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A Theory of Partial Sales and Underpricing in Privatizations

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Abstract

This paper develops a theory of gradual sales and underpricing of shares in privatizations. Gradual sales and underpricing are strategic devices used by policymakers to signal commitment when the capital markets are unsure about the government's type and attempt to infer it from observed performance. The model establishes for which firms the use of underpricing is optimal. Further, in the context of a multiple firm model, the question of when gradual sales can be abandoned is analyzed.

1 Introduction

Among other things, the Eighties will be remembered as the decade of privatization: the selling of state assets is taking place at the same time almost everywhere. Perhaps surprisingly, most privatization programs seem to display important similarities. It is, for instance, true that, during the initial stages of the program, governments frequently use partial sales. Although a number of reasons has been offered, ranging from seller inexperience to stock-market liquidity constraints, none of these provides a completely satisfactory explanation. Also, it has been reported that the underpricing of shares in privatizations is significantly and consistently larger than the average underpricing occurring in the private sector initial public offers. Jenkinson and Mayer (1988), measuring discounts of issue price in relation to offer prices at the end of the first trading day, note that only 5 out of 23 of the discounts in privatizations in France and the U.K. were in single figures, which are consistent with the underpricing in private IPOs for the two countries. This is particularly intriguing because state owned corporations are usually well known to the public, and the government, unlike shareholders of private corporations, should not have to pay the underwriters for risks that it is better placed to assume.1

In this paper we investigate why gradual sales and underpricing of shares may be optimal features in the design of privatization plans. Our explanation relies on the fact that governments, unlike any other seller of an asset, can interfere with the activities of the firm after privatization and change the ex-post contractual rights of the new owners.2 If investors were able to directly identify the nature of the government, they would

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1 We would like to thank Drew Fudenberg and Jean Tirole for their comments and suggestions.

2 Political reasons may provide satisfactory explanations. The oversubscription, due to underpricing, is often presented by the governments as a signal of the success of the privatization.

3 Interference can take many forms, from changes in the environment in which the firm operates subsequent to privatization, to tax changes, restrictions on sales in the secondary market (e.g. foreign takeovers) and even renationalization. The attempt by the Japanese government to breakdown NTT after it has been privatized is a typical example of discretionary interference. Other examples are either the decision by the British government forcing the Kuwait Investment Office to reduce its stake in BP acquired during privatization, or the overturn by the socialist administration in France of the Chirac government's allocation of shares in privatized firms among hard-core groups of investors.
pay a low price for companies sold under the rule of a policymaker unable to make a commitment of no interference, and a high price for companies sold by a policymaker capable of making binding commitments. In reality, however, investors are not able to discern the type of the government; instead, they try to infer it from observed actions. The whole issue becomes a matter of credibility since, each time interference is not observed, there is an upward revision of the government’s reputation, which in turn is reflected in higher prices for later privatizations.

In a world with incomplete information, uncommitted governments understand investors’ learning process and decide to match the actions that would be chosen by the committed type, as long as higher prices in future asset sales can compensate for the loss incurred in postponing interference with any corporation already privatized. The possibility of matching by the uncommitted affects the payoff of the committed, for the pooled price is less than the price received by the latter in a world with perfect information. The committed policymaker must then find a credible way of signalling his true type to the capital market. He can do so by taking advantage of the uncommitted’s impatience and organizing gradual sales; by slowing the process he hopes to induce separation. Indeed, partial sales become a strategic variable, for quicker sales serve the best interests of the uncommitted, in that the incentives to interfere increase as privatization approaches completion.

Partial sales have been extensively advocated in the financial press, in our opinion, for the wrong reasons. Often government officials and financial experts argue for gradual sales as a way of avoiding the ‘disruption’ of the stock-market. They fear that the sale of big corporations at one go might cause investors to suffer from indigestion. However, such fear has never been confirmed by events: quite the contrary, capital markets seem to be able to absorb large blocks of stock, and when a discount is perceived oversubscription appears to be the rule.

In some cases, however, a simple strategy of partial sales may not be sufficient to make the uncommitted government prefer to reveal its nature and deviate from the committed’s optimal fraction of the firm to be privatized. Then, underpricing, in addition to gradual sales, may be considered as a useful strategic device by the committed type. Rather than accepting the pooled price throughout the program, the committed type decides to set the offer for sale price below the price investors are willing to pay, if this action induces demarcation from the uncommitted. Sale prices set below market prices, at the beginning, can make sense when combined with perfect information prices in later sales. In fact, that seems to be the case when the level of reputation is relatively low and underpricing is not a signal of great cost. The result is consistent with the finding of lower discounts for later privatizations, as reported by Jenkinson and Mayer (1988).

The analysis surrounding strategic underpricing also highlights two other points. First, it shows that the proper measure of the magnitude of the discount in privatizations is the fraction of the firm sold times the difference between the market and the offer for sale price. By focusing just on the discount price, as it is frequently the case, it is possible to reach incorrect conclusions. This is because bigger unit discounts can be associated with smaller fractions sold and what may be taken to be a grand giveaway, is not so in reality. Second, if underpricing is an optimal action, committed governments choose offers for sale at a fixed price instead of tender offers, because sales by competitive auction are usually associated with lower discounts, and therefore cannot be used as a signalling device.

The paper is organized as follows: in the next section we analyze the privatization of a single firm in a simple model to highlight the main ideas of this research. The government, knowing that investors cannot discern its type directly, sets the share of the firm to privatize in the first period. If this is different from one, there will be a later sale for the remaining assets of the firm. Between sales, as well as after the complete

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3 Losses from not interfering are not just purely financial rents lost. In general, they represent desutility associated with the choice of no action.
4 Also, gradual sales have been suggested to avoid excessive underpricing (see Vickers and Yarrow (1988)). The argument here is that an auction for a small portion of the capital should take place first, as a way of resolving the disagreement among investors over the value of the company. However, the sole purpose of selling at a price that conveys information about the market valuation of the firm does not justify gradual sales, except if the demand by the participants in the auction were insufficient.
privatization, it decides whether or not to interfere. In section 3 a plan that combines gradual sales with underpricing is analyzed. In section 4 the model is extended to the case of multiple firms and the question of when gradual sales can be abandoned is studied. Section 5 concludes and discusses some extensions. All proofs are provided in an appendix.

2 The Basic Model with Partial Sales

In this section we introduce the basic model of the paper.\(^7\) There is one state-owned firm which the government has decided to privatize. The government has one of two types (committed or uncommitted) and knows it.\(^8\) The committed policymaker is capable of making the commitment of no interference with the firm after it has been privatized.\(^9\) The uncommitted policymaker, on the other hand, has no capacity to honor commitments and may decide to interfere, at his own discretion, once the firm has been privatized. Examples of interferences are any unanticipated actions by the sovereign power that change the contractual rights of the private owners: taxes, breaking down of a privatized monopoly, restrictions on managerial discretion imposed by regulatory bodies, renationalization, among others. Government interference will have a negative effect on the firm’s payoff, increases the uncommitted government’s payoff by a fraction of the firm’s payoff reduction and reduces the committed government’s payoff by a sufficiently large amount. For simplicity we model interference as a rent extracted from private shareholders, equal to a share \(\tau (0 < \tau < 1)\) of the firm’s profits. However, only a fraction \(\alpha (0 < \alpha < 1)\) of this rent is payoff to the uncommitted government.

The firm’s profit, when state owned, is constant and normalized to one per period. If the firm is privatized profits grow at a constant rate \(g (g \geq 0)\). A mixed firm has profit equal to the weighted average of the public and private profits, with weights equal to the relative ownership fractions.\(^10\) Future payoffs are discounted at rate \(r\) by the capital market and at a rate \(\rho (\rho \geq r)\) by the government. The higher government’s rate of time preference may result from electoral short-termism. The difference in discount rates between the private sector and the government implies that, even under complete information, the firm has different values for each agent.

There is a large number of risk-neutral investors, which are potential buyers of the firm’s assets. We assume that capital markets are competitive, so if the shares are sold by tender the price will be bid up to their market value. The market is not able to observe the government’s type and tries to infer it from past actions, using Bayesian updating.

The timing of the game, shown in figure 1, is as follows: at date zero there are two stages: first, nature draws the government’s type which is immediately learned by the government;\(^11\) second, the government announces the fraction \(\alpha\) of the firm to be privatized at date 1, leaving the remainder to be sold at date 2. At date 1 there are also two stages: first, investors competitively bid for the shares of the firm, which are then allocated, at the bid received, \(S_1\); second, the government decides whether to interfere or not. At date 2 there are two other stages: first, investors submit their competitive bids for the remaining shares, which are allocated at the submitted price, \(S_2\); second, the government decides whether or not to interfere.\(^12\) After this period no more actions can be taken, but the firm goes on. and, if interference has occurred in the last stage, it is assumed to remain thereafter.\(^13\)

The payoff to each player is the discounted value of the cash-flows per period, which include the firm’s dividend plus eventual transfers due to interference. The payoffs of a period are realized at the end of the

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\(^7\) A similar model was first explored in Branco and Mello (1989), a paper presented at the New University of Lisbon. A two period variant of the model was studied in Perotti (1990), with whom some ideas here in were shared.

\(^8\) Throughout the paper we will use the letters \(C\) and \(U\) to refer to the committed and uncommitted type, respectively.

\(^9\) In this model this commitment is guaranteed by the assumption that if a committed government interferes, it gets a very large negative payoff.

\(^10\) Our results do not qualitatively depend on the specific assumptions about the firms’ profit, in particular on its smooth dependence on the ownership structure of the firm. Such assumptions were made to facilitate the analysis.

\(^11\) Let \(p_0\) be the probability that a committed type is drawn.

\(^12\) Let \(\nu_x\) be the probability that interference is chosen at date \(t\) by the type \(\vartheta\) government (\(\vartheta \in \{C, U\}\)).

\(^13\) Actions at each stage are contingent on prior actions; however we do not explicitly write this dependence. A generic strategy profile, which we denote by \(\sigma\), is written as a vector \((\sigma_C, \nu_C, \nu_{C}^x), (\sigma_U, \nu_U, \nu_U^x, (S_1, S_2))\).
period. Let \( U^\theta(\sigma) \) and \( U^I(\sigma) \) be the payoff of the type \( \theta \) government and the investors, respectively, with \( \theta \in \{C, U\} \). From the above discussion it should be clear that these payoffs are:

\[
U^C(\sigma) = \begin{cases} 
\alpha^C S_1 + \frac{1 - \alpha^C}{1 + \rho} (1 + g \alpha^C + S_2) & \text{if } \lambda^C_1 = \lambda^C_2 = 0 \\
-\infty & \text{otherwise}
\end{cases} 
\]

(1)

\[
U^U(\sigma) = \alpha^U \left( S_1 + \frac{1 + g \alpha^U}{1 + \rho} \sigma \tau \lambda^U_2 \right) + \frac{1 - \alpha^U}{1 + \rho} (1 + g \alpha^U + S_2)
\]

\[+ \frac{(1 + g \alpha^U)(1 + g)}{(1 + \rho)(\rho - g)} \sigma \tau \lambda^U_2 \]

(2)

\[
U^I(\sigma) = \sum_{\theta \in \{C, U\}} \left\{ \alpha^\theta \left( \frac{1 + g \alpha^\theta}{1 + r}(1 - \tau \lambda^\theta_1) - S_1 \right) \\
- (1 - \alpha^\theta) \frac{S_1}{1 + r} + \frac{(1 + g \alpha^\theta)(1 + g)}{(1 + r)(\rho - g)} (1 - \tau \lambda^\theta_2) \right\} p^\theta_0 \]

(3)

where \( p^C_0 = p_0 \) and \( p^U_0 = 1 - p_0 \).

