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TRADING COSTS, LIQUIDITY, AND ASSET HOLDINGS

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1. **Introduction:**

In the last few years there has been a big surge in trading activity in financial markets across the world and this trend is likely to continue as the financial marketplace gets more globalized. As trading activity increases and as more and more investors are attracted to these markets, the liquidity and trading costs in such markets become issues of great importance. The objective of this paper is to examine the factors that affect the liquidity of asset markets and the costs of trading in them. An attempt is made to answer questions such as: why are some assets more liquid than others; when faced with liquidity needs or excess liquidity, how does a trader decide on the amounts of the various assets to be traded; and what role does informed trading play in influencing the cost of trading and trading volume in a security?

The prior research in this area has taken essentially two approaches. Under the first approach, following Demsetz's seminal work, researchers have modeled the cost of trading in an asset market as arising due to the inventory costs of dealers or specialists.¹ This paper is based on the second approach, which, following Bagehot (1971), views trading costs as an informational phenomenon.² Under this approach, trading costs arise as a result of the adverse selection problem that the market maker faces in trading with those who have better information than him. Trading cost is thus modeled as the tradeoff for the market maker between the losses to the informed traders versus the gains from the liquidity traders, who are

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¹See, for example, Garman (1976), Stoll (1978), Amihud and Mendelson (1980), and Ho and Stoll (1981).

²Examples of this approach are Copeland and Galai (1983), Glosten and Milgrom (1985), Kyle (1985), and Admati and Pfleiderer (1988).
willing to pay a price for the liquidity service. This approach is able to explain the existence of trading costs even when market makers are risk-neutral and act competitively and when all their transactions costs, both fixed and variable, are zero.

The major contribution of this paper is that it develops a multi-asset model which allows for interactions in trading costs arising amongst the various asset markets. These interactions have thus far been ignored in the prior research which has focused only on single-asset models. The basic idea why these interactions may be important is as follows. When trading costs can vary across assets, liquidity traders are likely to exercise discretion in deciding on how much to trade in the various assets. Naturally such traders would like to trade larger amounts in asset markets with more "depth" (the size of an order flow needed to move the price by a given amount). However, more liquidity trading in an asset may attract more informed trading which adversely affects the cost of trading in that asset.³ Thus, when (at least some) liquidity traders have discretion to allocate their liquidity needs across assets, the characterization of equilibrium is not clear-cut and it is not obvious if results derived from single-asset models that ignore interactions amongst the various asset markets are likely to hold. An interesting question is to examine what factors affect the depths (liquidity) of the various asset markets. Another item of interest is to see what kind of asset holdings by the liquidity traders emerge in equilibrium as a result of their optimizing behavior.

We consider a single period economy with many risky assets. There are three types of traders in this economy: noise traders, who have to trade a

³See also Admati and Pfleiderer (1988).
given number of shares in a particular asset, liquidity traders, each of whom wants to minimize his total cost of trading by allocating optimally his liquidity demands across the various assets, and a monopolistic informed trader in each asset, who collects information about the asset and chooses the quantity traded to maximize his trading profits. Our reasons for assuming three classes of traders are following. Some traders may have little or no discretion in selecting an asset for trading; they may, for example, be short or long on some particular risks and may thus be forced to trade in a particular asset. We model such trading in an asset as attributable to the actions of the noise traders and their trading behavior is treated as exogenous. A liquidity trader, often, however, has discretion about which assets to trade in, and we model such trading through the actions of the liquidity traders. Finally, there will be some trading in these markets by those who invest in information collection. Liquidity and noise traders provide a camouflage to the informed traders and depending on the level of the liquidity and noise trading in an asset, generally a certain level of informed trading can also be supported in that asset. We model the informed trading in each asset as trading done by a monopolistic informed trader in that asset. The market makers in the various asset markets are assumed to make zero expected profits.

Our model is based on Kyle (1985), and is similar in spirit to both Admati and Pfleiderer (1988), and Foster and Viswanathan (1987). In the latter two studies the focus is on examining intra- or inter-day variations in trading costs and trading volume; thus discretionary liquidity trading is analyzed, but in the context of trading in a single asset over time. In a multi-asset securities market, the optimizing behavior of liquidity traders is likely to lead to important interactions amongst the various asset
markets. We focus here on studying those interactions and allow the liquidity traders the discretion of allocating their liquidity needs across assets rather than over time.

