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THE TECHNOLOGICAL PROGRESS FUNCTION:
A NEW TECHNIQUE FOR FORECASTING*

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1. **BASIC ASSUMPTIONS:**

The core of technological trend forecasting has most often been based upon a plot of a technical parameter against time. This study concerns the implications for forecasting, when these technical parameters are plotted against cumulative production.

For the examples covered in this study, the relationships between technical characteristics and cumulative production all have the mathematical form:

\[ T_i = a(i)^b \]  

where \( T_i \) = value of parameter at the \( i^{th} \) unit, 
\( i \) = cumulative production, 
and \( a, b \) = constants.

This dependence of a technical value upon production is defined here as the technological progress function. This paper relates the development of that function, the implications of the math-
ematical form just mentioned for trend forecasting, and information compiled from a variety of case studies.*

It is of interest to note that the form of the technological progress function is similar to that of the common industrial "learning curve", as well as the learning curves of psychologists. The industrial "learning curve" relates cost to production (1, 2, 3).

\[ y_i = y_1(i)^{-b} \]

where \( y_i \) = cost of the \( i^{th} \) unit
\( i \) = cumulative production
\( y_1 \) = cost of 1st unit
\( b \) = constant .

and the psychological learning curve relates the efficiency of performing a task to the number of repetitions (4, 5, 6, 7)

\[ E_N = k(N)^b \]

where \( E_N \) = efficiency of performing \( N^{th} \) task
\( N \) = cumulative task number
\( k, b \) = constants.

* Studies completed included:

  - civil aircraft-speed
  - military aircraft-speed
  - turbojet engines- specific weight
  - turbojet engines- specific fuel consumption
  - automobiles- horsepower
  - electric lamps- lumens
  - computer programs- figure of merit
  - hovercraft- figure of merit
They are similar to the technological progress function, 
\[ T_i = a(i)^b, \]
because all three show the measure of some characteristic, for which improvement is desired, as an exponential function of the cumulative number of repetitions or production.

The environment in which technology develops is clearly of additional interest and appears to affect the rate of learning through discrete changes in the learning constant, \( b \). It must be specified here, that environment refers to the external economic factors which surround the production process. An example of this effect was the change in the government investment and market potential relating to the automotive industry in the late twenties (8, 9), which multiplied the rate of progress, \( b \), by a factor of 6.2.
2. **INTRODUCTION AND BACKGROUND INFORMATION:**

The development of the technological progress function, \( T_i \), characteristic of technological improvement, evolved from a consideration of:

1. problems and background factors inherent in industrial progress relationships,

2. indications from psychology that general phenomena of "learning" were present,

and

3. difficulties in existing techniques of forecasting.

2.1 **PROGRESS FUNCTIONS:**

Briefly, industrial progress relationships are functions, such as production costs/unit, maintenance costs/unit, and manufacturing costs/unit, which can be written as a function of costs associated with unit number one, the cumulative unit number, and a learning constant. They are termed progress functions because a reduction in costs indicates a gain in efficiency. These functions, as previously mentioned, have been found to have the precise form (1, 2, 10, 11):

\[
y_i = a(i)^{-b}
\]

(1)

When plotted on log-log paper, this gives a straight line, whose slope, \(-b\), is the rate of progress, that is, the rate at which efficiency is improving. It should be noted here
that, as previously mentioned, the technological progress function has the same form and expression, the difference being a positive rather than a negative slope.

Various studies of industrial progress functions have shown that there are a number of causes or factors which are common to most of the different types (1, 2, 11, 12, 13).

The more direct factors are:

1. engineering or design improvements;
2. servicing technician progress;
3. job familiarization by workmen;
4. job familiarization by shop personnel and engineering liaison;
5. development of a more efficient parts supply system;
and 6. development of a more efficient method of manufacture.

The innate factors are:

1. the inherent susceptibility of an operation to improvement;

and 2. the degree to which this can be exploited.

These functions can be called "micro-economic progress functions" (MEPF) because:

1. they are similar in mathematical form,
and 2. each relates to decisions of the "firm".

However, their applicability to development decisions is not
as simple as it would seem initially since the functions contain two areas of problems. Both areas are relevant to the subject of the technological progress function because the first helped to prompt its development and the second helps to explain its behavior.

The first area involves certain MEPF characteristics which present problems of decision that have been unsolvable because of their link with technological innovation. The problems begin when one attempts to determine the component costs of the first unit of production and finds that one must arbitrarily set a limit upon the background development expenses. A mechanism by which background and development costs would be linked to all of the generations of a product might help to alleviate this difficulty of first unit costs; of course, this would first require a model for technological change.