The structure of the payoffs prevents equilibrium interferences of a committed government; on the other hand, an uncommitted government may postpone interference to the extent that this may lead to higher prices in later sales; of course in the last stage, since no future actions will be taken by investors, it decides to interfere. From the assumption of competitive capital markets, the price paid in either period has to be equal to the firm’s market value. Moreover, these arguments do not depend on the information structure of the model and they would still be valid were the investors informed about the government’s type.

An important element in the determination of the equilibria of the game is the market value of the firm, \( V_I(\alpha, p_t) \). This is influenced by three factors. First, it depends on the current probability assessments of the market about the government’s type, \( p_t \); second, it depends on the fraction of the firm’s capital that is privatized at date 1, \( \alpha \); third, it depends on the expectation of future government interference. Let \( \lambda^\theta_t \) be the expected value of \( \lambda_t \) at the beginning of period \( t_0 \), i.e. \( \lambda^\theta_t = \lambda^\theta_t p_{t_0} + \lambda^\theta_t (1 - p_{t_0}) \).

Consider the value of the firm at date 2. If the investors assign probability \( p_2 \) to the committed government, the value is

\[
V_2(\alpha, p_2) = \sum_{t=2}^{\infty} \frac{(\alpha(1 + g) + (1 - \alpha))(1 + g)^{t-1}}{(1 + r)^{t-1}} (1 - \tau \lambda^\theta_2)
\]

\[= \frac{(1 + g \alpha)(1 + g)}{r - g} (1 - \tau \lambda^\theta_2). \]

(4)
The value at date 1 consists of the payoff in the first period and the discounted expected value of the firm at date 2.

\[
V_1(\alpha, p) = \frac{1 + g\alpha}{1 + r}(1 - \tau\lambda_{11}^1) + \frac{V_2(\alpha, p)}{1 + r}
\]

\[
= \frac{1 + g\alpha}{(1 + r)(r - g)}(1 + r - (r - g)\tau\lambda_{11}^1 - (1 + g)\tau\lambda_{21}^1).
\]  

(5)

From equations (4) and (5) we may conclude that the value of the firm increases with the probability that the government is committed, \(p\). Because the committed government does not interfere, if the market becomes more certain that the government is committed the value of the firm increases.

In order to see the role of the asymmetry of information, Proposition 1 describes the equilibrium if investors were fully informed of the government's type. Even though it cannot be directly transposed to the incomplete information case, this equilibrium can be used as a benchmark to evaluate the results of the model. Also, it prescribes the optimal actions of either agent once the investors have correctly inferred the government's type.

**Proposition 1** When investors observe the government's type, the unique subgame perfect equilibrium requires the government to fully privatize the firm at date 1, and immediately interfere if it is uncommitted; investors, then pay \(V_1(1, 1)\) to a committed government and \(V_1(1, 0)\) to an uncommitted government.

The equilibrium of the complete information game is the first best, because the firm is always privatized in the first date. Also, the government appropriates all the benefits from immediate privatization, while investors have a zero net payoff, since paying for what they get. This result does not hold in the incomplete information case for the uncommitted government would like to claim to be committed in order to sell at a high price. If investors cannot observe the government's type, the amounts \(S_1\) and \(S_2\) paid are only contingent on the government's actions: therefore, investors payoff is zero in expected terms, but not necessarily in both states of nature. More important, the firm may not be fully privatized in the first date, because partial sales allow the committed government to signal its type to the market.

A common feature of imperfect information games is the multiplicity of equilibria. Here we provide a partial characterization of the set of equilibria. Moreover, in order to make simple predictions, we select among equilibria using a two step refinement. The refinement combines forward induction arguments with the backward induction process. In a first step we collapse the game into a signaling game and apply the intuitive criterion (Cho and Kreps (1987)) to this reduced signaling game.\(^{13}\) We call forward induction equilibrium to a pair of strategies and beliefs surviving this refinement. If more than one equilibrium survives the first step, we select the outcomes that are not strictly dominated, among the equilibria surviving the first step, for each of the government's type. We call dominant forward induction equilibrium to an equilibrium that survives this two-step refinement.

In general, considering the reduced signaling game, our model may have three types of equilibria. First, there are separating equilibria, where each type of government follows a different action at date 0, i.e. they differ in their choice of \(\alpha\); hence, in equilibrium, the government's initial action completely reveals its type to the market. Second, there are pooling equilibria, where both types follow the same action at date 0, so that from the equilibrium date 0 action the market does not learn anything about the government's type. Third, there are semi-separating equilibria, where for the equilibrium choices of \(\alpha\) intermediate levels of learning may occur.\(^{14}\)

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\(^{13}\)The first step of the refinement works in the following way. Compute the equilibrium continuation actions for the stages corresponding to the sale of 1 - \(\alpha\) of the firm; the game formed by the remaining stages constitutes a signaling game (the government chooses the fraction \(\alpha\) and the market responds), referred as the reduced signaling game: for each terminal node in the reduced signaling game, let the payoff be equal to the expected payoff, in the original game, when that node is reached and the equilibrium continuation actions are played in the last stages; apply the intuitive criterion to the signaling game. In our model, this corresponds to the determination of forward induction equilibria (Cho (1987)), a refinement of sequential equilibria.

\(^{14}\)This categorization of the equilibria is based only on the share of the firm that is privatized at date 1. For example, we call pooling equilibrium to any equilibrium in which the committed and the uncommitted governments follow the same policy of partial sales, even though the uncommitted government may, in equilibrium, want to signal its type before completing the privatization, through interference.
We start by looking at separating equilibria, where the initial choice of \( \alpha \) by the committed government differs from the choice of the uncommitted government: \( \alpha \), then, reveals the government's type to the investors. Once the uncommitted government is identified, investors' decisions will be as described in Proposition 1: in that case the best strategy for the uncommitted government is to follow its equilibrium strategy in Proposition 1. In particular, the firm is fully privatized in the first date. However, the committed government cannot get the first best, because it must prevent the uncommitted government from mimicking its choice of \( \alpha \), so it must adopt a policy of partial sales. Proposition 2 establishes necessary and sufficient conditions for the existence of separating equilibria and proves that one of them is selected by our two step refinement.

**Proposition 2** A separating perfect Bayesian equilibrium, with the committed government following a policy of partial sales, exists if and only if

\[
\tau \leq \left( \frac{(1 + \rho)(1 + g) - (1 + r)}{(1 + \rho)(\rho - g) - (r - g)\rho} \right) \left( \frac{\rho - g}{1 + g} \right).
\]

In this case, the intuitive criterion predicts that the committed government uses partial sales while the uncommitted government fully privatizes the firm in the first period.

Proposition 2 says that the committed government may perfectly signal its type as long as the effect of interference on the firm's profitability is not too large. When the effect of interference is sufficiently large and the uncommitted government reveals its type, through a different choice of \( \alpha \), the uncommitted government substantially reduces its payoff; since this is not optimal, it follows that the committed government cannot ensure separation.

The second part of Proposition 2 predicts that when separating equilibria exist the committed government will follow a policy of partial sales to signal its type. The reasoning behind this result is the following: because the payoff function of the government is increasing in \( \alpha \) associated to each equilibrium we can determine the lowest level of \( \alpha \), denoted \( \alpha^U \), that would leave the uncommitted government indifferent between the equilibrium payoff and the payoff it would receive if deviating and taken as a committed government: in any equilibrium, however, the uncommitted government payoff must not be lower than the payoff it could get if it had decided to fully privatize the firm at date 1: therefore the lowest \( \alpha^U \) is associated with the separating equilibria. The payoff of the uncommitted government, in any case, is equal to the payoff of the committed government plus the gains from interference, which are increasing in \( \alpha \); hence, in order to achieve separation, the committed government ought to deviate to a level of \( \alpha \) slightly below \( \alpha^U \). Following the intuitive criterion, investors beliefs when observing a fraction \( \alpha \) slightly lower than \( \alpha^U \) must assign probability one to the possibility that the government is committed. Hence, any equilibrium where the committed government is not identified does not satisfy the first step of our refinement: the only equilibrium is a separating one where the uncommitted government is indifferent between the two equilibrium actions.

Proposition 2 leaves unanswered the question of which equilibrium is selected if no separating equilibria exist. Note that the argument used in the proof to show that no non-separating equilibria survives our refinement does not apply when a separating equilibrium does not exist, because the lower value \( \alpha^U \) may not be positive. In this case a non-separating equilibrium may survive the intuitive criterion. Proposition 3 characterizes the situations in which semi-separating equilibria survive the first step of our refinement.

**Proposition 3** Suppose there is no separating equilibria; then there exist semi-separating equilibria satisfying the intuitive criterion if and only if

\[
\left( \frac{(1 + \rho)(1 + g) - (1 + r)}{(1 + \rho)(\rho - g) - (r - g)\rho} \right) \left( \frac{\rho - g}{1 + g} \right) < \tau \left( \frac{(1 + \rho)(1 + g) - (1 + r)}{(p_0 + \rho)(\rho - g) - (r - g)\rho} \right) \left( \frac{\rho - g}{1 + g} \right).
\]

Moreover, in a semi-separating dominant forward induction equilibrium, if it exists, the committed government delays the privatization to date 2 while the uncommitted government randomizes between full privatization at date 1 and at date 2.
Proposition 3 may be seen as a natural extension of Proposition 2. If τ is sufficiently high to prevent the committed government from perfectly signal its type to the market, in equilibrium there is still an imperfect signal, as long as τ is not too high. Also it keeps the basic properties of Proposition 2 in the sense that, whenever possible, the committed government wants to incur the cost of partial sales (or even delaying the privatization).

Propositions 2 and 3 do not cover all the parameterizations of the model. In the remaining cases, the committed government cannot expect to signal its type to the market, because only pooling equilibria are reasonable.

**Proposition 4** Suppose that there are no separating equilibria; then there exist pooling equilibria satisfying the intuitive criterion. Moreover if

\[
\tau \geq \left( \frac{(1+p)(1+g)-(1+r)}{(p_0+p)(\rho-g)-(r-g)\rho} \right) \left( \frac{p-g}{1+g} \right)
\]

(8)

this is the only type of equilibrium that survives the intuitive criterion. In a pooling dominant forward induction equilibrium, either both types of government fully privatize the firm at the date 1, or both follow the same policy of partial sales.

Proposition 4 states that, as long as no separating equilibria exist, there are reasonable equilibria in which both types of government choose a given level of α at date 0. This level may or may not involve partial sales. In particular, it can be shown that if τ is sufficiently high the equilibrium requires full privatization of the firm in the first date; on the other hand, if τ is not so high, partial sales may be supported in a pooling equilibrium. This is the case when an increase in α by the uncommitted government leaves open the possibility for the committed government to signal its type to the market by following a sufficiently tight policy of partial sales (choosing a very small α).

If the inequalities (7) are satisfied there may be two forward induction equilibria, a semi-separating equilibrium and a pooling equilibrium. The following proposition formalizes this point.

**Proposition 5** There exists a cutoff level τ, satisfying (7) such that for all τ satisfying (7) we have

1. if τ < τ, both the semi-separating forward induction equilibrium from Proposition 3 and the pooling forward induction equilibrium from Proposition 4 exist;

2. if τ ≥ τ, only the pooling forward induction equilibrium from Proposition 4 exists.

