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TRANSACTIONAL RISK, MARKET CRASHES,
AND THE ROLE OF CIRCUIT BREAKERS

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Abstract

We develop a pair of models that illustrate how imperfections in transactional mechanisms can lead to a market crash. Neither market orders nor limit orders allow traders to condition their demands on the full information set needed to achieve a Walrasian outcome. When volume shocks are sufficiently large, the deviations from Walrasian prices and allocations are large also. Properly designed and implemented, circuit breakers may help to overcome some of these informational problems, and thereby improve the market’s ability to absorb large volume shocks.
I. Introduction

Many accounts of the stock market crash of October 1987 point to market microstructure factors as playing an important role in the dramatic events of those few days. For example, Grossman and Miller (1988) write:

"...both markets had by then become highly illiquid and virtually incapable of supplying immediacy at the low cost their users had come to expect ... That illiquidity was evidenced in the spot market by (1) the virtual impossibility of executing market sell orders at the bid quoted at the time of order entry, and (2) the delays in executing and confirming trades on Monday afternoon and again, after the opening on Tuesday." (p. 632)

In our interpretation of the Brady Task Force Report, (Greenwald and Stein (1988)), we also emphasized the breakdown of market-making as one of the key characteristics of the crash. We argued that this type of breakdown can prevent the normal Walrasian adjustment mechanism from working: under more ordinary circumstances, "value buyers," perceiving attractive opportunities, would prevent prices from falling too far. However, if the market-making system is failing, value buyers may be deterred from placing orders because they view the "transactional risk" as extremely high—they are very unsure as to the prices at which their orders will be executed. We concluded that such transactional risks may justify the use of "circuit breakers." According to this view, circuit breakers are not intended to prevent a rapid fundamental Walrasian price adjustment, but rather to facilitate it, by reducing transactional risks and thereby encouraging value buyers to bring their demands to market.

This paper presents a pair of market microstructure models designed to elaborate upon and clarify the informal arguments made in our earlier paper. The models appear to fit with many of the stylized facts about the crash, and they have quite specific implications for the design and implementation of circuit breakers.
The models are closely related to the one seen in Grossman and Miller. Like Grossman and Miller, they examine how an informationless supply shock is, in two stages, disseminated to the investing public (or "value buyers") as a whole. Because the entire public is not immediately prepared to receive the shock, it must in the first stage be temporarily absorbed by a smaller group of competitive, risk-averse, market makers. Only in the second stage, after a delay, does the public buy some of the new supply from the market makers.

There is one key respect in which the models here differ from Grossman and Miller, however. Their primary focus is on the first stage of this transmission sequence, and their second stage is, by assumption, completely Walrasian in nature: value buyers and market makers effectively submit complete demand curves to a hypothetical auctioneer, who then sets a market clearing price. Hence there is no scope for transactional risk—everyone is assured of receiving a price-quantity pair that they view as optimal. In contrast, the second stage of the models here is less efficient: value buyers must submit market orders that are uncontingent on the execution price. Consequently, there is transactional risk—there is a possibility that these orders will be executed at an unattractive price level.

The main point being made here is that a large volume shock in the first stage of transmission sequence will tend to increase the transactional risk in the second stage. As a result, value buyers will be reluctant to come to market to absorb some of the shock. This in turn feeds back to the market makers. They will shy away from large volumes in the first stage, knowing that such amounts will subsequently be very difficult to unload to the public. The end result is a "microstructure-induced crash"—the outcome of the two-stage transmission sequence is very different from what would happen in a hypothetical world where the value buyers were able to meet the volume shock instantaneously in a Walrasian fashion, without using the market-making mechanism as a temporary conduit.
In the context of our models, a market crash is really nothing more than an inefficient allocation of the volume shock among the potential risk-bearers in the economy. This inefficiency can be traced to an informational imperfection—the fact that market orders cannot be conditioned on an important variable, namely the execution price.\(^2\) The primary purpose of a circuit breaker mechanism should thus be to improve the information available to market participants at the time they submit their orders.

In Sections II and III, we develop our two simple models. In doing so, we assume that traders use only market orders—limit orders are not considered. This assumption makes the analysis tractable, and is consistent with the observed tendency of many large real world "value buyers" to make little use of limit orders. In Section IV, we use our first model to shed some light on the question of why limit orders do not play a more important role in market stabilization. In Section V, we discuss the implications of the models with respect to both the stylized facts about the October 1987 crash, and to the design and implementation of circuit-breaker trading mechanisms. Section VI concludes.

