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SIGNALLING, INFORMATION DISSEMINATION,
AND TIMING OF INFORMATION ACQUISITION

by

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Sloan School W.P. No. 1767–86 February 1986

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ABSTRACT

The effect of strategic signalling is multifold. It allows the firm to fully exploit the information advantage, incurs an additional gain or cost, and reallocates the information in later periods. The importance of each effect depends on the informativity of the signal as well as the nature of the uncertainty. The equilibrium information exchange in the multiperiod oligopoly is determined by the tradeoffs among these factors. As a result, firms ex ante prefer a fast information dissemination when uncertainty is about the firms' specific parameters and prefer a slow information dissemination when the uncertainty is about a common parameter if the signals are not very informative. The timing of information dissemination will have a delay in the private value game and a move-up in the common value game if the signals are sufficiently accurate.
1. INTRODUCTION

In this article, we study a two-period model of a Cournot duopoly in which firms have sources to acquire information about some unknown parameters of the industry. Prior to learning the private signals and making output decisions, firms have a choice between acquiring information in the first period and in the second period. A choice of acquiring information in the first period implies that the firm fully utilizes the information and meanwhile signals the private information to the rival via its first-period strategy which are assumed to be observable between periods. While a choice of acquiring information in the second period provides a measure to conceal firms' private information from the rival. We are interested in deriving the subgame-perfect equilibrium for this multi-stage game.

There is an alternative way of stating the problem. The question we ask is: How fast does the private information disseminate among firms through the equilibrium itself in a multi-period Cournot game? In each period, firms have a choice of signalling the private information to other firms while fully exploiting the informational advantage or simply ignoring the private information and hence preventing the dissemination.

Intuitively, firms are always better off with more exclusive information in a one-shot game. But fully exploiting this advantage may result in revealing the private information and hence changing the information allocation in the later periods. This reallocation may not be favorable to the duopolists as suggested by the recent work on information sharing in a one-period game (Clarke (1983), Gal-Or (1985), Shapiro (1984), Li (1985)).
All the papers assume that information is verifiable and study how the profitability is affected by various allocations of the information. The information effect alone is a complex matter. On the one hand, receipt of additional information improves the efficiency of decisions by the recipients and hence the profitability. On the other hand, the pooling of information alters the degree of correlation among the decisions by the oligopolists and hence increases or decreases the profits. The behavior in information exchange is based on these tradeoffs. Despite the ambiguous result that might arise in various circumstances, a systematic difference between the consequences of sharing information about a common parameter and the firm's specific parameters is discovered. Assuming the signals that firms possess are equally informative, perfect revelation is the unique equilibrium when the uncertainty is about a firm-specific parameter, while no information sharing is the unique equilibrium when the uncertainty is about a common parameter.

We will show that the last period subgame in the multiperiod model is the same as discussed in the early information sharing literature in which the information effect is the only decisive factor. Thus, based upon the second period alone, we may conjecture that the oligopolists favor a slow information dissemination and then delay the information acquisition when the uncertainty is about a common value, while a fast information dissemination is preferable when the uncertainty is about private values. However, an additional "cost" due to signalling effects will be incurred when information exchange is implemented as an equilibrium outcome in the multiperiod model.

The result in this paper reveals that the equilibrium information exchange is determined by the tradeoffs of these three factors: the signalling effect, the first- and second-period information effects. The first-period
information effect always favors an early information acquisition. The second-period information effect favors a fast information dissemination in the private value case and favors a slow information dissemination in the common value case as mentioned earlier. The signalling effect is more complicated. But the signalling effect together with the first-period information effect always tends to shift the equilibrium away from the conjectured timing based on the second period alone. Here the informativity of the signals or the correlation between the signals plays an essential role. In comparison with the conjectured timing, based on the second-period information effect alone, there will be an advance in information acquisition in the common value game and a delay in the private value game if the signals are sufficiently informative. In other words, the conjectured timing stays as an equilibrium only when the signals that firms may acquire are less informative, otherwise the contribution of the signalling effects and the first-period information effects will be dominating.

Several authors (Milgrom and Roberts (1982), Matthews and Mirman (1983), Parsons (1985), Gal-Or (1985b), etc.) analyze the equilibria in multiperiod duopoly with private information. The prominent feature of these models is the strategic signalling. This paper attempts to integrate the findings of the information sharing literature and the signalling models and to provide a more complete analysis on the issue under a tractable linear information structure employed by Li (1985a). As a result, we may have a better understanding of the implications of signalling and the process of information dissemination through signalling.
This work is also inspired by Gibbons (1985) on a totally different subject, namely incentives in internal labor markets. The paper argues that compensation schemes that fully reflect all available performance indicators may reveal information about workers' abilities too quickly and workers may be better off with coarser schemes that reveal information more slowly. We provide some other examples in which players prefer slower information revelation resulting from the equilibrium behavior of information transmission.

The body of the article is organized as follows. The discussions about two cases—common value versus private value—are paired in Section 2 and 3. In each case, we start by stating assumption and developing notation. Then we construct the equilibria for the two-period game representing various patterns of information dissemination and discuss in detail the informational and signalling effects. Finally, the equilibrium timing of information acquisition is characterized. Section 4 includes concluding remarks.
2. COMMON VALUE CASE

2.1 The Model

We first study the common value case in which the unknown parameter $\theta$ is a zero-mean random variable added to the intercept of the linear inverse demand, $p = a + \theta - bQ$, where $Q$ is the total output in the industry. The duopoly produces at a constant marginal cost $c$. The signal firm $i$ may acquire is $x_i$, $i=1,2$. We also assume the expectation of the unknown conditional on the signals is linear in signals. Formally,

