Strategies for Sizing Service Territories

Donald B. Rosenfield
Sloan School of Management
Massachusetts Institute of Technology

Israel Engelstein
Technology Research Center
Arthur D. Little, Inc.

David Feigenbaum
Technology Research Center
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Abstract

The major tradeoff in designing a service network, where facility-based drivers or servers travel to customer sites, is the cost of delivery versus the cost of facilities. Where the constraining factor is the capacity of the delivery vehicles, the costs are a straightforward computation. In many applications, however, the available time of day drives the system in that excess travel time, due to a smaller number of facilities, cuts down on the efficiency of delivery people, thereby requiring more such people. This article develops a model based on this concept, and presents some actual applications of the model.

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1. Introduction

This article focuses on the problem of determining service territories in a service business. A service territory consists of a central location, from which some type of delivery or visits are made to individual customer sites. The problem occurs in any type of repair organization such as the telephone repair operations, or a business requiring direct customer delivery of merchandise. It also occurs in point-to-point distribution problems such as in the delivery of mail from postal facilities to individual mail stops.

Since each service territory has a facility that serves as a central base of service, the problem of determining service territories requires determining the number and location of facilities. As more facilities are added and the organization becomes less centralized, the fixed-facilities costs go up. On the other hand, the delivery operators are generally closer to their customers, and thus labor efficiency goes up; this in turn allows for fewer operators and vehicles.

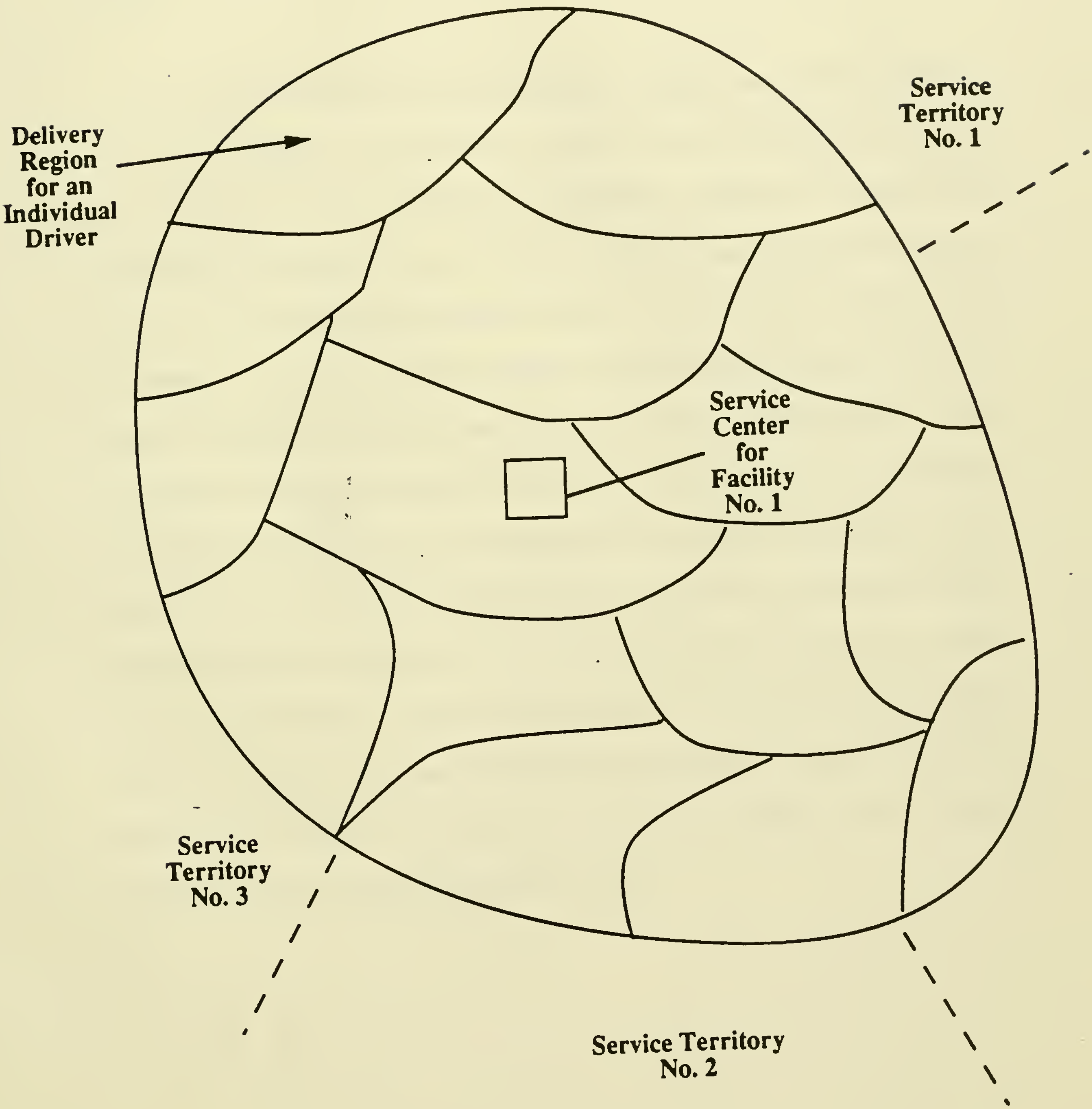
Burns et. al. [1985] treat a very similar problem when they examine the optimum size of <u>delivery regions</u> when delivery is made from a central location within a larger supplier region. The latter corresponds to what we call service territories (What Burns et. al. refer to as the delivery regions, a term that we also adopt, corresponds to the specific area a driver travels to). The <u>service territory</u> is the entire area that a group of drivers service from a single service center or

