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TAKEOVER DEFENSES AND SHAREHOLDER VOTING*

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Takeover Defenses and Shareholder Voting*

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Abstract

Why do shareholders vote for anti-takeover devices which apparently lower the value of their firm? We address this question by constructing an agenda-setting model in which rational, informed, and value-maximizing shareholders vote on requests for such devices made by a self-interested management with employment opportunities outside the firm. We find sufficient conditions for the value of the firm to decline as a result of a request, although it is approved by shareholders. In our model, the apparently paradoxical voting behavior occurs because the expected takeover premium would be reduced more by rejection of the request than by approval, so shareholders rationally choose approval.
Takeover Defenses and Shareholder Voting

1. Introduction

Why do shareholders vote for anti-takeover devices which apparently lower the value of their firm? We address this question by constructing a model in which rational, informed, and value-maximizing shareholders vote on requests for such devices made by a self-interested management with employment opportunities outside the firm. We describe conditions under which the value of the firm declines as a result of the request, although it is approved by shareholders. In our model, the apparently paradoxical voting behavior occurs because the expected takeover premium is reduced more by rejection of the request than by approval.

A large and increasing number of amendments to corporate charters are specifically designed to increase the cost of transferring control. DeAngelo and Rice (1983) report over 250 proposed amendments in 1974 through 1979. Linn and McConnell (1983) find generally increasing incidence of such proposals among NYSE firms over the period from 1960 to 1980. The Investor Responsibility Research Center lists over 200 in 1985 alone. While the use of anti-takeover charter amendments is increasing, there is considerable disagreement over their effects on shareholders and on the market for corporate control.

Two major hypotheses describing the motives for instituting anti-takeover charter amendments are commonly described as "management entrenchment" and "shareholder interests." According to the management entrenchment view, incumbents are interested in job security and seek protection from the takeover market, to the detriment of shareholders. However, a manager cannot change the corporation's charter without winning voted approval from shareholders. Two suggested explanations for shareholder approval of entrenching antitakeover devices are: (1) for a majority of shareholders, the costs of becoming informed about the effects of defensive charter amendments exceed any potential benefits, and uninformed shareholders consistently give their proxies to
management; and (2) large shareholders wish to maintain friendly relations with management to ensure the benefits of future business, and large shareholders control sufficient shares to be pivotal in the vote. Shareholder irrationality is sometimes offered as a third alternative. Note that any explanation for the observed voting behavior must describe individuals who control sufficient shares to be pivotal, that is, sufficient shares to determine the outcome of the vote.

The stockholder interests hypothesis recognizes a free-rider problem in collective action by shareholders (Grossman and Hart (1980), Jarrell and Bradley (1980)). Shareholders may have difficulty colluding to extract larger premia from takeover bidders, particularly if remedies to limit dilution of minority interests are imperfect. Anti-takeover devices may benefit shareholders by enforcing a level of collusion in takeover negotiations, to extract a higher premium. If this is the case, there is no inconsistency in rational shareholders voting for these devices. However, there is little evidence that shareholders benefit from such amendments. DeAngelo and Rice (1983) find statistically insignificant negative abnormal returns around the public announcement of proposed antitakeover amendments. Linn and McConnell (1983) find positive returns around the board meeting date at which amendments are proposed, and insignificant negative returns around the proxy mailing date. Jarrell, Poulsen and Davidson (1985) find negative returns accompanying the announcement of "shark repellents", viz. supermajority, classified board and authorized preferred amendments.

In the context of our model, the management entrenchment and shareholder interests hypotheses do not necessarily lead to different predictions about shareholder value. Anticipatory takeover defenses raise the costs of acquiring control of the firm, thereby entrenching management, but lead to a higher premium for shareholders if a bid succeeds. We presume that different potential managers contribute different value to a given firm, by virtue of skill or experience. Only managers capable of producing higher value than the incumbent can mount successful takeover bids. Whether shareholders gain or lose as a result of the incumbent's defenses depends upon the quality of the incumbent management relative to potential bidders, and the incumbent's
opportunities for employment outside the firm. We describe circumstances in which shareholders rationally vote for anti-takeover charter amendments, although their wealth is lower afterwards than it was before the amendments were proposed. We also describe different circumstances in which shareholders are made better off by the amendments.

The paper is organized as follows. In section 2 we provide a brief overview of the model. The formal model is developed in section 3, and we summarize and discuss the results in section 4. Section 5 concludes. All proofs are confined to Appendix A. Appendix B consists of an example that illustrates our results.

2. Overview

We construct a two-period model of a firm with risk-neutral shareholders, with a manager who has employment opportunities outside the firm, and with the prospect of a management change by takeover. We consider the capital structure and assets of the firm to be fixed, and focus on the change in value associated with a change in management. At the start of the first period no takeover bid is outstanding. During the first period, the incumbent manager may propose anti-takeover charter amendments, on which shareholders vote. During the second period, a takeover bid may be made. Depending upon the outcome of the first period and the manager's preferences for employment with the firm versus employment outside the firm, the manager may choose to erect additional defenses in response to the bid. If the takeover bid is successful, the shareholders sell their shares to the bidder and the manager leaves the firm for alternative employment. Both shareholders and the incumbent manager make decisions in the first period based upon rational expectations about what may occur in the second period.

A major implication of the results developed in sections 3 and 4 is that observations of a firm's value before and after the implementation of takeover defenses are not observations of the alternatives shareholders face when voting on these defenses. In our model, shareholders who approve defenses face an unobserved, inferior alternative if they choose rejection, and so rationally
choose approval. Two central features of the model drive the results. First, the manager sets the agenda on takeover defenses monopolistically, and shareholders may choose only to accept or to reject the managerial proposal. They are unable to amend the proposal or counter-propose before the vote. Second, the manager's request changes the subsequent decision-making environment.

The two critical features, monopolistic agenda-setting and the real effect of the request itself, generate the voting conundrum discussed in the Introduction. The monopolistic agenda-setting power of the manager ensures that, for a properly-chosen request, the shareholders can do no better than to approve. In certain circumstances, the value of the firm following approval of a managerial request for takeover defenses will be lower than the pre-proposal value. Nevertheless, the value would be no greater, and may be lower, if the request is rejected. The wedge between pre-proposal value and the post-rejection value is induced by the real effect of the request, described below. Previous research on the effects of the implementation of takeover defenses has compared the pre-proposal value of the firm with its post-proposal or post-vote value. In our model, the comparison for inferences about shareholder rationality in voting is, instead, the value under approval of the request versus the value under rejection.

Our model describes the manager as a self-interested intermediary between shareholders and potential acquirers. The model rests on a reduced form specification of an outside managerial labor market, our characterization of the mechanism that produces the real effect of the request for takeover defenses. Other mechanisms may exist with the same general property of inducing a change in the decision-making environment. We find the outside labor market a natural way to introduce this real effect, since it defines the manager's opportunities in the event of a successful takeover. In the model, the manager's value added to the firm and utility schedule are common knowledge at the time of the vote.

The common knowledge assumption is used here to emphasize that the voting conundrum can obtain even when shareholders are fully informed when they vote. In our model, shareholders recognize the implications of their vote: they do not make mistakes or have regrets with respect to
their decision. Note, however, that information must by conveyed to shareholders by the proposal for it to result in a change in value. At the start of the first period, shareholders are not perfectly informed about the manager's utility for employment, but they learn from the proposal. We assume that they learn everything, so that manager's utility schedule is known precisely at the time of the vote. This is clearly extreme, but we expect that retaining some information asymmetry at the time of the vote would increase the incidence of shareholders' approving measures which, ex post, leave them worse off. We return to this point in the fourth section. Similarly, the proposal would not result in a change in firm value if it were possible to eliminate incentive problems entirely using contingent claims contracts. Agenda control issues would not arise: shareholders would hedge themselves ex ante against the possibility of voting on value-reducing amendments. Given empirical observation of the phenomenon, it does not appear unrealistic to assume the non-existence of complete contingent claims markets. We assume that contracts to hedge against value-reducing behavior by management are impossible or too costly to write. This issue is considered further in section 5.

One final aspect of the model is worth noting at the outset. In some circumstances, the value of the firm will rise as a result of a request for takeover defenses. The empirical results to date on the effects of voted anti-takeover measures are mixed, admitting the possibility that some such measures increase shareholder wealth. We describe circumstances in which the takeover defenses are value-increasing. However, the focus of this paper is to describe circumstances in which the value of the firm will fall, to provide a rational choice answer to our opening question.