**Assumption 1:** $E[x_i|\theta] = \theta$, $i=1,2$.

**Assumption 2:** $E[\theta|x_1, x_2] = a_0 + a_1 x_1 + a_2 x_2$, where $a_i$ are constants.

**Assumption 3:** $x_1$ and $x_2$ are independent conditional on $\theta$.

As a result (see Lemma 1 in Li (1985a))

\[
E[r|x_1] = \frac{t_1}{R + t_1} x_1 \quad \text{and} \quad (1)
\]

\[
E[r|x_1, x_2] = \frac{1}{R + t_1 + t_2} (t_1 x_1 + t_2 x_2)
\]

where $R = 1/\text{Var}[\theta]$ and $t_i = 1/E[\text{Var}[x_i|$], $i=1,2$. For simplicity, we further assume that the signals are equally informative, i.e., $t_i = t$, $i=1,2$. Also notice that

\[
\rho = \frac{t}{R + t} \quad . \quad (2)
\]
where $\rho$ is the correlation coefficient between $x_1$ and $x_2$. The mapping from $t$ to $\rho$ is one-to-one and strictly increasing. Thus, we can use either the expected conditional precision of the signal, $t$, or the correlation coefficient between the signals, $\rho$, as a measure of the informativity in the common value case. To facilitate the comparison with the private value case, we use $\rho$ and rewrite (1) as

$$E[\theta|x_1] = \rho x_1$$

and

$$E[\theta|x_1, x_2] = \frac{\rho}{1 + \rho} (x_1 + x_2)$$

The joint distribution of $\theta$, $x_1$, $x_2$ is common knowledge.

Before learning their signals and making the output decisions, firms first decide whether they want to observe its exclusive information in the first period or in the second period. Then the first-period outputs are simultaneously selected based on the observed information, if any, depending on the earlier timing decisions. Between the first and second period, each firm observes the first-period outputs, and if it has not yet observed its private signal, it may do so then. Finally, the second-period output decisions are made again in a simultaneous move based on any additional information the firms have or can infer. This is a three-stage game. It can be solved backwards since we are only interested in the subgame-perfect equilibrium.

2.2 Two-Period Game Equilibrium

Give the first-stage decisions on the timing of information acquisition, it results in three possible cases:
Case 1. Both firms acquire information in the first period;
Case 2. One firm acquires information in the first period and the other in the second period.
Case 3. Both firms acquire information in the second period.

We proceed to solve the equilibrium strategies of the above three cases one by one.

Denote by \( q_{i1} \) the first-period strategy for firm \( i \) which may or may not depend on the private signal \( x_i \) according to his decision on when to observe the information. At the beginning of the second period, each firm observes the rival's output in the first period and now definitely his private signal \( x_i \), and then chooses the second-period output \( q_{i2} \) which may depend on the private signal \( x_i \) and the first-period strategies. By the definition of the subgame perfection, the equilibrium can be solved backwards.

Case 1: We assume in this case that \( q_{i1}, i=1,2, \) are invertable. Thus, in the second period, firm \( i \) can infer the rival's signal \( x_j \) by inverting \( q_{j1} \), i.e., \( x_j = q_{j1}^{-1}(y_j) \) if the observed first-period output of firm \( j \) is \( y_j \). Firm \( i \) chooses \( q_{i2} \) to maximize the profit based on \( x_i, q_{i1}^{-1} \) and \( q_{j2}^{-1}, j \neq i \). This is simply a one-period Cournot model with private information solved by Li (1985a) and others. There is a unique equilibrium to the subgame which is linear and the equilibrium strategies are

\[
q_{i2} = \frac{a-c}{3b} + \frac{\rho}{6b(1+\rho)} (3x_i - q_{i1}^{-1} + 2q_{j1}^{-1}) \quad \text{for } i=1,2, j \neq i. \quad (3)
\]

Note that if we let \( x_i = q_{i1}^{-1} \) in (3), then

\[
q_{i2} = \frac{a-c}{3b} + \frac{\rho}{3b(1+\rho)} (x_1 + x_2).
\]

This is nothing but a Cournot solution with pooled information by noticing
\[ E[\theta|x_1, x_2] = \frac{\rho}{1+\rho} (x_1 + x_2). \] And the second-period payoff to i conditional on \( x_i, q_{ij}^{-1}, \) and \( q_{ij}^{-1}, j \neq i \) is \( bq_{i2}^2 \).

Assuming the second-period equilibrium strategies will follow and the second-period profits are discounted by a factor \( \beta \ (0 < \beta \leq 1) \), the total expected payoff to firm i conditional on \( x_i \) is

\[
E[\pi_{i1}|x_i] + \beta E[\pi_{i2}|x_i] = q_{i1}(a-c+E[\theta|x_i] - bq_{i1} - bE[q_{j1}|x_i]) + \beta bE[q_{i2}^2|x_i], j \neq i.
\]

The first-order condition yields

\[
-2bq_{i1} - \frac{\beta \rho}{3(1+\rho)} \left( \frac{a-c}{3b} + \frac{\rho}{3b} x_i \right) \frac{1}{q_{i1}} + a-c + \rho x_i - bE[q_{j1}|x_i] = 0 \quad \text{for } i=1,2, j \neq i.
\]

System of differential equations (4) provides a necessary condition for the first-period equilibrium strategies. It is not easy to characterize all possible candidates. To maintain tractability, we look at linear strategies only. Let \( q_{i1} = \gamma_i + \alpha_i x_i \) be a linear strategy. Equations (4) imply \( \gamma_i, \alpha_i \) must satisfy

\[
\rho(1-b\alpha_j) - 2b\alpha_i - \frac{\beta \rho^2}{9b(1+\rho)} \cdot \frac{1}{\alpha_i} = 0, \quad \text{(5)}
\]

\[
a-c-b\gamma_j - 2b\gamma_i - \frac{(a-c)\beta \rho}{9b(1+\rho)} \cdot \frac{1}{\alpha_i} = 0, \quad \text{for } i=1,2, j \neq i. \quad \text{(6)}
\]