The model implies that noise trading is an important determinant of the liquidity of asset markets. Informed traders' profits result from the losses sustained by the liquidity and noise traders. If there are no noise traders in any asset, then the model implies that all asset markets except the one with the lowest value of private information will shut down. An informed trader's profits are positively related to the amount of information he has. Thus the liquidity traders can reduce their losses to the informed by shifting their trading into the asset in which the informed trader has the least amount of private information from which he can profit. The net result is that they concentrate all their trading in that asset.⁴

In general, there are likely to be noise traders in most assets. The presence of noise traders in other assets provides the incentives for liquidity traders to move some of their trading into these other assets also. This is because the informed traders in these other markets make some of their profits off the noise traders also, implying that liquidity traders have to pay less for trading in these markets than if there were no noise traders. Thus the additional losses due to higher value of private information in an asset are traded off against the sharing of losses by the noise traders in that asset. In other words, the presence of noise traders in an asset alleviates the adverse selection problem and thus provides the incentives for liquidity traders to trade in such assets also. If there is no noise trading in an asset, then the market breaks down in that asset.

⁴This result is similar to the concentrated trading patterns results that arise in Admati and Pfleiderer (1988).
The paper also provides a motivation other than risk reduction for diversification by investors. In our model all traders are risk-neutral. Hence risk reduction is not a motive for diversification. Still in our model the liquidity traders generally end up with diversified asset holdings. This occurs because liquidity traders want to minimize their trading costs which is best achieved by diversifying their trading across assets. Thus we have a positive theory for diversification even in the face of risk-neutrality.

The plan of the rest of the paper is as follows. In section 2 we discuss the model and its implications for the case when there is no noise trading. In section 3 we consider the case of noise trading. In section 4 we present a brief discussion followed by some concluding remarks in section 5.

2. The Basic Model:

We consider an economy in which M risky assets are traded. There is only one period in the economy and the end-of-period payoff on a share of the risky asset $i$ is denoted $\tilde{v}_i$. The random variable $\tilde{v}_i$ is assumed to be normally distributed with an expected value of 1 and variance $\sigma_i^2$. We also assume that all $\tilde{v}_i$'s are independent of each other. We first consider the case of no noise trading in any asset.

All the liquidity traders and the informed traders in this economy are assumed to be risk-neutral. We also assume that the market makers in the various assets make zero expected profits, i.e., market making is a competitive activity in each asset. Thus the market maker in asset $i$ sets a

\[ \text{If an asset's total expected payoff is } K \text{ then dividing this asset's total supply into } K \text{ shares implies that the expected payoff on each share of this asset is } 1. \]
price which equals the conditional expected value of the asset \( i \) given his information, which is the order flow in asset \( i \). The insider in asset \( i \) observes the true value of \( \tilde{v}_i \) and trades the quantity \( \tilde{X}_i \). The insider in asset \( i \) only observes \( \tilde{v}_i \) and thus \( \tilde{X}_i \) is only a function of \( \tilde{v}_i \).

There are \( T \) liquidity traders in the economy. The \( k \)-th such trader wants to trade \( y^k \) shares, which he wants to allocate across the various assets. The random variables \( y^k \)'s are normally and independently distributed of each other with mean zero and variance \( \Delta \) so that \( \tilde{Y} = \sum_{k=1}^{T} y^k \), the total liquidity demand in all assets, is distributed with mean zero and variance \( T \Delta \). We assume that the number of liquidity traders \( T \) is large.

Each of the liquidity traders wants to minimize his expected cost of trading and thus solves the problem of optimally choosing the fractions of the various assets to be traded. We assume that the random variables \( \tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_M, \tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_T \) are multivariate normal. Denote by \( \bar{\omega}_i \) the total order flow \( \tilde{X}_i + \tilde{Y}_i \) to the market maker in asset \( i \) where \( \tilde{Y}_i \) is the total quantity of the asset \( i \) traded by the liquidity traders.

Following Kyle (1985) and Admati and Pfleiderer (1988) we assume that the pricing strategies of the market makers and the order submission strategies of the informed traders are linear. Let \( D_i \) be the depth of the market in asset \( i \) in equilibrium, i.e., the order flow necessary to move the price by one dollar. For all the asset markets, lemma 1 (proved in appendix) provides the restriction that is imposed on \( D_i \) through the equilibrium behavior of the market maker and the informed traders.

\[ \text{Similar results obtain if we relax the assumption that the insider observes } \tilde{v}_i \text{ perfectly.} \]
Lemma 1: Consider the asset \( i \). In equilibrium, let \( \tilde{P}_i \) be the price of this asset set by the market maker and \( \tilde{X}_i \) be the order of the informed trader for it. Then

\[
\tilde{X}_i = D_i [\tilde{\nu}_i - E(\tilde{\nu}_i)] / 2
\]

and

\[
\tilde{P}_i = E(\tilde{\nu}_i) + \tilde{\omega}_i / D_i
\]

where \( D_i \) satisfies:

\[
D_i = (2/\sigma_i) \left[ \text{var}(\tilde{Y}_i) \right]^{1/2}.
\]