Propositions 2–5 completely characterize the first period action in any forward induction equilibrium of the model. For a generic parameterization of the model, if we allow τ to change from 0 to 1, there will be two cutoff values. Shown in figure 2, τ₁ < τ₂ such that:

1. for values of τ not higher than τ₁ the committed government perfectly signals its type through partial sales and the uncommitted government sells the firm at date 1;

2. for values of τ satisfying τ₁ < τ ≤ τ₂, there are two plausible equilibria: in one the committed government postpones privatization of the firm to date 2 while the uncommitted government randomizes between privatizing the firm at date 1 or date 2; in the other both types announce the same level of sales at date 1 (which may or may not require partial sales);

Figure 2: Structure of equilibria and effect of interference
3. for values of $\tau$ greater than $\tau_2$ both types of government follow the same action at date 1: it may or may not include partial sales; however, if it requires full privatization at date 1 for a given $\tau$, it also requires it for higher values of $\tau$.

Before outlining the characterization of the equilibrium continuation actions, it is of interest to analyze some particular cases, which help to make clear some of the implications of the previous propositions. The first studies the effect of the discount rates.

**Lemma 1** If the government’s discount rate equals the market discount rate, $r = \rho$, and a separating equilibrium exists, then $\tau < r$.

From Lemma 1 it is clear that partial sales ensuring separation would hardly occur if the government was as patient as the market. With higher government impatience quicker privatization represents an important source of funds to be used at the discretion of politicians in power.

Proposition 2 provides a bound for the interference parameter of firms in which a separating equilibrium exists. The following lemma relates that bound with the government’s discount rate, providing a simpler necessary condition for the existence of a separating equilibrium

**Lemma 2** For firms with $\tau > \frac{\rho r}{1 + \rho}$ the choice of $\alpha$ cannot perfectly reveal the government’s type, regardless of the values of $r, g$, and $\phi$.

Lemma 2 states that there may be state owned firms for which perfect revelation of the government’s type through partial sales is not possible. The set of firms meeting the condition in the lemma is larger for less patient governments; this is because for a less patient government the current payoff is highly valued and the uncommitted government cannot recover from a sale made at low price.

Another interesting case is the privatization of firms where there are no reasons to believe that private ownership provides efficiency rations, i.e. $g = 0$. In that case either type of government is interested in privatizing the firm insofar as $\tau > r$. Indeed, if both discount rates were equal, there would be no economic reason for the committed government to privatize.

**Lemma 3** If $g = 0$, existence of a separating equilibrium requires that

$$\tau \leq \frac{\rho - r}{1 + \rho - r \phi}.$$

The remainder of this section looks at how the parameters of the economy influence the optimal actions of the uncommitted government through time. We take as given that the committed government chooses to privatize a fraction $\alpha^*$ of the firm at date 1. The uncommitted government then has to decide whether to reveal its type. Because at date 1 the uncommitted government either chooses $\alpha = \alpha^*$ or $\alpha = 1$, we can summarize its optimal action with the probabilities $\lambda_0$ and $\lambda_1$, where $\lambda_0$ is the probability that it decides to fully privatize the firm at date 1, and $\lambda_1$ is, as before, the probability that it interferes after having privatized the first fraction of the firm, but before the privatization ends. Proposition 6 describes the best action at date 1.

**Proposition 6** When separation does not occur at date 0 the probability of separation before the privatization is completed, $\lambda_1$, is increasing in $\alpha^*$ and $\phi$, decreasing in $p_1$ and $g$, and independent of $\tau$.

The most interesting results of the lemma are those related to the effect of the endogenous parameters. The effect of $\alpha$ is clear. The amount of the reward (revenue from interference) is increasing in $\alpha$ while the amount of the punishment (decrease in price for the remaining privatization) is decreasing in $\alpha$. Therefore an increase in $\alpha$ increases the incentive for deviation soon after the first tranche is privatized.

The parameter $p_1$ is the level of reputation that the government has at the beginning of the second period, after the first sale of $\alpha$. The amount of the reward is independent of the reputation level while the amount of the punishment increases with the initial level of reputation. Therefore, an increase in reputation decreases the incentive for separation. Similar results can be extracted for the probability of separation at date 0.
Proposition 7 The probability that the uncommitted government separates at date 0, at the announcement of the first sale, $\lambda_0$, is decreasing in $\alpha$, $p_0$ and $\tau$ and increasing in $g$ and $\phi$.

The greater the optimal share of the firm privatized at time 1 by a committed government, the greater the incentive for the uncommitted to match this choice and get the pooled price — for a greater fraction of the firm — instead of the low price. But after the first sale is over, the greater the share of the firm already sold, the greater the cash-flows from interference and the smaller the losses associated with a low sale price for the remaining portion not yet privatized, $1 - \alpha^*$. This explains why $\lambda_0$ is inversely related to $\alpha^*$ while $\lambda_1$ is proportional to $\alpha^*$. Regardless of the stage of the privatization, the higher the level of government credibility, the lower the motivation for the uncommitted government to reveal its nature. Revelation implies price cuts for future sales that are proportional in size to the lost reputation. Also, since the choice for the uncommitted is between $\alpha = \alpha^*$ and $\alpha = 1$ the greater the $g$ the greater the incentive to set $\alpha = 1$ and benefiting from the efficiency gains.

It is worth noting that $\lambda_0$ decreases with $\tau$, while $\lambda_1$ seems independent of $\tau$. At the time of the first sale, the larger the effects expected from government interference, the better it is for the uncommitted government to conceal its identity and match the committed’s choice of $\alpha$ to get the pooled price. Consequently, separation will be quicker in privatizations of firms where $\tau$ is low, since in these cases the cost associated with revealing the true type must be lower. The results for $\phi$ are somewhat obvious. This parameter may be interpreted as the share of the rents extracted from the private sector from government interference that are not dissipated in the collection process. The more efficient is this collection, the more rewarding is the interference, and the greater must be the incentives for the uncommitted government to do so.

3 Underpricing of Shares

One of the most controversial aspects of privatization programs is the appreciable underpricing of the shares of the companies offered for sale. Several authors, i.e. Jenkinson and Mayer (1988) and Vickers and Yarrow (1988), report substantial premiums both in the U.K. and France when trade opens in the shares of the newly privatized firms. Discounts seem to be frequent and common throughout the world. Indeed, as Sir Alan Walters remarks, it seems that no privatization can escape the accusation of giveaways, perhaps with justification for the discounts in many privatizations are significantly higher in relation to the degree of underpricing typical on IPOs in private issues. The evidence is counter intuitive, because one would expect lower discounts in the sale of well known public companies, after expensive marketing campaigns, by a seller better positioned to bear the underwriting risks.

The discount of share issues in privatizations is related with the method chosen for the sale. Although the government can arrange for the sale to be done by tender at a much more accurate pricing, in most instances the preference is for offers for sale at a fixed and preannounced price. Why the apparent refusal by the government in letting the market set the prices, or in choosing to follow common capital market practice, as in the case of France where the large majority of IPOs on the official listing are tender offers?

In this section, a model is presented that explains significant discounts in privatizations as well as why the government decides to make offers for sale. The view taken here is consistent with an action of deliberate underpricing taken for strategic purposes. In a world of imperfect information, where investors are unable to observe directly whether or not the government is committed to a policy of no ex-post interference, underpricing of shares may be used to convey information about the government’s commitment, in addition to a policy of gradual sales. Indeed, cases are in which partial sales alone, when completed at market

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16 See "Privatisation in Britain: Comment" in Privatisation and State-Owned-Enterprises.
17 For instance, Vickers and Yarrow (1988) compute a weighted average of the price changes in privatizations of 18.4% of gross proceeds. Jenkinson and Mayer (1987) find that the average degree of underpricing of larger issues in the private sector is around 5%.
18 Apart from the fact that young private firms are often cash constraint at the time of the IPO, in many countries there is a ban on publicity about the company and its business imposed by the regulatory agencies.
19 The reader may note that underpricing affects both types of government in the same way, corresponding to burned money. Therefore, underpricing alone would never be a useful device to ensure separation.
prices, do not guarantee separation; separation may only be achieved when the weak signal provided by partial sales is combined with a second control variable, the issue price. In order to be able to use the price strategically, the government has to abandon the possibility of organizing an auction and decides instead to make an offer for sale at a fixed price.\(^{20}\)

The model herein belongs to the class of signalling models that have recently appeared in the literature explaining pricing of unseasoned new issues of common stock. In the works of Allen and Faulhaber (1989), Grinblatt and Hwang (1989) and Welsh (1989), when the issuer has superior information about the firm's prospects than outside investors, underpricing may be used in equilibrium as a signal of firm quality.\(^{21}\) Also, in these models there is an exogenous timing for the information to be revealed about the quality of the issuing firm. Never, in our setting, investors have an opportunity to observe directly the nature of the government. It should be noted, however, that our hypothesis for the underpricing does not contradict any of the above, nor the winner's curse explanation offered in Rock (1986) and Beatty and Ritter (1986). In a few cases it is even possible that the effects might interact to increase the degree of underpricing.

Let us introduce a few necessary modifications in the model described in Section 2 to analyze pricing decisions. The timing and information structure of the game remain basically the same. However, at date 0, when the government announces the share \(\alpha\) of the firm to be privatized, it also decides whether to sell at a preannounced price, \(\bar{S}\), or to organize a sale by tender; if the government decides to fix the sale price, the subsequent response by the capital market will be either to accept (the share \(\alpha\) of the firm is sold at unit price \(\bar{S}\)) or to reject (the firm remains state owned and is fully privatized at date 2); except for this action the game is similar to that described in the previous section.

We assume that an external constraint, of the form

\[
\frac{V_1(\alpha, p) - \bar{S}}{V_1(\alpha, p)} \leq \psi^\prime,
\]

limits the level of underpricing that the government may decide. This type of constraint may be motivated by the budget constraint of the government, or by laws requiring that measures that change the government position relatively to past rights and liabilities cannot have a current negative impact on the budget.\(^{22}\)

Since partial sales may be selectively used in conjunction with the price to signal commitment, in equilibrium the share of the firm, \(\alpha\), and the offer for sale price, \(\bar{S}\), must be determined simultaneously. Below, it is shown that:

a when partial sales can guarantee separation, if mispricings may result, it is never optimal to sell at a fixed price and tenders should be arranged;

b the underpricing is directly related to the fraction of the firm sold.

This last point is crucial for the correct evaluation of the wealth transfers resulting from deliberate discounting. The true measure of the underpricing is not obtained from relative discount (difference between the market price and the offer for sale price per share), but from absolute discount (relative discount times the fraction of the firm that is privatized).

By using both the share of the firm and the price to induce separation the government can obtain new results, not possible when \(\alpha\) is the unique choice variable. The additional outcomes must, though, be associated with setting the sale price below the market price, for if the sale was offered above the market price, the outcome would be equivalent to that of choosing to fully privatize the firm at date 2, a result also possible when \(\alpha\) is the only variable of choice.

\(^{20}\)To the extent that underpricing leads to expected oversubscription and subsequent rationing of shares our explanation is consistent with the frequently used argument that privatizations promoting wider share ownership make it more difficult for future interference of particular type by the government, i.e. renationalization.

\(^{21}\) Of these models, that of Allen and Faulhaber (1989) also includes learning from investors.

\(^{22}\) There is a problem of \(\psi\)-ante observability of \(V_1(\alpha, p)\). The constraint should be interpreted as a reduced form constraint, induced by some primitive rule.
In this section we restrict our attention to the existence of separating forward induction equilibria, still focusing on the reduced signaling game. In this setting, the pairs \( (a, S) \) that guarantee separation and survive the intuitive criterion satisfy the equality

\[
U^U((a, S, 0, 0), (1, 1, 1), (S_1, S_2)) = U^U((a, S, 0, 0), (a, S, \lambda^U, 1), (S_1, S_2)),
\]

where \( U((a, S, 0, 0), (a, S, \lambda^U, 1), (S_1, S_2)) \) is the payoff of the uncommitted government if it decides to match the committed choice and sell \( \alpha \) shares in the first date at a price \( S \), given that it interferes before completing the privatization with probability \( \lambda^U \). Expression (10) implies that the uncommitted government is indifferent between immediate separation and matching the choice \( (a, S) \) by the committed government, at date 0.