II. Model #1

a) Model Description

This model features two types of traders: competitive market makers and "value buyers," each of whom has constant absolute risk aversion utility with a risk aversion coefficient of one. The market makers are "myopic maximizers"—at each point in time, they only look one period ahead
in formulating their demands. (This simplifies the mathematics without changing anything substantive.) The timing of the model is diagrammed below:

<table>
<thead>
<tr>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply shock s of risky asset hits market, is absorbed by n₁ market makers. Price = P₁ emerges. This activates a random number n₂ of value buyers, each of whom submits a market order d. Fundamentals: E₁F = 0.</td>
<td>Competition among market makers for value buyer orders leads to price = P₂. News about fundamentals: E₂F = f₂.</td>
<td>Liquidation of asset, more news about fundamentals. Anyone holding asset gets F = f₂ + f₃.</td>
</tr>
</tbody>
</table>

At time 1, the group of n₁ risk-averse market makers must absorb the entire supply shock s. This leads to a price P₁. Since P₁ conveys information about s (indeed, it is perfectly informative about s in this model) it arouses the attention of the value buyers. They submit market orders at time 1 based on the magnitude of the price decline at time 1—the lower is P₁, the larger is the order placed by each value buyer. An important feature of the model is that the number of value buyers n₂ reacting to the price signal is a random variable. There are a variety of interpretations that can be given to n₂. The most literal one is that not all value buyers are monitoring the market at all times, so that a fraction of them are "asleep" and miss the price signal at time 1. Alternatively, it may be that not all buyers are able to place their orders with equal speed, and some are unable to get an order in before time 2. (The model developed in the next section expands on this idea, allowing for two separate times at which value buyer orders can be executed.)

The uncertainty surrounding the realization of n₂ leads to
transactional risk for value buyers placing orders at time 1. The larger is the realized \( n_2 \), the more total orders are placed, and the higher is the execution price \( P_2 \). It should be pointed out that there is no assumption here that large values of \( s \) imply more uncertainty about \( n_2 \)--the distribution of \( n_2 \) is taken to be independent of \( s \). However, in reality, it is likely that a high \( s \) does increase the conditional variance of \( n_2 \). This was certainly the case in October 1987, when larger order flows led to purely mechanical problems (such as telephone and computer backups) that made it more difficult to predict what proportion of orders placed at a given time would be executed promptly. If this relationship between the magnitude of \( s \) and the variance of \( n_2 \) were incorporated into the model, the results would just be accentuated.

Time 3 is introduced into the model only as a device for tying down the equilibrium. At this time, anybody holding a unit of the risky asset gets a payoff of \( F \). News about the value of \( F \) (i.e. news about "fundamentals") arrives slowly over the course of time--\( F \) is represented as \( f_2 + f_3 \), where \( f_2 \) is learned at time 2 and \( f_3 \) is learned at time 3. These news innovations are taken to be normally distributed, with mean zero. The date 2 innovation has variance \( \sigma_f^2 \). The date 3 innovation (which may be thought of as occurring over a longer interval of time) is normalized to have variance of unity.

b) Solution for Non-Stochastic \( n_2 \)

As a benchmark, it is useful to first work through the model for the case where \( n_2 \) is non-stochastic, and takes on the fixed value \( \bar{n}_2 \). The model can be solved backwards. At time 2, the market-makers will be left holding a total supply of \( (s - \bar{n}_2d) \)--the initial supply shock, less any that has been absorbed by the value buyers. The aggregate demand function for the market makers, denoted by \( n_1 m_2 \), is given by:
Setting this demand equal to the supply that is held by the market makers leads to the price that emerges at time 2:

\( P_2 = f_2 + \frac{[\bar{n}_2 d - s]}{n_1} \)

The next question to address is the determination of the market orders \( d \) submitted by the value buyers at time 1. The demand of an individual value buyer is given by:

\( d = \frac{E_1(F-P_2)}{V_1(F-P_2)} - \frac{[s-\bar{n}_2 d]}{n_1} \)

Note that there are two potential sources of uncertainty for value buyers—not only are they uncertain about the terminal value of the asset, they are also unsure about the price \( P_2 \) at which they will buy it. In this particular case, equation (2) tells us that the only uncertainty in \( P_2 \) concerns the time 2 fundamental news \( f_2 \). Everything else can be inferred by the value buyers from looking at \( P_1 \).

Equation (3) can be solved to yield the Nash equilibrium market order made by each value buyer, predicated on the conjecture that all other value buyers are behaving the same way:

\( d^* = \frac{s}{n_1 \cdot n_2} \)

Knowing how value buyers will behave, market makers can then formulate their strategies at time 1. Plugging value buyer demand back into equation (2) gives:

\( n_1 m_2 = n_1 \frac{(E_2 F - P_2)}{V_2(F)} - n_1 (f_2 - P_2) \)
At time 1, competition among market makers for the supply shock leads to a price of:

\[ P_1 = E_2 P_2 - \frac{S V_1(P_2)}{n_1} = -S \left[ \frac{1}{n_1 + n_2} + \frac{\sigma^2}{n_1} \right] \]

In the special case of a non-stochastic \( n_2 \), the use of market orders in this framework still leads to a Walrasian outcome by time 2: the price of \( P_2 = f_2 - \frac{S}{n_1 + n_2} \), as well as the distribution of the supply shock, \( n_1 n_2 = n_1 s \), are all exactly the same as would be attained in the idealized situation where both the market makers and the value buyers were able to submit their entire demand curves to an auctioneer at time 2.\(^3\) Intuitively, this is because value buyer demands are driven by the conditional mean and variance of \((F-P_2)\). When \( n_2 \) is non-stochastic, nothing new is learned about \((F-P_2)\) at time 2. Thus a value buyer can predict exactly ahead of time the single point on his Walrasian demand curve that will be relevant, and can submit that quantity as a market order. Consequently, the Grossman-Miller result is reproduced: there is a temporary price drop at time 1, when the market makers must briefly absorb all of the supply shock by themselves, but by time 2, the transmission is complete.

