WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

SOME COMMENTS CONCERNING
MUTUAL FUND VS. RANDOM PORTFOLIO PERFORMANCE

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In the current issue of this Journal,1 Friend and Vickers have criticized several points of the analysis of our paper on portfolio selection which was contained in an earlier issue of this Journal.2

This paper developed two multi-index models for portfolio selection and compared their ex ante and ex post performance with other portfolio selection models as well as with the ex post performance of randomly generated portfolios and the actual performance of some mutual funds primarily invested in common stocks. As a result of our analysis we raised two issues in regard to a previous study of mutual fund performance by Friend and Vickers.3 These criticisms have been labeled by Friend and Vickers as "incorrect in

*The authors wish to thank Professors Richard Bower and Edwin Kuh for helpful comments and suggestions.

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view of certain deficiencies"1 in our analysis and "gratuitous"2 in view of the irrelevance of our analysis to the points in their paper which we criticize.

Our present paper is organized into four parts. In the first section, we will consider the specific criticisms by Friend and Vickers of our earlier paper and indicate why we feel they are largely incorrect. In the second section, we shall restate our initial criticisms in somewhat more detail than was contained in the original footnote of our previous paper3 and indicate why we still feel they are relevant to the conclusions drawn by Friend and Vickers. Finally, in the third section we will acknowledge the "two caveats"4 and attempt to evaluate the significance of their conclusions about mutual fund performance in terms of them and certain other serious deficiencies in their analysis. The brief concluding section of the paper indicates our views that both our earlier paper and the papers of Friend and Vickers are largely irrelevant to the empirical issue of the quality and value of mutual fund management.

PART I: REPLY

It should be kept in mind that the random portfolios and mutual funds were considered in our earlier paper primarily to provide a basis for comparison with the ex post performance of the ex ante efficient portfolios.

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3Cohen and Pogue, op. cit., footnote 16, p. 188.
It was not our stated or implied purpose to provide a definitive test of the performance of mutual funds versus random portfolios. However, this does not preclude our observations as being used as contradictory evidence to the rather sweeping claims made in the earlier Friend and Vickers paper, as we shall discuss in Part II of this paper.

Friend and Vickers have challenged our use of the term "dominance." On the basis of Figures 4 and 5 of our earlier paper we made the following observation relating to the relative performance of the mutual fund and random portfolio groups: "The mutual funds, however, are not dominated ex post by either random portfolios or the model selected portfolios, but tend to be more conservative, accepting less risk and a lower return."¹

As Friend and Vickers have noted, the ex post performance of the group of 60 random portfolios and 78 mutual funds tend to have a 'first-third quadrant relationship.' This relationship, however, is not inconsistent with our statement that the observed mutual fund performance was not dominated by the ex post performance of the random portfolios.

While realizing that we stand in danger of belaboring some rather elementary concepts, we have included in this present paper Figures 1 and 2 to illustrate our arguments. Figure 1 illustrates a possible ex ante opportunity set of portfolios. The efficient frontier is spanned by efficient portfolios 1 through 6.² A possible ex post performance pattern is

¹Cohen and Pogue, op. cit., p. 186.

²Only in the case of a perfect capital market where investors have unlimited borrowing capacity at a pure rate of interest would the efficient frontier be confined to the line extending through portfolios 1, 2, and 3. This obviously is not the case in a real world situation and we are unwilling to make this assumption.
shown in Figure 2. Figure 2 illustrates an ex post situation not dissimilar to that found in Figures 4 and 5 of our earlier paper. The ex post performance of the portfolios numbered 1 through 8 has tended to fall in two distinct clusters, A and B. The performance of cluster A is not dominated ex post by the performance of cluster B, since the portfolios in A generally provide a lower average return and a lower average risk than the portfolios in B. Thus, some investors, given that they base their preference for portfolios solely on return and standard deviation of return, would prefer portfolios in group A, while other investors would prefer portfolios in group B. Our usage of terminology in this regard is quite standard and elementary, and the Friend and Vickers criticism is incorrect.