The first result of this section gives us conditions for the existence of separating equilibria.

**Proposition 8** If underpricing of shares is allowed, \( \psi \geq \tau \) is a sufficient condition for the existence of a separating equilibrium.

In what follows we will assume that the sufficient condition in Proposition 8 is satisfied, and will concentrate on the identifying when is underpricing used in equilibrium.

How is \( S \) related to \( a \)? To answer that, we first determine how much of the firm should be privatized if underpricing is used. Given that the actual discount is increasing in \( a \) it is not immediate to identify which effect dominates. From expression (10) above and because the right hand side is equal to the committed government’s payoff plus the proceeds from interference, which are increasing in \( a \), it follows that the share of the firm to be privatized at date 1 by the committed government, when underpricing is deliberate, should be the smallest possible that still allows the committed government to meet its objectives.\(^{23}\)

In order to analyze under what conditions underpricing should be used as a signaling device, it is important to identify what price should the government ask when, by following a policy of partial sales, it is possible to guarantee full separation. The following proposition states what is the degree of underpricing when such circumstances apply.

**Proposition 9** If full separation through partial sales is possible, in equilibrium underpriced shares are not offered for sale.

If partial sales alone allow a separating equilibrium the uncommitted government must be indifferent between deviating by choosing to fully privatize the firm and matching the committed government’s choice \( a = a^* \). If the committed government decides to sell a share \( \hat{a} \) bigger than \( a^* \), it will have to announce an offer for sale below the market price. \( \bar{S}(\hat{a}) < V_1(\hat{a}, 1) \). Proposition 9 states that this strategy is dominated and the costly signal with underpricing cannot improve the payoff of the committed government. Consequently, gradual sales should be organized at market prices. This can be best achieved by offers for sale by tender. Proposition 9 formalizes the argument previously made by others, that where tenders can be arranged they will reduce mispricings, and that these are best avoided by the sale of the assets by stages.\(^{24}\)

What Proposition 9 fails to give is an indication of the optimal strategy when gradual sales at market prices are not sufficient to guarantee separation. In that case underpricing may be an optimal signalling device.

**Proposition 10** If full separation is not possible when the committed government chooses the quantity \( a^* \) to be privatized, the use of the price as a strategic variable helps in achieving separation. Moreover, in equilibrium, partial sales will be used by the committed government.

\(^{23}\)In this model the lowest bound for \( a \) is then induced by the constraints on discounts. However, imperfections not considered in our model, such as economies of scale in advertising and flotation costs, may impose a lower bound on the share sold, and help to explain why underpricing is often associated with non negligible fractions of the firms being privatized.

The conditions needed for underpricing to be seen in equilibrium, presented in Proposition 10, imply that there are cases where offers for sale at a fixed price dominate the tender option. Only by means of an offer for sale the government can use the price as a signalling device, clearly impossible when market forces are used in the setting of prices. Furthermore, if the government decides to use the price, mispricings should necessarily be the result.\textsuperscript{25}

![Figure 3: Underpricing per Share](image)

The signalling schedule for the offering prices is shown in Figure 3. Each of the lines corresponds to a signalling schedule for fixed values of the parameters \( g, \phi, \tau, \rho \) and \( r \). The graph suggests cases where the underpricing per share, \( 1 - S/V_1 \), is increasing in \( \alpha \), and cases where this underpricing is decreasing in \( \alpha \). The ascending schedules correspond to the cases where a separating equilibrium exists without underpricing. They are reported to account for these situations whereby market frictions and institutional costs would force the sale of a fraction greater than the \( \alpha \) that would give separation. An increase in the proportion \( \alpha \) announced must come at a cost of larger discounts since a larger \( \alpha \) gives the uncommitted government a greater incentive to match the committed government’s action.

In other instances the uncommitted government is significantly better off if it follows the \( \alpha \) chosen by the committed, even if the sale is done at a (moderate) discount. A separating equilibrium is always possible for a large proportion \( \alpha \) sold at a price sufficiently close to the full information low. A small decrease in \( \alpha \) breaks this separating equilibrium, since the uncommitted government can do better by matching the committed’s actions and getting a high price at date 2 for a larger remainder of the firm. To induce uncommitted’s disclosure and ensure a separating equilibrium with a smaller \( \alpha \), the discount must, as a result, be increased. Consequently, the signalling schedules are decreasing in \( \alpha \) when the payoff of the uncommitted government is significantly greater under a pooling than in a separating equilibrium with gradual sales alone.

The fact that the signalling schedule can be increasing as well as decreasing in the proportion of the firm sold has an immediate implication on the way of measuring underpricing. Often, in the literature, discounts are reported on a per unit share price basis. This way of measuring underpricing is, however, inaccurate since it leaves out of the picture the proportion of the issue sold at the reduced price. Figure 4, graphing the degree of underpricing as the product of the fraction \( \alpha \) times the unit price discount \( 1 - S/V_1 \), shows that the largest

\textsuperscript{25}It is important to distinguish between true underpricing, the difference between market price and the offer for sale price, and the bias occurring from oversubscription once the market associates fixing the price with underpricing.
discount per share observed in Figure 3 does not correspond to the largest degree of underpricing offered. A closer look reveals that for the descending (ascending) signalling schedule the degree of underpricing is inversely (directly) related to the discount offered on the unit price. When measured correctly, it can be seen from Figure 4 that the underpricing is directly related to the fraction of the firm sold.

4 The Multiple Firm Case

In the last two sections, we examined the privatization of a single firm. There, we were able to show that a policy of gradual sales and deliberate underpricing of shares can be the result of rational actions. We now turn to the characterization of the optimal behavior when there is a finite number of firms to be privatized. The objective is to analyze whether there exists a time period $t$, after which the committed government can abandon gradual sales. At that point, capital markets are perfectly able to discern the government’s type and any firm still to be privatized at $t$, will then be sold in a single round.\textsuperscript{26}

The assumed uncertainty in this more general setting of multiple firms is of great importance. We consider two plausible and somewhat opposite cases. In the first, nature is assumed to independently draw the type of the government at the beginning of the privatization of each firm. This would correspond to the case of a volatile policymaker with a time and/or firm varying type, perhaps because he is likely to interfere with some firms and in particular periods and not to interfere in other instances. When the government is sometimes committed and other times uncommitted, the private sector does not learn the government’s type from actions taken in past privatizations. That is, investors are able to learn during each privatization, but unable to use effectively this knowledge between the privatization of different companies. In this sense the multiple firm multiple period case is a just a repeated version of the single firm model examined in the previous sections.

In the second case, a committed government will stay committed throughout the program, and so for an uncommitted. Investors try to infer the type of the government from observed actions during the whole period of the privatization program, as long as the government stays in power. As privatization moves

\textsuperscript{26}We do not focus on underpricing in this more general framework, because the basic question can be addressed within the simpler framework used before.
along and interference is not observed there is an upward revision of the government's reputation, which is reflected in higher prices for the firms still to privatize. The revelation by the uncommitted is, in this case, an absorption state: once investors notice one deviation, any remaining uncertainty about the government's type is immediately and definitively resolved; investors have a long memory and do not forgive. Because of that, this second case is relatively more disadvantageous for the uncommitted government than the first case, and one should expect this to be reflected in a greater propensity to match the committed's choices.

The two cases here considered may be seen as polar cases of a general model where the type of the government would correlatedly change from firm to firm. The analysis of the extreme cases is, however, rich in the wide range of the equilibrium behaviors allowed to the government.

The model retains the basic features described in Section 2, but now the number of firms is \( N \), and the number of periods is \( 2N \). All the firms are assumed to be equal,\(^{27}\) so we may refer to a representative firm.\(^{28}\) The sequence for the entire privatization program assumes that the next firm is privatized only after the government has finished to privatize the previous one, i.e. following the sequence \( \alpha_i, 1 - \alpha_i, \alpha_{i-1}, 1 - \alpha_{i-1}, \ldots, \alpha_1, 1 - \alpha_1 \), being the last firm to be privatized indexed by \( 1 \).\(^{29}\)

Given the assumptions, when the type of the government independently changes between firms the model becomes the repeated version of that in Section 2. The results in that section can, therefore, be extended to the dynamic framework.

**Proposition 11** If the government's type independently changes across firms, partial sales may either be seen at any moment during the privatization program, or never be implemented.

This result follows immediately from the intuition developed in Section 2 given that, in this case, the multiple period multiple firm problem is a sequence of single and sequentially unrelated privatizations. Because we assume identical firms, the equilibrium in the multiple firm model corresponds to the repetition of the equilibrium for the privatization of the representative firm.

The situation, however, differs when the type of the government is the same throughout the privatization program, as the following set of propositions clearly indicate.\(^{30}\) The first one states sufficient conditions for immediate separation, resulting in partial sales being abandoned right after the first firm is privatized:

**Proposition 12** Suppose that a separating equilibrium exists for the privatization of the representative firm. Then, in the multiple firm model only the first firm to be privatized has partial sales, and immediate separation results. Moreover, the share \( \alpha \) of the first firm is exactly that giving separation if this was the only firm to be privatized.

This is a powerful result. If a separating equilibrium exists for one of the firms, then no matter how many firms the government privatizes, the separating equilibrium in the multiple firm privatization requires partial sales only for the first firm in the sequence. All remaining firms can be sold under a first best policy.

The results derived in previous sections and the proof of Proposition 12 lead to the case of a more general result for the privatization of multiple firms. In Section 2 we saw that, if the values of the parameters are such that separating equilibria do not exist, there will exist semi-separating and pooling equilibria, some of them requiring partial sales. Indeed, this result can be generalized to the multifirm framework. It is possible to have equilibrium strategies, in this general setting, where the committed government follows a policy of partial sales while separation is not obtained, and the uncommitted government either pools or randomly chooses between the policy of partial sales and full privatization.\(^{31}\)

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\(^{27}\) Different firms complicate the model without necessity. Preliminary analysis with different firms seems to indicate that the results derived in this section would not have been affected. However, the ordering of the sale would have become endogenous and a function of the firms' characteristics.

\(^{28}\) When mentioning the privatization of the representative firm we refer to the privatization of a single firm with characteristics, as described in Section 2.

\(^{29}\) Many other rules are possible. The issue of analyzing the optimal sequence is beyond the scope of this paper.

\(^{30}\) In the remaining of this section we consider the case in which the government's type is drawn once at the beginning of the privatization program.

\(^{31}\) This general equilibrium is just an analogue, in our model, of the similar equilibria in the chain store model with asymmetric information in Kreps and Wilson (1982).
Proposition 13 Suppose that semi-separating equilibria, satisfying the intuitive criterion, exist for the privatization of the representative firm. Then, there exist forward induction equilibria, in the multiple firm model, where the committed government chooses \( \alpha^*_c < 1 \) until separation is achieved, and the uncommitted government randomizes between full privatization and \( \alpha^*_u \). Moreover, if

\[
\tau < \left( \frac{p-g}{1+g} \right) \left( \frac{(1+p)(1+g) - (1+r)}{(1+p)(p-g) - (r-g)\phi \left( \rho + \sum_{i=2}^{N} (1+g\alpha^*_i)(1+g)^{2i}(2+p+g) \right)} \right)
\]

(11)

separation is achieved with probability one, before the end of the privatization program.

Proposition 13 characterizes situations in which separation can be achieved at any date during the privatization program, because the committed government follows a policy that leaves the uncommitted government really indifferent between pooling or separating at any date during the program. However, as times elapses and more firms are privatized, the cost of delaying interference by the uncommitted government increases and eventually there is a firm for which it is better to interfere.

