SENESCENT INDUSTRY COLLAPSE REVISITED

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Abstract

One of the most robust empirical regularities in the political economy of trade is the persistence of protection. This paper explains persistent protection in terms of the interaction between industry adjustment, lobbying, and the political response. Faced with a trade shock, owners of industry-specific factors can undertake costly adjustment, or they can lobby politicians for trade protection and thereby mitigate the need for adjustment. The choice will depend on the returns from adjusting relative to lobbying. By introducing an explicit lobbying process, it can be shown that the level of tariffs is an increasing function of past tariffs. Since current adjustment diminishes future lobbying intensity, and protection reduces adjustment, current protection raises future protection. This simple lobbying feedback effect has an important dynamic resource allocation effect: declining industries contract more slowly over time and never fully adjust. In addition, the model makes clear that the type of collapse predicted by Cassing and Hillman (1986) is only possible under special conditions, such as a fixed cost to lobbying. The paper also considers the symmetric case of lobbying in growing industries.
1. INTRODUCTION

One of the most robust and least discussed empirical regularities in the political economy of trade is the persistence of protection. Empirical research by a number of authors suggests that past levels of protection are significant in explaining current protection levels in an industry. For instance, in their study of the pattern of protection that emerged in the U.S. following the Kennedy Round of the GATT, Marvel and Ray (1983) found that an industry was more successful in resisting liberalisation the higher was its level of protection preceding the liberalisation, after controlling for industry growth rates, industry concentration, comparative advantage, and buyer concentration. Similarly, in studies of both the Kennedy and Tokyo Rounds, Baldwin (1985) found that industries received more post-liberalisation protection the greater were pre-liberalisation levels of protection, even after controlling for labor force characteristics, growth rates, and import penetration ratios. In empirical tests explaining corporations' positions on 6 trade initiatives during the 1970s, Pugel and Walters (1985) found that the demand for protectionism was strongly increasing in the industry's initial tariff level, controlling for import penetration and variables reflecting exporting strength.

This paper offers an explanation for the persistence of protection in terms of the interaction between industry adjustment, lobbying, and protection. The story is simple, but the results are quite striking. Faced with a trade shock, owners of industry-specific capital can respond by undertaking costly adjustment. Alternatively, they can lobby politicians for trade protection, and thereby mitigate the need for adjustment. The choice between the two will reflect the relative profitability of adjusting versus lobbying. In equilibrium, the level of protection will depend on the intensity of lobbying and on the value politicians place on lobbying.
revenues relative to the welfare cost of the intervention, and will in turn affect firms’ marginal adjustment decisions. By introducing an explicit lobbying process similar to Grossman and Helpman (1992), it can be shown that the level of tariffs is an increasing function of past tariffs. This relationship works indirectly through the adjustment process: since current adjustment diminishes future lobbying effectiveness, and protection reduces current adjustment, current protection raises future protection. This simple lobbying feedback effect has an important dynamic resource allocation effect: declining industries contract more slowly over time and contract less than they would in the absence of protection.

Several papers have studied protection and lobbying in declining industries. Hillman (1982) examines political-support protectionist responses to declining industries by adapting the regulatory capture framework of Stigler (1971) and Peltzman (1976) to an international trade context. He shows that the derived protection does not fully compensate specific factors in the import-competing industry for the adverse terms-of-trade shock. Long and Vousden (1991) extend the analysis to a general equilibrium Ricardo-Viner framework, and discuss how this partial compensation result is affected by the degree of risk aversion of the specific factor owners and by the way tariff revenues are redistributed. Both papers share the feature that the analysis is static and the political support function is specified exogenously as a black box.

Dynamic aspects of protectionist policies have also been investigated. Several authors have analysed circumstances under which the adjustment path under protection is socially suboptimal in the absence of lobbying.¹ In contrast, Cassing and Hillman’s (1986) analysis of

¹ See Matsuyama (1987), Tornell (1991), and Brainard (1993). These results generally hinge on a dynamic inconsistency problem.
senescent industry collapse explicitly considers the effect of lobbying on industry dynamics. The Cassing, Hillman model differs from the one presented below in two important respects. First, the Cassing, Hillman analysis hinges on an ad hoc tariff response function, which is increasing in the level of labor in an industry, whereas we derive it explicitly from interaction between a politician and specific factor owners in an industry. Secondly, the Cassing, Hillman model predicts that initially resources will shift gradually out of an industry in response to an adverse trade shock, up to some point at which protection is abruptly terminated, and the industry collapses.\(^2\) This discontinuous adjustment behavior is attributable to an inflection point in the tariff response function, which is ad hoc. In contrast, the model presented below predicts a smooth path of decline in response to an adverse shock. In an extension, we show that results similar to those of Cassing and Hillman require an additional assumption, such as a per period fixed cost to lobbying.

The model is designed so that it can be applied symmetrically to the case of a growing industry. We examine this case in another extension to make the point that the disproportionate share of protection afforded to mature industries in countries such as the US is better explained by a bias in the political process than by pure economic differences. We also discuss the implications for general equilibrium.

The paper proceeds as follows. Section II constructs a general, two-period model of the interaction between a trade-impacted industry and a politician. Section III extends this model

\(^2\) Lawrence and Lawrence (1987) also develop a model in which labor adjustment in a declining industry is discontinuous. However, there is no political intervention in their model. The discontinuity is attributable to the combination of lumpy capacity reduction with monopoly behavior on the part of labor.
to a discrete time, infinite horizon framework by simplifying the interaction between the industry and the politician. Section IV extends the model to consider industry collapse. Section V considers growing industries and the implications for general equilibrium. Section VI concludes.

II. TWO-PERIOD MODEL

We start with an industry that is a price taker on international markets. There is a specific factor in the industry, which is fixed, and a variable factor that we will refer to as labour. The variable factor is supplied competitively, and any rents accrue to the owners of the specific factor. At the beginning of each period \( t \in \{1,2\} \), the stock of employment in the industry is given by \( y_t \). Each period, the industry chooses to adjust employment by some amount \( x_t \) (where \( x > 0 \) implies contraction). Production takes place after the adjustment has occurred. Given the fixed specific factor input, output is assumed equal to the net level of employment, \( q_t = y_t - x_t \), which is also just equal to the stock of employment at the beginning of the next period, \( y_{t+1} \). There is a cost of adjustment, \( \phi \), which is assumed to be a convex, increasing function of the amount of adjustment: \( \phi'(x_t) > 0 \) and \( \phi''(x_t) > 0 \). There is also a production cost, \( C \), which is assumed convex and increasing in output: \( C'(q_t) > 0 \), \( C''(q_t) > 0 \).

Domestic demand, \( D \), is a decreasing function of the domestic price, \( p_t \), which in the absence of intervention is just equal to the exogenously given world price, \( p_{w,t} \). We will assume that the international price is constant over time, with the exception of a discrete jump at time 0. Prior to that time, both the international price and the domestic price are constant at \( p_0 \), which is consistent with an equilibrium employment level, \( y_w^0 \). The employment level is chosen such that the marginal cost of production equals the price: \( C'(y_w^0) = p_0 \). At time 0, a permanent
shock in the international market causes a decline in the world price to \( p_w < p_0 \). Thus, the stock of employment at the outset of period 1, \( y_1 = y_w^0 \), exceeds the new long run equilibrium level, and the industry must contract in order to bring the marginal costs of production down to the new international price.

**Adjustment:**

In the absence of lobbying, the industry does not receive any protection, since there are no market imperfections. In this case, its adjustment program is defined as:

\[
(1) \quad \max_{x_1, x_2} \pi(p_w, y_1, x_1) + \rho \pi(p_w, y_2, x_2) \quad \text{with} \quad y_2 = y_1 - x_1
\]

where \( \rho \) is the discount factor of the industry, and profits in period \( t \) are defined:

\[
\pi(p, y, x) = p(y_t - x_t) - C(y_t - x_t) - \Phi(x_t)
\]

The first order conditions are:

\[
(2) \quad -p_w + C'(y_1 - x_1) + \rho [ -p_w + C'(y_1 - x_1 - x_2) ] = \Phi'(x_1)
\]

\[
-\frac{d}{dy_1} \Phi'(x_1)
\]

\[
-\frac{d}{dy_2} \Phi'(x_2)
\]

In each period the industry trades off the marginal return to adjustment against the marginal cost.