service facility. (see Figure 1). However, Burns et. al. do not consider the optimum size of supplier regions. Furthermore, the major constraint in their approach, which is typical of vehicle routing approaches (see Bodin et. al. [1983]) is in the aggregate size of a delivery load (e.g. weight or volume). We have observed that in many service delivery operations, the key constraints are work-load of the delivery person (which we refer to as the driver). For example, in delivering mail to customers, the major limitation on the number of stops a driver can make is his allotted time rather than the amount of mail he can put into his vehicle. While there superficially does not appear to be any difference between these two types of constraints (size or time), the time constraint is based on the time it takes the driver to drive to his delivery region while the size constraint is not. This type of time constraint yields an entirely different type of optimality condition for the number of facilities.

The precise trade-off between the facilities and the delivery cost is a matter of structure of the service offered. In many service operations, service requirements are probabilistic and each service demand is satisfied through dispatch from the nearest service center. The service representative travels from the service center to the site of the request and then returns. In this type of situation, response time and distance varies as the inverse square root of the number of service facilities (e.g. in Kolesar and Blum [1973]).

Figure 1

Service Territories and
Delivery Region

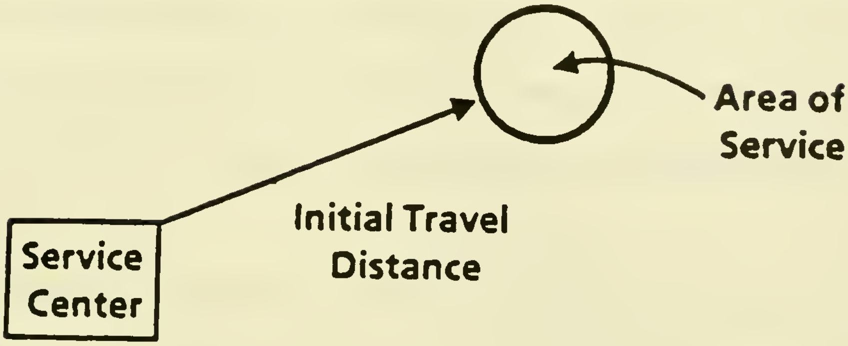


In other service organizations, at the beginning of each day, a set of demands or customer locations are assigned to each operator along with approximate daily vehicle routes (the sequences of stops) In this case the service representative leaves the service location at the beginning of the day and returns at the end of the day.

If one views, in this case, the delivery regions being preassigned, then the only variables with respect to the number of service centers are the initial and final travel time and the distance of the delivery vehicles (see Figure 2). Based on this preassignment assumption, we can develop a straightforward model to determine the economics of the number of service facilities. As centralization decreases and the number of service facilities increases, initial and final travel distances decrease. If the initial and final travel time is defined as unproductive time, and all the time in between as productive time, then the total amount of work time available per worker per day is some constant (such as eight hours) less the initial and final travel time. If this initial and final travel time goes down, then the productive time for a worker goes up, and the company will need fewer workers to service the workload. This is traded off against the fixed-facilities cost. (Of course, if the number of service facilities is changed, then the actual routes and regions will be changed. In this sense the delivery regions are not totally preassigned, but this is considered in the analysis).

Figure 2

The Service Distribution Problem



This article presents the analytic relationships for the various costs involved as the number of service facilities and territories are changed. These relationships can be used as the basis for determining the best number of service facilities. In relatively simple situations, the analytic relationships can be solved directly. In more complex situations, there may be fairly complicated relationships between, for example, fixed-facilities costs and the number of delivery workers that that facility is serving. In this type of situation, one can use the analytic relationships as a basis for computer simulations that include all the relevant costs.

The authors have applied the approach for three separate service situations. For a major telephone company, one of the authors, by simulating travel times on an actual network and by deducting travel times from daily productive labor, helped restructure the network. In this case, the facilities covered different size service territories and travel speeds varied, but the productive time within each service territory was reduced by the actual travel time. For an industrial gas distributor, by analyzing the trade-offs involved in traveling from the service center to the customer sites, one of the authors helped to convert service centers that were the origin of vehicle routes into simple sales operations.

In the most comprehensive application of the approach, all three of the authors worked on a project with the U.S. Postal Service (USPS) to develop a methodology to size postal service territories (called delivery units), those territories whose mail carriers operate from a central facility (also called a

delivery unit) and deliver mail to individual mail stops. The result of this effort was a computerized planning system that the USPS is implementing at the present time.

This article consists of three additional sections. In the next section, we present the analytic relationships for determining the optimum number of service territories. In Section 3, we discuss the application to the sizing of delivery units in the USPS. In the final section, we discuss a variation of the final model to cover the case of a varying density of service requests.