3. Model

We assume that, by virtue of skill, experience or effort, different managers can contribute different value added to the fixed capital structure and investments of the firm. There is a continuum $M$, which may be infinite, of possible firm values based on the value added by different possible managers. Denote by $m \in M$ the value of the firm under incumbent management at the
start of the first period of the two-period model, absent the possibility of a takeover. Of course, the value of the firm to shareholders will depend on both \( m \) and the expected premium to be received in the event of a successful takeover, as discussed below. The distribution \( F(\cdot) \) on \( M \) of firm values under different managers, with density function \( f(\cdot) \), is assumed to be common knowledge. For technical reasons, we assume that \( \lim_{t \to \infty} [1-F(t)]/f(t) < \infty \): this is not a strong restriction.

The firm is owned by a set of risk-neutral shareholders, interested only in maximizing the expected value of their shares. We presume that there is a corporate charter in place at the start of the first period, and that this charter does not prohibit dilution of shareholders' claims by potential acquirers. The corporate charter in place at the start of our model is not presumed to be tailored optimally to the current manager. This can be true because it is prohibitively costly for the initial shareholders to write contracts completely exhausting all possible eventualities. Further, in a diffuse-ownership corporation each current shareholder has little incentive to incur the costs of revising the charter.

Heuristically, the model of takeovers we have in mind for the second period is the following. In the second period, Nature draws a potential alternative manager from the set \( M \), who can choose whether or not to make a takeover bid for the firm. If a bid is made and is successful, then the new manager buys the firm and controls it thereafter. If there is no bid or if there is an unsuccessful bid, the incumbent remains in control. Since the life of the firm is only two periods, the value of the firm in this last period is determined by the value added by the second period manager.

Given this model of takeovers in the second period, the first period expected value of the firm is the sum of the value of the net assets under the incumbent management, \( m \), and the expected premium in the second period. Without loss of generality, let discount factors be equal to one for the manager and all shareholders. The expected premium depends on the probability of a successful bidder appearing in the second period, and the expected size of the premium paid by that bidder. In what follows, we first describe the role of takeover defenses, next explain more fully
the incumbent manager's utility for employment and the second period takeover process, and then work backwards to consider the incumbent's and shareholders' decisions in the first period.\textsuperscript{7}

3.1 Takeover Defenses

The manager is endowed with two control variables, \(x\) and \(y\), both takeover defenses. From a potential bidder's perspective, \(x\) and \(y\) are indistinguishable, and are simply increments in the minimum price necessary to acquire the firm. Thus, \(x\) and \(y\) are measured in the same units as \(m\), reduce the probability of a successful bid appearing, and increase the expected premium, conditional on a successful bid. From the perspective of the incumbent manager and the shareholders, however, \(x\) and \(y\) are quite distinct.

Defense \(x\) is anticipatory: it can be put in place only in the first period, prior to the potential bidder appearing. The manager is obliged to ask the shareholders to approve anticipatory defense \(x\). Majority voting among the shareholders determines whether or not the manager's request is granted. Amendments to the proposal are not permitted, so the shareholders can only accept or reject the manager's proposal. Examples of voted, anticipatory defenses include staggered board elections, super-majority voting provisions, "fair price" amendments, "poison pills" that take the form of amendments to the corporate charter, and establishment of classes of stock with differential voting rights.

For convenience, direct costs to the manager of implementing \(x\) are normalized to zero. However, the manager bears an indirect cost for requesting anticipatory defense, \textit{regardless} of whether the request is granted. This cost is in terms of his outside value -- the utility payoff he can expect to receive in the second period, conditional on being ousted from the firm. Our presumption is that efforts on the part of incumbent managers to erect takeover defenses in the absence of an explicit offer indicate that the manager is, for instance, more interested in personal security than the welfare of the shareholders; and this lowers his outside market value. Let \(X\) denote a requested level of anticipatory defense \(x\). If \(X\) is approved by shareholder vote, then the implemented level of
this defense is \( x = X \); and if it is not approved then \( x = 0 \). Evidently, \( X = 0 \) implies \( x = 0 \). Note however that the above discussion implies that the states of the world \( [x = 0 \mid X = 0] \) and \( [x = 0 \mid X > 0] \) are distinct: this is crucial to our model.

The second type of takeover defense available to the manager, \( y \), is \textit{responsive}. Responsive defenses are implemented only to fight an explicit takeover bid in the second period, conditional on a bid materializing. Unlike anticipatory defenses, the manager implements responsive defenses without obtaining shareholder approval. Examples of non-voted, responsive defenses include anti-trust lawsuits, targeted repurchases, "poison pills" implemented by the board of directors, private placements of stock with parties friendly to incumbent management, and counter-offers to acquire the would-be acquirer.

The incumbent bears direct costs, measured in units of his utility, for implementing responsive defenses. As described below, the manager receives utility from the activity of managing the firm. These direct costs of engaging in a takeover battle may be thought of as the disutility he receives from diverting time and effort from his usual managing activities into the less-preferred activity of fighting a takeover.

The distinction we make between anticipatory and responsive takeover defenses, that the former are voted by shareholders while the latter are not, is an abstraction. It is a useful one for our purposes, and does not do violence to empirical reality. From 1968 to 1979, the Williams Act required that a takeover bid be outstanding for at least ten business days. Currently, SEC's Rule 14e-1, adopted in 1979, sets this minimum time for a takeover bid at twenty business days. However, voting decisions by shareholders are governed by state corporation law requirements regarding disclosure and mailing of formal proxies. Delaware General Corporation Law requires that notice be mailed to shareholders of record eligible to vote at least twenty days prior to the meeting. While these requirements do not preclude the use of voted defenses as responsive tactics (the Martin Marietta - Bendix case is a counterexample), they surely make it more difficult. Indeed, most voted defenses are put in place when no explicit takeover threat exists.\textsuperscript{8}
3.2 The Second Period: The Incumbent's Utility for Employment

We suppose that there is no utility value to making a bid per se, so that, given the value $m$ of the firm under the incumbent manager, only managers with values $n > m$ find it worthwhile to make takeover bids. If there is any administrative cost to bidding (i.e., a cost other than the price of the firm's shares), then no manager $n \leq m$ will make an offer. Hereafter, we assume there is such a cost, but, to avoid cluttering the notation, suppose the cost constant and take it to be implicit in $m$. Hence, $n > m$ is a necessary, but not sufficient, condition for a successful takeover bid.

The condition $n > m$ is not sufficient for a successful takeover bid because of the possibility that the incumbent will erect takeover defenses, as described above. If takeover defenses are used, the potential bidder's value $n$ must exceed $m$ by at least the amount of takeover defenses. Given defenses $x$ and $y$, a takeover bid must be at least equal to $[m+x+y]$ in order to win. Will any more be paid? Because shareholders are interested only in maximizing wealth, and given that dilution is available to the acquirer, it is sufficient to pay $\epsilon > 0$ more than $[m+x+y]$ to persuade them to remove the incumbent. No acquirer wishes to pay any more than necessary to take over the firm, so in the limit epsilon will be zero. Hence, the premium above $m$ that the incumbent secures for the shareholders -- conditional on losing a takeover battle despite defenses $x$ and $y$ -- is precisely $[x+y]$. Hereafter, denote the premium by $\pi$.

To derive the expected premium, consider first the incumbent's objective function. The incumbent is surely in control of the firm in the first period, and the investment and financial structure of the enterprise are taken as given. Therefore, it is necessary only to examine his second period payoff under two possible employment alternatives: within the firm, or, if a bid succeeds, outside the firm.

If there is no takeover bid in the second period, the incumbent stays in control and receives utility $V(m)$ from managing the firm. If there is a takeover bid, the manager may choose to erect responsive defense $y$, as described above. Let $c(y)$ be the direct costs of implementing responsive
defense $y$, assumed increasing convex in $y$, $c' > 0$, $c'' > 0$, with $c(0) = 0$. For technical reasons, it is convenient to assume $\lim_{y \to 0} c'(y) = 0$: none of our principal results depend on this.\footnote{9}

Because of these costs, the incumbent's second period utility if he must fight a takeover bid which is ultimately unsuccessful is $v = V - c$.