Solutions of this system can be written as: \( x_1 = x_1^*(y_1, p_w) \) and \( x_2 = x_2^*(y_1 - x_1^*, p_w) = x_2^*(y_1, p_w) \). Solving the two first order conditions under the usual concavity and convexity assumptions establishes that first period adjustment, \( x_1^* \), is decreasing in the new international price level, \( p_w \), and increasing in the initial employment stock, \( y_1 \), with \( 0 < \frac{\delta x_1}{\delta y_1} < 1 \), and therefore the second period employment stock, \( y_2^* \), is increasing in \( y_1 \). In addition, second period adjustment, \( x_2 \), is an increasing function of \( y_2 \), and thus the equilibrium value of \( x_2^* \) is increasing in the initial stock of employment, \( y_2 \). The full impact of the world
price $p_\text{w}$ on $x_2^*$ is a priori ambiguous: the direct effect is negative, while the indirect effect through decreases in $x_1^*$ is positive. If the direct effect outweighs the indirect effect, then a smaller price shock results in lower adjustment in period two: $\frac{\partial x_t^*}{\partial p_\text{w}} < 0$. In the case of linear demand and quadratic costs, the direct effect dominates and the optimal adjustment levels are:

$$
\begin{align*}
  x_1 &= (p_0 - p_\text{w}) \frac{(1 + \phi)(1 + \rho)}{(1 + \phi)^2 + \rho \phi} \\
  x_2 &= (p_0 - p_\text{w}) \frac{\phi}{(1 + \phi)^2 + \rho \phi}
\end{align*}
$$

Thus, adjustment in each period is increasing in the size of the price shock.

**Lobbying:**

Next we assume the industry has the option of lobbying to influence the domestic price. The domestic price is the product of an endogenously determined ad valorem tariff, $\theta_\text{t}$, and the exogenous world price: $p_\text{t} = (1 + \theta_\text{t})p_\text{w}$. Following Helpman and Grossman (1992), we model the lobbying process as a contribution game where, in each period, the industry can influence the level of the tariff by offering a schedule of contributions to the incumbent politician as a function of the tariff, or equivalently, of the two prices, $F(p_\text{t}, p_\text{w})$. Given world price $p_\text{w}$, and employment adjustment, $x_\text{t}$, profits in period $t$ are:

$$
\pi(p_\text{t}, y_\text{t}, x_\text{t}) = p_\text{t}(y_\text{t} - x_\text{t}) - C(y_\text{t} - x_\text{t}) - \phi(x_\text{t}) - F_t(p_\text{t}, p_\text{w})
$$

We restrict consideration to differentiable lobbying contributions, and assume that the industry does not provide any contribution if there is no protection: $F_t(p_\text{w}, p_\text{w}) = 0$.

The policymaker values both social welfare, $W$, and lobbying contributions in different degrees. We assume that the utility function is linear in both elements:

$$
G(p_\text{t}, y_\text{t}, x_\text{t}) = \beta W(p_\text{t}, y_\text{t}, x_\text{t}) + (1 - \beta)F_t(p_\text{t}, p_\text{w})
$$
where the weight on welfare, $\beta$, lies between $1/2$ and $1$. Welfare is the sum of consumer surplus, industry profits and tariff revenue:

$$W(p, y, x) = \int_{p_1} D(u) du + p_1(y_1-x_1) - C(y_1-x_1) - \phi(x_1) + (p_1 - p) [D(p) - (y_1-x_1)]$$

This objective function is the partial equilibrium, dynamic analogue of that developed in Grossman and Helpman (1992). In the interests of simplicity, it ignores a host of interesting features of political interaction, especially competition between political parties. But, as Grossman, Helpman point out, evidence that lobby groups disproportionately make contributions to incumbents, and frequently contribute to candidates after they have won, suggests this simplification has some empirical credence.

The timing of the game is as follows:

<table>
<thead>
<tr>
<th>period 1</th>
<th>period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>Government</td>
</tr>
<tr>
<td>$F_1$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>Industry</td>
<td>Industry</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$p_2$</td>
</tr>
</tbody>
</table>

In each of the two periods, the industry first chooses its contribution function, $F_i$, the politician then chooses the domestic price, $p_i$, given the contribution schedule, and finally the industry chooses the level of adjustment, $x_i$.  

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3 The welfare weight parameter, $a$, in Grossman, Helpman is just equal to $\beta/(1-\beta)$ here.

4 Qualitatively similar results obtain if instead the government is assumed to choose a price schedule as a function of the contribution, and the firm then chooses its contribution and employment level simultaneously.
The second period lobbying game:

Solving backwards, the industry chooses \( x_2 \) to maximize \( \pi(p_2,y_2,x_2) \), yielding first order conditions:

(6) \[-p_2 + C'(y_2 - x_2) = \phi'(x_2)\]

which gives \( x_2 \) as a function of employment stock, \( y_2 \), and domestic price, \( p_2 \):

(7) \[x_2 = x_2^*(y_2,p_2) \text{ with } \frac{\partial x_2^*}{\partial p_2} < 0 \text{ and } 0 < \frac{\partial x_2^*}{\partial y_2} < 1\]

Second period adjustment is a decreasing function of the second period tariff (reflected in the domestic price level), and of first period adjustment, since \( y_2 = y_1 - x_1 \). Plugging the optimal adjustment from (7) into the profit function in (4), yields the indirect profit function, \( \pi^*(p_2,y_2) = \pi(p_2,y_2,x_2^*) \), which is decreasing in the employment stock, \( y_2 \), and increasing in the domestic price, \( p_2 \).

Fully anticipating the industry's employment response, the politician chooses the tariff to maximize \( G(p_2,y_2,x_2^*) \). The first order condition yields an expression relating the marginal industry contribution to the domestic price level:

(8) \[\frac{\beta}{1-\beta} (p_2 - p_w) \left[ D'(p_2) + \frac{\partial x_2^*}{\partial p_2} \right] + \frac{dF_2(p_2,p_w)}{dp_2} = 0\]

The first term in equation (8) is the marginal deadweight loss of the tariff weighted by \( \beta/(1-\beta) \). In order to induce the politician to choose the domestic price level, \( p_2 \), the trade-impacted industry has to propose a contribution schedule that exactly compensates on the margin the associated social welfare loss, weighted by the politician's preference for contributions. The level of tariff protection increases in the marginal contribution and decreases in the weight the politician assigns to welfare, under the assumption that the import demand function,
\[ M(y_2, p_2) = D(p_2) - y_2 + x_2(y_2, p_2), \]

is not overly convex in the domestic price. When the politician values only social welfare, the tariff is 0.