Quadratic equation (5) has two symmetric and one asymmetric solution. But only one symmetric solution satisfies the sufficient condition for an
equilibrium. This solution has the form

\[ a_i = \frac{\rho}{b(2+\rho)} (1-\mu), \quad i = 1, 2, \] (7)

where

\[ \mu(\beta, \rho) = \frac{1}{2} \left(1 - \sqrt{1 - 4\beta(2+\rho)/9(1+\rho)} \right). \]

It is easy to see that \( \mu(\beta, \rho) \) is increasing in \( \beta \) and decreasing in \( \rho \) and \( \mu(0, \rho) = 0, \mu(1, 0) = 1/3 \). Therefore, \( 0 < \mu < 1/3 \) for \( 0 < \beta \leq 1 \) and \( 0 < \rho < 1 \). Solving (6) with \( a_i \) in (7), we have

\[ \gamma_i = \frac{a-c}{3b} \left(1 - \frac{\beta(2+\rho)}{9(1+\rho)} \cdot \frac{1}{1-\mu} \right) = \frac{a-c}{3b} (1-\mu) \]

since \( \mu \) solves the quadratic equation

\[ \mu^2 - \mu + \frac{\beta(2+\rho)}{9(1+\rho)} = 0. \]

**PROPOSITION 1** In Case 1, there is a unique linear strategy equilibrium to the two-period game which is symmetric and the equilibrium strategies satisfy:

\[ q_{i1} = (1-\mu) q_i \]

where \( \mu = \frac{1}{2} \left(1 - \sqrt{1 - 4\beta(2+\rho)/9(1+\rho)} \right) \) and \( q_i = \frac{a-c}{3b} + \frac{\rho}{b(2+\rho)} \ x_i \),

\[ q_{i2} = \frac{a-c}{3b} + \frac{\rho}{6b(1+\rho)} \left(3x_i - q_{i1}^{-1} + 2q_{j1}^{-1}\right) \quad \text{for} \quad i = 1, 2, \quad j \neq i. \]

Moreover, \( 0 < \mu < 1/3 \) for \( 0 < \beta \leq 1 \) and \( 0 < \rho < 1 \) and \( \mu \) increases as \( \beta \) increases or \( \rho \) decreases.

Some interesting observations can be obtained from the proposition. Note that \( q_i \) is just the Bayesian strategy for a one-period Cournot game with private information \( x_i, \ i=1,2 \), held by each firm. Thus, the first-period equilibrium strategy in a two-period Cournot game with bilateral information transmission is just the one-period equilibrium strategy scaled down by a factor \( (1-\mu) \). The multiplier \( \mu \) measures the deviation due to the
signalling effects. Obviously, $\beta$ is positively related to the signalling effects. When $\beta = 0$, $\mu = 0$, $q_{11} = q_1$ and the game is simply reduced to the one period. Otherwise, the duopolists produce less in the first period than what would be produced in the one-period game. On the other hand, a high $\rho$ (better signals) enhances the importance of the information effects and hence the duopolists would rather stay closer to the strategies for the case in which the information effect presents solely. Also notice that the total expected first-period output $\frac{2(a-c)}{3} (1-\mu)$ is bounded above by the classic duopoly output $\frac{2(a-c)}{3}$ and below by $\frac{4(a-c)}{9b}$ which is lower than the monopoly output $\frac{a-c}{2b}$. The expected first-period price may exceed the monopoly price. This occurs when the discount factor is high and the informativity of the signal is low. This collusive-like behavior arises purely from the equilibrium information transmitting.

**Case 2.** Without loss of generality, it suffices to examine the case in which firm 1 observes $x_1$ in the first period and firm 2 observes $x_2$ in the second period. Following the same approach as employed in Case 1, we first solve the second-period game in terms of the private signals plus the additional information to firm 2 that he can infer. The unique equilibrium strategies are

\[
q_{12} = \frac{a-c}{3b} + \frac{\rho(2+\rho)}{4b(1+\rho)} x_1 - \frac{\rho(2-\rho)}{12b(1+\rho)} q_1^{-1}
\]

\[
q_{22} = \frac{a-c}{3b} + \frac{\rho}{2b(1+\rho)} x_2 + \frac{\rho(2-\rho)}{6b(1+\rho)} q_1^{-1}
\]

If we substitute $q_1^{-1}$ by $x_1$, then
This is again the Cournot-Bayesian equilibrium for a one-period game with firm 1 uninformed and firm 2 informed of l's type \( x_1 \). The \textit{ex post} second-period payoffs to firm 1 and 2 are \( bq_{12}^2 \) and \( bq_{22}^2 \), respectively. Therefore, in the first period, firm 1 chooses a \( q_{11} \) as a function of \( x_1 \) to maximize

\[
E [\pi_{11} | x_1] + \beta E [\pi_{12} | x_1]
\]

\[
= q_{11} (a-c + E[\theta | x_1] - bq_{11} - bE[q_{21} | x_1]) + \beta b E[q_{21}^2 | x_1]
\]

given \( q_{21} \). While firm 2 chooses a \( q_{21} \) so as to maximize

\[
E [\pi_{21}] = q_{21} (a-c-bq_{21} - bE[q_{11}])
\]

since firm 2 does not possess any private information and his first-period strategy has no effect on the second-period payoff. The first-order conditions yield,

\[
-2bq_{11} - \frac{8\beta (2-\rho)}{6(1+\rho)} \left( \frac{a-c}{3b} + \frac{\rho}{3b} x_1 \right) \frac{1}{q_{11}} + a-c + \rho x - bE [q_{21} | x_1] = 0 \tag{8}
\]

\[
-2bq_{21} + a-c - bE [q_{11}] = 0. \tag{9}
\]

Solving (8) and (9), we obtain the candidates for the equilibrium strategies and check against the second-order condition.