In equation (3) the quantity \( \text{var}(\tilde{Y}_i) \), is endogenous since it depends on the behavior of the liquidity traders and in particular on their choice of the fractions of their liquidity needs to be allocated to various assets. These fractions, in turn, depend on the depths of the various markets since the depth of a market affects the cost of trading in that market. The expected cost of trading in a given asset for a liquidity trader can be defined as the number of shares of this asset traded by him times the expected difference between the value of the asset and the price that he pays for it. The liquidity traders allocate their demands across the various assets to minimize their expected total cost of trading. With no noise trading, it turns out that the only equilibrium is for all trading to take place in the asset with the least return variability. Theorem 1 below characterizes this result.

Theorem 1: The liquidity traders do all their trading in the asset with the lowest return variability. There is a breakdown of market in all other assets.
Proof: Consider the liquidity trader $k$. Let $\tilde{y}_i^k$ denote the amount of asset $i$ traded by him. Then his expected cost of trading in asset $i$ is given by:

$$EC_i = E\{ (\tilde{P}_i(\tilde{\omega}_i) - \tilde{\nu}_i) \tilde{y}_i^k | y_i^k \}. \quad (4)$$

Substituting $E(\tilde{\nu}_i) + \tilde{\omega}_i/D_i$ for $\tilde{P}_i(\tilde{\omega}_i)$ from equation (2) and computing the expected value,

$$EC_i = (y_i^k)^2/D_i. \quad (5)$$

so that the marginal cost $MC_i$ (i.e., the expected cost of trading one more share) for asset $i$ is:

$$MC_i = 2 y_i^k / D_i. \quad (6)$$

Each trader will trade in such a fashion so as to equate the marginal cost of trading across all assets or if it is lowest in any asset for all values of $y_i^k$ then do all his trading in that asset.

Denote by $\alpha_i^k$ the fraction of his demand allocated to asset $i$ by trader $k$, i.e., $\alpha_i^k = y_i^k / y^k$. Assume first that all liquidity traders choose the same fraction of their demands to be allocated to asset $i$, i.e., $\alpha_i^k$ does not depend on trader $k$'s identity. Then dropping the superscript $k$ from $\alpha_i^k$, we have:

$$\text{var}(\bar{Y}_i) = \alpha_i^2 T \Delta$$

so that

$$D_i = (2/\alpha_i) \alpha_i (T \Delta)^{1/2} \quad (7)$$

and

$$\text{var}(\bar{Y}_i) = \alpha_i^2 T \Delta$$
\[ MC_i = 2 \alpha_i \frac{y^k}{(2/\sigma_i \alpha_i (T\Delta)^{1/2})} \]

or

\[ MC_i = y^k \sigma_i / (T\Delta)^{1/2}. \tag{8} \]

This implies that \( MC_i \) is proportional to \( \sigma_i \). Thus the marginal trading cost is lowest in the asset with the lowest \( \sigma_i \). Hence the solution is to choose \( \alpha_i = 1 \) for the smallest value of \( \sigma_i \) and \( \alpha_i = 0 \) for all other \( i \). The quantities \( \alpha_i = 0 \) for all other \( i \) implies that no trading takes place in any other asset. Hence all liquidity trading will take place in asset \( i \).

The only remaining question is to check if any liquidity trader has an incentive to deviate from this equilibrium. Assume that trader \( k \) chooses to deviate, i.e., \( \alpha_i^k \neq \alpha_i \). Denote by \( i \) the asset with the lowest \( \sigma_i \).

Consider asset \( j \neq i \) in which he trades the fraction \( \alpha_j^k \). Then \( \text{var}(\bar{Y}_j) = (\alpha_j^k)^2 \Delta \) since he is the only one trading in this asset. Thus

\[ MC_j = 2 \frac{y_j^k}{D_j} = 2 \alpha_j^k \frac{y^k}{(2/\sigma_j \alpha_j^k \Delta^{1/2})} = y^k \sigma_j / \Delta^{1/2}. \]

Since \( T \) is large, trader \( k \)'s deviation does not affect the depth of the market in asset \( i \), i.e., \( MC_i \) can still be obtained from equation (7) so that

\[ MC_i = y^k \sigma_i / (T\Delta)^{1/2}. \]

Since \( T \) is large (and hence > 1) and \( \sigma_j > \sigma_i \), therefore, \( MC_j > MC_i \). Hence it does not pay for any liquidity trader to try to shift some of his trading into other assets.