The lobbying contribution schedule of the industry in period 2 is then derived by integrating (8):

\[ F_2(p_2, p_w, y_2) = -\frac{\beta}{1-\beta} \int_{p_w}^{p_2} \left[ D'(u) + \frac{\partial x_2^*(u, y_2)}{\partial p_2} \right] du \]

Anticipating the politician's tariff response, the industry chooses its contribution in period 2 (or equivalently the domestic price) so that the marginal cost of lobbying just equals the marginal benefit to the industry from increased protection:

\[ (y_2 - x_2^*) = \frac{dF_2(p_2, p_w, y_2)}{dp_2} \]

which can be restated in the usual Ramsey form (Grossman, Helpman (1992)):

\[ \frac{(p_2 - p_w)}{p_w} = \frac{(1-\beta)}{\beta} \frac{(y_2 - x_2^*)}{M_2(p_2)} \frac{1}{\epsilon_{M_2}(p_2)} \]

where \( M_2(p_2) \) is the import demand in period 2 and \( \epsilon_{M_2}(p) = -M'(p)p/M(p) \) is the elasticity of import demand. Equation (10) defines the equilibrium second period domestic price, \( p_2 \), as a function of the stock of employment in that period \( p_2^*(y_2) \). For a program that is well-defined and concave in the price \( p_2 \), the sign of \( \partial p_2^*/\partial y_2 \) is the same as the sign of the expression:

\[ \frac{\partial p_2^*}{\partial y_2} \]

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5 In the Grossman, Helpman general equilibrium framework, each industry's contribution includes a constant in addition to this integral, which is equal to the difference between its return when it lobbies and when it does not, given the equilibrium contributions of the competing lobbies. Here, this term does not appear since there are no competing lobbies.
which is derived by totally differentiating the first order condition in (10). The first term in brackets represents the first order effect of $y_2$ on the output level, $q_2$, and is positive. The second term has an ambiguous sign, but is of second order for small tariffs. As long as the first term is sufficiently large relative to the second, the equilibrium level of protection in period two is an increasing function of the employment stock, $y_2$, and therefore a decreasing function of the adjustment undertaken in the previous period, $x_1$. In particular, in the linear-quadratic case it is straightforward that:

(12) \[ x_2^* = \frac{y_2 - p_2}{1 + \Phi} \quad \frac{dF(p_2, p_w)}{dp_2} = \frac{\beta}{1 - \beta} (p_2 - p_w) \]

(13) \[ p_2^* = \frac{\beta(1 + \Phi)p_w + \Phi(1 - \beta)y_2}{\beta(2 + \Phi) - 1} \]

In this case, the second term in equation (11) is identically equal to zero, the second period contribution, $F_2$, does not depend directly on $y_2$, and $\partial p_2^*/\partial y_2 > 0$. Second period protection is an increasing function of the stock of employment at the beginning of the period, and therefore a decreasing function of the adjustment undertaken in the previous period.

The first period lobbying game:

Continuing backwards, the industry's first period adjustment level is chosen, taking into

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6 It cannot be signed without some additional conditions on the derivative of marginal costs. In particular, it can be shown that for convex marginal production and adjustment cost functions, $\partial^2 x_1^*/\partial p_2 \partial y_2$ is positive. The larger the industry, the less sensitive is the adjustment to changes in domestic prices in period 2.
account its effect on the level of protection and adjustment in period two. The maximization program for the industry can be written as:

\[
\text{MAX}_{x_1} \; p_1(y_1 - x_1) - C(y_1 - x_1) - \Phi(x_1) + \rho [\pi^*(p_2^*(y_1 - x_1), y_1 - x_1)]
\]

Using the envelope theorem for the second period profit function and equation (7), the first order condition for \( x_1 \) can be written as:

\[
-p_1 + C'(y_1 - x_1) + \rho \left[ \phi'[x_2^*(y_1 - x_1)] + \frac{\partial F_2}{\partial y_2} \right] = \phi'(x_1)
\]

The first two terms represent the marginal gain from adjustment in period 1, while the third term is the marginal adjustment cost saved in period 2 due to adjustment in period 1. The last term represents the strategic impact of adjustment in period 1 on the lobbying contribution in period 2. Its sign depends on \( \partial^2 x_2^*/\partial p_2 \partial y_2 \). In the linear quadratic case, it is equal to zero. At the equilibrium, the marginal direct and indirect returns from adjustment are balanced against the marginal cost of adjustment in the current period. For a well-defined concave problem, this defines the optimal adjustment level, \( x_1^* \), as a function of the initial stock of employment and the domestic price, \( p_1 \):

\[
x_1 = x_1^*(y_1, p_1) \quad \text{with} \quad \frac{\partial x_1^*}{\partial p_1} < 0 \quad \text{and} \quad 0 < \frac{\partial x_1^*}{\partial y_1} < 1
\]

Combining equation (16) with the condition derived above that \( \partial p_2^*/\partial y_2 > 0 \) yields the result that the second period tariff is an increasing function of the first period tariff. The intuition is straightforward: the higher is the domestic price in the first period, the less the industry adjusts, and the lower is first period adjustment, the more the industry lobbies in period two.

Continuing farther backwards, the government chooses the domestic price, \( p_1 \), anticipating the effect of this price on the industry's subsequent adjustment and lobbying
behavior in period 2, and taking as given the contribution schedule, $F_1$, proposed in that period.

Assuming the policymaker has the same discount rate as that of the industry, the policymaker's optimization problem is:

$$
Max_{p_1} \beta W(p_1, y_1, x_1^*) + (1 - \beta) F_1(p_1, p_w) + \rho [\beta W(p_2^*(y_2); y_2, x_2^*) + (1 - \beta) F_2(p_2^*(y_2); p_w, y_2)]
$$

with $y_2 = y_1 - x_1^*(p_1, y_1)$

Using the envelope theorem with respect to the optimal price and adjustment in period 2 yields the first order condition for the politician:

$$
\delta F_1(p_1, p_w) \frac{dp_1}{dF_1} = -\frac{\beta}{(1 - \beta)} \left( p_1 - p_w \right) \left( D'(p_1) + \frac{\partial x_1^*}{\partial p_1} \right) - \rho \left( p_2^* - p_w \right) \left( 1 - \frac{\partial x_2^*}{\partial y_2} \right) \frac{\partial x_1^*}{\partial p_1} + \rho \frac{1}{(1 - \beta)} \frac{\partial F_2}{\partial y_2} \frac{\partial x_1^*}{\partial p_1}
$$

At the equilibrium protection level, the lobbying contribution in period 1 must balance three terms at the margin. The first term in equation (18) is the usual static deadweight loss. The second term reflects the loss of tariff revenue in period 2 due to the increase in production implied by reduced adjustment in period 1. The third term is the strategic impact of period 1 protection on the lobbying contribution in period 2.

Integrating equation (18) yields the optimal first period lobbying contribution schedule of the industry. The industry chooses the optimal tariff taking the contribution schedule as a constraint. Again, using the envelope theorem, the first order condition is:

$$
y_1 - x_1^*(p_1, y_1) = \frac{dF_1(p_1, p_w, y_1)}{dp_1}
$$

which defines the equilibrium first period protection level as a function of the initial employment stock, $y_1$. For a well-defined concave program, the sign of $dF_1/\partial y_1$ is the same as the sign of:

$$
\left[ 1 - \frac{\partial x_1^*}{\partial y_1} \right] - \frac{\partial^2 F_1(p_1, p_w, y_1)}{\partial p_1 \partial y_1}
$$
The first term, which represents the direct effect of the initial size of the employment stock on protection, is positive. The higher is the initial level of employment, the higher is output in period 1, and the higher is the tariff. The second term reflects the lobbying interaction between the declining industry and the policymaker. It is difficult to sign in general. However, in the linear-quadratic case and when the policymaker is relatively impatient this term is also positive. Thus, \( \partial p_1/\partial y_1 \) is positive, and protection in period 1 is increasing in the initial employment stock. The larger is the initial shock, the larger is the initial protection level and the larger is future protection.

III. INFINITE HORIZON MODEL

We now extend the model to the infinite horizon case, simplifying in two ways. First, in order to obtain explicit solutions, we focus on a linear-quadratic version of the model, with cost and demand functions given by:

\[
\phi(x_i) = \frac{\phi x_i^2}{2}; \quad C(q_i) = \frac{q_i^2}{2}; \quad Q(p) = \alpha - p_i
\]

Second, in order to derive an explicit path of adjustment for employment and domestic prices over an infinite horizon, we simplify the structure of the stage game. Each period, the industry is assumed to choose its adjustment level and its contribution schedule simultaneously. The politician then chooses a tariff level to maximize utility. Compared to the two-period framework, this timing assumption assigns greater commitment power to the industry’s action each period, but this is offset by the alternating sequence of moves in an infinite horizon context.

