the resolution of uncertainty over time either for a single investment or a portfolio of investments. Many models of the capital budgeting problem have assumed that cash flows from various investment projects in future time periods are random variables which are independent of previous realizations of associated random variables in earlier time periods. In this situation the problem of uncertainty resolution, although still present, is in a sense a simpler problem than in the general case. Some exceptions to the assumption of independence of intertemporal cash flows are the work by Byrne, Charnes, Cooper and Kortanek ([1] and [2]), Hillier [5], Naslund [6], Salazar and Sen [9], and Wilson [12], all of which allow for dependent relationships between random variables representing cash flows in different periods of time. However, for reasons to be stated subsequently, none of these models focuses explicitly on the resolution of uncertainty over time.[1] A paper by Hausman [3] relating to forecast revisions provides a framework for the specification of uncertainty resolution through time, but that framework involves a very special case in which revised forecasts of a single unknown quantity become available as time passes; it would appear difficult to adapt the typical capital budgeting problem to that special case.

Robichek and Myers [8] deal in part with the question of " ... the manner in which uncertainty is expected to be resolved over time"[2] for a firm, from the viewpoint of equilibrium in financial markets. These authors, apparently the first to coin the term "resolution of uncertainty," discuss the potential importance of the concept for an investor choosing a portfolio among securities which have different uncertainty-resolution patterns. The current

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1 References [1] and [2], however, discuss a variety of methods to include multi-dimensional aspects of risk in their models.

2 See reference [8], pp. 224-227.

paper follows the spirit of Robichek and Myers, exploring the potential importance and use of the concept "resolution of uncertainty" from the viewpoint of internal capital budgeting of a firm.

There are two recent papers which consider uncertainty resolution in capital budgeting problems. Van Horne [10] proposes what he terms an approximate method for analyzing how uncertainty is resolved over time in the case of a new product; we will subsequently present an analysis of his suggested methodology. Separately, Weingartner [11] argues that one virtue of the payback method of capital budgeting analysis is that it is a measure (albeit a very crude one) of the rate at which uncertainty is expected to be resolved. He emphasizes that knowledge of the expected pattern of uncertainty resolution gives managers some foresight in determining when they may be required to commit additional resources, alter managerial supervision, or take advantage of other investment opportunities which present themselves, ([11], p. 605). Weingartner cites these reasons for the continuing use of the payback concept by businessmen as one of several measures in the face of numerous articles pointing out its theoretical and practical weaknesses.[3] He does not argue that one should use the payback method for decision-making purposes; instead, he concludes that "... the problems which managers seek to attack by its use will not disappear simply by arguing that payback is not meaningful. Rather, it is necessary to face up to these problems, and to employ methods which solve them."[4] It is in this spirit that we now present an analysis of Van Horne's method of dealing with uncertainty resolution for a single investment.

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3 See Hertz [4] for a discussion of the inferiority of the payback criterion in analyzing a portfolio of investments.

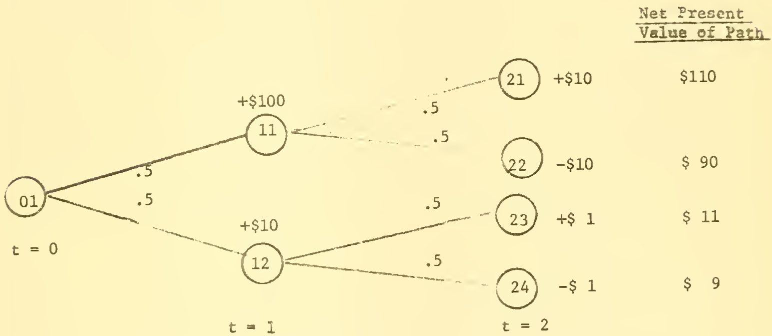
4 See reference [11], p. 606.

II. UNCERTAINTY RESOLUTION--AN EXAMPLE

We first demonstrate what is meant by the resolution of uncertainty with a simple numerical example (see Figure 1), in which all cash flows occurring at nodes have already been discounted back to time t = 0 using a default-free interest rate:

Figure 1

Probability Tree--Example 1



In the example, between periods 0 and 1 there is a .50 probability of moving to node (11) and obtaining a cash inflow whose present value is \$100, versus a .50 probability of moving to node (12) and obtaining a cash inflow whose present value is only \$10. Furthermore, in each case the probabilities and cash flows associated with period 2 are as stated in Figure 1. The expected net present value E[NPV] of the entire investment when viewed at time t = 0 is E[NPV] = \$55. Moreover, the "initial" variability of the investment at time t = 0 can be measured by the variance of discounted total path values (see Figure 1) about the mean: Var(NPV) = 2,075.5.