2. Analytic Model

The basic assumptions of the model are as follows:

- Service centers or facilities are located in the center of each service territory
- Stops are uniformly distributed
- Each driver travels at a constant speed
- The drivers have equal work days (that include traveltime as well as time performing their service)
- Demand can be random or predetermined, but routes are pre-assigned each day

Letting

A = Total area covered by all service territories

M = Total number of stops or customers

N = Number of service centers or facilities,

then

Area (i.e. size) of each service territory = Area serviced by each facility = A/N

The heart of the model is the relationship between the initial and final travel time (Burns et. al. refer to this as linehaul and backhaul distances) and the size of the service territory. As indicated in Figure 3, this travel time will be proportional to the linear distance of the surface area for each facility or the square root of the area. (See Kolesar and Blum [1973] for a complete discussion of this topic. The topic is also treated in Adoni and Larson [1981]). The mathematical relationship is based on the distance from the service facility to a set of uniformly distributed points. For a set of delivery routes, this implies that the starting and ending points of the routes are uniformly distributed. Simchi-Levi [1985] shows this asymptotically. In practice, this implies that delivery regions are layed out as in Figure 1, at varying distances from the service facility rather than like petals of a flower. Haimovich and Rinnooy Kan [1987], who analyse delivery region structure, provide some evidence of this. Newell and Daganzo [1986], and Langevin and Soumis [1989], for example, assume delivery

regions of the "ring-radial" form, in which case the regions are part of concentric rings. The first and last stop on each route is located close to the inner ring of each group of delivery regions. Furthermore, Langevin and Soumis indicate that the width of the regions is of the form $L = A - Br$, where r is the distance of the region from a centrally located service center. Hence, for a randomly located demand point that is r from the center (assumed to be uniformly distributed) the initial and final travel time corresponding to its route is proportional to $r - L/2 = r(1 + B/2) - A/2$. Integrating this over rings of the size $2\pi r dr$ from 0 to the radius R and dividing by the area R^2, the average initial final and travel time is of the form

$$KR - B$$

R of course is proportional to the square root of the area, indicating that the initial and final travel time does not strictly follow the square root law. However, in practice B will be relatively small and it asymptotically goes to zero. Furthermore, the existence of B does not change the form of the equation (3) for the total number of routes that follows. We will hence assume the square root relationship.

The actual constant of proportionality in the square root relationship will vary depending on the assumed shape of the service territory, e.g. circular versus square, the location of the service center within the territory, and the nature of travel to the delivery region, e.g. Manhattan metric versus direct distance (see Adoni and Larson [1981]). However, under any of these assumptions, the square

root relationship still holds. (In the application discussed in the following section, the value of the constant becomes important and we will discuss the topic further.)

Note that distances to the different delivery regions will vary (see Figure 1). The comparison is for different size service territories. While the distance within the delivery region, i.e. the local travel, will also change, the effect is not as significant as the effect on the initial and final travel time.

The total work time per day on each route can then be defined in terms of the number of delivery regions (or routes) X in all of the service territories. (Note that the actual number of drivers can exceed the number of routes, if backup drivers are needed).

For a given service territory, then, the initial and final travel time, linehaul and backhaul, averaged over the delivery regions in the service territory is

$$K_1 \sqrt{A/N} \qquad (1)$$

where K_1 is a constant

The time between stops can be based on the shortest Euclidean path connecting the stops (see Burns et. al. [1985], Bearwood et. al. [1959], Eilon et. al. [1971], and Stein [1978]):

$$K_2 \sqrt{(\text{Area of delivery region})\ (\text{Number of stops per route})}$$

(According to Burns et. al., K_2 is approximately .6 divided by the travel speed.)

We next compute the average of this time between stops. Suppose we denote the areas and number of stops for all delivery regions in all service territories as A_i and M_i respectively, $i = 1, \ldots, X$

Then

$$\sum_{i=1}^{X} M_i = M$$

$$\sum_{i=1}^{X} A_i = A$$

$\overline{M}$ = Average number of stops = M/X

$\overline{A}$ = Average area = A/X

$\overline{R}$ = Average of $\sqrt{A_i M_i}$

$$= \frac{1}{X} \sum_{i=1}^{X} \sqrt{A_i M_i}$$

Assuming further that stops are uniformly distributed then $A_i = CM_i$ for some C, and

hence

$$\bar{A} = A/X = \frac{1}{X} \sum_{i=1}^{X} A_i = \frac{1}{X} \sum_{i=1}^{X} CM_i = CM/X = C\bar{M}$$

and $$\bar{R} = \frac{1}{X} \sum_{i=1}^{X} \sqrt{A_i M_i} = \frac{1}{X} \sum_{i=1}^{X} \sqrt{CM_i^2} = \frac{1}{X} \sum_{i=1}^{X} = \frac{1}{X} \sum_{i=1}^{X} M_i \sqrt{C}$$

$$= \bar{M} \sqrt{C} = \sqrt{\bar{M}C\bar{M}} = \sqrt{\bar{M}\bar{A}} = \sqrt{MA/X^2}$$

$$= \sqrt{\frac{AM}{X}}$$

Hence the average for all delivery regions of the time between stops is

$$K_2 R = (K_2/X) \sqrt{AM}$$

Finally, the average processing time for the stops on a single route is

$$K_3 \bar{M} = K_3 \; (M/X)$$

where K_3 = processing time per stop

(Hausner [1975], in an empirical study, suggests that travel time between stops includes a fixed charge. K_3 could also reflect this). Thus, the total time for an average route per day is

$$K_1 \sqrt{A/N} + \left(K_2/X\right)\sqrt{AM} + K_3\left(M/X\right) \qquad (2)$$