Both the industry and the politician are assumed to employ Markov strategies. The state variable is the employment stock, whose evolution is described by the simple equation:
Then the politician's value function is:

\[
V_p(y) = \text{Max}_{p_t} \left[ \beta W(p_t, y_t) + (1-\beta)F_t(p_t, p_w) + \delta V_p(y_{t+1}) \right]
\]

Given the relationship between the domestic price and the contribution level embodied in the politician's first order conditions, the industry maximizes its value function by its choice of contribution and adjustment levels:

\[
V_f(y) = \text{Max}_{p_t, x_t} \left[ p_t(y_t - x_t) - \frac{(y_t - x_t)^2}{2} - \frac{\phi x_t^2}{2} - F_t(p_t, p_w) + \rho V_f(y_{t+1}) \right]
\]

With this timing structure and Markov strategies, the politician's protection decision affects only the contemporaneous levels of the contribution and employment adjustment. Therefore, the politician's problem can be simplified to a static maximization problem. Choosing the optimal level of protection as a function of contributions yields the same condition on the relationship between the marginal contribution and the tariff level as in equation (8).

Integrating yields \( F(p_t, p_w) = (p_t - p_w)^2/2a \), where \( a = (1-\beta)/\beta \).

Faced with this tariff response function, the industry's problem simplifies considerably, permitting the first order conditions to be expressed as a second order difference equation. Substituting \( y_t, y_{t+1} \) for \( x_t \), incorporating the politician's tariff response in the industry's value function, and differentiating with respect to the contribution yields the condition:

\[
y_{t+1} = \frac{(p_t - p_w)}{a}
\]

while differentiating with respect to the employment level yields:

\[
p_t = -\rho \phi y_{t+2} + (1 + \phi (1 + \rho)) y_{t+1} - \phi y_t
\]

Combining (25) and (26), and using the initial condition that the employment stock at time 0 is
$y_w$, (and setting the coefficient on the larger root to 0) yields the industry's optimal adjustment path equation:

\begin{equation}
    (27) \quad y_t = \left( p_o - \frac{p_w}{(1-a)} \right) b(a)^\prime + \frac{p_w}{1-a}
\end{equation}

where the root is defined:

\begin{equation}
    (28) \quad b(a) = \frac{1-a+\phi(1+\rho) - \sqrt{(1-a)^2 + 2(1-a)\phi(1+\rho) - 4\rho \phi^2 + \phi^2(1+\rho)^2}}{2\phi \rho}
\end{equation}

and $0 < b(a) < 1$ for the restrictions on $\beta$ adopted above, and $b'(a) > 0$.

This path can be contrasted with the adjustment path in the free market equilibrium, which is obtained by setting the politician's weight on welfare to 1 ($a=0$):

\begin{equation}
    (29) \quad y_t = (p_o - p_w) b(0)^\prime + p_w
\end{equation}

Comparing the two expressions reveals three channels through which lobbying affects the adjustment path. The equilibrium stock of employment is higher each period because lobbying reduces the rate of adjustment and reduces the cumulative amount of adjustment that takes place, and these two effects more than offset the decrease in the coefficient, $p_o - p_w/(1-a)$, due to the reduction in the price shock. The rate of adjustment is slower, and the long run equilibrium level of employment is higher, the greater is the politician's preference for contributions relative to welfare. With lobbying, employment is adjusted downward smoothly, at a decreasing pace,
eventually converging to a level that is permanently above the efficient level of employment. The two paths of adjustment are compared in Figure 1.

The path of the associated equilibrium tariff can be derived by combining the industry's first order conditions with the adjustment path of employment:

\[ \theta_t = b(a)^t \frac{p_0}{p_w} a + (1 - b(a)^t) \frac{a}{1-a} \]

The level of protection declines smoothly and gradually with employment over time, reflecting the effect of past protection through the current stock of employment. Further, the tariff is higher at each point of time, the larger is the initial adverse shock or shift of comparative advantage (measured by \( p_0/p_w \)). This is closely related to "the compensation effect" of Magee and Young (1989). The larger the initial shock, the more the industry must adjust in order to adapt to the new international environment, and the larger are the incentives to lobby for protection to mitigate the need for costly adjustment.

IV. SENESCENT INDUSTRY COLLAPSE

Our result differs markedly from that of Cassing, Hillman, who find that the industry declines smoothly up to some point, after which it suddenly loses protection and collapses. This result is attributable to the shape of their tariff response function, which switches from convex to concave at some threshold level of employment. By making the tariff formation process explicit, the framework above makes clear that some kind of discontinuity would be required in the industry's lobbying activities to yield a point of collapse. In particular, if participation in the lobbying process each period required the payment of a fixed cost, C, in addition to the variable contribution, then the industry would lobby only as long as the intertemporal return to
lobbying offsets the fixed cost. Such a fixed cost might be associated with operating an information network, maintaining political connections, or paying lobbyist's fees.

Recall that with a zero fixed cost, the industry always lobbies, and adjusts gradually to a level of employment, $p_a/(1-a)$ above the free market level, $p_a$. Intuitively, the effect of introducing a fixed per period cost is fairly clear. If the fixed cost is below the per period return to lobbying when adjustment has reached its steady state, $C < ap_a^2/2$, then the industry never finds it optimal to stop lobbying and receives protection permanently. There is a smooth adjustment process, which is identical to that in equation (27). If the fixed cost exceeds the difference between the return to lobbying and the free market return at time 0, when employment is at its maximum relative to the steady state value, the industry never lobbies and simply adjusts according to (29). For a fixed cost in an intermediate range between these two levels, the industry lobbies and receives protection for some finite number of periods, $\tau(y_0)$. Up to time $\tau(y_0)$, the industry lobbies and contracts gradually, cushioned by the resulting protection. The rate of contraction on this interval lies between that under permanent protection and the free market rate. At time $\tau(.)$, it is no longer worth paying the fixed cost, so the industry stops lobbying and loses its political influence. Domestic protection collapses to zero, and adjustment accelerates in a discontinuous manner.

The appendix proves these results and specifies the relationship between the time, $\tau$, the fixed cost, $C$, and the other parameters of the model. The proof proceeds by defining the value function for a single period of lobbying followed by no lobbying, and then solves forward recursively to determine the optimal number of periods of lobbying before the switch to no lobbying. This value function for optimal temporary lobbying is compared to the value functions
for permanent lobbying and for unprotected adjustment, and the resulting inequalities define the ranges for the fixed cost relative to the intertemporal return from lobbying.

V. GROWING INDUSTRIES AND IMPLICATIONS FOR GENERAL EQUILIBRIUM

Growing Industries

The case of growing industries may be accommodated quite simply in the above framework, with rather startling results. Start by assuming there is no fixed cost. Suppose that there is a permanent price shock at time \( t \), such that \( p_0 \) rises to some level, \( p_w \). In the free market economy, the industry will want to raise employment each period, to adjust to the steady state level \( y_w = p_w > y_0 \), so that employment adjustment will be positive in equilibrium. The analysis of lobbying in a growing industry is exactly symmetric to the case of decline, as are the maximizing levels of contributions and employment adjustment each period. Proceeding through the same steps as for the declining industry in the infinite horizon framework yields an expression for the equilibrium stock of employment in period \( t \):

\[
y_t = \frac{p_w}{(1-a)} - \left( \frac{p_w}{(1-a)} - p_0 \right) b(a)'
\]

where the characteristic root is defined as in equation (28). When it lobbies, a growing industry grows more rapidly than it would in the free market, at a rate increasing in the size of the price shock, and the steady state level of output exceeds that in the free market by an amount that increases with the politician's preference for contributions.