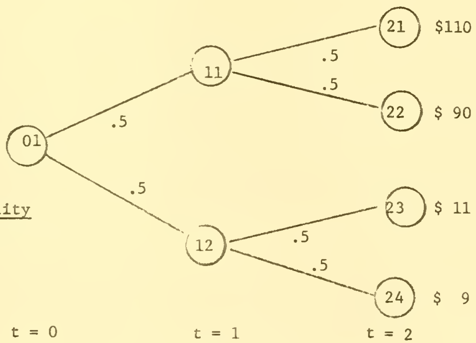
When period t = 1 occurs, either node (11) or node (12) will have been reached, each with probability .50, and some of the initial uncertainty concerning the discounted return from the investment will be resolved. Specifically, we will have learned (by time t = 1) whether the \$100 associated with node (11) will be obtained or not; and considering all the possible cash flows in Figure 1, most people would agree that the major uncertainty involves the \$100 cash flow.

Figure 1 can be transformed into a probability tree with positive discounted cash flows occurring only at the branch tips in the final period (see Figure 2).

Figure 2

Alternative Probability

Tree--Example 1



The actual resolution of uncertainty is identical in the two probability trees shown in Figures 1 and 2. With the investment illustrated in Figure 1, we know that if node (11) is reached, a minimum of $90 will be received; and with the investment shown in Figure 2, we know exactly the same thing.

Van Horne's Measure of Uncertainty Resolution

Van Horne has proposed the following as a measure of the multi-dimensional concept of uncertainty resolution[5]: $CV_t = S_t/E[NPV]$, where S_t is the square root of the weighted average of variance (about the conditional mean) of the various branches of the probability tree at period t. Thus the terms CV_t represent an "average" coefficient of variation.

In our example, Van Horne's measure would be computed as follows:

For $t = 0$, $S_0^2 = \sigma_{01}^2 = 2075.5$ (the total variance at time $t = 0$),

so that

$$CV_0 = S_0/E[NPV] = \sqrt{2075.5}/55 = 0.829.$$

For $t = 1$, first compute

$$\sigma_{11}^2 = 100 \text{ and } \sigma_{12}^2 = 1;$$

then

$$S_1^2 = .5(\sigma_{11}^2) + .5(\sigma_{12}^2) = .5(100) + .5(1) = 50.5$$

so that

$$CV_1 = S_1/E[NPV] = \sqrt{50.5}/55 = 0.129.$$

Finally, for $t = 2$ there is no remaining uncertainty, so $S_2 = 0$ and therefore $CV_2 = 0$.

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5 For details of Van Horne's procedure see [10], pp. 378-379.

Van Horne suggests that "...we can approximate the expected resolution of uncertainty..simply by plotting the CV_t over time and studying the pattern of relationship."[6] This is done in Figure 3 for our numerical example.

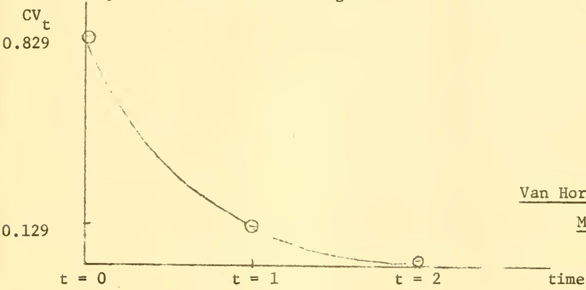

Figure 3

Van Horne's Uncertainty Resolution Measure--Example 1

As stated previously, the major uncertainty concerning the $100 discounted cash flow is resolved by period t = 1; and Figure 3 reflects this resolution. Thus in this example, Van Horne's measure appropriately reflects the resolution of uncertainty.

III. A CRITIQUE OF VAN HORNE'S MEASURE OF UNCERTAINTY RESOLUTION FOR A SINGLE INVESTMENT

We have stated that Van Horne's methodology is equivalent to dividing the square root of the weighted average of possible variances (about the relevant conditional means) at any time period t by the total expected discounted value of the investment. The question remains: Is this an adequate measure of uncertainty resolution for a single investment? In order to approach this problem, consider the probability tree in Figure 4, in which E[NPV] = $15:

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6 [10], p. 380.