The quantities \( \alpha_i = 1 \) for the smallest value of \( \sigma_i \) and \( \alpha_i = 0 \) for all other \( i \) imply that all liquidity trading takes place in the asset with the lowest \( \sigma_i \) and no trading takes place in any other asset. This completes the proof.

Equation (5) shows that the expected cost of trading in an asset rises as the square of the amount traded in that asset if the depth \( D_i \) were to
stay the same. If the market maker did not adjust the price to the order flow and charged a constant proportional spread then the expected cost of trading should rise linearly with the amount traded or the marginal cost $MC_i$ will be independent of the amount traded. However, a larger order to the market maker results in his charging a more adverse price and in fact the spread that he charges is proportional to the order flow [see equation (2)] so that the marginal cost $MC_i$ is proportional to the amount traded [equation (6)] if the depth $D_i$ were to stay the same. But the depth $D_i$ also changes when all traders decide to allocate more of their trading to an asset. A higher fraction of the liquidity demands allocated to an asset implies a higher variance of liquidity trading in that asset, thus reducing the adverse selection problem that the market maker faces. Hence he charges a less adverse price [see equation (3)]. $D_i$ rises (linearly) with $\sigma_i$ thus reducing the cost of trading. The overall effect is that the marginal cost of trading is lower in an asset with a lower $\sigma_i$.

These results are intuitive. Liquidity traders want to trade so as to minimize their losses to the informed traders. The informed traders' profits in an asset are an increasing function of the amount of information they can collect about it, which is proxied by $\sigma_i^2$, the return variability of the asset since we assume that the informed traders are perfectly informed about the risky assets' payoffs. In the limit, if $\sigma_i^2$ were to go to zero for an asset, i.e., one of the assets were to become riskless, or there was no information being collected about one asset, then all liquidity trading would take place only in that asset since liquidity traders' losses to the informed would be zero if they do all their trading in that asset. If there is no asset with $\sigma_i^2$ equal to zero then the informed traders' profits are the lowest in the asset with the lowest $\sigma_i^2$ which implies that
all liquidity trading takes place in this asset. Thus in the absence of noise trading, all liquidity trading will be concentrated in the asset with the lowest value of private information.

So far we have assumed that there is no noise trading in any asset. Are the results affected if there is some noise trading in some assets? We address this question next.

3. The Existence of Noise Traders:

We define noise trading as the trading done by traders who have inelastic demands. That is, a noise trader has to trade a given number of shares in a given asset during this period inelastically.\(^7\) We do not address the motives of such traders in any great detail here. Suffice it to say that such trading behavior can result if a trader is long or short on a particular risk and hence has to trade a given amount in a short span of time to cover the risk. Denote by \(\tilde{Z}_i\) the total quantity traded by noise traders in asset \(i\). We assume that \(\tilde{Z}_i\) is normally distributed with mean zero and variance \(\Gamma_i\). The rest of the set up of the model is the same as in section 2. The total order flow \(\tilde{\omega}_i\) to the market maker in asset \(i\) then becomes \(\tilde{X}_i + \tilde{Y}_i + \tilde{Z}_i\).

Equations (1) and (2) still represent respectively the trading strategy and the price setting strategy of the informed trader and the market maker in asset \(i\). However, since there is additional uninformed trading, equation (3) that gives the expression for depth \(D_i\), gets modified to:

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\(^7\)The assumption of inelastic demands may appear unrealistic at first. However, risk or other considerations may result in a noise trader’s demands being inelastic at least in a small price range and as long as the price stays in that range, this assumption is justified.
The expression in square brackets in equation (9) represents the variance of the total uninformed demands, i.e., the sum of the variances of the noise traders' demands and the liquidity traders' demands. As in section 2, the liquidity traders allocate their demands across the various assets to minimize their expected total cost of trading.

Equation (5) and (6) still give the expressions for \( \text{EC}_i \) and \( \text{MC}_i \), the expected and marginal cost of trading in asset \( i \) respectively. Assume again that \( \alpha_i^k \) is independent of the trader \( k \)'s identity so that superscript \( k \) can be dropped from it. Then from (9),

\[
D_i = \left( \frac{2}{\sigma_i} \right) \left[ \frac{\Gamma_i + \text{var}(\bar{Y}_i)}{\alpha_i^2 T \Delta} \right]^{1/2}.
\]

Substituting for \( D_i \) from this expression into equation (6),

\[
\text{MC}_i = \alpha_i \frac{y^k}{\sigma_i} \left[ \frac{\Gamma_i + \alpha_i^2 T \Delta}{\alpha_i^2 T \Delta} \right]^{1/2}.
\]

The fractions \( \sigma_i \)'s are chosen so as to equate the marginal cost across all assets or if the marginal cost is lowest in any asset for all values of \( \alpha_i \), then all liquidity trading is done in that asset.