**PROPOSITION 2.** There is a unique linear equilibrium to the two-period game in Case 2 given by
\[ q_{11} = \frac{a-c}{3b} (1-2\nu) + \frac{\rho}{2b} (1-\nu) x_1, \]  
\[ q_{21} = \frac{a-c}{3b} (1+\nu), \]  
\[ q_{12} = \frac{a-c}{3b} + \frac{\rho(2+\rho)}{4b(1+\rho)} x_1 - \frac{\rho(2-\rho)}{12b(1+\rho)} q_{11}, \]  
\[ q_{22} = \frac{a-c}{3b} + \frac{\rho}{2b(1+\rho)} x_2 + \frac{\rho(2-\rho)}{6b(1+\rho)} q_{11} \]

where \( \nu(\beta, \rho) = \frac{1}{2} (1 - \sqrt{1-4\beta(2-\rho)/9(1+\rho)}) \). And \( \nu \) increases as \( \beta \) increases or \( \rho \) decreases and \( 0 < \nu < 1/3 \) for \( 0 < \beta \leq 1 \) and \( 0 < \rho < 1 \).

Note that \( (q_1, q_2) \) such that

\[ q_1 = \frac{a-c}{3b} + \frac{\rho}{2b} x_1 \]  
\[ q_2 = \frac{a-c}{3b} \]  

is the unique equilibrium for the one-period Cournot game with asymmetric information, i.e., firm 1 observes \( x_1 \) and firm 2 has no private information. Comparing (12), (13) with (10), (11), we conclude that in the presence of unilateral information transmission, the firm which takes an active role in signalling (firm 1) produces less while the firm which passively receives the transmitted signal in the second period (firm 2) produces more than what would be produced in absence of signalling. Again, the multiplier \( \nu \) measures the deviation due to the signalling effects. And the higher \( \beta \) is or the lower \( \rho \) is, the larger the deviation \( \nu \) will be. In the expected values, the total first-period output \( \frac{a-c}{3b} (2-\nu) \) is less than that in absence of signalling \( \frac{2(a-c)}{3b} \), but higher than \( \frac{5(a-c)}{9b} \) which is bounded above the monopoly output \( \frac{a-c}{2b} \).
The expected first-period equilibrium price with unilateral information transmission is lower than the monopoly price. Moreover, the total output \( \frac{a-c}{3b} (2-\nu) \) is higher than that in case 1 and hence the expected first-period equilibrium price with unilateral information transmission is lower than that with bilateral information transmission.

**Case 3.** This case is trivial. Since there is no signalling between periods, the Cournot-Nash and Cournot-Bayesian solutions to first and second periods, respectively, constitute the equilibrium.

**Proposition 3.** There is a unique equilibrium to the two-period game in Case 3 given by

\[
q_{i1} = \frac{a-c}{3b}
\]

\[
q_{i2} = \frac{a-c}{3b} + \frac{\rho}{b(2+\rho)} x_i, \quad \text{for } i=1,2.
\]

### 2.3 Timing of Information Acquisition

We now derive the *ex ante* profits of the duopolists for the three cases. We use a superscript to indicate each case and keep the subscripts for the players and periods. For example, \( \pi^k_{ij} \) is the expected \( j^{th} \) period payoff to firm \( i \) in case \( k \). In order to identify the effects due to asymmetric information and signalling, we write each \( \pi^k_{ij} \) in terms of three parts. Denote by \( D \) the contribution in absence of private information which is just the duopoly profit in a deterministic model. Denote by \( I^k_{ij} \) the contribution of the information effect which is the gain with private
information but without signalling additional to $D$, and $S_{ij}^k$ represents the contribution of the signalling effect which is the further additional gain or loss because of signalling.

We start with the second-period payoffs which can be obtained from the results in Li (1985a) by making the transformation (2) to replace $t$ by $\rho$. The following proposition lists the results as a reference.

**PROPOSITION 4.** The expected second-period profits are:

$$\pi_{i2}^k = D + I_{i2}^k \quad i = 1,2 \quad k = 1,2,3$$

where

$$D = \frac{(a-c)^2}{9b},$$

$$I_{i2}^1 = \frac{2\rho}{9bR(1+\rho)} , \quad i = 1,2,$$

$$I_{i2}^2 = \frac{\rho}{9bR} , \quad I_{i2}^2 = \frac{\rho(5-\rho)}{18bR(1+\rho)} ,$$

$$I_{i2}^3 = \frac{\rho}{bR(2+\rho)^2} , \quad i = 1,2.$$

And

$$I_{i2}^2 > I_{i2}^3 > I_{i2}^1 > I_{i2}^3 \quad \text{for } i = 1,2 \text{ and } 0 < \rho < 1. \quad (14)$$

The profit ordering given by (14) establishes the basic result in the literature on information sharing about the common demand uncertainty, namely, no sharing of information is the unique stable situation. Let us briefly repeat the argument. First, the complete pooling of information (Case 1) is not an equilibrium since if one firm (firm 2) does not reveal his information,
the it turns out to be in Case 2 and firm 2 is better off \( (I_{22}^2 > I_{22}^1) \).

Second, the asymmetric sharing of information (Case 2) is not stable either since the firm which reveals the private information (firm 1) has incentive to reverse its decision and brings about Case 3 \( (I_{12}^3 > I_{12}^2) \). Thus no pooling (Case 3) is the unique equilibrium.

The second-period information effect alone suggests that the duopoly prefers a delay in information acquisition. However, there are other accounts such as the first-period information effect and the signalling effect which may play a more important role.