\begin{itemize}
  \item[c)] **Solution when \( n_2 \) is Stochastic**
  
  Things change quite dramatically when \( n_2 \) becomes stochastic. Denote the variance of \( n_2 \) by \( \sigma^2 \). The price at time 2 will now be given by:
Now there is more uncertainty for a value buyer trying to guess at
time t the price at which an order will be executed. With $\sigma^2_n > 0$, new
information about $(F-P_2)$ is revealed at time 2--specifically, the
realization of $n_2 d$, or the total number of value buyers. Thus it is no
longer possible for someone to guess at time t the single relevant point on
their time 2 Walrasian demand curve. A trader is likely to regret the size
of his time t market order upon learning the price at which it was
executed. The magnitude of this transactional risk depends on the size of
individual orders d. If not much is happening in the market and d is
small, transactional risk will be small too. On the other hand, when there
is a large supply shock that calls for large d’s, transactional risk will
be heightened.

The market order placed by an individual buyer can be calculated as:

$$d = \frac{E_t(F-P_2)}{V_t(F-P_2)} - \frac{[s - \bar{n}_2 d]}{n_t + (\sigma^2_n d^2/n_t)}$$

A comparison of equation (8) with equation (3) highlights the effect
of randomness in $n_2$. When $\sigma^2_n = 0$ (i.e., $n_2$ is nonstochastic), the two are
identical. However, when $\sigma^2_n$ is non-zero, increased responsiveness by any
given value buyer makes transacting more risky for his peers, and tends to
make them shy away from the market. In other words, for non-zero $\sigma^2_n$,
increased responsiveness by a given value buyer imposes a negative externality on all other potential value buyers.

This negative externality prevents value buyers as a group from
responding aggressively to a large volume shock. Equation (8) can be
rearranged to give an expression for equilibrium value buyer demands:

\[(8') \frac{\sigma_n^2 (d')^3}{n_1} + (n_1 + \bar{n}_2) d' = s\]

Figure 1A illustrates the dependence of \(d'\) on \(s\), and compares this relationship to the one seen in the Walrasian case. The limiting behavior of \(d'\) for large and small volume shocks can be summarized as follows:

**Proposition 1**:

\[(9) \text{ As } s \to 0, \quad d' \to \frac{s}{n_1 + n_2}\]

\[(10) \text{ As } s \to \infty, \quad d' \to s^{\frac{1}{3}} \left[ \frac{n_1}{\sigma_n^2} \right]^{\frac{1}{3}}\]

When the volume shock is relatively small, and \(n_2\) equals its mean value of \(\bar{n}_2\), (9) states that the outcome is very close to the Walrasian one seen with a non-stochastic \(n_2\). However, when the volume shock is very large, the transmission mechanism breaks down--rather than absorbing a constant proportion of the shock, value buyers wind up taking an amount that only increases with the cube root of \(s\). The much smaller group of market makers is thus forced to swallow the remainder--an inefficient allocation of risk-bearing. The price consequences of this misallocation can be obtained by plugging the equilibrium value of \(d'\) back into equation (7), thereby determining \(P_2\) as a function of the supply shock \(s\). Figure 1B plots \(E(P_2)\) against \(s\), and again compares the relationship to the one that obtains without transactional risk. As can be seen, large volumes are associated with increased deviations of prices from "fundamental" Walrasian levels.
III. Model #2

In the above model, the relationship between volume and transactional risk is driven solely by the fact that greater volume implies more uncertainty as to the total value buyer order flow at time 2. Market liquidity—the extent to which a unit of time 2 order flow moves prices—is not affected by volume. The next model demonstrates that when not all value buyers hit the market at once, liquidity may also be impaired by large volume shocks, exacerbating the allocational inefficiencies. The timing of the model is outlined below:

<table>
<thead>
<tr>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
<th>Time 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply shock of s, absorbed by market makers, leading to P₁. This activates n_f &quot;fast&quot; value buyers who submit market orders d_f, and n_s &quot;slow&quot; value buyers who submit market orders d_s. E₁F=0.</td>
<td>Orders of fast value buyers arrive, are executed at P₂. E₂F= f₂.</td>
<td>Orders of slow value buyers arrive, are executed at P₃. E₃F= f₂+f₃.</td>
<td>Asset pays F=f₂+f₃+f₄.</td>
</tr>
</tbody>
</table>

Once again, a random number of value buyers submit market orders at time 1. But now, there is a further complication, because not all of the uncertainty concerning the number of value buyers is resolved right away—n_f of them are "fast" and submit their orders by time 2, while n_s are "slow," and don’t get their orders in until time 3. For simplicity, it is assumed that n_f and n_s are uncorrelated, with means of n_f and n_s, and a common variance of σ_n². It is also assumed that buyers know whether they are fast or slow, and therefore can submit demands conditional on this knowledge, d_f or d_s. This assumption is unrealistic—it is more likely
that the investors themselves face some uncertainty about the exact time their trades will be executed, and that this uncertainty adds further transactional risk. However, allowing value buyers to know their execution time simplifies the analysis, and by ignoring another type of risk, only makes the argument for a crash scenario excessively conservative.