Equation (10) shows that unlike section 2, there are no trivial solutions to this problem. In section 2, the marginal cost was lowest in the asset with the lowest \( \sigma_i^2 \) for all values of \( \alpha_i \) but such is not the case here. There will be non-zero liquidity trading in all assets with non-zero levels of noise trading. Assets with no noise trading in general will also have no liquidity trading and consequently markets will break down for such assets. These results are summarized in theorem 2 below:

**Theorem 2:** There is a unique equilibrium characterized by the following:
(a) In general, there is no trading in those assets which have no noise trading in them (i.e., assets with \( \Gamma_i = 0 \)). In other words, there is a market breakdown in such assets.

(b) For all assets with non-zero levels of noise trading, in equilibrium there would be non-zero levels of liquidity trading also in such assets.

Proof: For all assets which have no noise trading in them, using equation (10), the marginal cost \( MC_i \) equals \( \sigma_i \frac{y^k}{(T\Delta)^{1/2}} \). Hence for these assets, \( MC_i \) is not a function of \( \sigma_i \), which clearly implies no trading in all such assets except the one with the lowest \( \sigma_i \) (call it asset 1). A positive level of trading in any of these assets other than asset 1 implies lower trading cost can be achieved by shifting trading from other assets into asset 1. Thus markets break down in all assets with zero levels of noise trading but possibly not in asset 1.

Now consider the assets with positive levels of noise trading. Equation (10) then implies that \( MC_i \) for all such assets is an increasing and concave function of \( \sigma_i \), being equal to zero at \( \alpha_i = 0 \) and its maximum value is equal to \( \frac{y^k \sigma_i}{(\Gamma_i + T\Delta)^{1/2}} \) which is achieved at \( \alpha_i = 1 \). Denote by asset 2 the asset with the lowest \( \sigma_i \) in this category. Then if \( \sigma_1 > \sigma_2 \), it is easily seen that \( MC_1 > \max (MC_2) \). Since the equilibrium marginal cost has to be less than or equal to \( \max (MC_2) \), no liquidity trading will ever occur in asset 1. This is equivalent to a market break down in asset 1 also. Furthermore, since \( MC_i \) is an increasing and concave function of \( \alpha_i \) with a lower bound of zero for all assets in this category, and since liquidity traders are trading so as to equate marginal costs across assets, \(^8\) it

\(^8\)Or else, they want to be at a corner, i.e., do all trading in the asset with the lowest MC if such an asset were there but that is not the case here.
Immediately follows that in equilibrium, \( \alpha_i \neq 0 \) for any of these assets. Thus there are positive levels of liquidity trading in all such assets.

If \( \sigma_2 > \sigma_1 \), then it is possible the that market does not break down in asset 1. This can happen if \( MC_1 \) is very low. To make this notion precise, let \( \alpha_i^* \) be the fraction for asset \( i \) at which the marginal cost \( MC_i \) equals \( MC_1 \). If these fractions \( \alpha_i^* \)'s sum to less than 1 then equilibrium marginal cost will equal \( MC_1 \) and the liquidity traders will trade fraction \( \alpha_i^* \) in asset \( i \) and \( \alpha_i^* \) will be non-zero for all assets in this category. The remaining fraction \( (1 - \Sigma \alpha_i^*) \) will be traded in asset 1. In this case, the market in asset 1 will not break down as well as there will be positive levels of liquidity trading in all assets with \( \Gamma_i \neq 0 \). If these \( \alpha_i^* \)'s sum to greater than 1, then the equilibrium \( MC \) will be less than \( MC_1 \) and market in asset 1 will break down and there will be positive levels of liquidity trading in all assets with positive \( \Gamma_i \).

Finally, there is no incentive for any liquidity trader to deviate from the above trading policies because any deviation will only increase his trading cost. This completes the proof of the theorem.

Thus the existence of noise trading has a significant impact on the behavior of liquidity traders. Now their trading behavior is to diversify across assets. The presence of noise trading in an asset has the effect of lowering the marginal cost of trading in that asset compared to the case of no noise trading. Thus the liquidity traders have an incentive to shift

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9 The occurrence of this situation depends on the number of assets with noise trading, the levels of noise trading in them and the relative magnitude of \( \sigma_1 \) compared to other \( \sigma_i \)'s. If there is a large number of these assets, or there is large enough noise trading going on, this situation is very unlikely to occur.
some of their trading from the least \( \sigma_i^2 \) asset into assets with noise trading in them. Essentially, the presence of noise traders in an asset makes the adverse selection problem less severe in that asset. This reduces a liquidity trader's cost of trading in that asset compared to the case when there were no noise traders in that asset. The additional losses due to higher return variability of such an asset are traded off against the reduction in trading cost that is achieved by the presence of noise traders.