In the event of a successful takeover bid, the manager's second period utility is the utility he receives from his employment alternative outside the firm, net of the costs borne in engaging in the takeover battle:

$$\omega(x, y \mid X) = W(\pi, X) - c(y).$$

$W$ is the manager's gross outside value, or utility for alternative employment. We assume the incumbent is rewarded by the outside labor market for extracting a higher premium for shareholders, given he is ousted from the firm. His outside value is increasing concave in the takeover premium he is able to extract from the acquirer, $W_1 > 0$, $W_{11} < 0$; he is rewarded at a diminishing marginal rate. As described above, the manager suffers a decline in outside value, $W_2 < 0$, for requesting anticipatory defenses when no explicit takeover threat exists. We assume $\lim_{X \to \infty} W_2 = -\infty$, so that requests for guaranteed job security result in extreme loss of outside value. We further assume the cross-effects of the decline due to the request on the increase due to extracting the premium are non-increasing, $W_{12} \leq 0$. That is, the two effects may be separable, but if they are not then they dampen, not amplify, each other. To make the problem nontrivial, we suppose that at the start of the first period, before any request is made, the manager prefers employment in the firm to his outside alternative, $V(m) > W(0, 0)$.

The above assumptions imply that the manager's utility for alternate employment is, for large enough approved requests ($x = X$), strictly decreasing in $x$. This does not exclude the possibility that $W$ is everywhere decreasing in approved requests. This decrease is the indirect cost to the manager of requesting anticipatory defenses, mentioned above.

Conditional on anticipatory defenses $x$, the manager can raise his outside value in the event of a successful bid by using responsive defenses $y$ to increase the premium received by shareholders.
Given the anticipatory defenses in place after the first period, $X \geq 0$ and $x \in [0, X]$, and observing a bidder with value $n$, the incumbent chooses responsive defense $y$ in the second period to maximize his utility. Define $y^*$ as the response which maximizes the incumbent's net utility if the bid is successful and the manager leaves the firm:

$$(2) \quad y^*(x \mid X) = \text{argmax}_{y \in \mathbb{R}^+} \omega(x, y \mid X).$$

From (1), we obtain first- and second-order conditions:

$$(3) \quad \partial \omega / \partial y = W_1 - c';$$

$$(4) \quad \partial^2 \omega / \partial y^2 = W_{11} - c'' < 0, \forall y.$$  

Setting $\partial \omega / \partial y = 0$ implicitly defines $y^*$. By (4), $y^*$ is unique for all $X, x \geq 0$.

We now define the incumbent's net utility from fighting and retaining his job, that is, fighting a bid which fails. Observing a bidder with value $n \geq m+x$, the incumbent will win if he chooses $y \geq n-m-x$. Since $y$ is costly, the incumbent's best winning response is $y = n-m-x$. Thus $v$, the incumbent's net utility from fighting an unsuccessful bid can be written as a function of $n$ and $x$:

$$(5) \quad v(n, x) = V(m) - c(n-x-m).$$

The manager will fight to win as long as his net utility from employment with the firm $v$ exceeds his net outside utility $\omega$. Once $\omega$ dominates $v$, however, he will fight only to the extent that doing so increases his outside utility; then he will leave to collect $\omega$.

### 3.3 The Second Period: Takeover Bids

We argued above that, if $x$ and $y$ are fixed and known, only a potential bidder with value $n > [m+x+y]$ can make a successful offer. Moreover, only bids which can succeed will be made, as long as there is any bidding cost, and no utility value to making unsuccessful bids. At the beginning of the second period, the anticipatory defense $x$ is indeed known. However, since defense $y$ is responsive, the incumbent's selection of $y$ depends on the particular $n$ drawn by Nature in the second period. By the second period, we suppose the incumbent's utility and cost schedules, $W, V,$ and $c$, are common knowledge, along with his contribution to firm value $m$. 
Likewise, once Nature has made her draw, the value $n$ of the potential acquirer is common knowledge. Therefore, the potential acquirer is capable of calculating the incumbent manager's credible response $y$ to any bid. A response $y$ to a bid $n$ is credible if, when $n$ is bid, the manager's utility-maximizing response is $y$. If this utility-maximizing response is sufficient to beat the bidder's best offer (that is, if $m+x+y > n$), then -- because of the bidding cost -- the potential acquirer makes no offer at all. If the potential acquirer is capable of topping the incumbent's best response ($m+x+y < n$), then she makes the smallest offer necessary to win control of the firm. Though the incumbent loses surely, by definition of credibility he prefers to bear the costs of fighting, $c$, and the winner pays the premium $\pi = x+y$.

The minimum bid which can succeed against the incumbent is defined by examining the manager's choice between $v$ and $\omega$. The manager faces one of two scenarios prior to the takeover battle. His highest attainable gross outside utility, $W^* = W(x+y, X)$, is either less than his utility for employment inside the firm, $V$, or it is not. In what follows, we discuss the process which determines bids and responsive defenses in each of these two cases in turn. Formally, denoting the minimum bid which can succeed by $n$, and defining $\omega^* = W^* - c(y^*)$:

\[
\begin{align*}
[\inf n \mid v = \omega], & \quad \text{if } W^* \geq V; \\
(6) \quad n(x \mid X) = \{ & \quad \text{if } W^* \leq V. \\
[\inf n \mid v = \omega^*], & \quad \text{otherwise.}
\end{align*}
\]

In the first case, where $W^* \geq V$, the minimum bid which can succeed will be below the one that allows the manager to attain his maximum outside utility. To see this, note that at the beginning of the second period, absent a takeover bid, the incumbent prefers his job inside the firm to outside employment, or $V > W(x+0, X)$. Since by hypothesis $V \leq W^*$, then it must be that, for some $y$ between zero and $y^*$, $V = W$ or equivalently $v = \omega$. Above this point, the incumbent prefers outside employment to employment with the firm, and so prefers to surrender control. Thus, a bidder whose value $n$ is slightly above the point where $v = \omega$ can win. This case is illustrated in Figure 1(a).
In this case, the premium paid by the successful bidder may depend on the bidder's value \( n \).

Define \( n^* \) to be the bid corresponding to the incumbent's highest attainable outside utility:

\[
(7) \quad n^*(x | X) = m + x + y^*,
\]

where \( y^* \) is defined as above. Note that \( n^* > n \) if and only if \( W^* > V \). Consider Figure 1(a) and first let \( n \in (n, n^*) \). The incumbent's best response is to fight to extract all the bidder's willingness-to-pay for the firm by choosing \( y = n - m - x \), and then to leave the firm to collect his net outside value, \( \omega \): this is clearly credible.

Now let the bidder's value \( n \) exceed \( n^* \). Observing this, the incumbent optimizes by choosing \( y = y^* \) and then leaving the firm to obtain his outside value, \( \omega^* \). Any bidder with value \( n > n^* \) pays only \( n^* \) to acquire the firm. That is, the bidder does not pay all that she is willing to pay for the firm. Although the firm may attain the value \( n \) in the second period by virtue of the new manager's value added, the new manager is assumed to consume the surplus, \( n - n^* \). That is, we presume that the anticipatory defenses \( x \) put in place in the first period, along with the responses chosen by the manager in the second period, set the minimum acquisition price but do not prevent acquirers from capturing benefits above this price.

Now consider the other case, where \( W^* < V \). Here the manager is willing to undertake costly responsive defenses \( y \) until the cost of doing so drives his net utility from continued employment with the firm below his maximum attainable outside utility. In Figure 1(b), observing any \( n \leq n \), the incumbent prefers employment in the firm, for which he receives utility \( v \), to his outside alternative which nets him \( \omega \). Thus, for any \( n < n \), the manager's utility-maximizing action is to fight and defeat the bidder. Any successful bidder, with \( n > n \), will pay \( n^* \) regardless of individual value added. This is true because the incumbent, observing a bidder who can contribute a firm value of \( n > n \), recognizes defeat, but chooses \( y = y^* \) to maximize his personal outside value.

From the above discussion, the premium \( \pi \) paid by the successful bidder of value \( n \), above the value \( m \) contributed by the incumbent, can be defined:

\[
(8) \quad \pi(n, x | X) = \min\{n - m, n^* - m\}.
\]
Inspection of (6) and Figure 1 reveal that, for bids \( n \leq n \), it is credible that the incumbent will fight and win. On the other hand, any potential acquirer who can bid \( n > n \) will be successful. Therefore, the probability of a successful bid appearing in the second period is \( [1 - F(n)] \). The premium she will pay, described by (8), depends on \( n \) and on the incumbent's utility function. Together, these two pieces of information determine the expected second-period premium, evaluated by shareholders and the manager in making their first period choices.