At time 4, the asset is liquidated, and everyone holding it receives the fundamental value \( F \). As in the previous model, some news about \( F \) arrives in each period, so that holding the asset is always risky to the market makers. The time 2 and time 3 innovations, \( f_2 \) and \( f_3 \), both have variance \( \sigma^2 \). Once again, the variance of the final piece of news, \( f_4 \), is normalized to unity.

b) Solution

The same backward solution technique applied in the previous section can be used here. At time 3, all of the orders of \( (n_f d_f + n_s d_s) \) will have arrived, so market-maker inventory will be \([s - (n_f d_f + n_s d_s)]\). Consequently, the price will be given by:

\[
(11) \quad P_3 = f_2 + f_3 + \frac{[n_f d_f + n_s d_s - s]}{n_1}
\]

At time 2, market-maker inventory is \([s - n_f d_f]\). Thus \( P_2 \) can be determined as:

\[
(12) \quad P_2 = E_2(F_3) - \frac{[s - n_f d_f] V_2(F_3)}{n_1}
\]

In order to evaluate (12), one needs to find the conditional mean and variance of \( P_3 \) as of time 2. To do so, recall that the conditional mean and variance of \( n_s \) are given by \( \bar{n}_s \) and \( \sigma^2 \). One source of risk to market makers at time 2 is that they only have partial information at this time about the total value buyer order flow. If the number of slow orders, \( n_s \), turns out to be unexpectedly small, \( P_3 \) will be unexpectedly low.
Taking these facts into consideration leads to:

\[
(P_2 - f_2) \cdot \left( \frac{n_f d_f + n_s d_s - s}{n_1} \right) - \frac{[s - n_f d_f] [\sigma_f^2 + (\sigma_s^2 d_s^2/n_1^2)]}{n_1}
\]

The last term on the right hand side of (13) represents the risk premium built into \(P_2\) by market makers for holding the supply \([s - n_f d_f]\) at time 2. As suggested above, this risk premium depends not only on fundamental risk, \(\sigma_f^2\), but also on the remaining uncertainty about future order flow, \(\sigma_s^2\).

Now consider things from the point of view of a value buyer deciding how large an order to place at time 1. If he is a slow trader, he faces the same type of transactional risk as was seen in Model #1—the execution price \(P_3\) depends on the magnitude of the uncertain order flow \(n_f d_f + n_s d_s\).

However, if he is a fast trader, things are even worse. The price at time 2 also depends on an uncertain order flow, \(n_f d_f\). But the price at time 2 is much more sensitive to variations in the order flow than the price at time 3. This can be seen explicitly by comparing the derivatives \(dP_3/d(n_s d_s)\) and \(dP_2/d(n_f d_f)\), which can be thought of as measures of market liquidity at times 3 and 2 respectively:

\[
(14) \quad \frac{dP_3}{d(n_s d_s)} = \frac{1}{n_1}; \quad \frac{dP_2}{d(n_f d_f)} = \frac{[1 + \sigma_f^2 + (\sigma_s^2 d_s^2/n_1^2)]}{n_1}
\]

The reason that prices at time 2 are much more sensitive to variations in order flow is because market makers still face another period of order flow risk after time 2, and hence are very unwilling to smooth prices. This is a feature that was absent from Model #1, where market makers were only hit with one uncertain order flow.
Thus transactional risk at time 2 is even more pronounced than it was in Model #1. Now there are two adverse externality effects working simultaneously: 1) as before, uncertainty about \( n_f \) means that aggressive fast trader response (i.e. a high \( d_f \)) imposes risks on other fast traders; and 2) in addition, uncertainty about \( n_s \) means that aggressive slow trader response (i.e. a high \( d_s \)) makes the market "illiquid" at time 2, and thereby exacerbates the effect of \( n_fd_f \)-type risk.

These effects can be seen explicitly when the demand of fast traders is calculated:

\[
(15) \quad d_f = \frac{E_s(F-P_s)}{V_1(F-P_s)} - \frac{n_1 [(s-n_fd_f) (1+\sigma_f^2 + (\sigma_n^2 d_f^2/n_1^2)) - \bar{n}_d d_s]}{n_1^2 (1 + \sigma_f^2 + \sigma_n^2 d_f^2 (1 + \sigma_f^2 + (\sigma_n^2 d_f^2/n_1^2)))^2}
\]

Now \( d_f^2 \) and \( d_s^2 \) terms both appear in the denominator of (15), reflecting the fact that aggressive responses by either type of value buyer have a negative externality effect that blunts the demand of fast value buyers. This results in the following limiting behavior:

**Proposition 2:**

(16) As \( s \to \infty \), \( d_f^2 \to k_1 s^{\frac{1}{3}} \), where \( k_1 \) is a constant.