This suggests that the empirical evidence that declining industries receive a disproportionate share of protection in countries such as the US would be better explained by a bias in the political process than by pure economic differences. There are a variety of reasons
why the political process may be biased against growing industries. First, there may be
important differences in the cost of lobby formation for fledgling as opposed to mature
industries. In the model above, we simply assume that an industry lobbies whenever the
intertemporal return is positive, thereby ignoring the critical issue of lobby formation. However,
research in political science suggests that industries are more likely to overcome the free rider
problems of lobby formation when they have large committed resources and established unions.
In addition, growing industries are characterized by rapidly changing market structures and a
high likelihood of future entry, while declining industries are more likely to have stable market
structures with a reduced threat of domestic entry. The greater risk that future rents will be
dissipated with entry may make lobby formation more difficult in growing industries than in
decreasing industries with clearly identified players and more predictable rents.

Secondly, in the presence of imperfect capital markets\textsuperscript{7}, liquidity constraints on lobbying
activities may be more binding in a growing industry than in a declining industry. This would
be the case if incumbent domestic firms in mature industries have more accumulated cash
reserves from past retained profits relative to investment opportunities than do firms in emerging
industries. To illustrate the effect of a liquidity constraint, assume that firms must finance both
investment and contributions from current profits. The industry’s maximization program is
modified to take into account the liquidity constraint as follows:

\textsuperscript{7} Imperfect capital markets may exist because either it is not possible to borrow to finance
lobbying, or investors are less optimistic about an industry’s future growth path than are industry
participants due to asymmetric information.
where \( \lambda_i \) is the multiplier on the liquidity constraint. Since the unconstrained equilibrium profit level rises monotonically over time, the constraint must bind initially and over a continuous interval, if it binds at all. When the constraint binds, it affects the equilibrium stock of employment only through the characteristic root:

\[
\begin{align*}
(32) & \quad V_i(y_i, \lambda_i) = \max_{p_t, y_{t+1}} \left[ \left( p y_{t+1} - \frac{y_{t+1}}{2} \phi (y_{t+1} - y_t) - \frac{(p_t - \rho)^2}{2a} \right) (1 + \lambda_t) + V_j(y_{t+1}, \lambda_{t+1}) \right] \\
& \quad \text{where } \lambda_i \text{ is the multiplier on the liquidity constraint. Since the unconstrained equilibrium profit level rises monotonically over time, the constraint must bind initially and over a continuous interval, if it binds at all. When the constraint binds, it affects the equilibrium stock of employment only through the characteristic root:}
\end{align*}
\]

\[
(33) \quad b(a, L_i) = \frac{1 - a + \phi (1 + \rho L_i) - \sqrt{[(1 - a) + \phi (1 + \rho L_i)]^2 - 4 \phi^2 \rho L_i}}{2 \phi \rho L_i} \quad \text{where } L_i = \frac{(1 + \lambda_{t+1})}{(1 + \lambda_t)}
\]

Thus, a binding constraint lowers the industry growth rate relative to the unconstrained lobbying path. However, the growth rate remains above the free market rate, and the long run equilibrium value is the same as that for unconstrained lobbying. The equilibrium adjustment paths for the unconstrained lobbying equilibrium, constrained lobbying equilibrium, and unconstrained free market equilibrium are compared in Figure 2.

In addition, a fixed cost in the lobbying process might create a bias against growing industries. Suppose, as above, that the fixed cost does not depend on the size of the industry. If an industry ever starts to lobby, it will not subsequently stop lobbying, since the return to lobbying never decreases in a growing industry. Thus, an industry chooses how many periods to wait before it starts lobbying. If the per period return to lobbying exceeds the fixed cost in the first period, it lobbies permanently, and conversely if the fixed cost exceeds the return to

---

8 It is not possible to solve for \( L_t \) explicitly. However, by combining (33) with the liquidity constraint it can be shown that \( b \) is declining in \( L \) and that \( L \) is rising over time under sensible conditions, such that \( b \) is decreasing over time.
lobbying at the steady-state level of employment under lobbying, then the industry never starts lobbying. If the fixed costs lies in some intermediate range, then the industry waits for some optimal number of periods until it is large enough that the return to lobbying exceeds the fixed cost, and begins lobbying.

Implications for General Equilibrium

These results ignore potential spillover effects of lobbying across sectors, which is a central consideration in understanding the effect of lobbying on dynamic resource allocation. In a general competition for protection, there are a variety of channels whereby the equilibrium pattern of protection might result in a diversion of resources away from infant industries toward industries with declining competitiveness. The results derived by Grossman and Helpman (1992) in a static general equilibrium framework, where protection spills over between interest groups through consumption, suggest that mature sectors would gain protection at the expense of infant industries in a competition for protection if the infants were less well organised than mature industries for any of the reasons cited above or if the infants initially were smaller. Similarly, in a model where there is a limited supply of a common factor of production, or the import-competing sector produces an input used by the exporting sector, the equilibrium pattern of protection might result in resources being directed away from the growing industries to mature industries, distorting adjustment in both directions. If the distortions were sufficiently great, growing industries would grow more slowly in a lobbying equilibrium than in the free market equilibrium.
VI. **CONCLUSION**

Motivated by the strong empirical regularity that the best predictor of future protection is past protection, this paper has analyzed the adjustment path in declining industries under endogenous protection. By introducing an explicit political objective function similar to that developed in a static framework by Grossman and Helpman into a dynamic model with convex adjustment costs, the paper shows that the level of tariffs is an increasing function of past tariffs. In this model, industry adjustment and lobbying are substitutes: the more an industry lobbies, the greater the protection it receives and the less it adjusts, and the less the industry adjusts the more effective it is in lobbying next period. Lobbying is an increasing function of the initial price shock, or equivalently of the gap between the initial level of employment and the long run equilibrium level.

The paper finds that in the absence of nonconvexities in the lobbying process, the paths of lobbying and adjustment are smooth. The industry contracts employment gradually over time to a level that is permanently above the free market level by an amount that increases in the value the politician places on lobbying contributions relative to welfare. This result contrasts sharply with the Cassing and Hillman finding that there is a point of collapse, which corresponds to an inflection point in an ad hoc tariff response function. Here, we derive a similar collapse in the path of protection by introducing a per period fixed cost into the lobbying function. When the fixed cost lies in a range defined by the difference in returns between lobbying and not lobbying at the initial level of employment and at the steady-state level under protection, the industry lobbies and receives protection for some finite number of periods and then abruptly stops lobbying, resulting in a collapse in protection and accelerated adjustment. The rate of
adjustment under temporary lobbying lies between the adjustment rates under permanent lobbying and no lobbying, and the associated long run employment level is just the free market equilibrium.

Ultimately, the question of whether adjustment and protection are smooth or discontinuous is empirical. To the best of our knowledge, there have been no systematic investigations comparing adjustment paths across industries. Several articles that investigate a small number of industries do not address this issue directly. However, our purpose was to investigate the conditions that determine the adjustment path rather than to establish the validity of a particular path.

In addition, the paper shows quite clearly that in a partial equilibrium framework, growing industries will grow faster under endogenous protection unless there is some bias against growing industries in the political process that makes lobby formation costly. These results are suggestive for general equilibrium, where such a bias in combination with a resource constraint or a vertical relationship between infant and mature industries would result in a dynamic misallocation of resources between sectors.
Figure 1

ADJUSTMENT TO NEGATIVE SHOCK

Stock of Employment

0 1 2 3 4 5 6 7 8 9 10
Time

a2 = 0.85
pw/(1 - a2)

a = 0
a1 = 0.5
pw/(1 - a1)
pw
Figure 2

ADJUSTMENT TO POSITIVE SHOCK

Stock of Employment

\[
pw/(1-a)
\]

a=0.5

Constrained

pw

a=0

Time

25
Appendix

Assume that in order to offer a contribution schedule \( F(p_0, p_n) \) to the politician, the industry must pay a fixed cost, \( C \), in each period. Assume also that if the industry does not pay the fixed cost at any time \( t \), then it cannot lobby in any period after \( t \). We start by defining the value functions of the industry when it lobbys permanently, when it does not lobby, and when it lobbies for some finite number of periods and then stops.