The next Friend and Vickers criticism of our analysis is contained in the statement, "A rational investor's choice between them [the random portfolios and the mutual funds] can not be ordered solely on the basis of the statements made to this point. The investor's preference between low risk, low return portfolios or high risk, high return portfolios clearly depends on the form of his utility function defined over the relevant arguments, and on that question Cohen and Pogue have made no statement at all." This statement illustrates some lack of understanding of the

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1Cohen and Pogue op. cit., p. 185.
2The portfolios within each cluster, however, are by no means equally desirable ex post.
4Added by the present authors.
Figure 1

Possible Ex Ante Opportunity Set

Figure 2

Possible Ex Post Performance Pattern
utility concepts inherent in the assumption that the investor is risk averse and bases his choices among portfolios solely on their expected return and standard deviation. In effect, we have said a great deal about the form of the investors preference function, namely that it is quadratic.¹ The particular parameter values of the investors preference function completely determine his preference for risk-return combinations. Thus, the criticism of Friend and Vickers in this regard is also incorrect.²

Friend and Vickers have further suggested the use of a measure of relative dispersion (the "reward-to variability" ratio) as a measure of investors' preferences for portfolios. However, in their exuberance to assist our efforts they have perhaps overlooked some of the assumptions inherent in this measure if it is to be interpreted meaningfully. At best, the R/V ratio is an equivalent device for summarizing, in one dimension, the return-risk relationships that we presented as two-dimensional diagrams in our earlier paper. More seriously, however, the R/V measure in effect assumes the existence of a perfect capital market. However, under the assumption of an imperfect capital market³ the R/V ratio cannot be used to order a rational investor's preference for portfolios in either the ex ante or ex post frame of reference. This is clearly indicated in Figures 1 and 2.

¹For a proof of this statement refer to H. M. Markowitz, Portfolio Selection, Cowles Foundation Monograph 16, Wiley 1959, pp. 286-288.

²The above statement by Friend and Vickers indicates that they do not understand the utility theory implications behind the criterion used in their own Journal of Finance paper. "We shall, therefore, assume for the purposes of this paper that the objective of . . . investors . . . is to maximize average return, but only for a given risk." (Friend and Vickers, Journal of Finance, op. cit., p. 394) The latter was then defined as the standard deviation of return.

³Specifically, when borrowing becomes progressively more costly and price-volume related transactions costs of large investors are considered.
In the ex ante case, portfolio 7 has a higher R/V ratio than portfolio 6, but portfolio 7 would not be acceptable to a rational investor because it is not efficient. In the ex post case a similar discussion applies to portfolios 3 and 5.¹

We now consider the regression relationships of Figure 7 of our earlier paper. These regression lines are based on the data contained in Figures 4 and 5 of our earlier paper (and similar diagrams for the other portfolio selection models). Figure 7, however, as has been noted by Friend and Vickers, tends to compress the available information by not showing the dispersion of the source data points in relation to the individual regression lines. On the basis of these regression lines the mutual funds appear to dominate the random portfolios over a wide range of ex post returns (by virtue of the fact that the mutual fund risk-return line lies entirely "south-east" of those for the two random portfolio groups).² Considering Figures 4, 5, and 7 together the mutual funds tend to dominate the random portfolios below about 16 percent ex post return. Above 16 percent the situation is unclear due to the lack of comparable data points. In retrospect, however, the main observation we would like to carry away from the mutual fund vs. random portfolio comparisons (which resulted as a by-product of our main comparisons) is that the two groups provided different ex post performances. The ex post performance of the group of mutual funds con-

¹In fact, in order to hold the portfolio with the highest ex post R/V ratio in our example would have required an irrational ex ante decision (i.e. selecting a non-efficient portfolio).

²It should be noted, however, that the R/V ratio cannot be computed solely from the slopes of the regression lines in Figure 7 for obvious reasons. Namely, that they do not have a common intercept at the pure rate of interest.
sidered was not found to be dominated by the ex post performance of the random portfolios. This conclusion is, of course, conditioned by our experimental design.\(^1\) We have not previously, nor will we now, make any attempt to generalize our conclusion to the performance of mutual funds generally vs. random portfolios. However, we do feel that our results provide conflicting evidence to the rather broad conclusions made by Friend and Vickers, and we now proceed to relate our evidence to these conclusions.