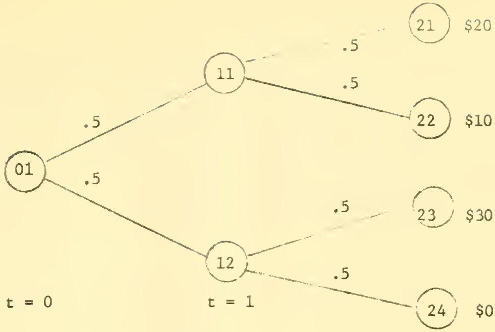

Figure 4

Probability Tree--Example 2

Computations obtained from application of Van Horne's method would produce $CV_0 = \sqrt{125}/15$ and $CV_1 = \sqrt{125}/15$, indicating that there has been no resolution of uncertainty at time period t = 1 as compared to time period t = 0. However, a study of Figure 4 reveals that some knowledge will have been learned as of time period t = 1; namely, whether one faces the less risky lottery[7] involving $20 and $10 or the more risky lottery involving $30 and $0. This information is of value since knowing which node we have attained changes the amount of uncertainty remaining, and may allow the firm to take improved actions in the face of the impending events.

In the example being described the amount of uncertainty remaining may be either increased or decreased, depending on which node is reached. If node (12) is reached, the variance being faced has a value of 225, while if node (11) is reached the remaining variance is only 25. Contrast these variances with the initial total variance of 125, and we see that the knowledge which we will have gained by t=1 is that either the remaining risk is "small" or that it is even larger than the so-calle initial risk! On the average the variance is still 125, but at time 1 we will know which node is attained so the average no longer applies.

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[7] As an extreme case the "lottery" from node (11) onward could be $15 with probability one.

Considering the investment of Figure 4, the fact that Van Horne's analysis fails to note any change in uncertainty from time 0 to time 1 is a deficiency. This deficiency will occur whenever the weighted average of the node variances at a period in time equals the weighted average of the node variances of the preceding time period. (This occurs whenever the nodes have identical conditional expected returns.)

Instead of trying to collapse the multi-dimensional concept of uncertainty resolution into a single vector CV_t, $t=1,\ldots,N$, one might simply list[8] the relevant conditional means and conditional variances which may occur, together with their probabilities of occurrence, for each time period. The difficulty here is that higher moments of the distribution of outcomes may be relevant in determining the utility of the investment (see [7]). Consider Figure 5.

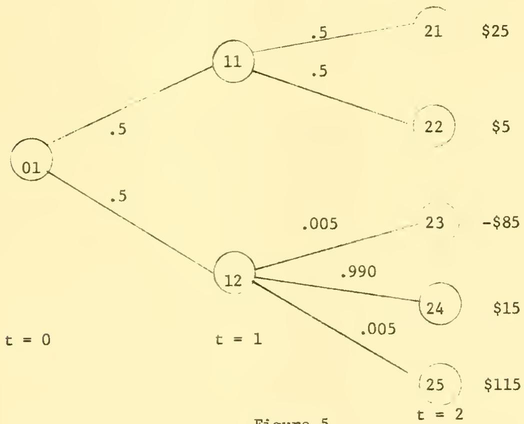

Figure 5

Probability Tree[9]--Example 3

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[8] The entire conditional probability distribution supplies more relevant information but is even less of a summary.

[9] K.O. Kortanek is to be credited with a variant of Figure 5 above.

Here the conditional means and variances at nodes(11) and (12) are both $15 and 100 respectively; yet node(12) exhibits a much wider range of possible outcomes than does node(11). Thus the conditional means and variances do not fully describe the data-generating process of Figure 5, and higher moments (or, as stated previously, the entire probability distribution) need to be included for full information.

We conclude that for a single investment the Van Horne measure of uncertainty resolution does not provide complete information. However, since most capital budgeting problems involve portfolios of investments, and since most of those problems involve continuing sequential decisions concerning investment opportunities which arise in the future, two questions can be raised: Is the concept of uncertainty resolution important in the portfolio-of-investments case, and does Van Horne's method of assessing it have possible use in that situation? We will address ourselves to each of these questions in turn.

IV. RESOLUTION OF UNCERTAINTY IN THE PORTFOLIO INVESTMENT CASE

Van Horne [10] and Byrne et al. [1] (among many others) have extended their frameworks to include the portfolio approach to capital budgeting. These two works have relevance for the concept of portfolio uncertainty resolution. Van Horne deals directly with this concept and Byrne et al present a portfolio-type payback requirement which, as stated previously, may be viewed as a crude measure of the rate at which uncertainty is resolved over time for a collection of investments in real assets.