Since there is a breakdown of markets in all assets with no noise trading except possibly the one with the lowest \( \sigma_i^2 \), and with enough noise trading going on in other assets, the market is likely to break down in this asset also, in the analysis that follows we make the assumption that markets are active only in assets which have some noise trading in them.\(^{10}\) Renumber these assets from \( i =1, \ldots, P \). Then the equilibrium condition that \( MC_i = MC_j \) implies from equation (10) that

\[
\frac{a_i \sigma_i}{[\Gamma_i + \alpha_i^2 T\Delta]^{1/2}} = \frac{a_j \sigma_j}{[\Gamma_j + \alpha_j^2 T\Delta]^{1/2}} = K_1
\]  \( (11) \)

where \( K_1 \) is a constant. Equation (11) and the fact that \( \sum a_i = 1 \) together imply that for \( i =1, \ldots, P \),

\[
a_i = \left[ \frac{K_1 \Gamma_i}{\sigma_i^2 - K_1 T\Delta} \right]^{1/2} \quad (12)
\]

where \( K_1 \) must satisfy:

\[
P \sum_{i=1}^{P} \left( \frac{K_1 \Gamma_i}{\sigma_i^2 - K_1 T\Delta} \right)^{1/2} = 1. \quad (13)
\]

\(^{10}\) This assumption is made only for ease of expression and to reduce notational burden. None of the results in any way rely on this assumption.
$K_1$ is thus an economy-wide constant and is a solution to a complicated equation involving the various constants $\sigma_i^2$, $\Gamma_i$, and $\Delta$.

Equation (12) is a useful equation. First, it shows that the fraction of liquidity needs allocated to an asset is proportional to the standard deviation of the total noise demands (measured by $[\Gamma_i]^{1/2}$) in that asset. Thus, ceteris paribus, a liquidity trader will trade twice as much in an asset which has a standard deviation of the total noise demands twice as that of another. Second, the equation shows that $\alpha_i$ varies inversely with the return variability of the asset.

Equations (11) and (9) together imply that $\frac{\alpha_i}{D_i}$ equals a constant for $i=1, \ldots, P$ so that the equilibrium depth in an asset is proportional to the fraction traded in that asset. Furthermore, from equation (5), it follows that the percentage cost of trading (measured here by the cost of trading a share since the expected payoff from the share of any asset is one) varies inversely with the depth of the market in that asset. Hence one can easily obtain results for these quantities also. We summarize these results in theorem 3.

**Theorem 3:** (1) The fraction of their liquidity demands allocated by liquidity traders to an asset is proportional to $[\Gamma_i]^{1/2}$, the standard deviation of the total noise demands in that asset, and varies inversely with the variance of its return $\sigma_i^2$; (2) the depth of the market in an asset is proportional to $[\Gamma_i]^{1/2}$ and varies inversely with $\sigma_i^2$; and (3) the percentage cost of trading in an asset is inversely proportional to $[\Gamma_i]^{1/2}$ and varies directly with $\sigma_i^2$.

**Proof:** Part (1) of the proof immediately follows from equation (12). Part (2) follows from equations (9) and (11) and part (1) of this proposition.
Part (3) follows from part (2) of this proposition and the fact that the percentage cost of trading equals the cost of trading one share (since the expected payoff from the share of any asset is one) and thus equals the reciprocal of the depth of the market in that asset [see equation (5)].

The positive relation between depth and standard deviation of the noise demands is intuitive: more variable noise trading leads to a deeper market by alleviating the adverse selection problem. The inverse relation between depth and return variance is also intuitive. The market maker in an asset sets the share price to equal the conditional expected value. For an asset with low variance of return, the conditional expected value cannot deviate much from the true value and hence the percentage trading cost will be low (or the depth will be high).

Assets with high return variability have higher value of information in our set up since here the informed traders know the return perfectly. The relation between return variability and the value of information need not be monotonic when traders get only imperfectly informed about the returns. However, our results can be easily interpreted in terms of the relation between trading cost and the value of information to the informed. Thus our results imply an inverse relation between depth or the fraction traded by liquidity traders in an asset and the value of private information about that asset. Assets in which the informed can find out little will have higher depths because for such assets the market maker's conditional expectation about its value will not deviate much from the true value, thus reducing the losses of liquidity traders to the informed.