I. Definitions and notations

1) Define \( V_d(y) \) as the value function of the industry without lobbying:

\[
V_0(y) = \max_{0 \leq z \leq y} \left( p_c z - \frac{z^2}{2} - \frac{\phi (y-z)^2}{2} + \rho V(z) \right)
\]

with \( z = y-x \). Because the function \( h(z, y) = p_c z - \frac{z^2}{2} - \frac{\phi (y-z)^2}{2} \) is strictly concave and quadratic, results from Lucas and Stokey (1989) establish that there is a unique, continuous, and strictly concave value function \( V_0(.) \) defined on the interval \([0, y_0]\) that satisfies this equation.

2) Further, define \( V_s(y) \) as the value function of the industry with permanent lobbying on the interval \([0, y_0]\).

After optimization on \( p \), this value function is equivalent to:

\[
V_s(y) = \max_{0 \leq z \leq y} \left[ p_c z - \frac{(1-a) z^2}{2} - \frac{\phi (y-z)^2}{2} - C + \rho V_s(z) \right]
\]

The same result establishes that \( V_s(y) \) is a well-defined, continuous and strictly concave function on \([0, y_0]\). Moreover, \( V_s(y) = V_s^0(y) - C/(1-\rho) \), where \( V_s^0(y) \) is the value function of permanent lobbying with zero fixed costs.

It is also useful to define an operator, \( T_s \), that associates to any continuous function \( V(.) \) on \([0, y_0]\) the new function \((T_s V)(.)\) defined by:

\[
(T_s V)(y) = \max_{0 \leq z \leq y} \left[ p_c z - \frac{(1-a) z^2}{2} - \frac{\phi (y-z)^2}{2} - C + \rho V(z) \right]
\]

\( T_s V(.) \) is the value of lobbying in the current period followed by the value function \( V(.) \) in the following period. It is clear that \( V_s(.) \) is the unique fixed point of this operator on the set, \( C[0, y_0] \), of the continuous functions on \([0, y_0]\).
Define \( (T_k V)_v = V_k \). Define \( \{T_k\}^k \) as the \( k \)th iterate of the operator \( T_k \): \( T_k = T_o o T_o o T_o \ldots o T_o \). It is well known that for any initial continuous function \( V \) on \([0, y^0]\), the sequence of functions \( ((T_k)^k V) \) converges uniformly to \( V_o(.) \).

3) In addition, define the two "unconstrained" operators, \( T_0^\ast \) and \( T_1^\ast \), on the set of all functions from \( R \) to \( R \) as:

\[
(T_0^\ast V)(y) = \max \left\{ p_w z - \frac{z^2}{2} - \frac{\phi(y-z)^2}{2} + \rho z \right\}
\]

\[
(T_1^\ast V)(y) = \max \left\{ \frac{(1-a)z^2}{2} - \frac{\phi(y-z)^2}{2} - C + \rho z \right\}
\]

Again drawing on results from Lucas and Stokey (1989, Theorem 4.14 and p. 95), because the functions \( h(z, y) \) defined above and \( h(z, y, a) = p_w z - (a)z^2/2 - \phi(y-z)^2/2 \) are strictly concave, quadratic in \((z, y)\) for all \( 0 \leq a < 1 \), the operators \( T_0^\ast \) and \( T_1^\ast \) have unique fixed points, \( V_0^\ast(.) \) and \( V_1^\ast(.) \) respectively, defined on \( R \), which are well-defined, quadratic, strictly concave functions in \( y \). The optimal adjustment paths \( z(0, y) \) and \( z(a, y) \) associated with \( V_0^\ast \) and \( V_1^\ast \) are linear and increasing in \( y \), with a slope less than unity, such that:

\[
\forall y \geq p_w, \ p_w \leq z(0, y) \leq y; \ \forall y \leq p_w, \ y \leq z(0, y) \leq p_w.
\]

\[
\forall a \in [0, 1) \text{ and } \forall y \geq \frac{p_w}{1-\alpha}, \ \frac{p_w}{1-\alpha} \leq z(a, y) \leq y; \ \forall y \leq \frac{p_w}{1-\alpha}, \ \frac{p_w}{1-\alpha} \leq z(a, y) \leq y.
\]

Consider the restriction \( V_0^\ast \) of \( V_0^\ast \) to \([p_w, y^0]\). It is clear that the function:

\[
V(y) = \begin{cases} 
\frac{2p_w y - y^2}{2(1-\rho)} & \text{if } y \in [0, p_w] \\
V_0^\ast(y) & \text{if } y \in [p_w, y^0] 
\end{cases}
\]

satisfies the fixed point property of \( V_0 \) on \([0, y^0]\) and is therefore equal to \( V_0 \). Similarly, defining \( V_1^\ast(y) \) as the restriction of \( V_1^\ast \) on \([p^*/1-\alpha, y_0]\), it is a simple matter to verify that:

\[
V_1^\ast(y) = \begin{cases} 
\frac{2p_w - (1-a)y^2 - 2C}{2(1-\rho)} & \text{if } y \in [0, \frac{p_w}{(1-\alpha)}] \\
V_1^\ast(y) & \text{if } y \in \left[\frac{p_w}{(1-\alpha)}, y_0\right]
\end{cases}
\]
\( V_0(\cdot) \) and \( V_*(\cdot) \) are therefore differentiable, and using the envelope theorem yields:

\[
\frac{dV_0(y)}{dy} = \begin{cases} 
\frac{p_w - y}{1 - \rho} & \text{for } y \in [0, p_w] \\
-\phi(y - z(0, y)) & \text{for } y \in [p_w, y_0]
\end{cases}
\]

and

\[
\frac{dV_*(y)}{dy} = \begin{cases} 
\frac{p_w - \rho y(1 - a)}{1 - a} & \text{for } y \in [0, \frac{p_w}{1 - a}]
\\
-\phi(y - z(a, y)) & \text{for } y \in \left[\frac{p_w}{1 - a}, y_0\right]
\end{cases}
\]

From equation (8) it is clear that \( V_*(y) > V_0(y) \) for all \( y \in [0, y_0] \), and \( V_*(y) - V_0(y) \) is increasing in \( y \) on this interval. It is also clear that \( V_*(y) > V_0(y) \) on this interval. Namely, with zero fixed costs it is always better for the industry to lobby permanently than not to lobby.

4) Consider the function \((T_\delta V_0)(\cdot)\), the \( T_\delta \) operator applied to \( V_0(\cdot) \). As before, we may consider the unconstrained program:

\[
(T_\delta V_0)(y) = \max_{z \geq 0} \left[ h(z, y, a) - C + \rho V_0(z) \right]
\]

Define \( z_\delta^*(y) \) as the solution of this program, where the superscript refers to the number of iterates of the \( T_\delta \) operator. \( z_\delta^*(y) \) is determined by:

\[
z_\delta^*(y) = \{ z \mid p_w - z(1 - a) + \phi(y - z) = -\rho V_0^\prime(z) \}
\]

Because \( V_0(y) \) is quadratic, (10) establishes that for all \( y \) in \( [0, y_0] \), \( z_\delta^*(y) \) is linear and increasing in \( y \) with a slope less than unity. Also, one can verify that \( z_\delta^*(0) > 0 \). Then there is a unique point \( y_*^* \) such that \( z_\delta^*(y) \leq y \) if and only if \( y \geq y_*^* \). Hence one may rewrite \((T_\delta V_0)(y)\) as:

\[
(T_\delta V_0)(y) = \begin{cases} 
p_w y - \frac{(1 - a)y^2}{2} - C + \rho V_0(y) & \text{if } y \in [0, y_*^*] \\
(T_\delta V_0)(y) & \text{if } y \in [y_*^*, y_0]
\end{cases}
\]