3.4 The First Period: Shareholder Voting on the Incumbent's Proposal

We now specify the actions of the incumbent and the shareholders in the first period, which are based on their expectations about the second period. Viewed from the first period, the incumbent's expected second period payoff is a function of anticipatory defenses \( x \). This expected payoff is his utility from retaining his job times the probability no bidder exceeding \( n \) appears, plus his utility from alternative employment times the associated probability of a successful bidder appearing:

\[
U(x \mid X) = F(n) \cdot V + \delta \cdot [F(n^*) - F(n)] \int_0^n \omega(x, n - m - x \mid X) \cdot f(n) dn
\]

\[
+ \{\delta \cdot [1 - F(n^*)] + (1 - \delta) \cdot [1 - F(n)]\} \cdot \omega(x, n^* - m - x \mid X),
\]

where \( \delta = 1 \) iff \( n < n^* \) (equivalently, iff \( V < W^* \), and \( \delta = 0 \) otherwise. As described above, the incumbent's best response may depend on the bidder's value \( n \) if \( V < W^* \).

The first period expected value of the firm under management \( m \) is the sum of \( m \) and the expected premium:

\[
S(x \mid X) = m + \delta \cdot [F(n^*) - F(n)] \int_0^n [n - m] \cdot f(n) dn
\]

\[
+ \{\delta \cdot [1 - F(n^*)] + (1 - \delta) \cdot [1 - F(n)]\} \cdot [n^* - m]
\]

with \( \delta \) defined as above.

Since anticipatory defenses \( x \) require shareholder approval, the manager's first period optimization problem is to choose a request, \( X \geq 0 \), to maximize his expected utility, subject to a constraint of shareholder approval:
\begin{equation}
\max U(x \mid X) \\
\text{subject to: } S(X \mid X) \geq S(0 \mid X).
\end{equation}

The constraint says simply that shareholders will vote rationally on the manager's request. Clearly, if the manager asks for no anticipatory defense, $X = 0$, then the constraint trivially binds. However, if $X > 0$ then the constraint says that approval of the request must result in an expected value of the firm that is no less than its value if the request is rejected. Because the incumbent manager's outside value in the second period is reduced by his request, $X$, in the first period -- even if it is not granted -- we have in general that $S(0 \mid 0) \neq S(0 \mid X)$ for any $X > 0$. In other words, the alternative that rational shareholders compare against the manager's request is not $S(0 \mid 0)$, their situation prior to any request, but is $S(0 \mid X)$, the result if they reject the request. This notion is central to the results on the voting conundrum.

If the constraint binds at some $X > 0$, then shareholders are indifferent between accepting and rejecting the request. Since managers are not indifferent, they can insure acceptance in such circumstances by slightly perturbing the request, $X$, to induce strict preference on the part of shareholders: as Appendix A shows, such a perturbation is always available. From a game-theoretic perspective, this induces a unique subgame perfect voting rule for the shareholders (Banks and Gasmi (1987)): vote for the management's proposal with probability one if and only if $S(X \mid X) \geq S(0 \mid X)$, and vote against the proposal with probability one otherwise (i.e. $S(X \mid X) < S(0 \mid X)$). Therefore, without loss of generality, we can adopt the convention that in cases of shareholder indifference, shareholders always "vote with the management".

We turn now to some results.\textsuperscript{12}

4. Results

We now analyze the voting conundrum discussed in the Introduction in the context of the model described in section 3. The manager requests anticipatory defenses in the first period, and shareholders vote on the request. The outcome of the vote can affect both the probability of a
takeover bid appearing in the second period, and the premium shareholders receive if a takeover bid succeeds. In part, these effects are driven by the fact that the manager has responsive defenses available to counter a particular takeover bid, and his utility maximizing use of responses will be affected by the outcome of the first period. In all cases, the probability of a bid appearing in the second period falls as a result of a request for anticipatory defenses, and in all cases the value of the firm to shareholders is lower if they reject the proposal than it would have been if no defenses had been requested. The value of the firm to shareholders if they accept the proposal may be greater or less than than it would have been if no defenses had been requested. The voting conundrum occurs if the value under acceptance is larger than the value under rejection, so rational voting implies acceptance; but the value under acceptance is lower than it would have been if no defenses had been requested. We provide a sufficient condition for this to occur.

There are three possible conditions at the beginning of the second period: (1) the manager requested no anticipatory defenses, (2) the manager made a request which was voted down by shareholders, and (3) the manager's request was approved. Proposition 1 puts an ordering on $y^*$, the second period response to a takeover bid which maximizes the manager's outside value, conditional on the first-period outcome.

**Proposition 1:** $y^*(0 | 0) \geq y^*(0 | X) > y^*(X | X) > 0$.

**Corollary 1:** $X + y^*(X | X) > y^*(0 | X)$.

If a first-period request is rejected by shareholders, the second-period maximizing response is strictly larger than if the request is approved. However, from Corollary 1, total defenses (anticipatory plus responsive) are smaller if the request is rejected. Since total defenses determine the premium received in a successful bid, this implies that shareholders can expect a smaller premium if they reject the manager's request and a successful bidder emerges.
An interesting feature evident from Proposition 1 is that the second-period maximizing response is never larger after a request, even if the request is denied, than if no request is made. Thus, the request carries no implicit threat of "scorched earth" responses if shareholders deny it. Rather, as we show below, the request alters the manager's incentives to fight for a higher premium if a successful bidder appears.

Anticipatory and responsive defenses are substitutes in their effects on \( n \), the minimum bid required to acquire the firm. Generally, the higher the level of anticipatory defense approved by shareholders in the first period, the lower will be the manager's choice of responsive defenses. Comparative statics on \( y^* \), the maximizing second-period response (Appendix A, (a3) - (a7)) show that this response is non-increasing in the size of the first-period request.

In evaluating how to vote, value-maximizing shareholders consider both the size of the premium they can expect from a successful bid, and the probability that such a bid will emerge. Proposition 2 describes the probability of a successful takeover bid, as a function of requested anticipatory defenses and shareholder approval of the defenses.

**Proposition 2:**

(a) \( n(x \mid X) \) is strictly increasing in \( X, x \in \{0, X\} \);  
(b) \( \forall X > 0, n(X \mid X) \geq n(0 \mid X) > n(0 \mid 0) \).

Proposition 2 states that the minimum bid which can succeed against incumbent management increases with the level of anticipatory defense requested. Since the probability that a successful bid will emerge, \( [1 - F(n)] \), falls as the minimum successful bid rises, part (a) of Proposition 2 implies that the probability of takeover declines with increases in the level of requested anticipatory defenses, whether or not these are approved. Part (b) implies that the probability of takeover is largest if no anticipatory defenses are requested, declines if there is a request, and may decline more if the request is approved. Notice that part (b) is not an immediate consequence of part (a): the
change in state from \([x = X | X > 0]\) to \([x = 0 \mid X > 0]\) is not incremental because of the "take it or leave it" nature of the shareholders' decision.

The value to shareholders from anticipatory takeover defense depends on both the premium if a successful bid is made, which generally increases with the level of anticipatory defense, and on the probability of a successful bid, which generally decreases. Proposition 3 shows that shareholders are unambiguously worse off if they reject management's request for anticipatory defenses than they were before the request.

**Proposition 3:** For \(X > 0\), \(S(0 \mid 0) > S(0 \mid X)\).

Proposition 3 follows directly from part (b) of Proposition 2, along with the first inequality of Proposition 1. The probability that a successful bid will appear in the second period is lowered after rejection of the manager's proposal, because the minimum bid which can succeed against the incumbent is now higher. Moreover, for any successful bid the premium received by shareholders will be no higher than it would have been if no request had been made, and may be lower. These effects are driven by the manager's utility for employment outside the firm, which declines as a result of the request for anticipatory defenses.

The voting (incentive compatibility) constraint in the model requires that approval of a request leave shareholders at least as well off as rejection. Proposition 3 states that shareholders are worse off if they reject the request than they would have been if no request had been made. (Note that this latter circumstance, no request, is not available to shareholders deciding how to vote on a request.)