Further and more fundamental complications become apparent when the development of new generations of equipment is considered, as opposed to the difficulties of measuring cost for any one generation. Technical changes and design modifications are part of the inherent forces that cause the MEPF to behave as it does, but there are no provisions for setting a limit on these changes; the engineering change procedure of firms being arbitrary in nature. That is to say that, after
a certain number of modifications in production procedure
or design, the product in reality will be a "new" product
and will be so labeled by sales and management. This is often
the case with developments resulting from both conscious and
unconscious defensive research, which is designed to improve
a product's competibility.

The locating of an "old-new" product line or determining
where a major change begins are questions posed by users of
the MEPF and seem to beg a progress function which cuts
across individual generation lines. The technological pro-
gress function is such a relationship and contains the same
factors of engineering learning and motivation of the MEPF,
which not only cause problems in its use but also provide a
major stimulus for technological advancement.

The other problem area relates to deviations from normal
(linearity on log-log paper) MEPF behavior. The explanations
of these differences are important, since they help to clarify
the effects of external factors of the environment on tech-
nological progress. Some of these deviations are due
to changes in the psychological motivation of the personnel
involved, as would be the case where a product line is sud-
denly scheduled for discontinuance or where the success of a
new development is announced to the employees (1, 2, 14).
Others would be due to changes in design or the introduction of new people to the job. Finally, if the product or system is being constructed on parallel assembly lines or shifts, the addition or subtraction of new lines, perhaps with changes in demand, will cause anomalies in the MEPF (14). All of these deviations cause a change in the slope of the progress function. Thus, it is implied that changes in the rate of progress (i.e., the slope of the progress function) are not random but are a function of critical parameters forming part of the external environment.

2.2 PSYCHOLOGY:

In addition to the information supplied by analysis of progress functions, further insight has been gained from psychological learning theory. Since there is considerable dispute over exact definitions of what learning is or is not, let me point out that, here, psychological learning refers to perceptible gains in performing given tasks. These tasks in academic studies have ranged from solving puzzles and going through mazes to simple studies of response time for given stimuli. It has been noted in a variety of projects concerning both animals and people, that the efficiency of performing a
given tasks increases with the cumulative number of repetitions (4, 6, 7, 14, 15, 16). It should also be pointed out that none of the studies differentiate between the frequency of the repetitions as long as the frequency was within the maximum retention interval. The studies found that,

\[ E_N = K(N)^a \]  

(2)

where \( E_N \) = efficiency in performing the \( N^{th} \) task,

\( N \) = the cumulative task number,

and \( K, a \) = constants.

Such a relation was particularly apparent in a 1934 study with children by Melcher (7) and in a 1955 study with dogs by Bush and Mosteller (4). A similar but different relationship regarding repetitious activity can be derived from work presented by Frank Logan in his book, \textit{Incentive} (16). Logan noticed of his subjects, that they, "...behave in such a way as to maximize reward while at the same time minimizing effort."

Such behavior is not only identical with the aims of man's economic endeavors, but is also a causal factor of technological progress. Since the progress functions discussed previously show a similar dependency on repetition of tasks, it would appear that the same type of learning discussed above is involved.

2.3 FORECASTING:

The third area that has contributed to the present development of the technological progress function is technological
forecasting. The primary methods already available, such as Delphi, trend extrapolation, trend correlation, and growth analogy, do not allow for the easy or precise handling of environmental factors (17). This provided further incentive to find a progress function that would be as competent as other techniques under constant forces and yet allow for environmental change.
3. **TECHNIQUES USED IN ANALYSIS:**

The work involved in investigating and defining the technological progress function was completed through two separate lines of reasoning. Both paths were developed while taking note of:

(1) an observation, through analysis of the MEPF, that the independent variable should be the cumulative production number;

(2) an implication in learning theory that the form of the function should be similar to that of the MEPF and the general structure characteristic of improvement functions; and

(3) a need of forecasting techniques for a term through which environmental change could be introduced.

The paths chosen for development are common to most scientific endeavors. They were those of theoretical derivation from known relationships and empirical model building from real world data.
The theoretical side bases itself on the equivalence, for at least certain areas, of the rate of patent output with technological growth or progress. Through relationships derived or shown by Schmookler (18) and Villers (19), the rate of patent output was related to investment and expected profit functions. From this point, substitutions were made from investment/profit functions, denoting technical advance.

The result was a relationship that equated the level of technological improvement to a constant multiplied by an increasing quantity dependent function raised to a positive exponent, i.e.,

$$T_i = K(f(i))^c$$  \hspace{1cm} (3)

where $T_i =$ the level of technology at the $i^{th}$ unit of production, 
$f(i) =$ a function which varies with the production number, $i$, and 
$K,c =$ constants.
Although it seemed to substantiate earlier hypotheses, it could only be taken as a further indication that this might be the correct approach, since it was not precise enough in nature to stand by itself.