(17) As \( s \to \infty \), \( d_s^2 \to k_2 s^{\frac{1}{9}} \), where \( k_2 \) is a constant.

In words, the limiting behavior of slow buyers is as described in Model #1. However, the fast buyers face even more risk, and their ability
IV. Why Don’t Value Buyers Use Limit Orders?

Thus far, the formal analysis has simply assumed that value buyers are restricted to placing market orders that are uncontingent on the execution price. And it is the transactional risk inherent in these market orders that drives the results of the models. Hence a natural question to ask is this: do limit orders represent a way for value buyers to circumvent the sorts of transactional risk described above, thereby enabling them to transmit their demands to the market more forcefully?

Other authors have argued that limit orders will not be appropriate for certain types of traders. Grossman and Miller, as well as Rock (1987) point out that since limit orders carry a risk of non-execution, they will be avoided by traders with a high demand for immediacy, or by those with valuable but perishable inside information. However, neither of these papers suggests that limit orders will not be used by the sorts of value buyers in our models.

In order to motivate the use of market orders by our value buyers, we focus on a different drawback of limit orders: the fact that they leave traders exposed to innovations in fundamentals that occur between the time an order is placed and the time it is executed. Before proceeding to the formal argument, we should emphasize that we are not attempting to do violence to observed reality. Major institutional investors, (e.g., pension and mutual funds) who comprise much of the economy’s risk absorption capacity, do not appear to make much use of limit orders on a regular basis. Furthermore, we are not aware of any evidence that suggests that limit orders played a meaningful stabilizing role during the October 1987 crash. What follows is meant simply to be a theoretical rationalization of these observations.
The tradeoff between market and limit orders is easiest to see in the context of Model #1. Recall equation (7), which gives the execution price received by a trader placing a market order:

\[ P_2 - f_2 + \frac{[n_2 d - s]}{n_1} \]

Market orders are risky because \( n_2 \) is uncertain—traders do not know at the time an order is placed how many other orders will be executed along with theirs. However, market orders also have an advantage, in that \( P_2 \) impounds the time 2 news innovation \( f_2 \) perfectly. This is a direct consequence of the competition among market makers for these orders.

Limit orders face a different sort of risk. If, at time 1, a trader submits a limit order to buy at a price \( P_L \), he can be sure that no matter what the realization of \( n_2 \), he will not pay more than that price. However, his execution price may be too high \textit{relative} to time 2 fundamentals. A trader cannot replicate a time 2 Walrasian demand curve with limit orders, because the limit orders are not conditional on time 2 fundamental news. Consequently, limit orders suffer from an adverse selection problem: a buy order is more likely to be executed at the limit price \( P_L \) when bad news has arrived between time 1 and time 2. Or said differently, a limit buy order involves giving away a put option on fundamentals.\(^6\)

Limit orders are thus relatively attractive when the variance of fundamentals \( \sigma_f^2 \) is low. Conversely, market orders are preferred when there is little uncertainty regarding the number of traders arriving at any given time—i.e., when \( \sigma_n^2 \) is low. The appendix proves the following proposition:

\textbf{Proposition 3:}

For any \( P_L \), and any volume shock \( s \), there exists a value of \( \sigma_f^2 \) sufficiently large so that a limit buy order at price \( P_L \) yields negative expected profits.
The proposition formalizes the notion that limit orders become unprofitable for large values of $\sigma_t^2$. As $\sigma_t^2$ increases, so does the range of prices for which rational traders will be unwilling to leave limit orders on specialists' books. This logic may help explain the relative paucity of limit orders in day-to-day trading.

It is more difficult to understand why limit orders should not play an augmented role in a crash situation. If fundamental risk stays constant while the execution price risk associated with market orders rises, one might expect this to tip the balance in favor of using limit orders. However, there are a number of reasons to believe that fundamental risk also effectively rises during times like those seen in October 1987. Enough volume, even if it is known not to contain private information, can raise questions about such fundamentals as the adequacy of clearinghouse collateral and the stability of other financial institutions. Furthermore, a more realistic model than ours might allow for the fact that market participants do not know for sure that all the volume is informationless. Duffee (1989) presents such a model, wherein large volume signals the possibility that a lot of news has arrived. In Duffee's model, large volume shocks are associated with increased uncertainty about fundamentals. Finally, it should be noted that what matters for limit orders is fundamental risk over a given interval of time—the time between when an order is placed and when it can be removed, or reconditioned on new information. If, as was the case in October 1987, large volume causes backups and delays throughout the system, this can increase the exposure of those using limit orders.

Of course, none of this is meant to argue that limit orders play no role in accommodating large volume shocks. However, the arguments presented above do suggest that limit orders alone may not easily solve all the problems that are evident in our models with just market orders. To
the extent that limit orders are used, it is quite possible that they will be used timidly, with limit buy orders placed at levels well below current prices.