Based on the explicitly calculated equilibrium strategies, the first-period payoffs are rather easy to obtain by noticing that

\[
\pi_{11} = E \left[ q_{11} (a-c + \theta - b(q_{11} + q_{21})) \right]
\]

and

\[
E[\theta x_1] = E[x_1, x_2] = \frac{1}{R}, \quad E[x_1^2] = \frac{1}{R^2}, \quad i=1,2.
\]

**PROPOSITION 5.** The expected first-period payoffs are

\[
\pi_{11}^k = D + I_{11}^k + S_{11}^k, \quad i=1,2, \quad k=1,2,3,
\]

where

\[
D = \frac{(a-c)^2}{9b},
\]

\[
I_{11}^1 = \frac{\rho}{bR(2+\rho)^2},
\]

\[
S_{11}^1 = \mu(1-2\mu)D + \mu (\rho-(1+\rho)\mu) I_{11}^1,
\]
\[ I_{11}^2 = \frac{6}{4}\mu^2, \quad S_{11}^2 = -\nu(1+2\nu)D - 2\nu^2 I_{11}, \]
\[ I_{21}^2 = 0, \quad S_{21}^2 = \nu(2+\nu)D, \]
\[ I_{11}^3 = S_{11}^3 = 0. \]

And
\[ I_{11}^2 > I_{11}^1 > I_{21}^2 = I_{11}^3. \]  
(15)

Notice that \( D \) is the Cournot duopoly profit with no private information and \( D + I_{11}^k \) is the profit to firm \( i \) in a one-period Cournot game with the structure of private information indicated by \( k \) (Case \( k \)). One way to check it is to let \( \beta \) be zero. Then \( \mu = \nu = 0, S_{11}^k = 0 \) for \( i=1,2 \) and \( k=1,2,3 \), and the game is reduced to one period. Thus, we think of \( I_{11}^k \) as the contribution of the information effect and \( S_{11}^k \) the contribution of the signalling effect.

Regarding the information effect, the contribution of the information effect ranks in a complete reversed order in (15) comparing with that in second period (14). The firm has to pay the information losses in the first period in order to gain the second-period advantage. The profit ordering (15) also coincides with our intuition that firms are always better off by possessing more private information. The contributions of the signalling effects are a bit more complicated. In the asymmetric signalling (Case 2), firm 1 which acquires information early and signals to firm 2, suffers a loss due to signalling \( (S_{11}^2 < 0) \), while firm 2 which delays information acquisition enjoys a gain due to the signalling effect \( (S_{21}^2 > 0) \). However, firm 1 grasps a first-period information advantage over firm 2 \((I_{11}^2 < I_{21}^2 = 0)\). In the case of symmetric signalling (Case 1),
the contribution of signalling is ambiguous. The contribution will be positive if \( \rho \) is big or \( \beta \) is small. The equilibrium behavior is based on these tradeoffs within one period and across periods.

Denote by \( \pi^k_i \) the total \textit{ex ante} payoff to firm \( i \) in Case \( k \), i.e.,

\[
\pi^k_i = \pi^k_{i1} + \beta \pi^k_{i2}.
\]

**PROPOSITION 6.** \( \pi^1_i > \pi^3_i \). That is, the firms are better off when both acquire their information in the first period in comparison with a simultaneous delay.

**Proof:**

\[
\pi^1_i - \pi^3_i = \pi^1_{i1} + S^1_{i1} - \beta \pi^1_{i2} - \beta S^3_{i2} = \mu(1-2\mu) \frac{a-c^2}{9b} + (1-\mu) \frac{\rho}{br(2+\rho)^2} + \frac{\rho(1+\sigma)}{br(2+\rho)^2} (1-\mu) + \frac{2b\rho}{9br(1+\sigma)} - \frac{\beta \rho}{br(2+\rho)^2} .
\]

Note that \( \mu(1-\mu) = \frac{\sigma(2+\sigma)}{9(1+\sigma)} \).

We have \( \pi^1_i - \pi^3_i = \mu(1-2\mu)D + (1-\mu) \frac{\rho(1+2\rho+3\rho)}{9br(1+\rho)(2+\rho)^2} > 0 \) Q.E.D.

This result turns out contrary to our conjecture based on the one-period information exchange which says that a duopoly facing an uncertain common demand prefers keeping their information to publicizing it. In a two-period game, the contribution of the informational effects if observing it early (Case 1) is so big that it outweighs not only the contribution of information effects in Case 3 but also the possible cost due to signalling.
However, either case can be an equilibrium depending on the informativity of the signals.

**PROPOSITION 7.** There exist two critical numbers $\rho_1^*$ and $\rho_2^*$ such that $0 < \rho_1^* < 1$ and $0 < \rho_2^* \leq 1$. For $\rho \geq \rho_1^*$, that both firms acquire information in the first period is an equilibrium. For $\rho \leq \rho_2^*$, that both firms acquire information in the second period is an equilibrium. And for $\rho_2^* \leq \rho \leq \rho_1^*$, that one firm acquires information earlier and the other later is an equilibrium.

**Proof:**

$$
\Delta_{12}(\rho) \equiv \frac{\pi_1}{1} - \frac{\pi_2}{2} = \frac{I_1}{11} + \frac{S_1}{11} + \beta I_{12} - \frac{S_2}{21} - \beta I_{22}
$$

$$
= (\mu(1-2\mu) - \nu(2+\nu))(a-c)^2 + (1-\mu)^2 \frac{\rho}{bR(2+\rho)}^2 + \frac{6\rho}{18bR} .
$$

Let $h(\rho, \beta) = \mu(1-2\mu) - \nu(2+\nu)$. The term $h$ is the only one in the expression which may be negative. But

$$
\frac{\partial h}{\partial \rho} = \frac{8}{9R(1+\rho)^2} \left( \frac{6(1+\nu)}{1-2\nu} - \frac{1-4\mu}{1-2\mu} \right)
$$

$$
> \frac{8}{9R(1+\rho)^2} \cdot \frac{5}{1-2\mu} > 0 \text{ since } \mu > \nu.
$$

Also $\mu$ is decreasing in $\rho$. Thus $\Delta_{12}$ is increasing in $\rho$. When $\rho=0$, $\mu=\nu$, and

$$
\Delta_{12} = -\mu(1+3\mu)(a-c)^2 \frac{9b}{9b} < 0.
$$

Notice that $h(1,0) = 0$, $h(1,1) = (3\sqrt{3} + 9\sqrt{7} - 29)/18 > 0$, and $h$ is concave in $\beta$.