It is clear that \((T_\delta V_0)(\cdot)\) is differentiable, strictly concave, piecewise quadratic on \([0, y_0]\). Using the envelope theorem yields:
\((T_\alpha V_\alpha)(y) = \begin{cases} p_\alpha - (1 - \alpha) y + \rho V_0'(y) & \text{if } y \in [0, y_0^*] \\ -\phi(y-z_\alpha^*(y)) & \text{if } y \in [y_0^*, y_0] \end{cases} \)

From (4) and (5), unconstrained optimal adjustment without lobbying, \(z(0, y)\), and with permanent lobbying, \(z(a, y)\), are determined respectively by:

\begin{align*}
(13) & \quad z(0, y) = \{z \mid p_\alpha - z + \phi(y-z) = -\rho V_0^\phi(z)\} \\
(14) & \quad z(a, y) = \{z \mid p_\alpha - z(1 - \alpha) + \phi(y-z) = -\rho V_\alpha'(z)\}
\end{align*}

It follows directly from inspection of (10), (13), and (14) and the fact that on \([0, y_0] \ V_\alpha'(y) > V_0'(y)\) that:

- \(z(a, y) > z_\alpha^*(y) > z(0, y)\), for all \(y \in [0, y_0] \)

Hence, \(p_\alpha/(1 - \alpha) > y_0^* > p_\alpha\). Using this, and comparing the expressions of \(V_\alpha'(y)\), \((T_\alpha V_\alpha)'(y)\) and \(V_\alpha'(y)\) obtained in (8) and (12), one concludes that:

- \(V_\alpha'(y) > (T_\alpha V_\alpha)'(y) > V_0'(y)\) for all \(y \in [0, y_0] \)

In particular, we conclude that \((T_\alpha V_\alpha)(y) - V_\alpha(y)\) is increasing in \(y\).

5) Now consider the \(t\)th iterate of the \(T_\alpha\) operator applied to \(V_\alpha\). For all \(t \geq 1\) and all \(y \in (0, y_0]\), one may construct \((T_\alpha)^t V_\alpha(y)\) recursively, given that \((T_\alpha)^t V_\alpha(y)\) is a differentiable, strictly concave, piecewise quadratic function on \([0, y_0]\). Let \(z_t'(y)\) be the solution of the following unconstrained program:

\begin{equation}
(15) \quad (T_\alpha)^t V_\alpha(y) = \text{Max}_{z \geq 0} \{h(z, y, a) - C + \rho (\alpha)^{-1} V_\alpha(z)\}
\end{equation}

Then \(z_t'(y)\) is the solution of the following equation:

\begin{equation}
z'_t = \{z \mid p_\alpha - z(1 - \alpha) + \phi(y-z) = -\rho (\alpha)^{-1} V_\alpha'(z)\}
\end{equation}

Because \((T_\alpha)^t V_\alpha(y)\) is concave and piecewise quadratic, (16) shows that for all \(y \in [0, y_0]\), \(z'_t(y)\) is a linear, increasing function in \(y\) with a slope less than unity. In addition, \(z'_t(0) > 0\). So there is a unique point, \(y_\alpha^*\), such that \(z'_t(y) \leq y\) if and only if \(y \geq y_\alpha^*\). Hence, one may rewrite \((T_\alpha)^t V_\alpha(y)\) as:
Using the envelope theorem, it is a simple matter to see that \((T_\nu^*V_\nu')\) is differentiable, strictly concave, and piecewise quadratic on \([0,y_\nu]\) and:

\[
(T_\nu^*V_\nu')(y) = \begin{cases} 
    p_\nu' y - \frac{(1-a)y^2}{2} - C + \rho((T_\nu)'^{-1}V_\nu)(y) & \text{if } y \in [0, y_\nu^*] \\
    (T_\nu^*V_\nu)(y) & \text{if } y \in [y_\nu^*, y_\nu]
\end{cases}
\]

Now we show by forward recursion the following property for \(\tau \geq 1\):

- \(z_{\tau+1}'(y) < z_\tau'(y) < z(a, y)\) and \((T_\nu)'^{-1}V_\nu)'(y) < (T_\nu)'V_\nu)'(y) < V_\nu'(y)\) for all \(y \in [0,y_\nu]\) and all \(t \leq \tau\).

Our discussion of \((T_\nu V_\nu)\) showed that this property is true for \(t=1\) (where \(z(0,y) = z_\nu(y)\) with the previous notation). Assume the property is also true for \(t > 1\). Then it is clear that for all \(t \leq \tau\), \(y_{\tau+1}^* < y_\tau^* < p_\nu/(1-a)\). To show that the property is true for \(t+1\), we need only show that:

- \(z_{\tau}'(y) < z_{\tau+1}'(y) < z(a, y)\) and \((T_\nu)'V_\nu)'(y) < (T_\nu)'V_\nu)'(y) < V_\nu'(y)\).

The first part of the assertion follows directly from \((T_\nu)'V_\nu)'(y) < (T_\nu)'V_\nu)'(y) < V_\nu'(y)\), equation (16), and the fact that \((T_\nu)'V_\nu)(y)\) is a strictly concave function in \(y\). The second part follows directly from (18).

Thus, the previous discussion establishes:

**Lemma 1:** For all \(y \in [0,y_d]\), for all \(t \geq 1\),

- **i)** The value function \((T_\nu^* V_\nu)(y)\) is differentiable and strictly concave.
- **ii)** \((T_\nu^* V_\nu)(y) - (T_\nu^* V_\nu)(y)\) is increasing in \(y\).
- **iii)** \(z_{\tau}'(y) < z_{\tau+d}'(y) < z(a, y)\) for all \(y \in [0,y_d]\) and \(t > 0\).

**II. The problem of lobbying with fixed costs**

We are now equipped to solve the problem of lobbying with fixed costs. In any period when the industry
has lobbied in the previous period, the industry chooses between lobbying and not lobbying. The value to an industry with an initial size \( y_0 \) of \( t \) periods of lobbying followed by no lobbying is simply \((T_t)\!V_0(y_0)\). The basic problem then is:

(A) \[
\text{Max}_{t \geq 0} \ (T_t)\!V_0(y_0)
\]

If the argmax of this problem is \( \infty \), then permanent lobbying will prevail. Otherwise, the industry stops lobbying after a finite number of periods and loses protection. We start by showing the following lemma:

**Lemma 2:** For all \( y_0 > p_{-1}, \)

i) If for some \( t \) \( \geq 1 \), we have \((T_{t+k})V_0(y_0) \leq (T_{t+k-1})V_0(y_0)\), then for all \( k > 0 \), we also have \((T_{t+k})V_0(y_0) < (T_{t+k-1})V_0(y_0)\).

ii) If for some \( t \) \( \geq 1 \), we have \( V_0(y_0) \leq (T_{t+k})V_0(y_0)\), then for all \( k > 0 \), we also have \( V_0(y_0) < (T_{t+k})V_0(y_0)\).