V. Implications of the Models

A. The Crash of October 1987

Taken together, the two models suggest a couple of reasons why large volume shocks may tend to increase transactional risks and possibly result in a "crash" of the normal transmission mechanism: 1) Large volume shocks lead to more uncertainty about the aggregate value buyer response at any given point in time, so that even if market-maker behavior is unchanged, an investor faces more risk with respect to the execution price of a given order, and 2) Large volume shocks can also make it more dangerous for market makers to smooth prices. This "illiquidity" effect makes prices fluctuate more with given variations in order flow, thereby exacerbating the first problem.

The models appear to accord closely with the events of Monday and Tuesday October 19 and 20, 1987. In the last hour of trading on Monday, heavy selling volume pushed prices down approximately 12%, resulting in an overall market decline of almost 23% for the day. These price levels would seem to have indicated an attractive opportunity for value buyers, and indeed, many placed market orders at the end of the day. However, the execution price for most of these orders must have come as an unpleasant surprise--because of the large number of overnight orders, the market opened up on Tuesday some 12% above Monday's close.

The market subsequently reversed direction, so that by noon it had returned to about the same level as Monday's close. (To put these reversals in perspective, note that each 12% leg is roughly comparable in magnitude to the previous record one day fall of 12.8% on October 28,
1929.) Throughout Tuesday morning, traders submitting orders virtually contemporaneously received "fills" at vastly differing prices, both on the exchanges and with over-the-counter stocks. Even more than the overall magnitude of Monday's drop, it is the reversals and the variability in transactions prices that seem impossible to reconcile with a Walrasian model of adjustment to changing fundamentals.

As investors began to realize the tremendous risks inherent in transacting, buying support evaporated. The market appeared to be in danger of collapsing completely before many corporations announced buybacks on Tuesday afternoon, thereby setting themselves up as "value buyers of last resort."

In addition to the dramatic market-wide reversals, a variety of other evidence supports a view of the crash as at least partly non-Walrasian in nature. For example, Blume, MacKinlay and Terker (1989) find that on Monday October 19, NYSE stocks in the S&P 500 index declined an average of seven percentage points more than NYSE stocks not in the index. They attribute this difference to greater sell-side imbalances for the index stocks.

B. The Role of Circuit Breakers: Timely vs. "Full Information" Order Execution

Our models have two sorts of implications for market architecture. First, the models illustrate that some level of transactional risk is present in all trading, and that this transactional risk leads to inefficient allocations and prices. Thus one important consideration in the design of an everyday trading environment should be the tradeoff between the costs and benefits of reducing transactional risk. This argument is also made by Ho, Schwartz and Whitcomb (1985) and Bronfman and Schwartz (1990). As these papers suggest, there are a number of dimensions
along which the tradeoff might take place. One example is the degree to which trading is allowed to take place continuously. Another is the extent to which trading takes place in a spatially fragmented setting, as opposed to the centralized one considered here. The fragmentation/centralization issue is in turn related to questions about automation: To what degree should automated trading systems be used to aggregate data across spatially separated markets, and to enable traders to condition their orders on better information?

The second important feature of our models is that the magnitude of transactional risk can increase dramatically when the market is confronted with abnormally large volume. This implies that a trading mechanism that is "optimal" under everyday circumstances may no longer be well-suited for handling extremely large volumes. A system of continuous trading with market and limit orders may represent a good balance between transactional risk and other desirable characteristics (e.g., immediacy) on a day-in, day-out basis, but it can nonetheless involve too much transactional risk when volume is very high. At such times, it may make sense to switch to an alternative, "circuit breaker" trading mechanism.

The basic purpose of any circuit breaker should be to reduce transactional risk, in an effort to stimulate value buyer responsiveness. In essence, the trading halt should be designed as a time during which something much closer to the hypothetical full-information Walrasian process occurs. Thus it is inappropriate to think of circuit breakers as "sand in the gears" that aims to slow down a fundamental price adjustment; rather, their aim is to patch up the transmission apparatus so that Walrasian price adjustment can be facilitated.

According to Model #1, one goal of a circuit breaker system should be to help make potential value buyers aware of the response of other traders to large shocks. On the exchanges, this could be accomplished by having
specialists halt trading briefly and open their order books for general inspection.

Model #2 further suggests that sequential execution of trades at rapidly oscillating prices may not be the best way for the market to digest a large volume shock. It may instead be preferable to "pool" trades—that is, have an open period during which orders could be submitted, with a promise that all orders submitted during that period would be executed at the same price.

These types of alternative trading mechanisms are, of course, not wholly unknown to stock markets. On the NYSE, a specialist can delay the opening of his stock if he is faced with a large order imbalance. During the trading halt, he may announce trial clearing prices in an effort to attract additional buy or sell orders. Some European stock markets use periodic "batch" auctions to clear accumulated orders. In both cases, the net result is to diminish the importance of the market makers as a transmission device, and to shift the emphasis to direct trade between end buyers and sellers.

The contrast between these alternative mechanisms and the continuous trading typically seen in U.S. equity markets illustrates a fundamental tradeoff: that between timely execution and full information execution. Under normal circumstances, the informational loss due to using uncontingent market orders is probably relatively small: traders can anticipate with a good deal of precision their execution prices, simply by looking at the prices that prevail when their order is submitted. Thus there may be little to be gained from using an alternative trading mechanism. However, under extreme circumstances, the informational problems associated with continuous trading are much greater: current prices no longer provide very accurate information about subsequent execution prices. When this is the case, it may be worthwhile to accept
some loss in timeliness in order to allow each trader to condition his responses on the responses of others.  