(Asymptotically, as the number of stops per route increases, the first term dominates the other travel term, as shown by Simchi-Levi: [1985])

Since (2) must be equal to some constant D (e.g. 8 hours per day) we get the following relationship for X.

$$K_1 \sqrt{A/N} + \left(K_2/X\right)\sqrt{AM} + K_3\left(M/X\right) = D$$

Solving for X,

$$X = \frac{K_2\sqrt{AM} + K_3 M}{D - K_1 \cdot \sqrt{A/N}} \qquad (3)$$

In examining (3), the numerator represents the total daily processing time for all stops (actual processing time plus inter-stop time) once drivers reach their delivery regions. The denominator represents the available daily work time once travel to and from the delivery region is deducted.

Expression (3) represents the worker and associated costs as a function of N. As N increases, these costs decrease, but facility related costs (i.e. service center costs) increase. In actual applications, the relationships can be complex (e.g. nonlinear relationships between facility costs and number of routes serviced). However, for a simple functional form for facility costs, one can develop a relationship for the optimum number of facilities as follows:

Let B = Fixed cost per facility

F = Cost per route

$C = K_2\sqrt{AM} + K_3M$

(Note that variable costs of facilities are not affected by the number of facilities and can be left out).

Then the total cost is

$$\frac{CF}{D-K_1\sqrt{A/N}} + BN \tag{4}$$

which is equivalent to

$$\frac{CF\ N}{D\sqrt{N} - K_1\sqrt{A}} + BN \tag{5}$$

To find the minimum of (3) we take its derivative and set it to zero (see Figure 4).

$$\frac{.5CFN^{-1/2}}{DN-K_1\sqrt{A}} - \frac{CFN^{1/2}(.5DN^{-1/2})}{(DN^{1/2}-K_1A)^2} + B = 0$$

which yields the following cubic equation for $\sqrt{N}$

$$BDy^3 - 2K_1BDA^{1/2}y^2 + K_1^2ABy - .5CFD_1A^{1/2} = 0 \tag{6}$$

where

$y = \sqrt{N}$

The next level of sophistication is a more general model of facilities costs. An

Figure 3

Transit Distance and Area

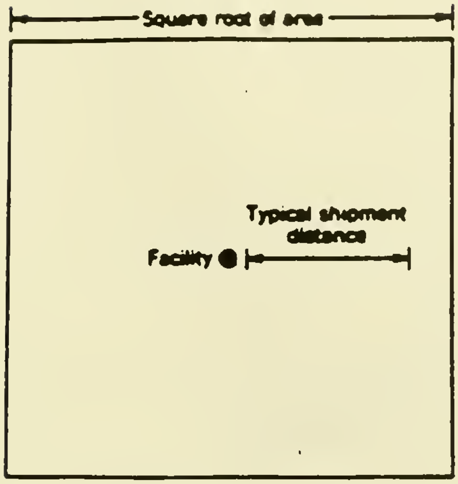
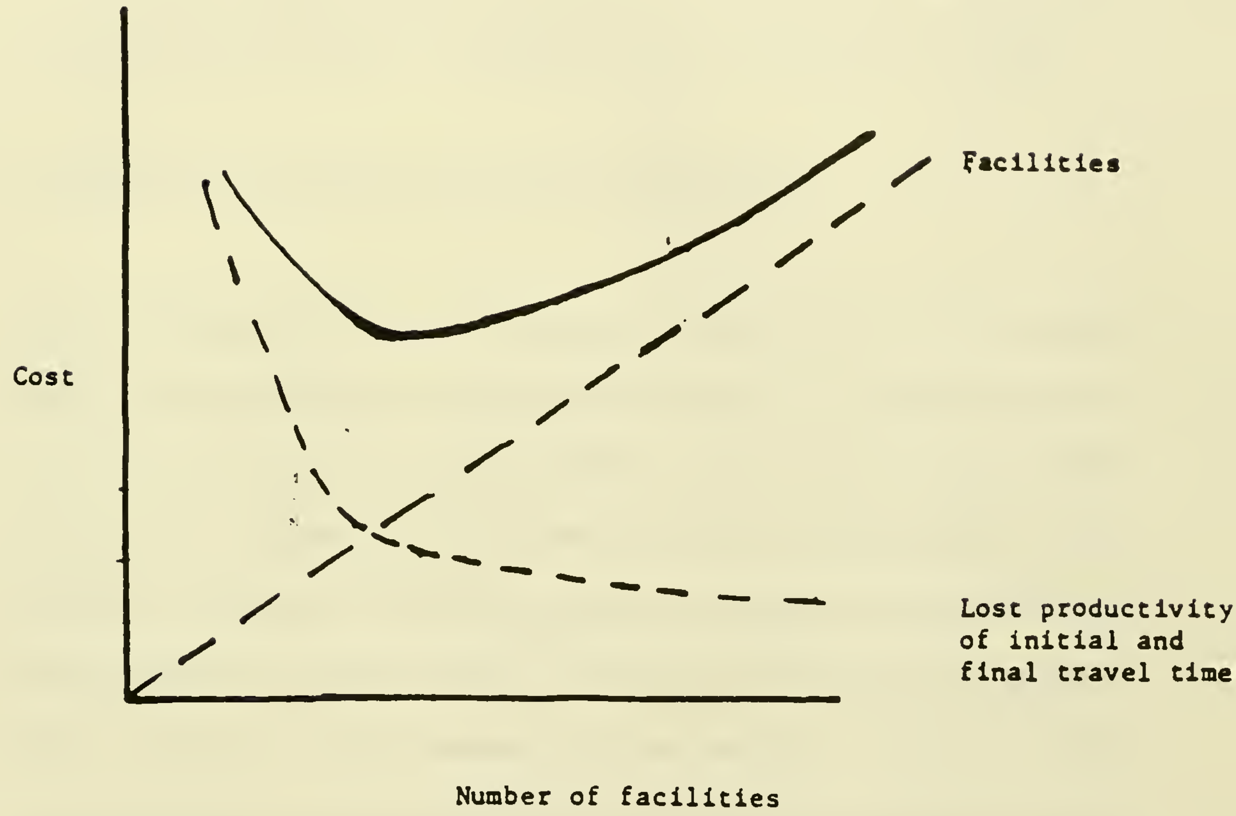