So $h(1,\beta) > \min(h(1,0), h(1,1)) \geq 0$. Therefore $\Delta_{12}(1) > 0$. Given any other parameters, there is a unique $\rho_1^*$ such that $\Delta_{12}(\rho_1^*) = 0$, $\Delta_{12}(\rho) < 0$ if $\rho < \rho_1^*$, and $\Delta_{12}(\rho) > 0$ if $\rho > \rho_1^*$. 
Similarly calculate:

\[
\Delta_{23}(\rho) = \pi_1^2 - \pi_1^3 = I_{11}^2 + S_{11}^2 + 8I_{12}^2 - 8I_{12}^3
\]

\[
= -v(1+2v)\frac{(a-c)^2}{9b} + (1-v)(1-3v) \frac{\rho}{4bR} + \frac{6\rho(1+\rho-\rho^2)}{3bR(1+\rho)(2+\rho^2)}
\]

Again, \( \Delta_{23} \) is increase in \( \rho \) since \( v \) is decreasing in \( \rho \). And

\[
\Delta_{23}(0) = -v(1+2v)\frac{(a-c)^2}{9b} < 0.
\]

But the sign of \( \Delta_{23}(1) \) depends on other parameters. For example, \( \Delta_{23}(1) \) may be positive if \( b \) is small, \( a-c \) is small or \( b \) is big. Nevertheless, there is a \( \rho_2^* \) (maybe 1) such that \( \Delta_{23}(\rho) < 0 \) if \( \rho < \rho_2^* \), \( \Delta_{23}(\rho) > 0 \) if \( \rho_2^* < \rho < 1 \), and \( \Delta_{23}(\rho_2^*) > 0 \). Q.E.D.

Generally, in the common value case, the earlier information observation results in a first-period advantage and a second-period disadvantage due to information effects and a possible cost of signalling, especially when the signals are less informative. The equilibrium behavior is determined by these tradeoffs. When the signals are less informative, firms cannot afford the signalling cost and hence delay the information acquisition. When the signals are accurate, the information effect is dominating and reduces the signalling cost and firms prefer the earlier information acquisition. The second-period information effect correctly predicted by the one-period model of information sharing always favors a delay, but this effect diminishes as the signals become more and more informative.
3. **PRIVATE VALUE CASE**

3.1 **The Model**

Consider a similar Cournot duopoly facing a deterministic inverse demand \( p = a - bQ \). Each firm produces a homogeneous good at a constant cost \( c_i \) with mean \( c \). Firm 1 can only acquire a signal \( x_i \) about its own cost \( c_i \), i.e., \( x_i = c_i - c \). And the signals have the same precision \( R = 1/\text{Var}[x_i] \).

We still restrict our discussion to a class of distributions such that the conditional expectations obey a linear property. Equivalently,

**Assumption 4.** \( E[x_i|x_j] = \rho x_j, \forall i, \) where \( \rho \) is the correlation coefficient between \( x_i \) and \( x_j \) and \( 0 \leq \rho < 1 \).

The joint distribution of \( (x_1, x_2) \) is assumed to be common knowledge.

At the very beginning, firms decide whether to observe the exclusive information in the first period or in the second period. Following the same approach in the previous section, we first solve the two-period game for each first-stage decision and then study the equilibrium behavior for the full game assuming the equilibrium strategies follow in the subsequent stages.

3.2 **Two-Period Game Equilibrium**

We now solve the equilibrium strategies for three cases resulting from the first-stage timing decisions, namely, both firms acquire information in the first period (Case 1), firm 1 acquires information in the first period and
firm 2 in the second period (Case 2), and both acquire information in the second period (Case 3).

**PROPOSITION 8.** There is a unique linear equilibrium for each case and the equilibrium strategies, \((q_{i1}, q_{i2}), i=1,2\), satisfy:

**Case 1.**

\[ q_{i1} = \frac{a-c}{3b} \left( 1 + \frac{1}{2-\rho} \mu \right) - \frac{1}{b(2+\rho)} (1+\mu) x_i \]

\[ q_{i2} = \frac{a-c}{3b} - \frac{1}{6b} q_{i1}^{-1} + \frac{1}{3b} q_{j1}^{-1} - \frac{1}{2b} x_i, \quad j \neq i, \quad i=1,2, \]

where \(\mu = \frac{1}{2} \left( \sqrt{1+4\theta (4-\rho^2)/9} -1 \right)\). Moreover \(0 < \mu < \frac{1}{3}\), \(\mu\) is increasing in \(\beta\) and decreasing in \(\rho\) and \(\frac{1}{2-\rho}\) \(\mu\) is increasing in both \(\beta\) and \(\rho\).

**Case 2.**

\[ q_{i1} = \frac{a-c}{3b} \left( 1 + \frac{2}{2-\rho} \upsilon \right) - \frac{1}{2b} (1+\upsilon) x_1, \]

\[ q_{21} = \frac{a-c}{3b} \left( 1 - \frac{1}{2-\rho} \upsilon \right), \]

\[ q_{12} = \frac{a-c}{3b} - \frac{2-\rho}{12b} q_{11}^{-1} - \frac{2-\rho}{4b} x_1, \]

\[ q_{22} = \frac{a-c}{3b} + \frac{2-\rho}{6b} q_{11}^{-1} - \frac{1}{2b} x_2, \]

where \(\upsilon = \frac{1}{2} \left( \sqrt{1+4\theta (2-\rho)^2/9} -1 \right)\). Both \(\upsilon\) and \(\frac{1}{2-\rho}\) \(\upsilon\) are increasing in \(\beta\) and decreasing in \(\rho\) and \(0 < \upsilon < 1/3\).