A number of criticisms have been raised concerning circuit breakers. One is the "free trade" objection: traders enter the market voluntarily, so how can shutting it down possibly make anybody better off? Our emphasis on the negative externalities present in a market crash provides an answer to this question. Even though rational agents will continue to trade in the absence of a circuit breaker, the aggregate outcome is an inferior one. Intervention that leads to a broader distribution of the volume shock can improve efficiency.

Another objection is that the very existence of circuit breakers might somehow cause large declines to feed on themselves. According to this "gravitational", or "magnet" theory, if it is known that the market will be shut down after, say, a 10% drop, an initial fall can lead to further selling pressure, as investors try to get out before the shutdown leaves them in a temporarily illiquid position. Empirically, however, there is not much support for this theory. Kuserk et al (1989) study transactions data for Treasury bond and commodity futures markets where price limits were in effect, and find no significant evidence of gravitational effects.

Finally, some observers have suggested that rather than adopting circuit breakers, the exchanges should force market makers to dramatically increase their capital positions. While this would have no direct effect on the magnitude of transactional risk, it would presumably lessen its impact by reducing the need to quickly mobilize value buyers to absorb large shocks. However, such an approach may be very inefficient. As Grossman and Miller (1988) point out, there is an opportunity cost to devoting capital to market-making activities, and the appropriate amount of such capital depends (loosely speaking) on average demands for immediacy. It
makes no sense for market makers to set aside enough capital to easily absorb the very largest volume shocks, if such capital is unnecessary 99% of the time. Circuit breakers have the advantage of only coming into play when there are extremely large shocks, without imposing any cost at other times.

The Brady report advocated the installation of circuit breakers, although it did not go into detail about their design. In May of 1988, the Working Group on Financial Markets (composed of officials from the Treasury, the Federal Reserve, the CFTC, and the SEC) also endorsed the adoption of circuit breakers in their interim report to the President. The Working Group report is more specific than the Brady Report. It recommends coordinated, cross-market trading halts after Dow Jones declines of 250 and 400 points. Consistent with the logic outlined above, the bulk of the Working Group's discussion centers on procedures for active dissemination of information prior to reopening: Order imbalances for all major stocks are to be publicized. Tentative quote indications are then to be provided. Further orders can then be submitted, and subsequent quote indications will be made available as needed. (Appendix A, page 2) The Working Group's recommendations were subsequently adopted, and the circuit breakers described above remain in place today. Circuit breakers were also recently endorsed by the NYSE's Market Volatility and Investor Confidence Panel (1990). Like the Working Group report, the NYSE study also emphasizes the importance of information dissemination during trading halts.
VI. Conclusions

A system of continuous trading involving market and limit orders sacrifices some degree of informational efficiency for timeliness. Under ordinary circumstances, the informational loss is small--prices evolve gradually, and traders can thus predict with confidence the levels at which their orders will be executed. However, when volume shocks are large, the informational problem worsens and the deviation from Walrasian efficiency is magnified. When this occurs, it may make sense to temporarily switch to an alternative transactional mechanism--a "circuit breaker"--that allows orders to be made contingent on a larger information set, even if this switch compromises the ability of the market to provide immediacy to all participants.
Appendix

Proof of Proposition 3:

The proposition is proven in the context of Model #1. A similar argument applies for Model #2. Assuming that all other traders use market orders, an individual trader faces a time 2 pricing function of:

\[(A.1) \quad P_2 = f_2 + \frac{[n_2d - s]}{n_1}\]

This is just equation (7) in the text. Using the results of Proposition 1, this can be rewritten as:

\[(A.2) \quad P_2 = f_2 + \alpha n_2 - \beta,\]

where \(\alpha = \alpha(s)\) and \(\beta = \beta(s)\) are both increasing functions of the volume shock \(s\). The expected return to a limit order at price \(P_L\), denoted by \(R_L\), can be expressed as:

\[(A.3) \quad R_L = E(f_2 - P_L | P_L \geq P_2)\]
That is, a limit buyer has his order executed at \( P_L \) if and only if \( P_L \geq P_2 \). This happens when \( f_2 \leq P_L + \beta - \alpha n_2 \). Thus \( R_L \) can be written as the following double integral:

\[
R_L = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [f_2 - P_L] h(f_2) df_2 g(n_2) dn_2,
\]

where \( h(\cdot) \) and \( g(\cdot) \) are the density functions of \( f_2 \) and \( n_2 \) respectively. This integral can be broken into two parts:

\[
R_L = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [f_2 - P_L] h(f_2) df_2 g(n_2) dn_2 - P_L \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [f_2] h(f_2) df_2 g(n_2) dn_2.
\]

The first term is negative and decreases without bound as \( \sigma_f^2 \) is increased. The second term is the limit price times the probability that the order will be executed. Since this probability cannot exceed unity, the second term must be smaller in absolute magnitude than \( P_L \). Thus for a fixed \( P_L \), as \( \sigma_f^2 \) is increased, the first term will eventually dominate the second, ensuring that \( R_L \) is negative. This verifies the proposition.
1. Limit orders are discussed in Section IV of the paper.