Figure 4

Cost Components in Service Business
With Customer Delivery



example might be

$$\text{Facility costs} = B_1 N + B_2 \sqrt{N} + B_3 \tag{7}$$

The square root term might represent spare parts inventories (see Magee et. al. [1985], for example). This will also yield a cubic equation.

In practice, however, the relationship between various costs and the number of routes is more complex, as exemplified by the case study from the USPS.

3. Optimizing Delivery Units for the U.S. Postal Service

The USPS operates service facilities called delivery units (DUs) from which mail is delivered to delivery points as denoted in Figure 1. Mail to the delivery units arrives from a mail processing facility, called a sectional center facility (SCF), which is responsible for a large service area that encompass many delivery units. Mail is delivered by carriers (i.e. drivers) who work part of their time in the office to case and sequence the mail. After sequencing the mail they leave the office and deliver the mail to customers in preassigned routes. The responsibility of planning delivery units in a given service area lies with the management of the processing facility.

To assist postal planners in developing the network of delivery units in a specific given area and to deal with problems of space deficiency, the authors

developed a spreadsheet based cost model, called Delivery Unit Planning Model, or DUPM, based on the relationships described in the previous section. This section reviews the DUPM.

Postal delivery unit costs include four components:

- o Carriers and their vehicles
- o Supervisor and support staff
- o Facilities, including maintenance and utilities
- o Transportation (to delivery units)

Figure 5 shows the trends in costs when there is a shift toward decentralization (i.e., from a few large DUs to many small ones). Carrier costs decrease because carriers spend less time traveling from the delivery unit to their routes. Facility plus supervisor and support staff costs increase because some portion of these costs are fixed, even for very small delivery units. Transportation costs increase because of the increase in the number of locations that must be served.

As indicated in relationship (2), the available work time for a postal carrier was computed by subtracting travel time. This available work time then became the basis for many of the other calculations. To calculate the initial and final travel time from (1), we used the general formula