**Case 3.**

\[ q_{i1} = \frac{a-c}{3b} \]

\[ q_{i2} = \frac{a-c}{3b} - \frac{1}{b(2+\rho)} x_i, \quad i=1,2. \]

In contrast to the common value case, the firms who are active in information transmitting (firm 1 and 2 in Case 1 and firm 1 in Case 2) always
produce more in the first period than what should be produced in absence of signalling, whereas the passive firm (firm 2 in Case 2) produces less. Parameters \( \mu \) and \( \nu \) may be viewed as a measure of strategic deviation due to the signalling effect. For example, in Case 1, for \( \mu = 0 (\beta=0) \)

\[
q_{i1} = \frac{a-c}{3b} - \frac{1}{b(2+p)} x_i, \quad i=1,2.
\]

This is just the one-period Cournot-Bayesian solution provided that each firm possesses some private signal \( x_i \). And the deviation is smaller if the second-period effect carries less weight (a smaller \( \beta \)) or the signals are more informative (a higher \( \rho \)). It is noteworthy that there is one exception. That is, the deviation of the average output in the case of bilateral information exchange (Case 1), \( \frac{1}{2-p} \mu \), is larger if \( \rho \) is higher.

To see the magnitude of deviation in the expected equilibrium price, we denote by \( Q_1^k \) the total first-period output in case \( k \). Then

\[
\frac{2(a-c)}{3b} < E[Q_1^1] = \frac{2(a-c)}{3b}(1 + \frac{1}{2-p} \mu) < \frac{(3+\sqrt{21})(a-c)}{9b},
\]
\[
\frac{2(a-c)}{3b} < E[Q_1^2] = \frac{(a-c)(2 + \frac{1}{2-p} \nu)}{3b} < \frac{13(a-c)}{18b},
\]
\[
E[Q_1^3] = \frac{2(a-c)}{3b}.
\]

Also note that \( 13(a-c)/18b < 3(a-c)/4b < (3+\sqrt{21})(a-c)/9b \) and \( 3(a-c)/4b \) is the three-firm oligopoly output in expected value without signalling. Thus, on average, the first-period equilibrium price with unilateral information exchange is lower but bounded above the three-firm oligopoly price. But equilibrium price with bilateral exchange of information can be lower than the three-firm oligopoly price. Also note that \( E[Q_1^1] > E[Q_1^2] \) since \( \mu > \nu \).
By substituting $\frac{q_{i1}}{q_{i1}}$ with $x_1$, we can easily identify that the second-period strategies are nothing but the one-period Cournot-Bayesian strategies given the information available to each firm. For example, in Case 1,$$
abla_{12} = \frac{a-c}{3b} - \frac{1}{3b} (2x_1 - x_j) \quad j \neq i, i=1,2,$$
and in Case 2,$$
abla_{12} = \frac{a-c}{3b} - \frac{2-p}{3b} x_1 ,$$
$$
abla_{22} = \frac{a-c}{3b} + \frac{2-p}{6b} x_1 - \frac{1}{2b} x_2 .$$

3.3 Timing of Information Acquisition

We employ the same notation as in Section 2 and derive the ex post profits period by period.

**Proposition 9.** The expected second-period profits are

$$\Pi_{12}^k = D + \Pi_{12}^k \quad i=1,2, \quad k=1,2,3,$$

where

$$D = \frac{(a-c)^2}{9b},$$

$$\Pi_{12}^1 = \frac{5-4p}{9bR} , \quad i=1,2,$$

$$\Pi_{12}^2 = \frac{(2-p)^2}{9bR} , \quad \Pi_{22}^2 = \frac{(1-2p)^2}{9bR} + \frac{1-p^2}{4bR} ,$$

$$\Pi_{12}^3 = \frac{1}{bR(2+p)^2} , \quad i=1,2.$$

and,

$$\Pi_{12}^1 > \Pi_{12}^2 > \Pi_{22}^2, \quad \Pi_{12}^1 > \Pi_{12}^2 > \Pi_{12}^3 .$$

(16)
The second-period information effect is just the same as the one-period model discussed in Li (1985a) where the profit ordering (16) implies that firms like to share the information about the private costs since complete information sharing is much like a dominant strategy equilibrium noticing that 

\[ I_{22}^1 > I_{22}^2 \text{ and } I_{12}^2 > I_{12}^3. \]

This observation implies that the duopolists tend to move up the information acquisition in order to obtain a second-period information advantage if the uncertainty is about the private costs.

**PROPOSITION 10.** The expected first-period payoffs are

\[
\pi_i^k = D + I_{i1}^k + S_{i1}^k \quad i=1,2, \quad k=1,2,3
\]

where

\[
D = \frac{(a-c)^2}{9b},
\]

\[
I_{i1}^1 = \frac{1}{bR(2+\rho)^2},
\]

\[
S_{i1}^1 = -\frac{1}{2-\rho} \mu (1+\frac{2}{2-\rho} \mu) D - \mu(\rho + (1+\rho)\mu) I_{i1}^1,
\]

\[
I_{i1}^2 = \frac{1}{4bR}, \quad S_{i1}^2 = \frac{1}{2-\rho} \nu(1-\frac{2}{2-\rho} \nu)D - \nu^2 I_{i1}^2,
\]

\[
I_{i1}^3 = S_{i1}^3 = 0.
\]

and

\[ I_{i1}^1 > I_{i1}^2 > I_{21}^2 = I_{i1}^3. \] (17)
The ordering of the information contribution (17) tells us that a firm is always better off if the signal is acquired in the first period. Note that this effect is the same as that of common value case. But it works here in the same direction as the second-period information effect, i.e., it favors observing information earlier. However, the signalling effects tend to jeopardize the favorable situation suggested by the information effect. Firms pay a strict cost \( S_{11}^1 > 0 \) for signalling information to each other. In the unilateral information transmitting, the active firm (Firm 1) transfers part of the cost \( S_{21}^2 < 0 \) to the passive firm (Firm 2) but in total, the signalling is still costly \( S_{11}^2 + S_{21}^2 < 0 \). It is this cost transference that makes Case 3 unstable.