The previous proposition suggests a more general situation. Likewise the model in Section 2, it is possible that the cost of separation at the beginning of the privatization program is so high that the uncommitted government does not take the risk of being identified, and decides to pool with the committed government for any choice of \( \alpha \). Under these circumstances, both types of government will follow the same policy for the first group of firms being privatized; afterwards, the equilibrium actions are similar to those described in Proposition 13.

Proposition 14 Suppose that

\[
\tau \geq \left( \frac{(1+p)(1+g) - (1+r)}{(p_0+p)(p-g) - (r-g)\phi \rho \left( \rho + \sum_{i=2}^{N} (1+g\alpha^*_i)(1+g)^{2i}(2+p+g) \right)} \right) \left( \frac{p-g}{1+g} \right)
\]

(12)

then in any forward induction equilibrium both types of government will follow the same policy for the initial privatizations. If conditions in Proposition 13 are met at any time during the privatization program, the equilibrium actions must then allow for separation with positive probability at any date.

The intuition behind Proposition 14 is clear. When the effect of interference is very high the uncommitted government wants to conceal its type from the investors and there is no way for the committed government to successfully reveal its type through partial sales at the beginning of the privatization program. However, as more and more firms are privatized, the relative costs and benefits of interference by the uncommitted government change, increasing its incentives for interference. Hence, at a later stage, the uncommitted government randomizes between interference and no interference. Finally, when a significant number of firms have already been privatized, if the uncommitted government has not yet revealed its type, it decides to interfere.

We can now discuss the importance of the assumed evolution of the government's type. In one case, the government's type was assumed constant over time and, in general, partial sales could be observed in the privatization of the initial firms but, eventually, separation could be achieved before the end of the privatization program. In another case the government's type was assumed to change with the firm as well as with time and the conclusion was that gradual sales might appear at any moment in time, even after some firms had been privatized without using partial sales. Both contexts are plausible ones and the question of how and in what conditions partial sales should be used and abandoned in the multiple firm program has been addressed. It can, however, be presumed that the conclusions would somewhat differ if the government's type followed a different process, but we can expect that partial sales would still occur in equilibrium.
5 Conclusions

This study provides an explanation for two phenomena often observed in privatizations: the gradual sale of the firms' capital (usually in two phases) and the underpricing of shares. When the private sector is unclear about the government's type, investors try to learn it from the actions the government takes. This provides an opportunity for a committed government to signal its intention of not interfering with the private ownership after the firm has been privatized. Gradual sales may induce a more impatient uncommitted government to reveal its nature, as it does the underpricing of shares since a greater discount reduces the cost of identity disclosure for the uncommitted. Gradual sales and underpricing can thus be optimal signalling devices used in equilibrium by rational governments.

The model can be extended in several directions. One is to analyze the optimal partial sale sequence. There might be considerable difficulties in doing it, however, given the number of different alternatives available and the increasing complexity of the payoff schedules. A less ambitious project but of no less interest is to examine which firms should be privatized first, whether large or small firms, monopolies or competitive firms. Finally, the question of why an uncommitted government decides to privatize deserves some thought, since there is always the possibility of retaining a sizable share in the firm and combining it with adequate managerial incentives. Perhaps time will suffice to answer this and many other related questions, as more is learned about the effects of privatizing and of deregulating the economy.
Appendix

Proof of Proposition 1

The game is finite, so the subgame perfect equilibrium can be computed solving the game backwards. In the last stage the government, depending on its type, chooses \( \lambda_C^f = 0 \) and \( \lambda_U^f = 1 \). In the first stage of date 2, investors pay the value of the firm, \( S_2 = V_2(\alpha, 1) \) if the government is committed and \( S_2 = V_2(\alpha, 0) \) if the government is uncommitted. In the second stage of period one, a committed government does not interfere, \( \lambda_C^l = 0 \); because later actions of the investors are independent of current government’s actions, the uncommitted government maximizes the current period payoff and interferes, \( \lambda_U^l = 1 \). The situation of investors at date one is similar to that at date two; therefore, they pay \( S_1 = V_1(\alpha, 1) \) to the committed government and \( S_1 = V_1(\alpha, 0) \) to the uncommitted government. Considering the equilibrium continuation actions and equations (1) and (2), either government payoff is an increasing function of \( \alpha \); therefore, the government fully privatizes the firm at the date 1, \( \alpha_C = \alpha_U = 1 \). ☐

Proof of Proposition 2

Let us first prove the condition for the existence of separating equilibria. From Proposition 1, in a separating perfect Bayesian equilibrium the uncommitted government is required to fully privatize the firm in the first date, after which it starts to interfere, i.e. \( \alpha_U = 1 \), \( \lambda_U^f = 1 \) and \( \lambda_U^l = 1 \). The committed government privatizes \( \alpha^* \) at date 1 (with \( \alpha^* < 1 \)), and does not interfere. The investors will have the strategy \( (S_1(\alpha|\alpha^*), S_2(\alpha|\alpha^*)) \) with

\[
S_1(\alpha|\alpha^*) = \begin{cases} 
V_1(\alpha, 0) & \text{if } \alpha > \alpha^* \\
V_1(\alpha, 1) & \text{if } \alpha \leq \alpha^* 
\end{cases}
\]

\( (A.1) \)

\[
S_2(\alpha|\alpha^*) = \begin{cases} 
V_2(\alpha, 0) & \text{if } \alpha > \alpha^* \text{ or the government interfered at } t = 1 \\
V_2(\alpha, 1) & \text{otherwise.}
\end{cases}
\]

\( (A.2) \)

where the out of equilibrium actions are consistent with the beliefs that a choice of \( \alpha \) below \( \alpha^* \) is made by a committed government and a choice above is made by an uncommitted government.\(^{32}\)

Let \( \sigma^* \) be the above strategy profile. The payoffs are

\[
U^C(\sigma^*) = \frac{1 + g \alpha^*}{r - g} \left( 1 - (1 - \alpha^*) \frac{\rho - r}{1 + \rho} \right)
\]

\( (A.3) \)

\[
U^U(\sigma^*) = \frac{1 + g}{r - g} (1 - \tau) + \frac{1 + g}{\rho - g} \alpha \tau
\]

\( (A.4) \)

\[
U^I(\sigma^*) = 0.
\]

\( (A.5) \)

The profile \( \sigma^* \) is an equilibrium as long as no player wants to deviate. It is immediate that the investors do not want to deviate. The binding condition for the committed government is \( U^C(\sigma^*) \geq U^C \left( (1, 0, 0), (1, 1, 1), (V_1(1, 0), V_2(1, 1)) \right) \), because if it lowers \( \alpha \) it will lower the payoff and if it increases \( \alpha \) it will be mistakenly viewed as an uncommitted government and the highest payoff is achieved when maximizing the choice of \( \alpha \); the condition can be written as:

\[
(1 + g \alpha^*) \left( 1 - (1 - \alpha^*) \frac{\rho - r}{1 + \rho} \right) \geq (1 + g)(1 - \tau)
\]

\( (A.6) \)

The left hand side of \( (A.6) \) is increasing in \( \alpha^* \), and the inequality is strict for \( \alpha^* = 1 \). Let \( \alpha_C \) be the value of \( \alpha^* \) which gives equality in \( (A.6) \). Then in a separating equilibrium \( \alpha^* \geq \alpha_C \).

\(^{32}\)We could have different assumptions about the investors out of equilibrium beliefs without affect the results.
The uncommitted government, on the other hand, may deviate to one of two different strategies, each requiring it to privatize a share $\alpha^*$ at date 1. In fact, deviating to lower $\alpha$ is dominated by one of the above deviations and deviating to $\alpha$ in the interval $(\alpha^*,1)$ is dominated by the initial strategy. After choosing $\alpha = \alpha^*$ it may decide to reveal its type before or after completing the privatization. It chooses to reveal its type immediately after the sale of $\alpha^*$ if
\[
(1 - \alpha^*)V_2(\alpha^*, 1) \leq (1 - \alpha^*)V_2(\alpha^*, 0) + \alpha^*(1 + g\alpha^*)\phi \tau ,
\]  
which implies
\[
\alpha^* \geq \frac{1 + g}{\phi(r - g) + 1 + g} .
\]
Let $\tilde{\alpha}$ denote this bound, which is always positive. Define $\tilde{\alpha}^U_1$ as the solution to the equality
\[
U^U(\alpha^*) = U^U \left( (\tilde{\alpha}^U_1, 0, 0), (\tilde{\alpha}^U_1, 0, 1), (V_1(\tilde{\alpha}^U_1, 1), V_2(\tilde{\alpha}^U_1, 1)) \right)
\]
which represents the value of $\alpha$ giving indifference between full separation and completing pooling, when the investors believe the government is committed when they see $\tilde{\alpha}^U_1$. Similarly, let $\tilde{\alpha}^U_2$ be the solution to the equality
\[
U^U(\alpha^*) = U^U \left( (\tilde{\alpha}^U_2, 0, 0), (\tilde{\alpha}^U_2, 1, 1), (V_1(\tilde{\alpha}^U_2, 1), V_2(\tilde{\alpha}^U_2, 1)) \right)
\]
representing the value of $\alpha$ which gives indifference between full separation and deviating immediately after the first privatization, and the investors believe the government is committed when they see $\tilde{\alpha}^U_2$. From (A.7), (A.9) and (A.10) it follows that $\tilde{\alpha}^U_1 = \tilde{\alpha}^U_2$ at $\tilde{\alpha}$. Moreover they both lie on the same side of $\tilde{\alpha}$. If they are greater than $\tilde{\alpha}$, $\tilde{\alpha}^U_1 < \tilde{\alpha}^U_2$; if they are lower than $\tilde{\alpha}$, $\tilde{\alpha}^U_2 > \tilde{\alpha}^U_1$. Hence, $\tilde{\alpha}^U = \min \{\tilde{\alpha}^U_1, \tilde{\alpha}^U_2\}$; $\tilde{\alpha}^U$ is the upper bound for $\alpha^*$, otherwise the uncommitted government matches the choice of $\alpha^*$.

Next, we prove that $\tilde{\alpha}^C < \tilde{\alpha}^U$. Use (A.9) and substitute $\tilde{\alpha}^U_1$ for $\tilde{\alpha}^C$ in the right hand side; then subtract it from the left hand side to get
\[
\frac{1 + g}{\rho - g} \phi \tau \left( 1 - \frac{1 + g\tilde{\alpha}^C}{1 + \rho} \right)
\]
which is positive, since $g\tilde{\alpha}^C < \rho$. Therefore
\[
U^U(\alpha^*) = U^U \left( (\tilde{\alpha}^C, 0, 0), (\tilde{\alpha}^C, 0, 1), (V_1(\tilde{\alpha}^C, 1), V_2(\tilde{\alpha}^C, 1)) \right)
\]
and $\tilde{\alpha}^U_1 > \tilde{\alpha}^C$. Similarly, use (A.10) and substitute $\tilde{\alpha}^U_2$ for $\tilde{\alpha}^C$ in the right hand side; then subtract it from the left hand side to get
\[
\left( \frac{g}{\rho - g} + \frac{1 + g\tilde{\alpha}^C}{1 + \rho} \right) (1 - \tilde{\alpha}^C)\phi \tau + \left( \frac{1 - \tilde{\alpha}^C}{1 + \rho} (V_2(\alpha, 1) - V_2(\alpha, 0)) \right),
\]
being clear that both terms are positive. Therefore,
\[
U^U(\alpha^*) > U^U \left( (\tilde{\alpha}^C, 0, 0), (\tilde{\alpha}^C, 1, 1), (V_1(\tilde{\alpha}^C, 1), V_2(\tilde{\alpha}^C, 0)) \right)
\]
and $\tilde{\alpha}^U_2 > \tilde{\alpha}^C$. Hence, $\tilde{\alpha}^C < \tilde{\alpha}^U$.