**PART II: INITIAL CRITICISMS**

It was our opinion that we were *generous*, rather than "disingenuous," in not previously indicating our views on the so-called "two caveats" acknowledged by Friend and Vickers. In Part III of this paper we will consider these caveats and indicate why, in terms of these and other serious deficiencies in the Friend and Vickers analysis, their *Journal of Finance* paper has little relevance to the question of actual mutual fund vs. random portfolio performance.

The evidence that we obtained in our research regarding the relative performance of common stock mutual funds and random portfolios, while admittedly limited, was considered relevant to the issue for the following reason.

1. We considered the actual investment performance achieved by the mutual funds during the test period, rather than the returns achieved by pseudo-mutual fund portfolios which have little obvious connection to the actual portfolios (Part III of the present paper includes a more complete discussion of this point).

\(^1\)Which included the mutual funds considered, the method of selecting random portfolios, the test period, etc.
2. Our random portfolio returns were based on random portfolios of individual common stocks rather than random portfolios based on indexes of common stocks.

3. We attempted to avoid the artificial and questionable practice of segmenting the mutual fund portfolio into two parts, common stocks and defensive issues, and inferring from the former the performance of the whole. To circumvent this problem we attempted to select basically common stock mutual funds which held low proportions of debt issues.

Friend and Vickers have criticized our procedures on a number of counts, most of which we do not accept.

First, it is not clear that at any given time institutional investors select their portfolios from a universe composed of many thousands of securities. To the contrary, real mutual funds (apparently as apposed to pseudo-mutual funds) have significant information and decision-making costs which naturally tend to reduce the number of investment alternatives concurrently pursued to a subset\(^1\) which is far smaller than the universe of possible investment possibilities.

In our paper we selected our random portfolios from universes of 75 and 150 securities.\(^2\) It was observed (see Table in this paper) that the group performance mean \((\bar{u}, \bar{s})\) for the random portfolios tended to move further into the first quadrant in relation to the group mean for the 78 mutual funds as the universe size was increased.

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\(^1\)Over time, however, the investment universe considered will in most cases change significantly with respect to composition as individual securities are added or dropped from consideration.

\(^2\)The 75 security universe was a randomly chosen subset of the 150 security universe.
Table 1

EX POST PERFORMANCE GROUP MEANS

<table>
<thead>
<tr>
<th>Group</th>
<th>Average Return $\bar{u}$</th>
<th>Average $\bar{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>78 Mutual Funds</td>
<td>14.87</td>
<td>18.46</td>
</tr>
<tr>
<td>60 Random Port. (75 sec. universe)</td>
<td>15.38</td>
<td>21.90</td>
</tr>
<tr>
<td>60 Random Port. (150 sec. universe)</td>
<td>17.97</td>
<td>22.64</td>
</tr>
</tbody>
</table>

Now given the fact that distribution of the security yields tends to be positively skewed, and that we use telescoping universes (i.e. each security universe is a randomly chosen subset of the next largest universe), the average return $\bar{u}$ and average standard deviation $\bar{\sigma}$ for the group of random portfolios could both be expected to be non-decreasing as the number of securities in the investment universe was increased. If this is the case, then the performance of the 78 mutual funds during the test period would continue to be preferred by a certain subset of investors, even when the random portfolios are selected from universes equivalent in size to those of the mutual funds.

It is also unclear to us that our method of selecting random portfolios is inappropriate. Given that institutional investors tend to pre-select, either explicitly or implicitly, a security universe for detailed consideration from a total universe of all investment opportunities, then we consider our 'each security equally likely' criterion to be an appropriate
basis for a null test.¹

Friend and Vickers have also criticized the fact that our random portfolios were held unchanged during the test period, thus providing the mutual funds with an additional degree of freedom. The question might be raised as to the value of making random changes to random portfolios. However, another issue must be considered. If we wish to compare the performance of mutual funds against comparison portfolios, whether ex ante efficient or random, which are managed during the test period, whether optimally in the first case or randomly in the second, then we must take into consideration the transactions costs² involved in modifying these portfolios. The mutual funds in managing their portfolios are forced to trade off between expected gains to be made from moving to a more efficient portfolio with the costs involved in making the desired transition. In our opinion the performance of randomly managed random portfolios would suffer because of transactions costs to the extent that they would not be desirable as the basis for a null test.