2. Ho, Whitcomb and Schwartz (1985) and Bronfmen and Schwartz (1990), using models quite different from the one presented here, also demonstrate that transactional risk can affect traders' order placement decisions and lead to inefficient outcomes. A key distinction between our work and these others is that we focus on the relationship between large volumes and transactional risk, rather than simply on the existence of transactional risk per se in everyday trading.

3. Note that since all traders (including market makers) are identical, efficient risk sharing involves $m_2 = d$.

4. One piece of evidence concerning the relative unimportance of limit orders comes from the brokerage commissions of NYSE specialists. A specialist acts as a broker when other traders leave limit orders in the specialist's book that are subsequently executed. In 1986, specialists participated as commission earning brokers in only 12.7 percent of twice total NYSE volume (the sum of all purchases and sales.) See the Report of the Presidential Task Force on Market Mechanisms (1988), (page VI-5).

5. Given the large price declines that occurred in the last hour of trading on Monday October 19, and the apparent consensus that much of these last-hour drops were due simply to an exhaustion of market-maker capital (as opposed to news about fundamentals) one might have expected that a huge flow of limit orders would have been submitted overnight, thereby stabilizing prices at or above Monday closing levels. In actuality, this does not appear to have occurred. The Task Force report notes that "the vast majority of orders to buy at the market's open were market orders..." (page III-22). This observation is consistent with the price patterns seen on many stocks. A typical example is provided by the NYSE stock denoted "number 11" in the report (page VI-38). That stock fell 9% in the last hour of trading on Monday. It recovered this entire loss at the open on Tuesday morning, opening up almost 11%. However, there was not enough buying support at this level to sustain the price, and it fell almost 24% by 11:30 a.m., to a point roughly 13% below Monday's close. Evidently, there were not enough limit orders on the specialist's books to stabilize the price anywhere near Monday's close.

6. This assumes that when the market price falls, outstanding limit buy orders are executed at their limit prices, rather then at the more favorable new market price. Although this assumption is generally true, there is an important exception. NYSE Rule 127 provides some protection for limit orders that are executed as part of a block transaction crossed on the floor of the exchange. The Task Force Report describes the rule as follows (page VI-10):

The rule requires that unless (i) the trade is to be executed at a price no more than one eighth below the bid or one eighth above the offer, and (ii) both sides of the cross consist solely of public customers, then the member with the block cannot execute part of it by selling to or buying from the specialist's book at limit prices away from the cross price. For instance, if the stock price is
currently bid at 20 and the firm intends to cross a block of stock at $19^{1/2}$ and limit orders to buy are on the specialist's book at $19^{7/8}$, $19^{3/4}$ and $19^{5/8}$, the firm intending to cross the block cannot execute part of the order by selling stock to the specialist's book at prices from 20 down to $19^{5/8}$. Thus, the person with a limit order on the book at or near the market cannot suffer an immediate paper loss, as he would if his order was executed as part of a series of transactions immediately preceding a cross occurring at a price away from the market. The person with the order on the book will benefit by generally receiving an execution at the cross price.

7. Thus an immediate corollary of the proposition is that in a world where value buyers use only limit orders, volume shocks can generate large deviations from Walrasian prices, provided $\sigma^2$ is big enough.

8. Bronfman and Schwartz contrast the transactional risk properties of call vs. continuous markets.

9. See Cohen, Maier, Schwartz and Whitcomb (1986) for detailed treatment of these questions.

1 Because circuit breakers do involve some cost in terms of timeliness, their use is justified only to the extent that the extra time really does enable transactional risk to be reduced in a way that cannot be accomplished in a continuous trading setting. Some observers have reacted to the events of October 1987 and October 1989 by suggesting improvement that would make the continuous trading system work in a moreWalrasian fashion. Miller's (1990) proposal for "contingent limit orders" can be thought of in this spirit. To the extent that such proposals can be successful in reducing transactional risk without a halt in trading, the case for circuit breakers is weakened.

11. See however, McMillan (1990) who argues that a gravitational effect was at work on the S&P 500 futures market prior to a trading halt during the October 13, 1989 "minicrash".

12. These 250 and 400 point thresholds have not yet been reached. However, there are other smaller "circuit breaker" type mechanisms in place that have come into play. More modest declines have lead to temporary shutdowns of futures trading on the Chicago Mercantile Exchange, as well as to the diversion of program trades in S&P stocks to "sidecar" files. It is not clear that the latter technique meets any of the criteria for an effective circuit breaker discussed above.
References


Market Volatility and Investor Confidence Panel, report to the Board of Directors of the New York Stock Exchange, June 1990.


Figure 1A

Value Buyer Demand as a Function of Supply Shock

--- Walrasian case
--- Transactional risk case
Figure 1B
Prices as a Function of Supply Shock

--- Wairasian case
--- Transactional risk case