To complete our description of the apparent paradox that shareholders vote for defenses which leave them worse off, we require a comparison of shareholder value if the request is approved with shareholder value if no request is made: \(S(X \mid X)\) versus \(S(0 \mid 0)\). Proposition 4 gives a sufficient condition for the apparent paradox to occur.
Proposition 4: \( V(m) < W(y^*(0\,\|\,0), 0) \)
\[ \Rightarrow \exists X \in (0, \infty): S(0\,\|\,0) > S(X\,\|\,X) \geq S(0\,\|\,X). \]

According to Proposition 4, if the manager's utility from employment with the firm is less than his maximum gross outside utility before any request for anticipatory defenses, then requests exist which shareholders will approve, but which leave them worse off than they were before. It is important to observe that this maximum gross outside utility is available only if, in the second period, Nature draws a bidder with value \( n \geq n^*(0\,\|\,0) \). Recall that we assumed the incumbent's utility from employment in the firm was larger than his initial gross outside utility: that is, his utility before defenses \( x \) or \( y \) are considered. Proposition 4 concerns the incumbent's maximum gross outside utility: that is, considering his optimal choice of \( y \) at \( X = 0 \). The situation is pictured in Figure 1(a), by setting \( x = X = 0 \).

With Proposition 4, we have shown the existence of requested levels of anticipatory defense which result in the voting behavior we hoped to describe. That is, rational, value-maximizing shareholders who are fully informed at the time of the vote, will approve the defenses but are made worse off by them. An artifact of our assumption of identical, risk-neutral shareholders is that all votes will be unanimous. Empirically, shareholders will differ: for example, managers who own shares, and holders of large blocks may differ from holders of small positions who have well-diversified portfolios. The results will apply to non-identical shareholders as long as the pivotal voter is risk-neutral and value-maximizing.

The sufficient condition for the result in Proposition 4 depends on the manager's utility in current and alternative employment. In Appendix B, we construct an example of a manager with utility satisfying this condition, whose optimizing choice of anticipatory defense will be approved by shareholders, and will leave them worse off than if no request had been made. The example is in no sense pathological, and demonstrates that these levels of anticipatory defense could be requested by managers with utilities satisfying the antecedent of Proposition 4.
It is worth noting that under some circumstances, shareholders can be made better off by implementing some anticipatory defense. By Proposition 3, a necessary condition for this to occur is that the manager's request, $X$, be strictly interior to the constraint set; i.e. $S(X \mid X) > S(0 \mid X)$. It is not, however, sufficient. In our example in Appendix B, shareholders would like some anticipatory defense, but the manager prefers more and, despite the constraint not binding, the request results in a decline in shareholder wealth.

We remarked in section 2 that, in order for the stock price to fall as a result of a request, some information must be conveyed by the request. The information conveyed concerns the manager's utility from employment outside the firm. In the circumstances described by Proposition 4, it is clear that any probability weighting of $S(0 \mid 0)$ with $S(X \mid X)$ or $S(0 \mid X)$ implies a downward revision when the first possibility is eliminated. In these circumstances, as long as shareholders place some prior probability on the outcome that the manager will prefer no anticipatory defense, there exist requests $X > 0$ which are associated with a drop in the value of the firm.

The primary implication of our model for interpreting the empirical work which has preceded it is that inferences about whether shareholders vote rationally cannot be made from a comparison of shareholder wealth before and after the vote. We describe the alternative to voting with management as an unambiguous drop in shareholder value, not as a return to the pre-proposal status quo. Therefore, pre-proposal shareholder wealth is not the correct benchmark for determining shareholder rationality. Our model suggests that empirical investigation of shareholder voting and takeover defenses should consider the manager's outside employment opportunities, and the manager's skill relative to potential bidders.

5. Conclusion

We have provided a rational choice model of shareholder voting on anticipatory takeover defenses. In our model, it is feasible for informed, value-maximizing shareholders to approve measures which leave them worse off than they were before the measures were requested. What
drives this result is that a manager, in requesting such measures, lowers his outside market value. Consequently, the manager's optimal response to an actual takeover bid is different if the request is rejected than if he had made no request. Shareholders recognize this in evaluating how to vote.

An open question is why shareholders allow the manager to act first to request anti-takeover devices, and allow him discretion over the level of responsive defenses. Shareholders will not generally want to prohibit the use of responsive defenses as we have characterized them. This is true because, although their existence reduces the probability of a successful bid appearing, responsive defenses allow shareholders to receive higher premia from some bidders. In fact, if a bidder appears with sufficiently high value added \( (n > n^*) \), shareholders would like the incumbent to use more responsive defense than he will choose to maximize his own utility. Since shareholders cannot know \textit{ex ante} which bidder will appear in the future, and may have difficulty discerning the utility schedules of current and future managers, prohibiting takeover defenses may preclude value-increasing actions as well as value-reducing ones. The empirical fact that managers are given control over voting agendas, and are not prohibited by shareholders from engaging in responsive tactics suggests that complete contingent contracts are costly to write. However, we offer as an observation that recent charter amendments which prohibit the use of "greenmail," or targeted repurchases, may be viewed as shareholders' prohibiting a particular responsive defense which, if implemented, is unlikely to result in a higher premium for them.

Finally, we offer two remarks. First, it is frequently asserted (cf. Easterbrook and Fischel (1983)) that the alienability of ownership claims protects shareholders from detrimental management entrenchment tactics. However, unless shareholders anticipate the proposal of takeover defenses by management, they cannot sell a voting claim in response to the proposal. SEC proxy mailing requirements demand that the record date for shareholder voting precede the proposal date. Once the proposal is announced, the constituency is fixed. Our model suggests that any drop in share value should occur with the announcement of the proposal, not with the vote.

Second, our model implies that changes in shareholder wealth associated with voting on
takeover defenses may be positive or negative, depending on the manager's value added and utility schedule. In cases where shareholders' wealth is reduced, the reduction should occur at the date of the manager's request. If there is any detectable change at the date of the vote, it should be positive. This follows from Proposition 3 and the voting constraint: the first states that shareholders' wealth is unambiguously reduced if they reject a request; the second ensures that shareholders' wealth under rejection is no larger than their wealth under approval. On this interpretation, empirical results which find shareholders' wealth declines following implementation of anticipatory defenses are not evidence of shareholder irrationality or ignorance, but reflect an informed choice of the lesser of two declines in value.
Appendix A: Proofs

Proposition 1: Let $X > 0$. Then: $y^*(0 \mid 0) \geq y^*(0 \mid X) > y^*(X \mid X) > 0$.

Proof:

(i) $y^*(X \mid X) > 0$ follows from assuming $W_1 > 0$ everywhere, and $\lim_{y \to 0} c'(y) = 0$.

(ii) $y^*(0 \mid 0) \geq y^*(0 \mid X)$

Suppose $y^*(0 \mid 0) < y^*(0 \mid X)$. Then, by $W_{11} < 0$, $W_{12} \leq 0$ and $X > 0$:

$W_1(y^*(0 \mid 0), 0) - W_1(y^*(0 \mid X), X) > 0$.

Use (3) to obtain:

(a.1) $W_1(y^*(0 \mid 0), 0) - W_1(y^*(0 \mid X), X) = c'(y^*(0 \mid 0)) - c'(y^*(0 \mid X))$.

But $c'' > 0$ implies the RHS(a.1) < 0: contradiction.

(iii) $y^*(0 \mid X) > y^*(X \mid X)$

Suppose $y^*(0 \mid X) \leq y^*(X \mid X)$, and again use the first order condition (3) to get:

(a.2) $W_1(X + y^*(X \mid X), X) - W_1(y^*(0 \mid X), X) = c'(y^*(X \mid X)) - c'(y^*(0 \mid X))$.

Then $c'' > 0$ implies RHS(a.2) $\geq 0$.

But $X > 0$, so that $W_{11} < 0$ implies LHS(a.2) $< 0$: contradiction. \(\parallel\)

Remark 1: $y^*(0 \mid 0) = y^*(0 \mid X)$ iff $W_{12} \equiv 0$. Hence, $x = X > 0$ and $W_{12} \equiv 0$ imply:

$[X + y^*(X \mid X)] > y^*(0 \mid 0)$.

Corollary 1: Let $X > 0$. Then: $[X + y^*(X \mid X)] > y^*(0 \mid X)$.

Proof: Use (3) and Proposition 1 with $W_{11} < 0$ and $c'' > 0$. \(\parallel\)

Comparative Statics:

(a.3) $dy^*(X \mid X)/dX = -[W_{11} + W_{12}]/[W_{11} - c''] < 0$.

(a.4) $dy^*(0 \mid X)/dX = -W_{12}/[W_{11} - c''] \leq 0$, with the inequality strict iff $W_{12} < 0$. 
(a.5)  \(\frac{d\omega(X, y \mid X)}{dX} = W_1 + W_2\), which a priori has ambiguous sign.