Since the evidence from our earlier paper regarding the relative performance of random portfolios and mutual funds is admittedly fragmentary, and that from the Friend and Vickers Journal of Finance paper, in our opinion,³

¹There is no requirement for micro-consistency at the level of each mutual fund to produce macro-consistency at the level of the market. It might also be noted that our view that institutional investors pre-select a sub-universe would tend to support the aggregate consistency requirement, i.e., mutual funds tend to invest in companies with sufficient capitalization so that their trading actions will have minimal effect on market prices.

²For large institutional investors a significant component of the transactions cost is the price spread involved in buying or selling large quantities of stocks.

³Our detailed justification for this remark is contained in Part III of this paper.
virtually non-existent, we shall not pursue this discussion further.

We now come to the second comment that we originally made in regard to the Friend and Vickers *Journal of Finance* paper. It was focussed on a number of statements, which are characterized by the following excerpt from their conclusion. "This paper, in addition, points up the dangers of using past measures of return and variance as a basis for portfolio selection, or of assuming that the procedures for portfolio selection outlined by Markowitz provide any clues to future investment performance."¹ In our paper we stated that our evidence was contradictory to this strong conclusion.²

Friend and Vickers have stated that our criticism is without foundation, They further state "Some of the same points presented in connection with our discussion of their first criticism of our paper indicate why we do not find their analysis completely convincing with respect to the ex post performance of the ex ante efficient versus random portfolios."³

We reject both of these criticisms and consider the second to be absolutely unjustified. While the comparison of the random portfolios of the mutual funds was in effect a by-product analysis, our experimental design for evaluating the relative ex post performance of the ex ante efficient and random portfolios is completely appropriate. The random portfolios

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were drawn from the same size universe as the efficient portfolios and evaluated over the same ex post period.\(^1\) In addition our 'each security equally likely' criterion would appear to be an appropriate rule for selecting random portfolios, particularly in view of the fact that market composition was not a factor taken into consideration by the portfolios selection models. The results of our empirical tests indicated that the ex post performance of the efficient portfolios clearly dominates the performance of the random portfolios during the 1958 to 1964 test period (see Table 2 in this paper). Our conclusions, which are based on the above empirical evidence, can in no way be considered as without foundation.

Table 2

EX POST PERFORMANCE MEANS

<table>
<thead>
<tr>
<th>Security Universe</th>
<th>Efficient Portfolios</th>
<th>Random Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\bar{u})</td>
<td>(\bar{\sigma})</td>
</tr>
<tr>
<td>75</td>
<td>18.8</td>
<td>20.0</td>
</tr>
<tr>
<td>150</td>
<td>20.3</td>
<td>21.4</td>
</tr>
</tbody>
</table>

Friend and Vickers next state that our criticism completely misses the mark. They state a so-called "basic theoretical proposition" that they would not expect ex ante efficiency to be correlated highly with ex post performance. This opinion is then supported by a further opinion

\(^1\)The test with which we were concerned was related to the ability of the portfolio selection models to select portfolios which would outperform ex post naive selection rules (as represented by the random portfolios) applied to the same security universes.
contained in footnote 9 of their present paper. Unfortunately, Friend and Vickers have neglected to supply the results of their tests upon which these proclamations are based.

While we consider using unweighted averages of past yields and unweighted measures of past yield dispersions probably one of the more naive ways of going about predicting future yield and dispersion measures, even these measures appear to have some predictive powers when a statistically relevant test is performed. Fortunately, at this point we have some empirical results to consider. Pratt, in his doctoral thesis, tested the hypothesis that common stocks characterized by relatively high degrees of risk, as measured by instability of their rates of return in the past, have provided their holders with higher rates of return, on the average and over considerable periods of time, than have common stocks characterized by relatively lower degrees of risk, measured by instability of returns. A risk factor was computed for each common stock listed on the New York Stock Exchange for each of 372 "base dates," at monthly intervals from January 31, 1929 to December 31, 1959. At each of the 372 base dates the common stocks listed on the New York Stock Exchange were divided into portfolios based on the values of the risk factor. The return of each portfolio was then

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3The measure of risk used in his study is the standard deviation of the natural logarithms of the quarterly investment performance relatives for a series of quarters immediately preceding the point in time at which it is desired to measure the relative degree of risk associated with various stocks.
computed for one, three, five and seven years forward from each of the base dates. The findings indicated that during the 30 years covered by the study, the average return increased with increasing risk. The findings also supported the proposition that stocks whose returns had been unstable in the past tended to continue to be risky in the future. This evidence stands in disagreement with the basic proposition announced by Friend and Vickers.