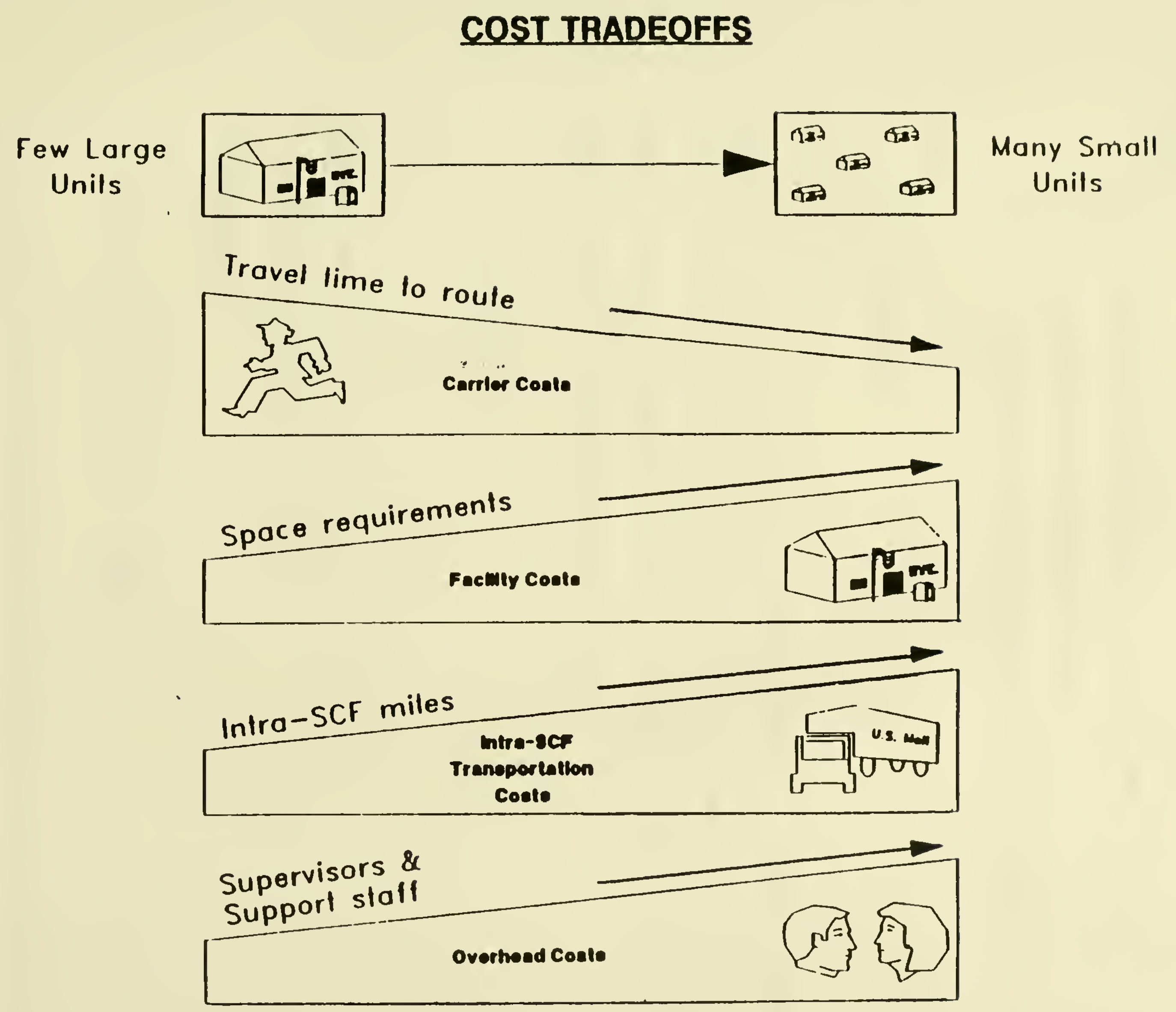


Figure 5. Cost Tradeoffs in the U.S. Postal Service

$$ATT = \left(\frac{2K_4}{speed}\right) x \sqrt{A/N} \tag{8}$$

where

K_4 = parameter relating one-way distance to square root of area

ATT = average initial and final travel time

speed = average speed which is input by the user of DUPM

That is

$$K_1 = 2K_4/speed \tag{9}$$

K_4 can be computed both theoretically and empirically. For example, consider a delivery unit that is circular in shape. Assume that the facility is located in the center and there is a uniform distribution of routes. It then follows that the average distance to a randomly selected route is

$$\frac{\int_0^R r(2\pi r dr)}{R^2} = 2R/3$$

Since the maximum radius R is related to area by

$$Area = \pi R^2 \Longrightarrow R = \sqrt{Area/\pi}$$

Then distance is

$$\overline{Distance} = \frac{2}{3} \frac{Area}{\pi} \Longrightarrow K_4 = \frac{2}{3\sqrt{\pi}} = .38$$

Of course, the area may not be circular, routes may not be distributed uniformly, the facility may not be located at the center, and travel may not be

direct. Larson and Odoni [1981] present a series of models that examine variations, including the "Manhattan metric" of right-angle travel. Values for K_4 range from .38 to 0.75. Based on data from a USPS survey, the authors selected a value of K_4 of 0.6.

Given K_1 according to (8) and (9), the number routes in DUPM is computed by a variant of (3). The variant is due to the fact that each daily route consists of route time plus what the post office refers to as internal time spent at the facility sorting and performing other route-dependent tasks. This time depends on the route time less travel time, and the effect, is that the time visiting stops in the denominator of (3) is multiplied by a constant (.53 is the default in DUPM). Based on the number of routes and the initial and final travel time ATT, the model calculates the other costs as follows:

Carrier and Vehicle Costs - The number of vehicles is equal to the number of routes. The number of carriers exceeds the number of routes, because of backups and weekends. Based on a survey conducted by the authors on number of carrier and routes in post offices and regression analysis of the data in these surveys, the number of carrier hours as a function of the number of routes can be estimated to be

$$\text{Carrier hours} = 8.822 \times \text{routes} + .0097 \times \text{Routes}^2 \qquad (10)$$

Carrier costs are then computed based on the total number of hours and vehicle requirements.

Facilities Cost

Facilities costs consist of space, utilities, and maintenance. Space requirements, according to postal guidelines are

$$\text{square feet} = 2048 + 296 \times \text{routes} \tag{11}$$

The lower bound of 2048 results from the fact that each facility, no matter how few routes it contains, must have a minimum amount of space for washrooms, cafeteria, docks, heating and cooling, etc. The cost per square foot, which is rent if the building is leased and amortized construction and land if owned, typically varies by location and is generally user input. Utility and maintenance costs are based on a survey of postal facilities, consist of annual fixed and variable (with square footage) costs. DUPM uses default values reflecting the survey averages.