Van Horne's approach is an extension of his methodology for a single investment. He suggests computing the expected standard deviation remaining (about the conditional mean), taking into account covariances by means of the expected

correlation between different investments.[10] This approach, while properly including covariances, has the same deficiency noted earlier by the example in Figure 4. The Byrne et al approach involves a chance-constrained model; specifically, there is a probabilistic constraint on the portfolio payback. This approach deals with uncertainty resolution only in a limited sense, and will not be pursued further here.

We now distinguish between two different assumptions concerning knowledge about future potential investments. In the first case we assume that all future (as well as present) investment opportunities are known[11] at the present time. This type of assumption implicity characterizes the work previously cited (see [1], [2], [5], [6], [9] and [12]). Under this situation one has full knowledge of his future as well as present investment alternatives for each set of possible events, and may conceptually imbed them in a common probabilistic sequential framework to achieve an integrated solution. In such circumstances it seems neither useful nor necessary to have a measure of uncertainty resolution for each individual investment. If one wishes to maintain, as Van Horne indicates, a particular type of risk posture over time (in expected value - variance terms),[12] one could, under the stated assumptions, develop the portfolio resolution of uncertainty directly from the raw data associated with the portfolio. Thus we conclude that in cases in which all future investment opportunities are known, a measure of uncertainty resolution is not needed for individual investments. If desired, it can be constructed for any portfolio of investments.[13]

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10 [10], p. 381.

11 The investments are still risky; however, at time t = 0 we have full knowledge of all the (risky) investment opportunities available to us both now and in the future. Certain characteristics of future investment alternatives may be dependent on sets of future events.

12 As suggested by Van Horne [9], pp. 384,385. See footnote 14 for a comment on the appropriateness of this goal.

13 A referee has pointed out that in practice, manipulating measures of uncertainty resolution for individual investments may be a practical way to estimate portfolio uncertainty resolution.

Now consider the alternative situation in which future investment opportunities are not known with certainty. The most well-defined version of this case would involve postulating some data-generating process for future investment opportunities, taking proper account of relevant dependencies among those investments as well as those currently being considered. Very little work has been done on such models, for the reason that to be at all realistic such models would necessarily be quite complex and unwieldy. Retreating from this type of model, we consider the qualitative, vague and ill-defined case which fits the situation most businessmen face: the occurrence, timing, and characteristics of future investment opportunities are difficult to predict. Under these circumstances, one cannot build a rigorous model of this situation when the decision-maker is not even willing to (or feels he is not able to) state probabilistically his future investment alternatives. This is where the concept of uncertainty resolution may be of some use. In such a case the concept of portfolio uncertainty resolution can indicate how uncertainty due to the current portfolio of investments will be resolved, at least in an approximate sense, at various points in time. Such information could be of some use to a manager who has a basic feeling for the general manner in which new investment opportunities may occur for his particular company as time passes. Even though he is not willing to state precisely the probabilistic mechanism for generation of investment opportunities over time, it seems reasonable to presume that he has some information about this process and can therefore use portfolio uncertainty resolution information in his decision-making.

As an example, suppose that two alternative portfolios of current investment opportunities generate Van Horne-type measures of uncertainty resolution as shown in Figure 6.

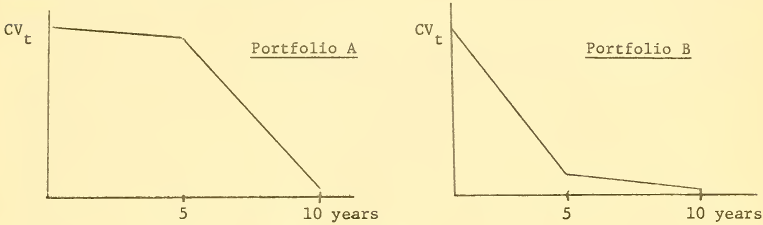

Figure 6

Both of the portfolios in Figure 6 contain the same initial amount of uncertainty, but portfolio B resolves most of the uncertainty by five years, whereas this is not true of portfolio A. Recognizing that the future portfolio variance is a function of both the variance of each investment and the covariances between each pair of investments, if the covariances are held constant, then portfolio A would not allow for much acceptance of future, risky investment alternatives until years 6 through 10, whereas portfolio B allows for acceptance of some risky investments (adding net risk to the total portfolio) from year 1 onward.