**PROPOSITION 11.** \( \pi_1^3 < \pi_1^2 \). That is, both firms' acquiring information in the second period is not equilibrium.

Proof:

\[
\Delta_{23} = \pi_1^2 - \pi_1^3 = I_{11}^2 + S_{11}^2 - \beta_{12}^2 - \beta_{12}^3 \\
= \frac{1}{2-\rho} \nu(1 - \frac{2}{2-\rho} \nu)D + (1-\nu^2)I_{11}^2 + \frac{\beta(1-\rho^2)(7-\rho^2)}{9br(2+\rho)^2} > 0 \quad Q.E.D.
\]

**PROPOSITION 12.** There is a critical number \( \rho^* \) such that \( 0 < \rho^* < 1 \). For \( \rho < \rho^* \), that both firms acquire information in the first period is an equilibrium and for \( \rho^* < \rho < 1 \), that one firm acquires information in the first period and the other in the second period is an equilibrium.
Proof:

\[ \Delta_{12}(\rho) \equiv \pi_1 \Delta - \pi_2 \]

\[ = \frac{(a-c)^2}{9b} \left( \frac{1}{2-\rho} \nu(2- \frac{1}{2-\rho} \nu) - \frac{1}{2-\rho} \mu(1+\frac{2}{2-\rho} \mu) \right) \]

\[ + \frac{1}{bR(2+\rho)^2} (1 - \mu(\rho+(1+\rho)\mu)) + \frac{7b(1-\rho^2)}{36bR} \]

(18)

Notice that

\[ \frac{3}{\partial \rho}(1-\mu(\rho + (1+\rho)\mu)) = - \frac{3b(1-\rho^2)}{9} - \frac{8(2\mu(1-\rho) + 1 - 2\rho\mu)}{9(2\mu + 1)} < 0. \]

(19)

All terms in (18) are decreasing in \( \rho \) since as a function of \( \rho \), \( \frac{1}{2-\rho} \nu \) is decreasing, \( \frac{1}{2-\rho} \mu \) is decreasing and the two positive multipliers of the second term are decreasing (by 19). Thus \( \Delta_{12}(\rho) \) is decreasing. Also,

\[ \Delta_{12}(0) = \frac{(a-c)^2}{18b} \mu(1-\frac{3}{2} \mu) + \frac{1}{4bR}(1-\mu^2) + \frac{7b}{36bR} > 0, \]

and

\[ \Delta_{12}(1) = \frac{(a-c)}{9b} \nu(2-\nu) - \mu(1+2\mu) \]

\[ + \frac{1}{9bR} (1-\mu(1+2\mu)). \]

The only term in \( \Delta_{12} \) that may be negative is

\[ h(\rho, \beta) \equiv \frac{1}{2-\rho} \nu(2 - \frac{1}{2-\rho} \nu) - \frac{1}{2-\rho} \mu (1+\frac{2}{2-\rho} \mu) \]

which is concave in \( \beta \) and \( h(\rho, 0) = 0 \), \( h(1,1) = (9 \sqrt{13} + 3 \sqrt{7} - 50)/18 < 0 \). So there is a \( \beta^* \) \((0 < \beta^* < 1)\) such that \( h(1, \beta^*) = 0 \), \( h(1, \beta) > 0 \) if \( \beta < \beta^* \) and \( h(1, \beta) < 0 \) if \( \beta > \beta^* \). Therefore, the only case for a negative \( \Delta_{12}(1) \) occurs when \( \beta > \beta^* \) and \( a-c \) or \( R \) is relatively large. In this case, there is a \( \rho^* \) \((0 < \rho^* < 1)\) such that \( \Delta_{12}(\rho^*) = 0 \), \( \Delta_{12}(\rho) > 0 \) if \( \rho < \rho^* \) and \( \Delta_{12}(\rho) > 0 \) if \( \rho > \rho^* \). Otherwise, \( \Delta_{12}(\rho) > 0 \) for any \( \rho \in [0,1) \). Q.E.D.
One more comparison we can draw between the common value case and the private value case is that the first- and second-period information effects work in the same direction in the private value case which favors the earlier information acquisition but in opposite directions in the common value case. As a result, acquiring information in the second period by both firms is never an equilibrium when uncertainty is about private costs.
4. CONCLUSION

Intuitively, firms are always benefited from observing more private information and "timing of information acquisition" sounds meaningless. However, exploiting the information advantage takes the risk of revealing the information to the rival and hence changes the information allocation in later period. Signalling itself incurs an additional cost or gain and the resulting reallocation of information may or may not be favorable, depending on the nature of the game (e.g., common value versus private value) as well as the informativity of the signals. These tradeoffs make the timing of the equilibrium information exchange not only nontrivial but also a complex matter. The paper sheds light on the issue through two examples.

We conclude the paper with some more remarks. First, we may let firms have a choice of not observing the signal at all. But this is an obvious inferior strategy because there is no risk of revealing the private information in the last period (second period) and then more private information is always beneficial. Second, though we examine two-period models only, the approach is applicable to any finite period models. There, an early information revelation results in a stronger effect due to information reallocation while a delay of the information utilization suffers an information loss in the early periods. Finally, the extension to n-firm oligopoly is not an easy job. However, our conjecture is that the signalling effect diminishes with the information effect (see Li (1985a) and Palfrey (1985)) when the number of firms becomes large and the competitive equilibrium will be reached in the limit.
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