Supervisory and Support Staff

Supervisory and support staff, based on the same survey, increase with the number of routes:

$$\text{Supervisors} = 1.274 + .0374 \times \text{routes} + .00024 \times \text{routes}^2 \qquad (12)$$

$$\text{Support staff hours} = 4 + 0.3 \times \text{routes} + .00112\ \text{routes}^2$$

DUPM has these relationships built in and multiplies the terms by appropriate input cost factors.

Delivery of Mail to Delivery Units

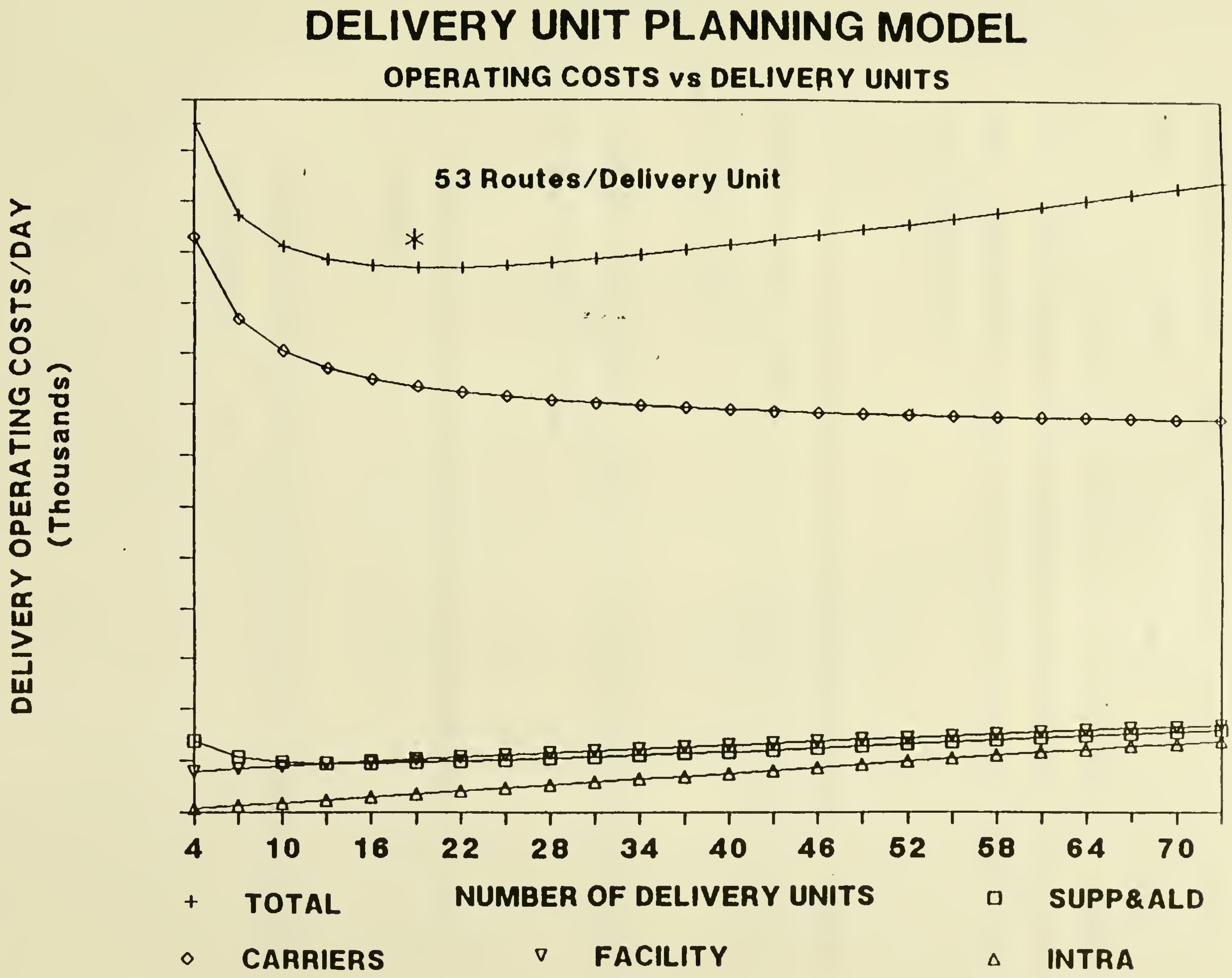
Mail is delivered from the sectional center facility to each of the delivery units. Assuming that the cost to deliver mail to each set of two or three delivery units is proportional to the distance to those units (each two or three delivery units requires a separate trip) then it follows that the cost of delivery for all units is proportional to the square root of the total area of all delivery units. DUPM uses this proportionality and a constant of proportionality based on the same survey data.