Maintenance of A Given Risk Posture

If the firm's entire set of investments, while just risky enough at time t=0, had complete uncertainty resolution at time t=1, then from that point on, if nothing else were done, the firm would be theoretically riskless, and would be drastically altered from its initial risk posture. At the other extreme, if each investment of the firm, while bringing in risky returns from year to year, is never resolved but continues to maintain "identical" risk, then the firm

precisely maintains its risk posture over time and may be said to be in a type of equilibrium. Such a firm would only accept new investment opportunities which could be appended to the existing portfolio without a significant change in the relative risk posture.

More realistically, most investment opportunities may typically be characterized by some decline in uncertainty over their lives. Under these circumstances, if the firm wishes to maintain a given expected value - variance risk posture it must continually seek and accept new investment opportunities which, when grafted on to the existing portfolio (with its declining uncertainty), approximately maintain the desired risk posture. For financial market stability it may be reasonable for a firm to seek an approximately constant risk posture[14] over time, one which is obtained by adding new, initially risky investments to an existing portfolio which exhibits declining risk over time.

The maintenance of an approximately constant risk posture need not involve (and realistically, cannot involve) a constant level of uncertainty over time from the existing portfolio of investments. This is why it is important to have some knowledge of the types of investment opportunities which will arise in the future. If, for some reason, the opportunities we expect to arise are "less" risky (after taking covariances into account), then it may behoove us to select a current portfolio in which expected net present value is somewhat larger and uncertainty persists longer than usual. On the other hand, if a series of "more" risky investment opportunities are expected to occur, then we may wish to select a current portfolio with a lower expected net present value but a more rapid resolution of uncertainty.

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[14] The blind adherence to a constant risk posture may be an inappropriate goal for the firm; the desired risk posture may be viewed as a flexible constraint.

In the ill-defined situation being considered, the concept of portfolio uncertainty resolution seems to have some value. It can be used, as indicated above, to allow the firm to approximately maintain its desired risk complexion over time if the firm is willing to make some guesses about the relative net riskiness (and the pattern of uncertainty resolution) of investment opportunities which will arise in the future.

Stability of Net Present Value

In discussing these points, Van Horne states that "... If the uncertainty ... is expected to be resolved very quickly, and the firm periodically takes on new products that result in similar patterns of uncertainty resolution, the net-present value of the firm is likely to fluctuate considerably over time ... If the uncertainty resolution pattern of the firm were more spread out, the firm would have a greater opportunity to balance the risk of its various products so as to stabilize the trend of net-present value over time."[15] We question this conclusion as follows: If the actual portfolio risk posture (expected value - variance tradeoff) is the same from one year to the next, and is expected to be the same out into the future, it makes no difference whether this phenomenon is caused by a set of investments which never resolve their uncertainty (although they do give rise to cash flows from one time period to the next) or by a set of "new" one-year investments each year, each of which fully resolve their uncertainty each year. In either case the prospective purchaser of a share of stock in the firm would face a given expected value - variance tradeoff and would justifiably presume that the same tradeoff would persist in the future. If the stockholder's well-being is affected only by the stated risk posture, the two situations described above should be valued equally by him.

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15 [10], p. 385.

We conclude this section by emphasizing that the stability or lack thereof of net-present value is a function of the variance of the portfolio of investments. As long as the firm plans to maintain an approximately constant risk posture over time, the particular combination of declining uncertainty from the current portfolio and the uncertainty addition from future investment opportunities at future points in time is not important (as long as such balancing is feasible).

V. CONCLUSIONS AND FURTHER RESEARCH

Van Horne and others [10,8,11,1,2] have emphasized the potential importance of the resolution of uncertainty through time. The Van Horne measure of uncertainty resolution for a single investment has deficiencies, and in situations in which all future investment opportunities are known, a measure of uncertainty resolution for individual investments is not needed for systematic analysis of the portfolio investment problem. However, for situations in which future investment opportunities cannot be precisely known, the concept and Van Horne's suggested measure for portfolio resolution of uncertainty do seem to have some usefulness in aiding the firm in its attempts to maintain a given risk posture. Firms may try to maintain an approximately constant risk posture over time; however, the mixture of "old" and "new" investments required to produce a particular level of variability does not seem crucial.

More questions have been asked than answered here. Normative analysis of the concept of uncertainty resolution, involving intertemporal tradeoff of risk, requires a model or framework containing a data-generating process for future investment opportunities. One thing is clear: Much further research is needed before uncertainty resolution can be incorporated in a systematic manner into a framework for investment decisions.

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