Finally, in order to ensure existence of a separating equilibrium, it must be that $[\tilde{\alpha}^C, \tilde{\alpha}^U] \cap [0, 1]$ is non empty. A necessary and sufficient condition for that to be true is $\tilde{\alpha}^U > 0$. Recall that $\alpha > 0$, therefore we only need that $\tilde{\alpha}^U_1 > 0$. From (A.9) we know that this is so if
\[
U^U(\alpha^*) \geq U^U \left( (0, 0, 0), (0, 0, 1), (V_1(0, 1), V_2(0, 1)) \right)
\]
(A.11)
which requires that
\[
\tau \leq \frac{(1 + \rho)(1 + g) - (1 + r)\rho - g}{(1 + \rho)(\rho - g) - (r - g)\phi \rho + g}.
\]  

(A.12)

In the unique separating equilibrium that survives the Cho-Kreps intuitive criterion the committed government sells \( \bar{\sigma}^U \) of the firm at date 1, because the separating equilibria with lower values of \( \alpha^* \) can only be supported with beliefs that assign positive probability to the uncommitted government for values of \( \alpha \) in the interval \( (\alpha^*, \bar{\sigma}^U) \), which does not satisfy the intuitive criterion.

To prove the second part of the proposition it is sufficient to show that, if separating equilibria exist, no other equilibrium survives the Cho-Kreps intuitive criterion. In a nonseparating equilibrium there is at least one value of \( \alpha \) that can be chosen by either government with positive probability. Fix a nonseparating equilibrium and let \( \alpha(1), \ldots, \alpha(n) \) be the values of \( \alpha \) that can be chosen with positive probability, labeled in such a way that \( \alpha(1) < \cdots < \alpha(n) \), let the equilibrium strategy of the investors be \( (S_1(\alpha(1), \ldots, \alpha(n)), S_2(\alpha(1), \ldots, \alpha(n))) \). Let \( \tilde{\alpha} \) be the smallest value of \( \alpha \) that can be chosen with positive probability by both types of government. A necessary condition for equilibrium is that
\[
U^U\left((\tilde{\alpha}, 0, 0), (\tilde{\alpha}, \lambda^U(\tilde{\alpha}), 1), (S_1(\tilde{\alpha}|\alpha(1), \ldots, \alpha(n)), S_2(\tilde{\alpha}|\alpha(1), \ldots, \alpha(n)))\right) \geq U^U\left((\tilde{\alpha}, 0, 0), (1, 1, 1), (S_1(1|\alpha(1), \ldots, \alpha(n)), S_2(1|\alpha(1), \ldots, \alpha(n)))\right)
\]
so that the uncommitted government does not strictly prefer to fully privatize the firm. Moreover
\[
U^U\left((\tilde{\alpha}, 0, 0), (1, 1, 1), (S_1(1|\alpha(1), \ldots, \alpha(n)), S_2(1|\alpha(1), \ldots, \alpha(n)))\right) \geq U^U(\sigma^*) \tag{A.13}
\]
because with the strategy profile \( \sigma^* \), if the firm is fully privatized at date 1, the uncommitted government is perfectly identified. Then define \( \tilde{\alpha}_1 \) as the solution to
\[
U^U\left((\tilde{\alpha}_1, 0, 0), (1, 1, 1), (S_1(1|\alpha(1), \ldots, \alpha(n)), S_2(1|\alpha(1), \ldots, \alpha(n)))\right) = U^U\left((\tilde{\alpha}_1, 0, 0), (\tilde{\alpha}_1, 0, 1), (S_1(1|\tilde{\alpha}_1), S_2(1|\tilde{\alpha}_1))\right) \tag{A.14}
\]
and \( \tilde{\alpha}_2 \) as the solution to
\[
U^U\left((\tilde{\alpha}_2, 0, 0), (1, 1, 1), (S_1(1|\alpha(1), \ldots, \alpha(n)), S_2(1|\alpha(1), \ldots, \alpha(n)))\right) = U^U\left((\tilde{\alpha}_2, 0, 0), (\tilde{\alpha}_2, 0, 1), (S_1(1|\tilde{\alpha}_2), S_2(1|\tilde{\alpha}_2))\right) \tag{A.15}
\]
Define \( \bar{\alpha} = \min\{\tilde{\alpha}_1, \tilde{\alpha}_2\} \); from (A.9), (A.10), and (A.13)–(A.15) it follows that \( \tilde{\alpha} \geq \bar{\alpha} \geq \tilde{\sigma}^U \). Thus the intuitive criterion requires investors to assign probability one to the committed government when \( \alpha < \tilde{\alpha} \) is seen, which can not be the case in this equilibrium; otherwise the committed government would prefer to deviate from \( \tilde{\alpha} \) to a value slightly below \( \tilde{\alpha} \). Hence no nonseparating equilibrium survives the first step of our procedure, and the unique equilibrium surviving it is the separating equilibrium where \( \alpha^* = \bar{\sigma}^U \) is chosen by the committed government. ■

**Proof of Proposition 3**

The first step is to characterize the structure of the semi-separating equilibria of the model. It is not possible for both types of government to assign positive probability to two different values of \( \alpha \), say \( \alpha_1 \) and \( \alpha_2 \); the uncommitted government must be indifferent between choosing \( \alpha_1 \) and \( \alpha_2 \), and the committed government’s payoff then imply that the committed government strictly prefers the lowest value. Moreover it is not possible that one type assigns positive probability to more than two values of \( \alpha \). Hence there are only

\[33\text{It is easy to prove that a finite set of values of } \alpha \text{ may be chosen.}\]
the following semi-separating equilibria: two values of \( \alpha \) are chosen with positive probability, one by one type and the other by both types. However, semi-separating equilibria where the uncommitted government chooses \( \alpha_1 \) and the committed government randomizes between \( \alpha_1 \) and \( \alpha_2 \) (it must be that \( \alpha_2 < \alpha_1 \)) can only be supported if the investors assign positive probability to the uncommitted when a value of \( \alpha \) slightly above \( \alpha_2 \) is seen, which violates the Cho-Kreps intuitive criterion, because the indifference of the committed government between \( \alpha_1 \) and \( \alpha_2 \) implies that the uncommitted government strictly prefers \( \alpha_1 \). Therefore the only semi-separating equilibria that may survive the refinement have the committed government choosing \( \alpha^* < 1 \) and the uncommitted government randomizing between \( \alpha^* \) and \( \alpha = 1 \) (an equilibrium choice of \( \alpha \) that reveals the uncommitted government’s type must be \( \alpha = 1 \)).

Given a parameterization of the model we may compute the level of reputation \( p_1(\alpha) \) that must be assigned to the committed government if a value \( \alpha < 1 \) is seen, to leave the uncommitted government indifferent between matching \( \alpha \) or full privatization of the firm at date 1. Because the government’s payoff is increasing in \( \alpha \) and \( p_1(\alpha) \) is a decreasing function; moreover, because separating equilibria do not exist for \( p_1(0) < 1 \). Hence, any \( \alpha \) such that \( p_1(\alpha) > p_0 \) can be supported in a semi-separating equilibrium that survives the intuitive criterion. The necessary and sufficient condition for the existence of such semi-separating equilibria is thus \( p_0 < p_1(0) < 1 \), which can be written as

\[
\frac{(1 + \rho)(1 + g) - (1 + r)}{(1 + \rho)(\rho - g) - (r - g) \sigma \rho} \left( \frac{\rho - y}{1 + y} \right) < \tau < \left( \frac{(1 + \rho)(1 + g) - (1 + r)}{(p_0 + \rho)(\rho - g) - (r - g) \sigma \rho} \right) \left( \frac{\rho - y}{1 + y} \right). \tag{A.16}
\]

In all these semi-separating equilibria, the uncommitted government gets the same payoff, which is equal to the payoff from full privatization at date 1. Hence the second step of our refinement, applied to the semi-separating equilibria selects the one maximizing the committed government’s payoff, that can be written as

\[
U^C\left( (\alpha^*, 0, 0), (\alpha^*, \lambda_1^+, \alpha^*), 1, \left( V_1(\alpha^*, p_1(\alpha^*), 1), V_2(\alpha^*, p_2(\alpha^*)) \right) \right) = \quad U^U\left( (\alpha^*, 0, 0), (\alpha^*, \lambda_1^+, \alpha^*), 1, \left( V_1(\alpha^*, p_1(\alpha^*), 1), V_2(\alpha^*, p_2(\alpha^*)) \right) \right) - \frac{(1 + g \alpha^*)(1 + g)}{(\rho - g)(1 + \rho)} \sigma \tau - \frac{\alpha^*}{1 + \rho} \left( V_2(\alpha^*, 0) + (1 + g \alpha^*) \sigma \tau - V_2(\alpha^*, p_2(\alpha^*)) \right).
\]

Because the payoff of the uncommitted government is constant, and lower proceeds from intervention are associated with lower sales at date 1, the committed government ought to prefer delaying the privatization to date 2, i.e. choosing \( \alpha = 0 \). ■

**Proof of Proposition 4**

Pooling equilibria always exist; to support them it is sufficient that the investors believe that the sales policy does not convey any information about the government’s type. Such beliefs do not satisfy the intuitive criterion when separating equilibria exist. On the other hand, if separating equilibria do not exist, pooling equilibria satisfying the intuitive criterion always exist. Consider that the committed government privatizes \( \alpha^* \) at date 1, the uncommitted government privatizes \( \alpha^* \) and then follows the optimal policy of intervention, \( (\lambda_1^+, 1) \), with the investors following the strategy \( (\Sigma_1, \Sigma_2) \) with

\[
\Sigma_1(\alpha | \alpha^*) = \begin{cases} V_1(\alpha, p_0) & \text{if } \alpha = \alpha^* \\ V_1(0, 1) & \text{if } \alpha = 0 \\ V_1(\alpha, 0) & \text{otherwise} \end{cases} \tag{A.17}
\]

\[
\Sigma_2(\alpha | \alpha^*) = \begin{cases} V_2(\alpha, p_2) & \text{if } \alpha = \alpha^* \text{ and the government did not interfere at } t = 1. \\ V_2(0, p_2) & \text{if } \alpha = 0 \text{ and the government did not interfere at } t = 1. \\ V_2(\alpha, 0) & \text{otherwise}. \end{cases} \tag{A.18}
\]
As long as
\[
U^U \left( (\alpha^*, 0, 0), (\alpha^*, \lambda^U, 1), (\xi_1(\alpha^*|\alpha^*), \xi_2(\alpha^*|\alpha^*)) \right) \geq \\
U^U \left( (\alpha^*, 0, 0), (1, 1, 1), (\xi_1(\alpha^*|\alpha^*), \xi_2(\alpha^*|\alpha^*)) \right)
\] (A.19)
and
\[
U^U \left( (\alpha^*, 0, 0), (\alpha^*, \lambda^U, 1), (\xi_1(\alpha^*|\alpha^*), \xi_2(\alpha^*|\alpha^*)) \right) \geq \\
U^U \left( (\alpha^*, 0, 0), (0, 0, 1), (\xi_1(0|\alpha^*), \xi_2(0|\alpha^*)) \right)
\] (A.20)
the given strategy profile is an equilibrium and it satisfies the intuitive criterion. The non existence of separating equilibria ensures that whenever (A.20) is satisfied (A.19) will be satisfied. Let \( \overline{\alpha} \) be the value of \( \alpha \) that satisfies (A.20) with equality; then \( \overline{\alpha} = \min\{\overline{\alpha}, 1\} \) gives an upper bound for \( \alpha \). If
\[
\tau \geq \left( \frac{(1 + \rho)(1 + g) - (1 + r)}{(p_0 + \rho)(p - g) - (r - g)\phi\rho} \right) \left( \frac{\rho - g}{1 + g} \right),
\]
from Propositions 2-3, only pooling equilibria survive the intuitive criterion.