(a.6)  \(\frac{d\omega(0, y \mid X)}{dX} = W_2 < 0\).

Note that at any \(y\), \(|(a.5)| < |(a.6)|\).

Since \(n^*(x \mid X) = x + y^*(x \mid X) + m\), and \(y^*(x \mid X)\) is differentiable in \(X\), \(n^*(x \mid X)\) is differentiable in \(X\). If \(x = 0\), then \(\partial n^*(0 \mid X)/\partial X\) is given by (a.4). If \(x = X\), then \(\partial n^*(X \mid X)/\partial X = 1 + dy^*(X \mid X)/\partial X\). Substituting from (a.3) and collecting terms, we obtain:

(a.7)  \(\frac{\partial n^*(x \mid X)}{\partial X} \geq (\leq) 0\) as \(c''(y^*(x \mid X)) \geq (\leq) -W_{12}(x+y^*(x \mid X), X)\).

Lemma 1: Let \(X \geq 0\), \(x \in \{0, X\}\). Then: \(n(x \mid X)\) is differentiable in \(X\); and \(\frac{\partial n(x \mid X)}{\partial X} > 0\).

Proof:

(i) \(n(x \mid X)\) is differentiable.

First, recall from section 3 that \(n(x \mid X) > x + m\), \(\forall x, X \geq 0\). Let \(x = X \geq 0\), and define:

\[\Delta^* = [v(n^*(X \mid X), X) - \omega(X, y^*(X \mid X) \mid X)]\]

Differentiating \(\Delta^*\) with respect to \(X\) at \(x = X\), and using (3) gives:

\[\frac{\partial \Delta^*}{\partial X} = -W_2(X+y^*(X \mid X), X) > 0\]

Therefore, \(\Delta^*\) can change sign at most once as \(x\) increases, and only from negative to positive.

Suppose there exists an \(X\) such that \(\Delta^* = 0\). Then \(X\) is unique and:

\[V(m) \leq (>) W(X+y^*(X \mid X), X) \text{ iff } X \leq (>) X\]

So by (6): \(n(X \mid X) \leq (>) n^*(X \mid X) \text{ iff } X \leq (>) X\).

Case α: Consider \(X < X\).

By (6), \(n(X \mid X)\) is implicitly defined by: \(V(m) - W(n(X \mid X) - m, X) = 0\).

In this case, \(n(X \mid X)\) is differentiable because \(W\) is differentiable. In particular, \(\forall X < X\),

(a.8)  \(\frac{\partial n(X \mid X)}{\partial X} = -W_2(n(X \mid X) - m, X)/W_1(n(X \mid X) - m, X)\)

and,

(a.9)  \(\lim_{X \to X^-} \frac{\partial n(X \mid X)}{\partial X} = -W_2(n^*(X \mid X) - m, X)/W_1(n^*(X \mid X) - m, X)\).

Case β: Consider \(X > X\).
By (6), \( n(X \mid X) \) is implicitly defined by: 
\[
V(m) - c(n(X \mid X) - X) - \omega(\omega, y^*(X \mid X) \mid X) = 0.
\]
Differentiability of \( n(X \mid X) \) follows from differentiability of \( c \) and \( W \). In particular, using (3):

(a.10) \[
\frac{\partial n(X \mid X)}{\partial X} = [c'(n(X \mid X) - X) - W_1^*(X) - W_2^*(X)]/c'(n(X \mid X) - X),
\]
where \( W_i^*(X) \equiv W_i(n^*(X \mid X) - m, X), i = 1, 2 \). Hence,

(a.11) \[
\lim_{X \to X^+} \left[ \frac{\partial n(\cdot)}{\partial X} \right] = \\
\left[ c'(n^*(X \mid X) - X) - W_1^*(X) - W_2^*(X) \right]/c'(n^*(X \mid X) - X) = \\
-W_2(n^*(X \mid X) - m, X)/W_1(n^*(X \mid X) - m, X);
\]
the second equality follows from another application of (3). Together, (a.9) and (a.11) complete the argument that \( n(x \mid X) \) is differentiable everywhere when \( x = X \).

(ii) \( \frac{\partial n(x \mid X)}{\partial X} > 0 \)

\textbf{case} \( \alpha \): \( x = X \leq X \)

The proof of this case is immediate from (a.8) and (a.9).

\textbf{case} \( \beta \): \( x = X > X \)

Consider (a.10). By (3), \( W_1^*(X) = c'(y^*(X \mid X)) = c'(n^*(X \mid X) - X) \). Since \( \Delta > 0, n^*(X \mid X) < n(X \mid X) \). Therefore, \( c'' > 0 \forall y \) implies,

(a.12) \[
c'(n(X \mid X) - X) > W_1^*(X) > 0.
\]
Moreover, \( W_1 + W_2 < W_1 \). Therefore, the numerator of (a.10) is strictly positive. Since \( c' > 0 \), this completes the proof of the Lemma for \( x = X \).

Now suppose \( x = 0 \) and \( X > 0 \). Similar reasoning as before gives \( n(0 \mid X) \) differentiable in \( X \), and \( \frac{\partial n(0 \mid X)}{\partial X} > 0 \) follows on implicit differentiation of (6) for \( X \leq X \) and for \( X > X \). \( \square \)

\textbf{Remark 2:} (a.7) and Lemma 1 imply that \( U(x \mid X) \) and \( S(x \mid X) \) are differentiable in \( X \).

\textbf{Lemma 2:} \( n(X \mid X) \geq n(0 \mid X) \);

\textbf{Proof:}

\textbf{case} \( \alpha \): \( V(m) < W(X + y^*(X \mid X), X) \)

\textbf{Remark 2:}
By (6) and the premise, \( V(m) = W(n(X | X) - m, X) \). There are then two possibilities:

(a.1) \( V(m) \leq W(n^*(0 | X) - m, X) \)

\[ \Rightarrow V(m) = W(n(0 | X) - m, X) \text{, by (6)} \]

\[ \Rightarrow W(n(X | X) - m, X) = W(n(0 | X) - m, X) \]

\[ \Rightarrow n(X | X) = n(0 | X). \]

(a.2) \( V(m) > W(n^*(0 | X) - m, X) \)

\[ \Rightarrow V(m) = [\omega(0, y^*(0 | X) | X) + c(n(0 | X) - m)], \text{ by (6)} \]

\[ \Rightarrow W(n(X | X) - m, X) - c(n(0 | X) - m) = \omega(0, y^*(0 | X) | X). \]

By definition of \( y^*(0 | X) \), \( \omega(0, y^*(0 | X) | X) > W(n(0 | X) - m, X) - c(n(0 | X) - m) \).

Therefore, \( W(n(X | X) - m, X) - c(n(0 | X) - m) > W(n(0 | X) - m, X) - c(n(0 | X) - m) \)

\[ \Rightarrow n(X | X) > n(0 | X). \]

This proves the proposition for case (a).

case B: \( V(m) \geq W(X+y^*(X | X), X) \)

\[ \Rightarrow V(m) = \omega(X, y^*(X | X) | X) + c(n(X | X) | X - m), \text{ by (6)} \]

By Corollary 1 and \( X > 0 \), \( W(X+y^*(X | X), X) > W(y^*(0 | X), X) \).

Hence, \( V(m) > W(n^*(0 | X) - m, X) \). So by (6),

(a.13) \( V(m) = \omega(0, y^*(0 | X) | X) + c(n(0 | X) - m) \).

Therefore,

(a.14) \( \omega(X, y^*(X | X) | X) - \omega(0, y^*(0 | X) | X) = c(n(0 | X) | X - m) - c(n(X | X) | X - m). \)

By Proposition 1 and Corollary 1, LHS(a.14) is strictly positive; hence, the RHS(a.14) must likewise be strictly positive. Since \( c' > 0 \) and \( x = X > 0 \), this implies,

(a.15) \( n(0 | X) > n(X | X) - X. \)

Now implicitly differentiating (a.13), we obtain \( \forall X \geq 0: \)

\[ \partial n(0 | X) / \partial X = -W_2(y^*(0 | X), X) / c'(n(0 | X) - m) > 0. \]

Using (a.10), we have \( \forall X \geq 0: \)

(a.16) \[ [\partial n(X | X) / \partial X - \partial n(0 | X) / \partial X] = [1 - \{W_1(X+y^*(X | X), X) / c'(n(X | X) - X - m)])

+ [W_2(y^*(0 | X), X) / c'(n(0 | X) - m)] - [W_2(X+y^*(X | X), X) / c'(n(X | X) - X - m))]. \]
By (a.12), the first term of (a.16) in \( \{ \cdot \} \) is nonnegative \( \forall X > 0 \). Consider the second term in \( \{ \cdot \} \).