The behavior of liquidity traders leads to complex interactions amongst the various markets. Liquidity trading ties together all the markets: each
market has influence on the others and in turn gets affected by them. In general, it is not possible to get a closed-form solution for the fraction \( \alpha_i \)'s traded in the various assets. However, there is one special case, that of equal return variances, where there is a closed form solution and we consider it as an example next. This special case provides more intuition on how the optimizing behavior of liquidity traders results in inter-relationships among the different markets.

**AN EXAMPLE:**

A closed form solution for \( \alpha_i \)'s can be found when the return variances for all the assets are equal. Let this return variance be denoted \( \sigma^2 \). For this case, equation (13) simplifies to:

\[
K_1 = \frac{\sigma^2}{\left[ \sum_{i=1}^{P} (\Gamma_i)^{1/2} \right]^2 + T\Delta}.
\]  

(14)

Substitution of this expression for \( K_1 \) into equation (12) and simplification leads to:

\[
\alpha_i = \frac{(\Gamma_i)^{1/2}}{\left( \sum_{j=1}^{P} (\Gamma_j)^{1/2} \right)}, \quad i=1, 2, \ldots, P.
\]  

(15)

Equation (15) implies that the fraction traded in an asset is equal to the standard deviation of noise trading in this asset divided by the sums of the standard deviations of noise trading in all assets. More variable noise trading in an asset implies a proportionately higher fraction of the liquidity demands allocated to that asset. Again this result is intuitive. More noise trading in an asset implies more depth, ceteris paribus, and hence attracts more liquidity trading. Equation (15) also clearly
demonstrates that liquidity traders diversify their trading across all assets with non-zero levels of noise trading.

For this case, equation (10), which gives \( MC_i \), simplifies to:

\[
MC_i = \frac{y^k c}{[\left( \sum_{j=1}^{P} (\Gamma_j)^{1/2} \right)^2 + T\Delta]^1/2}}, \quad i=1, 2, ..., P.
\] (16)

For an asset which has no noise trading in it, equation (10) implies that the marginal cost \( MC_i \) would equal \( \sigma y^k/(T\Delta)^{1/2} \). Equation (16) shows that the equilibrium \( MC_i \) for a liquidity trader is less than this number. Thus no liquidity trading would ever occur in an asset with no noise trading since liquidity traders are better off in trading in assets with non-zero levels of noise trading. Equation (16) also shows that the presence of noise traders in the various markets reduces the trading costs for liquidity traders. The equilibrium marginal cost \( MC_i \) given by equation (16) is less than \( \sigma y^k/(T\Delta)^{1/2} \), which would be the equilibrium marginal cost if there were no noise traders in any asset.

4. Discussion:

In the previous section, we showed that the existence of noise traders in the various markets implies that liquidity traders diversify their trading across assets. Thus if a liquidity trader starts with positive liquidity demands, i.e., with net buying needs, he will end up with a portfolio of all the risky assets in which there are markets (i.e., all assets with non-zero levels of noise trading). The paper thus provides a motivation other than risk reduction for diversification by investors. In our model all liquidity traders are risk-neutral. Hence risk reduction is not a motive for diversification. Still in our model the liquidity traders
generally end up with diversified asset holdings. This occurs because liquidity traders want to minimize their trading costs which is best achieved by diversifying their trading across assets. The paper thus leads to a positive theory for diversification even in the face of risk-neutrality.

In the set up that we consider, we do not have a riskless asset in which traders can trade. However, we can generalize our results to allow for riskless borrowing and lending with the riskless lending rate smaller than the expected return on holding a risky asset and the riskless borrowing rate greater than the expected return on holding a risky asset.

Consider, for example the case when riskless lending rate is less than the expected return on risky assets by $\delta$ and the riskless borrowing rate exceeds the expected return on risky assets by $\delta$. Then the marginal cost of using the riskless asset for borrowing or lending as opposed to using risky assets is $\delta$, i.e, this marginal cost is a constant equal to the difference between the expected return on risky assets and the riskless lending (or borrowing) rate. This situation is exactly analogous to the scenario in theorem 2 where the marginal cost of investing in asset 1 (the lowest $\sigma_i$ asset with no noise trading) also equalled a constant. Thus following the same logic, it is easily seen that liquidity traders will always trade in all assets with non-zero levels of noise trading. Whether or not they will use the riskless asset will depend on how the opportunity cost of using it compares with the costs of trading in risky assets, which in turn depend on a number of factors such as the levels of noise trading in risky assets, the amount of information possessed by informed traders etc.