The second step of the refinement applied to the pooling equilibria that survived the first step, must select the highest possible \( \alpha \), i.e. \( \overline{\alpha}^* \), which does not have to be equal to 1, because either type of government increases the payoff with a larger \( \alpha \). Hence, either both types of government fully privatize the firm at date 1 or both types choose partial sales, giving the uncommitted government a payoff that makes it indifferent between partial sales and a policy of delaying the privatization, when the investors think that only the committed government would delay the privatization.

**Proof of Proposition 5**

From the proof of Propositions 3 and 4 we know that in any pooling equilibria that survives the Cho-Kreps intuitive criterion, the uncommitted government has a payoff no lower than the payoff from any semi-separating equilibrium, also surviving the intuitive criterion. Therefore, the second step of refinement cannot rule out the pooling equilibrium referred to in Proposition 4, where both governments choose \( \alpha = \overline{\alpha}^* \). As \( \tau \) approaches the bound in (6) from above, the difference between the payoffs of the uncommitted government in the semi-separating equilibrium and in the pooling equilibrium goes to 0, which, because the committed government's choice of \( \alpha \) in the semi-separating equilibrium is lower than in the pooling equilibrium, implies that the committed government has a higher payoff in the semi-separating equilibrium. Defining \( \tau \) as the value of \( \tau \) for which the committed government is indifferent between the two equilibria, the results of the proposition immediately follow.

**Proof of Lemma 1**

From Proposition 2, if \( \rho = r \) separating equilibria exist if and only if \( \tau \leq \frac{\phi(1 + \rho)}{(1 + \phi)(1 + g)} \). This bound is increasing in \( \phi \). If we let \( \phi = 1 \) then it becomes \( \tau \leq \frac{\phi(1 + r)}{1 + g} \), which is smaller than \( r \).

**Proof of Lemma 2**

Consider again the bound for \( \tau \) in Proposition 2. The numerator of the first fraction of the bound is limited from above, because
\[
(1 + \rho)(1 + g) - (1 + r) < \rho(1 + g),
\]
and the denominator is bounded from below, because
\[
(1 + \rho)(\rho - g) - (r - g)\phi\rho \geq (1 + \rho)(\rho - g).
\]
Hence if \( \tau > \frac{\rho}{1 + \rho} \), a separating equilibrium does not exist.
Proof of Lemma 3
The bound for the existence of separating equilibria is derived directly from Proposition 2. ■

Proof of Proposition 6
Define $\hat{p}_2$ as the level of reputation that leaves the $U$-type government indifferent between immediate revelation after the sale of $\alpha^*$ and complete pooling throughout the privatization. Then $\hat{p}_2$ solves the equation

$$(1 - \alpha^*)(V_2(\alpha^*, \hat{p}_2) - V_2(\alpha^*, 0)) = \alpha^*(1 + ga^*)\phi\tau$$

giving

$$\hat{p}_2 = \frac{\alpha^*\phi(r - g)}{(1 - \alpha^*)(1 + g)}.$$  

Let $\lambda_1$ be given by

$$\lambda_1 = \frac{\hat{p}_2 - p_1}{p_2(1 - p_1)}.$$

which is consistent with Bayes rule for values of $\hat{p}_2$ between $p_1$ and 1. Then the probability of separation before the privatization is completed is

$$\lambda^U = \begin{cases} 0 & \text{if } \lambda_1 < 0 \\ \lambda_2 & \text{if } \lambda_1 \in [0, 1] \\ 1 & \text{if } \lambda_1 > 1 \end{cases}$$

It is immediate that $\lambda^U$ is nondecreasing with $\lambda_1$, therefore all the results follow directly from the derivatives of $\lambda_1$ on the parameters:

$$\frac{\partial \lambda_1}{\partial \alpha^*} = \frac{1}{\alpha(1 - \alpha) p_2 (1 - p_1)} \frac{p_1}{p_2(1 - p_1)}$$

$$\frac{\partial \lambda_1}{\partial \phi} = \frac{p_1}{\phi p_2 (1 - p_1)}$$

$$\frac{\partial \lambda_1}{\partial \tau} = 0$$

$$\frac{\partial \lambda_1}{\partial p_1} = -\frac{1 - p_2}{p_2 (1 - p_1)}$$

$$\frac{\partial \lambda_1}{\partial g} = -\frac{(1 + \tau)p_1}{(r - g)p_2 (1 - p_1)}.$$  

Proof of Proposition 7
Consider $\hat{p}_2$ as defined in the proof of Proposition 6. Let $p_1$ be defined in a similar way, i.e. $\hat{p}_1$ is the level of reputation that leaves the uncommitted government indifferent between revelation at the date 0 or pooling on the choice of $\alpha^*$, following the optimal policy thereafter. Then let $\lambda_0$ be given by

$$\lambda_0 = \frac{\hat{p}_1 - p_0}{\hat{p}_1 (1 - p_0)}.$$  

(A.22)

Given that the optimal continuation strategy depends on the value of $\hat{p}_2$ we introduce an algorithm for the construction of $\hat{p}_1$ and $\lambda_0$. For the purpose of the algorithm let $p_0 = p_0$.  

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1. If $\hat{p}_2 < 1$ go to 2; otherwise $\lambda_1^U = 1$ and let $p_1$ be defined by
   \[
   \frac{1 + g}{r - g} (1 - \tau) + \frac{1 + g}{\rho - g} \phi \tau = U^U \left( (\alpha^*, 0, 0), (\alpha^*, 1, 1), (V_1(\alpha^*, p_1), V_2(\alpha^*, 0)) \right). \quad (A.23)
   \]

2. If $\hat{p}_2 < p_0$ go to 3; otherwise assume that the optimal choice is $\lambda_0^U \in (0, 1)$. Then we can define $\hat{p}_1$ by
   \[
   \frac{1 + g}{r - g} (1 - \tau) + \frac{1 + g}{\rho - g} \phi \tau = U^U \left( (\alpha^*, 0, 0), (\alpha^*, 0, 1), (V_1(\alpha^*, p_1), V_2(\alpha^*, \hat{p}_2)) \right)
   \]
   with
   \[
   \lambda_1 = \frac{\hat{p}_2 - \hat{p}_1}{\hat{p}_2 (1 - p_1)}.
   \]
   (a) If $\hat{p}_1 > \hat{p}_2$, go to 3.
   (b) If $\hat{p}_1 < p_0$ redefine $\lambda_1$ as
   \[
   \lambda_1 = \frac{p_2 - p_0}{p_2 (1 - p_0)}.
   \]

3. Define $\hat{p}_1$ by
   \[
   \frac{1 + g}{r - g} (1 - \tau) + \frac{1 + g}{\rho - g} \phi \tau = U^U \left( (\alpha^*, 0, 0), (\alpha^*, 0, 1), (V_1(\alpha^*, p_1), V_2(\alpha^*, \hat{p}_1)) \right).
   \quad (A.25)
   \]

The value $p_1$ constructed in this way, induces a $\lambda_0$, through (A.22) that allow us to get $\lambda_0$ by

\[
\lambda_0 = \begin{cases} 
0 & \text{if } \lambda_0 < 0 \\
\lambda_0 & \text{if } \lambda_0 \in [0, 1] \\
1 & \text{if } \lambda_0 > 1
\end{cases}
\]

Noting that $\lambda_0$ is a nondecreasing function of $\lambda_0$, the comparative static results for $\lambda_0$ can be obtained directly from comparative static for $\lambda_0$, which are similar to the analysis in Proposition 6. ■

**Proof of Proposition 8**

From equation (10) we conclude that the offer for sale price needed to induce separation when the firm is fully privatized at date 1 is $S = (1 - \tau)V_1(1, 1)$. For some parameterizations of the model, the locus $(\alpha, S)$ implicitly defined by (10) may be upward sloping. Hence the sufficient condition $\tau \leq \psi$. ■

**Proof of Proposition 9**

First note that if partial sales are sufficient to ensure separation when the committed government chooses $\alpha = \alpha^*$ to be privatized at date 1, the use of underpricing cannot help it in choices $\alpha \leq \alpha^*$. Hence we only consider the values $\alpha > \alpha^*$. When a separating equilibrium exists, the equilibrium strategies satisfy

\[
U^U \left( (\alpha^*, 0, 0), (1, 1, 1), (S_1(1|\alpha^*), S_2(1|\alpha^*)) \right) = \quad (A.26)
\]

\[
U^U \left( (\alpha^*, 0, 0), (\alpha^*, \lambda_1^U, 1), (S_1(\alpha^*|\alpha^*), S_2(\alpha^*|\alpha^*)) \right).
\]

Similarly the locus $(\alpha, S(\alpha))$ surviving the Cho-Kreps intuitive criterion, satisfies the condition that the uncommitted government is indifferent between deviating and fully privatizing the firm at date 1 and matching the date 0 decision of the committed government

\[
U^U \left( (\alpha, S(\alpha), 0, 0), (1, 1, 1), (S_1, S_2) \right) = \quad (A.27)
\]

\[
U^U \left( (\alpha, S(\alpha), 0, 0), (\alpha, S_1(\alpha) \lambda_1^U(\alpha), 1), (S_1, S_2) \right),
\]
where $\lambda_1^U(\cdot)$ is the equilibrium probability of interference at date 1 as given in the proof of Proposition 2.

The payoff in (A.26) can be written as

$$
U^C\left((\alpha^*, 0, 0), (1, 1, 1), (S_1(\alpha^*|\alpha^*), S_2(\alpha^*|\alpha^*))\right) + \frac{1 + g\alpha^*}{\rho - g} \frac{1 + g}{1 + \rho} \phi T
$$
$$
+ \frac{\alpha^*}{1 + \rho} \left(V_2(\alpha^*, 0) + (1 + g\alpha^*)\phi T - V_2(\alpha^*, 1)\right) \lambda_1^U(\alpha^*)
$$

(A.28)

and the payoff in (A.27) can be written as

$$
U^C\left((\alpha, \bar{S}(\alpha), 0, 0), (1, 1, 1), (S_1, S_2)\right) + \frac{1 + g\alpha}{\rho - g} \frac{1 + g}{1 + \rho} \phi T
$$
$$
+ \frac{\alpha}{1 + \rho} \left(V(\alpha, 0) + (1 + g\alpha)\phi T - V(\alpha, 1)\right) \lambda_1^U(\alpha)
$$

(A.29)

From (A.26) and (A.27) we conclude that expressions (A.28) and (A.29) must be equal. Because $\alpha$ in (A.29) is greater than $\alpha^*$ in (A.28), and the terms added to $U^C(\cdot)$ are increasing in $\alpha$ and $\alpha^*$ respectively, we conclude that

$$
U^C\left((\alpha^*, 0, 0), (1, 1, 1), (S_1(\alpha^*|\alpha^*), S_2(\alpha^*|\alpha^*))\right) > U^C\left((\alpha, \bar{S}(\alpha), 0, 0), (1, 1, 1), (S_1, S_2)\right)
$$