By Corollary 1, \( W_2 < 0 \), and \( W_{12} \leq 0 \),

\[
(a.17) \quad 0 > W_2(y^*(0 \mid X), X) \geq W_2(X + y^*(X \mid X), X).
\]

By (a.15) and \( c'' > 0 \),

\[
(a.18) \quad 0 < c'(n(X \mid X) - X - m) < c'(n(0 \mid X) - m).
\]

Together, (a.17) and (a.18) imply that the second term of (a.16) in \( \{ \cdot \} \) is also positive \( \forall X > 0 \).

When \( X = 0 \), both terms in \( \{ \cdot \} \) vanish. Therefore, \( \forall X \geq 0 \):

\[
[n(X \mid X) - n(0 \mid X)] = \int_0^X \left[ \partial n(r \mid r) / \partial r - \partial n(0 \mid r) / \partial r \right] dr \geq 0,
\]

as required (with equality iff \( X = 0 \)). \( \square \)

**Remark 3:** Notice that Lemma 2 is not implied by Lemma 1: this is because the change in state from \( [x = X \mid X > 0] \) to \( [x = 0 \mid X > 0] \) is not incremental.

**Remark 4:** Together, Remark 2 and Lemma 2 justify the claim made in section 3 that, if \( x = X \) and \( S(X \mid X) = S(0 \mid X) \), then there exists a perturbation in \( X \) -- say, \( X' \) -- such that \( S(X' \mid X') > S(0 \mid X') \).

**Lemma 3:** For any \( X > 0 \), \( n(0 \mid X) > n(0 \mid 0) \).

**Proof:** Given \( x = 0 \), \( v(n, x) = V(m) - c(n-m) \). Since \( c' > 0 \), \( v(n, x) \) is strictly decreasing in \( n \). By \( W_2 < 0 \), \( W(y, X) < W(y, 0), \forall y \geq 0 \). So by (6), \( n(0 \mid X) > n(0 \mid 0), \forall X > 0 \). \( \square \)

**Proposition 2:** (a) \( n(x \mid X) \) is strictly increasing in \( X, x \in \{0, X\} \); and,

(b) \( \forall x = X > 0, n(X \mid X) \geq n(0 \mid X) > n(0 \mid 0) \).

**Proof:** The proposition follows from Lemmas 1, 2 and 3. \( \square \)
Proposition 3: For $X > 0$, $S(\|0\|0) > S(\|0\|1)$.  

Proof: By Lemma 3, and the fact that $F$ is a c.d.f. defined on a continuous variable $n$:

\begin{equation}
(\text{a.19}) \quad [1-F(n(\|0\|1))] < [1-F(n(\|0\|0))].
\end{equation}

By Proposition 1, $n^*(0 \| X) \leq n^*(0 \| 0)$, with the inequality strict iff $W_{12} < 0$ at any $y \leq y^*(0 \| 0)$.

\textit{case }$\alpha$: $n \geq n(0 \| X)$.

By (8):

\begin{equation}
(\text{a.20}) \quad \pi(n, 0 \| 0) \geq \pi(n, 0 \| X).
\end{equation}

If $n^*(0 \| X) < n^*(0 \| 0)$, the inequality in (a.20) is strict $\forall n > n^*(0 \| X)$.

\textit{case }$\beta$: $n \in (n(0 \| 0), n(0 \| X))$.

By an argument in section 3, only managers with value $n > n(-\|)$ will make takeover bids. Therefore:

\begin{equation}
(\text{a.21}) \quad \pi(n, 0 \| 0) > \pi(n, 0 \| X) \equiv 0.
\end{equation}

\textit{case }$\gamma$: $n \leq n(0 \| 0)$.

By the same argument as case $\beta$:

\begin{equation}
(\text{a.22}) \quad \pi(n, 0 \| 0) = \pi(n, 0 \| X) \equiv 0.
\end{equation}

Together, (10) and (a.19) - (a.22) yield the desired result. $\blacksquare$

Lemma 4: $\lim_{X \to \infty} S(X \| X) = m$.

Proof:

By assumption, $W_{11} < 0$ and $\lim_{X \to \infty} W_2 = -\infty$. Therefore, for sufficiently large $X$, say $X > X_1$, $V(m) > W(X+y^*(X \| X), X)$. Hence, by (10), $\forall X \geq X_1$,

$S(X \| X) = m + [1-F(n(X \| X))]-[n^*(X \| X)-m].$

By Remark 2, $S(\cdot \| \cdot)$ is differentiable in $X$. In particular, $\forall X \geq X_1$,

\begin{equation}
(\text{a.23}) \quad \frac{\partial S(X \| X)}{\partial X} = [1-F(n(X \| X))]-\frac{n^*(X \| X)}{\partial X} - \{f(n(X \| X))\partial n(X \| X)/\partial X - [n^*(X \| X)-m].
\end{equation}

We prove the Lemma by first showing $\partial S(X \| X)/\partial X < 0$ for all finite $X \geq X_2$, $X_2$ sufficiently
large. By Lemma 1 and the assumption that \( V(m) > W(0, 0) \), the term in \( \{ \cdot \} \) on the RHS(a.23) is strictly positive for all finite \( X \). By (a.7), the first term on the RHS(a.23) is of ambiguous sign.

**case A:** \( \partial n^*(\cdot)/\partial X \leq 0 \) for all \( X \geq X_2 \).

Then, \( \partial S(X \mid X)/\partial X < 0 \) for all finite \( X \geq X_2 \).

**case B:** \( \partial n^*(X \mid X)/\partial X > 0 \) for all \( X \geq X_2 \).

By (7) and (a.3), \( \sup[\partial n^*(X \mid X)/\partial X] = 1 \). For sufficiently large \( X \), \( \partial S(X \mid X)/\partial X < 0 \) if:

(a.24) \[ \frac{1-F(n(X \mid X))}{f(n(X \mid X))} < \partial n(X \mid X)/\partial X \cdot [n^*(X \mid X) - m]. \]

By assumption, \( \lim_{t \to \infty} [(1-F(t))/f(t)] < \infty \). Therefore, by Lemma 1 and \( x = X \), \( \lim_{X \to \infty}[\text{LHS(a.24)}] < \infty \). Also by assumption, \( W_{11} < 0 \) and \( \lim_{X \to \infty} W_2 = -\infty \). From (a.10), therefore, \( \partial n(X \mid X)/\partial X > 1 \) for sufficiently large \( X \). By hypothesis, \( \partial n^*(X \mid X)/\partial X > 0 \), for all \( X \geq X_2 \). Hence, (7) and \( y \geq 0 \) imply that \( \lim_{X \to \infty}[\text{RHS(a.24)}] = \infty \). Therefore, there exists a sufficiently large value of \( X \), \( X_2 \), such that \( \partial S(X \mid X)/\partial X < 0 \) for all \( X \geq X_2 \). Since \( \lim_{t \to \infty} [1-F(t)] = 0 \), the Lemma follows from (10). \( \Box \)

**Proposition 4:** \( V(m) < W(y^*(0, 10), 0) \)

\[ \Rightarrow \exists X \in (0, \infty) : S(0 \mid 0) > S(X \mid X) \geq S(0 \mid X). \]

**Proof:** By Corollary 1, \( \exists X'' > 0 \) such that both \( V(m) < W(X'' + y^*(X'' \mid X''), X'') \), and \( V(m) \leq W(y^*(0 \mid X''), X'') \) obtain. Then by case (a.1) of Lemma 2, \( n(X'' \mid X'') = n(0 \mid X'') = n \). Hence:

(a.25) \[ 1-F(n(X'' \mid X''))] = [1-F(n(0 \mid X''))] = [1-F(n)]. \]

By Corollary 1, \( n^*(X'' \mid X'') \geq n^*(0 \mid X'') \). From the premise of the Proposition and the choice of \( X'' \), \( n^*(0 \mid X'') > n \). By an argument of section 3, only managers with value \( n > n \) will make a takeover bid. Using this and (8):