The above results obviously depend on the assumption about the lending and borrowing rates in relation to the expected return on the risky assets.
However, it appears that the general conclusions that in the presence of asymmetric information and noise trading, trading costs vary across assets and hence liquidity traders have incentives to diversify their trading across risky assets should be robust to such assumptions.

5. Conclusions:

In this paper we have presented a model, with many assets and with both uninformed and informed traders, to examine cross-sectional variations in trading costs and asset liquidity. The proposed model is an attempt to account for the interactions that arise in trading costs in the various asset markets. The model suggests that the level of noise trading is an important determinant of liquidity of an asset market and leads to several other interesting implications.

The model has several limitations some of which are listed below. We assume that the number of liquidity traders is the same for all assets, i.e., liquidity traders can trade in any asset. One of the model's predictions is that they trade in all assets with non-zero levels of noise trading. However, in practice, one never observes such trading behavior. The number of liquidity traders is different for different assets. Most traders, and even institutional investors, do not hold or want to hold all assets. Furthermore, trading costs have fixed components too and are unlikely to be proportional to the size of order. Hence liquidity traders, even if they held all assets, would not want to trade in all assets to satisfy their liquidity needs. Also, by assuming risk neutrality on part of all traders, we abstract away from portfolio diversification considerations resulting from risk aversion, which may also be relevant in the decision of which assets to trade.
In the paper we model informed trading through the actions of a monopolist informed trader in each asset. In general, the number of informed traders in the various assets would also be endogenous and amongst other things, would also depend on the relative costs of information collection in the various assets. Thus, the equilibrium trading costs and the liquidity in different markets would also depend on the relative costs of information collection in the various assets.

We believe that despite these limitations, the model does provide several useful insights and future research in this area, which can consider extensions along the lines of eliminating some of these limitations, is likely to enhance our understanding of how asset markets function and the inter-relationships that exist in such markets.
References


Appendix

Proof of Lemma 1: The proof immediately follows from theorem 1 in Kyle (1985). Consider asset i. Let $\gamma_i$, $\beta_i$, $\mu_i$, and $\lambda_i$ be constants such that

$$\widetilde{X}_i = \gamma_i + \beta_i \tilde{v}_i \text{ and } \widetilde{P}_i = \mu_i + \lambda_i \tilde{w}_i,$$

Given the linear pricing rule, the expected profits $\pi_i$ of the informed trader are:

$$\pi_i = E[\tilde{v}_i - \tilde{P}_i(\tilde{\omega}_i)X_i|\tilde{v}_i = v_i] = (v_i - \mu_i - \lambda_i x_i)x_i.$$  

Profit maximization implies $v_i - \mu_i - 2\lambda_i x_i = 0$, which means

$$\gamma_i = \mu_i/(2\lambda_i), \quad \beta_i = 1/(2\lambda_i). \quad (A.1)$$

Consider now the problem of the market maker in asset i. The total order flow $\tilde{\omega}_i$ to him is: $\widetilde{X}_i + \widetilde{V}_i$. Since he makes zero expected profits, he must set $\tilde{P}_i = E[\tilde{v}_i|\omega_i]$ or,

$$\mu_i + \lambda_i \omega_i = E[\tilde{v}_i|\gamma_i + \beta_i \tilde{v}_i + \widetilde{V}_i = \omega_i].$$

The multivariate normality of the variables makes the regression linear and it implies

$$\lambda_i = \frac{\beta_i \sigma_i^2}{\beta_i^2 \sigma_i^2 + \text{var}(\tilde{Y}_i)}, \quad \mu_i - E(\tilde{V}_i) = -\lambda_i [\gamma_i + \beta_i E(\tilde{V}_i)]. \quad (A.2)$$

Substituting for $\gamma_i$ and $\beta_i$ from (A.1) into (A.2) and imposing the second order condition that $\lambda_i > 0$, one gets:

$$\mu_i = E(\tilde{V}_i) \text{ and } \lambda_i = \left[\sigma_i^2/\text{var}(\tilde{Y}_i)\right]^{1/2}.$$

Then using (A.1), $\gamma_i = -E(\tilde{V}_i)/(2\lambda_i)$ and $\beta_i = 1/(2\lambda_i)$. Since the depth $D_i$ of the market in asset i is just the inverse of $\lambda_i$, one has:

$$\widetilde{X}_i = D_i[\tilde{v}_i - E(\tilde{V}_i)]/2 \text{ and } \tilde{P}_i = E(\tilde{V}_i) + \tilde{\omega}_i/D_i \text{ where } D_i \text{ satisfies:}$$

$$D_i = (2/\sigma_i^2)[\text{Var}(\tilde{Y}_i)]^{1/2}.$$
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