(A.30)

and the committed government should not use underpricing. ■

**Proof of Proposition 10**

The fact that underpricing will allow the committed government to achieve separation is an immediate consequence of Proposition 8. To prove that partial sales will be used in equilibrium we prove that when a separating equilibrium without underpricing does not exist, the locus $(\alpha, \bar{S})$ defined implicitly by (10) slopes upwards in a neighborhood of $\alpha = 0$. Start from equation (10), and because in a neighborhood of $\alpha = 0$, $\lambda_1^U = 0$, we can write

$$
\frac{1 + g}{r - g} (1 - \tau) = \alpha \bar{S} + \alpha \left(1 + g + \frac{(1 + g\alpha)(1 + g)}{r - g} \right) \phi T.
$$

From this expression we can compute the limit of the discount per share as $\alpha$ goes to 0.

$$
\lim_{\alpha \to 0} \left\{ \alpha \left( \frac{V_1(\alpha, 1) - \bar{S}}{V_1(\alpha, 1)} \right) \right\} = \frac{(1 + r) + (1 + g)\phi T}{1 + \rho} - (1 + g)(1 - \tau).
$$

(A.31)

The right hand side is negative, because there does not exist a separating equilibrium without underpricing. Therefore $\bar{S}$ goes to $+\infty$. Hence, by continuity, there must exist a value $\alpha^*$, bounded away from zero, such that

$$
\frac{V_1(\alpha^*, 1) - \bar{S}}{V_1(\alpha^*, 1)} = \psi.
$$

In equilibrium the committed government sells $\alpha^*$ at date 1, at price $\bar{S} = (1 - \psi)V_1(\alpha, 1)$. On the other hand, the uncommitted government fully privatizes the firm at date 1. ■
Proof of Proposition 11

Because the government's type is independent of its previous and future types, the privatization of any firm is formally analogous to the privatization of the representative firm. Hence, the equilibrium in the multiple firm model is given by the repetition of the equilibrium in the representative firm model. The possibility of having partial sales in equilibrium depends on its presence in the equilibrium of the representative firm. If partial sales appear as an equilibrium action in the representative firm model, partial sales may be seen in the privatization of any firm; otherwise, if partial sales are not an equilibrium action in the representative firm model, they will not be seen in the privatization of any firm.

Proof of Proposition 12

The proof is based on the backward resolution of the model. Consider the privatization of the last firm, firm 1. If separation has been achieved before, the first best policy, characterized in Proposition 1, can be followed; otherwise, the committed government ensures separation through partial sales by selling $\alpha_1^*$ while the uncommitted government fully privatizes firm 1. To prove that this is the equilibrium action for firm 1 we follow a process analogous to the proof of Proposition 2. First $\alpha_1^*$ must satisfy

$$(1 + g\alpha_1^*) \left( 1 - (1 - \alpha_1^*) \frac{\rho - r}{1 + \rho} \right) \geq (1 + g)(1 - \tau). \quad (A.32)$$

The left hand side of (A.32) is increasing in $\alpha_1^*$, and the inequality is strict for $\alpha_1^* = 1$. Let $\alpha_1^C$ be the value of $\alpha_1^*$ which gives equality in (A.32). Then for $\alpha_1^*$ to be an equilibrium action $\alpha_1^* \geq \alpha_1^C$.

The uncommitted government may deviate to one of two different strategies. Each requires it to privatize a share $\alpha_1^*$ at date $2N - 1$. In fact, deviating to lower $\alpha_1$ is dominated by one of the above deviations and deviating to $\alpha_1$ in the interval $(\alpha_1^*, 1)$ is dominated by the initial action $\alpha_1 = 1$; after choosing $\alpha_1 = \alpha_1^*$ it may decide to reveal its type before or after completing the privatization. It chooses to reveal its type immediately after the sale of $\alpha_1^*$ if

$$(1 - \alpha_1^*)V_2(\alpha_1^*, 1) \leq (1 - \alpha_1^*)V_2(\alpha_1^*, 0) + \left( \alpha_1^*(1 + g\alpha_1^*) + \sum_{i=2}^{N} (1 + g\alpha_i^*)(1 + g)^{2i-2} \right) \tau. \quad (A.33)$$

which implies that

$$\alpha_1^* \geq \frac{1 + g - \sum_{i=2}^{N} (1 + g\alpha_i^*) (1 + g)^{2i-2} (r - y)\phi}{\phi(r - g) + 1 + g}. \quad (A.34)$$

Let $\bar{\alpha}_1$ be the value of $\alpha_1^*$ which gives equality in (A.34). Define $\bar{\alpha}_{11}'$ as the solution to the equality

$$U^U(\sigma^*|1) = U^U \left( (\bar{\alpha}_{11}', 0, 0), (\bar{\alpha}_{11}', 0, 1), (V_1(\bar{\alpha}_{11}', 1), V_2(\bar{\alpha}_{11}', 1))|1 \right), \quad (A.35)$$

which represents the value of $\alpha_1$ giving indifference between separation at date $2N - 1$ and complete pooling, when by observing $\bar{\alpha}_{11}$ the investors believe the government is committed. Similarly, let $\bar{\alpha}_{12}'$ be the solution to the equality

$$U^U(\sigma^*|1) = U^U \left( (\bar{\alpha}_{12}', 0, 0), (\bar{\alpha}_{12}', 1, 1), (V_1(\bar{\alpha}_{12}', 1), V_2(\bar{\alpha}_{12}', 1))|1 \right) \quad \text{(A.36)}$$

representing the value of $\alpha_1$ which gives indifference between separation at date $2N - 1$ and separation at date $2N$ when by observing $\bar{\alpha}_{12}$ the investors believe the government is committed. From (A.33), (A.35) and (A.36) it follows that $\bar{\alpha}_{11}' = \bar{\alpha}_{12}'$ at $\bar{\alpha}_1$. Moreover they both lie on the same side of $\bar{\alpha}_1$. If they are greater than $\bar{\alpha}_1$, $\bar{\alpha}_{12}' < \bar{\alpha}_{11}'$; if they are lower than $\bar{\alpha}_1$, $\bar{\alpha}_{12}' > \bar{\alpha}_{11}'$. Hence, $\bar{\alpha}_1' = \min\{\bar{\alpha}_{11}', \bar{\alpha}_{12}'\}$: $\bar{\alpha}_1'$ is the upper bound for $\alpha_1^*$, since otherwise the uncommitted government matches the committed government's choice of $\alpha_1^*$. 

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Next, we prove that $\alpha_1^C < \alpha_1^U$. First we write (A.35) as

$$U^C((1, 0, 0), (1, 1, 1), (V_1(1, 0), V_2(1, 1)|1)) = \left(\sum_{i=2}^N \frac{(1 + g\alpha_i)(1 + g)^{2i-3}(1 + \rho)}{\rho - g} + \frac{1 + g}{\rho - g}\right)\phi =$$

$$U^C((\alpha_1^U, 0, 0), (\alpha_1^U, 0, 1), (V_1(\alpha_1^U, 1), V_2(\alpha_1^U, 1)|1)) = \left(\sum_{i=2}^N \frac{(1 + g\alpha_i)(1 + g)^{2i-3}(1 + \rho)}{\rho - g} + \frac{(1 + g\alpha_i^C)(1 + g)}{(1 + \rho)(\rho - g)}\right)\phi,$$

substitute $\alpha_1^U$ for $\alpha_1^C$ in the right hand side; then subtract it from the left hand side to get

$$\left(\sum_{i=2}^N \frac{(1 + g\alpha_i)(1 + g)^{2i-3}(1 + \rho)}{\rho - g} \left(1 - \frac{1 + g}{1 + \rho}\right)^2 + \frac{1 + g}{\rho - g}\left(1 - \frac{1 + g\alpha_i^C}{1 + \rho}\right)\right)\phi >$$

which is positive. Therefore $\alpha_1^U > \alpha_1^C$.

Similarly, use (A.36) and substitute $\alpha_1^U$ for $\alpha_1^C$ in the right hand side; then subtract it from the left hand side to get

$$\left(\sum_{i=2}^N \frac{(1 + g\alpha_i)(1 + g)^{2i-3}(1 + \rho)}{\rho - g} \left(1 - \frac{1 + g}{1 + \rho}\right) + \frac{g}{\rho - g} + \frac{1 + g\alpha_i^C}{1 + \rho}\right)\phi + \frac{1 - \alpha_1^C}{1 + \rho} (V_2(\alpha_1^C, 1) - V_2(\alpha_1^C, 0))$$

being clear that both terms are positive. Therefore, $\alpha_1^U > \alpha_1^C$. Hence, $\alpha_1^C < \alpha_1^U$.

Finally, in order to ensure existence of this separating action, it must be that $[\alpha_1^C, \alpha_1^U] \cap [0, 1]$ is non empty. A necessary and sufficient condition for that is $\alpha_1^U > 0$, which requires that

$$U^U(\sigma^*|1) \geq U^U((0, 0, 0), (0, 0, 1), (V_1(0, 1), V_2(0, 1)|1))$$

(A.37)

implying that

$$\tau \leq \frac{(\rho - g)}{(1 + g)} \left(\frac{(1 + \rho)(1 + g) - (1 + \rho)}{(1 + \rho)(\rho - g) - (\rho - g)\phi} + \frac{\sum_{i=2}^N (1 + g\alpha_i^C)(1 + g)^{2i+3}(2 + \rho + g)}{(1 + \rho)(\rho - g) - (\rho - g)\phi}\right).$$

(A.38)

This condition is satisfied, because of the assumption that a separating equilibrium for the privatization of the representative firm exists and because of Proposition 2.

Consequently, if separation has not been achieved before, the equilibrium strategy requires the committed government to privatize $\alpha_1^C$ of firm 1 at date $2N - 1$, while the uncommitted government fully privatizes the firm.

Moving back to date $2N - 2$, right after the privatization of firm 2, it must be that the uncommitted government interferes; no interference will gain it nothing since it will for sure be identified at date $2N - 1$ and will forego the proceeds from interference in the previous period.

Consider now the privatization of firm $i$. If separation has been achieved before, the first best policy, characterized in Proposition 1, can be followed; otherwise, if separation has not been achieved before date $2(N - 1)$, as the induction hypothesis, we assume that the continuation equilibrium strategy requires the uncommitted government to separate at date $2(N - i + 1)$. Thus, the committed government ensures
separation through partial sales, selling \( \alpha_i^* \) while the uncommitted government fully privatizes firm \( i \). To prove that this is the equilibrium action for firm \( i \) it is sufficient to follow the above steps for firm 1, to get

\[
\tau \leq \left( \frac{\rho - g}{1 + g} \right) \left( \frac{(1 + \rho)(1 + g) - (1 + r)}{(1 + \rho)(\rho - g) - (r - g)\phi (\rho + \sum_{j=1}^{N} (1 + g\alpha_j^*)(1 + g)^{2(i-j)+1}(2 + \rho + g))} \right) \quad (A.39)
\]

as the necessary and sufficient condition for separation to be an equilibrium outcome in the privatization of firm \( i \). This condition is satisfied, because of the assumption that a separating equilibrium for the privatization of the representative firm exists and because of Proposition (2). Consequently, if separation was not achieved before, the equilibrium strategy requires the committed government to privatize \( \alpha_i^* \) of firm \( i \) at date \( 2(N - i) + 1 \), while the uncommitted government fully privatizes the firm.

We can then conclude that, in equilibrium, there will be separation for firm \( N \), at date 1, with the committed government using partial sales and the uncommitted government fully privatizing the firm. Afterwards, each government fully privatizes all remaining firms.

**Proof of Proposition 13**

The proof follows from the proofs of Propositions 2–4 and 12. If separation is not possible in the privatization of the representative firm, then separation is not obtained in the privatization of firm \( N \), and partial sales may be used in equilibrium for the privatization of the first group of firms. However, if condition 11 is satisfied there will be separation at least for the last firm.

**Proof of Proposition 14**

The proposition follows directly from Propositions 4, 12 and 13.
References


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