However, we have slighted the efforts of Friend and Vickers to some extent. They have provided us with an illustration to support their proposition. We will now consider this illustration in some detail to expose a major fallacy in their analysis which will render a majority of their empirical results of questionable value.

Farrar\(^1\) found that the ex ante performance of certain mutual fund portfolios to be close to optimal (i.e. grouped near the efficient frontier). Friend and Vickers subsequently, in their *Journal of Finance* article, reported on the evaluation of mutual fund performance over subsequent test periods using holding periods of up to 6 years beginning in 1958, and in subsequent tests beginning in 1959. Based on their test results\(^2\) that indicated lower average return for the mutual funds but substantially higher risks\(^3\) in all cases, the following conclusion was reached, "The results of these further tests\(^4\) confirm, therefore, the general conclusions based on


\(^3\)Using a cross-sectional measure of yield dispersion.

\(^4\)Friend and Vickers, *Journal of Finance*, op. cit., Tables 4 and 5. (Footnote added by the present authors.)
the earlier time period\(^1\) analysis: namely, even if portfolio selection by mutual funds is close to optimal in the Farrar sense, i.e. on the basis of historical return and variance characteristics, there is no logical reason to believe that such a portfolio possesses any inherent qualities which will result in better than random performance over a period of time in the future.\(^2\)

It is, however, amazing to us that Friend and Vickers do not realize that their cross-sectional "risk" measures are effectively meaningless. The "risk" measures used in Tables 1 through 5\(^3\) do no more than illustrate the statistical "law of large numbers."

The random portfolios are in effect drawn from the same distribution and have the same a priori expected yield value in each period. The realized yields will differ only because of the inability to distribute a finite number of securities in the exact proportions of the relative dollar importance of the 27 index groups used. However, for a reasonably large number of securities in each random portfolio, the yield differences will tend to be small, and thus the cross-sectional dispersion will be small. In fact, the cross-sectional dispersion will approach zero and the number of issued in each portfolio becomes large.\(^4\)

This, however, is not the case for the mutual funds. The mutual funds

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\(^1\)Friend and Vickers, *Journal of Finance*, **op. cit.**, Tables 1, 2, and 3. (Footnote added by the present authors.)


\(^3\)And in other of their tables, as we shall discuss in Part III.

\(^4\)For example, if every dollar of each random portfolio was independently allocated.
represent basically dissimilar portfolios (or groups of portfolios). In this case the cross-sectional risk measure will tend to be large, accurately portraying the greater cross-sectional dispersion of mutual fund yields. This "risk"measure can be made larger (or smaller) by a judicious choice of mutual funds with dissimilar (or similar) investment strategies.

When the above fallacy is realized, many surprising results which were carefully rationalized away by Friend and Vickers are seen to be nothing more than statistical artifacts. For example, consider the following in terms of the law of large numbers. "A more troublesome result of the tests summarized in Table 1 is the significantly larger dispersion of the average return associated with investment in mutual funds as compared with the special type of random investment."1 Also--"Investment in small funds may frequently earn higher average return, but at a higher risk of return."2

On the basis of the above discussion we propose that the Friend and Vickers illustration is meaningless and the conclusions based upon it (and similar data) are vacuous.

Returning to the initial excerpt from the Friend and Vickers conclusion,3 we would propose that their criticism of Markowitz portfolio selection techniques is somewhat unsubstantiated.

PART III: "THE TWO CAVEATS"

When we consider the acknowledged caveats and certain other deficiencies in the Friend and Vickers analysis, the relevance of their Journal of

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Finance study to the issue of mutual fund performance must be seriously questioned.

We shall confine our discussion to the following four points.

1. The question of using portfolios of indexes as opposed to individual securities.

2. The question of dividing the portfolio and inferring from a part the performance of the whole.

3. The question of inferring mutual fund performance from holding a 'point in time' portfolio over a number of subsequent periods.