The results of the theoretical derivation cleared away doubts from the proposed directions of the empirical phase of the study. At this stage, it was clear that the work should attempt to correlate technological improvement with the cumulative quantity of production. In addition, thoughts of incorporating the results with other progress functions motivated the gathering of data designed for more extensive correlations.

Real data were then sought to provide information on technical parameters, production, and costs with background information to be provided where possible. One could then observe the behavior of technological parameters with respect to production under the sets of conditions forming the background environment.

However, before describing the case studies and conclusions some of the difficulties involved in this work should be pointed out. The most difficult problem was that of obtaining accurate data, particularly with regard to cost information. This was
solved by combining data from several sources and where possible having the material validated by someone familiar with the field.

Other problems arose concerning the choice of technical parameters. In this case decisions were made from background information and observation of changes in the parameter with respect to cost changes.

4. PRESENTATION OF DATA:

The empirical studies discussed here verified both the preliminary hypotheses and the theoretical derivation and were based upon analyses of data regarding the aircraft industry, the electric lamp industry, and computer programming.

4.1 AIRCRAFT DATA:

The data from the aircraft industry concerned turbo-jet engine development over a period of nearly twenty years and was synthesized from two Air Force Institute masters theses (20,21), supporting information supplied by the Pratt & Whitney Division of United Aircraft (22), and Aviation Facts and Figures 1958 (23). From these sources, production cost/unit, production by year, cumulative production, and the technological parameters were observed.

The results have shown, in Figure 1, that technological progress, as represented by specific weight (dry engine weight/
pounds of thrust) and specific fuel consumption (pounds of fuel/hr./pound of thrust) is log-linear to a logarithmic quantity axis, where quantity indicates the cumulative production of engines within the turbo-jet family. It also appears that an arithmetic time axis may be used in place of a logarithmic quantity axis on many occasions for the same accuracy, whenever production undergoes constant percentage increases with respect to time*.

4.2 ELECTRIC LAMP DATA:

The Electric lamp industry data was obtained by combing information from James Bright's book Automation and Management, (24) and Arthur A. Bright's book, The Electric Lamp Industry, (25) for 60 watt lamps. These sources enabled the study to be concerned with technological progress as represented by the output of the lamp in lumens, production by year, and cumulative production. The many development changes made it unnecessary to examine each individual generation of lamp.

The results confirm the existence of a technological progress function behaving in accordance with the generalized MEPF principles. Figure 2, where this is illustrated, actually

* The bracketed points at the end of the "specific weight" curve were obtained from current advertising information of Pratt & Whitney presented in Aviation Week during March, 1969.
ELECTRIC LAMP LUMEN OUTPUT
(60 Watts)

Cumulative Units in millions

Progress Slope
no. 2
no. 1

1920  1930  1940
indicates the existence of two rates of progress. One extends from 1912 until 1920 and the second from 1920 until 1940. This would imply that the external environment affecting the lamp industry underwent a shift. Since the production which had been increasing at a steady 9.39% per year throughout the first period concurrently suddenly shifted downward to a rate of 2.74% growth at the same transition point (Figure 3), it may be concluded that one or more changes in the environment increased the value of the exponent \( b \) in the technological progress equation, while at the same time decreasing the rate of growth of production. It is interesting to note that forecasting results with an arithmetic time axis are as good as they have been because of the substitution effect. That is, an arithmetic time axis may be substituted for a logarithmic quantity axis, when production quantities increase in reasonably constant percentages with time, as in Figure 3.

4.3 **COMPUTER PROGRAMMING DATA:**

An additional example has been drawn from a specific experience in programming. Data for this example was made
ELECTRIC LAMP PRODUCTION VS TIME

Cumulative units in millions

Electric Lamps

1915 1920 1925 1930 1935 1940

figure 3
ELECTRIC LAMP LUMEN OUTPUT

(60 Watts)

Figure 4

Trend line for
data to
1915
1925-1940
1920-1925
available through the help of John Cleckner, a student at The Johns Hopkins University. The example represents the rate of progress in the development of a single computer program developed solely by him under special arrangement with an area firm. A figure of merit was determined which would best represent his own aims while developing the program. This figure of merit equals the lines of output divided by the running time. The computer pro-
gram dealt with an estimation of bakery goods to be sold according to the day of the week, store, and particular item.

In an analogy to the development of products, it was decided to regard the last best figure of merit (Figure 5) for a particular run as the figure of merit to be processed. This may or may not be the true figure of merit for that run, but it would be an accurate representation of the figure of merit of the best program available at any given point in its total development.

The results illustrate several areas of interest. The first is the applicability of the technological progress function to the area of programming. There is also the observance of different slopes corresponding to different development phases. Finally, since the work was done by one person, the analysis indicates the possible role of psychological "learning" theory in technological progress.