(a.26) \[ Vn \leq n, \pi(n, X'' \mid X'') = \pi(n, 0 \mid X'') \equiv 0; \]

\[ \forall n \in (n, n^*(0 \mid X''))] , \pi(n, X'' \mid X'') = \pi(n, 0 \mid X'') = n - m; \]

\[ \forall n > n^*(0 \mid X''), \pi(n, X'' \mid X'') \geq \pi(n, 0 \mid X''). \]

Given (10), (a.25) and (a.26) imply that \( S(X'' \mid X'') \geq S(0 \mid X''). \) Therefore,
\[ C = \{ X > 0 \mid S(X \mid X) \geq S(0 \mid X) \} \neq \emptyset. \]

Let \( x = X \). There are now two cases:

(i) \( \exists X \in C : X < \infty \& S(X \mid X) = S(0 \mid X) \]
\[ \Rightarrow [S(0 \mid 0) > S(X \mid X)], \text{by Proposition 3}; \]

(ii) \( \forall X \in C : X < \infty , S(X \mid X) > S(0 \mid X) \]
\[ \Rightarrow [\exists X' \in C : X' < \infty \& S(0 \mid 0) > S(X' \mid X')], \text{by Lemma 4 and } S(0 \mid 0) > m. \]

To check this last inequality, recall \( V(m) < W(y^*(0 \mid 0), 0) \) by hypothesis. Hence, by (6) and \( y^*(0 \mid 0) > 0 \) (Proposition 1), \( m < n(0 \mid 0) < \infty \) and \( n^*(0 \mid 0) > m \).
Appendix B: Example

We claim in section 4 of the text that there exist managerial utilities satisfying the sufficient condition of Proposition 4, under which the manager would ask for, and shareholders approve, a level of anticipatory defense, $X$, such that $S(0 \mid 0) > S(X \mid X)$. In this appendix, we justify our claim with an example.

In the interests of computational simplicity, some technical assumptions of the model are violated in the example: viz. the support of $F$ is not the whole real line, and $W_1$ is zero at $x = y = 0$. The example therefore shows that the main result (Proposition 4) does not depend on these assumptions. Also, there are choices of the distribution function $F$ and the outside utility $W$, that do satisfy all the assumptions of the text, and which are arbitrarily closely approximated by the functional forms exploited in the example.

Let $F$ be uniform on the closed interval $[-2, 2]$, and assume the value added by the incumbent manager is $m = 0$.

Let:

$V(m) = 1.57,$

$W(\pi, X) = 2 \cdot [x + y]^{1/2} - (X^2)/2 + m,$

$c(y) = (2/3)y^{(3/2)}.$

For this specification, $U(\cdot)$ is strictly concave. Denote by $X^*$ the point where

$\partial U(X \mid X)/\partial X = 0,$

that is, the incumbent's unconstrained optimizing choice of $X$. The incumbent's utility function $U(x \mid X)$ is as defined in equation (9) of the text. Given this specification, we obtain the following results, illustrated in Figure B.1. For the manager:

$X^* = 0.35$

$U(0 \mid 0) = 1.3623$

$U(X^* \mid X^*) = 1.3766$

$\partial U(X^* \mid X^*)/\partial X = 0.0$
For the shareholders:

\[ S(0 \mid 1 \mid 0) = 0.2592 \]

\[ S(X^* \mid 1 \mid X^*) = 0.2582 \]

\[ S(0 \mid 1 \mid X^*) = 0.2570 \]

so the condition \( S(0 \mid 1 \mid 0) > S(X^* \mid 1 \mid X^*) > S(0 \mid 1 \mid X^*) \) holds, and \( S(X^* \mid 1 \mid X^*) \) is the outcome for shareholders. That is, by the second inequality, shareholders are worse off if they reject the request than they are if they approve it, but by the first inequality, the final outcome \( S(X^* \mid 1 \mid X^*) \) is worse than their starting value \( S(0 \mid 1 \mid 0) \).

In this example, at \( x = X = 0 \):

\[ \partial U(0 \mid 1 \mid 0)/\partial X = 0.095 > 0, \] and  

\[ \partial S(0 \mid 1 \mid 0)/\partial X = 0.024 > 0, \]

so both the manager and the shareholders prefer a strictly positive level of \( x \). However, the manager's most-preferred \( x \) is strictly greater than the shareholders'.

[FIGURE B.1 ABOUT HERE]

Notice that, for this parameterization, the manager's unconstrained optimal choice of anticipatory defense, \( X^* \), is low enough that the constraint, \( S(X \mid 1 \mid X) \geq S(0 \mid 1 \mid X) \), does not bind. If the manager's unconstrained optimum were approximately 0.4 or above in this example, then the constrained optimum \( X^* \) would be approximately .39, where \( S(X \mid 1 \mid X) = S(0 \mid 1 \mid X) \). (We can obtain this alternate result by raising \( V(m) \), the incumbent's utility for current employment.) In this circumstance, shareholders are indifferent between approving or rejecting. To ensure that \( X \) is accepted we could introduce the convention that, when indifferent, shareholders always vote with management. Alternately, as described at the end of section 3 (and justified more formally in Remark 4 of Appendix A), the manager can reduce his request \( X \) by \( \varepsilon > 0 \) to induce strict preference by the shareholders.
Footnotes.

1. DeAngelo and Rice (1983) provide a thorough discussion of these hypotheses.

2. SEC disclosure requirements regarding matters put to shareholder vote specify that the management mail proxy materials to shareholders eligible to vote. Thus, the record date for determining who may vote is prior to the proxy mailing date. The proxy materials include, in the case of charter amendments, the wording of the proposed change and a proxy form with which shareholders may vote in absentia, indicating approval, rejection, and sometimes abstention. Therefore, at the proxy mailing date, both the constituency for the vote and the proposal are fixed. This type of voting arrangement, termed "take it or leave it" agenda control, has been extensively studied in a political context by Romer and Rosenthal (1978, 1979).

3. We consider infinite $M$ for this exposition. This is not essential to our arguments. In Appendix B, we construct an example in which the voting conundrum obtains when $M$ is uniform on a finite interval.

4. We consider the value added by the incumbent to be fixed. Scharfstein (1987) considers circumstances in which the threat of takeovers can induce the manager to work harder, and consequently, to increase the value of the firm.

5. For example, the uniform, normal, gamma and exponential distributions all satisfy this restriction.

6. Grossman and Hart (1980) have shown that if dilution is prohibited, no takeover bids will be made. Thus, some dilution is necessary for the circumstances modelled here to be of interest. Generally, dilution is possible unless specifically precluded by provisions of the corporate charter or state corporation law. In the Grossman and Hart framework, our model can be interpreted as describing mechanisms by which shareholders and incumbent managers set the level of allowable dilution and acquirer-specific acquisition costs.

7. Although our exposition is not explicitly game-theoretic, the model is a sequential move game in which the manager, the shareholders, Nature, and potential bidders are players. It is
straightforward to describe the formal model as an extensive-form game. The equilibrium concept we employ is subgame perfect Nash (in pure strategies), and it is this which motivates solving for agents' optimal behavior by working backwards from the final stage of the process to the first (Selten, 1975).

8. See Gilson (1986) for discussion of these and other aspects of takeover law.

9. The role of the assumption that \( \lim_{y \to 0} c'(y) = 0 \), is to insure that some strictly positive level of \( y \) will always be chosen by the manager, whatever the value of \( x \). Consequently, we can use the calculus in our analysis. Without the assumption, we have to consider the corner case explicitly: this adds very little and does not substantively alter our main results. Similarly, insisting on the cost function being strictly convex in \( y \) is not essential for our results. It does, however, make the arguments cleaner.

10. The concept of credibility is simply that of subgame perfection: cf.fn.7.

11. This follows from the assumption that the incumbent prefers employment with the firm to the outside alternative at the start of the first period, and that he rationally chooses \( X \).

12. Proofs of all results are in Appendix A. These proofs also establish the existence of a unique subgame perfect equilibrium in pure strategies to the model.

13. The derivations of these results are available from the authors upon request.
References


Figure 1(a)

![Graph showing V versus ω with W(X, X) and n(X | X)-m-X, n*(X | X)-m-X on the x-axis]

Figure 1(b)

![Graph showing V versus ω with W(X, X) and n*(X | X)-m-X, n(X | X)-m-X on the x-axis]
Figure B.1

\[ U(X \mid X) \]

\[ S(010) \]

\[ S(01X) \]

\[ S(X \mid X) \]

0

\[ X^* \]

\[ x, X \]