4. The relevancy of cross-sectional measures of "risk" and related law of large numbers problems.

(i) Indexes versus Individual Securities  The transformation from a portfolio of individual securities to a portfolio of indexes is non order preserving in either the yield or risk dimensions. The transformation tends to compress the differences between portfolios and possibly distort the relative performance picture for portfolios with differing investment objectives. This is particularly true when broad index classifications are used. ¹ Thus the pseudo mutual funds portfolios used by Friend and Vickers have no well defined relationship to actual mutual fund portfolios.

(ii) Portfolio Dissection and Performance  When a portfolio is composed of a significant percentage of defensive securities, the performance of the total fund can not in general be accurately inferred from the performance of the common stock segment alone. This is because, obviously, many

¹The ultimate absurdity is the use of one index. It would then be difficult to find any differences in portfolio performance.
of the important (small or negative) covariance relationships\(^1\) which reduce the risk of the total portfolio have not been considered.

(iii) **Holding Periods and Management Performance** Friend and Vickers have stated: "If it is desired to know how well portfolio managers are performing their jobs, it can be asked how wisely or how well they have selected the portfolios which they hold at a given time. But the test of their effectiveness in this regard would have to be based on how, over a given subsequent period of time, such portfolios actually performed. Considerable significance then attaches to comparing what in actual fact would have been the performance results of such portfolios, and what results could have been realized over the same time period by selecting portfolios by some other specified means, such as for example, a suitable designed random process.\(^2\)

It is, in our opinion, neither necessary nor desirable to evaluate mutual fund performance in the manner proposed by Friend and Vickers. Mutual fund management is a dynamic, ongoing process. A procedure which attempts to infer anything about mutual fund performance from the performance over several years of an arbitrarily selected 'point in time' portfolio is of questionable value (either theoretically or heuristically). Again, there is no well-defined connection between the performance of real mutual funds and the pseudo mutual fund portfolios considered by Friend and Vickers.

(iv) **Risk Measures and Related Problems** The cross-sectional "risk" meas-

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\(^1\)Relationships which may have been carefully considered by the mutual fund management.

ures used are, of course, dependent on the composition of the mutual fund group considered. We, in no way, see how these provide any relevant measure of the risk associated with a mutual fund. Relevant measures of risk must be intertemporal in nature.

A more serious difficulty with the cross-sectional "risk" measure was discussed in Part II of this paper. This problem presents some major difficulties for the Friend and Vickers analysis since almost all of their empirical tests use this procedure. In effect, the "risk" measures in Tables 1, 2, 3, 4, 5, 6, and 9 have no relevance to the rather strong conclusions based upon them.

It is our opinion that, when these limitations of the Friend and Vickers study are considered, it becomes quite clear that their paper has little to contribute to the question of mutual fund performance.

PART IV: CONCLUDING REMARKS

It is our opinion that the Friend and Vickers criticisms of our earlier analysis have resulted, for the most part, from what appears to be some lack of understanding of the fundamentals of ex ante optimal portfolio selection and evaluation. With regard to their Journal of Finance paper, we find it impossible, on the basis of some rather severe restrictions in their analysis (both acknowledged and otherwise), to accept any relevance of their results to the issue of mutual fund performance.

We would like explicitly to make the point that we take no position in

1Friend and Vickers, Journal of Finance: Tables 7 and 8 develop more relevant intertemporal risk measures for five and six year mutual fund and random portfolio holding periods.
the controversy regarding mutual fund versus random portfolio performance. Our experimental design was much too restricted to permit generalizations, on the basis of our fragmentary data, about the ability of mutual fund managers in general to perform better than random portfolios. Such conclusions require a much fuller commitment of resources than was evidenced in our by-product comparisons of mutual fund and random portfolio performance.

If future studies of mutual fund performance are to be meaningful, then it appears obvious to us that the actual investment performance of an appropriately selected test group of mutual funds must be used as the standard of performance for the efficiently generated and managed or randomly selected portfolios. This requirement results in no unsolvable computational problems when random portfolios are generated from large security universes. Computational difficulties arise, however, when we wish to compare actual mutual fund performance with efficiently selected and managed portfolios, due to the optimization requirements of this approach.¹