**Proof:**

i) Consider \( t \geq 1 \), such that \((T_{t+k})V_0(y_0) \leq (T_{t+k-1})V_0(y_0)\). Since \((T_{t+k})V_0(y_0)-(T_{t+k-1})V_0(y_0)\) is increasing on \([p^*, y_0]\), we conclude that for all \( y \) in \([p^*, y_0]\), \((T_{t+k})V_0(y) < (T_{t+k-1})V_0(y)\). Therefore, as \( z^{t+k}(y) > p^* \) for \( y \) in \([p^*, y_0]\), we get:

\[
(T_{t+k})V_0(y) = h(Min[z^{t+k}(y), y], y, a) - C + \rho(T_{t+k}V_0(Min[z^{t+k}(y), y]))
\]

\[
< h(Min[z^{t+k}(y), y], y, a) - C + \rho(T_{t+k-1}V_0(Min[z^{t+k}(y), y])) \leq (T_{t+k-1})V_0(y)
\]

Hence, \((T_{t+k})V_0(y) < (T_{t+k-1})V_0(y)\). Using a similar argument, we can show by recursion that for all \( k > 0 \) we also have for all \( y \) in \([p^*, y_0]\), \((T_{t+k})V_0(y) < (T_{t+k-1})V_0(y)\). Result i) follows immediately.

ii) Consider \( t \geq 1 \), such that \( V_0(y_0) \leq (T_{t+k})V_0(y_0)\). As \( V_0(y) - (T_{t+k})V_0(y) \) is increasing in \([p^*, y_0]\), we conclude that for all \( y \) in \([p^*, y_0]\), \( V_0(y) < (T_{t+k})V_0(y)\). Therefore, since \( z(a, y) > z^*(y) > p^* \) for \( y \) in \([p^*, y_0]\), we get:

\[
V_0(y) = h(Min[z(a, y), y], y, a) - C + \rho V_0(Min[z(a, y), y])
\]

\[
< h(Min[z(a, y), y], y, a) - C + \rho(T_{t+k}V_0(Min[z(a, y), y])) \leq (T_{t+k})V_0(y)
\]

Hence, \( V_0(y) < (T_{t+k})V_0(y)\). Using a similar argument, we can show by recursion that for all \( k > 0 \) we also have for all \( y \) in \([p^*, y_0]\), \( V_0(y) < (T_{t+k})V_0(y)\), and result ii) follows immediately.
Lemma 2 i) implies that if problem (A) has a finite solution, there are at most two points $\tau$ and $\tau + 1 < \infty$ that can be the solution. Hence, unless $y_0$ belongs to a set of isolated points $y$ (such that $([T_0]'V_0)(y) = ([T_0]'^{+1})V_0(y)$) there is at most one finite solution $\tau(y_0)$ to problem (A). Note that Lemma 2 i) holds for all $C > 0$.

Defining $T_0'$ as the operator $T_0$ associated with zero fixed costs, and recalling that $V_0'$ is the value of permanent lobbying with zero fixed costs, we rewrite $([T_0]'V_0)(y)$ as $([T_0]'^+V_0)(y) - C[1-p']/[1-p]$, and $V_0(y)$ as $V_0'(y) - C/[1-p]$. Temporary lobbying will arise if and only if:

- $\exists \tau < \infty$ such that $([T_0]'V_0)(y_0) > V_0(y_0)$

or equivalently:

- $\exists \tau < \infty$ such that $C/[1-p] > [V_0'(y_0) - ([T_0]'V_0)(y_0)]/p'$.

It is clear that $V_0(y) < V_0'(y)$ for all $y > 0$. Then by recursion $([T_0]'V_0)(y) < V_0'(y)$ for all $y > 0$. Also, since $V_0(0) = (T_0V_0)(0) = 0$ and $([T_0]'V_0)(y) - V_0(y)$ is increasing in $y$, it is clear that $V_0(y) < ([T_0]'V_0)(y)$ for all $y > 0$.

By recursion one can also see that $([T_0]'V_0)(y) < ([T_0]'^{+1}V_0)(y)$ for all $y > 0$. Finally, we can conclude that the sequence of points $([T_0]'V_0)(y_0)$ is monotonically converging to $V_0'(y_0)$ from below.

Let us define $u_\tau = [V_0'(y_0) - ([T_0]'V_0)(y_0)]/p'$. Then the condition, $\exists \tau < \infty$ such that $([T_0]'V_0)(y_0) > V_0(y_0)$, is equivalent to the condition: $\exists \tau < \infty$ such that $C/[1-p] > u_\tau$.

**Lemma 3:** The sequence of points $(u_\tau)$ is a decreasing sequence (ie. $u_\tau \geq u_{\tau+1}$ for all $\tau \geq 0$).

**Proof:** Suppose the contrary: $\exists \tau$ such that $u_\tau < u_{\tau+1}$. Then one can choose a level of fixed cost $C$ such that $u_\tau < C/[1-p] < u_{\tau+1}$. This implies that $\exists \tau < \infty$ such that: $([T_0]'V_0)(y_0) > V_0(y_0)$ and $([T_0]'^{+1}V_0)(y_0) < V_0(y_0)$. This contradicts Lemma 2 ii).

Since it is a decreasing, positive sequence, $(u_\tau)$ converges towards a limit $u \geq 0$. The following proposition guarantees that this limit is strictly positive:
Proposition 1: If $V_d(p^*) > V_d(p^*)$, i.e. $C < a p^* / 2$, then there is permanent lobbying and permanent protection. In this case, the adjustment path is given by $y_{t+1} = z(a, y_t)$ with the initial condition $y_0$.

Proof: Given that $V_y(y) > V_y(y)$ for all $y \in [p^*, y_0]$, $V_y(p_\omega) \geq V_y(p_\omega)$ implies that $\forall y \in [p^*, y_0]$, $V_y(y) \geq V_y(y)$. Therefore, by recursion, $\forall y \in (p^*, y_0]$, $V_y(y) > (T_tV_0)(y)$ for all $t \geq 1$. Thus, for all $t \geq 0$, $V_y(y_0) > (T_tV_0)(y_0)$, which says that it is always beneficial for the industry to continue lobbying. Consequently, we conclude that for any $y_0 \geq p_\omega / (1-a)$, there is permanent lobbying and protection. The adjustment process $y_t$ and domestic protection $p_t$ are such that $y_t = z(a, y_{t-1})$, and $p_t = p^* + a y_t(t-a)$.

A corollary follows directly from this proposition:

Corollary 1: For all $t \geq 0$ and all $C \leq a p^* / 2$, $u_t > C/[1-p]$. Hence $u = \lim u_t \geq a p^* / 2/[1-p] > 0$.

We are now able to state our main result:

Proposition 2:

i) If the fixed cost $C$ is such that $C/[1-p] \leq u$, then there is permanent lobbying and protection never collapses.

ii) If the fixed cost $C$ is such that $C/[1-p] > u$, then there exists a unique time $\tau(y_0) \geq 0$, such that the industry enjoys temporary protection for $\tau(y_0)$ periods, after which it stops lobbying and protection collapses.

iii) When protection is temporary, the adjustment path during the lobbying periods always lies between the free trade and permanent lobbying adjustment paths.

Proof:

i) If $C$ is such that $C/[1-p] \leq u$, then for all $t$, $C/[1-p] < u$, which is equivalent to $V_y(y_0) > (T_tV_0)(y_0)$.

Hence, permanent lobbying is optimal.

ii) If $C$ is such that $C/[1-p] > u$, then, because $u$, is decreasing (except in the case where $y_0$ belongs to a set of isolated points such that $(T_tV_0)(y) = (T_t^{t+1}V_0)(y))$, there exists a unique $t$ such that:
\[ u_t = \{V_s(y_0) - ([T_s']V_o)(y_0)\}/\rho ' = C/[1-\rho] \text{ and } C/[1-\rho] > u_{t+1} = \{V_s(y_0) - ([T_s']V_o)(y_0)\}/\rho '. \]

Then for all \( t' > t + 1 \),
\[ ([T_s']V_o)(y_0) > V_d(y_0). \]
Moreover, since the sequence \( ([T_s']V_o)(y_0) \) converges to \( V_d(y_0) \), there is a point, \( \tau(y_0) > t \), that reaches the sup \( ([T_s']V_o)(y_0) \). By Lemma 2 this point, \( \tau(y_0) \), is unique for almost every \( y_0 > p_\omega/(1-\alpha) \).

Obviously, then it is optimal for the industry to lobby for \( \tau(y_0) \) periods, and then to stop lobbying. Thus, there is temporary protection for \( \tau(y_0) \) periods.

iii) This follows immediately from Lemma 1 iii) and the fact that at time \( t \) along the adjustment path,
\[ y_{t+1} = \min(y_t, z_s'(y_t)). \]

An immediate corollary is:

**Corollary 2:** If \( ([T_{sa}]V_o)(y_0) < V_d(y_0) \), then there is no lobbying and no protection along the adjustment path.
References