Applying the Model

Applying the model with default values on the cost parameters yields the types of curves presented in Figure 6. The lowest curve, marked Δ———Δ, represents the cost of delivery of mail to delivery units. The next curve, marked ∇———∇ represents facility costs. This includes space plus maintenance and utility costs. This curve increases as delivery units are added due to the constant term in equation (11). In addition, smaller buildings typically cost more per square foot to construct than do larger buildings. (This is imbedded in a default cost per foot function.) However, these two factors are offset by the fact that, as the number of delivery units increase, the total square footage of building space required in the application area decreases since fewer total routes are required. Therefore, facility costs increase at a decreasing rate as the number of routes increase.

The third curve, marked □———□, represents the cost of supervisors and support staff. It initially decreases, reaches a minimum, and then increases in proportion to the number of delivery units. There are two factors that cause these costs to behave this way. The first is that very large units have lower manpower productivity than do medium or small sized units (equation 12). This causes costs to increase more than proportionately as the number of units decrease. However, as the number of units increases, and their size decreases, a minimum number of supervisors and support staff hours are required in each delivery unit (equation 10), causing costs to increase in proportion to the number of delivery units.

Figure 6


DELIVERY UNIT PLANNING MODEL
OPERATING COSTS vs DELIVERY UNITS
53 Routes/Delivery Unit
*
DELIVERY OPERATING COSTS/DAY
(Thousands)
4
10
16
22
28
34
40
46
52
58
64
70
NUMBER OF DELIVERY UNITS
+ TOTAL
◇ CARRIERS
▽ FACILITY
□ SUPP&ALD
△ INTRA

The fourth curve, marked +——— +, represents the cost of carriers and their vehicles. It decreases as the number of delivery units increase. There are also two factors affecting this cost. Like supervisors and support labor, carriers also show reduced productivity in very large units (equation 10). The second factor is that of travel time to and from the routes, which decreases with an increased number of delivery units. Both of these factors have less effect as more and more delivery units are added, causing this curve to almost flatten out with very large numbers of delivery units.

The top-most curve is the sum of the four individual cost factors. It generally decreases, reaches a minimum and then increases as delivery units are added to the application area.

Application of the model to a variety of sectional center facilities in the country led to the following conclusions:

- The dominant part of the operating costs are the labor costs for the carriers themselves

- Changes in facility cost, such as from rent modifications, have little impact on the solution

- There is a fairly wide range of values for the number of delivery units over which costs are close to minimal.

- Very large or very small delivery units are inefficient, both in terms of their implications on the facility and their infrastructure (e.g. supervisors) and for the case of large units, in terms of the effect on all labor costs.

The authors undertook a set of field studies with a number of postal service sectional center facilities. In each one, the model was applied using the parameters for each locale (average speed area, etc.). The model proved to be an extremely useful approach to evaluating the current network and understanding whether decentralization or consolidation would be advisable.

Sizing Service Areas with Varying Densities

The model of Section 2 and the DUPM both assumed a uniform distribution of routes. If some parts of a set of service territories are denser than others, should the service territories corresponding to them be larger or smaller? This section addresses this issue.

Suppose we enumerate the individual service territories by the index i. Let the area of each be A_i the average route density be D_i, and the travel velocity be V_i. We expect that velocity V_i will decrease as density increases. (From a theoretical

view, velocity decreases as vehicle density increases, which might increase with route density. Huber (1982) present several alternative models, for the velocity and vehicle density relationships.) Suppose we can model velocity as being proportional to

$$D_i^a$$

We would not expect a strong relationship. For example, if a were between 0 and .5, then we would expect no more than a 41% increase in velocity if the density were cut in half.

Then it follows that:

Average distance to a route is

$$\text{Constant} \cdot A_i^{1/2}$$

Average time to a route is distance/velocity or

$$\text{Constant} \cdot A_i^{1/2} \cdot D_i^a$$

The objective to minimize is the total time, which is equivalently the weighted sum of times:

$$\text{Constant} \cdot N_i \cdot A_i^{1/2}\ D_i^a$$

where N_i is the number of routes in service territory. However,

$$N_i = D_i\ A_i$$

Hence the objective is:

$$\text{Constant} \cdot D_i^{(1+a)}\ A_i^{3/2}$$

This function is minimized when the separate terms are equalized. (See Zangwill [1969]). Hence,

$$D_i^{(1+a)} A_i^{3/2} = K_5 = \text{a constant}$$

$$A_i^{3/2} = K_5(D_i)^{-(1+a)}$$

or

$$A_i \sim D_i^{2/3(1+a)}$$

Furthermore, since the number of routes in area i is

$$D_i A_i$$

then

$$\text{Routes}_i = D_i A_i \sim D_i D_i^{-2/3(1+a)} \text{ or routes}_i \sim D_i^{1/3-2a/3} \quad (13)$$

If A varies between 0 and .5 then the number of routes should vary between a power of the route density between zero and one. From a practical point of view, the number of routes is not very sensitive to density and in many applications, service territories should have roughly the same number of routes. This general guideline was suggested to the USPS in applying the DUPM model.

Final Comments

The concept of a fixed amount of daily worker travel time obviously has a significant effect on network design and cost in a service network. This article will hopefully lead to additional extensions providing for the insight on service network design.

Aknowledgment

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