ADDITIONAL EXAMPLES:

Other cases studied demonstrated similar technological
Cumulative average figure of merit = \( \sum FM / R \)

Area of basic bug elimination and background

Development of auxiliary support program

Concentration begun on program efficiency
progress functions, that is, linearity on log-log paper when the technical progress parameter is plotted against production. Those cases not discussed here are civil aircraft-speed, military aircraft-speed, automobile-horsepower, hovercraft-figure of merit, and an office machine development-figure of merit.

One could get the impression, from the data just discussed, that the trends observed were basically limited to items of hardware, but this was not found to be the case, as evidenced by studies of the totally different area of agricultural efficiency. From information pertaining to the pounds of rice produced per acre in Japan over a period of twelve hundred years, three log linear trends were noticeable (26,27). The two major lines were a "long term" or ancient trend line and a modern trend line of much steeper slope. By calculating backwards, it was found that they intersected at a period of great external change-- the "opening of Japan". The third line was a moderate decrease in slope, which prevailed during the period of World War II.
THE TECHNOLOGICAL PROGRESS FUNCTION:

Thus, if we assume that initial studies are correct, technological progress functions exist and are of the same general nature as other micro-economic progress functions. It is then further implied, that technological progress, as denoted by a positively increasing function is logarithmically linear with respect to a logarithmic quantity axis under constant external forces. Over periods of time during which external forces vary, the slope of the line, that is, the rate of progress, may undergo discrete shifts. Such a function is denoted as,

\[ T_i = a(i)^b, \]  

where \( T_i \) = the value of the technological parameter,
\( i \) = the cumulative production number,
\( a \) = a constant associated with unit number one,
\( b \) = the rate of progress, a variable, which is a function of the external environment.
5.1 MATHEMATICS:

A brief glance at the mathematics shows that where $b$ is a constant:

$$\log (T_i) = b \log (i) + \log (a)$$

and regarding the differential of (5)

$$\frac{dT_i}{T_i} = b \frac{di}{i}$$

Equation (6) indicates that, $b$, being constant, the percentage change in a technical parameter is a linear function of the percentage change in cumulative production.

In addition, it should be noted that:

if $$i = (\text{constant}) e^{Kt},$$

$$\log (i) = Kt + \log (\text{constant})$$

and $$\frac{di}{i} = Kdt.$$  

Equation (9) represents a percentage change in production as a linear function of time, that is, what has been referred to as a "substitution effect".

Where $$\frac{di}{i} = Kdt,$$

$$\frac{dT_i}{T_i} = bKdt,$$

which is the traditional trend forecasting relationship (28).

5.2 RATE OF PROGRESS:

From analysis of background information involved in the work discussed above and current studies, the rate of pro-
grass, \( b \), has been designated as a function of:

\( L \), the effective size of the technical labor force;

\( \alpha \), an intelligence factor-average educational level;

\( \beta \), a pre-learning factor-average experience level;

\( I \), the level of investment;

\( \lambda \), the rate of change of the level of investment;

\( \delta \), a maturation factor-durability of item-compatibility;

\( m \), the anticipated rate of change of market demand-slope of sales curve;

and

\( c \), the communication or diffusion rate.

Currently, work is being done to define the exact nature of the function just described within the inherent constraints that \( b \) is greater than, or equal to, zero and never infinity and that if \( L, \alpha, \delta, I, \) or \( m \) go to zero, then \( b \) also goes to zero.

\section*{SUMMARY}

This study has been concerned with the technological progress function. The study proposes that the technological progress function may be based on the cumulative production unit number on a logarithmic scale, as opposed to the arithmetic time series base used for most forecasts. In this form, the technological progress function allows for environmental changes to act upon the rate of progress in a precise manner, that is, discrete changes in the slope of the function. However, it should be noted, that no less care must be taken when employing this
technique in addition to or instead of existing methods, particularly with regard to physical constraints imposed upon the system.

7. FUTURE WORK

The technological progress function is a tool to be used with other techniques by the foresighted corporate and military development planner, who can no longer overlook the effect of technological change upon his proposals for future development. Although, it is doubtful that a true "Newton's Law" for predicting technological advancement will ever be made, it is believed that the development of the technological progress function is a step towards more effective predictive methods.

Considerable work remains to be done in exploring the nature and anomalies of the technological progress function itself. There is, of course, the need for a precise definition of the rate of progress function. In addition, there are avenues of research concerning macro-economic implications and ways in which the function might be of maximum value to technologically oriented firms.
REFERENCES


(22) N. Turkowitz, letter to author, November 5, 1968.


BIBLIOGRAPHY

BOOKS


**ARTICLES**

Bright, J. R., "Can We Forecast Technology?", *Industrial Research*, March 5, 1968.


Part I- Summary Report  AD 659169


"The Dynamics of Automobile Demand", General Motors Corp.,
New York, 1939.


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